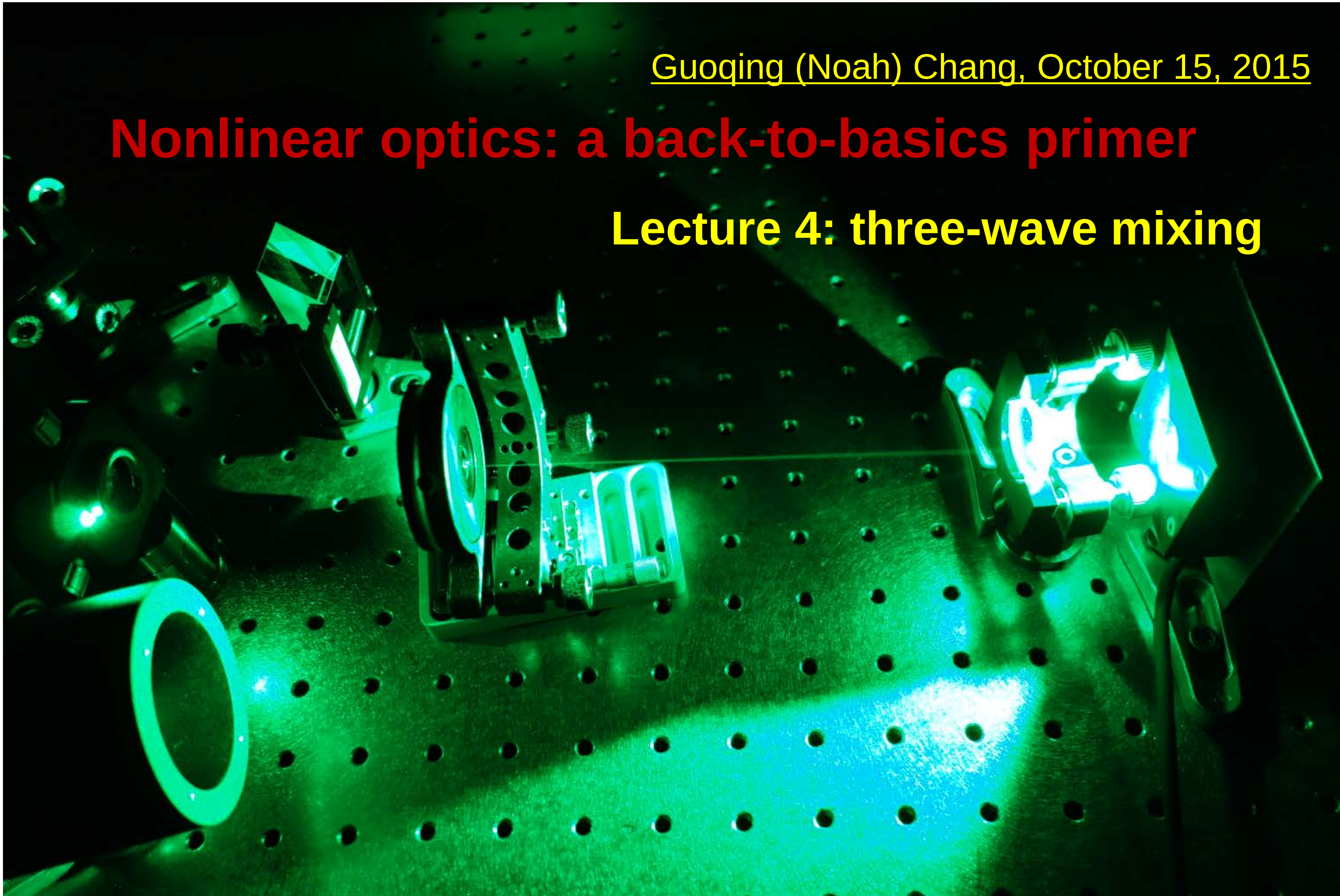
Guoqing (Noah) Chang, October 15, 2015

Nonlinear optics: a back-to-basics primer

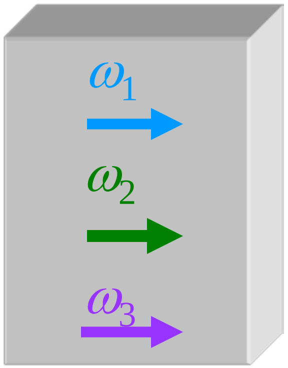
Lecture 4: three-wave mixing



2nd order nonlinearity: optical rectification

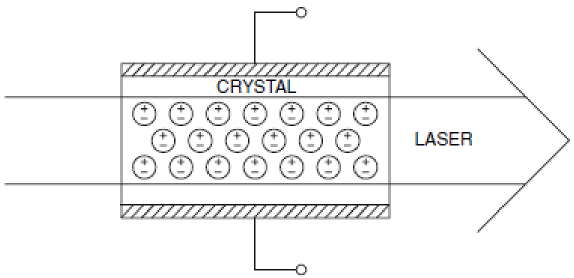
Example: three wave mixing at three different frequencies, $\omega_1, \omega_2, \omega_3$ and $\omega_3 = \omega_1 + \omega_2$

$$P_i^{(2)}(\omega_3, \omega_1, \omega_2) = \varepsilon_0 \chi_{ijk}^{(2)} E_j(\omega_1) E_k(\omega_2)$$

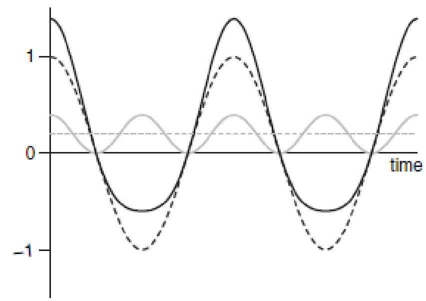


optical rectification $\quad 0 = \omega + (-\omega) \quad E_k(-\omega) = E_k^*(\omega)$

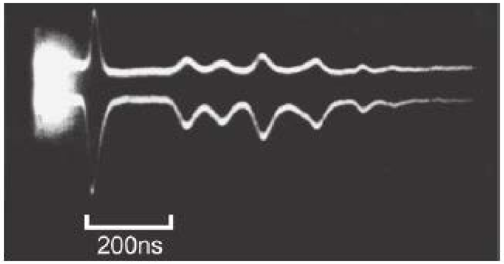
$$\boxed{P_i^{(2)}(0, \omega, -\omega) = \varepsilon_0 \chi_{ijk}^{(2)} E_j(\omega) E_k^*(\omega)}$$



Positive and negative charges in the nonlinear medium are displaced, causing a potential difference between the plates.



Response of a medium with 2nd-order nonlinearity to a single frequency light wave. It generates a DC term.



Optical rectification signal (top trace) and laser power (lower trace). It can be used for THz wave generation.

Geoffrey New, Introduction to nonlinear optics, chapter 1

2^{nd} order nonlinearity: Pockels effect

Pockels effect $\omega = \omega + 0$ $\boxed{P_i^{(2)}(\omega) = 2\varepsilon_0 \chi_{ijk}^{(2)} E_j(\omega) E_k^{DC}}$

Note that $P_i^{(1)}(\omega) = \varepsilon_0 \chi_{ij}^{(1)} E_j(\omega)$

$$D_i = \varepsilon_0[\delta_{ij} + \chi_{ij}^{(1)} + 2\chi_{ijk}^{(2)} E_k^{DC}]E_j(\omega) = \varepsilon_0[\varepsilon_{ij} + \underbrace{2\chi_{ijk}^{(2)} E_k^{DC}}_{\text{Pockels effect}}]E_j(\omega)$$

In linear optics as we have reviewed in lecture 1, an anisotropic medium is associated with an index ellipsoid.

$$\begin{bmatrix} \varepsilon_{xx} & 0 & 0 \\ 0 & \varepsilon_{yy} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{bmatrix} = \begin{bmatrix} n_x^2 & 0 & 0 \\ 0 & n_y^2 & 0 \\ 0 & 0 & n_z^2 \end{bmatrix} \Rightarrow$$

$n_x = n_y = n_o$ $n_z = n_e$

$$\frac{x^2}{n_o^2} + \frac{y^2}{n_o^2} + \frac{z^2}{n_e^2} = 1$$

Index ellipsoid for uniaxial crystal

Pockels effect leads to a change for the dielectric constant, and therefore modifies the index ellipsoid.

$$\Delta\varepsilon_{ij} = 2\chi_{ijk}^{(2)} E_k^{DC}$$

2nd order nonlinearity: Pockels effect

$(\varepsilon^{-1})_{ij}$ is more convenient to calculate the index ellipsoid.

$$(\varepsilon^{-1})_{ij} = \begin{bmatrix} n_x^{-2} & 0 & 0 \\ 0 & n_y^{-2} & 0 \\ 0 & 0 & n_z^{-2} \end{bmatrix} \longrightarrow \begin{bmatrix} x & y & z \end{bmatrix} \begin{bmatrix} n_x^{-2} & 0 & 0 \\ 0 & n_y^{-2} & 0 \\ 0 & 0 & n_z^{-2} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 1$$

$$\Delta(\varepsilon^{-1})_{ij} \equiv \sum_k r_{ijk} E_k^{DC}$$

$(\varepsilon^{-1})_{ij}$ is symmetric and thus the order of the indices ij is irrelevant, a similar contracted notation can be introduced:

$$\Delta(\varepsilon^{-1})_p = \Delta\left(\frac{1}{n^2}\right)_p = \sum_n r_{pn} E_n^{DC} \quad (n = 1-3,\ p = 1-6)$$

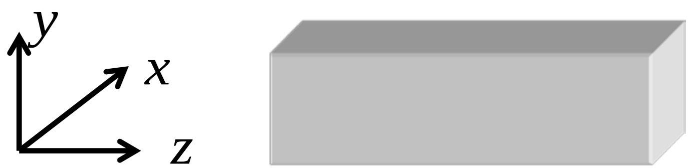
From $\Delta\varepsilon_{ij} = 2\chi_{ijk}^{(2)} E_k^{DC}$ we can find

$$r_{pn} = r_{ijk} = r_{jik} = -\frac{2\chi_{ijk}(\omega,\omega,0)}{\varepsilon_{ii}\varepsilon_{jj}}$$

Pockels effect: linear electro-optic effect

Take KDP crystal as an example:

$$r_{pn} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & r_{41} & 0 \\ 0 & 0 & r_{63} \end{bmatrix}$$



$$E^{DC} = \begin{bmatrix} 0 \\ 0 \\ V/L \end{bmatrix}$$

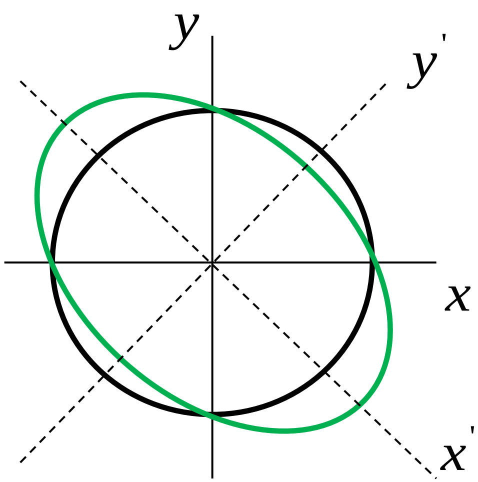
$$\Delta(\varepsilon^{-1})_6 = \Delta(\frac{1}{n^2})_6 = \frac{r_{63}V}{L}$$

$$\Delta(\frac{1}{n^2})_{xy} = \Delta(\frac{1}{n^2})_{yx} = \frac{r_{63}V}{L}$$

Assume an ordinary wave polarized in the x-y plane propagating in the +z-direction. Without applying a DC field, the light wave maintain its polarization. Now we add a DC voltage V between the exit and entrance surfaces. The crystal's length is L.

