IMPRS: Ultrafast Source Technologies

Lecture V: March 4, 2014: Ultrafast Optical Sources

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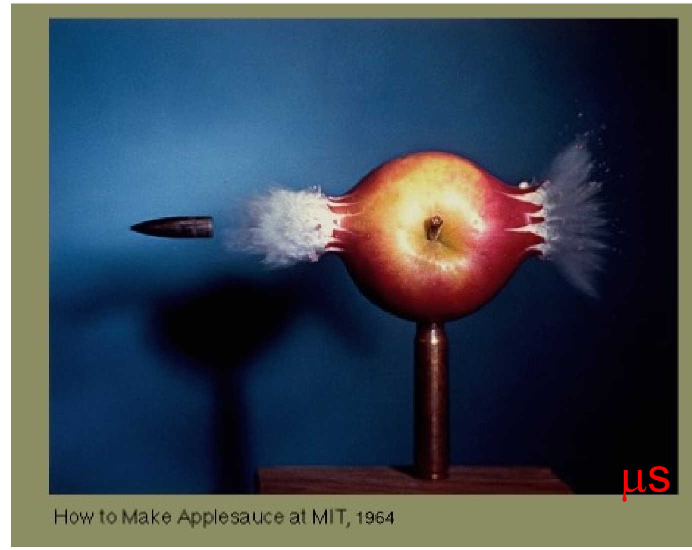


Is there a time during galloping, when all feet are off the ground? (1872) Leland Stanford

Eadweard Muybridge, * 9. April 1830 in Kingston upon Thames; † 8. Mai 1904, British pioneer of photography

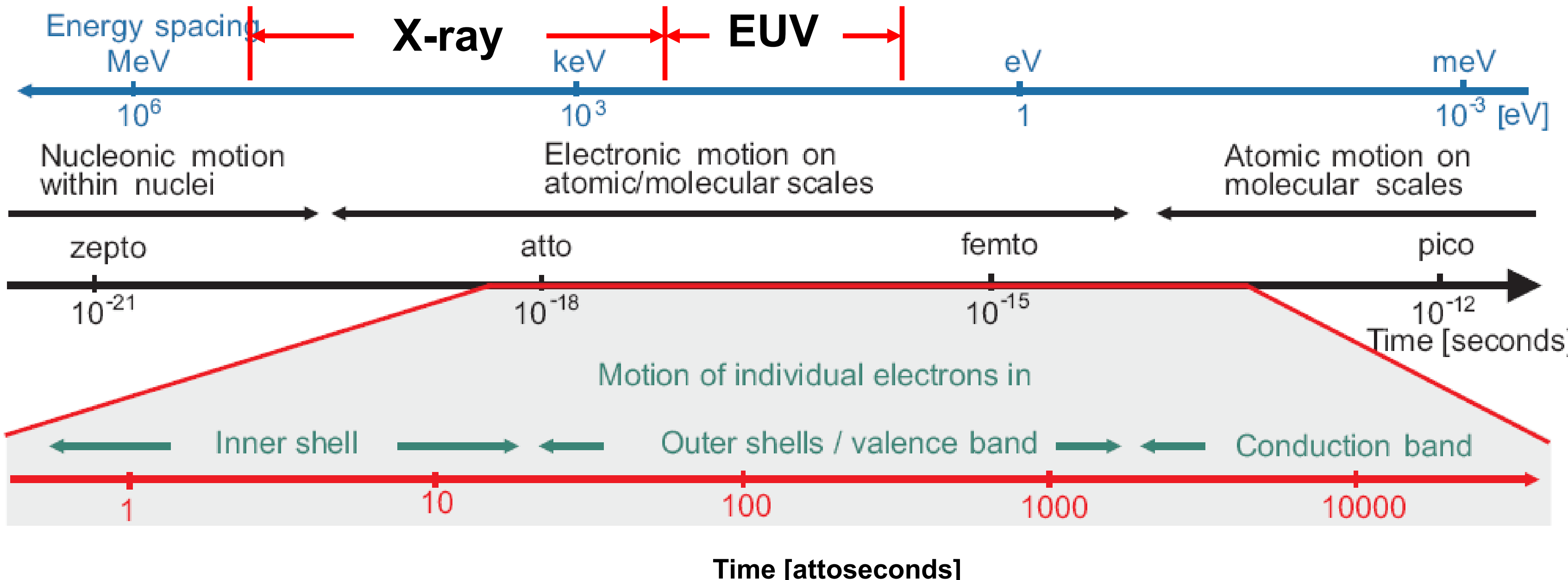
What happens when a bullet rips through an apple?

Harold Edgerton, * 6. April 1903 in Fremont, Nebraska, USA; † 4. Januar 1990 in Cambridge, MA, american electrical engineer, inventor strobe photography.

http://www.eadweardmuybridge.co.uk/

http://web.mit.edu/edgerton/

Physics on femto- attosecond time scales?



Light travels:

A second: from the moon to the earth

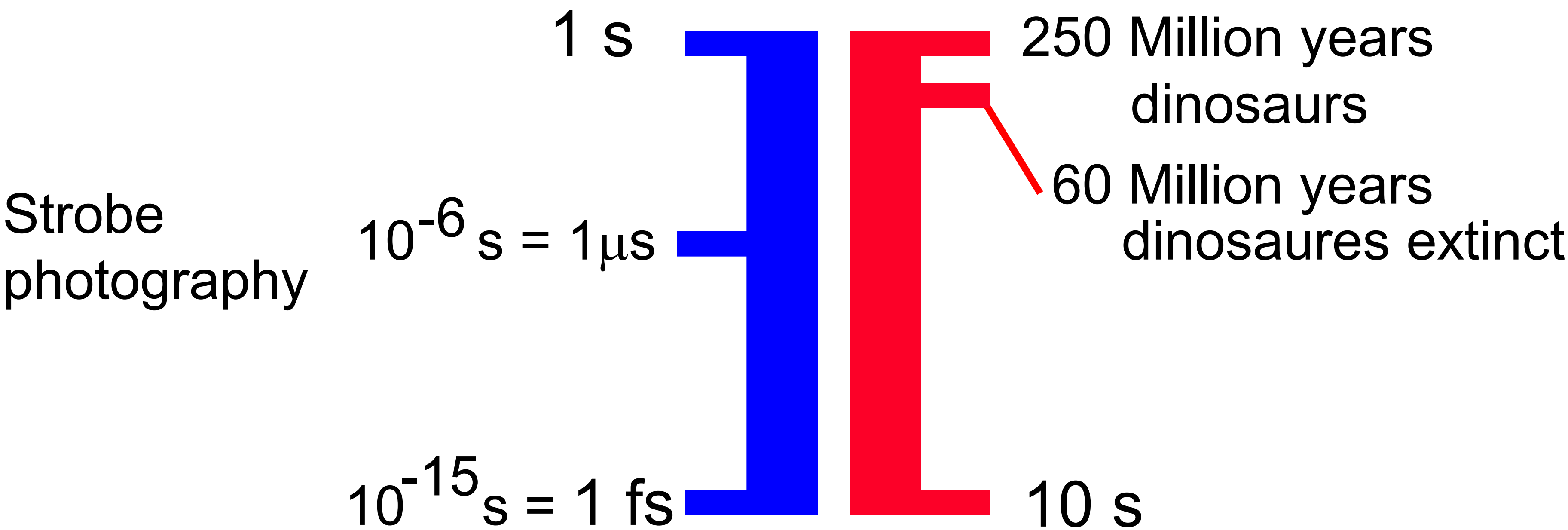
A picosecond: a fraction of a millimeter, through a blade of a knife

A femtosecond: the period of an optical wave, a wavelength

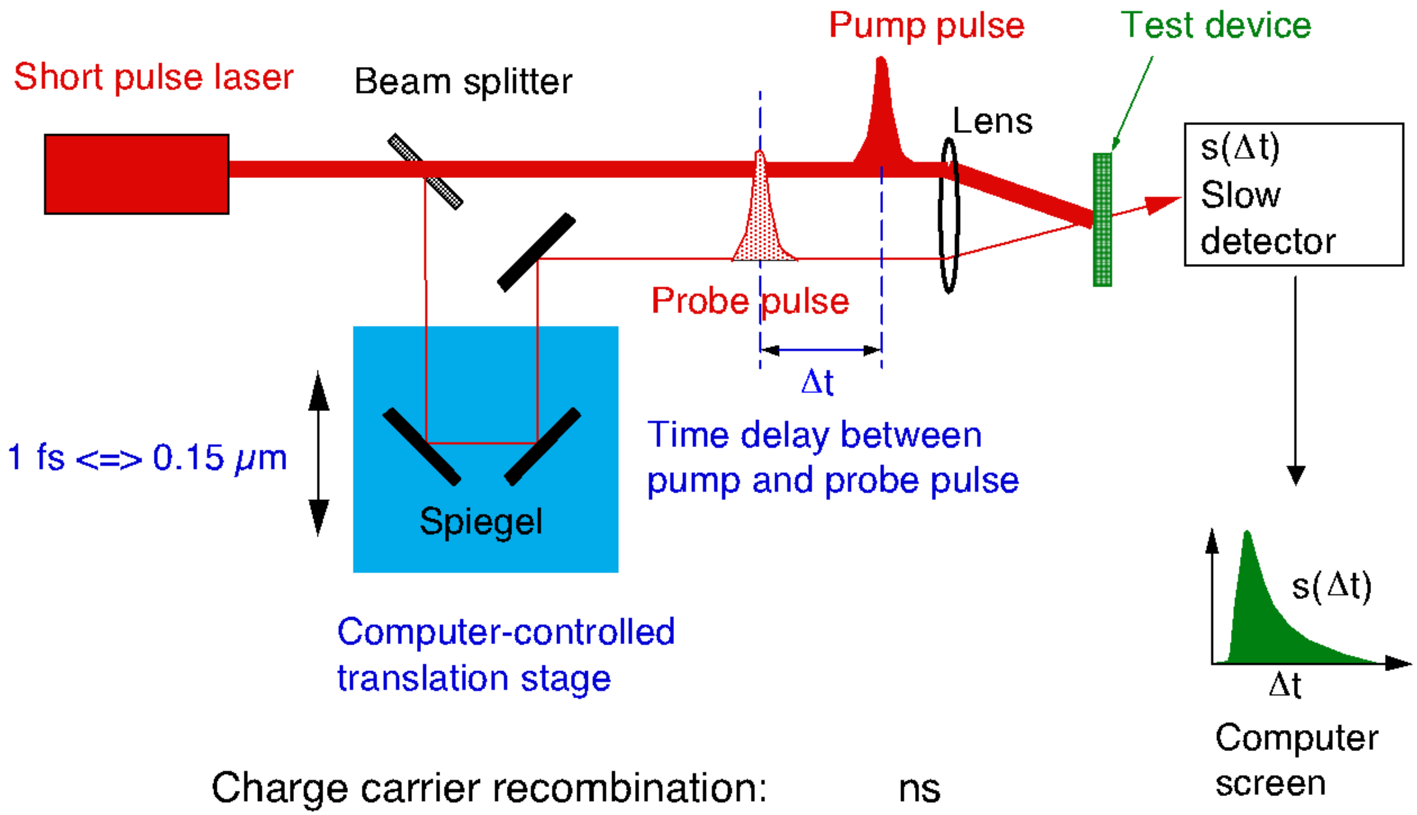
An attosecond: the period of X-rays, a unit cell in a solid

***F. Krausz and M. Ivanov, Rev. Mod. Phys. 81, 163 (2009)**

How short is a Femtosecond



Pump - Probe Measurements



Charge carrier recombination: ns

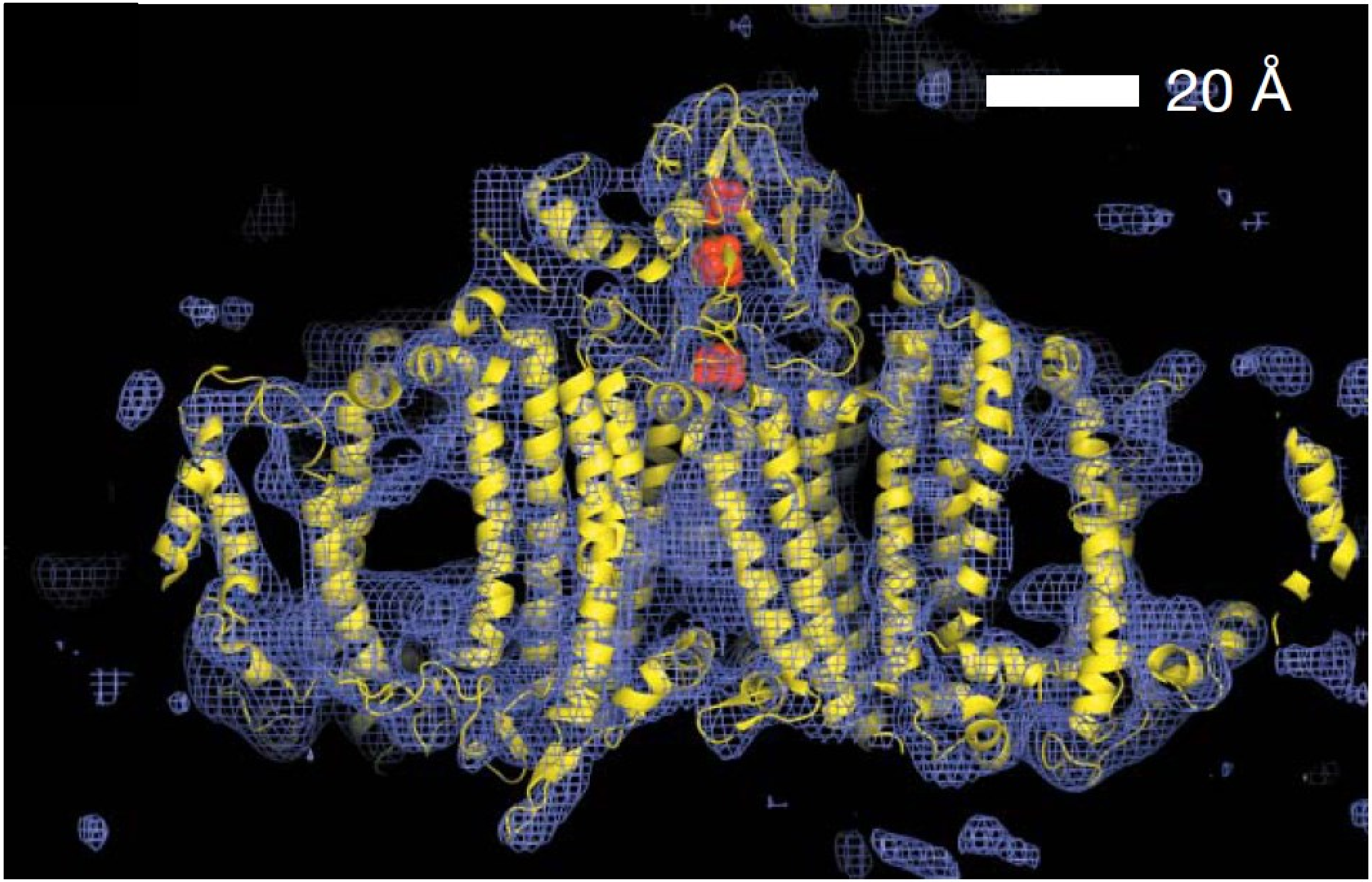
Thermalization electrons with lattice: ps

Thermalization electron gas: 10 -100 fs

Todays Frontiers in Space and Time

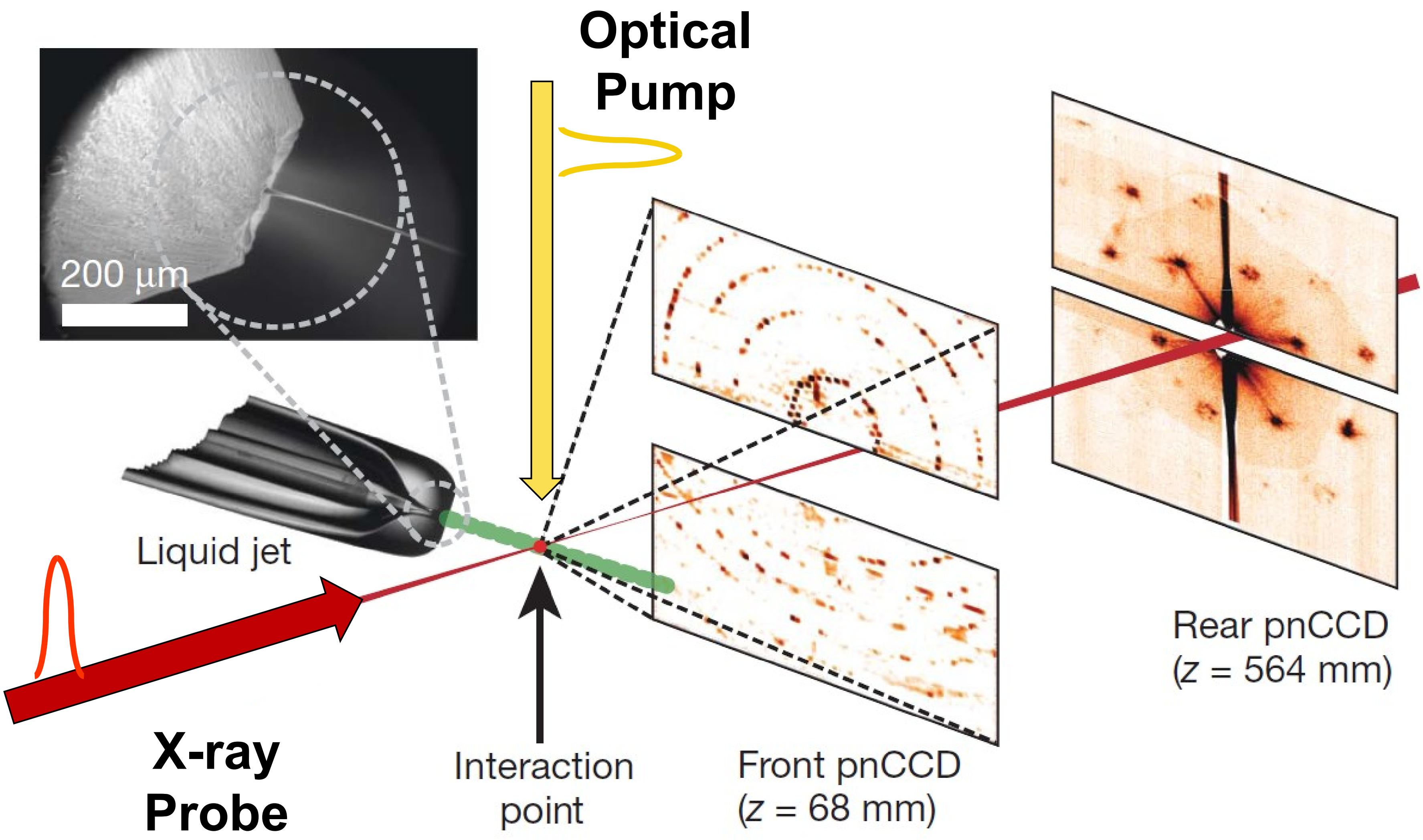
Structure, Dynamics and Function of Atoms and Molecules

Struture of Photosystem I



Chapman, et al. Nature 470, 73, 2011

X-ray Imaging (Time Resolved)



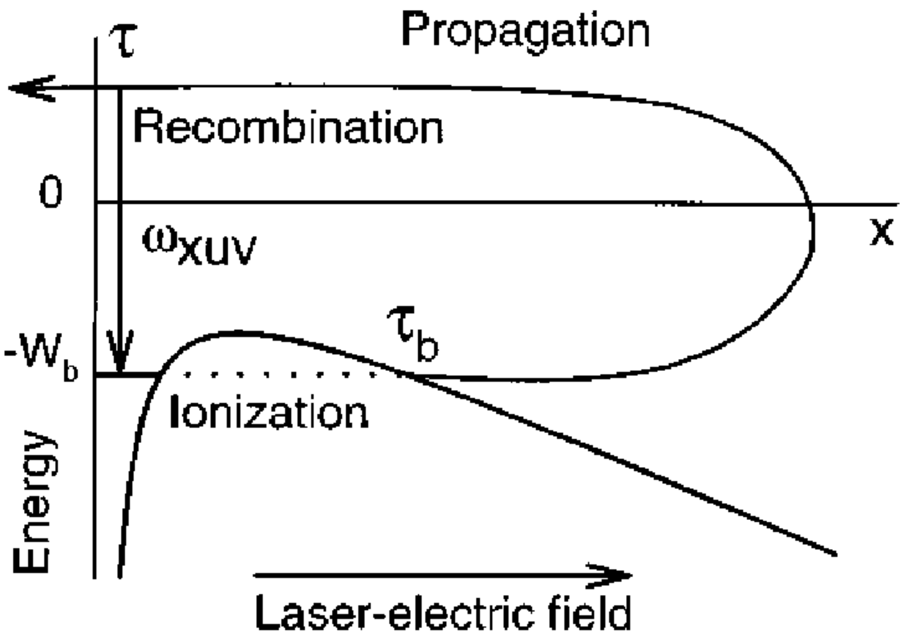
Imaging before destruction: Femtosecond Serial X-ray crystalography

Chapman, et al. Nature 470, 73, 2011

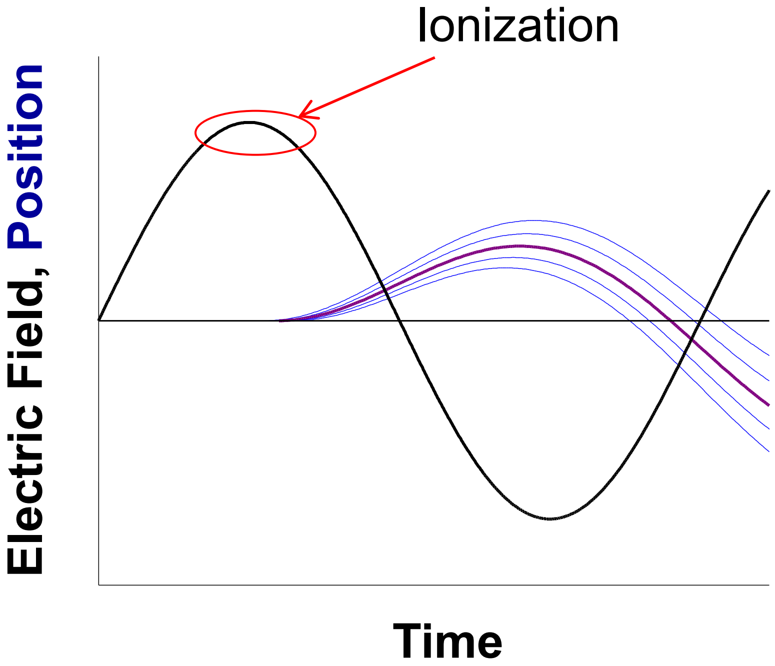
Attosecond Soft X-ray Pulses

Three-Step Model

Corkum, 1993



Trajectories



First Isolated Attosecond Pulses: M. Hentschel, et al., Nature 414, 509 (2001)

Hollow-Fiber Compressor: M. Nisoli, et al., Appl. Phys. Lett. 68, 2793 (1996)

High - energy single-cycle laser pulses!

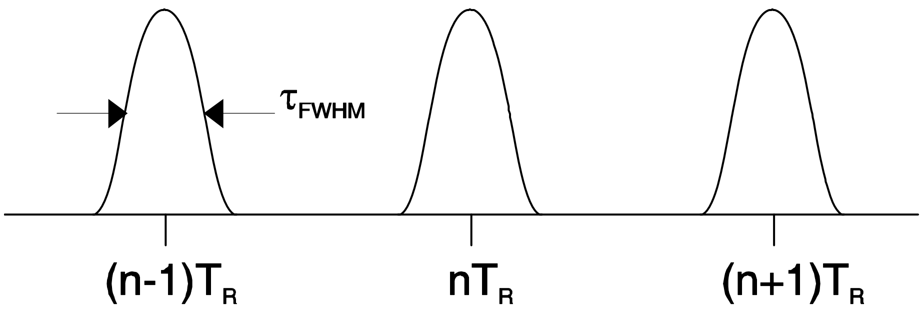
How do we generate them?

