University of Hamburg, Department of Physics

Nonlinear Optics

Kärtner/Mücke, WiSe 2017/2018

Problem Set 2

Issued: 02.11.'17 Due : 09.11.'17

1. An arbitrary wave polarized in the x-y-plane is described by

$$\vec{E}(z,t) = \frac{1}{2}\left[E_x(\omega)e^{j(\omega t - kz)} + c.c.\right]\hat{\mathbf{x}} + \frac{1}{2}\left[E_y(\omega)e^{j(\omega t - kz)} + c.c.\right]\hat{\mathbf{y}}$$

and propagates through an instantaneously reacting isotropic and lossless medium that shows a third-order nonlinearity.
Consider the generated nonlinear polarization at the third harmonic $P_x^{(3)}(3\omega)$.

   (a) Show that the nonlinear polarization at the third harmonic can be expressed as

$$P_x^{(3)}(3\omega) = \frac{\varepsilon_0}{4}\chi_{xxxx}E_x^3(\omega) + \frac{\varepsilon_0}{4}\left[\chi_{xxyy} + \chi_{xyxy} + \chi_{xyyx}\right]E_y^2(\omega)E_x(\omega).$$

   (b) Show that in the isotropic medium the following relation is valid

$$\chi_{xxxx} = \chi_{xxyy} + \chi_{xyxy} + \chi_{xyyx}.$$

   (Hint: consider a general rotation around the z-axis). And show that the nonlinear polarization at the third harmonic can be expressed as

$$P_x^{(3)}(3\omega) = \frac{\varepsilon_0}{4}\chi_{xxxx}\left[E_x^3(\omega) + E_y^2(\omega)E_x(\omega)\right].$$

   (c) Show that circularly polarized light ($E_y = \pm jE_x$) can not generate third-harmonic light in this medium.

   (d) In the following we consider a third-order process, which generates a signal at the input frequency, $P_x^{(3)}(\omega)$. This process induces a change in refractive index for the x-component of the field, which is generated by both components. Show that in an instantaneous, isotropic and lossless medium, $P_x^{(3)}(\omega)$ is given by the following expression:

$$P_x^{(3)}(\omega) = \frac{1}{4}\varepsilon_0\chi_{xxxx}\left[3\left|E_x\right|^2 E_x + 2\left|E_y\right|^2 E_x + E_y^2 E_x^*\right]$$

(e) Derive the analogous expression for $P_y^{(3)}(\omega)$.

(f) Derive an expression for $P_{\pm}^{(3)}(\omega)$ for the case of circularly polarized light. Circularly polarized light is described by

$$E_{\pm} = \frac{1}{\sqrt{2}} \left(E_x \pm jE_y\right).$$

Give expressions for $P_{\pm}^{(3)}(\omega)$.

(g) The intensity-dependent refractive index n_2 is defined by

$$n = n_0 + n_2 I.$$

Give an expression for n_2 for the case of linearly and circularly polarized light.

(h) Give a physical interpretation of the expressions derived in problems e-g?

2. Periodically poled $LiNbO_3$ (PPLN) is used to achieve quasi-phase matching (QPM) for a nonlinear process. We want to describe a parametric process, including a pump wave that generates a signal and an idler field. The crystal is pumped at $\lambda_p = 532nm$ and generates a signal beam at $\lambda_s = 950nm$.

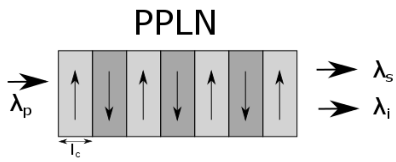

Figure 1: Scheme of periodically poled $LiNbO_3$

(a) Find the relations of frequency and wavelength between the three involved waves (pump, signal and idler) and determine the wavelength of the idler beam.

(b) What component of the polarization should be used if we want to make use of the high nonlinear coefficient $d_{33}/\chi_{zzz}^{(2)}$ Write down the full expression for that component of the polarization.

(c) Evaluate the phase mismatch between the three waves in the $LiNbO_3$ crystal. You can look up the refractive indices at http://refractiveindex.info/. Remember what polarization we preferred from (b) when choosing ordinary or extraordinary refractive index. Calculate the coherence length $l_c = \frac{\pi}{\Delta k}$ for the given setup.