Now the index ellipsoid becomes

$$\frac{x^2+y^2}{n_o^2} + \frac{z^2}{n_e^2} + \frac{2xyr_{63}V}{L} = 1$$



The plane through the origin and perpendicular to the propagation direction (+z) cut the new ellipsoid into an ellipse. The lengths of its semi-axes correspond to the refractive indices of the two orthogonally polarized waves:

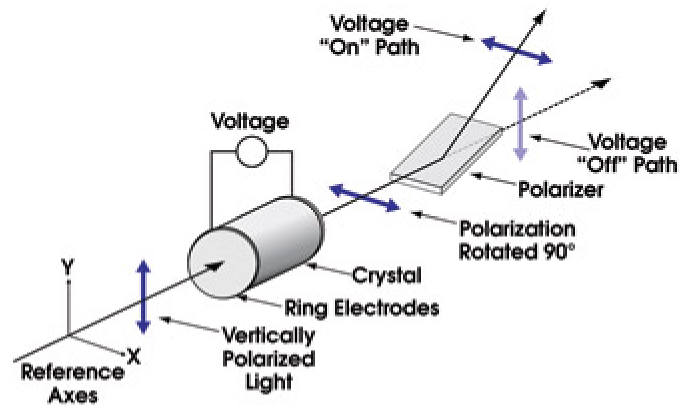
$$n_{\pm} \cong n_0(1 \pm \frac{1}{2} n_0^2 r_{63} V / L)$$

Pockels effect cause a refractive index change linearly proportional to the electric field. So it has another name—linear electro-optic effect. r_{63} is called the linear electro-optic coefficient.

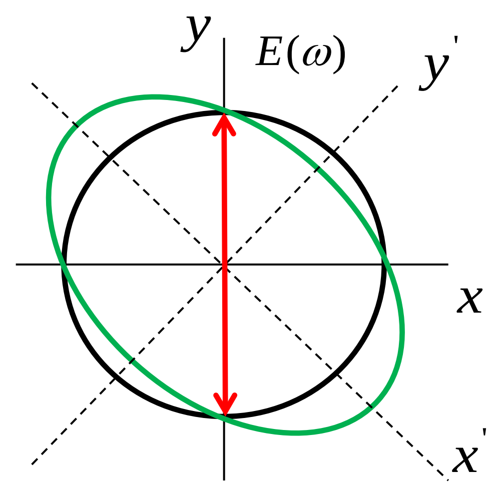
Pockels effect: application examples

Pockels cell: Electrically control the direction of an optical beam.

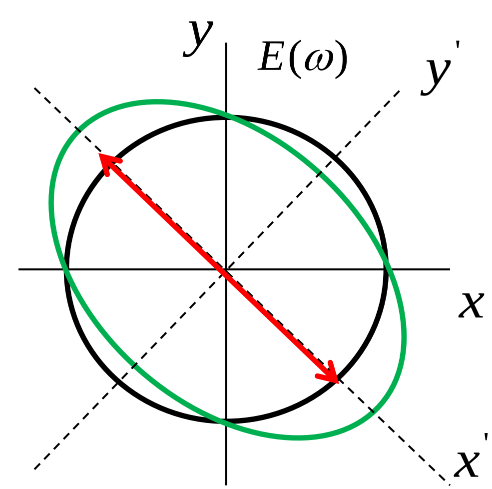
As the voltage varies with time, the optical power on the Voltage "off" path changes accordingly. In this scenario, the device functions as an EO intensity modulator. That is, we can use a low frequency RF or microwave signal to modulate an optical signal.



http://www.photonics.com/Article.aspx?AID=25650



$$n_{\pm} \cong n_0 (1 \pm \frac{1}{2} n_0^2 r_{63} V / L)$$



EO phase modulation: phase modulation of an optical beam can be achieved if we align the input light field with its polarization along the x' axis.

The Pockels effect provides a means to interact a low frequency electromagnetic wave (RF, microwave, or THz wave) with an optical wave. EO modulator enabled by this effect is one of the most important devices in modern optics.

Who is Pockels?

Friedrich Carl Alwin Pockels (1865–1913) was a German physicist. From 1900 to 1913 he was professor of theoretical physics at the University of Heidelberg. In 1893 he discovered that a steady electric field applied to certain birefringent materials causes the refractive index to vary, approximately in proportion to the strength of the field. This phenomenon is now called the Pockels effect.

-- From Wiki

Friedrich Carl Alwin Pockels (1865–1913)

B. G. TEUBNERS SAMMLUNG VON LEHRBÜCHERN
AUF DEM GEBIETE DER
MATHEMATISCHEN WISSENSCHAFTEN
MIT EINSCHLUSS IHRER ANWENDUNGEN
BAND XIX

LEHRBUCH
DER KRISTALLOPTIK

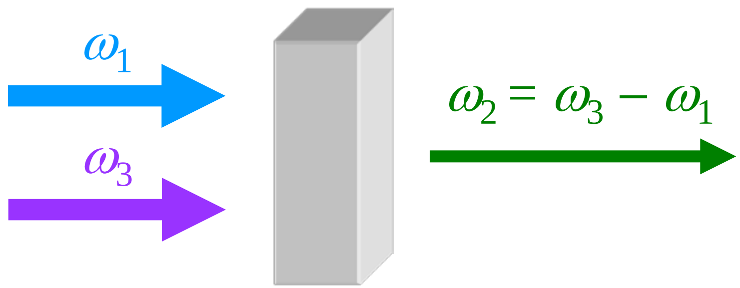
VON

DR. F. POCKELS

MIT 168 FIGUREN IM TEXT UND 6 DOPPELTAFELN

LEIPZIG UND BERLIN
DRUCK UND VERLAG VON B. G. TEUBNER
1906

Difference frequency generation (DFG)



$$\frac{dA_1}{dz} = -j\frac{2\omega_1 d_{eff}}{n(\omega_1)c_0} A_3 A_2^* e^{j\Delta kz} \quad (1)$$

$$\frac{dA_2}{dz} = -j\frac{2\omega_2 d_{eff}}{n(\omega_2)c_0} A_3 A_1^* e^{j\Delta kz} \quad (2)$$

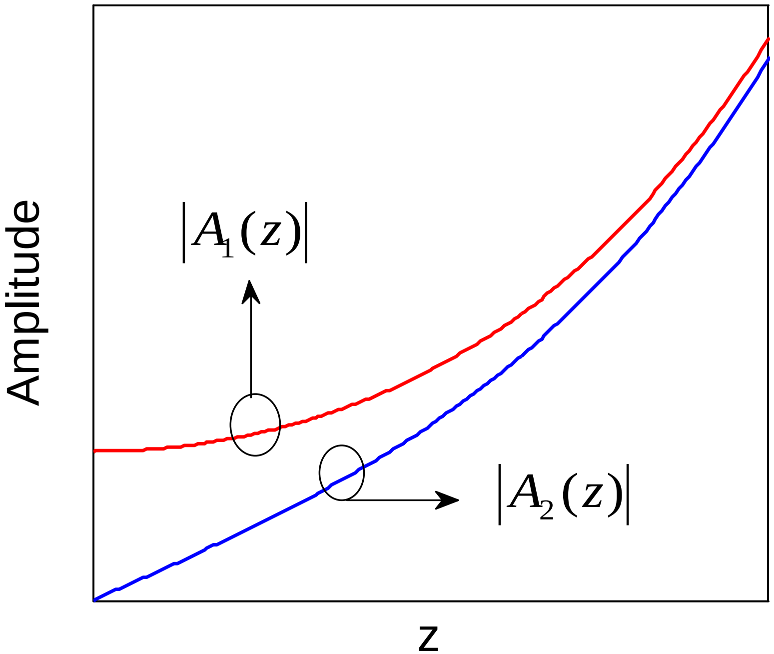
$$\frac{dA_3}{dz} = -j\frac{2\omega_3 d_{eff}}{n(\omega_3)c_0} A_1 A_2 e^{-j\Delta kz} \quad (3)$$

We assume that A_3 is nondepleted (i.e., constant) and $\Delta k = 0$. Differentiating both sides of Eq. (2) and using Eq. (1)(3), we can get

$$\frac{d^2 A_2}{dz^2} = \frac{4\omega_1\omega_2 d_{eff}^2}{n(\omega_1)n(\omega_2)c_0^2}|A_3|^2 A_2 \equiv \kappa^2 A_2$$

$$\kappa^2 = \frac{4\omega_1\omega_2 d_{eff}^2}{n(\omega_1)n(\omega_2)c_0^2}|A_3|^2$$

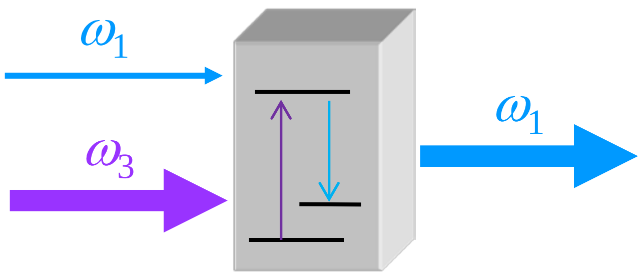
$$\boxed{\begin{aligned} A_1(z) &= \frac{1}{2}A_1(0)(e^{\kappa z} + e^{-\kappa z}) \\ A_2(z) &= -\frac{j}{2}\left[\frac{n(\omega_1)\omega_2}{n(\omega_2)\omega_1}\right]^{1/2}\frac{A_3}{|A_3|}A_1^*(0)(e^{\kappa z} - e^{-\kappa z}) \end{aligned}}$$



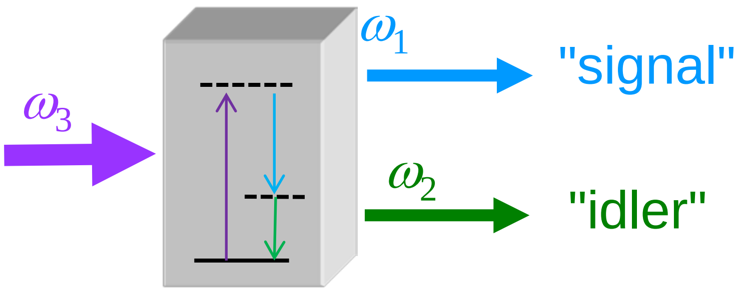
DFG: optical parametric generation, amplification, oscillation

For $\kappa z >> 1, \quad A_1(z) = \frac{1}{2} A_1(0) e^{\kappa z}$ corresponding to an exponential growth.