Short Pulse Laser Systems

- **Laser Oscillators (nJ), cw, q-switched, modelocked: Semiconductor, Fiber, Solid-State Lasers**
- **Laser Amplifiers: Solid-State or Fiber Lasers**
  - **Regenerative Amplifiers**
  - **Multipass Amplifiers**
  - **Chirped Pulse Amplification**
  - **Parametric Amplification and Nonlinear Frequency Conversion**

Content

1. Basics of Optical Pulses



T_R : pulse repetition rate
W : pulse energy
$P_{ave} = W/T_R$: average power
τ_{FWHM} : Full Width Half Maximum pulse width

P_p : peak power

Peak Electric Field: $$E_p = \sqrt{2Z_{F_0}\frac{P_p}{A_{\text{eff}}}}.$$

$$P_p = \frac{W}{\tau_{\text{FWHM}}} = P_{ave}\,\frac{T_R}{\tau_{\text{FWHM}}}$$

A_{eff} : effective beam cross section
Z_{Fo} : field impedance, Z_{Fo} = 377 Ω

average power:

$$P_{ave} \sim 1W - 1kW$$

repetition rates:

$$T_R^{-1} = f_R = \mathrm{mHz} - 100\,\mathrm{GHz}$$

pulse energy:

$$W = 1\mathrm{pJ} - 1\mathrm{kJ}$$

pulse width:

$$\tau_{\mathrm{FWHM}} = \begin{array}{ll} 5\,\mathrm{fs} - 50\,\mathrm{ps}, & \mathrm{modelocked} \\ 30\,\mathrm{ps} - 100\,\mathrm{ns}, & \mathrm{Q-switched} \end{array}$$

peak power:

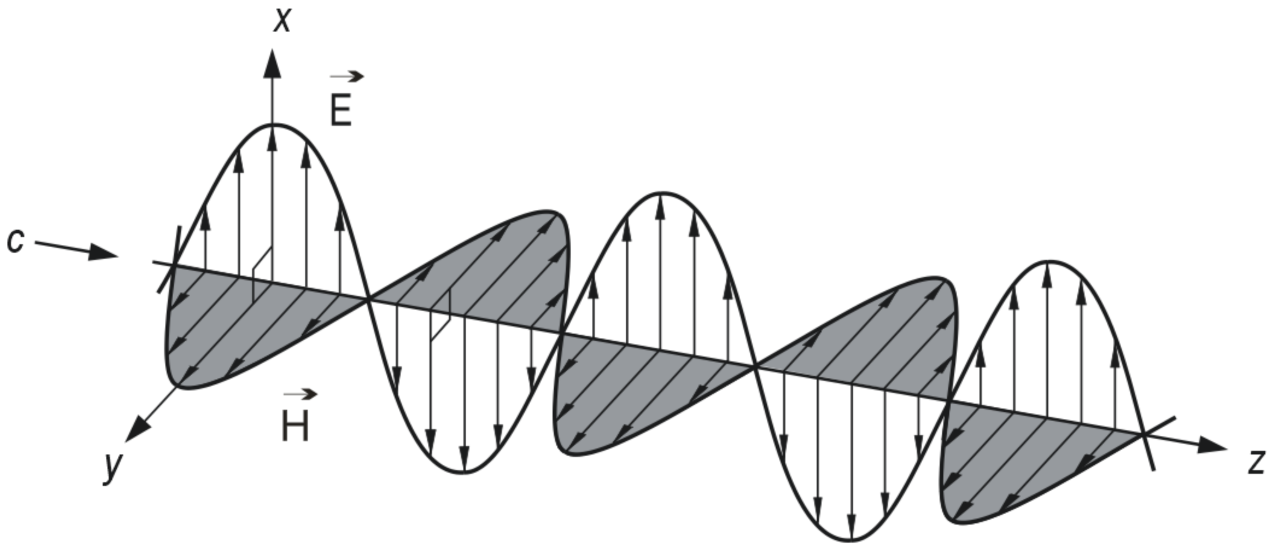
$$P_p = \frac{1\,\mathrm{kJ}}{1\,\mathrm{ps}} = \frac{1\,\mathrm{J}}{1\,\mathrm{fs}} \sim 1\,\mathrm{PW},$$

Typical Lab Pulse:

$$P_p = \frac{10\,\mathrm{nJ}}{10\,\mathrm{fs}} \sim 1\,\mathrm{MW}$$

$$E_p = \sqrt{2 \times 377 \times \frac{10^6 \times 10^{12}}{\pi \times (1.5)^2}}\,\frac{\mathrm{V}}{\mathrm{m}} \approx 10^{10}\,\frac{\mathrm{V}}{\mathrm{m}} = \frac{10\,\mathrm{V}}{\mathrm{nm}}$$

Time Harmonic Electromagnetic Waves



Transverse electromagnetic wave (TEM) (Teich, 1991)

See previous class: Plane-Wave Solutions (TEM-Waves)

Optical Pulses (propagating along z-axis)

$$\vec{\underline{E}}(\vec{r},t) = \int_0^\infty \frac{d\Omega}{2\pi} \underline{\tilde{E}}(\Omega) e^{j(\Omega t - K(\Omega)z)}\ \vec{e}_x$$

$$\vec{E}(\vec{r},t) = \frac{1}{2}\left(\vec{\underline{E}}(\vec{r},t) + \vec{\underline{E}}(\vec{r},t)^*\right)$$

$$\vec{H}(\vec{r},t) = \frac{1}{2}\left(\vec{\underline{H}}(\vec{r},t) + \vec{\underline{H}}(\vec{r},t)^*\right)$$

$|\underline{\tilde{E}}(\Omega)|e^{j\varphi(\Omega)}$: Wave amplitude and phase

$K(\Omega) = \Omega/c(\Omega) = n(\Omega)\Omega/c_0$: Wave number

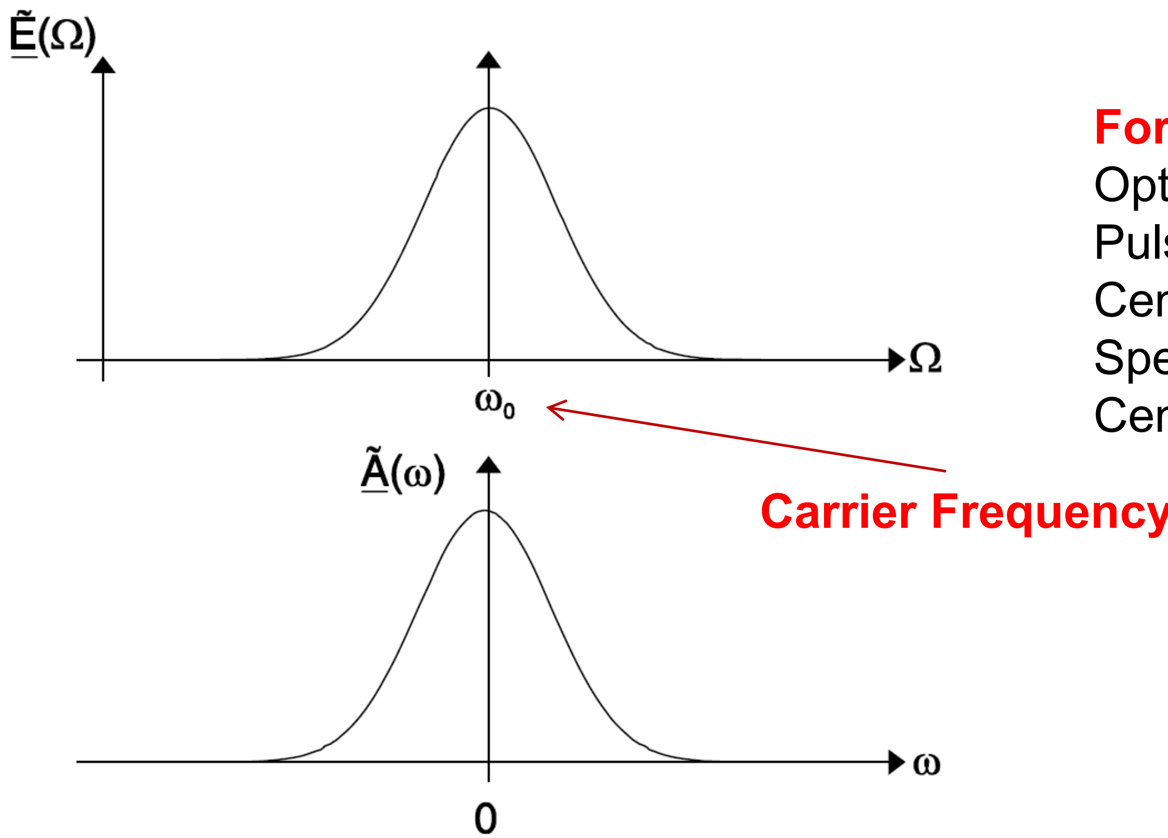
$c(\Omega) = \dfrac{c_0}{n(\Omega)}$: Phase velocity of wave

$$\tilde{n}^2(\Omega) = 1 + \tilde{\chi}(\Omega)$$

Absolute and Relative Frequency

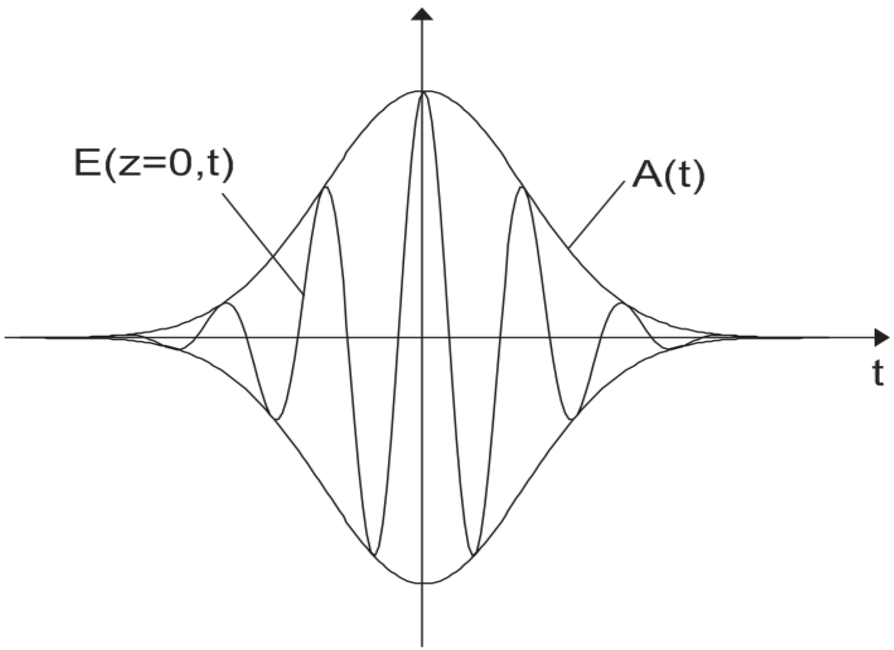
At z=0

$$\underline{E}(z=0,t)=\frac{1}{2\pi}\int_0^\infty \underline{\tilde{E}}(\Omega)e^{j\Omega t}d\Omega$$



For Example:
Optical Communication; 10Gb/s
Pulse length: 20 ps
Center wavelength : λ=1550 nm.
Spectral width: ~ 50 GHz,
Center frequency: 200 THz,

Spectrum of an optical pulse described in absolute and relative frequencies



Electric field and envelope of an optical pulse

Pulse width: Full Width at Half Maximum of $|A(t)|^2$

Spectral width : Full Width at Half Maximum of $|\underline{\tilde{A}}(\omega)|^2$

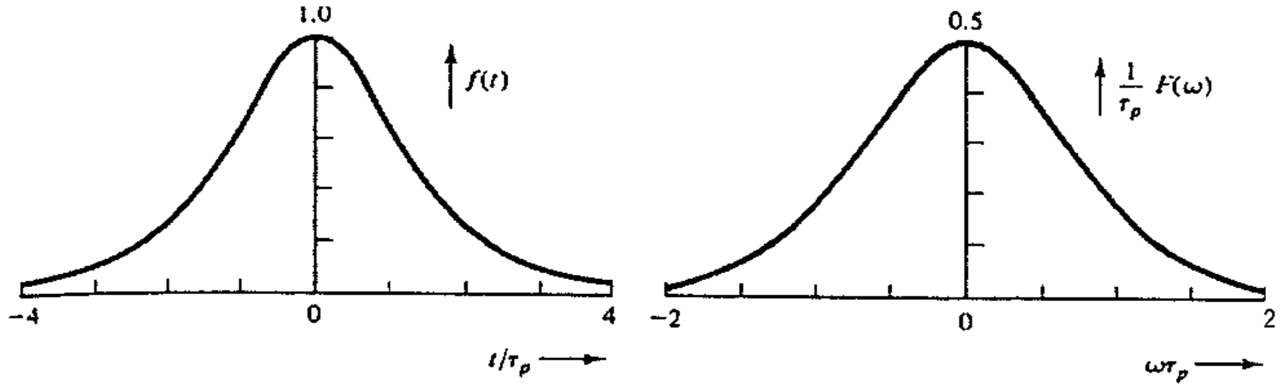
Often Used Pulses

Pulse Shape	Fourier Transform	Pulse Width	Time-Band-width Product
$\underline{A}(t)$	$\underline{\tilde{A}}(\omega)=\int_{-\infty}^{\infty} a(t)e^{-j\omega t}dt$	Δt	$\Delta t \cdot \Delta f$
Gaussian: $\mathrm{e}^{-\frac{t^2}{2\tau^2}}$	$\sqrt{2\pi}\tau \mathrm{e}^{-\frac{1}{2}\tau^2\omega^2}$	$2\sqrt{\ln 2}\tau$	0.441
Hyperbolic Secant: $\mathrm{sech}(\frac{t}{\tau})$	$\frac{\tau}{2}\,\mathrm{sech}\left(\frac{\pi}{2}\tau\omega\right)$	$1.7627\ \tau$	0.315
Rect-function: $\begin{cases} 1, & \lvert t\rvert \leq \tau/2 \\ 0, & \lvert t\rvert > \tau/2 \end{cases}$	$\tau\frac{\sin(\tau\omega/2)}{\tau\omega/2}$	τ	0.886
Lorentzian: $\frac{1}{1+(t/\tau)^2}$	$2\pi\tau \mathrm{e}^{-\lvert\tau\omega\rvert}$	$1.287\ \tau$	0.142
Double-Exp.: $\mathrm{e}^{-\lvert\frac{t}{\tau}\rvert}$	$\frac{\tau}{1+(\omega\tau)^2}$	$\ln 2\ \tau$	0.142

Table 2.2: Pulse shapes, corresponding spectra and time bandwidth products.

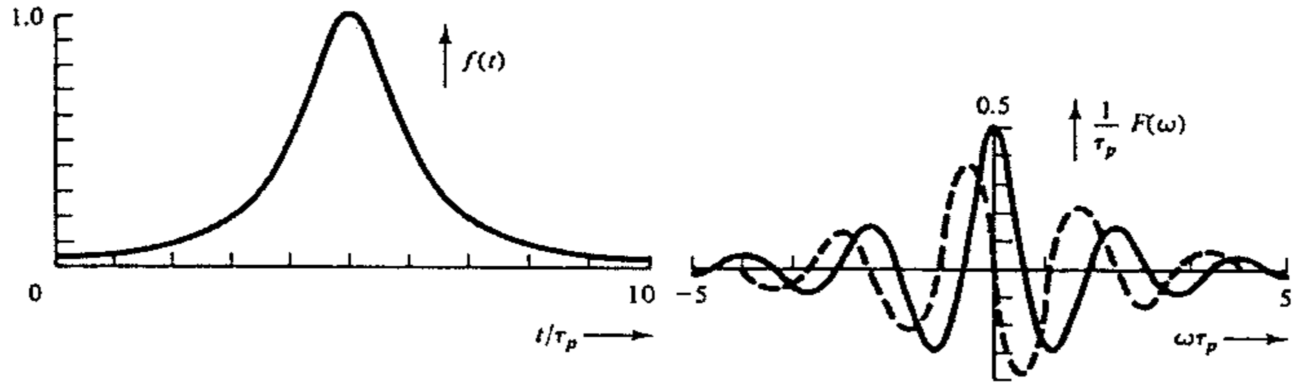
Pulse width and spectral width: FWHM

Fourier transforms to pulse shapes listed in table 2.2 [16]



$f(t) = \operatorname{sech}(t/\tau_p)$

$\frac{1}{\tau_p} F(\omega) = \frac{1}{2} \operatorname{sech}\left[\frac{\pi}{2}\omega\tau_p\right]$

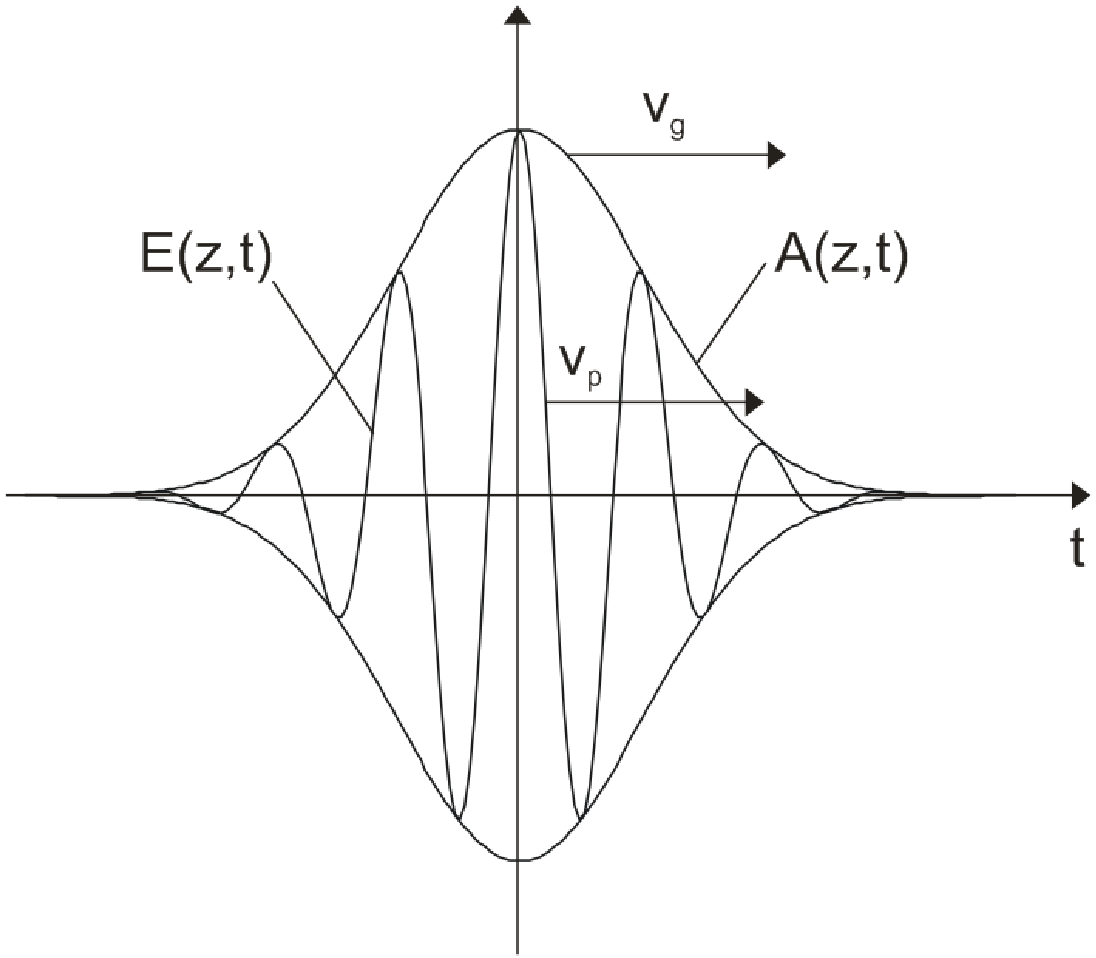


$$f(t) = \frac{1}{1 + \left[\frac{t - t_0}{\tau_p}\right]^2} \qquad t_0 = 5\tau_p$$

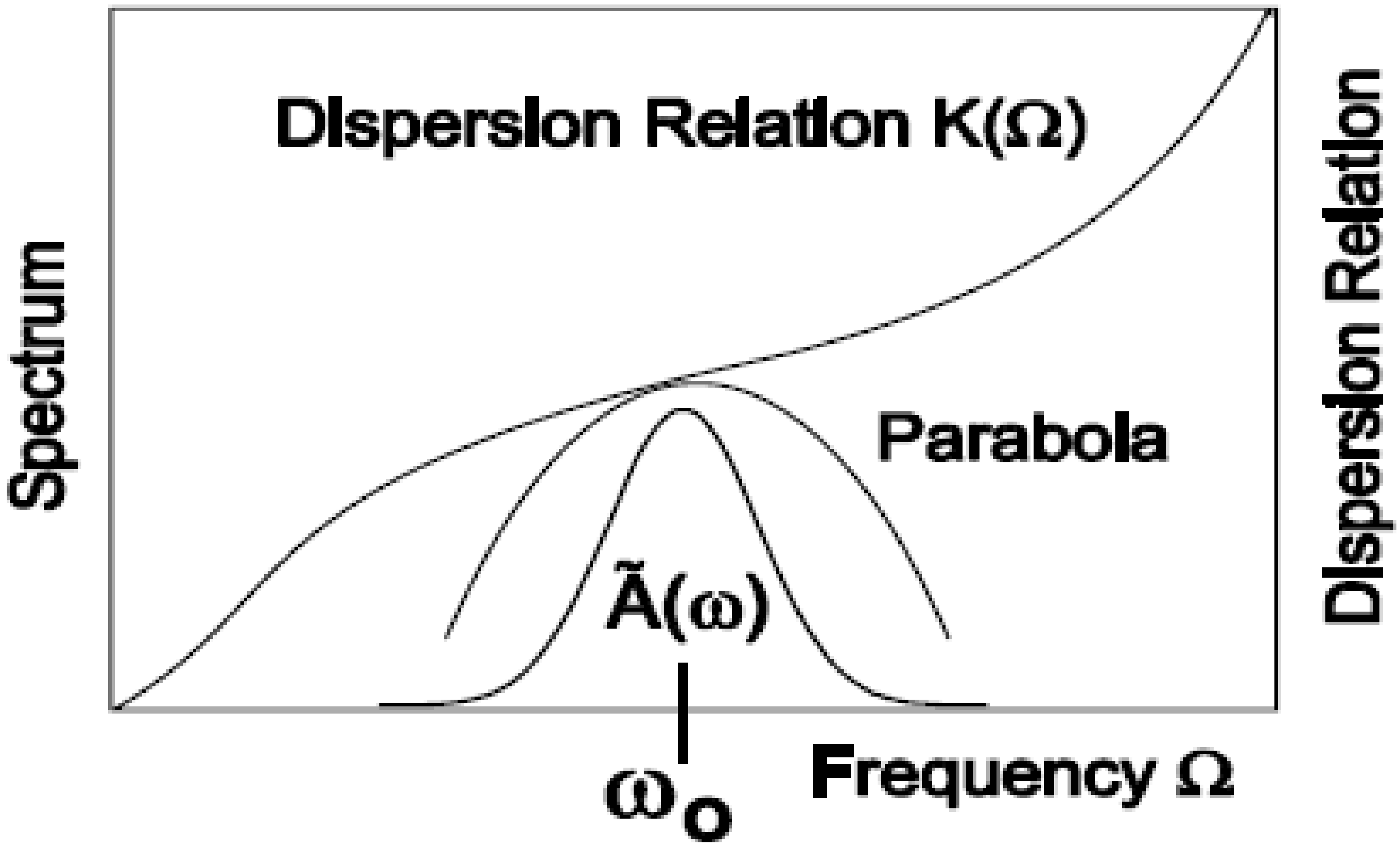
$$\operatorname{Re}\left[\frac{1}{\tau_p} F(\omega)\right] = \frac{1}{2} \cos \omega t_0 e^{-|\omega\tau_p|}$$

$$\operatorname{Im}\left[\frac{1}{\tau_p} F(\omega)\right] = -\frac{1}{2} \sin \omega t_0 e^{-|\omega\tau_p|} \; \ldots$$

Electric field and pulse envelope in time domain



Taylor expansion of dispersion relation at pulse center frequency



1.1 Dispersion

In the frequency domain:

$$\underline{\tilde{A}}(z,\omega) = \underline{\tilde{A}}(z=0,\omega)e^{-jk(\omega)z}$$

Taylor expansion of dispersion relation:

$$k(\omega) = k'\omega + \frac{k''}{2}\omega^2 + \frac{k^{(3)}}{6}\omega^3 + O(\omega^4)$$

i) Keep only linear term:

$$k(\omega) = k'\omega + \cancel{\frac{k''}{2}}\omega^2 + \cancel{\frac{k^{(3)}}{6}}\omega^3 + \cancel{O}(\omega^4)$$

$$\underline{\tilde{A}}(z,\omega) = \underline{\tilde{A}}(z=0,\omega)e^{-jk'\omega z}$$

Time domain: $$\underline{A}(z,t) = \underline{A}(0, t - z/\upsilon_{g0})$$

Group velocity: $$\upsilon_{g0} = 1/k' = \left(\left.\frac{dk(\omega)}{d\omega}\right|_{\omega=0}\right)^{-1} = \left(\left.\frac{dK(\Omega)}{d\Omega}\right|_{\Omega=\omega_0}\right)^{-1}$$

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Compare with phase velocity:

$$v_{p0} = \omega_0 / K(\omega_0) = \left(\frac{K(\omega_0)}{\omega_0} \right)^{-1}$$

