Problem 5.1 Active Mode-Locking and Gaussian Pulse Analysis

In this Problem, the steady-state pulse width in an actively mode-locked laser using Gaussian pulse analysis will be obtained.

The master equation for active mode locking by pure loss modulation is given by the following expression

$$TR \frac{\partial A}{\partial T} = \left[ g - l + D_g \frac{\partial^2}{\partial t^2} - M_s t^2 \right] A$$

(2)

where the loss modulator is already described in parabolic approximation. In the same way as the Split-Step Fourier transform method to simulate the NSE, we can use it to simulate the dynamics of the actively mode-locked laser. We start from a Gaussian pulse

$$A_1(t) = e^{-\Gamma_1 t^2/2},$$

(3)

with the complex Γ-parameter, $\Gamma_1 = a + jb$. Propagation through the gain and loss parts of the system within one round trip can be described in the frequency domain by the operation

$$\hat{A}_2(\omega) = e^{(g-l-D_g \omega^2)} \hat{A}_1(\omega),$$

(4)

which is again a Gaussian pulse in the frequency and time domain

$$A_2(t) = e^{-\Gamma_2 t^2/2}.$$ 

(5)

Propagation through the modulator is most simply described in the time domain by

$$A_3(t) = e^{-M_s t^2} A_2(t) = e^{-\Gamma_3 t^2/2}$$

(6)

(a) Calculate the complex Gamma coefficients $\Gamma_2$ and $\Gamma_3$ as a function of $\Gamma_1$, $D_g$ and $M_s$. Assume $D_g \Gamma_1 \ll 1$.

(b) What is the condition for steady-state pulse after one round-trip?

(c) Derive the Γ-parameter for the steady-state pulse. How do the stationary pulse width and chirp depend on the system parameters?