2021 Dec 15

NLO #19

- **Coupled mode equation** (Reminder)
- **Microresonator (dissipative) solitons** (Reminder)
- **Soliton crystals and molecules**
- **Synchronization with external pulsed pump**
- **Applications**
- **Noise in microresonators**
- **Other nonlinear processes:**
  - **THG**
  - **SRS and SBS**
  - **SHG**
  - **Optical parametric oscillation**

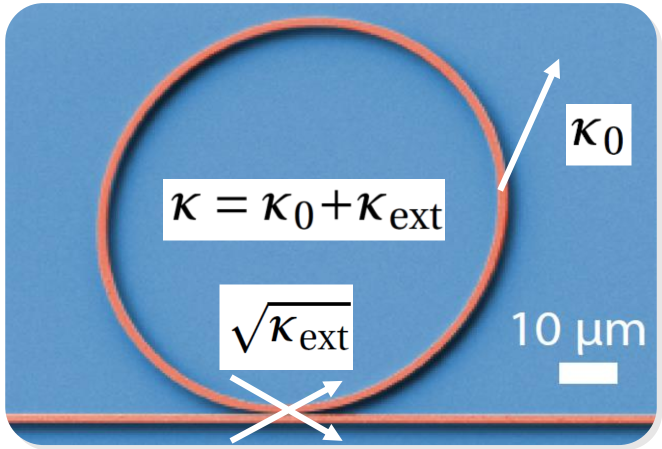
Coupled mode equations

loss input pump detuning

$$\frac{\partial A_\mu}{\partial t} = -\frac{\kappa}{2} A_\mu + \delta_{\mu 0} \sqrt{\kappa_{\text{ext}}} s_{\text{in}} e^{-i(\omega_p - \omega_0)t} + ig \sum_{\mu',\mu'',\mu'''} A_{\mu'} A_{\mu''} A^*_{\mu'''} e^{-i(\omega_{\mu'} + \omega_{\mu''} - \omega_{\mu'''} - \omega_\mu)t}$$

nonlinear coupling

$$\mu' + \mu'' - \mu''' - \mu = 0$$



when transforming into an frequency-equidistant grid of modes centered on pump frequency:

$$\frac{\partial}{\partial \tau} a_\mu = -(1 + i\zeta_\mu) a_\mu + i \sum_{\mu',\mu'',\mu'''} a_{\mu'} a_{\mu''} a^*_{\mu'''} + \delta_{0\mu} f$$

$$\mu' + \mu'' - \mu''' - \mu = 0$$

$$\tau = \kappa t/2$$

$$\zeta_\mu = 2(\omega_\mu - \omega_p - \mu D_1)/\kappa$$

$$f = \sqrt{8\eta g/\kappa^2} s_{\text{in}}$$

Dissipative temporal cavity soliton properties

$$\omega_\mu = \omega_0 + D_1\mu + \underline{\tfrac{1}{2}D_2\mu^2}$$

$$D_2 = 2\pi\ \mathrm{FSR}(\mu) = -\frac{c}{n_g}D_1^2\beta_2$$

$$d_2 = D_2/\kappa$$

Modulation instability can occur and dissipative soliton can exist when $D_2 > 0$ (anomalous dispersion)

Dissipative soliton properties

Frequency domain field:

$$\Psi(\omega - \omega_p) = \sqrt{d_2/2}\,\mathrm{sech}((\omega - \omega_p)/\Delta\omega)$$

$$\Delta\omega = \frac{2D_1}{\pi}\sqrt{\frac{\zeta_0}{d_2}}$$

$$d_2 = D_2/\kappa$$

Time domain:

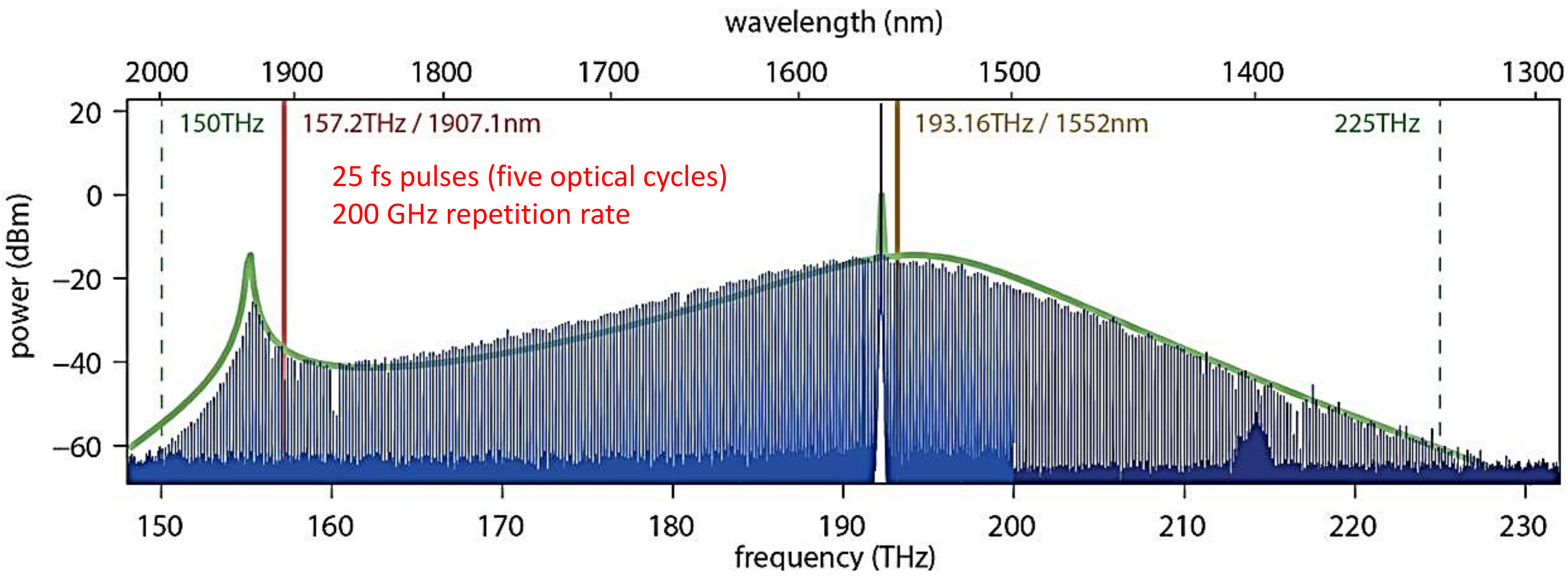
$$\Psi(t) = \sqrt{2\zeta_0}\,\mathrm{sech}(t/\Delta t)$$

$$\Delta t = \frac{1}{D_1}\sqrt{\frac{d_2}{\zeta_0}}$$

$$\Delta t_{\mathrm{min}} = \frac{2}{\sqrt{\pi}}\sqrt{\frac{-\beta_2}{\gamma F P_{\mathrm{in}}}}$$