Retarded time: $t' = t - z/v_{g0}$

$$\underline{A}(z,t) = \underline{A}(0,t')$$

ii) Keep up to second order term:

$$k(\omega) = k'\omega + \frac{k''}{2}\omega^2 + \cancel{\frac{k^{(3)}}{6}}\omega^3 + \cancel{O}(\omega^4)$$

$$\frac{\partial \underline{A}(z,t')}{\partial z} = \mathrm{j}\frac{k''}{2}\frac{\partial^2 \underline{A}(z,t')}{\partial t'^2}$$

Gaussian Pulse:

$$\underline{E}(z = 0,t) = \underline{A}(z=0,t)e^{j\omega_0 t}$$

$$\underline{A}(z = 0, t=t') = \underline{A}_0 \exp\left[-\frac{1}{2}\frac{t'^2}{\tau^2}\right]$$

Pulse width

$$\underline{A}(z,t') = A_0 \left(\frac{\tau^2}{(\tau^2 + jk''z)}\right)^{1/2} \exp\left[-\frac{1}{2}\frac{\tau^2 t'^2}{(\tau^4 + (k''z)^2)} + j\frac{1}{2}k''z\frac{t'^2}{(\tau^4 + (k''z)^2)}\right]$$

z-dependent phase shift

determines pulse width

chirp

FWHM Pulse width:

$$\exp\left[-\frac{\tau^2(\tau'_{FWHM}/2)^2}{(\tau^4 + (k''z)^2)}\right] = 0.5$$

Initial pulse width:

$$\tau_{FWHM} = 2\sqrt{\ln 2}\ \tau$$

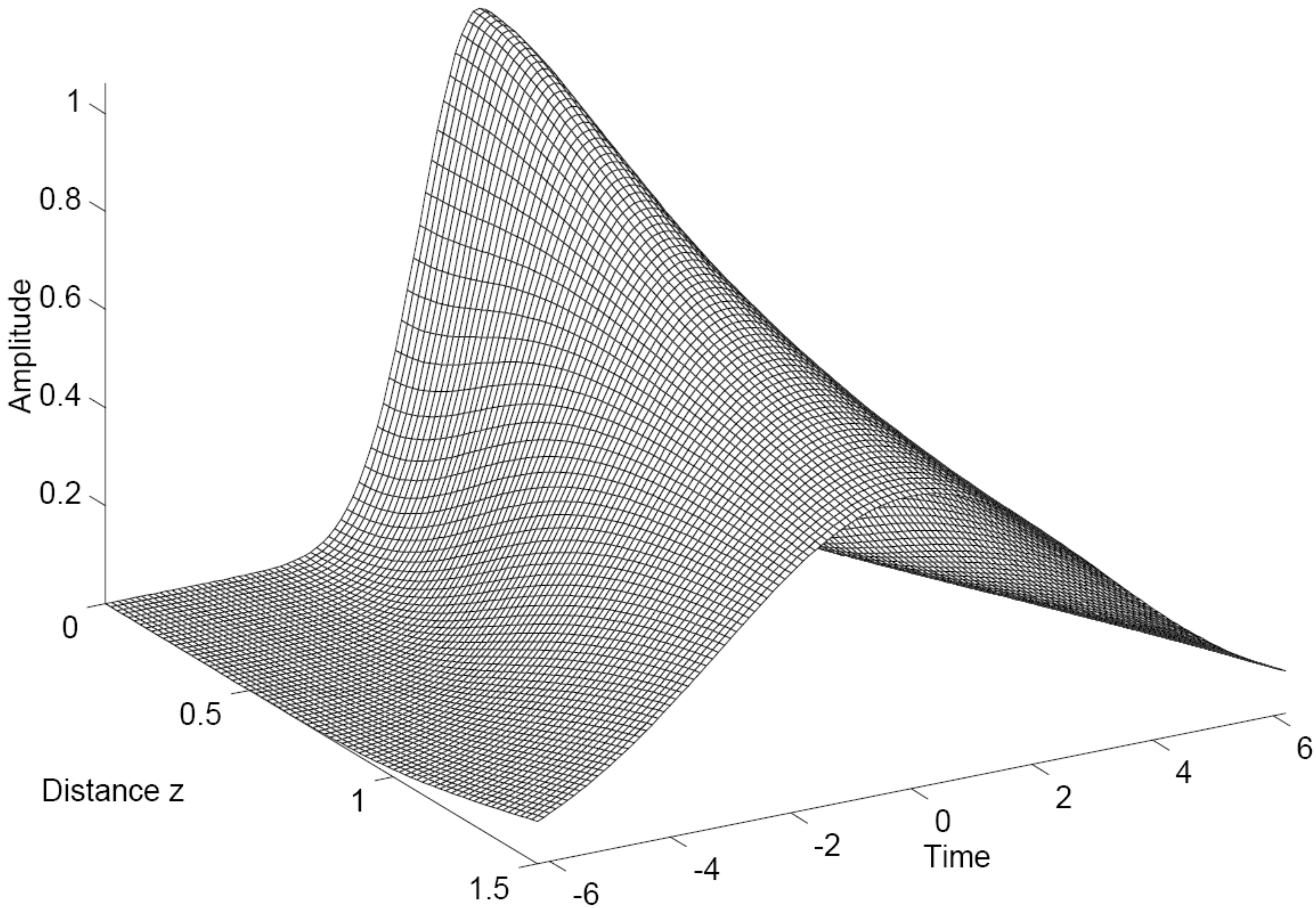
z = L

$$\tau'_{FWHM} = 2\sqrt{\ln 2}\ \tau\sqrt{1 + \left(\frac{k''L}{\tau^2}\right)^2}$$

$$= \tau_{FWHM}\sqrt{1 + \left(\frac{k''L}{\tau^2}\right)^2}$$

$$\tau'_{FWHM} = 2\sqrt{\ln 2}\ \left|\frac{k''L}{\tau}\right| \quad \text{for} \quad \left|\frac{k''L}{\tau^2}\right| \gg 1$$

Magnitude Gaussian pulse envelope, |$\underline{A}$(z, t')|, in a dispersive medium



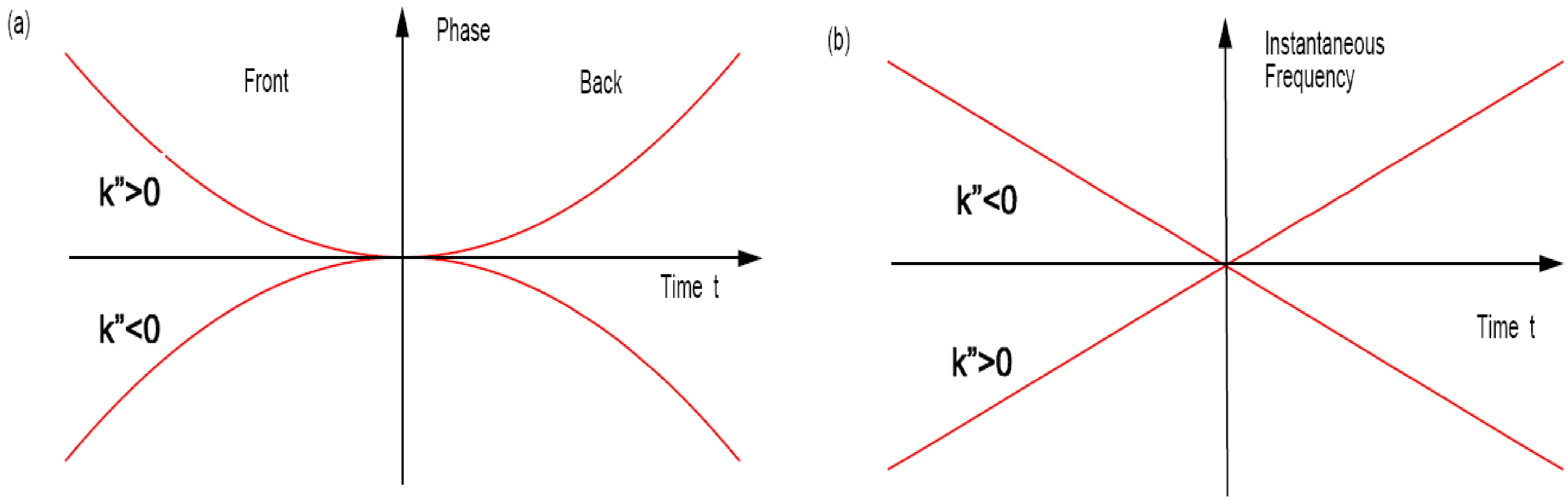
(a) Phase and (b) instantaneous frequency of a Gaussian pulse during propagation through a medium with positive or negative dispersion

Phase:

$$\phi(z=L,t') = -\frac{1}{2}\arctan\left[\frac{k''L}{\tau^2}\right] + \frac{1}{2}k''L\frac{t'^2}{\left(\tau^4+(k''L)^2\right)}$$

Instantaneous Frequency:

$$\omega(z=L,t') = \frac{\partial}{\partial t'}\phi(L,t') = \frac{k''L}{\left(\tau^4+(k''L)^2\right)}t'$$



k''>0: Postive Group Velocity Dispersion (GVD), low frequencies travel faster and are in front of the pulse

Sellmeier Equations

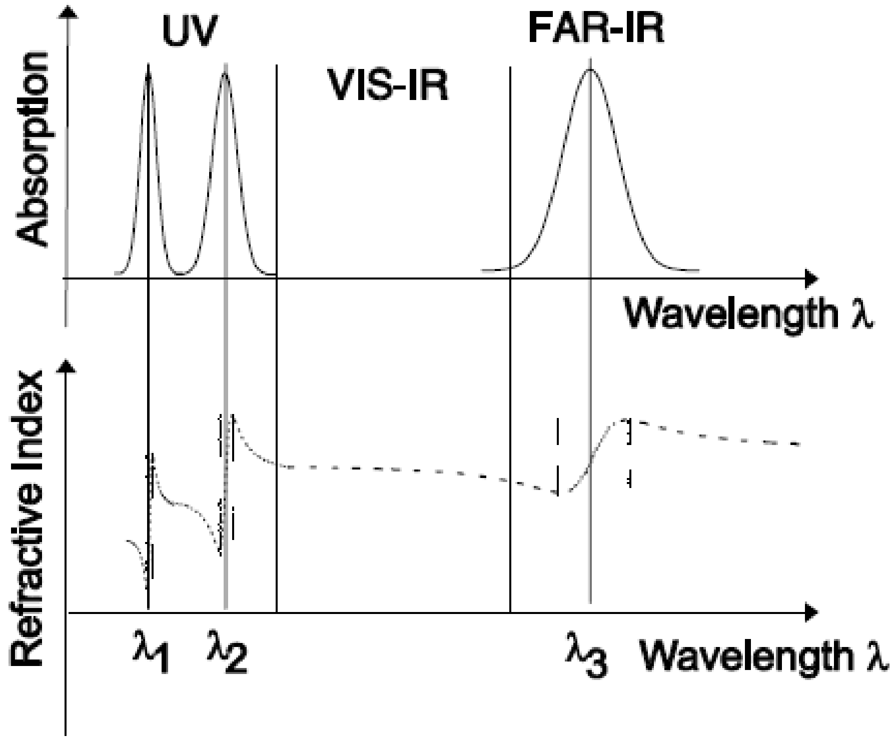
$$n^2(\Omega) = 1 + \underbrace{\sum_i A_i \frac{\omega_i}{\omega_i^2 - \Omega^2} = 1 + \sum_i a_i \frac{\lambda}{\lambda^2 - \lambda_i^2}}_{\chi_r(\Omega)}$$

Example: Sellmeier Coefficients for Fused Quartz and Sapphire

	Fused Quartz	Sapphire
a_1	0.6961663	1.023798
a_2	0.4079426	1.058364
a_3	0.8974794	5.280792
λ_1^2	$4.679148 \cdot 10^{-3}$	$3.77588 \cdot 10^{-3}$
λ_2^2	$1.3512063 \cdot 10^{-2}$	$1.22544 \cdot 10^{-2}$
λ_3^2	$0.9793400 \cdot 10^{2}$	$3.213616 \cdot 10^{2}$

Table 2.3: Table with Sellmeier coefficients for fused quartz and sapphire.

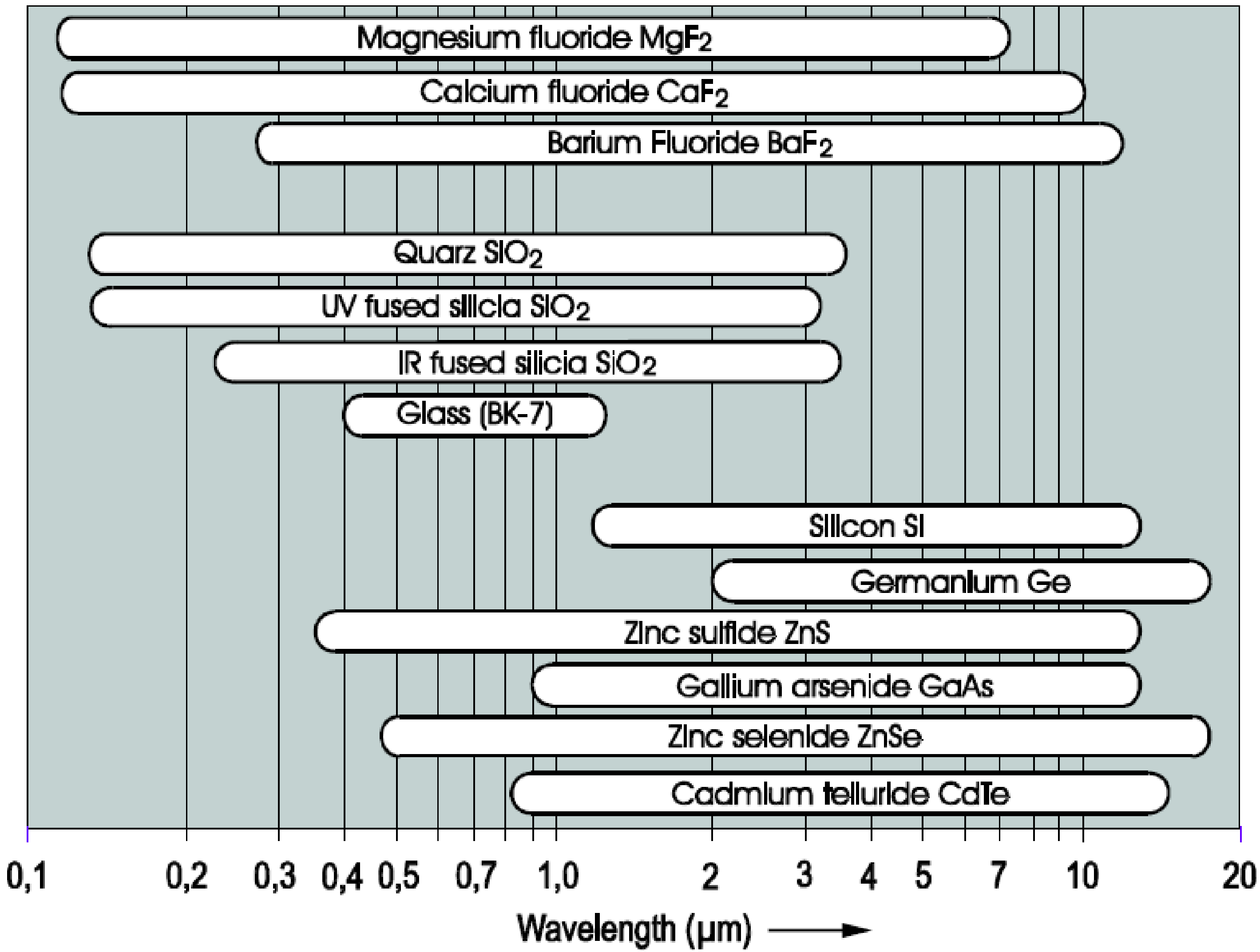
Typical distribution of absorption lines in medium transparent in the visible.



$$\frac{dn}{d\lambda} < 0 : \text{normal dispersion (blue refracts more than red)}$$

$$\frac{dn}{d\lambda} > 0 : \text{abnormal dispersion}$$

Transparency range of some materials, Saleh and Teich, Photonics p. 175.



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Group Velocity and Group Delay Dispersion

$$GVD = \left.\frac{d^2k(\omega)}{d\omega^2}\right|_{\omega=0} = \frac{d}{d\omega}\left.\frac{1}{v_g(\omega)}\right|_{\omega=0}$$

$$GDD = \left.\frac{d^2k(\omega)}{d\omega^2}\right|_{\omega=0} L = \frac{d}{d\omega}\left.\frac{L}{v_g(\omega)}\right|_{\omega=0} = \frac{d}{d\omega} T_g(\omega)\Big|_{\omega=0}$$

Group Delay: $T_g(\omega) = L/v_g(\omega)$

Dispersion Characteristic	Definition	Comp. from $n(\lambda)$
medium wavelength: λ_n	$\frac{\lambda}{n}$	$\frac{\lambda}{n(\lambda)}$
wavenumber: k	$\frac{2\pi}{\lambda_n}$	$\frac{2\pi}{\lambda} n(\lambda)$
phase velocity: v_p	$\frac{\omega}{k}$	$\frac{c_0}{n(\lambda)}$
group velocity: v_g	$\frac{d\omega}{dk}; d\lambda = \frac{-\lambda^2}{2\pi c_0} d\omega$	$\frac{c_0}{n}\left(1 - \frac{\lambda}{n}\frac{dn}{d\lambda}\right)^{-1}$
group velocity dispersion: GVD	$\frac{d^2k}{d\omega^2}$	$\frac{\lambda^3}{2\pi c_0^2}\frac{d^2n}{d\lambda^2}$
group delay: $T_g = \frac{L}{v_g} = \frac{d\phi}{d\omega}$	$\frac{d\phi}{d\omega} = \frac{d(kL)}{d\omega}$	$\frac{n}{c_0}\left(1 - \frac{\lambda}{n}\frac{dn}{d\lambda}\right) L$
group delay dispersion: GDD	$\frac{dT_g}{d\omega} = \frac{d^2(kL)}{d\omega^2}$	$\frac{\lambda^3}{2\pi c_0^2}\frac{d^2n}{d\lambda^2} L$

Table 2.4: Table with important dispersion characteristics and how to compute them from the wavelength dependent refractive index $n(\lambda)$.

1.2 Nonlinear Pulse Propagation

The Optical Kerr Effect

Without derivation, there is a nonlinear contribution to the refractive index:

$$n = n(\omega, |A|^2) \approx n_0(\omega) + n_{2,L}|A|^2$$

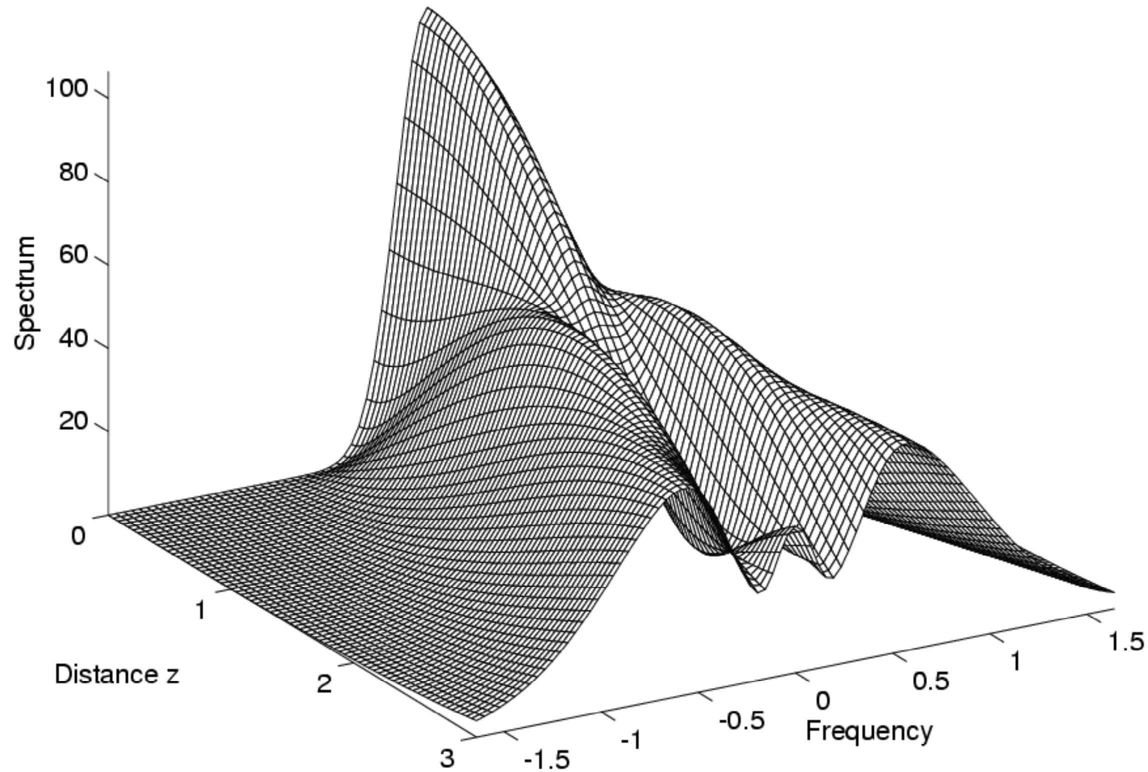
Polarization dependent

Material	Refractive index n	$n_{2,L}[cm^2/W]$
Sapphire (Al_2O_3)	1.76 @ 850 nm	$3{\cdot}10^{-16}$
Fused Quarz	1.45 @ 1064 nm	$2.46{\cdot}10^{-16}$
Glass (LG-760)	1.5 @ 1064 nm	$2.9{\cdot}10^{-16}$
YAG ($Y_3Al_5O_{12}$)	1.82 @ 1064 nm	$6.2{\cdot}10^{-16}$
YLF ($LiYF_4$), n_e	1.47 @ 1047 nm	$1.72{\cdot}10^{-16}$
Si	3.3 @ 1550 nm	$4{\cdot}10^{-14}$

Table 3.1: Nonlinear refractive index of some materials

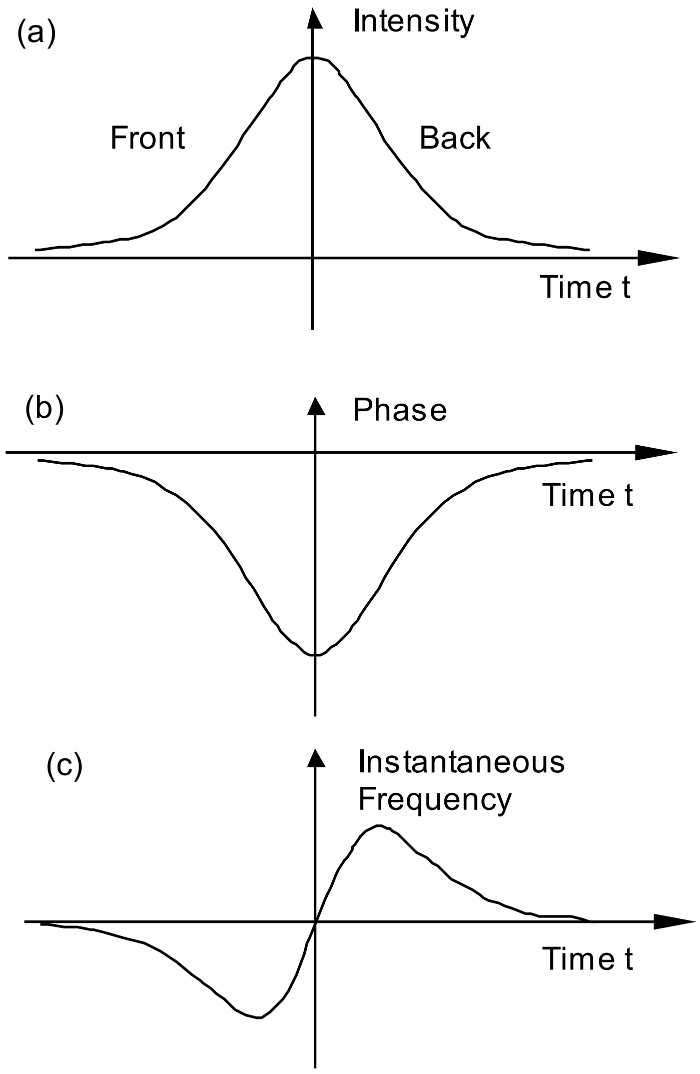
Self-Phase Modulation (SPM)

$$\frac{\partial A(z,t)}{\partial z} = -jk_0 n_{2,L}|A(z,t)|^2 A(z,t) = -j\delta|A(z,t)|^2 A(z,t).$$