So such a DFG process can be used to amplify a weak field. But it is completely different from a laser amplifier based on stimulated emission.

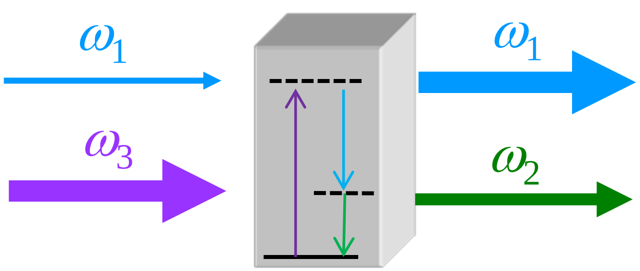


Laser amplifier involves electron transition and the amplifier stores energy by absorbing pump photons. Signal is amplified by stimulated emission. Only work at certain wavelengths limited by gain materials.

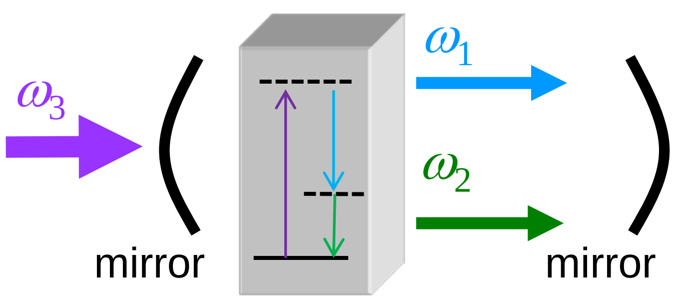


Optical Parametric Generation (OPG)

By convention:
$\omega_{signal} > \omega_{idler}$

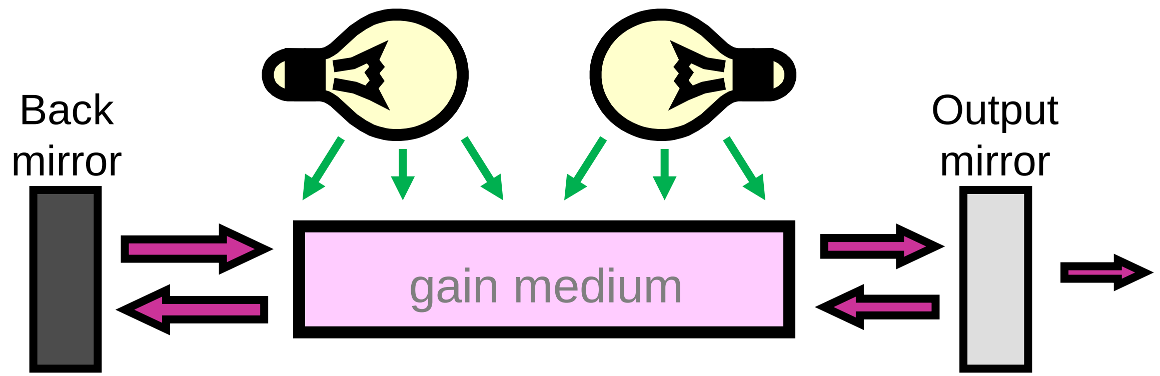


During optical parametric amplification (OPA) , the crystal does not absorb photons and no energy is stored in the crystal. Signal is amplified by wave mixing. Working wavelength is limited by crystal transmission.

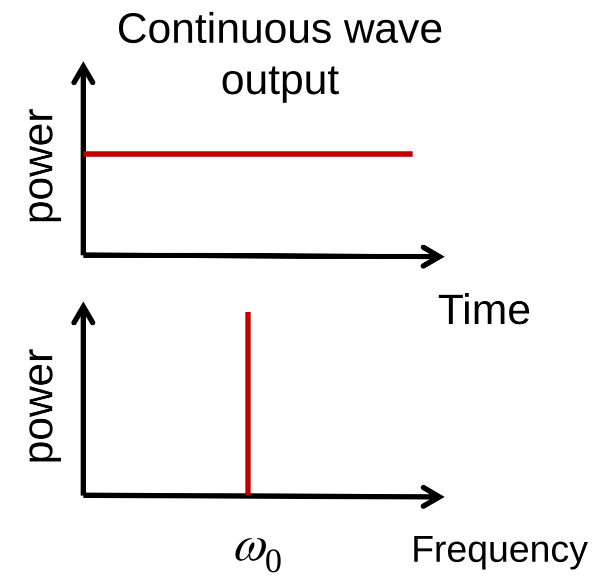


Optical Parametric Oscillation (OPO)

How a laser works



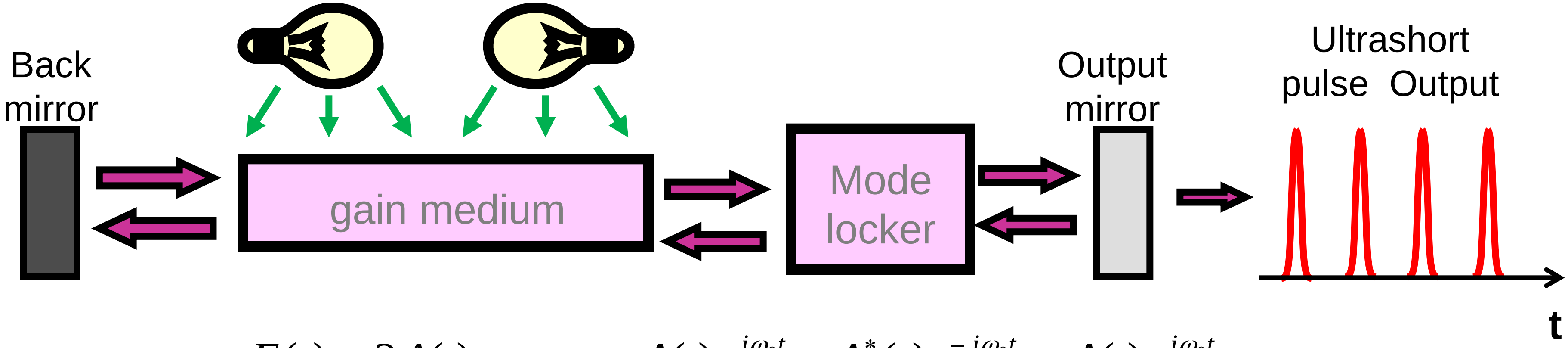
Continuous wave (CW) laser



$$E(t) = 2\mathrm{E}\cos\omega_0 t = \mathrm{E}e^{j\omega_0 t} + \mathrm{E}^* e^{-j\omega_0 t} = \mathrm{E}e^{j\omega_0 t} + c.c.$$

- <u>Gain medium</u>
  - Enable stimulated emission to produce identical copies of photons
  - Determine the light wavelength
- <u>Pump</u>
  - Inject power into the gain medium
  - Achieve population inversion
- <u>Resonator cavity</u>
  - Make light wave oscillating to efficiently extract energy stored in the gain medium
  - Improve directionality and color purity of the light

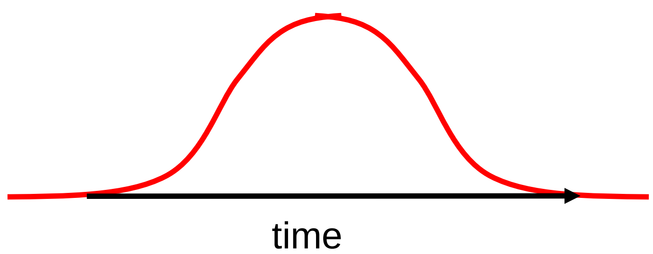
How an ultrafast laser works

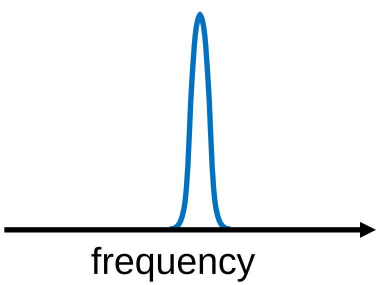


$$E(t) = 2A(t)\cos\omega_0 t = A(t)e^{j\omega_0 t} + A^*(t)e^{-j\omega_0 t} = A(t)e^{j\omega_0 t} + c.c.$$

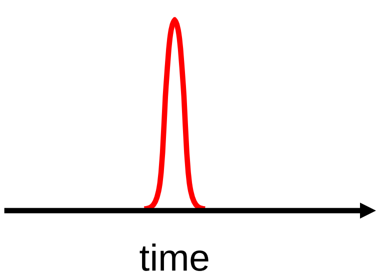
$$A(t) \rightarrow A(\omega) \qquad\qquad A(t)e^{j\omega_0 t} \rightarrow A(\omega - \omega_0)$$

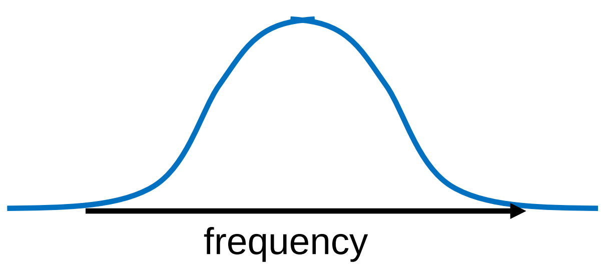
Longer pulse corresponds to narrower spectrum.



