Universität Hamburg
Physics Department
Ultrafast Optical Physics II
SoSe 2019

Problem Set 3

Issued: **May 3, 2019** Due: **May 10, 2019**

Instruction: Please write your answer to each problem on separate paper sheet. If you are using programming language to do numerical simulations, attach the original code with your answers.

Problem 3.1: Rate Equations for Four-Level System (30 points in total)

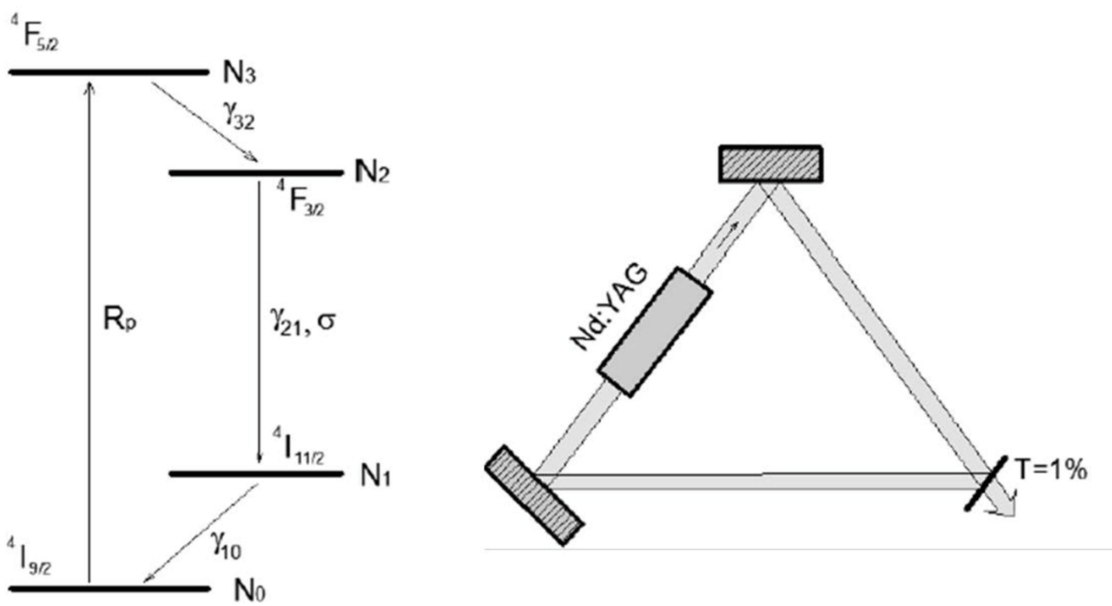

Figure 1: (a) Transitions in Nd:YAG (b) Ring laser with Nd:YAG gain medium.

As an example for a four level laser we consider the transition at $\lambda_0 = 1.064\ \mu m$ of the Nd:YAG laser (see figure above). The laser is optically pumped with diode lasers at $\lambda_P = 0.8\ \mu m$ from the ground state ($^4I_{9/2}$) into the ($^4F_{5/2}$) level. From that level the fast relaxation processes lead to transfer into the upper laser level ($^4F_{3/2}$). This level is long-living. Again an excitation in the lower laser level ($^4I_{11/2}$) relaxes very quickly into the ground state ($^4I_{9/2}$), such that the lower laser level can be considered empty. Thus the relaxation rate $\gamma_{32}, \gamma_{10} \gg \gamma_{21}$. The interaction cross-section for stimulated emission between levels 2 and 1 is $\sigma_{21} = 6.5 \cdot 10^{-19} cm^2$. The effective cross-section of the beam is $5000\ \mu m^2$ and the cavity length is $L = 1\ m$.

(a) Write down the rate equations for the four level laser in terms of

- Level population, i.e. the total number of atoms in each level N_0, N_1, N_2 and N_3;
- Intensity I of the resonator mode at wavelength $\lambda_0 = 1.064\ \mu m$ of the ring laser in Fig. 2(b) containing the Nd:YAG crystal as the gain medium;
- Pumping rate R_p, i.e. the number of atoms transferred by the pump from the ground level $\left({}^4I_{9/2}\right)$ to the third level $\left({}^4F_{5/2}\right)$ per unit time;

Assume, that only one mode of the ring laser can reach threshold and that the ring laser is running unidirectional. **(6 points)**

(b) Determine the differential equation for the time-dependent intensity of the resonator mode. For simplicity assume that both the mode cross-section A_{eff} and the intensity I are constant over the resonator. Eliminate the fast relaxing populations of level 3 and 1 by putting $dN_1/dt = dN_3/dt = 0$. The result should be

$$\frac{dN_2}{dt} = -\gamma_{21}N_2 - \frac{\sigma}{h\nu_0}N_2 I + R_p \tag{1}$$

$$\frac{dI}{dt} = -\gamma_{ph}I + \frac{\sigma v_g}{V}N_2\left(I + \frac{v_g h\nu_0}{V}\right) \tag{2}$$

where

- $V = A_{eff} \cdot L$ is the volume of the gain medium;
- $v_g = c/2$ is the group velocity at $\lambda_0 = 1.064\ \mu m$;
- $\nu_0 = c/\lambda_0$ is the laser frequency;
- γ_{ph} is the decay rate of photons in the cavity (equal to the inverse photon lifetime). **(6 points)**

(c) The upper state lifetime of Nd:YAG is $1/\gamma_{21} = 250\ \mu s$. It accounts for spontaneous emission in all possible modes of free space and nonradiative transitions. If the laser is below threshold, derive an expression for the ratio of number of transitions from the upper to the lower laser level to the number of spontaneously emitted photons into the laser mode. Identify the system parameter that dominates this ratio and discuss why it is this system parameter. **(4 points)**

For the following question (d), (e), (f) and (g), we can neglect the ~~spontaneous~~ emission into the laser mode.

(d) What is the minimum pump power to reach the threshold for lasing for a 1% output coupler? Assume no internal losses. **(4 points)**

(e) What is the maximum differential efficiency of the laser that can be achieved? **(4 points)**

In the following we assume that the laser crystal is not perfect and generates additional 2% of internal losses at the laser wavelength per roundtrip.

(f) What is the optimum output coupling T_{opt} to maximize the output power if the maximum available pump power is $1\ W$? What is the corresponding output power? **(3 points)**

(g) Repeat part (f) for the case without any internal loss. Compare the maximum power that can be achieved with and without internal losses given $1\ W$ pump power. **(3 points)**