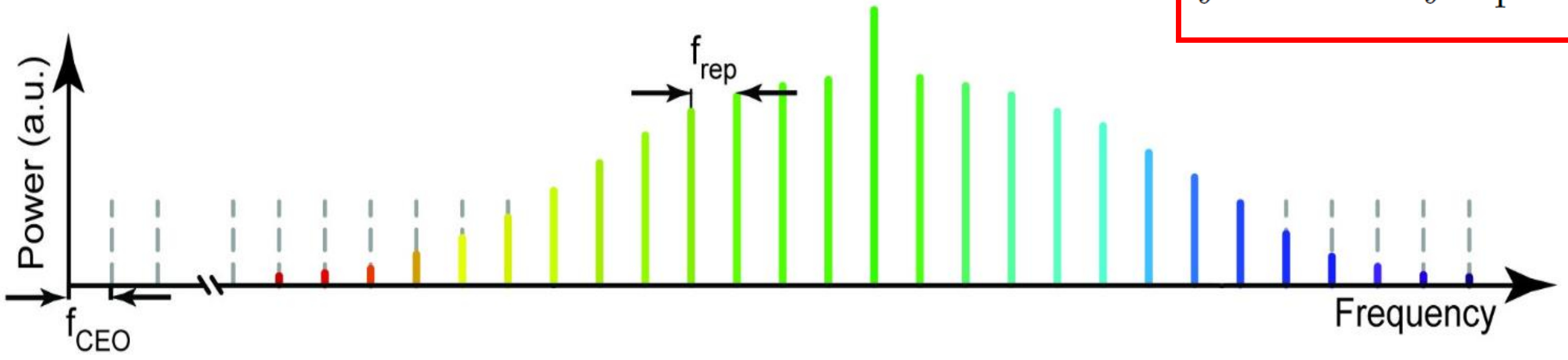
After finding the maximal detuning

Dissipative soliton frequency combs



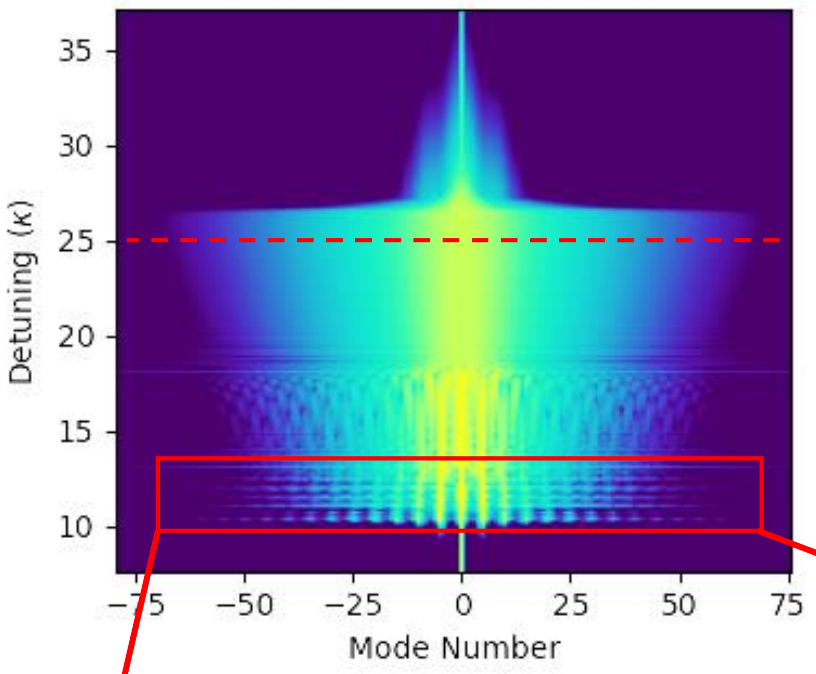
Frequency combs:

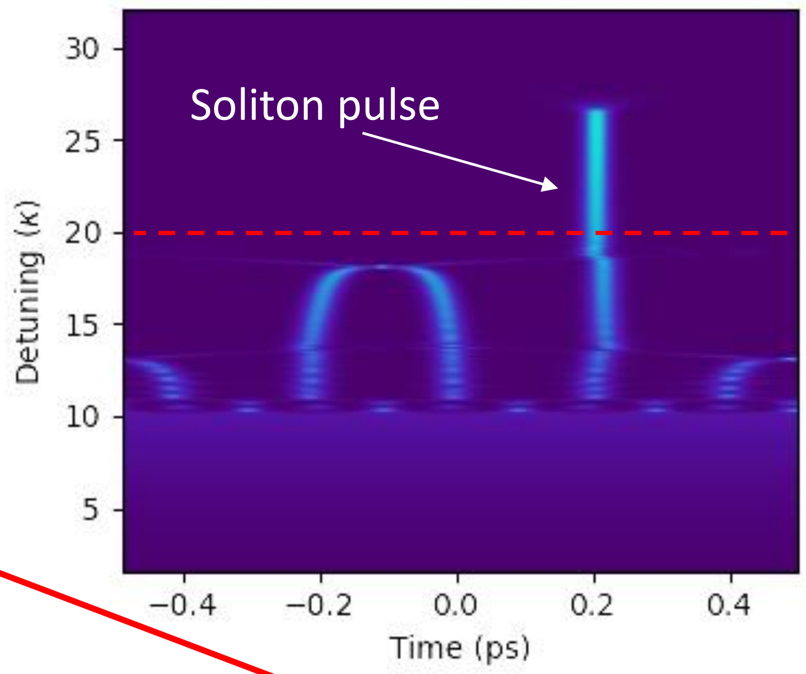
$$f_n = n \cdot f_{\text{rep}} + f_{\text{CEO}}$$

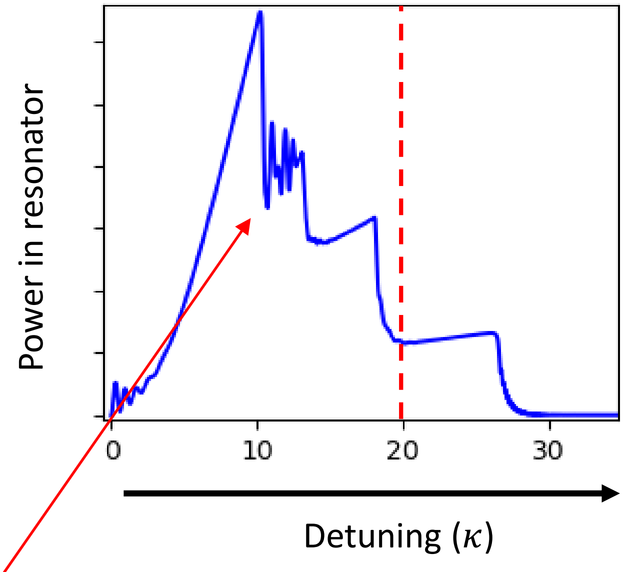


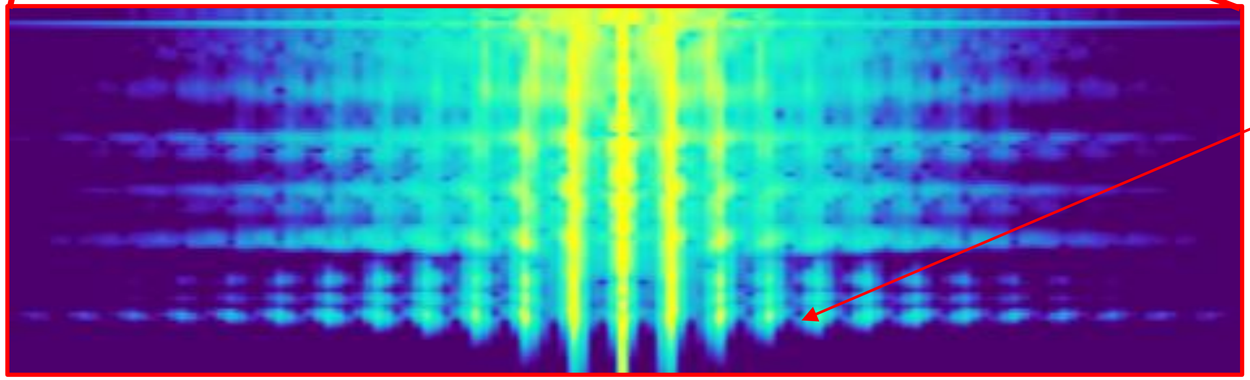
n^{th} Comb Line at

Soliton formation dynamics

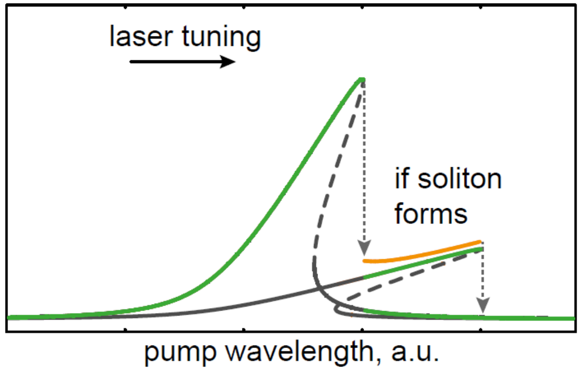








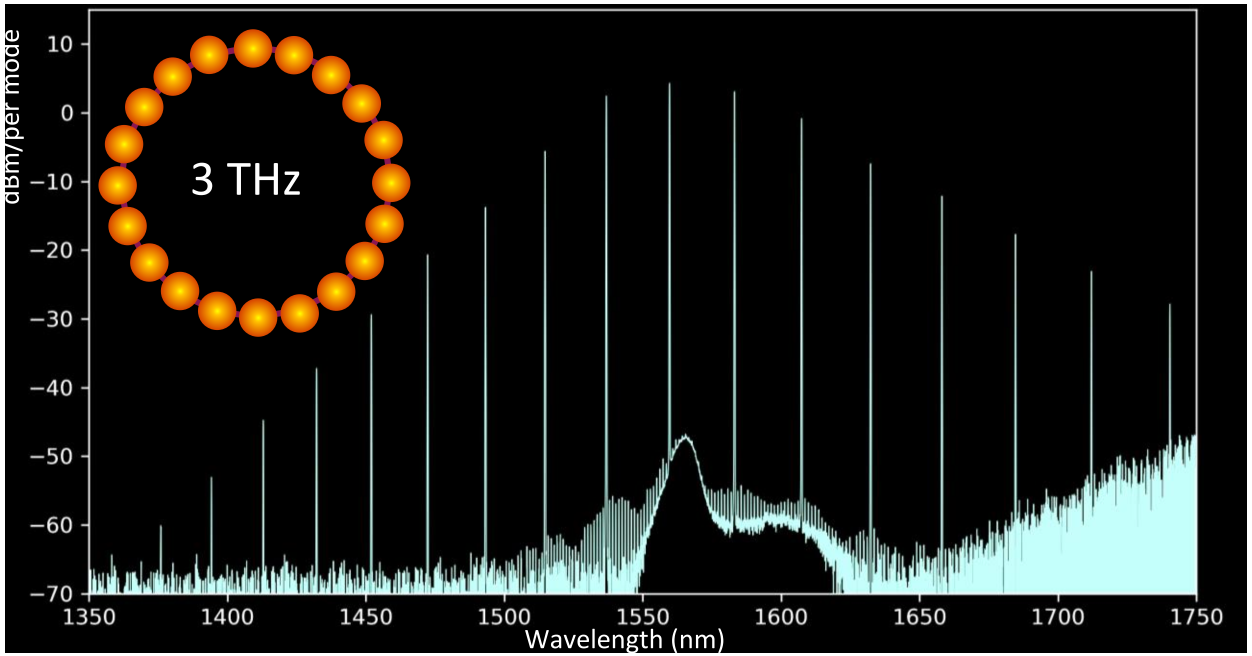
Modulation instability and four-wave mixing