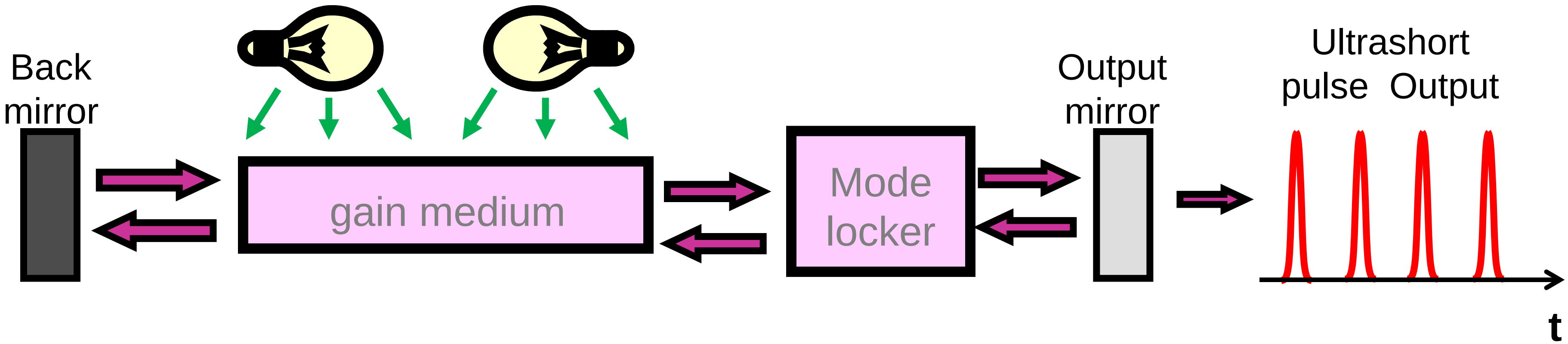


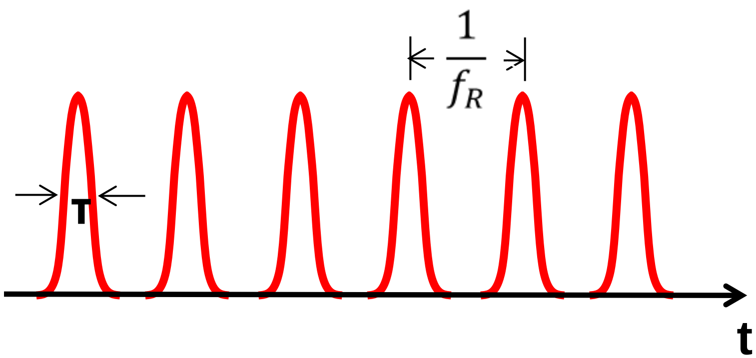
Shorter pulse corresponds to broader spectrum.





How an ultrafast laser works

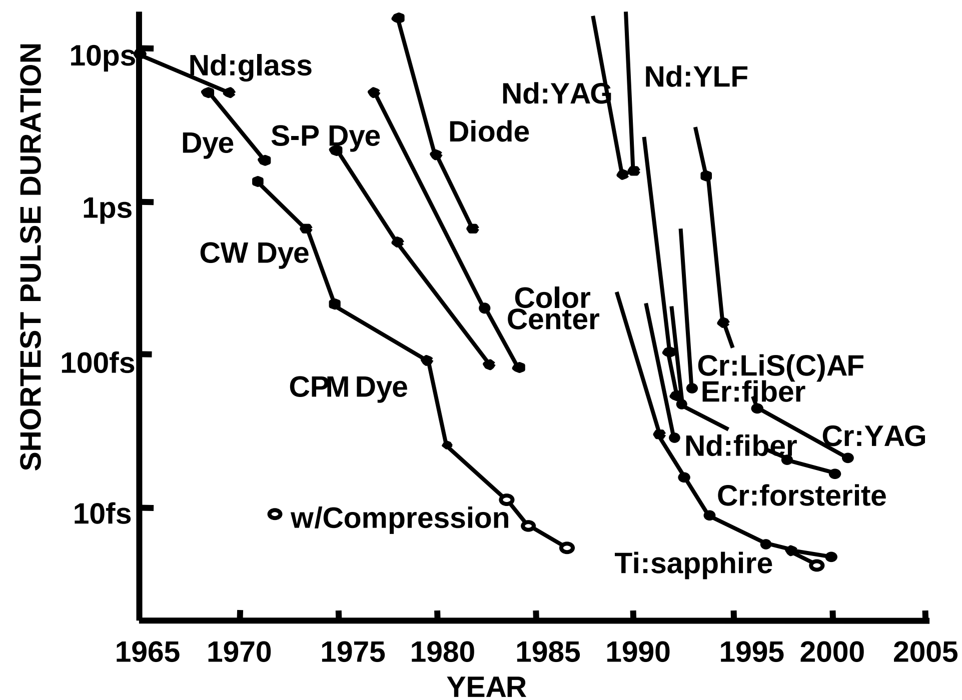




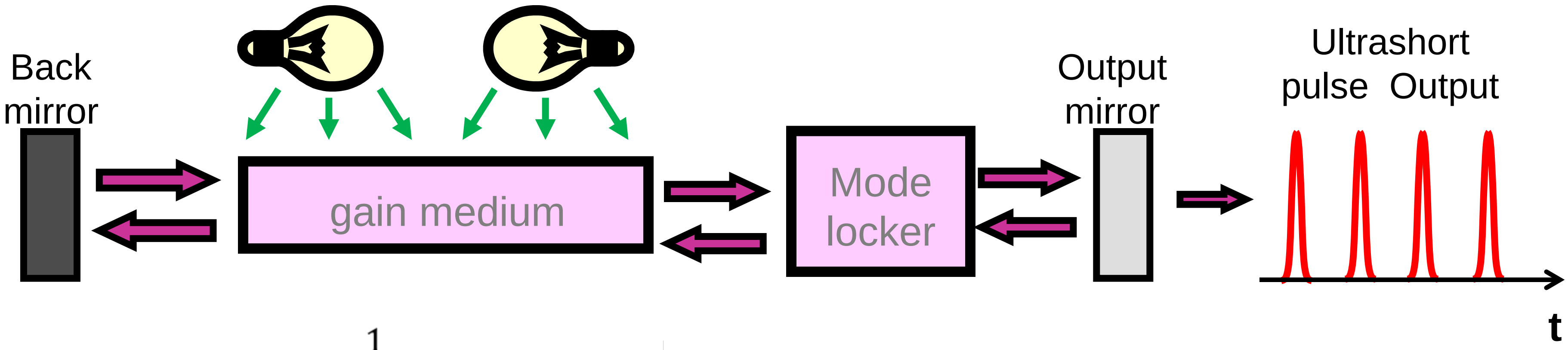
World shortest pulse: 67 attoseconds. The center wavelength is 20 nm. It is generated by high harmonic generation.

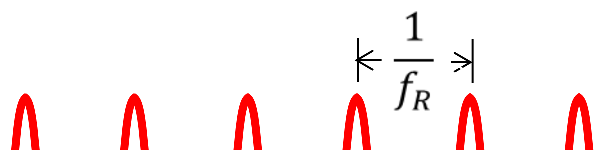
K. Zhao et al., "Tailoring a 67 attosecond pulse through advantageous phase-mismatch," Opt. Lett. 37, 3891 (2012)

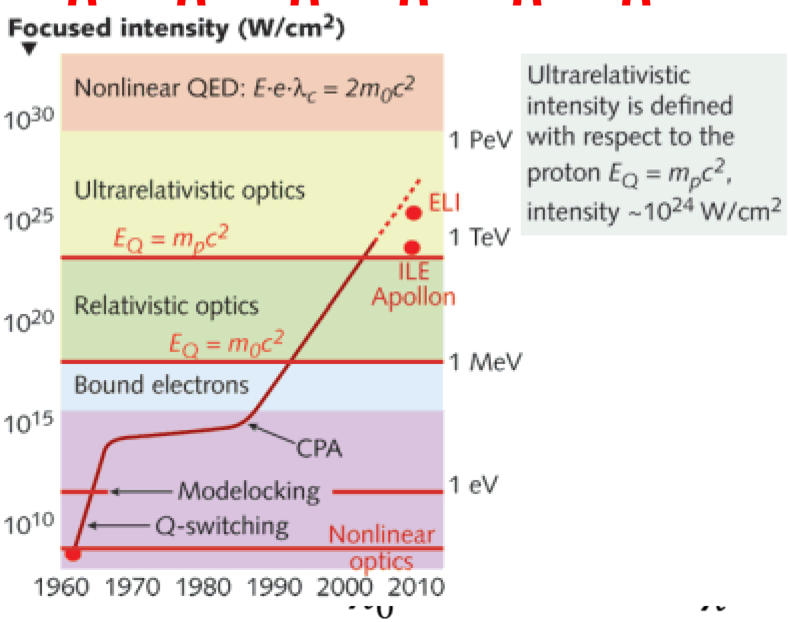
- Pulse duration T (fs – ps)



Ultrafast laser: the 4th element—mode locker

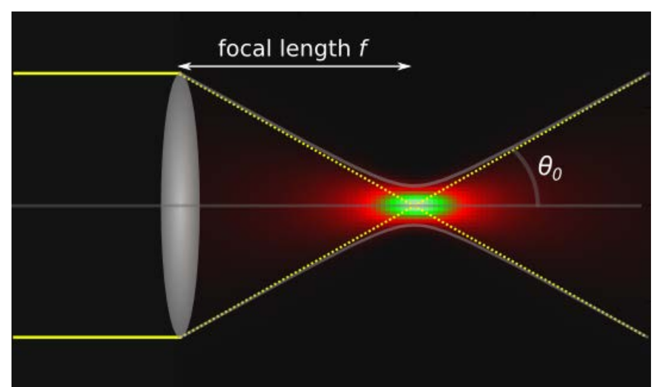






- Pulse duration T (fs – ps)
- Pulse energy E (pJ – mJ)
- Peak power P_p (1 kW – 1 PW)

  $P_p \approx E/T$ (e.g., 1 nJ, 100 fs pulse leads to 10 kW peak power.)