Spectrum of a Gaussian pulse subject to self-phase modulation

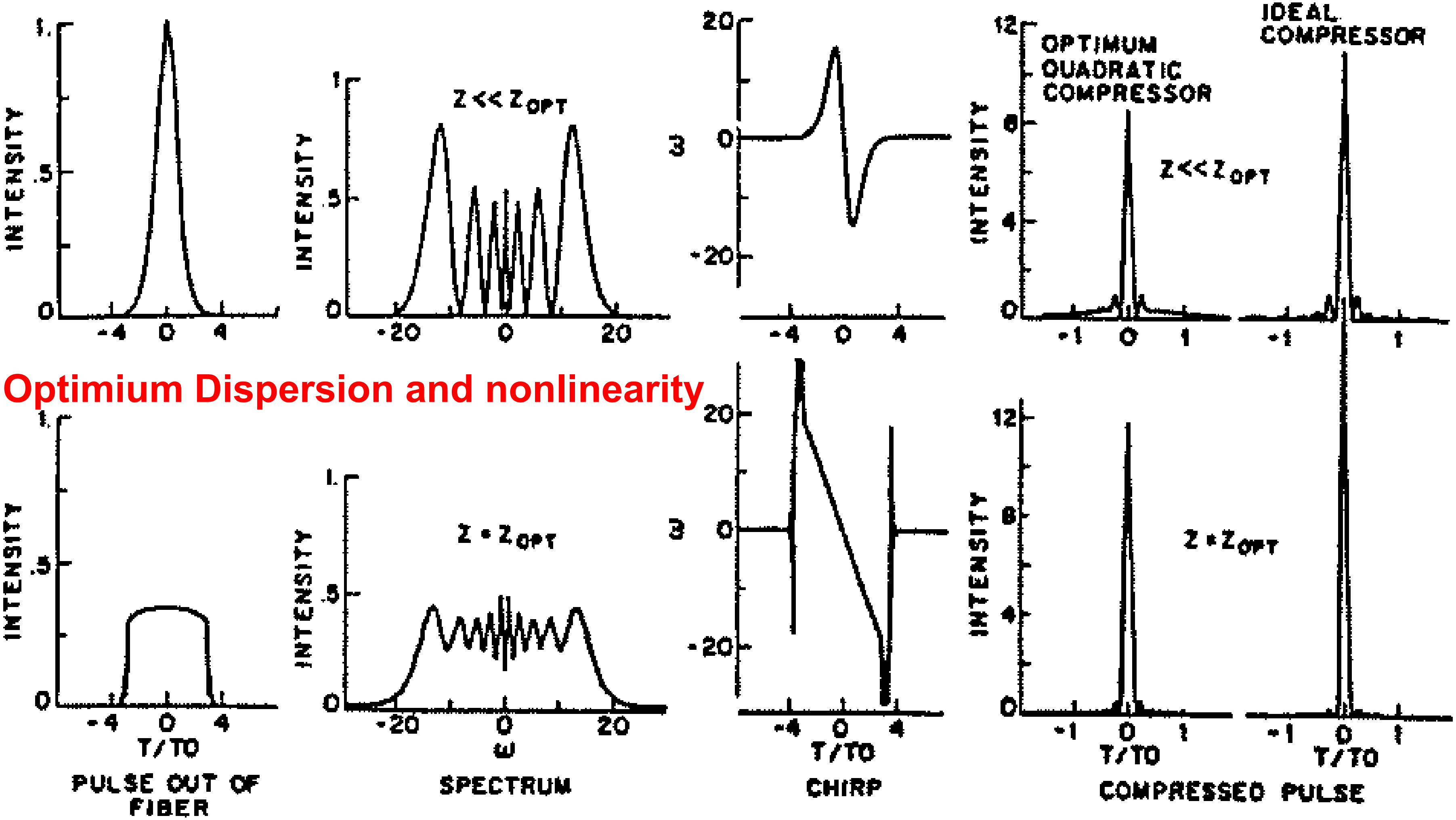
(a) Intensity, (b) phase and c) instantaneous frequency of a Gaussian pulse during propagation



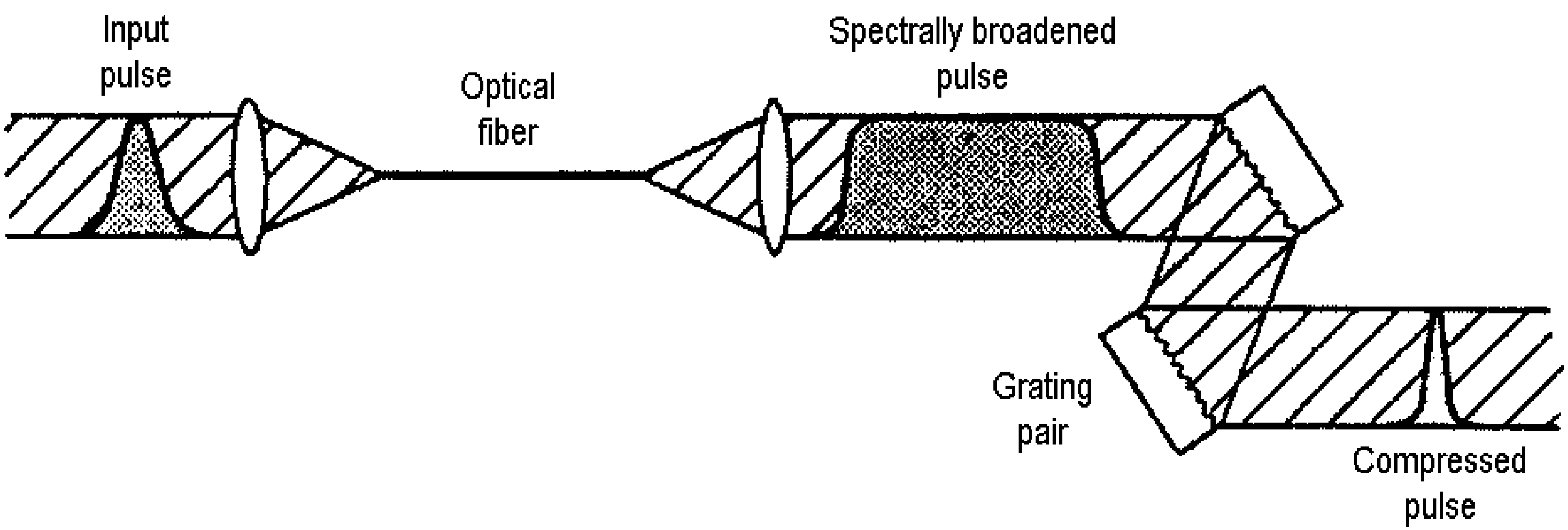
1.3 Pulse Compression

Dispersion negligible, only SPM

Optimium Dispersion and nonlinearity



Spectral Broadening with Guided Modes and Compression



Fiber-grating pulse compressor to generate femtosecond pulses

Pulse Compression:

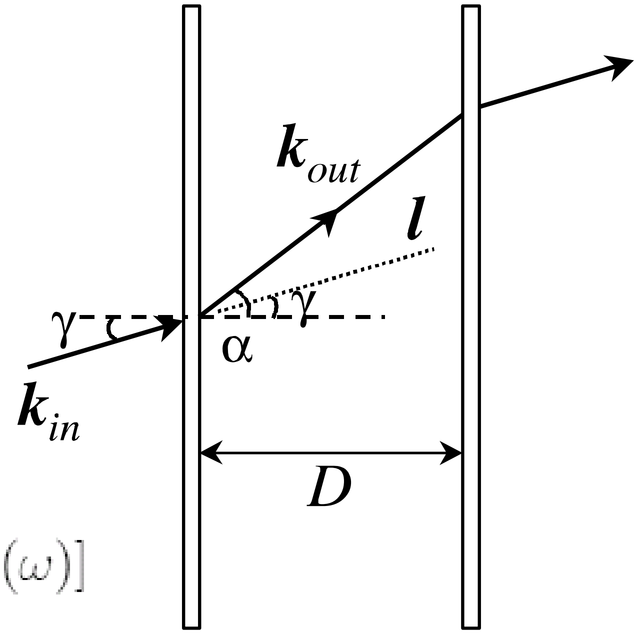
$$\phi''(\omega_0) = \phi''_{modulator} + \phi''_{compresssor} = 0$$

$$\phi'''(\omega_0) = \phi'''_{modulator} + \phi'''_{compresssor} = 0$$

Grating Pair

Phase difference between scattered beam and reference beam"

$$\phi(\omega) = \mathbf{k}_{out}(\omega) \cdot \mathbf{l}.$$



$$\phi(\omega) = \frac{\omega}{c}|\mathbf{l}|\cos[\gamma - \alpha(\omega)] = \frac{\omega}{c}\frac{D}{\cos(\gamma)}\cos[\gamma - \alpha(\omega)]$$

$$m\frac{2\pi c}{\omega d} = [\sin\alpha(\omega) - \sin\gamma]$$

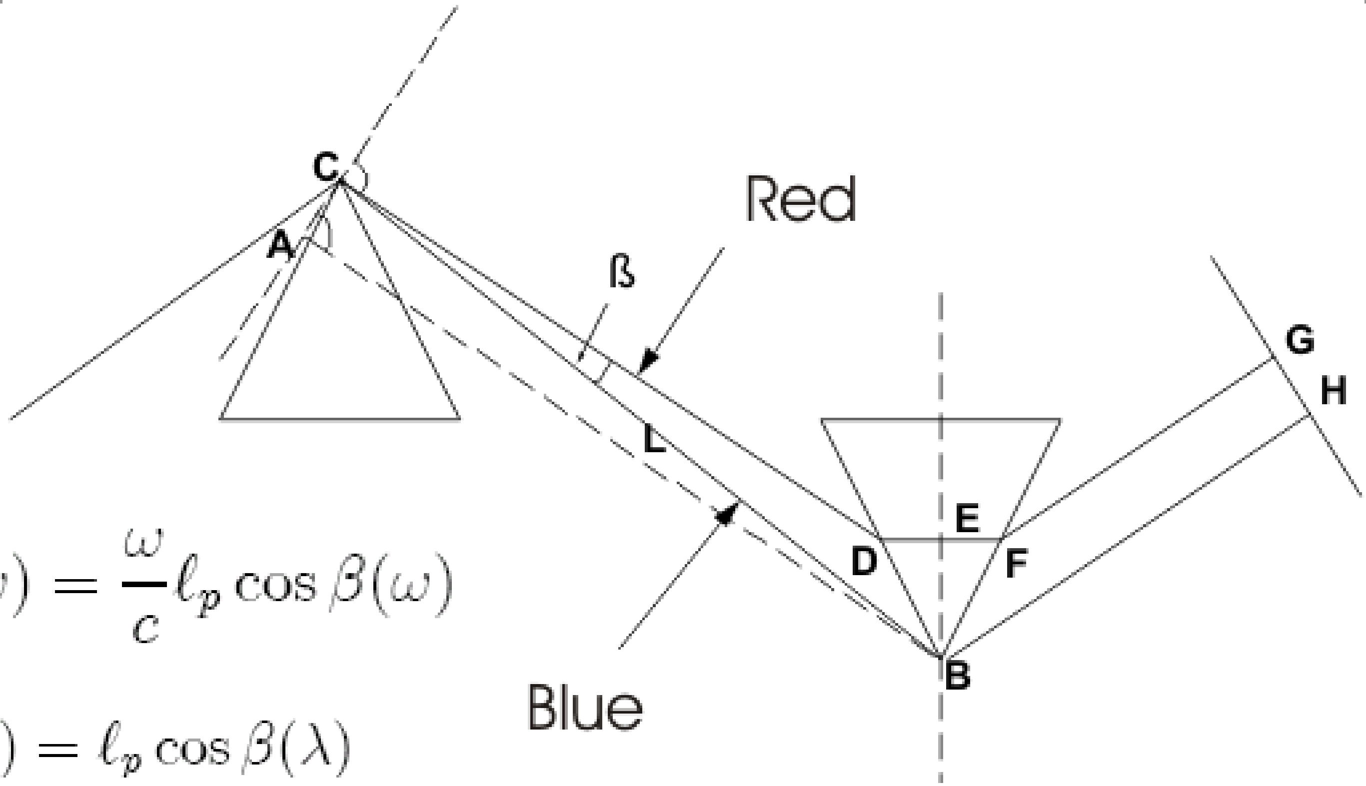
$$\phi''(\omega) = -\frac{4\pi^2 cD}{\omega^3 d^2 \cos^3\alpha(\omega)}m^2$$

$$\cos\alpha(\omega)\frac{d\alpha}{d\omega} = -\frac{2\pi c}{\omega^2 d}m$$

$$\phi'''(\omega) = \frac{12\pi^2 cD}{\omega^4 d^2 \cos^3\alpha(\omega)}\left(1 + \frac{2\pi c \sin\alpha(\omega)}{\omega d \cos^2\alpha(\omega)}\right)m^3$$

Disadvantage of grating pair: Losses ~ 25%

Prism Pair



$$\phi(\omega) = \frac{\omega}{c}\ell_p \cos\beta(\omega)$$

$$P(\lambda) = \ell_p \cos\beta(\lambda)$$

$$\phi''(\omega) = \frac{\lambda^3}{2\pi c^2}\frac{d^2P}{d\lambda^2}$$

$$\frac{d^2P}{d\lambda^2} = 2[n'' + (2n - n^{-3})(n')^2]\ell_p \sin\beta - 4(n')^2\ell_p\cos\beta$$

$$\phi'''(\omega) = -\frac{\lambda^4}{4\pi^2c^3}\left(3\frac{d^2P}{d\lambda^2} + \lambda\frac{d^3P}{d\lambda^3}\right)$$

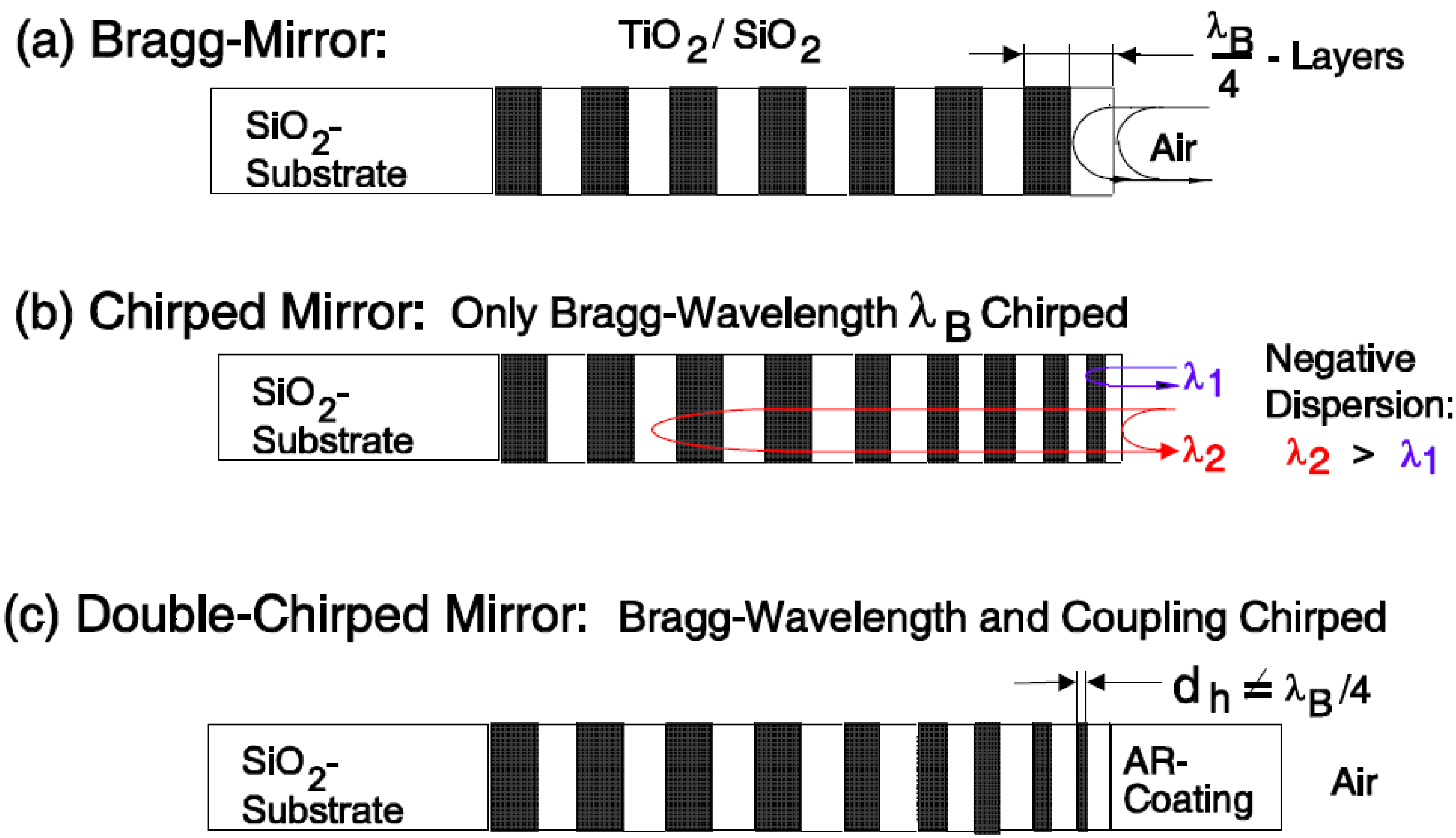
$$\frac{d^3P}{d\lambda^3} = [6(n')^3(n^{-6} + n^{-4} - 2n^{-2} + 4n^2) + 12n'n''(2n - n^{-3})$$

$$+2n''']\ell_p \sin\beta + 12[(n^{-3} - 2n)(n')^3 - n'n'']\ell_p\cos\beta$$

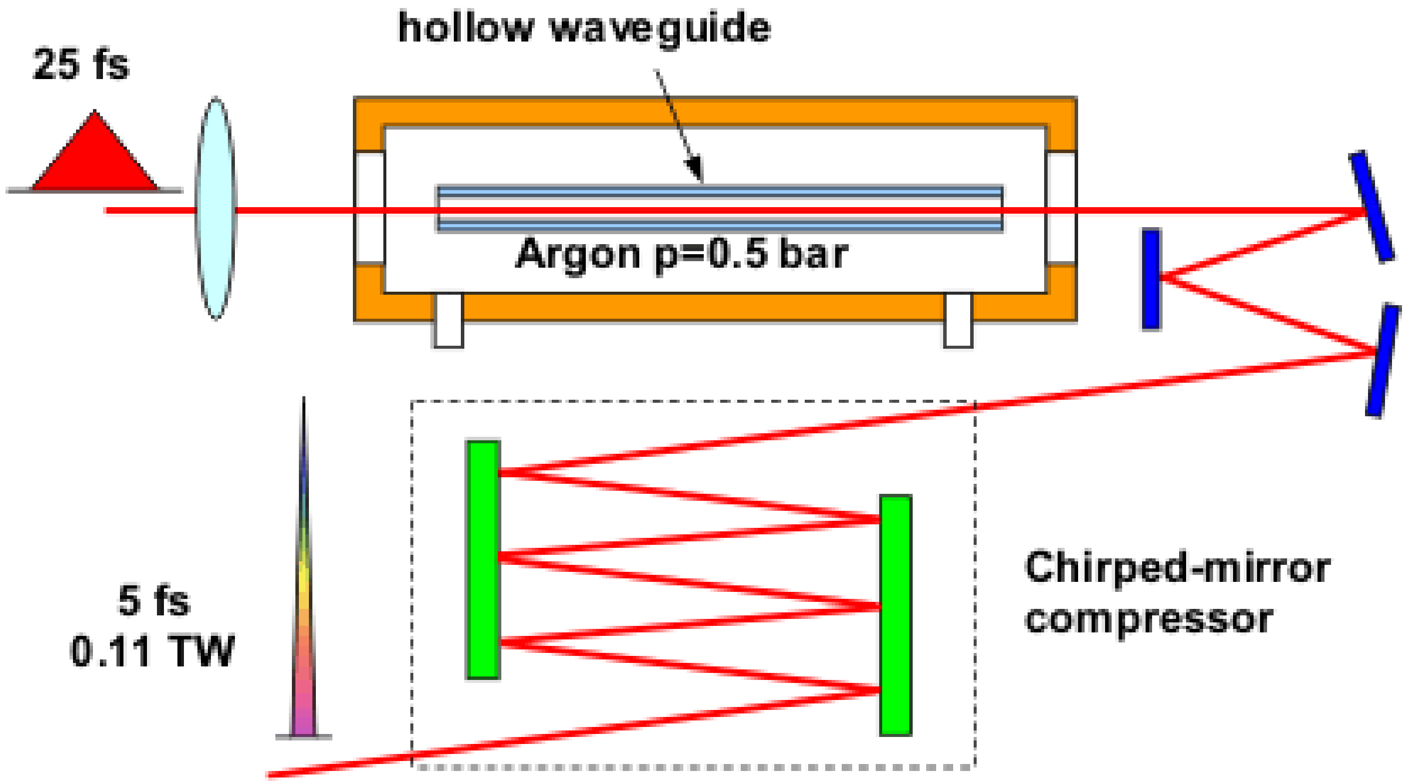
UHH DESY CFEL SCIENCE

3.7.4 Dispersion Compensating Mirrors

High reflecitvity bandwidth of Bragg mirror: $r_B = \dfrac{\Delta f}{f_c} = \dfrac{n_H - n_L}{n_H + n_L}$



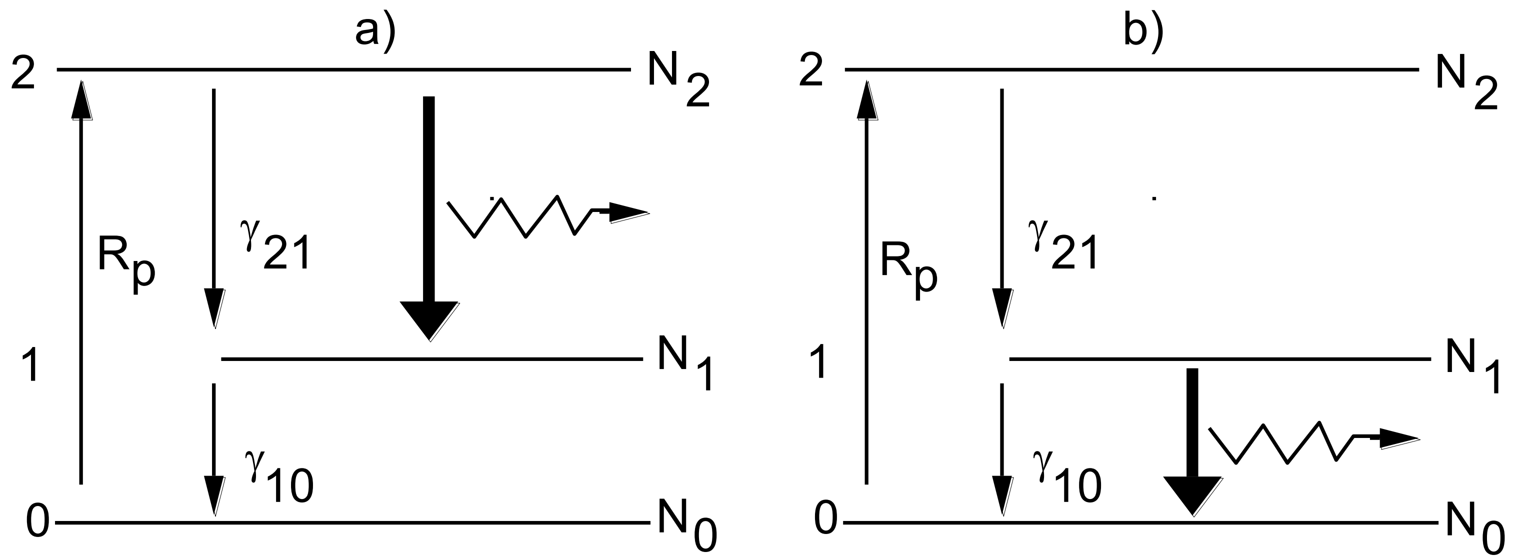
3.7.5 Hollow Fiber Compression Technique



Hollow fiber compression technique

2.1 Laser Rate Equations

How is inversion achieved? What is T_1, T_2 and σ of the laser transition? What does this mean for the laser dyanmics, i.e.for the light that can be generated with these media?