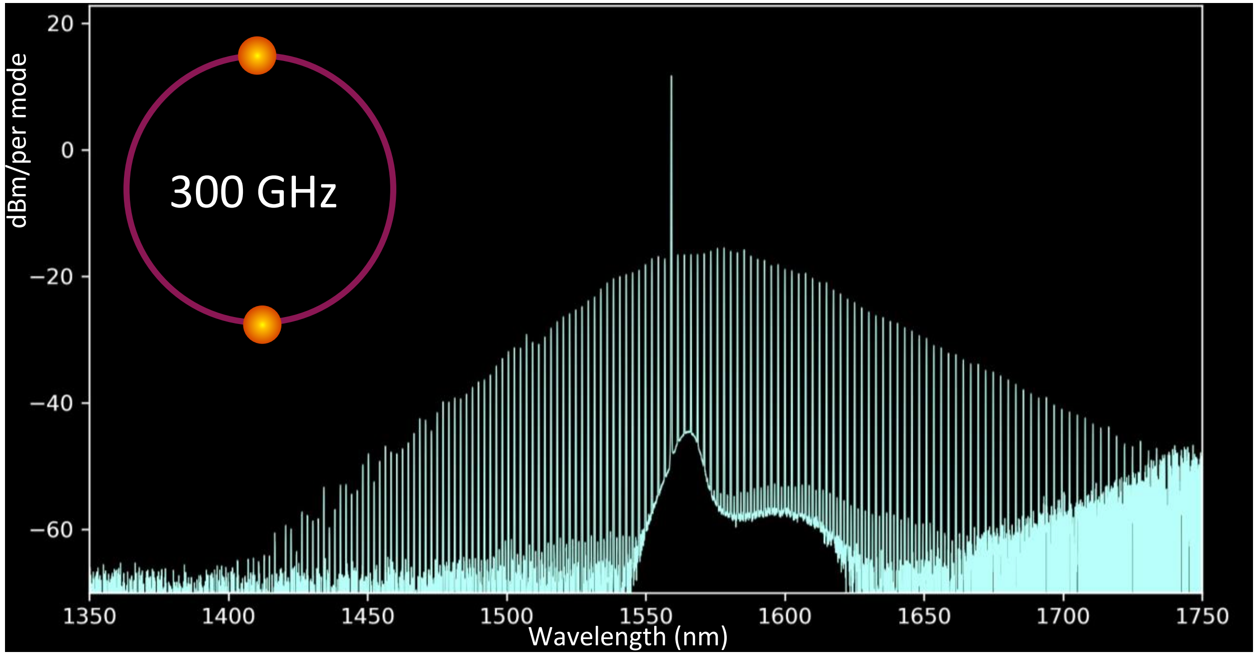


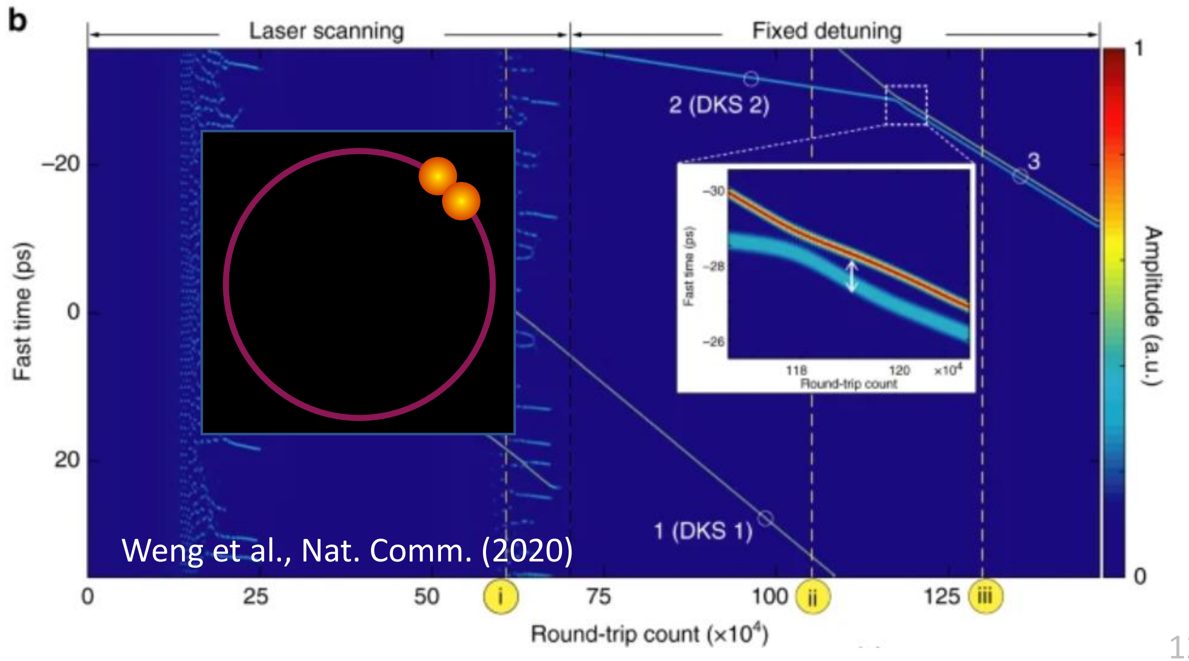
Soliton crystals and molecules

Field envelope for multiple solitons

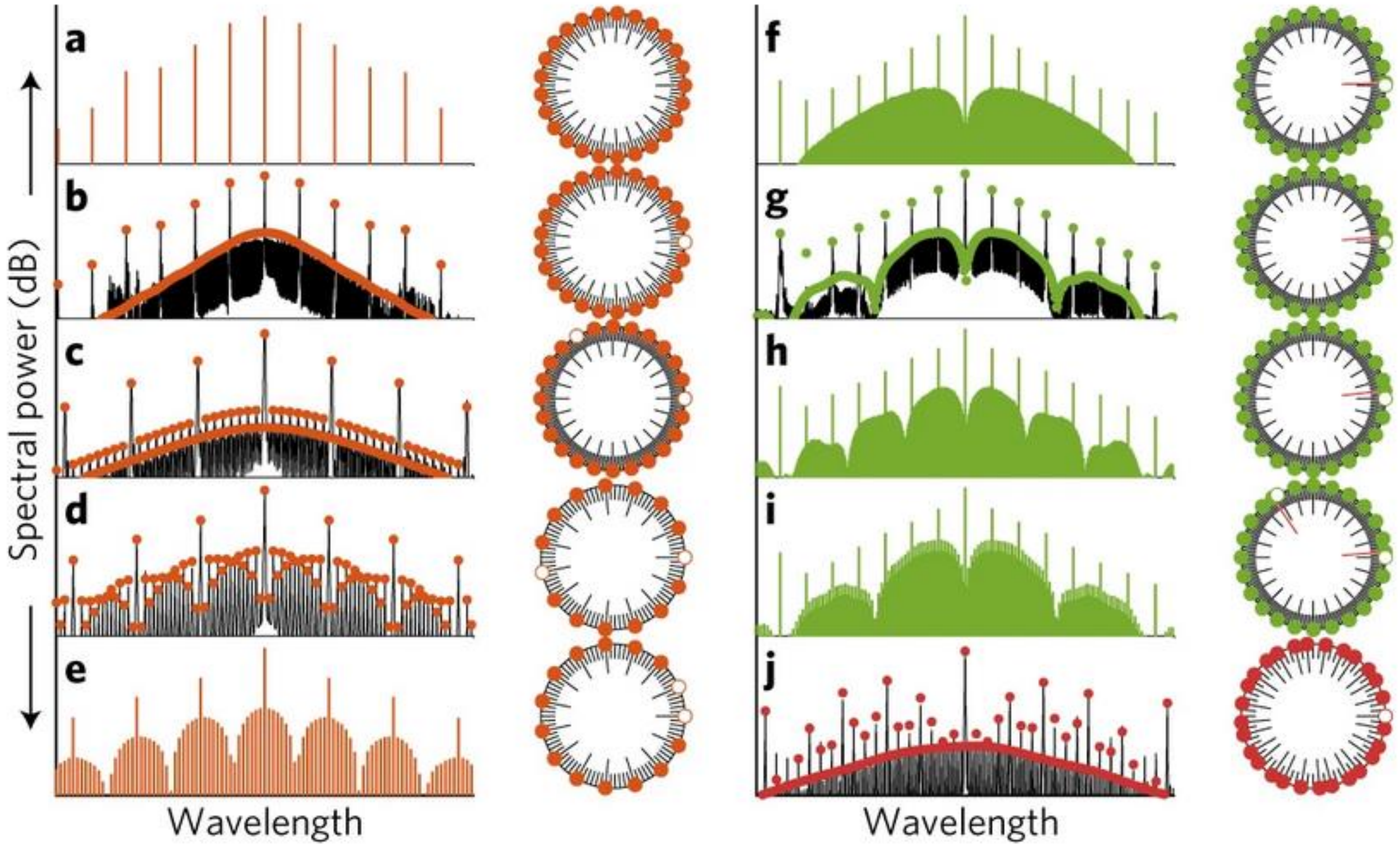
$$\Psi(\phi) \simeq C_1 + C_2 \cdot \sum_{j=1}^{N} \operatorname{sech}\left(\sqrt{\frac{2(\omega_0 - \omega_p)}{D_2}}(\phi - \phi_j)\right)$$





