University of Hamburg, Department of Physics

Nonlinear Optics

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Problem Set 3

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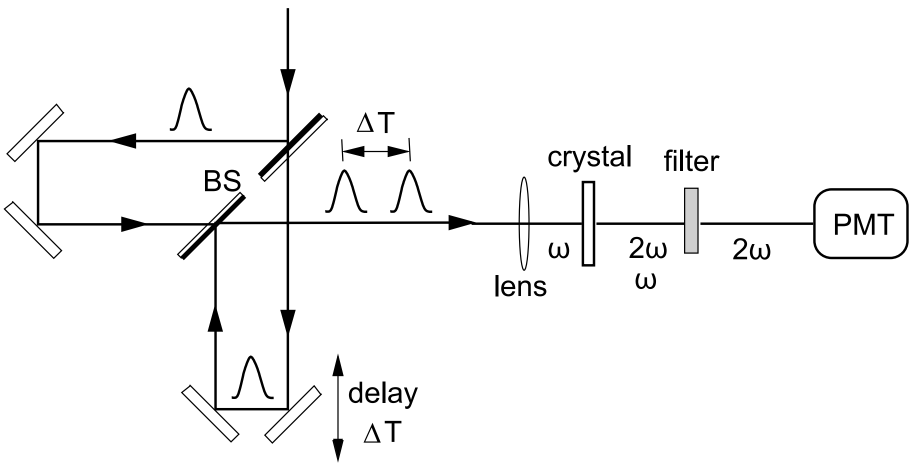
Pulse duration measurement by interferometric autocorrelation

One important application of frequency doubling is the characterization of ultrashort pulses. Transient optical signals on a picosecond and femtosecond time scale are too short to be characterized by direct electronic detection. In these cases, the short laser pulse itself is used to perform its own temporal characterization. A typical measurement scheme looks as follows:



The incident laser pulse is divided into two identical replica pulses (each with approximately half the original intensity) with a relative time delay ΔT by means of two beam splitters (BSs) rotated by 180°. Beam splitters for this purpose are typically thin (∼100-300 μm, to avoid dispersive pulse broadening) glass substrates with a metallic coating (e.g., Cr-Ni) on one side. The reflection occurs on the metallic coating. If one would use only a single beam splitter to implement an interferometer, one of the pulses would pass through the substrate only once, however, the other one would traverse the substrate three times. For very short pulses, a significant measurement error would occur. In the symmetric setup displayed above, both pulses travel identical optical paths.

The time delay ΔT is achieved using a scanning delay line consisting of two mirrors. Moving the delay stage by 3 μm leads to a delay of 2×10 fs. Practically, the pathlength difference can be varied with a piezo-mounted delay stage and (not shown in the figure above) permanently monitored employing an incoupled, copropagating

HeNe laser with well-defined wavelength, that at the interferometer's drop-port creates interferences, that can simultaneously be measured using a second detector. The resulting two replica pulses are afterwards focused into a nonlinear crystal. The generated second-harmonic signal is then detected (e.g., using a photomultiplier tube (PMT)) in a temporally integrated way. The pulse duration can be characterized by measuring the second-harmonic signal as function of delay ΔT. In other words, this setup measures the temporal overlap of both pulses versus ΔT. We assume that the $\chi^{(2)}$ nonlinearity has an instantaneous response, i.e., it is frequency independent in the relevant spectral range, thus it follows

$$E_2(t, \Delta T) = CE^2(t, \Delta T)$$

with

$$E(t, \Delta T) = E_1(t) + E_1(t + \Delta T).$$

Here, E_1 and E_2 denote the electric field envelopes at the fundamental and the generated second harmonic.

1. Derive an expression for the time-integrated intensity of the generated second-harmonic light
$$I_2(\Delta T) = \int_{-\infty}^{+\infty} |E_2(t, \Delta T)|^2 \, dt$$
This expression contains parts oscillating as function of ΔT, that originate from the phase delay of both replica pulses.

2. Assume that the fast oscillating parts mentioned above are averaged out. Show that the resulting averaged signal (after normalization) can be expressed as
$$\begin{aligned} S(\Delta T) &\sim 1 + 2G(\Delta T) \\ G(\Delta T) &\equiv \int_{-\infty}^{+\infty} I_1(t) I_1(t + \Delta T) dt \left(\int_{-\infty}^{+\infty} I_1^2(t) dt \right)^{-1}, \end{aligned}$$
with $G(\Delta T)$ being the autocorrelation function of the pulse intensity.

3. Consider a Gaussian pulse of the form
$$E_1(t) = E \exp\left(\frac{-t^2}{2\tau^2}\right),$$
where the pulse duration $t_p = 2\sqrt{\ln 2}\tau \approx 1.66\tau$ is the full width half maximum (FWHM) of the pulse intensity profile $I(t)$. Derive an expression for the autocorrelation function $G(\Delta T)$ and the interferometric autocorrelation $I_2(\Delta T)$.

4. **The effective nonlinear coefficient of BBO**

Consider the nonlinear crystal β-barium borate (BBO), which belongs to the crystal class 3m with m⊥y. Assume the crystal is negatively uniaxial. Calculate the effective nonlinear coefficient d_{eff} for type-I phase matching as function of the angles ϑ and φ in analogy to the lecture. Here, ϑ is the phase-matching angle between the z-axis of the crystal and of the $\vec{k}$ vector, and φ is the angle between the ordinary beam and the x-axis of the crystal. Assume that the birefringence is weak enough that $\vec{E}$ and $\vec{D}$ fields also for the extraordinary beam is approximately parallel. Simplify as much as possible.write the final polarization at $x' - y' - z'$ coordinate .

hint:
1.The $\vec{k}$ vector is along the z' axis. The electric filed E is along x' axis.(x' is in the $x - y$ plane since the input E satisfy the type I phase matching, e.g should be o-polarized).
2. The rotation of the coordinate from $x - y - z$ to $x' - y' - z'$ can be done by two separate rotation operation.