Universität Hamburg - Physics Department
Ultrafast Optical Physics II SoSe 2019

## Problem Set 7

Issued: June 7th, 2019.
Due: June 14th, 2019.

## Exercise 7.1: Interferometric autocorrelation

Consider a Michelson-type interferometer that produces two delayed replicas of an ultrashort pulse:

$$
\begin{equation*}
E(t)=E_{0}(t) e^{i \omega t} \tag{1}
\end{equation*}
$$

Both pulses are focused collinearly into a nonlinear crystal and give rise to secondharmonic generation (SHG).

- Which setup is it?
- What are the experimental parameters to be considered in order to measure the duration of few-fs laser pulses (resolution of the delay stage, vibrations, thermal effects)?
- Derive the expression of the SHG signal.
- For both cases of intensity and interferometric autocorrelation $\left(A_{I}(\tau)\right.$ and $A_{\text {interf }}(\tau)$ respectively), determine the ratio between the SHG-signal at zero delay and at large delays:

$$
\begin{gather*}
\frac{A_{\text {inter } f}(0)}{A_{\text {inter } f}( \pm \infty)}=?  \tag{2}\\
\frac{A_{I}(0)}{A_{I}( \pm \infty)}=? \tag{3}
\end{gather*}
$$

- Determine the same ratio for the intensity term in the interferometric autocorrelation. What do we learn from this?


## Exercise 7.2: Intensity autocorrelation

Consider a gaussian pulse $E(t)=e^{\frac{-t^{2}}{2 \sigma^{2}}}$ with a duration of 10 fs (FWHM).

- Determine the duration ( $\sigma$ or FWHM) of the intensity autocorrelation $A_{I}(\tau)$. Derive the duration of $A_{I}(\tau)$ by using the general expression of the integral of a gaussian function:

$$
\begin{equation*}
\int_{-\infty}^{+\infty} e^{-a x^{2}+b x+c} d x=\sqrt{\frac{\pi}{a}}{ }^{\frac{b^{2}}{4 a}+c} \tag{4}
\end{equation*}
$$

- Consider the general cross-correlation integral $I_{c c}=\int_{-\infty}^{+\infty} I_{1}(t) I_{2}(t-\tau) d t$. Determine the duration of the cross-correlation, $\sigma_{c c}$, with respect to the duration of $I_{1}(t)$ and $I_{2}(t)$, i.e. $\sigma_{1}$ and $\sigma_{2}$ respectively.
- Consider the 2 n -order autocorrelation $A_{I}^{2 n}(\tau)=\int_{-\infty}^{+\infty} I^{n}(t) I^{n}(t-\tau) d t$. Determine the duration of $A_{I}^{2 n}(\tau)$, i.e. $\sigma_{A_{I}^{2 n}}$, with respect to the duration of $I(t)$. For which value of $n$ has $A_{I}^{2 n}(\tau)$ the same duration as $I(t)$ ?


## Challenge: Single-cycle pulses

Consider the pulse $E(t)=E_{0} e^{\frac{-t^{2}}{2 \sigma^{2}}} \cos \left(\omega_{0} t+\phi\right)$. The electric field oscillates periodically, with a period $T: \omega_{0} T=2 \pi$. When $2 \sigma \sim T$, the pulse envelope contains mainly one cycle of the electric field. At the limit of extremely short pulses, the oscillatory part of the electric field is rather small and one could write the expression for such a pulse as:

$$
\begin{equation*}
E(t)=E_{0} e^{\frac{-t^{2}}{2 \sigma^{2}}} \tag{5}
\end{equation*}
$$

- Is the the expression of (5) physical? How to prove it?