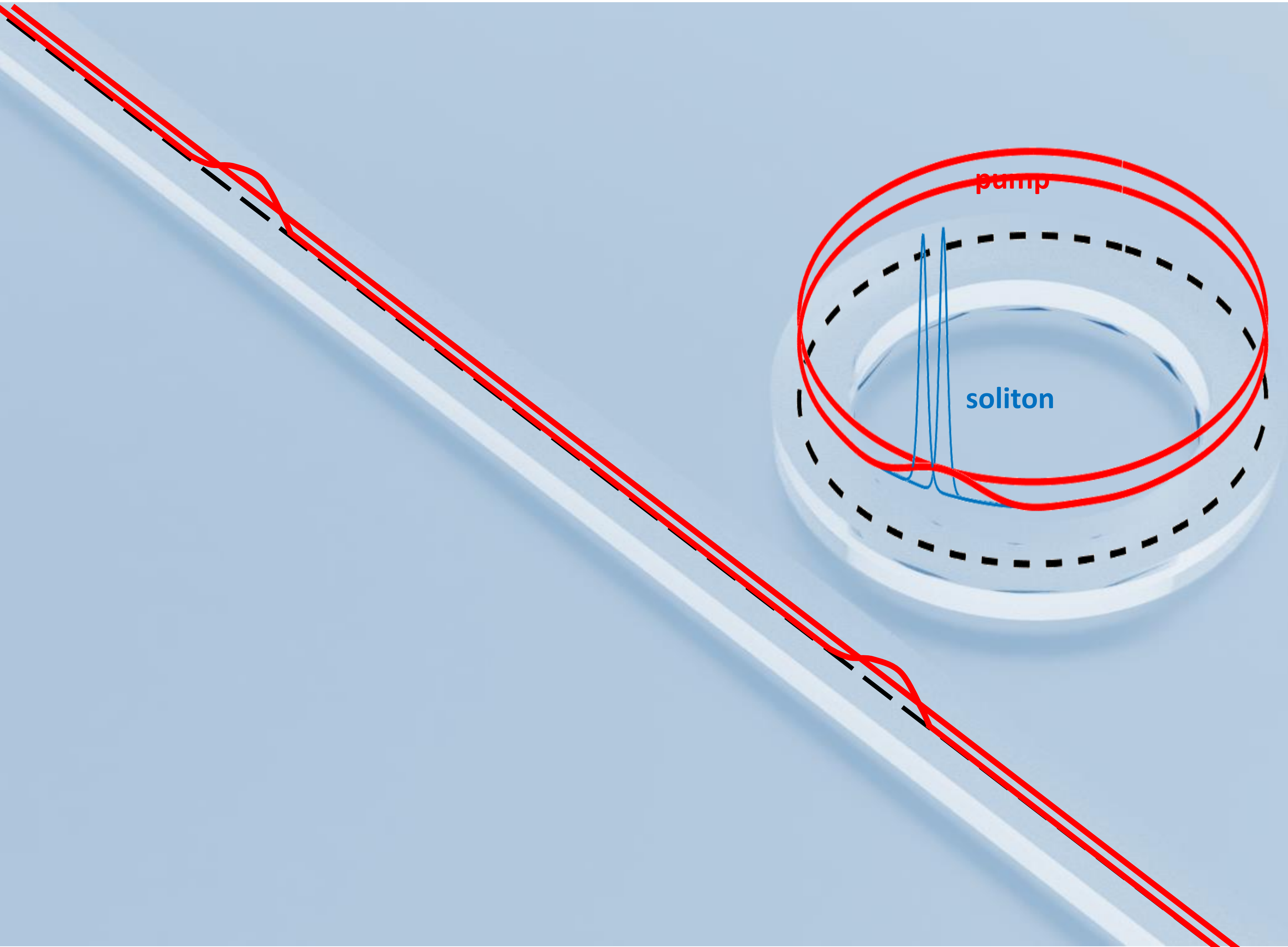


Soliton crystals



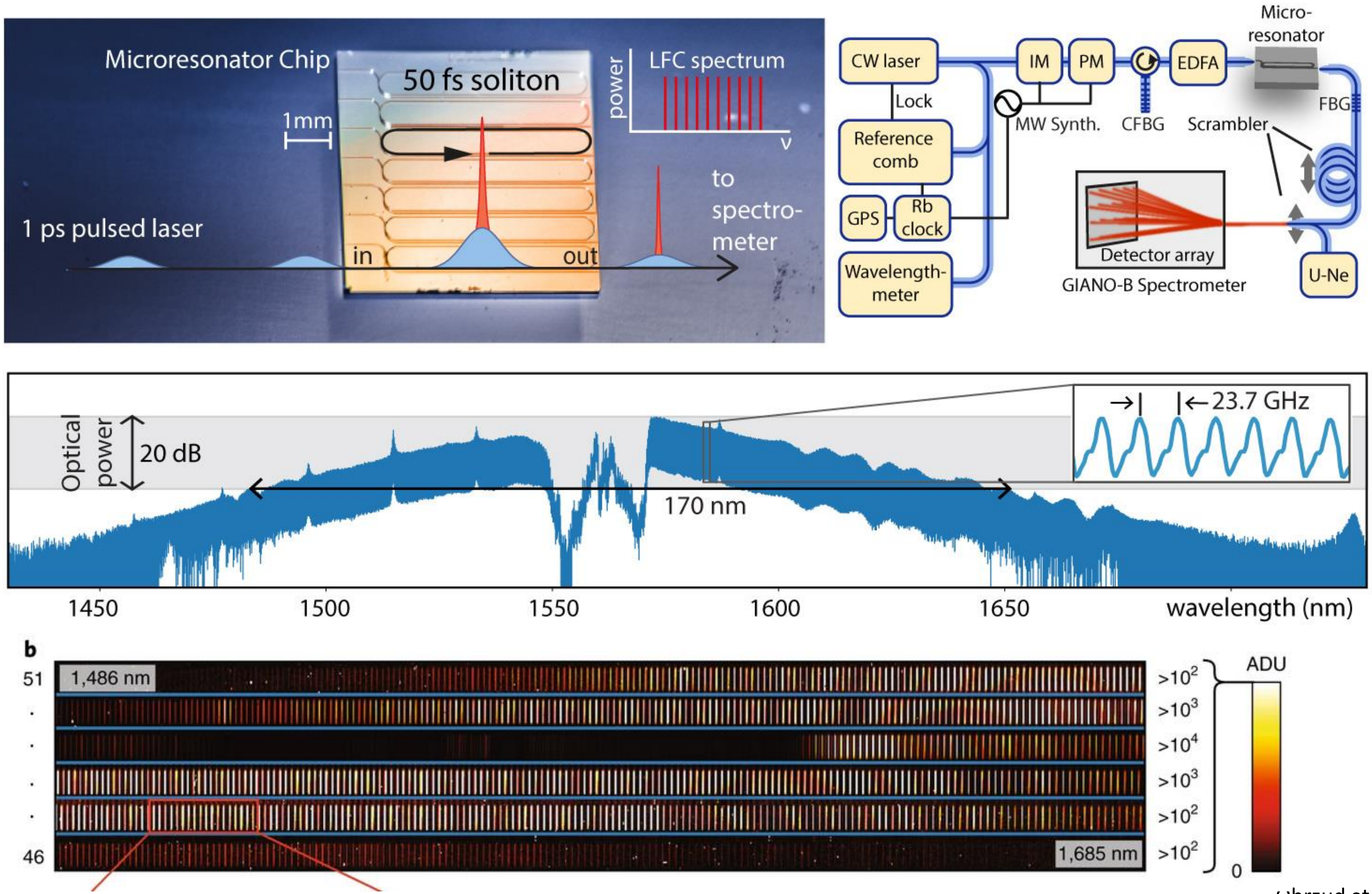
Cole et al. Nat. Phot. (2017)