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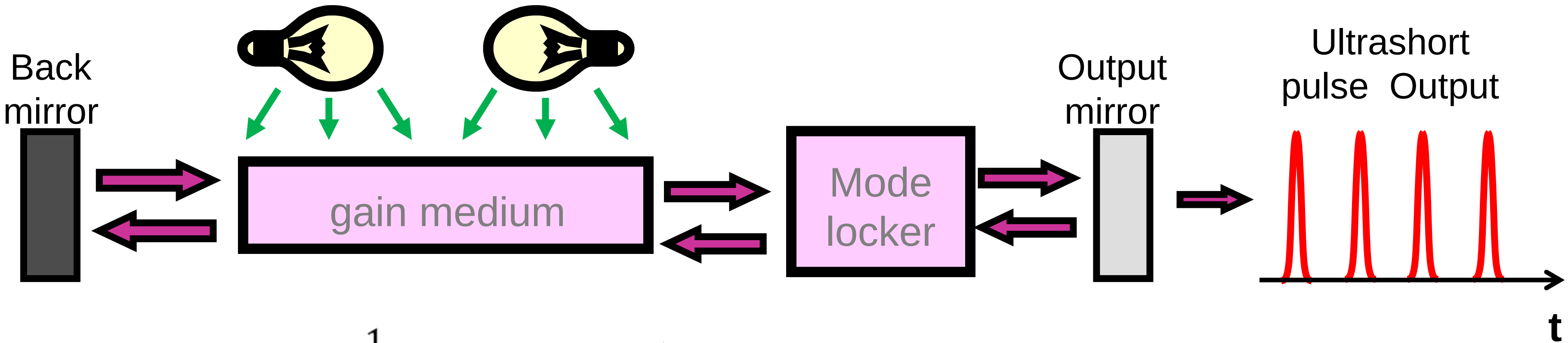
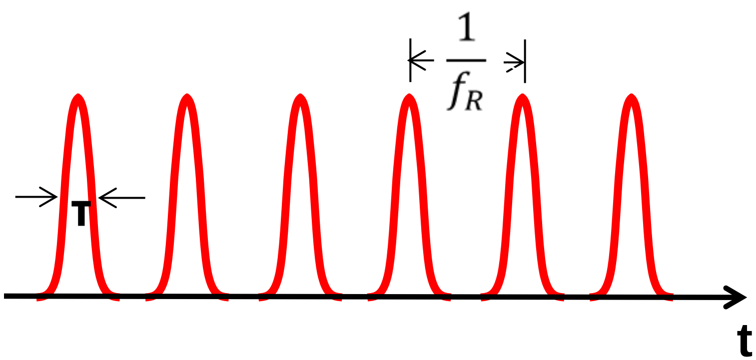
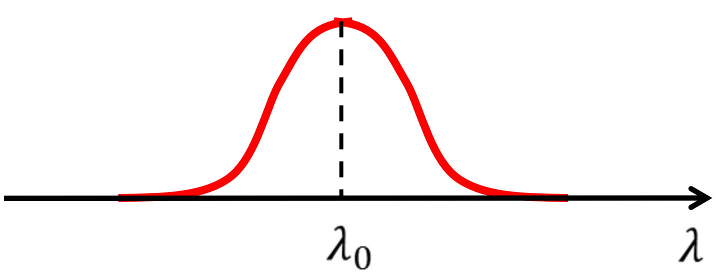






- Pulse duration T (fs – ps)
- Pulse energy E (pJ – mJ)
- Peak power P_p (1 kW – 1 PW)
  $P_p \approx E/T$ (e.g., 1 nJ, 100 fs pulse leads to 10 kW peak power.)
- Repetition rate f_R (10 MHz – 10 GHz)
- Average power P (10 mW – 100 W)
  $P = E \times f_R$ (e.g., 1 nJ, 100 MHz rep-rate laser produces 100 mW average power.)
- Center wavelength λ_0 (700 nm – 2000 nm)

The metric system

We meet with both small and large numbers when we talk about femtosecond pulses—they are incredibly short with the powers and intensities incredibly high.

Prefixes:

Small		Big	
Milli (m)	10^{-3}	Kilo (k)	10^{+3}
Micro (μ)	10^{-6}	Mega (M)	10^{+6}
Nano (n)	10^{-9}	Giga (G)	10^{+9}
Pico (p)	10^{-12}	Tera (T)	10^{+12}
Femto (f)	10^{-15}	Peta (P)	10^{+15}
Atto (a)	10^{-18}	Exa (E)	10^{+18}

Adapted from Rick Trebino's course slides

Some physical quantities

Average power: $P_{ave} \sim 1W - 1kW$

Repetition rate: $T_R^{-1} = f_R = \mathrm{mHz} - 100\,\mathrm{GHz}$

Pulse energy: $W = 1\mathrm{pJ} - 1\mathrm{kJ}$ (Average power = Rep-rate X Pulse energy)

Pulse width (duration):
$$\tau_{\mathrm{FWHM}} = \begin{array}{cc} 5\,\mathrm{fs} - 50\,\mathrm{ps}, & \mathrm{modelocked} \\ 30\,\mathrm{ps} - 100\,\mathrm{ns}, & \mathrm{Q-switched} \end{array}$$

Peak power: $P_p = \frac{1\,\mathrm{kJ}}{1\,\mathrm{ps}} = \frac{1\,\mathrm{J}}{1\,\mathrm{fs}} \sim 1\,\mathrm{PW}$ (Peak power = Pulse energy / duration)

(peak) Intensity: $$I = \frac{(Peak)\ power}{beam\ area}$$

If an optical beam with 1 PW peak power is focused to 1 um^2 area, the peak intensity is 10^{23} W/cm^2.

The Gaussian pulse

For almost all calculations, a good first approximation for any ultrashort pulse is that the pulse has a pulse envelope of Gaussian shape.