$\gamma_{10} \rightarrow \infty$

$w = N_2$

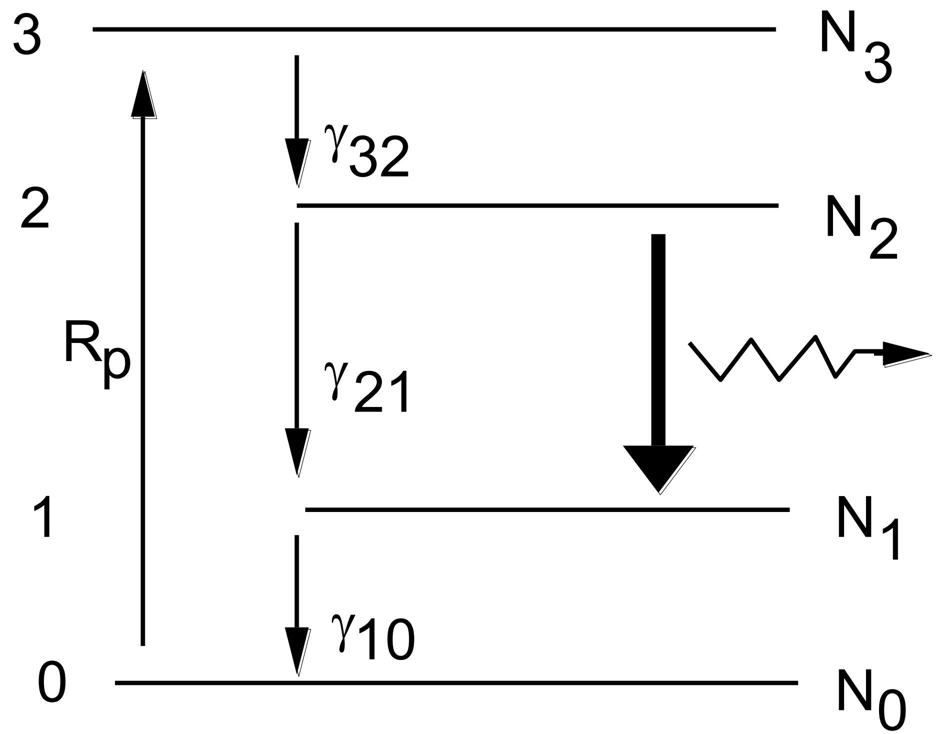
$R_p \rightarrow \infty$

$N_0=0$

$w = N_1$

Figure 4.5: Three-level laser medium

Four-level laser

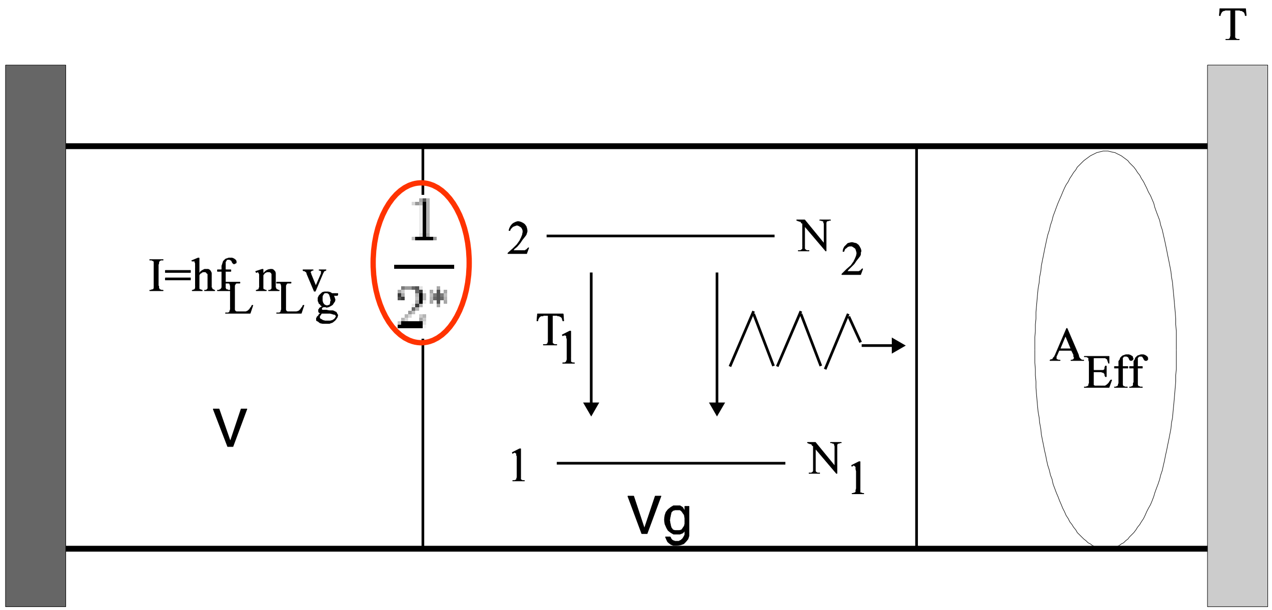


$\gamma_{10} \rightarrow \infty$

$\gamma_{32} \rightarrow \infty$

$w = N_2$

Rate Equations and Cross Section



Rate equations for a laser with two-level atoms and a resonator.

V:= ***A$_{eff}$ L*** **Mode volume**
f$_L$: **laser frequency**
I: **Intensity**
V$_g$: **group velocity at laser frequency**
N$_L$: **number of photons in mode**
w: **inversion**
σ: **interaction cross section**

$$I = hf_L \frac{N_L}{2^* V} v_g = \frac{1}{2^*} hf_L n_L v_g$$

$$\dot{w}\big|_{induced} = -\sigma w I_{ph} = -\frac{w}{T_1 I_s} I$$

$$\sigma = \frac{hf_L}{I_s \tau_L}$$

Laser Rate Equations:

Intracavity power: P

Round trip amplitude gain: g

$$P = I \cdot A_{eff} = h f_L \frac{N_L}{T_R}$$

$$g = \frac{\sigma v_g}{2V} N_2 T_R.$$

Output power: P_{out} $\quad P_{out} = T \cdot P.$

$$\frac{d}{dt} g = -\frac{g - g_0}{\tau_L} - \frac{gP}{E_{sat}}$$

$$\frac{d}{dt} P = -\frac{1}{\tau_p} P + \frac{2g}{T_R} (P + P_{vac})$$

$$E_{sat} = \frac{h f_L V}{\sigma v_g T_R} = \frac{1}{2^*} I_s A_{eff} \tau_L$$

$$P_{sat} = E_{sat} / \tau_L$$

$$P_{vac} = h f_L / T_R$$

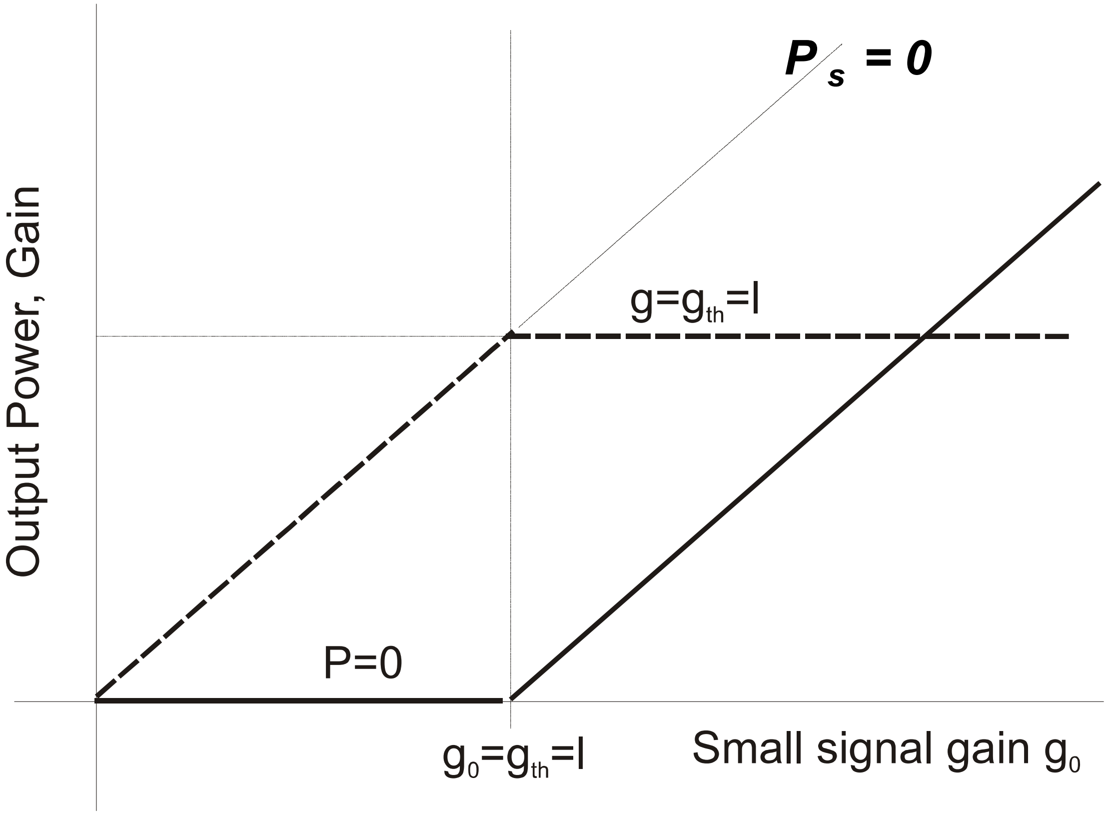
$$g_0 = 2^* \frac{R_p}{2 A_{eff}} \sigma \tau_L,$$

small signal gain ~ $\sigma\tau_L$ - product

2.2 Continuous Wave Operation

$P_{vac} = 0$

Steady State: d/dt = 0



Case 1:

$g_s = g_0$

$P_s = 0$

Case 2:

$$g_s = \frac{g_0}{1+\frac{P_s}{P_{sat}}} = l$$

$$P_s = P_{sat}\left(\frac{g_0}{l} - 1\right)$$

$$g_{th} = l,$$

$$R_{p,th} = \frac{2lA_{eff}}{2^{*}\sigma\tau_L}$$

Output power versus small signal gain or pump power

Lasers and Its Spectroscopic Parameters

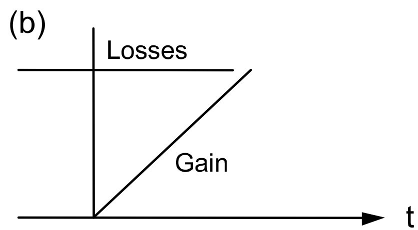
Laser Medium	Wave-length $\lambda_0 (nm)$	Cross Section σ (cm^2)	Upper-St. Lifetime τ_L (μs)	Linewidth $\Delta f_{FWHM} = \frac{2}{T_2}$ (THz)	Typ	Refr. index n
Nd^{3+}:YAG	1,064	$4.1 \cdot 10^{-19}$	1,200	0.210	H	1.82
Nd^{3+}:LSB	1,062	$1.3 \cdot 10^{-19}$	87	1.2	H	1.47 (ne)
Nd^{3+}:YLF	1,047	$1.8 \cdot 10^{-19}$	450	0.390	H	1.82 (ne)
Nd^{3+}:YVO_4	1,064	$2.5 \cdot 10^{-19}$	50	0.300	H	2.19 (ne)
Nd^{3+}:glass	1,054	$4 \cdot 10^{-20}$	350	3	H/I	1.5
Er^{3+}:glass	1,55	$6 \cdot 10^{-21}$	10,000	4	H/I	1.46
Ruby	694.3	$2 \cdot 10^{-20}$	1,000	0.06	H	1.76
Ti^{3+}:Al_2O_3	660-1180	$3 \cdot 10^{-19}$	3	100	H	1.76
Cr^{3+}:LiSAF	760-960	$4.8 \cdot 10^{-20}$	67	80	H	1.4
Cr^{3+}:LiCAF	710-840	$1.3 \cdot 10^{-20}$	170	65	H	1.4
Cr^{3+}:LiSGAF	740-930	$3.3 \cdot 10^{-20}$	88	80	H	1.4
He-Ne	632.8	$1 \cdot 10^{-13}$	0.7	0.0015	I	$\sim$1
Ar^+	515	$3 \cdot 10^{-12}$	0.07	0.0035	I	$\sim$1
CO_2	10,600	$3 \cdot 10^{-18}$	2,900,000	0.000060	H	$\sim$1
Rhodamin-6G	560-640	$3 \cdot 10^{-16}$	0.0033	5	H	1.33
semiconductors	450-30,000	$\sim 10^{-14}$	$\sim$ 0.002	25	H/I	3 - 4

Spectroscopic parameters of selected laser materials

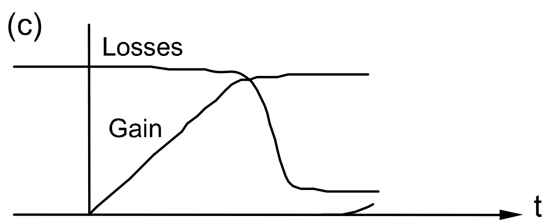
Here active Q-switching

$$\tau_L \gg T_R \gg \tau_p$$

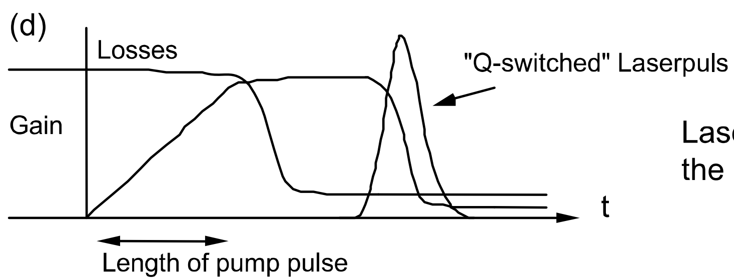
High losses, laser is below threshold



Build-up of inversion by pumping

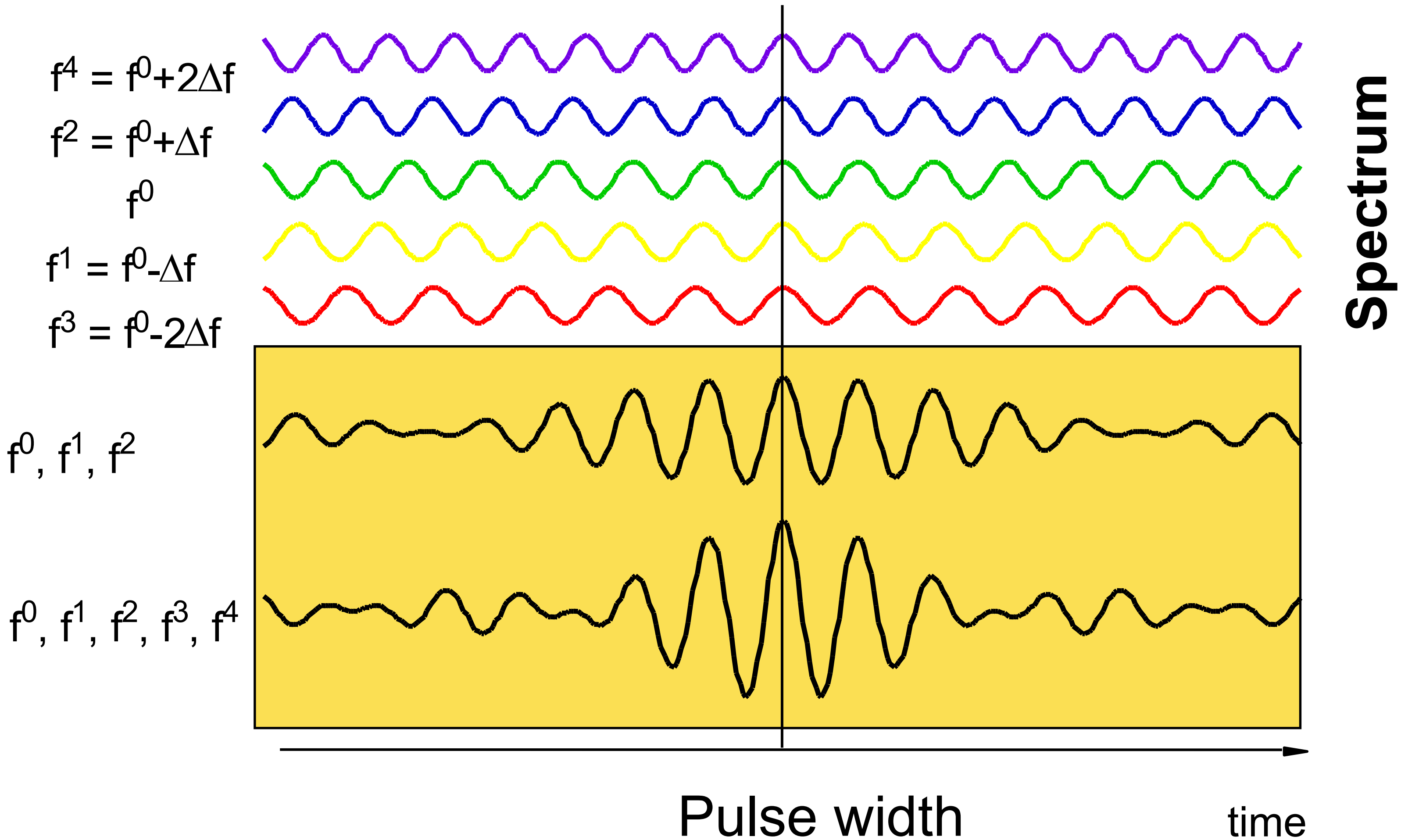


In active Q-switching, the losses are reduced, after the laser medium is pumped for as long as the upper state lifetime. Then the loss is reduced rapidly and laser oscillation starts.



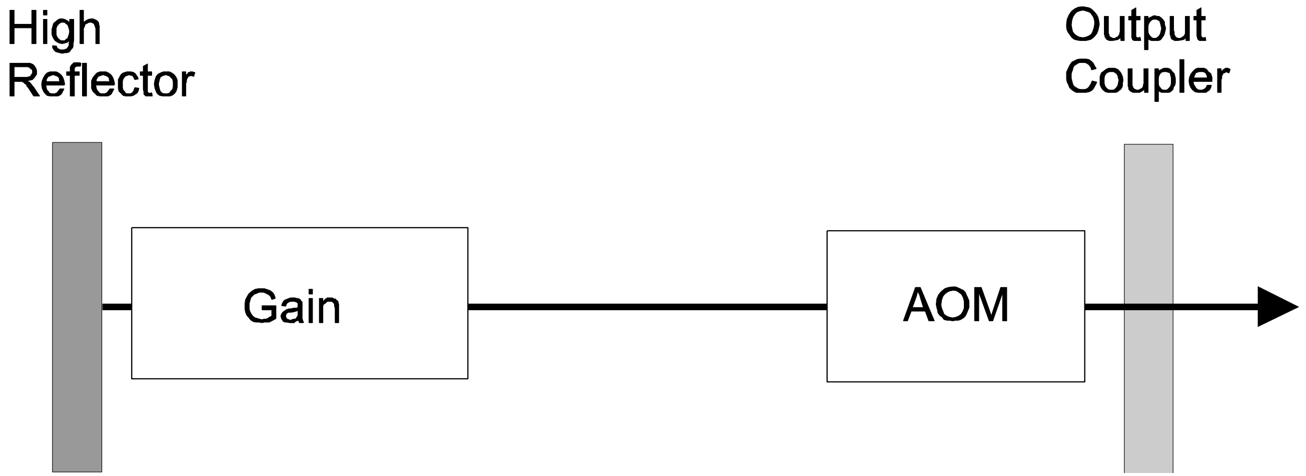
Laser emission stops after the energy stored in the gain medium is extracted.

4. Modelocked Lasers



4.1 Active Mode Locking

Actively modelocked laser



Master Equation:

$$T_R \frac{\partial A}{\partial T} = \left[g(T) + D_g \frac{\partial^2}{\partial t^2} - l - M\left(1 - \cos(\omega_M t)\right) \right] A$$

loss modulation

Parabolic approximation at position where pulse will form;

$$T_R \frac{\partial A}{\partial T} = \left[g - l + D_g \frac{\partial^2}{\partial t^2} - M_s t^2 \right] A$$

$$D_g = \frac{g}{\Omega_g^2},$$

$$M_s = \frac{M \omega_M^2}{2}$$

Compare with Schroedinger Equation for harmonic oscillator

$$A_n(T,t) = A_n(t)e^{\lambda_n T/T_R},$$

$$A_n(t) = \sqrt{\frac{W_n}{2^n\sqrt{\pi}n!\tau_a}}H_n(t/\tau_a)e^{-\frac{t^2}{2\tau_a^2}}$$

with

$$\tau_a = \sqrt[4]{D_g/M_s}.$$

Eigen value determines roundtrip gain of n=th pulse shape

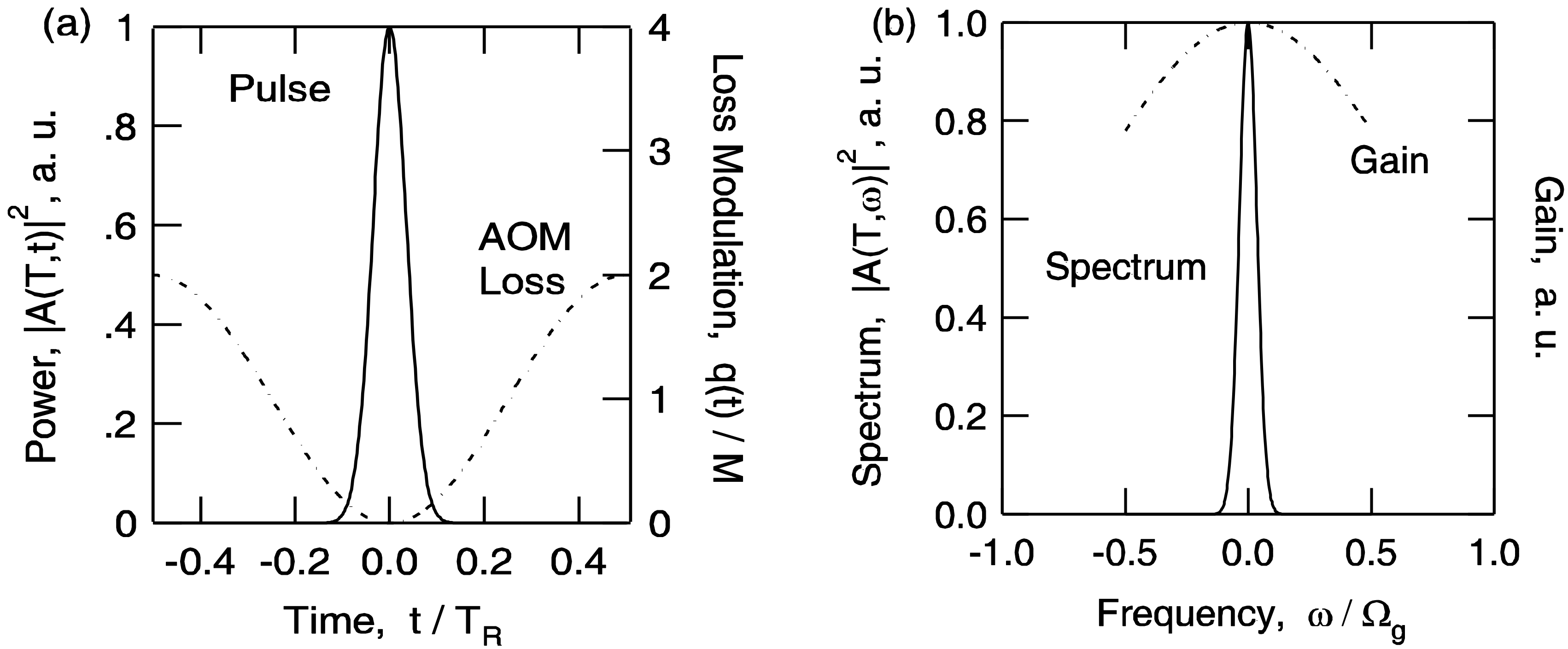
$$\lambda_n = g_n - l - 2M_s\tau_a^2(n + \frac{1}{2}).$$

Pulse shape with n=0, lowest order mode, has highest gain.

This pulse shape will saturate the gain and keep all other pulse shapes below threshold.

Pulse width: $$\Delta t_{FWHM} = 2\ln 2\tau_a = 1.66\tau_a$$

$$\Delta f_{FWHM} = \frac{1.66}{2\pi\tau_a} \qquad \Delta t_{FWHM} \cdot \Delta f_{FWHM} = 0.44$$



Pulse shaping in time and frequency domain.

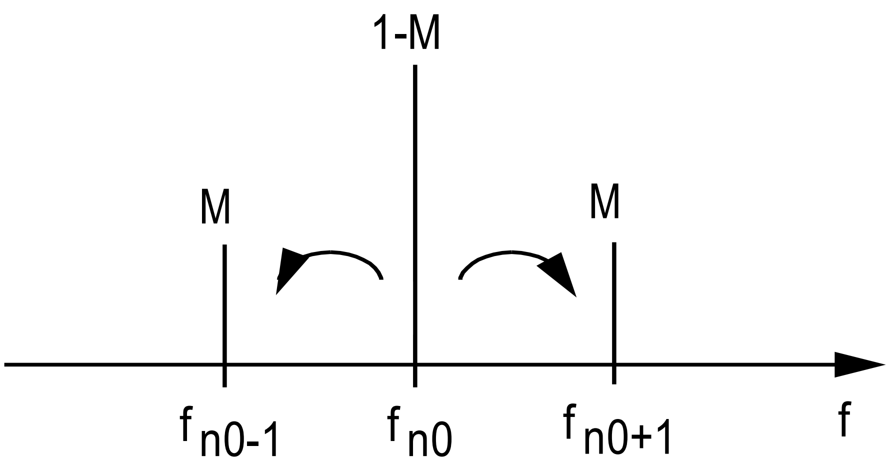
For example: Nd:YAG; $2l = 2g = 10\%$, $\Omega_g = \pi \Delta f_{FWHM} = 0.65$ THz
$M = 0.2$, $f_m = 100$ MHz, $D_g = 0.24$ ps^2, $M_s = 4 \cdot 10^{16} s^{-1}$, $\tau_p \approx 99$ ps.