Synchronization and multiplication of pulse repetition rate



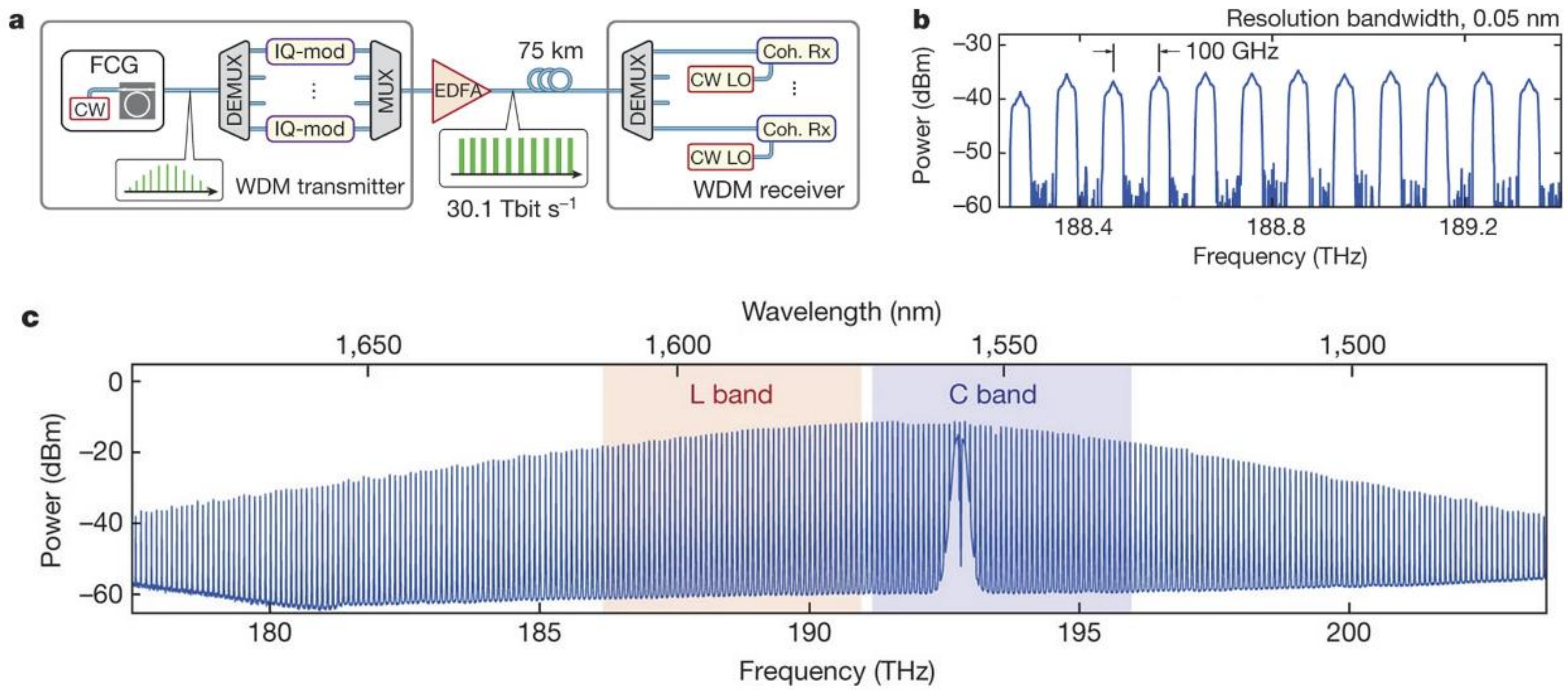
Obrzud et al. *Nature Photonics* (2017)
Brasch et al. Optica (2020)

Application: Astronomy



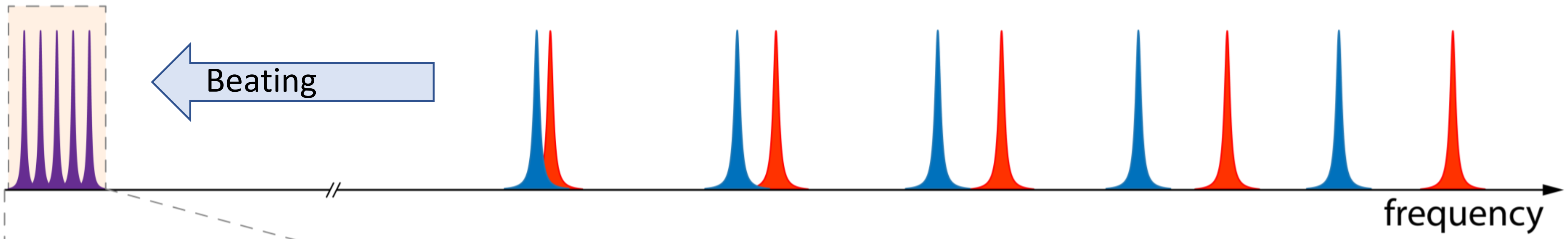
Obrzud et al., Nature Phot. (2019)

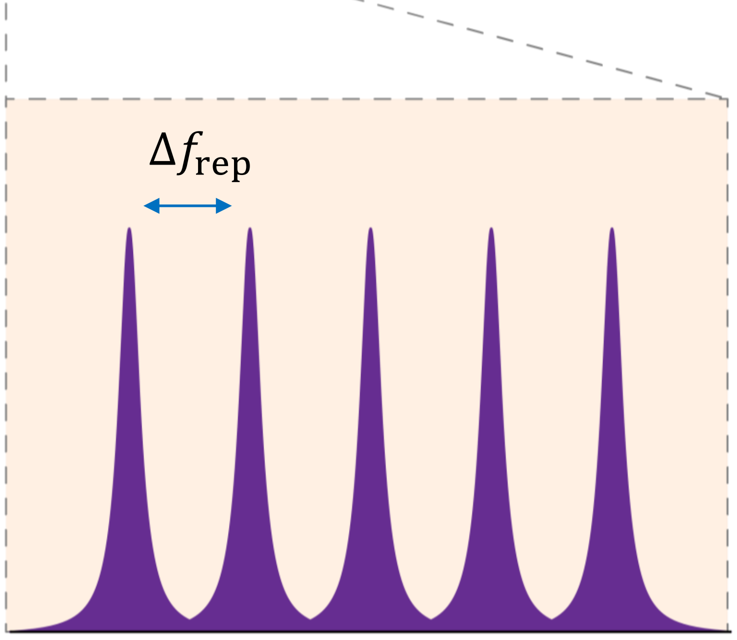
Application: Optical data transfer



Marin-Palomo et al. Nature (2017)