$$A(t) = A_0 \exp(-\frac{t^2}{2T_0^2})$$

The intensity is: $I(t) \propto |A_0|^2 \exp(-\frac{t^2}{T_0^2})$

Intensity full-width-half-maximum

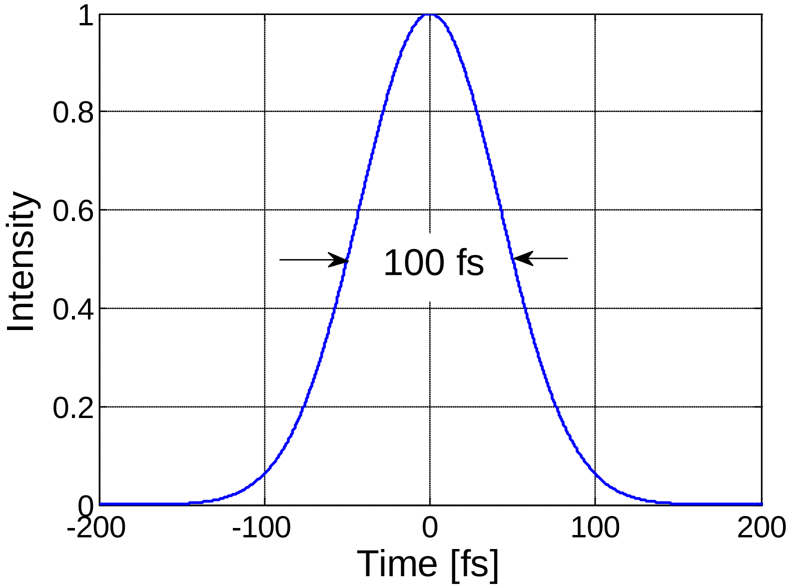
$$T_{FWHM} = 2(\ln 2)^{1/2} T_0$$

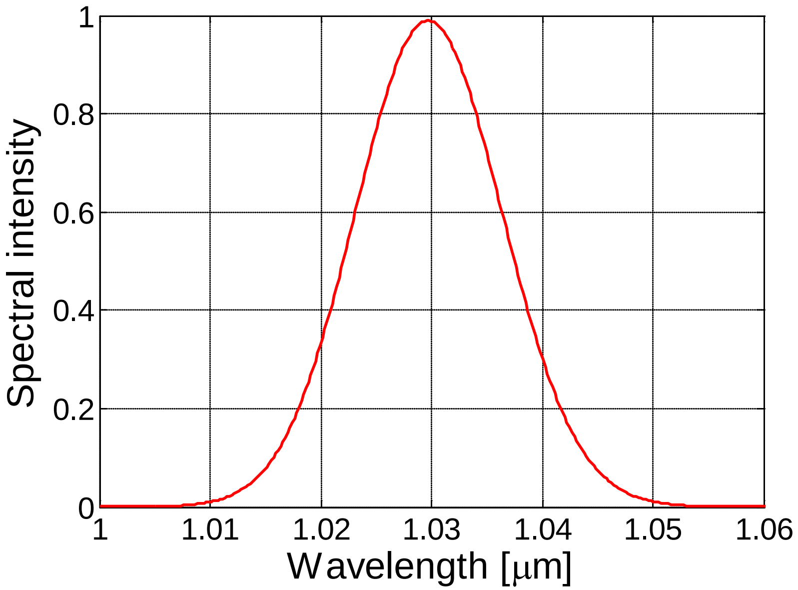
The optical spectrum

$$A(\omega) = (2\pi T_0^2)^{1/2} A_0 \exp(-\frac{T_0^2 \omega^2}{2})$$

Spectral bandwidth

$$\Delta\omega_{FWHM} = \frac{2(\ln 2)^{1/2}}{T_0}$$

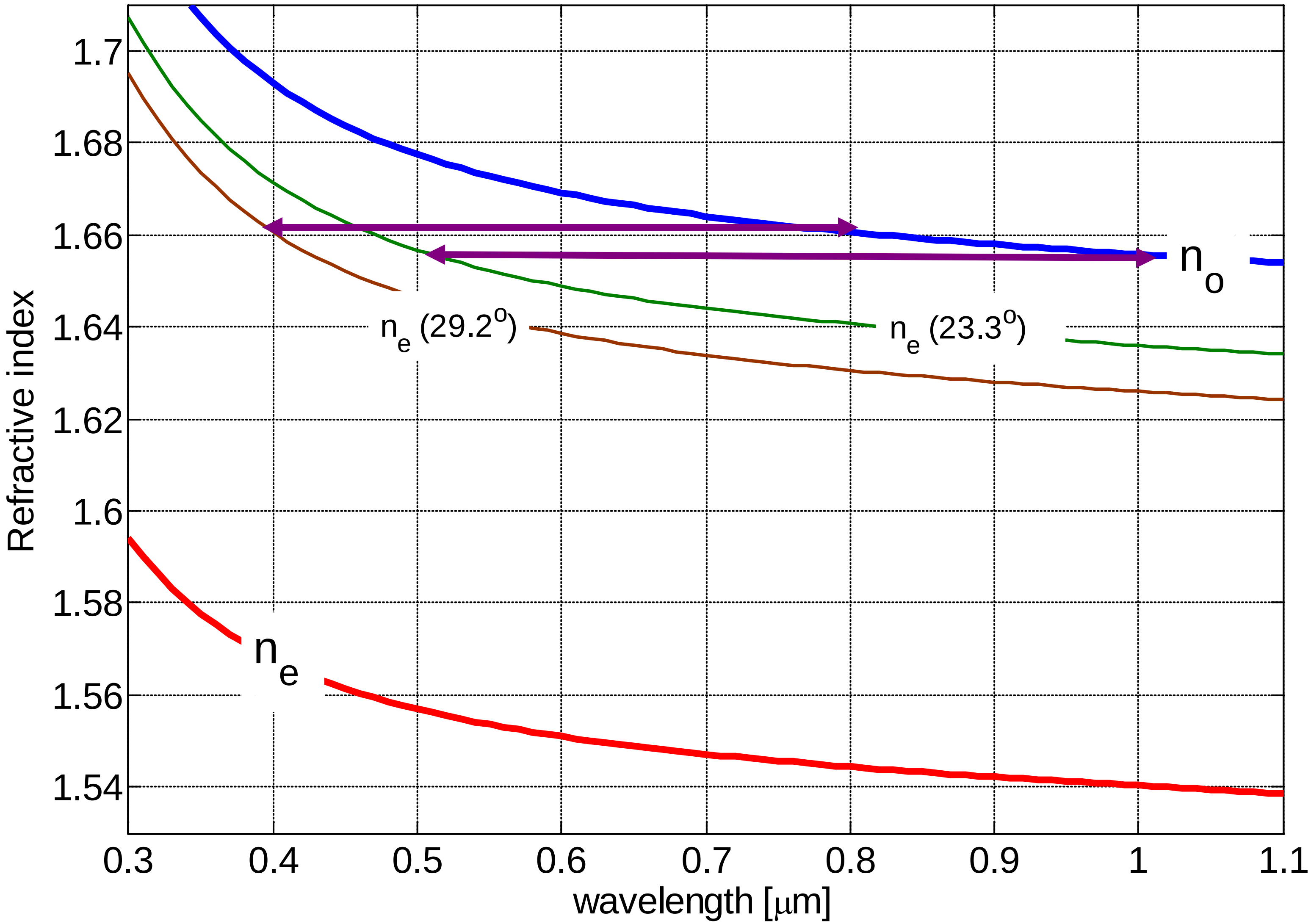




Phase matching for o→e SHG in BBO

$n_e(0.515um, \theta = 23.3^\circ) = n_o(1.03um)$

$n_e(0.4um, \theta = 29.2^\circ) = n_o(0.8um)$



Phase matching bandwidth for a pulse

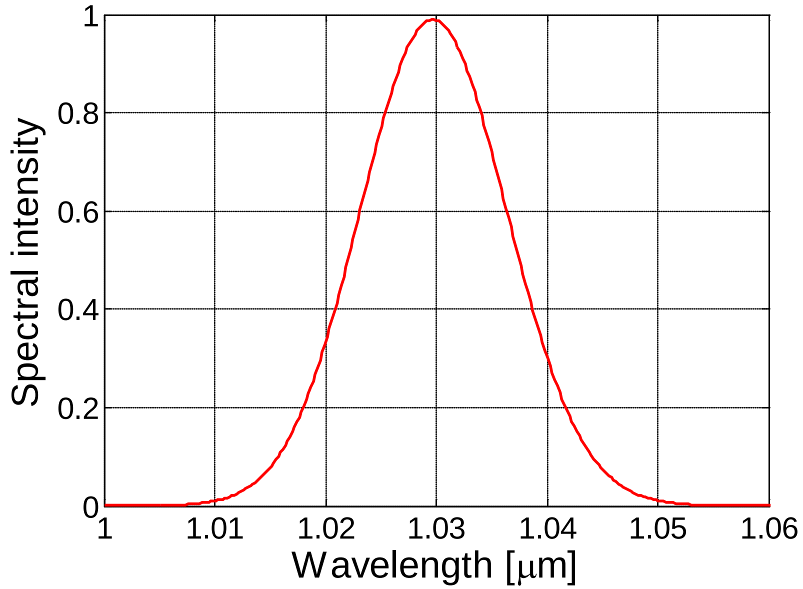
For type I SHG from a femtosecond pulse centered at $1.03um$, the center wavelength experiences a perfect phase matching if the BBO crystal is cut at $\theta = 23.3^\circ$. That is, the coherence length at the center wavelength is infinite.

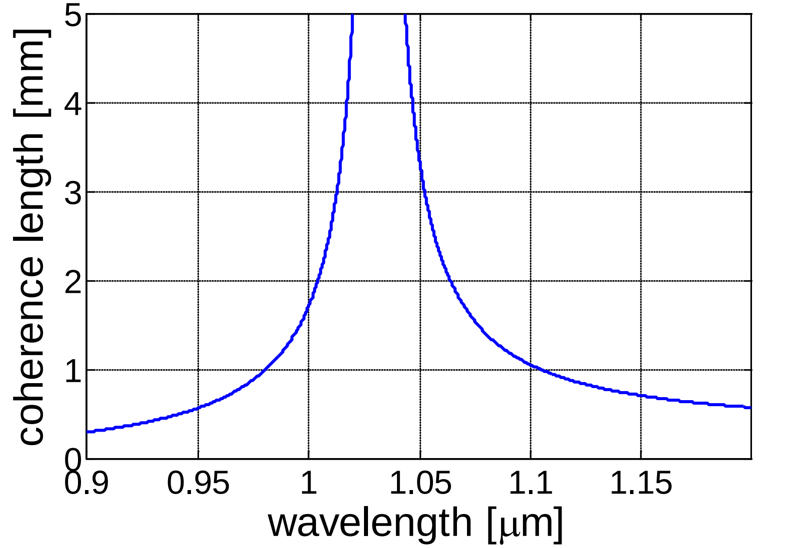
How about the coherence length at other wavelength at this angle?

$$L_c = \frac{\pi}{\Delta k} = \frac{\pi c_0}{2\omega[n_o(\omega) - n_e(2\omega, 23.3^\circ)]}$$

Roughly speaking, 2 mm BBO corresponds to 60 nm phase-matching bandwidth, and 1 mm BBO 220 nm.

In general, using thinner crystal leads to broader phase matching bandwidth at the expense of conversion efficiency.





Linear propagation of a pulse

$$(\nabla^2 - \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2})E = \mu_0 \frac{\partial^2 P}{\partial t^2}$$

$$E(z,t) = A(z,t)e^{j(\omega_0 t - k_0 z)}$$

Fourier transform

$$E(z,\omega) = A(z,\omega - \omega_0)e^{-jk_0 z}$$

$$k_0 = \frac{\omega_0 n(\omega_0)}{c_0}$$

Take the Fourier transform of both sides of the wave equation:

$$P(\omega) = \varepsilon_0 \chi^{(1)}(\omega)E(\omega)$$

$$(\nabla^2 + \frac{\omega^2}{c_0^2})E = -\omega^2 \mu_0 P(\omega) \longrightarrow (\nabla^2 + \frac{n^2(\omega)\omega^2}{c_0^2})E(\omega) = 0$$

Plane wave propagating in +z direction:

$$E(z,\omega) = A(z,\omega - \omega_0)e^{-jk_0 z}$$

$$[\frac{d^2}{dz^2} + k^2(\omega)]E(z,\omega) = 0$$

$$k(\omega) = \frac{n(\omega)\omega}{c_0}$$

$$\frac{d^2 A}{dz^2} - 2jk_0 \frac{dA}{dz} + [k^2(\omega) - k_0^2]A = 0$$

$$\frac{d^2 A}{dz^2} - 2jk_0 \frac{dA}{dz} + \underbrace{[k(\omega) + k_0]}_{\approx 2k_0}[k(\omega) - k_0]A = 0$$

Slowly varying amplitude approximation

$$\left|\frac{d^2 A}{dz^2}\right| << \left|2k_0 \frac{dA}{dz}\right|$$

$$\frac{dA}{dz} = -j[k(\omega) - k_0]A$$

Group velocity Vs phase velocity

$$\frac{dA(z,\omega-\omega_0)}{dz} = -j[k(\omega)-k_0]A(z,\omega-\omega_0) \qquad k(\omega) = k_0 + \left.\frac{dk}{d\omega}\right|_{\omega=\omega_0}(\omega-\omega_0) + \frac{1}{2}\left.\frac{d^2k}{d\omega^2}\right|_{\omega=\omega_0}(\omega-\omega_0)^2 + \ldots$$

Let's first take a look at the effect of the first two terms $\quad k(\omega) = k_0 + k^{(1)}(\omega-\omega_0) \quad k^{(1)} = \left.\frac{dk}{d\omega}\right|_{\omega=\omega_0}$

$$\frac{dA(z,\omega-\omega_0)}{dz} = -jk^{(1)}(\omega-\omega_0)A(z,\omega-\omega_0) \longrightarrow A(z,\omega-\omega_0) = A(0,\omega-\omega_0)e^{-jk^{(1)}(\omega-\omega_0)z}$$

$$E(z,\omega) = A(z,\omega-\omega_0)e^{-jk_0z} = A(0,\omega-\omega_0)e^{-jk(\omega)z} = A(0,\omega-\omega_0)e^{-j[k_0+k^{(1)}(\omega-\omega_0)]z}$$

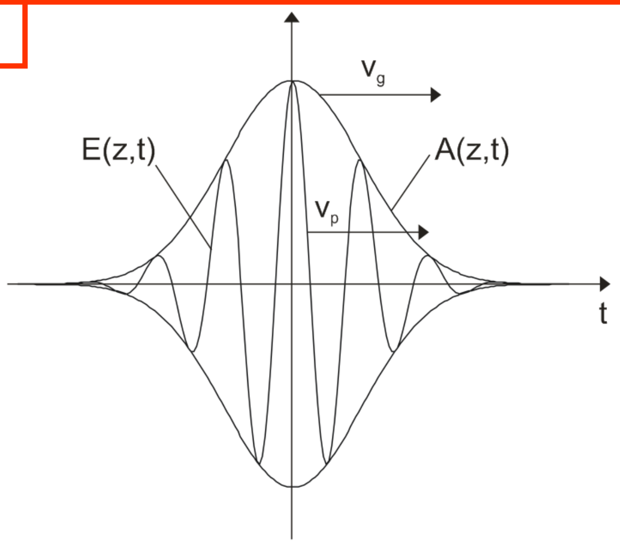
$$\boxed{E(z,t) = A(0,t-k^{(1)}z)e^{j(\omega_0t-k_0z)} = A(0,t-\frac{z}{V_g})e^{j\omega_0(t-\frac{z}{V_p})}}$$

Group velocity: travelling speed of the pulse envelope.

$$V_g = \frac{1}{k^{(1)}} = 1/\left.\frac{dk}{d\omega}\right|_{\omega=\omega_0} = \frac{c_0}{n(\omega_0)+\omega_0\left.\frac{dn(\omega)}{d\omega}\right|_{\omega=\omega_0}}$$

Phase velocity: travelling speed of the carrier wave under the pulse envelope.