Pulse width depends only weak on gain bandwidth.

10-100 ps pulses typical for active mode locking!

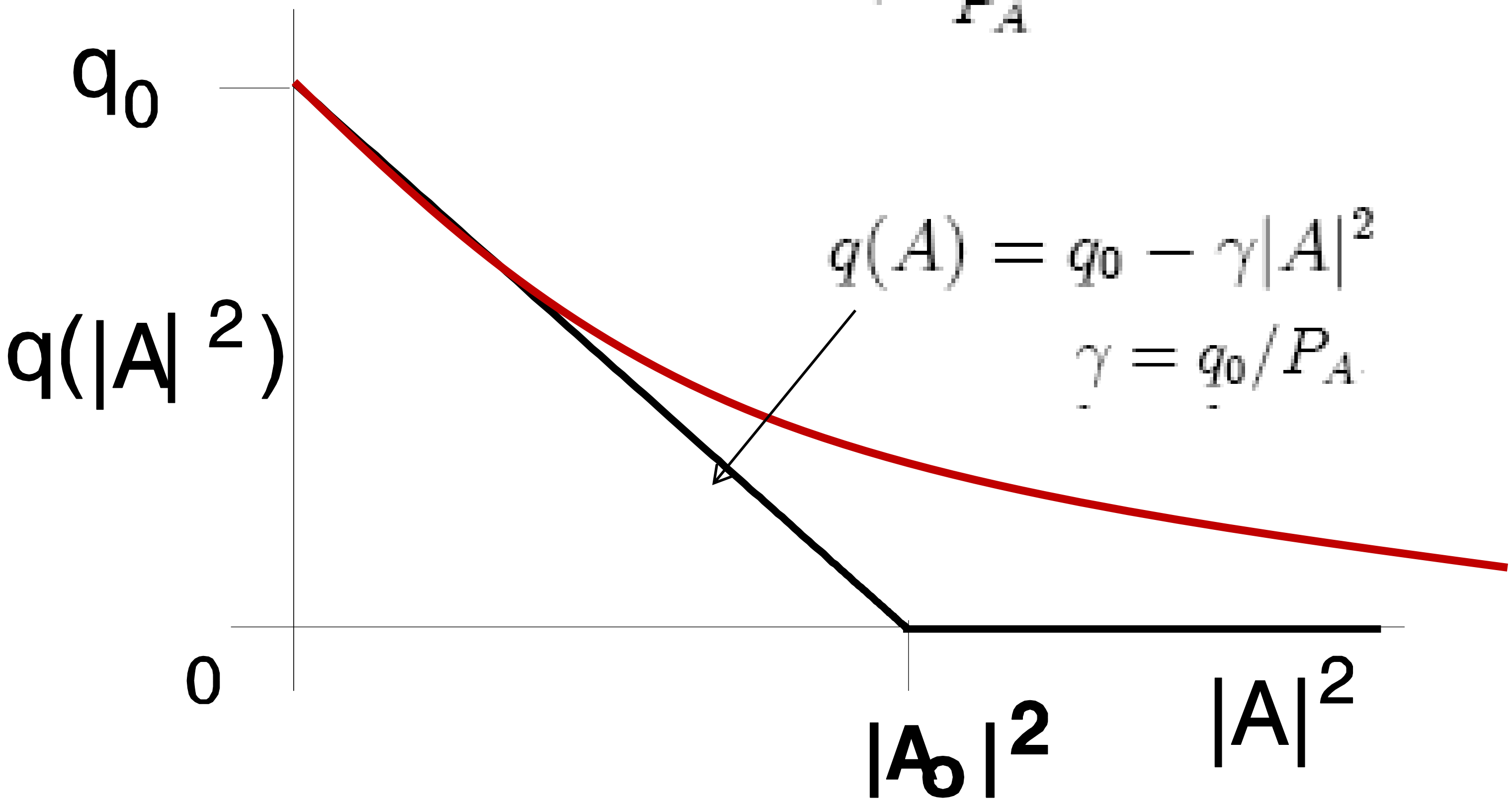
Active mode locking can be understood as injection seeding of neighboring modes by those already present.

$$
\begin{aligned}
& -M\left[1-\cos(\omega_M t)\right]\exp(j\omega_{n_0} t) \\
= \; & -M\left[\exp(j\omega_{n_0} t) - \frac{1}{2}\exp(j(\omega_{n_0} t - \omega_M t)) - \frac{1}{2}\exp(j(\omega_{n_0} t + \omega_M t))\right] \\
= \; & M\left[-\exp(j\omega_{n_0} t) + \frac{1}{2}\exp(j\omega_{n_0-1} t) + \frac{1}{2}\exp(j\omega_{n_0+1} t)\right]
\end{aligned}
$$



4.2 Passive Mode Locking

$$q(A) = \frac{q_0}{1 + \frac{|A|^2}{P_A}};$$



Saturation characteristic of an ideal saturable absorber and linear approximation.

Fast Saturable Absorber Modelocking

$$T_R \frac{\partial A(T,t)}{\partial T} = \left[g - l_0 + D_g \frac{\partial^2}{\partial t^2} + \gamma |A|^2 \right] A(T,t)$$

$$l_0 = l + q_0$$

Saturable absorber provides gain for the pulse

There is a stationary solution:

$$A_s(T,t) = A_s(t) = A_0 \mathrm{sech}\left(\frac{t}{\tau}\right)$$

$$\frac{D_g}{\tau^2} = \frac{q_0}{2},$$

Easy to check with:

$$\begin{aligned} \frac{d}{dx}\mathrm{sech}x &= -\tanh x \,\mathrm{sech}x, \\ \frac{d^2}{dx^2}\mathrm{sech}x &= \tanh^2 x \,\mathrm{sech}x - \mathrm{sech}^3 x \\ &= \left(\mathrm{sech}x - 2\,\mathrm{sech}^3 x\right). \end{aligned}$$

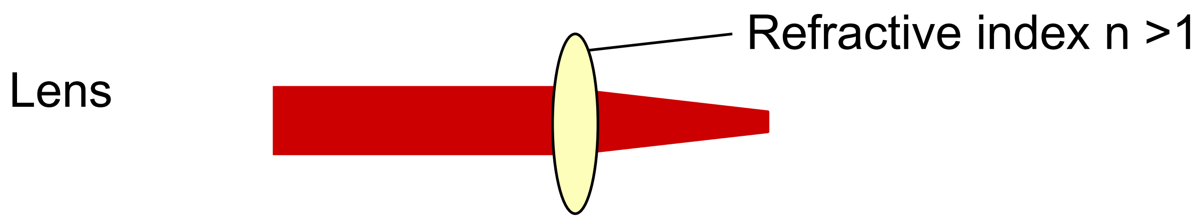
Shortest pulse:

$$\tau = \sqrt{\frac{2g_s}{q_0}} \frac{1}{\Omega g}. \quad \tau_{\min} = \frac{1}{\Omega_g}$$

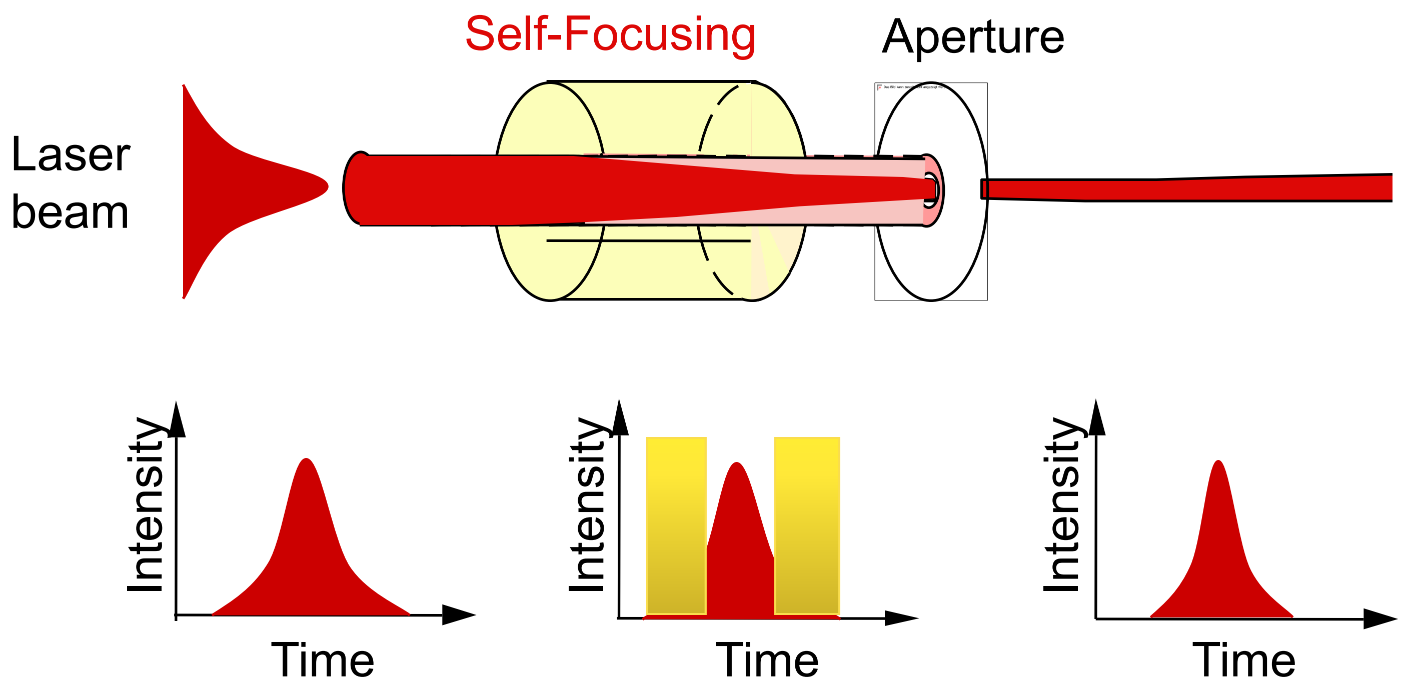
For Ti:sapphire

$$\tau_{FWHM} = 6.5 \text{ fs}$$

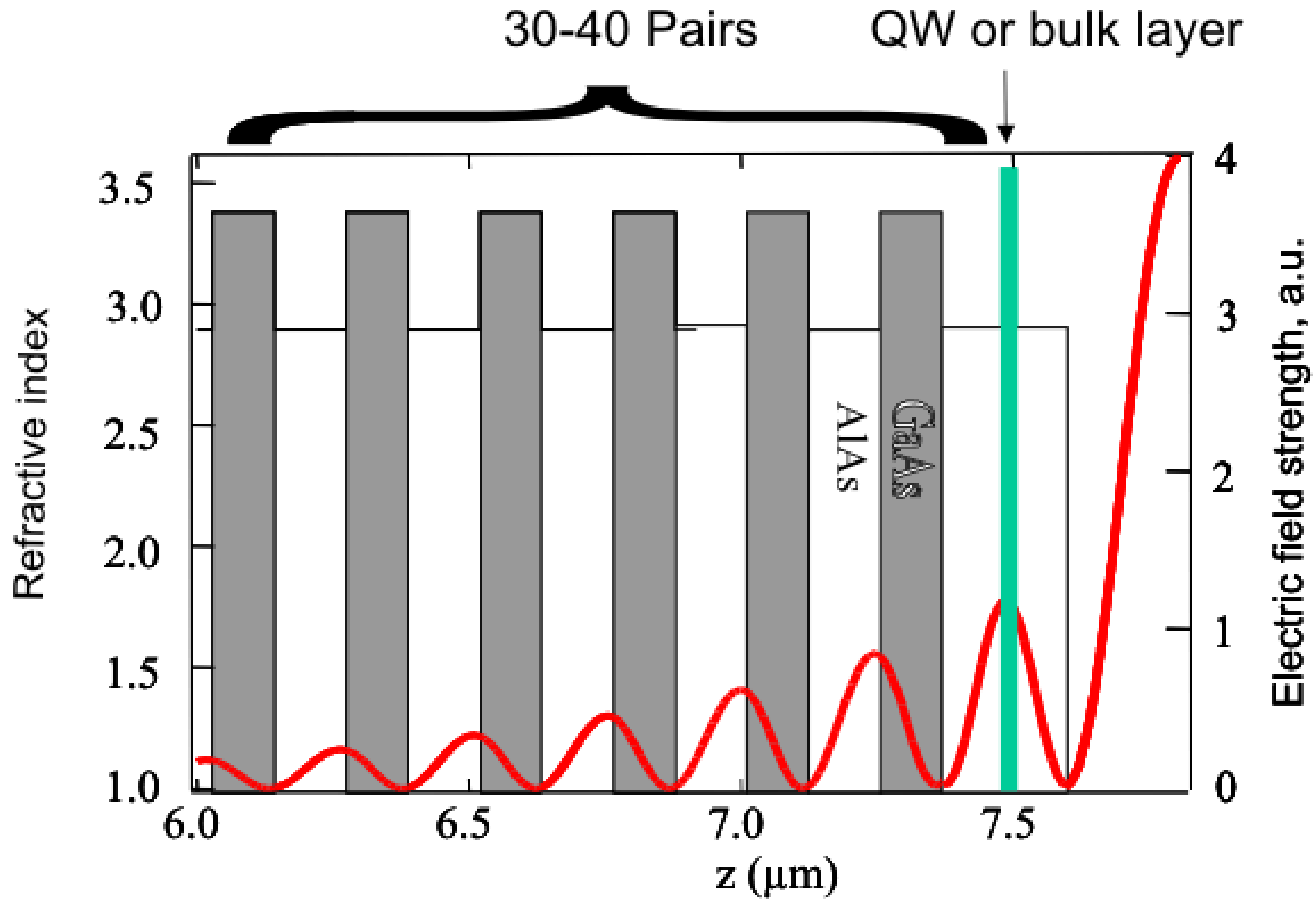
Kerr Lens Modelocking



Intensity dependent refractive index: "Kerr-Lens"

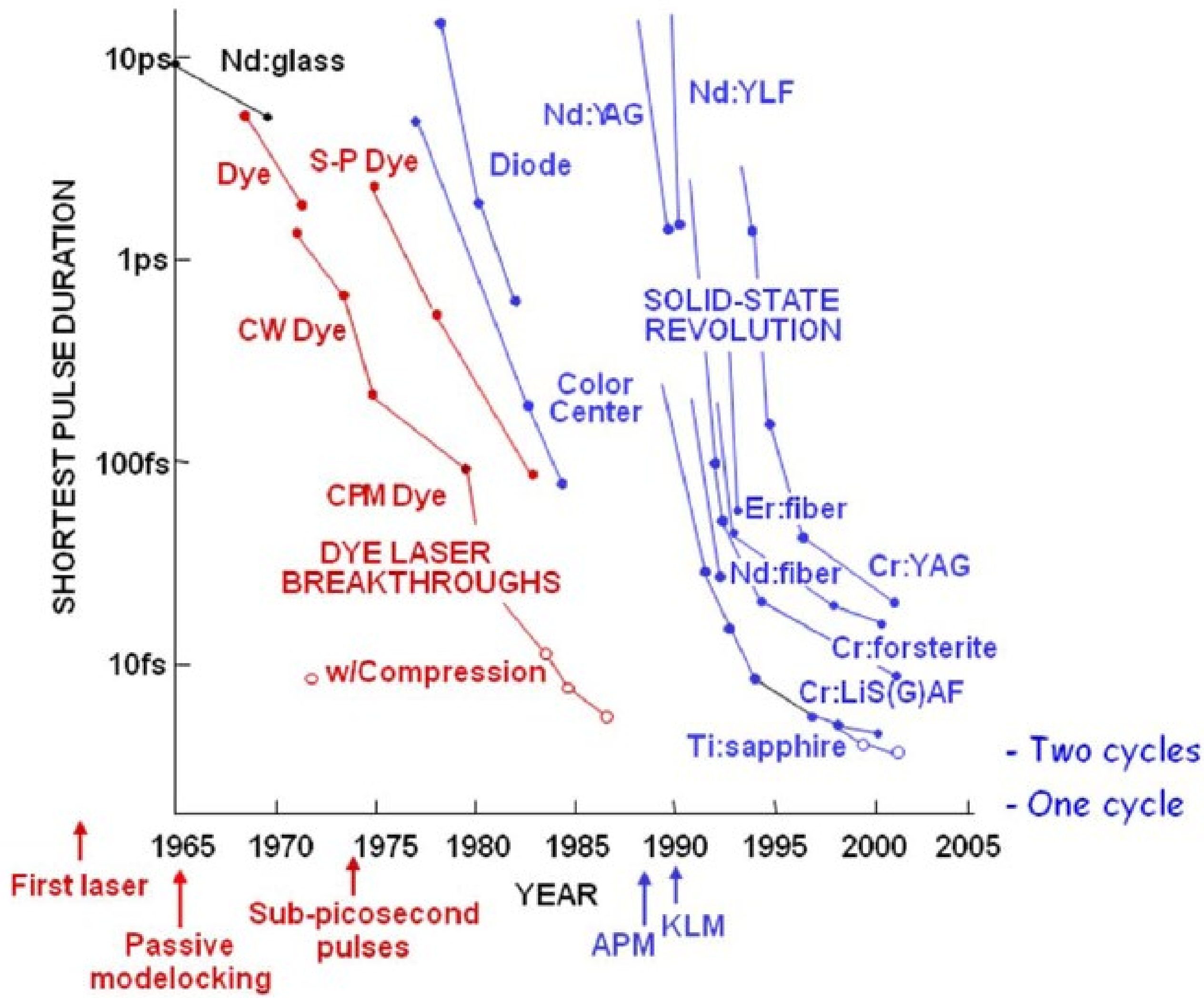


Semiconductor Saturable Absorbers



Semiconductor saturable absorber mirror (SESAM) or Semiconductor Bragg mirror (SBR)

Modelocking: Historical Development



Pulse width of different laser systems by year.

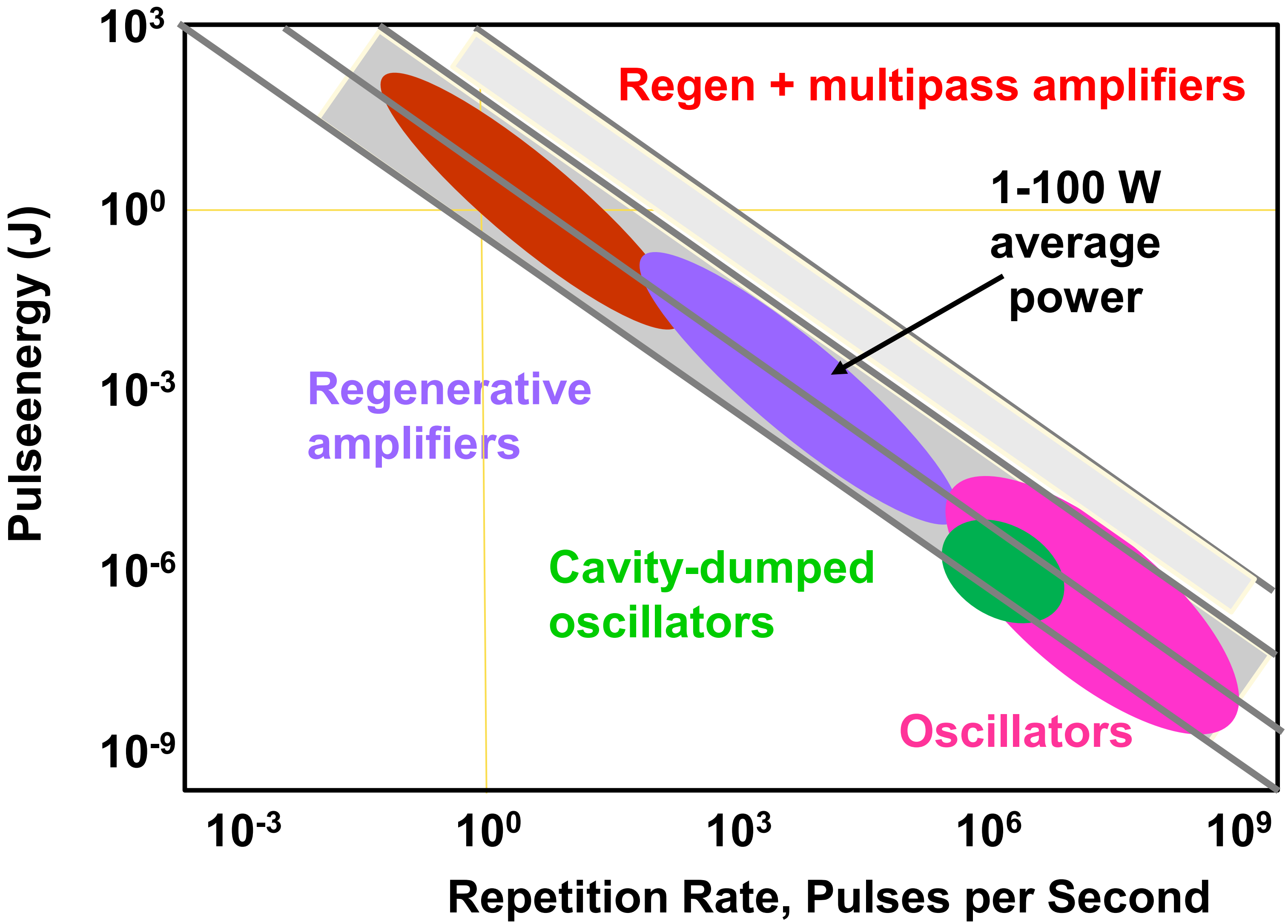
5. Laser Amplifiers

5.1 Cavity Dumping

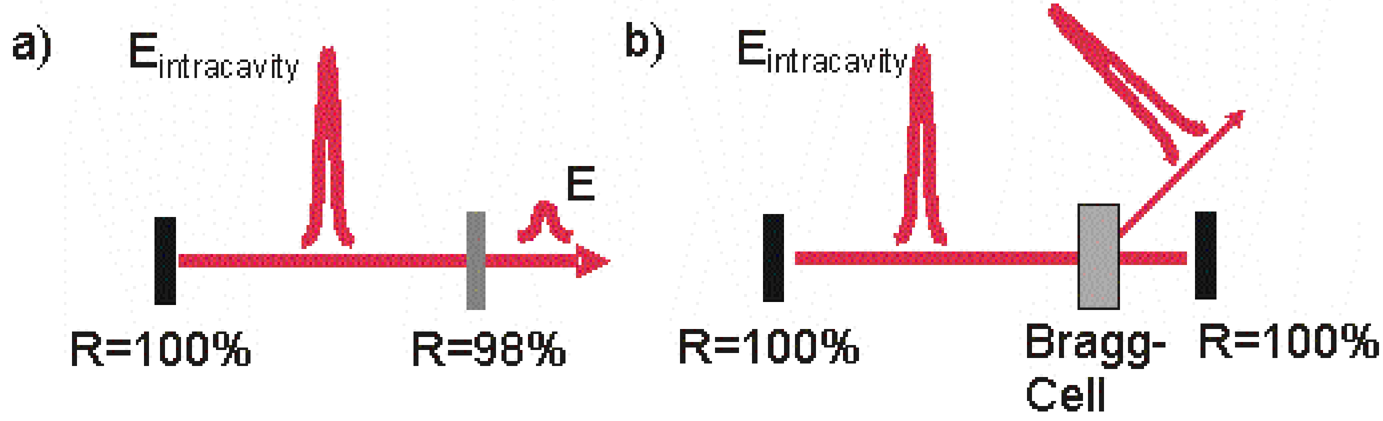
5.2 Laser Amplifiers
5.2.1 Frantz-Nodvick Equation
5.2.2 Regenerative and Multipass Amplifiers