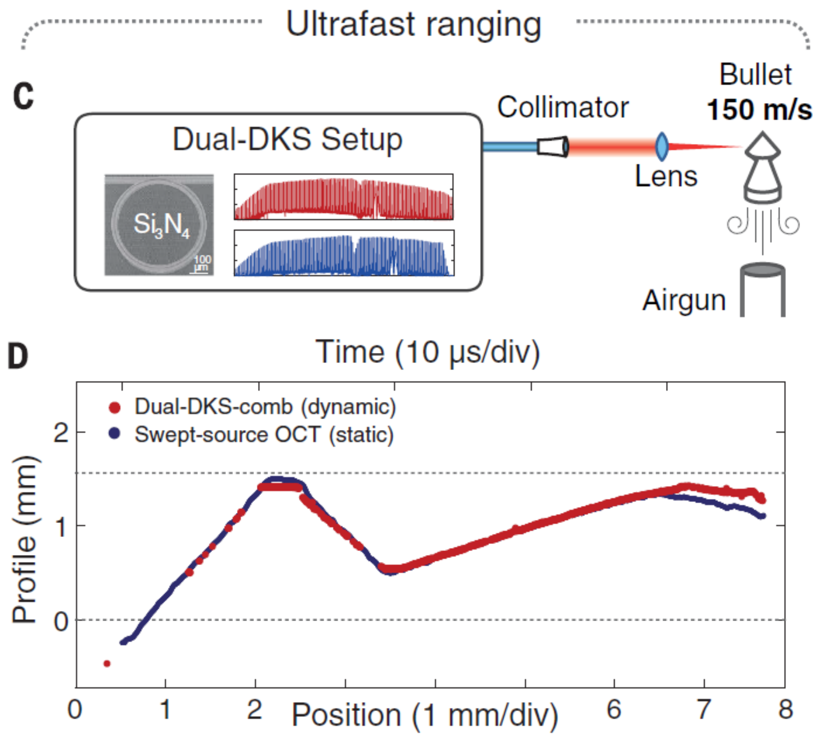
Application: Fast ranging





$f_{\text{rep}} \approx 100$ GHz

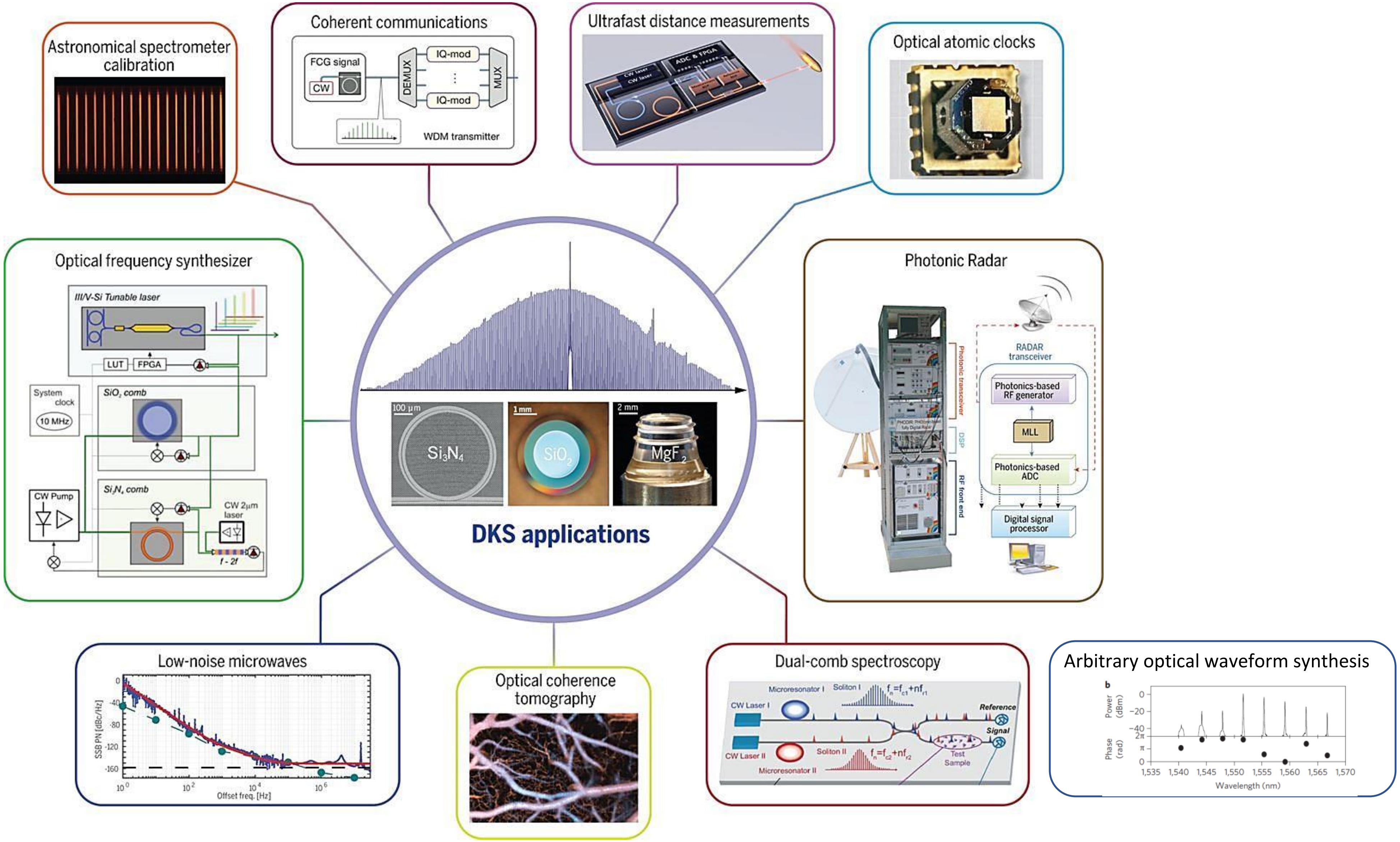
$\Delta f_{\text{rep}} \approx 100$ MHz



Trocha/Karpov et al. Science (2018)

Can measure phase between comb lines
-> **multi-wavelength interferometry**

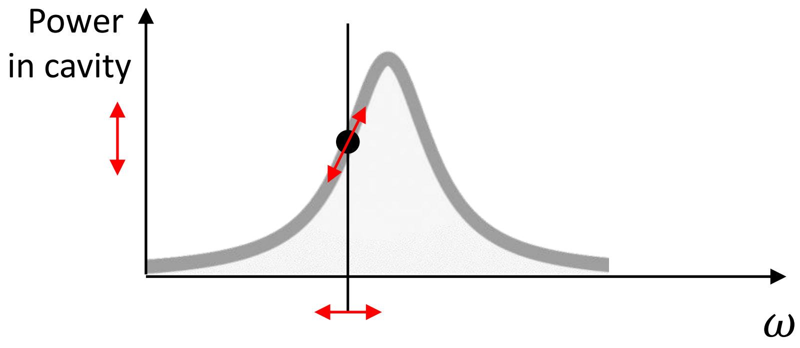
Microresonator solitons
Applications

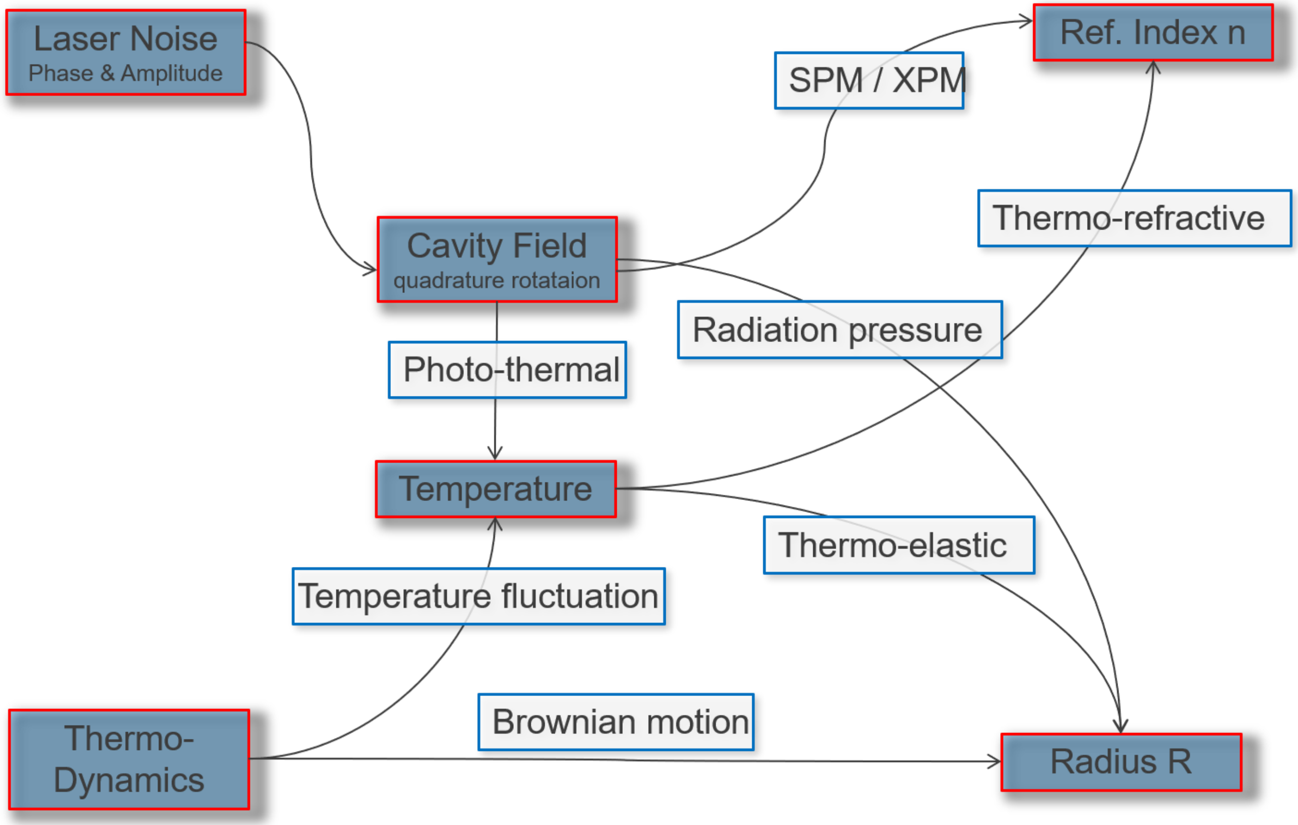


Review: Kippenberg, et al. Science (2018)