$$V_p = \frac{\omega_0}{k_0} = \frac{c_0}{n(\omega_0)}$$



Electric field and pulse envelope in time domain

Group velocity Vs phase velocity

In vacuum

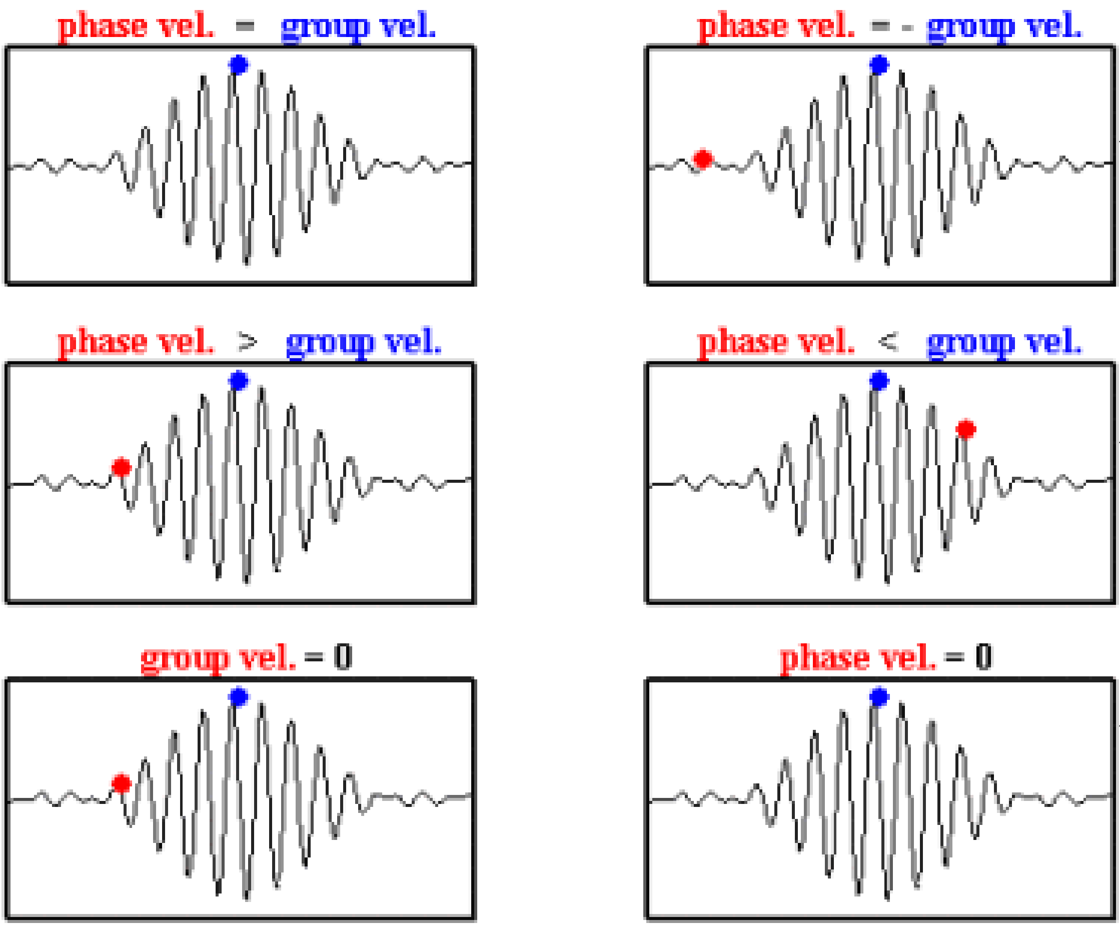
Unrealistic

Most common case

Possible

Unrealistic

Unrealistic



Adapted from Rick Trebino's course slides

Time domain description

$$\frac{dA(z,\omega-\omega_0)}{dz} = -jk^{(1)}(\omega-\omega_0)A(z,\omega-\omega_0) \xrightarrow[\text{Fourier transform}]{} \frac{\partial A(z,t)}{\partial z} + k^{(1)}\frac{\partial A(z,t)}{\partial t} = 0$$

Now take into account nonlinearity:

Coupled wave equations for three-wave mixing of CW waves

$$\frac{dA_1}{dz} = -j\frac{2\omega_1 d_{eff}}{n(\omega_1)c_0}A_3A_2^* e^{j\Delta kz}$$

$$\frac{dA_2}{dz} = -j\frac{2\omega_2 d_{eff}}{n(\omega_2)c_0}A_3A_1^* e^{j\Delta kz}$$

$$\frac{dA_3}{dz} = -j\frac{2\omega_3 d_{eff}}{n(\omega_3)c_0}A_1A_2 e^{-j\Delta kz}$$

Coupled wave equations for three-wave mixing of ultrashort pulses

$$\frac{\partial A_1(z,t)}{\partial z} + \frac{1}{V_{g,1}}\frac{\partial A_1(z,t)}{\partial t} = -j\frac{2\omega_1 d_{eff}}{n(\omega_1)c_0}A_3A_2^* e^{j\Delta kz}$$

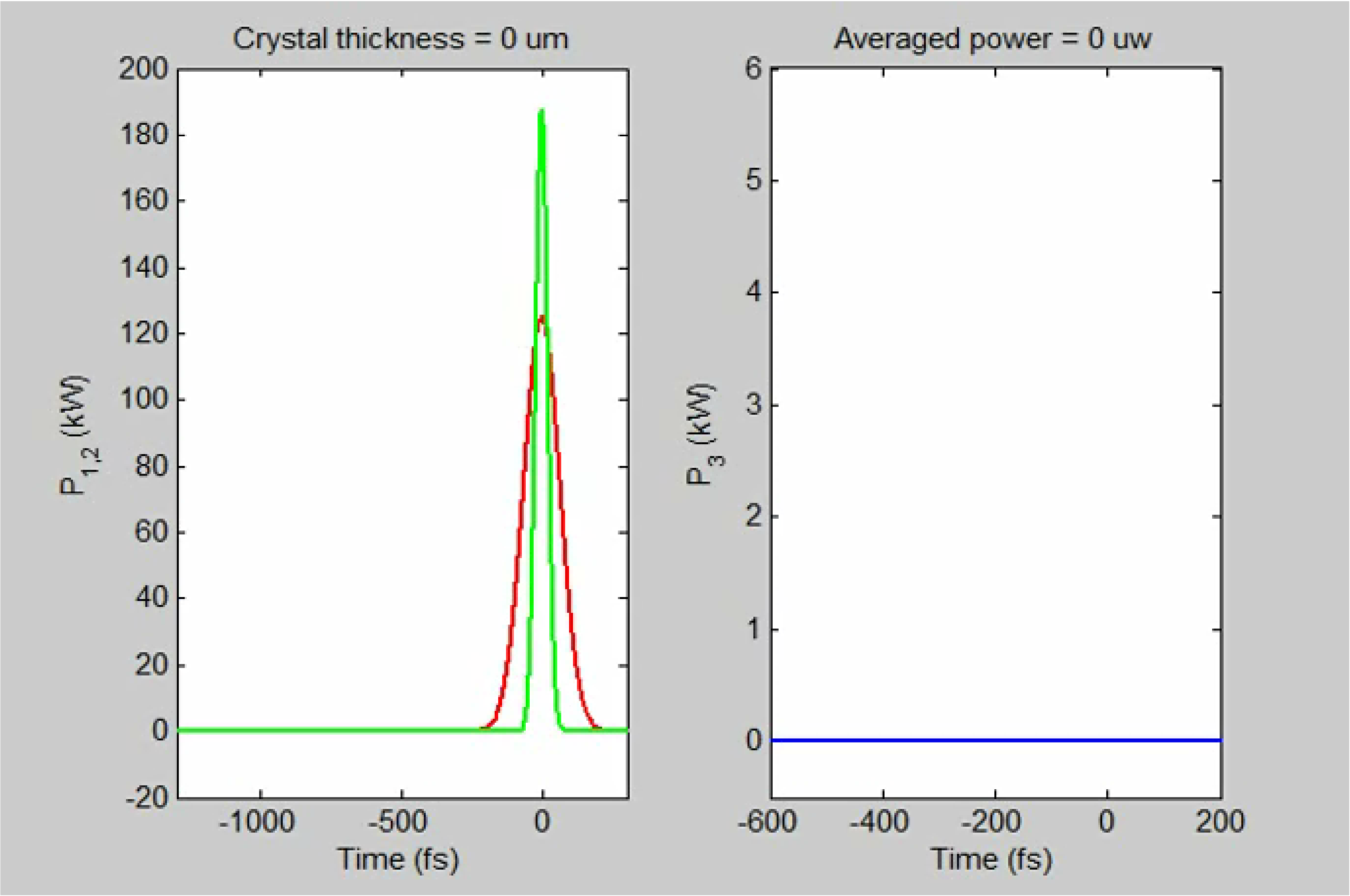
$$\frac{\partial A_2(z,t)}{\partial z} + \frac{1}{V_{g,2}}\frac{\partial A_2(z,t)}{\partial t} = -j\frac{2\omega_2 d_{eff}}{n(\omega_2)c_0}A_3A_1^* e^{j\Delta kz}$$

$$\frac{\partial A_3(z,t)}{\partial z} + \frac{1}{V_{g,3}}\frac{\partial A_3(z,t)}{\partial t} = -j\frac{2\omega_3 d_{eff}}{n(\omega_3)c_0}A_1A_2 e^{-j\Delta kz}$$