5.3 Chirped Pulse Amplification
5.4 Stretchers and Compressors
5.5 Gain Narrowing

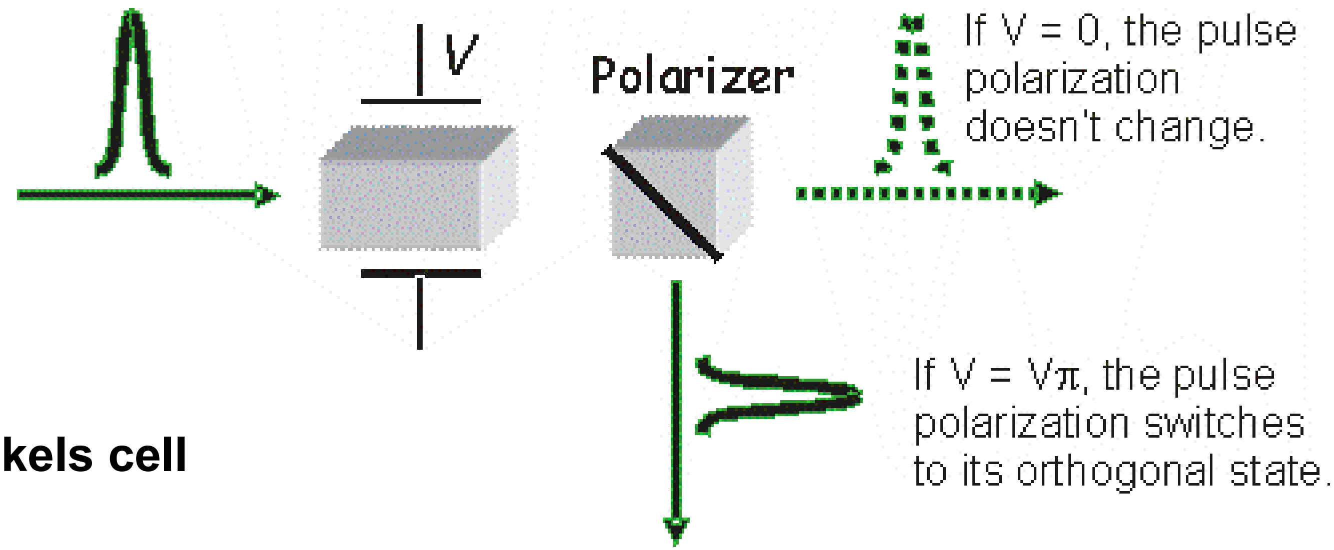
Pulse energies from different laser systems



5.1 Cavity Dumping

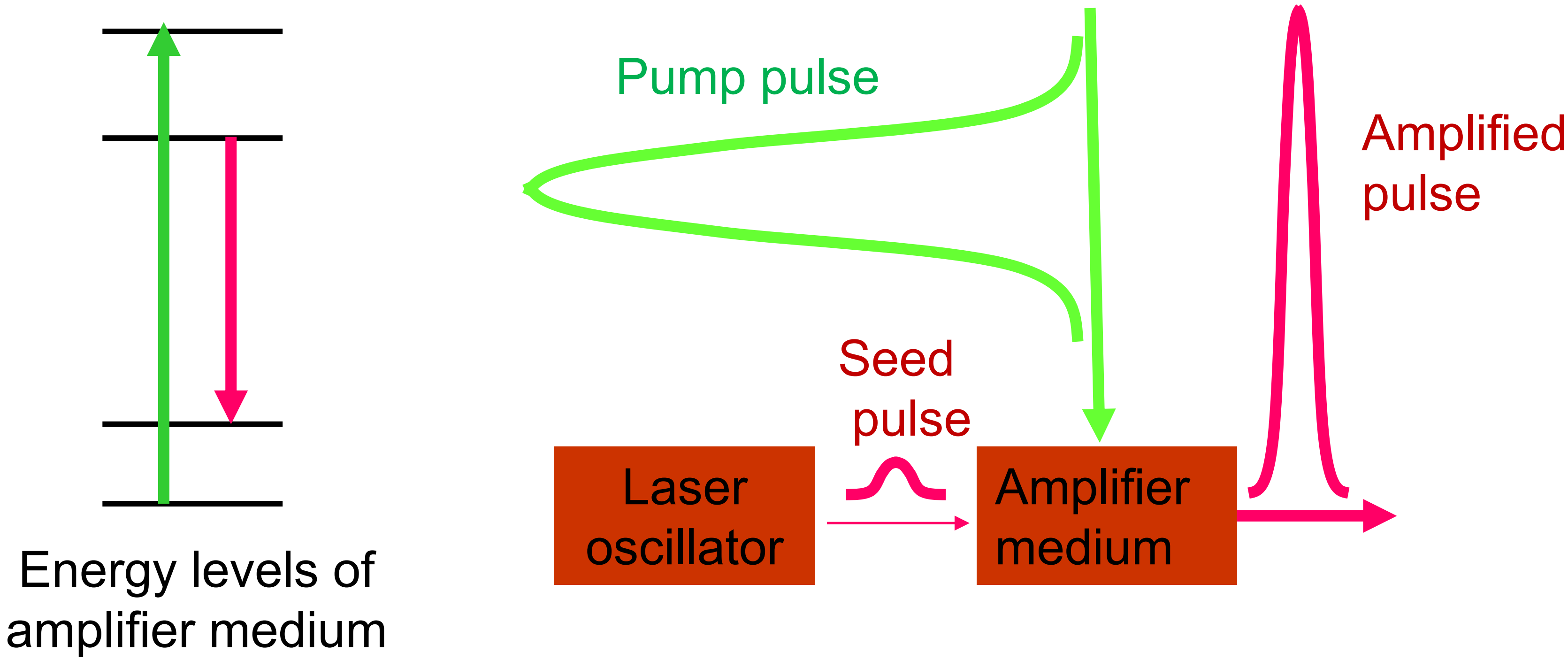


With Bragg cell



With Pockels cell

5.2 Laser Amplifiers



Laser amplifier: Pump pulse should be shorter than upper state lifetime. Signal pulse arrives at medium after pumping and well within the upper state lifetime to extract the energy stored in the medium, before it is lost due to energy relaxation.

5.2.1 Franz-Nodvik Equations

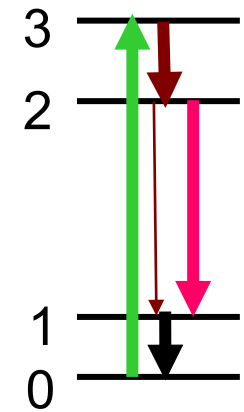
Multi-pass gain and extraction

$$\text{Fluence}: F(z) = \int_{-\infty}^{\infty} I(z,t)\,dt$$

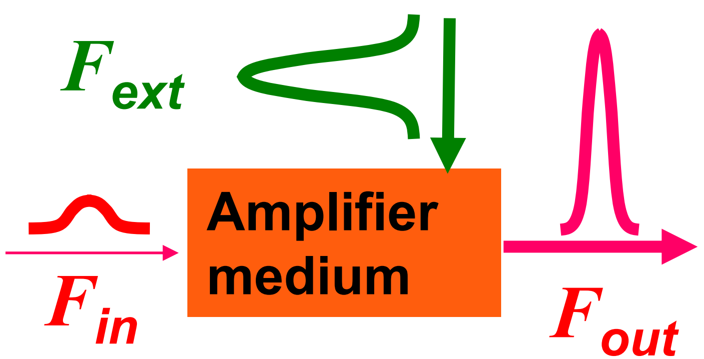
$$\text{Small signal gain}: G_o = \exp\left[\int_0^L g_0(z,t)\,dz\right]$$

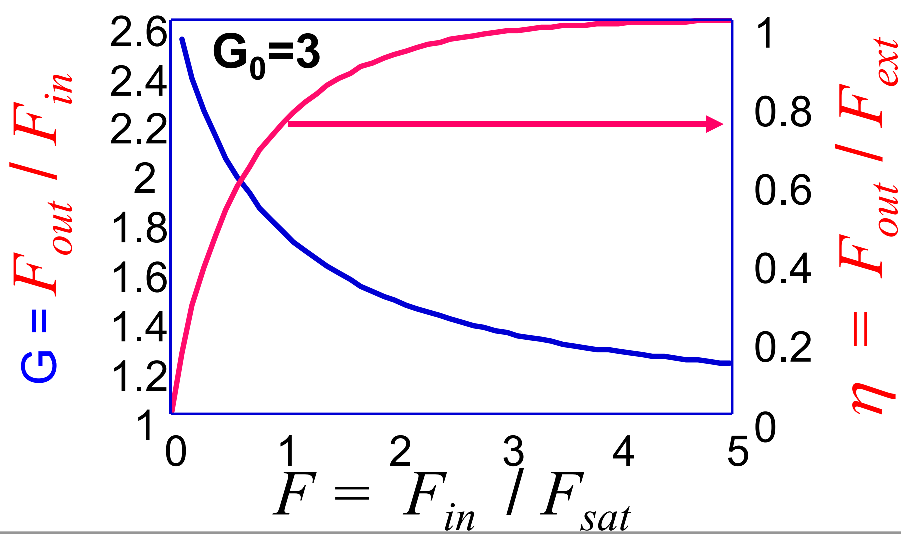
$$\text{Fluence after roundtrip } i: F_i = F_{sat} \ln\left[1 + G_o\left(e^{F_{i-1}/Fsat} - 1\right)\right]$$

$$\text{Gain after roundtrip } i: G_i = \left[1 - e^{-F_{i-1}/Fsat}\left(1 - 1/G_{i-1}\right)\right]^{-1}$$



***F*: Fluence =** $\frac{\text{Energy}}{\text{Area}}$

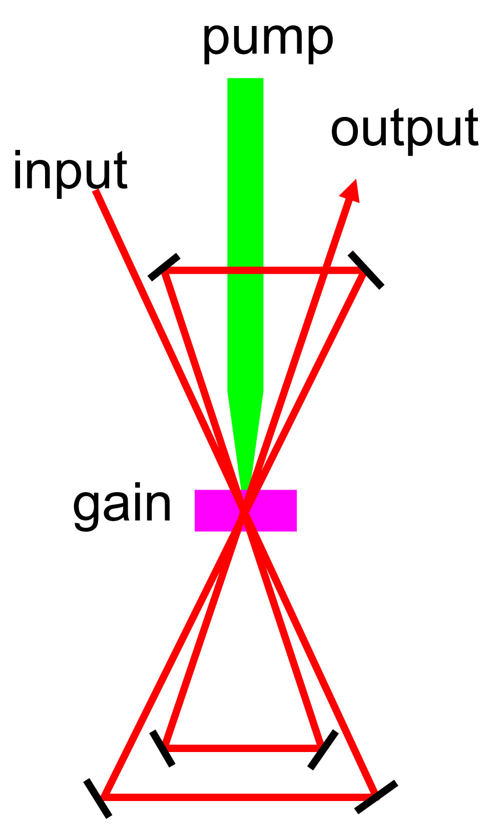




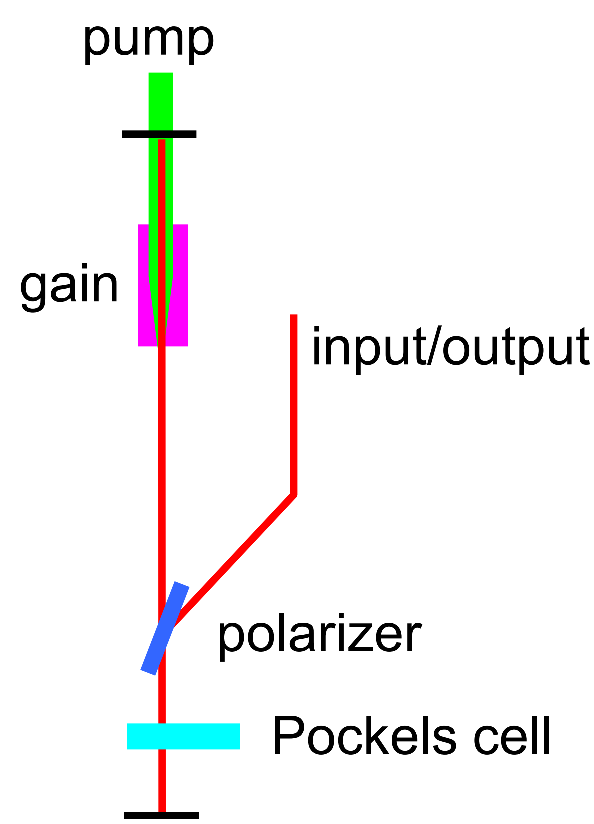
$$\text{extractable energy}: F_{ext} = F_{pump} \cdot \frac{f_L}{f_p} \cdot \text{pump efficiency}$$

5.2.2 Basic Amplifier Schemes

a) Multi-pass amplifier

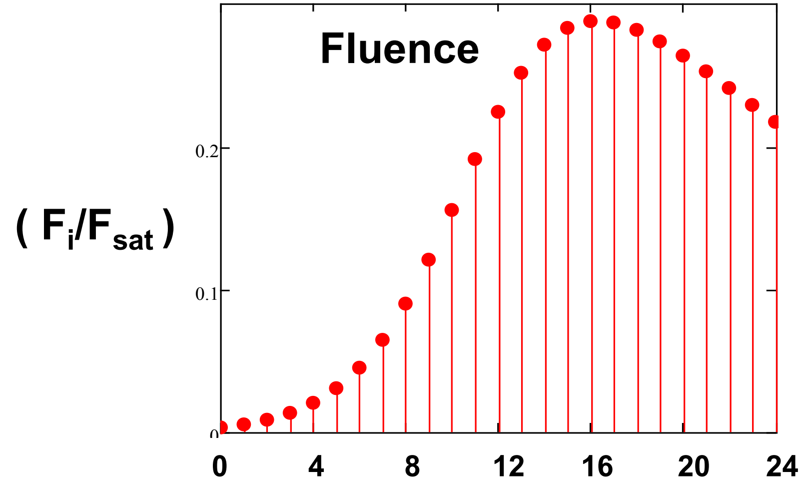


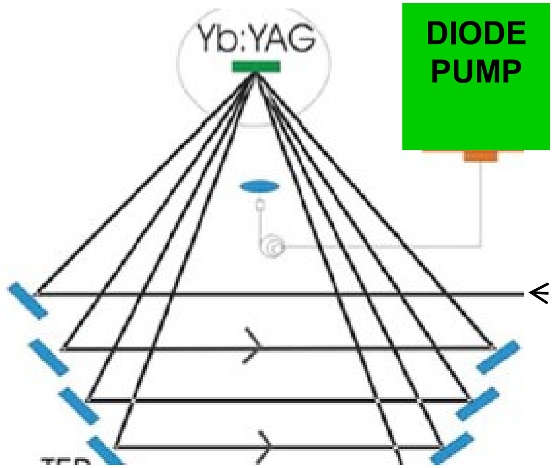
b) Regenerative amplifier



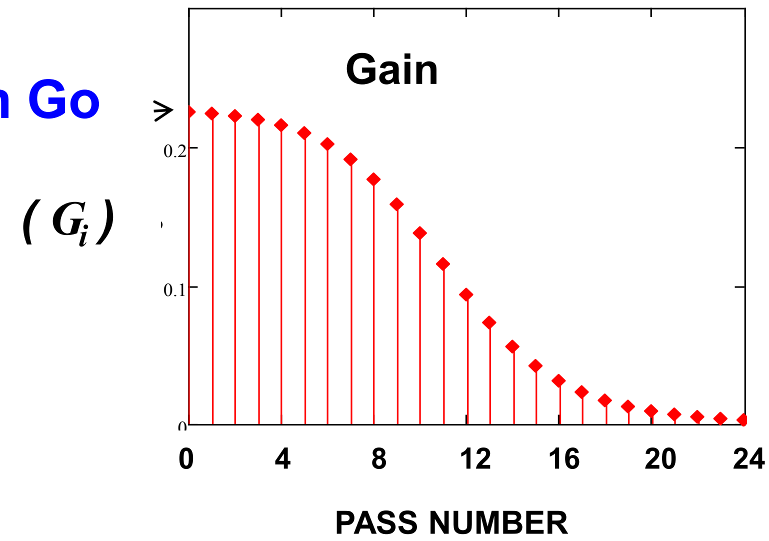
Multipass amplifier pulse-growth/gain-extraction

Pulse growth dynamics dictated by the system's GAIN and LOSS ratio



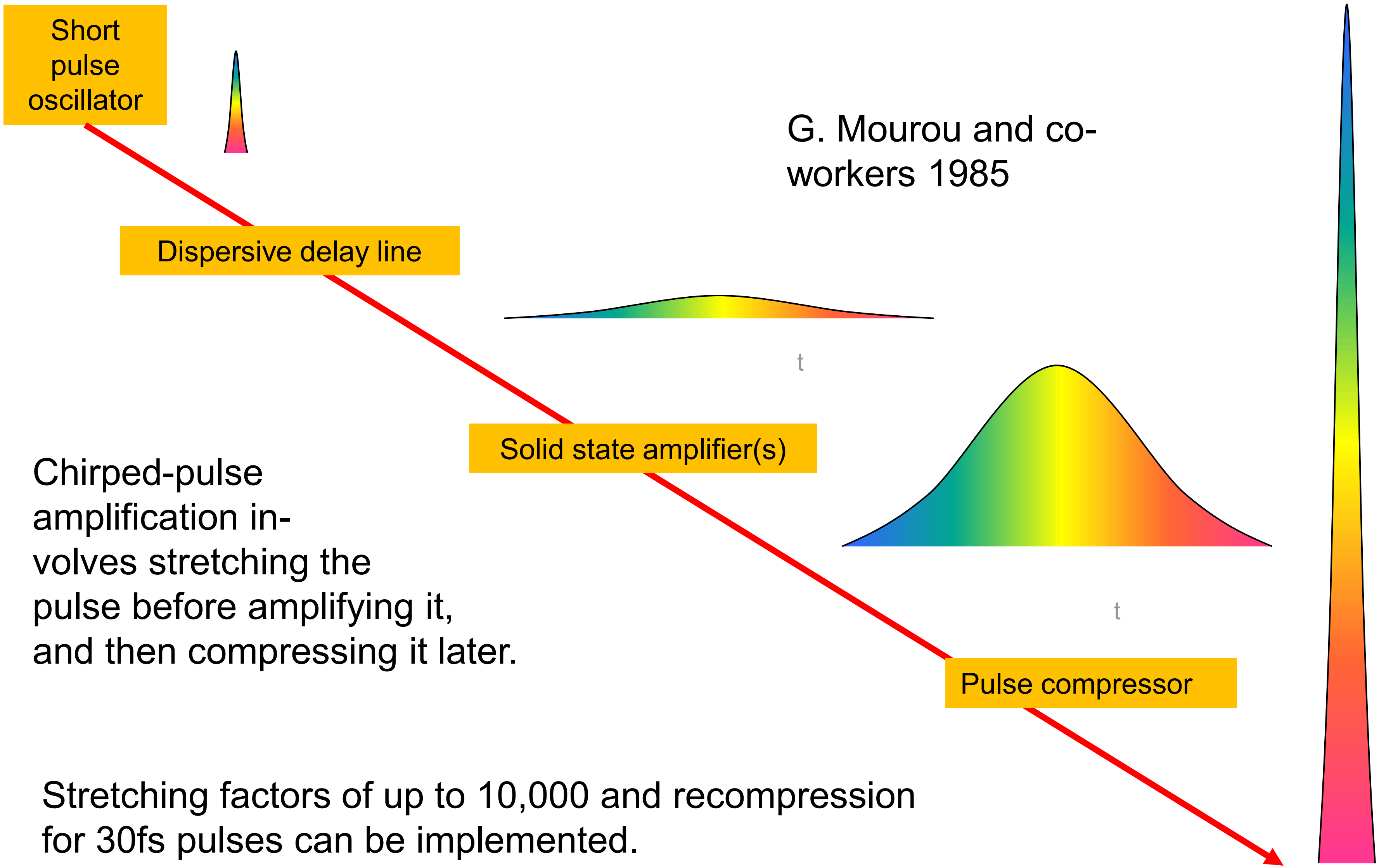


Initial gain Go



Round-trip transmission T

5.3 Chirped-Pulse Amplification



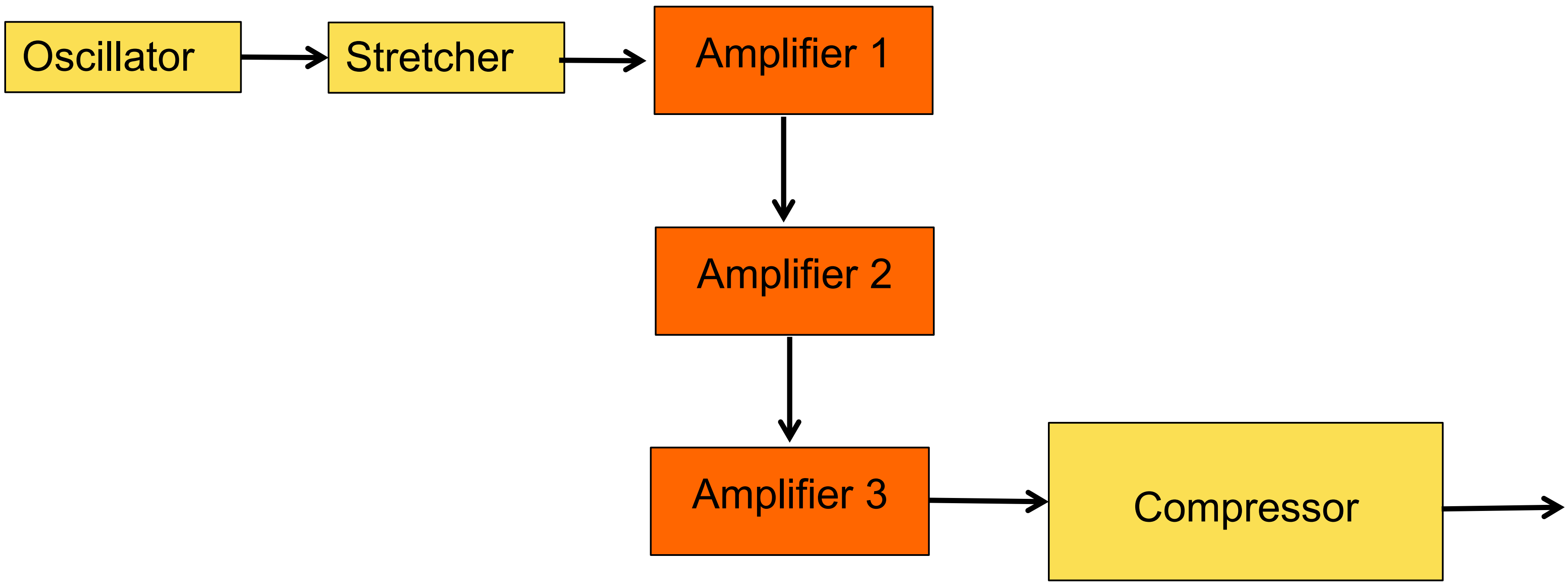
G. Mourou and co-workers 1985

Chirped-pulse amplification involves stretching the pulse before amplifying it, and then compressing it later.

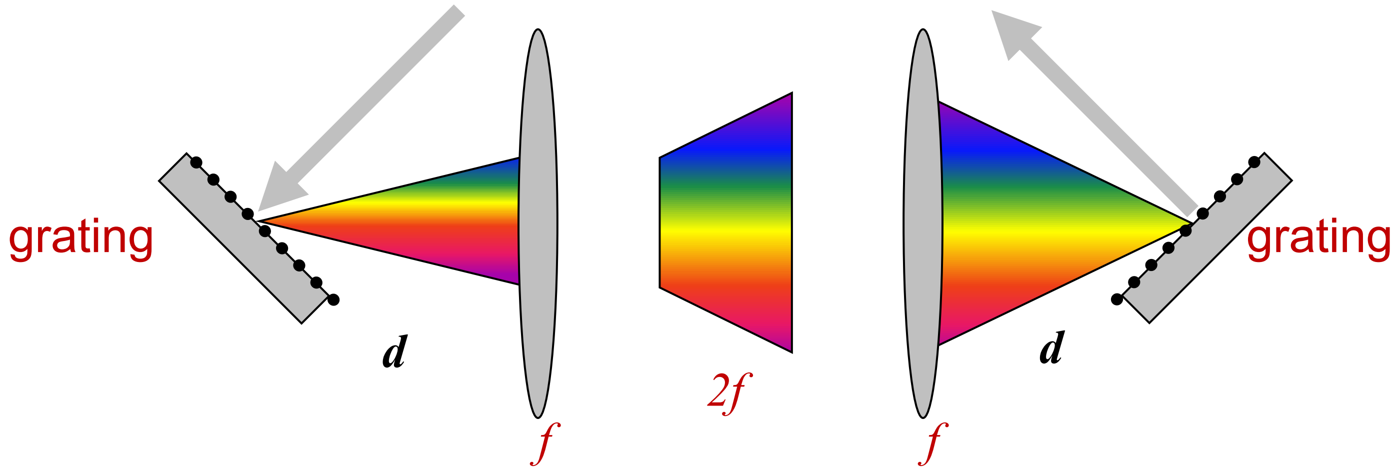
Stretching factors of up to 10,000 and recompression for 30fs pulses can be implemented.