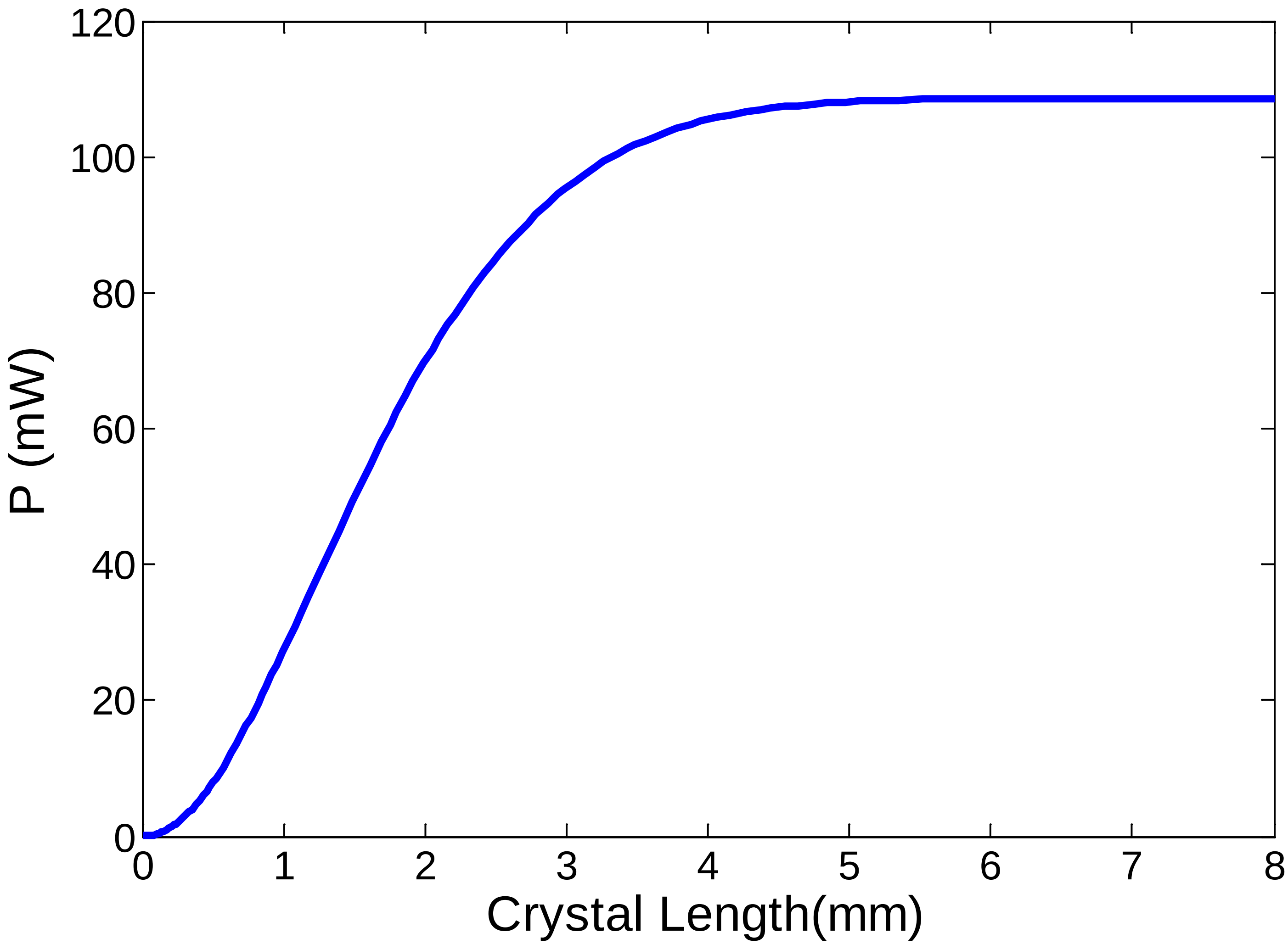
$$V_{g,i} = \frac{1}{k^{(i)}}, i = 1,2,3$$

In general, $V_{g,1} \neq V_{g,2} \neq V_{g,3}$. It leads temporal walk-off of these three pulses.

Numerical example: SFG between 1.5 um and 0.8 um pulses

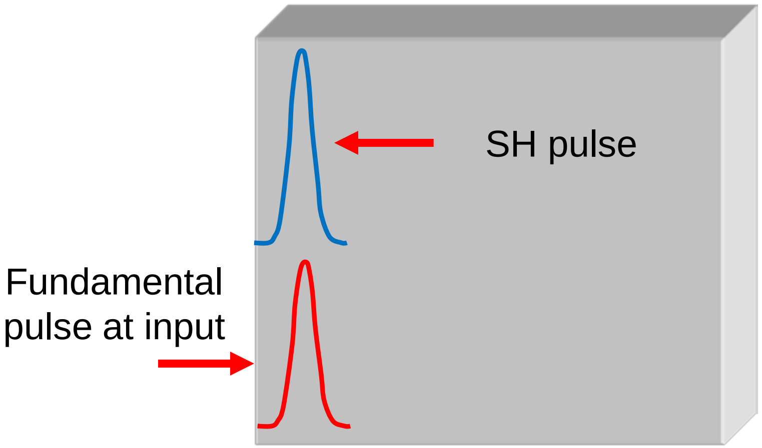


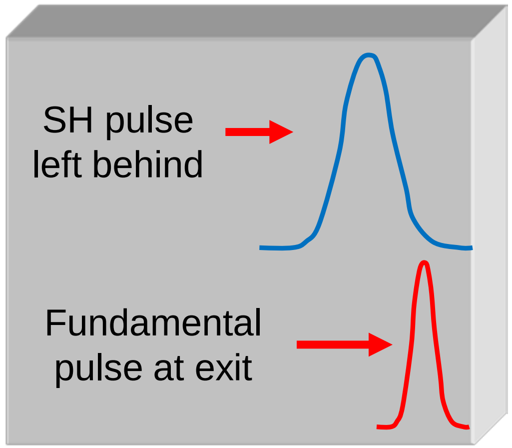
Saturation of the SFG pulse power due to temporal walk-off



Group-velocity mismatch: take SHG as an example

Fundamental pulse and SH pulse propagate at different group velocity, which introduce a walk-off between them and decrease the conversion efficiency.





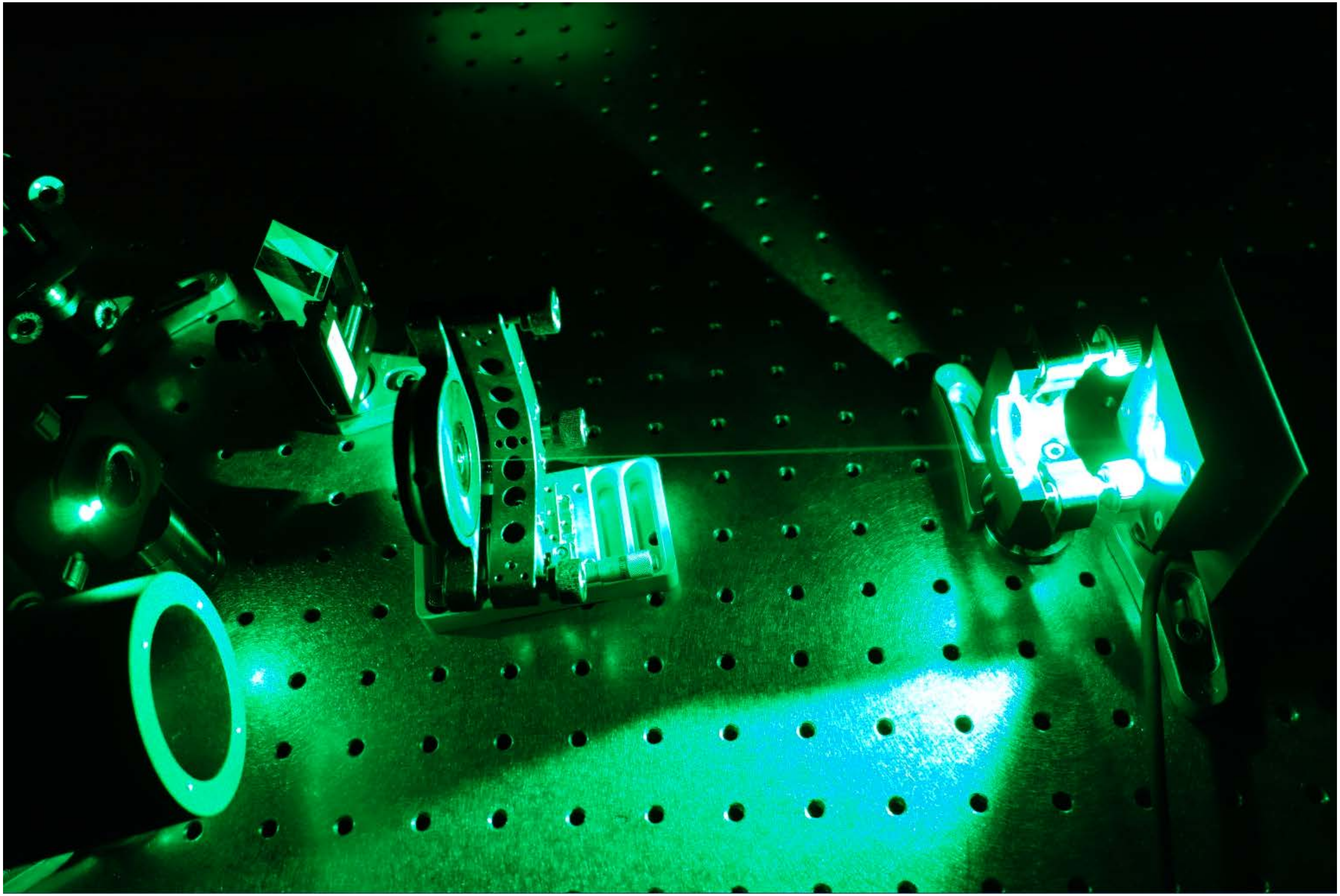
$$GVM \equiv \frac{1}{V_g(2\omega)} - \frac{1}{V_g(\omega)}$$

Calculated values of inverse group-velocity mismatch for SHG process in BBO

Interacting wavelengths [μm]	θ_{pm} [deg]	β [fs/mm]
SHG, $o + o \Rightarrow e$		
$1.2 \Rightarrow 0.6$	21.18	54
$1.1 \Rightarrow 0.55$	22.28	76
$1.0 \Rightarrow 0.5$	23.85	104
$0.9 \Rightarrow 0.45$	26.07	141
$0.8 \Rightarrow 0.4$	29.18	194
$0.7 \Rightarrow 0.35$	33.65	275
$0.6 \Rightarrow 0.3$	40.47	415
$0.5 \Rightarrow 0.25$	52.34	695
SHG, $e + o \Rightarrow e$		
$1.2 \Rightarrow 0.6$	29.91	103
$1.1 \Rightarrow 0.55$	31.46	130
$1.0 \Rightarrow 0.5$	33.73	164
$0.9 \Rightarrow 0.45$	36.98	210
$0.8 \Rightarrow 0.4$	41.67	276
$0.7 \Rightarrow 0.35$	48.74	373
$0.6 \Rightarrow 0.3$	60.91	531

Group-velocity mismatch results in a longer SH pulse.

20 W, 40 fs green light via SHG in 0.5 mm BBO crystal



Take-home message

- Linear EO effect is a second-order nonlinear effect.
- OPA—a special case of DFG—can be used to amplify weak signal.
- Ultrafast lasers generate femtosecond pulses.
- Thinner crystals are required to achieve broadband phase matching for nonlinear phenomena involving shorter pulses.
- Group velocity mismatch reduces the conversion efficiency and shapes the generated pulse.

Suggested reading

Three wave mixing

-- Geoffrey New, *Introduction to nonlinear optics*, chapter 1,2

-- George Stegemann and Robert Stegemann, *Nonlinear optics*, chapter 6

-- Robert Boyd, *Nonlinear optics*, chapter 2

Dispersion and ultrashort pulse

-- Geoffrey New, *Introduction to nonlinear optics*, chapter 6