Chirped Pulse Amplifier System

Oscillator – Stretcher – Multiple Amplifiers - Compressor Chain



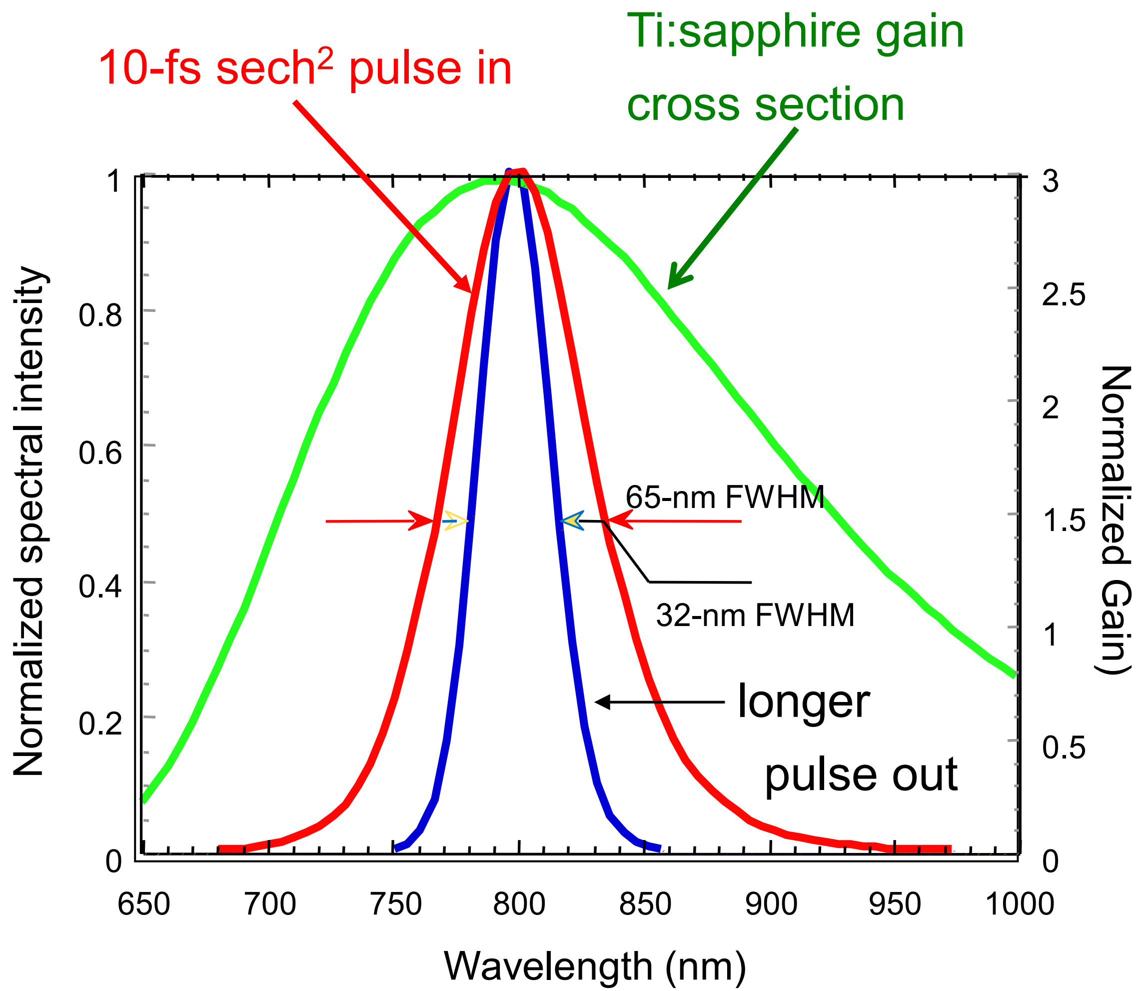
5.4 Stretchers and Compressors



Okay, this looks just like a “zero-dispersion stretcher” used in pulse shaping. But when d ≠ f, it’s a dispersive stretcher and can stretch fs pulses by a factor of 10,000!

With the opposite sign of d-f, we can compress the pulse.

5.5 Gain Narrowing



Influence of gain narrowing in a Ti:sapphire amplifier on a 10 fs seed pulse

In general when applying gain G with bandwidth $\Delta\lambda_{Fluo}$

to a pulse with input bandwidth $\Delta\lambda_0$

the output bandwidth is

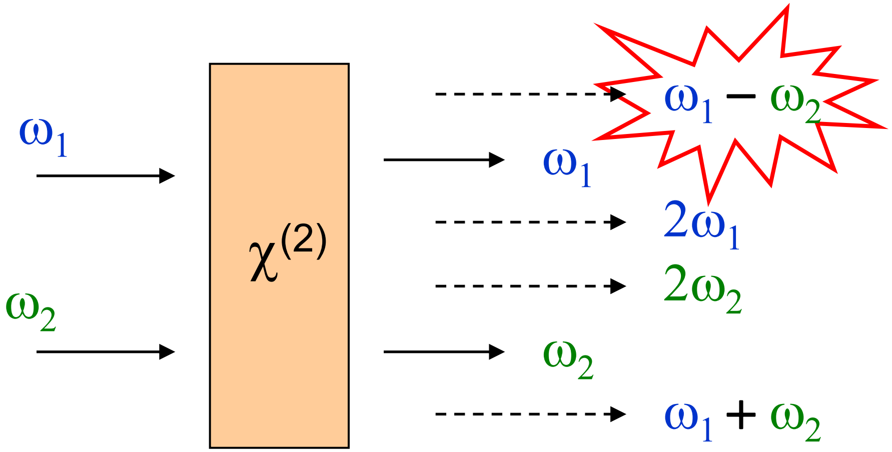
$$\Delta\lambda = \frac{\Delta\lambda_0}{\sqrt{1+\ln(G)\left(\frac{\Delta\lambda_0}{\Delta\lambda_{Fluo}}\right)^2}}$$

Rouyer *et al.*, Opt. Lett. 18, 214 (1993).

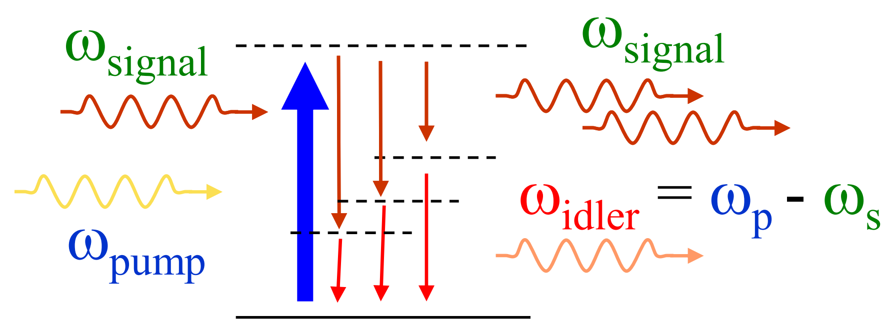
6. Optical Parametric Amplifiers

Non-linear polarization effects

$$P = \varepsilon_0 \chi^{(1)} E + \varepsilon_0 \chi^{(2)} E^2 + \varepsilon_0 \chi^{(3)} E^3 + \dots$$

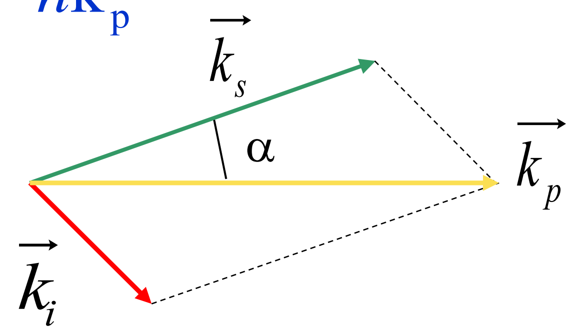


Optical Parametric Amplification (OPA)



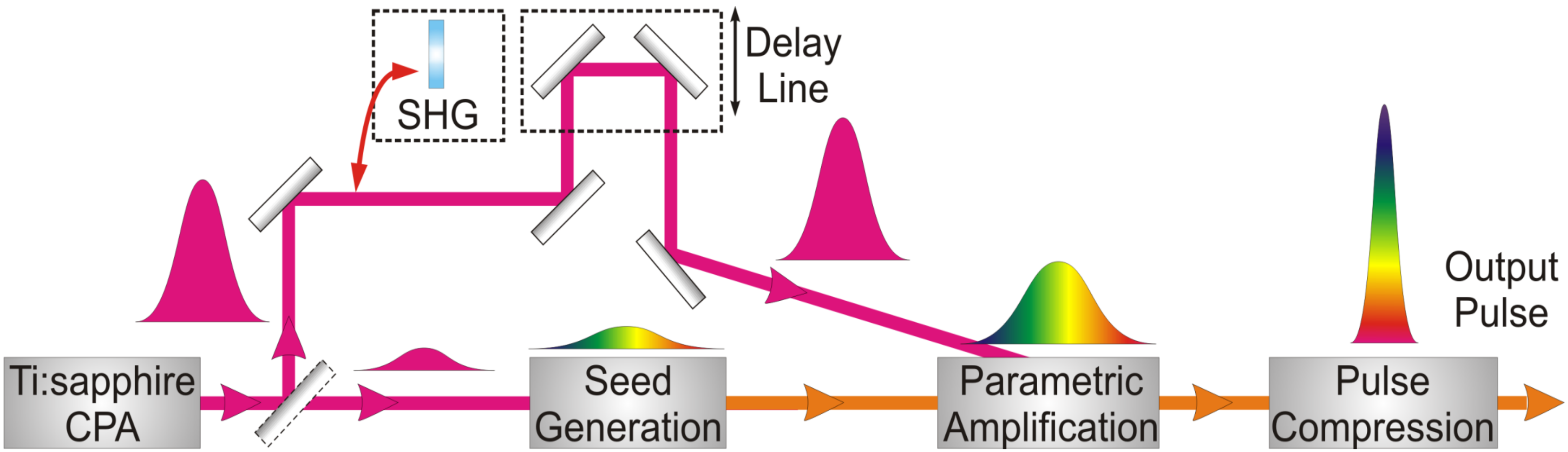
Energy conservation: $\hbar\omega_s + \hbar\omega_i = \hbar\omega_p$

Momentum conservation (vectorial): $\hbar\vec{k}_s + \hbar\vec{k}_i = \hbar\vec{k}_p$
(also known as **phase matching**)



$\Rightarrow$ **Broadband gain medium!**

Ultrabroadband Optical Parametric Amplifier



- **Broadband seed pulses can be obtained by white light generation**

- **Broadband amplification requires phase matching over a wide range of signal wavelengths**

G. Cerullo and S. De Silvestri, Rev. Sci. Instrum. **74**, 1 (2003).

Phase matching bandwidth in an OPA

If the signal frequency ω_s increases to $\omega_s+\Delta\omega$, by energy conservation the idler frequency decreases to $\omega_i-\Delta\omega$. The wave vector mismatch is

$$\Delta k = -\frac{\partial k_s}{\partial \omega}\Delta\omega + \frac{\partial k_i}{\partial \omega}\Delta\omega = \left(\frac{1}{v_{gs}} - \frac{1}{v_{gi}}\right)\Delta\omega$$

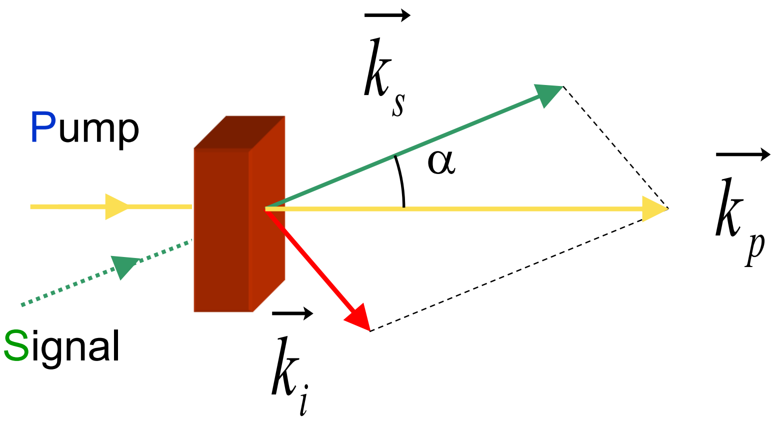
The phase matching bandwidth, corresponding to a 50% gain reduction, is

$$\Delta\nu \cong \frac{2(\ln 2)^{1/2}}{\pi}\left(\frac{\gamma}{L}\right)^{1/2} \frac{1}{\left|\dfrac{1}{v_{gs}} - \dfrac{1}{v_{gi}}\right|}$$

$\Rightarrow$ the achievement of broad gain bandwidths requires group velocity matching between signal and idler beams

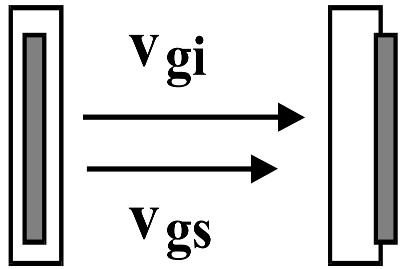
Broadband OPA configurations

- $v_{gi} = v_{gs}$: Operation around degeneracy $\omega_i = \omega_s = \omega_p/2$
  - ✓ Type I, collinear configuration
  - ✓ Signal and idler have same refractive index
- $v_{gi} \neq v_{gs}$: Non-collinear parametric amplifier (NOPA):
  - ✓ Pump and Signal at angle α

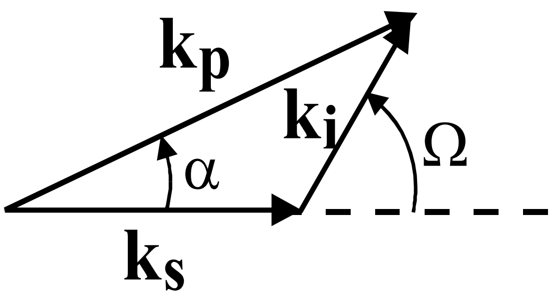


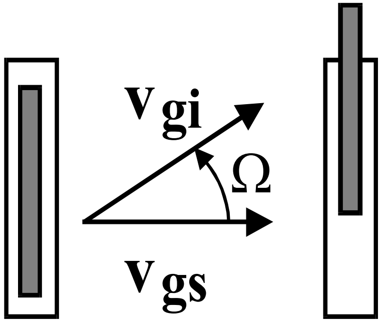
Noncollinear phase matching: geometrical interpretation

In a collinear geometry, signal and idler move with different velocities and get quickly separated



In the non-collinear case, the two pulses stay temporally overlapped





$$v_{gs} = v_{gi} \cos\Omega$$

Note: this requires $v_{gi} > v_{gs}$ (not always true!)

UH
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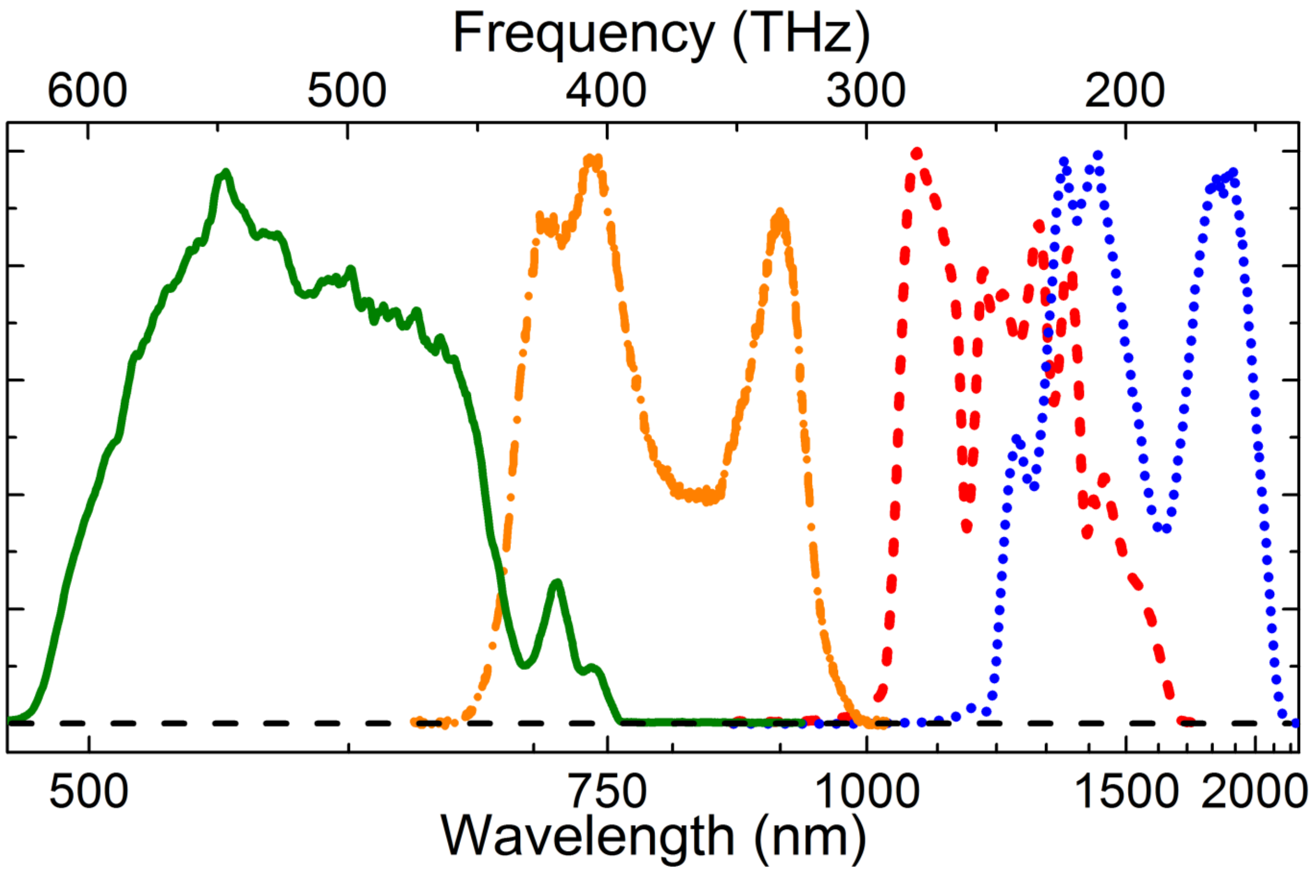
Broadband OPA configurations

Pump wavelength	NOPA	Degenerate OPA
400 nm (SH Ti:sapphire)	500-750 nm	700-1000 nm
800 nm (Ti:sapphire)	1-1.6 μm	1.2-2 μm

OPAs should allow to cover nearly continuously the wavelength range from 500 to 2000 nm (two octaves!) with few-optical-cycle pulses

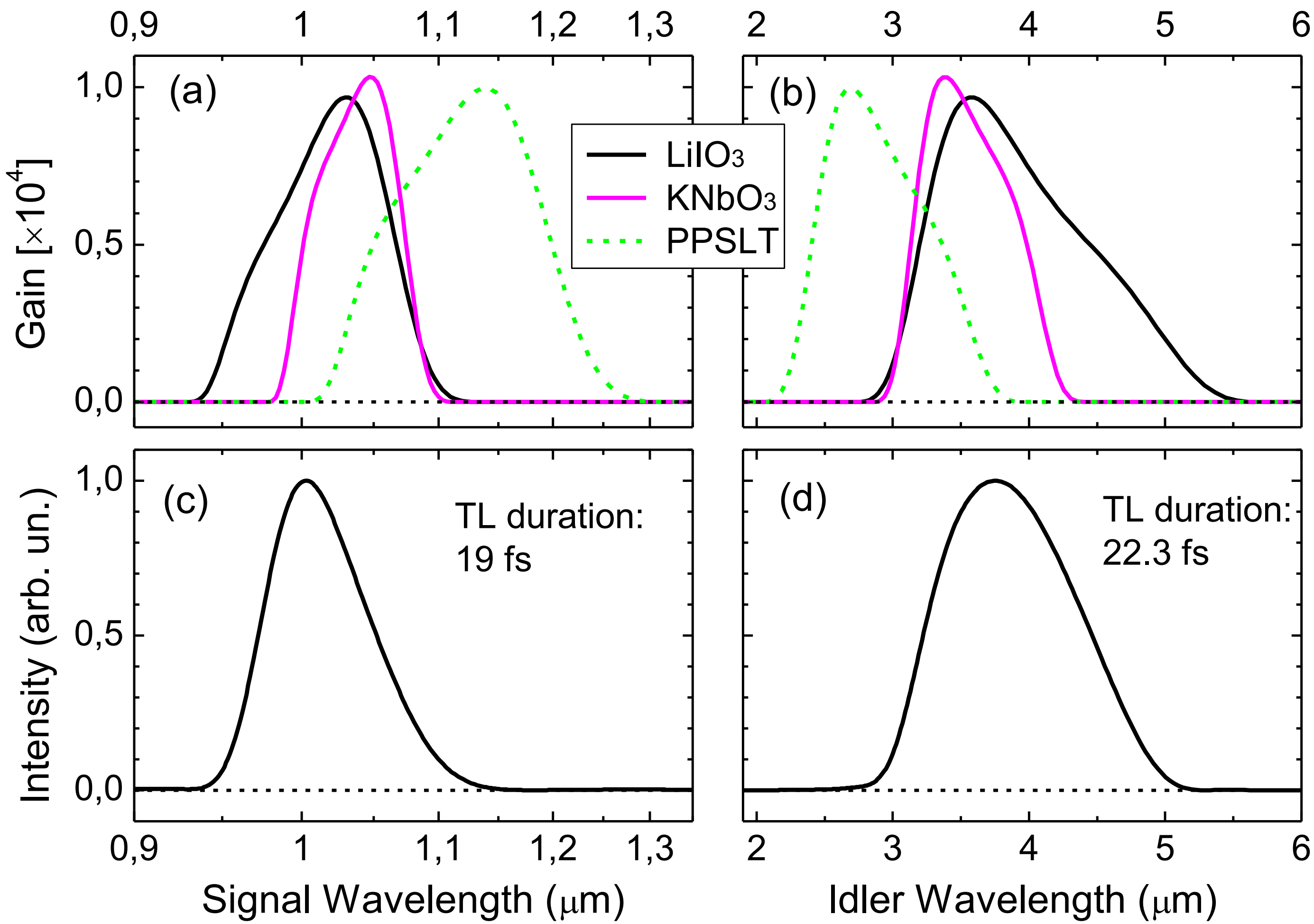
Tunable few-optical-cycle pulse generation



D Brida *et al.*, J. Opt. **12**, 013001 (2010).

Can we tune our pulses even more to the mid-IR? Yes, using the idler!

Broadband pulses in the mid-IR



- **Simulations confirm the generation of broadband idler pulses, with 20-fs duration (≈2 optical cycles) at 3 μm**

References

B.E.A. Saleh and M.C. Teich, "Fundamentals of Photonics," John Wiley and Sons, Inc., 1991.
P. G. Drazin, and R. S. Johnson, ”Solitons: An Introduction,”Cambridge University Press, New York (1990).

R.L. Fork, O.E. Martinez, J.P. Gordon: Negative dispersion using pairs of prisms, Opt. Lett. 9 150-152 (1984).
M. Nisoli, S. Stagira, S. De Silvestri, O. Svelto, S. Sartania, Z. Cheng, M. Lenzner, Ch. Spielmann, F. Krausz: A novel high energy pulse compression system: generation of multigigawatt sub-5-fs pulses, Appl. Phys. B 65 189-196 (1997).

J. Kuizenga, A. E. Siegman: "FM und AM mode locking of the homoge- neous laser - Part II: Experimental results, IEEE J. Quantum Electron. 6, 709-715 (1970).
H.A. Haus: Theory of mode locking with a slow saturable absorber, IEEE J. Quantum Electron. QE 11, 736-746 (1975).
K. J. Blow and D. Wood: "Modelocked Lasers with nonlinear external cavity," J. Opt. Soc. Am. B 5, 629-632 (1988).
J. Mark, L.Y. Liu, K.L. Hall, H.A. Haus, E.P. Ippen: Femtosecond pulse generation in a laser with a nonlinear external resonator, Opt. Lett. 14, 48-50 (1989)·
E.P. Ippen, H.A. Haus, L.Y. Liu: Additive pulse modelocking, J. Opt. Soc. Am. B 6, 1736-1745 (1989).
D.E. Spence, P.N. Kean, W. Sibbett: 60-fsec pulse generation from a self-mode-locked Ti:Sapphire laser, Opt. Lett. 16, 42-44 (1991).
H. A. Haus, J. G. Fujimoto and E. P. Ippen, ”Structures of Additive Pulse Mode Locking,” J. Opt. Soc. Am. 8, pp. 2068 — 2076 (1991).
U. Keller, ”Semiconductor nonlinearities for solid-state laser modelock- ing and Q-switching,” in Semiconductors and Semimetals, Vol. 59A, edited by A. Kost and E. Garmire, Academic Press, Boston 1999.

Lecture on Ultrafast Amplifiers by Francois Salin, http://www.physics.gatech.edu/gcuo/lectures/index.html.
D. Strickland and G. Morou: "Compression of amplified chirped optical pulses," Opt. Comm. 56,219-221,(1985).

G. Cerullo and S. De Silvestri, "Ultrafast Optical Parametric Amplifiers," Review of Scientific Instr. 74, pp. 1-17 (2003).