Noise in microresonator combs

$$\frac{1}{\omega}\frac{d\omega}{dT} = -\left(\frac{1}{n}\frac{dn}{dT} + \frac{1}{L}\frac{dL}{dT}\right)$$





Thermorefractive noise (TRN)

Variance of fundamental statistical temperature fluctuation of a volume V:

$$\langle \delta T^2 \rangle = \frac{k_B T^2}{\rho C V}$$

density (ρ) — Specific heat capacity (C)

$\sqrt{\langle \delta T^2 \rangle}$ can reach $\sim 10\ \mu K$

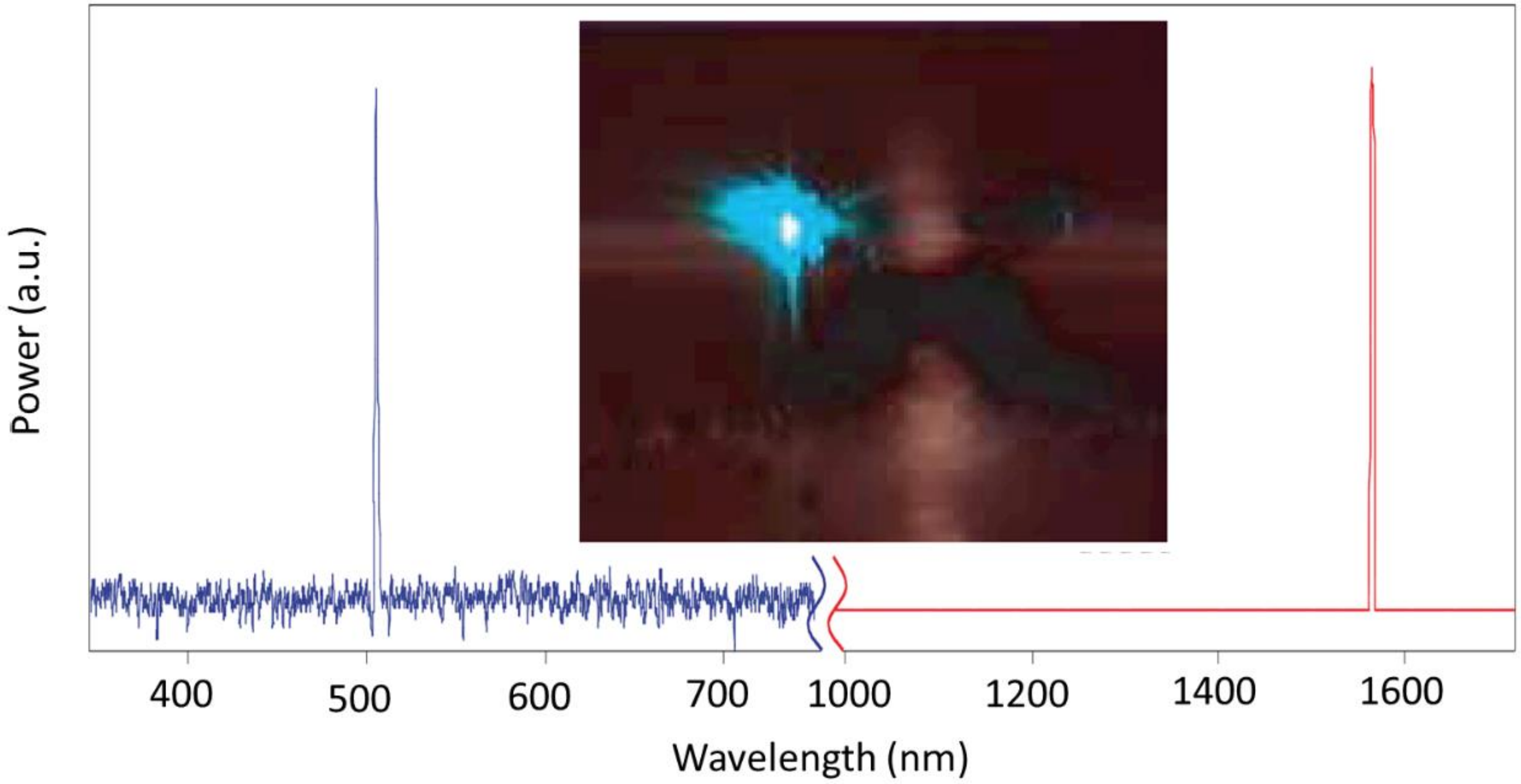
$$\frac{\delta\omega}{\omega} = -\frac{1}{n}\frac{\mathrm{dn}}{\mathrm{dT}}\sqrt{\langle \delta T^2 \rangle}$$ can reach $\sim 10^{-9}$

e.g. for a chip integrated Si3N4 microresonator expect **resonance frequency fluctuation of up to 100 kHz** which can be almost **1% of the resonance width**.

FSR of a 1 THz resonator can change by 1 kHz, impacting repetition rate of soliton combs.

Third harmonic generation

Carmon et al. Nat. Phys. (2007)



$$\frac{\partial}{\partial \tau} a_{3\mu} = -(1 + i\zeta_{3\mu})a_{3\mu} + 2iga_{\mu}a_{\mu}^{*}\, a_{3\mu} + iga_{\mu}a_{\mu}\, a_{\mu}$$

- Huge detuning for fundamental (Phase mismatch)
- Process only possible with higher order mode

Stimulated Raman Scattering (SRS)

- SRS gain is broadband, can easily overlap with a resonance.
- Especially relevant in normal dispersion regime (no MI) and in certain materials (e.g. diamond)

From lecture on stimulated Raman scattering:

$$\frac{\partial I_s}{\partial z} = g_R I_p I_s \quad \Rightarrow \quad \frac{\partial I_s}{\partial t} = g_R I_p I_s \frac{c}{n}$$

Loss:

$$\frac{\partial I_s}{\partial t} = -\frac{\kappa_s}{2} I_s$$

Raman gain > loss when:

$$g_R I_p \frac{c}{n} > \frac{\kappa_s}{2}$$

Intensity enhancement:

$$I_p = \frac{F_p}{\pi} I_{p0} = \frac{2c}{nL\kappa_p} I_{p0}$$

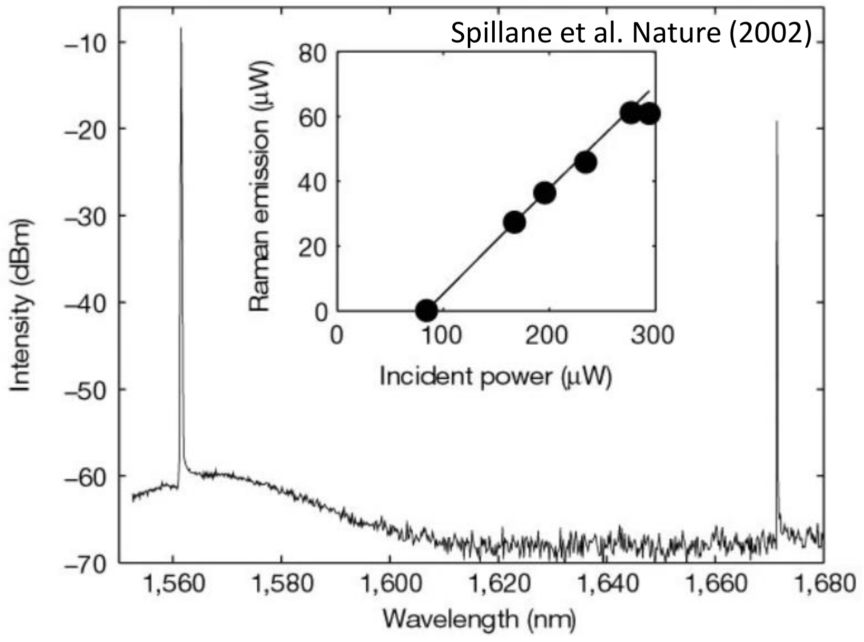
Threshold pump intensity:

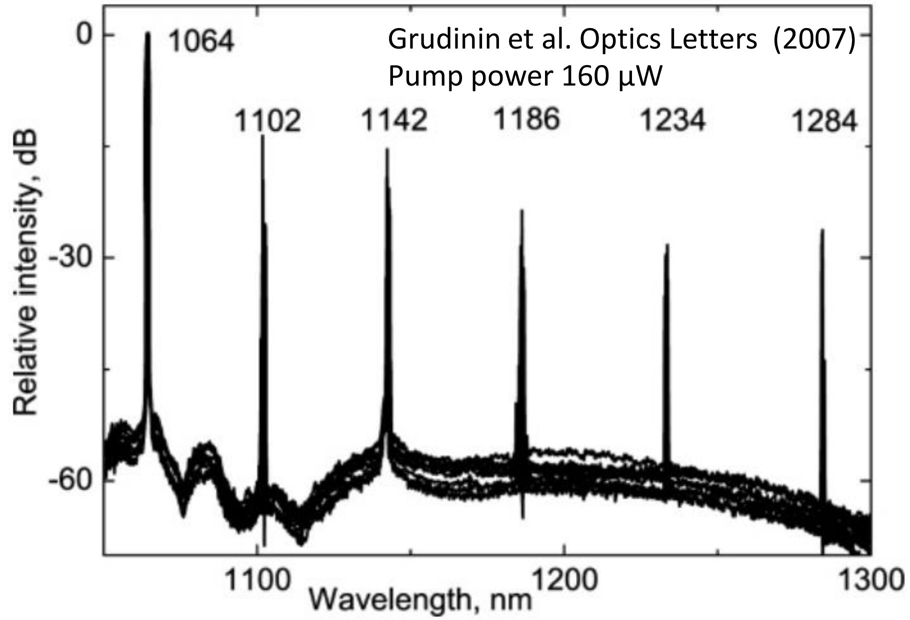
$$I_{p0} > \frac{\kappa_s \kappa_p L n^2}{4 g_R c^2}$$

Threshold pump power:

$$P_{p0} > \frac{\kappa_s \kappa_p V_{\text{eff}} n^2}{4 g_R c^2}$$

L: length of cavity,
$V_{\text{eff}} = A_{\text{eff}} \, L$: effective mode volume





Stimulated Brillouin Scattering (SBS)

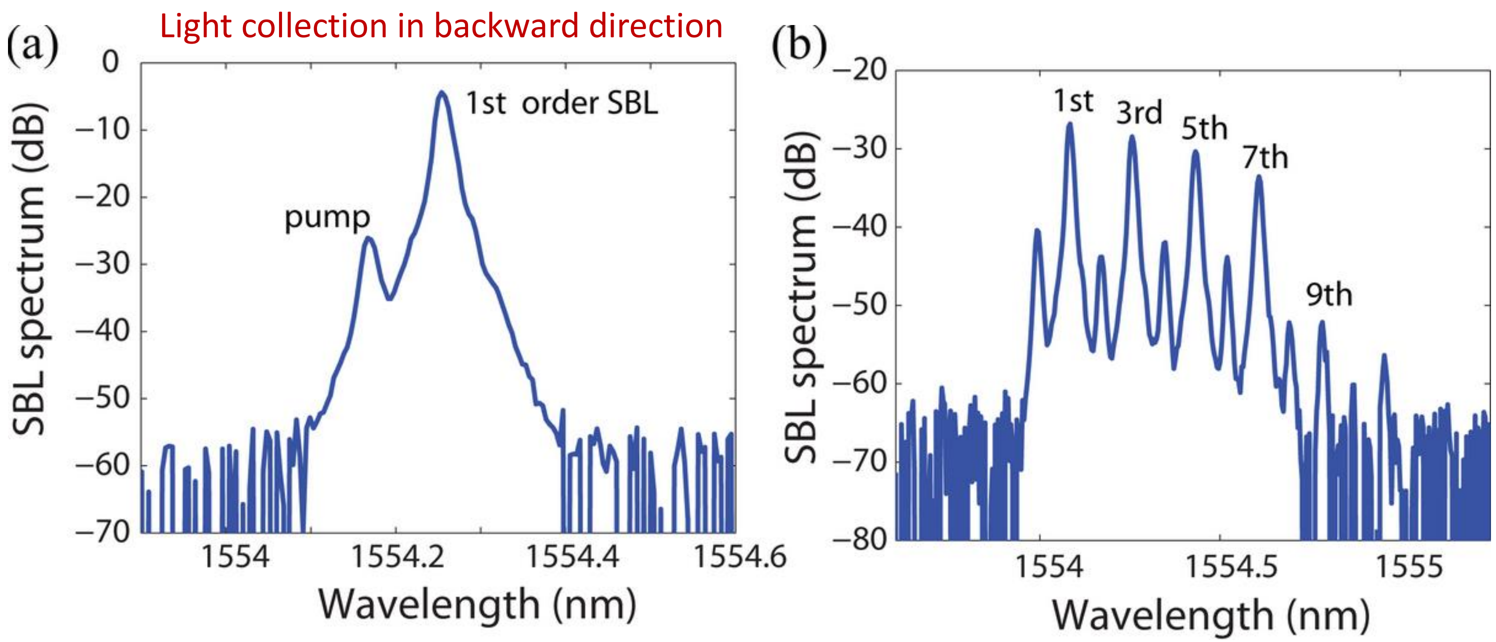
From lecture on stimulated Brillouin scattering:

$$\frac{\partial I_s}{\partial z} = g_B I_p I_s \quad \Rightarrow \quad \frac{\partial I_s}{\partial t} = g_B I_p I_s \frac{c}{n}$$

Threshold pump power:

$$P_{p0} > \frac{\kappa_s \kappa_p V_{\text{eff}} n^2}{4 g_B c^2}$$

- SBS gain is narrowband and can only be resonant when FSR ca. 10 GHz
- If resonant, SBS will dominate over MI/FWM and SRS (2 orders of magnitude larger gain)



Li Optics Express (2012)

Mixed effects

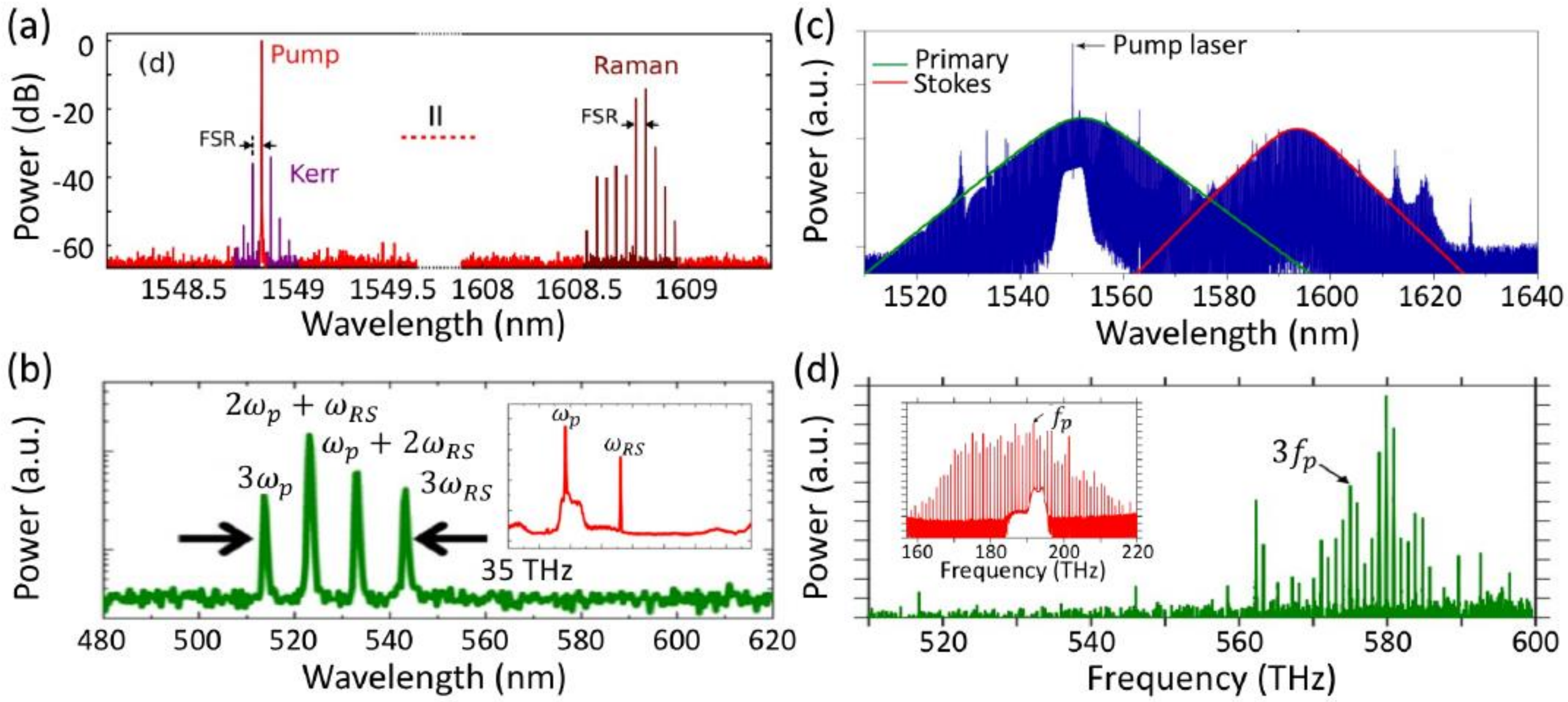


Figure from Yang et al. https://arxiv.org/abs/1809.04878

SHG in microresonators with $\chi^{(2)}$-nonlinearity

Quasi-phase matching | Non-critical phase matching | Cyclic phase matching

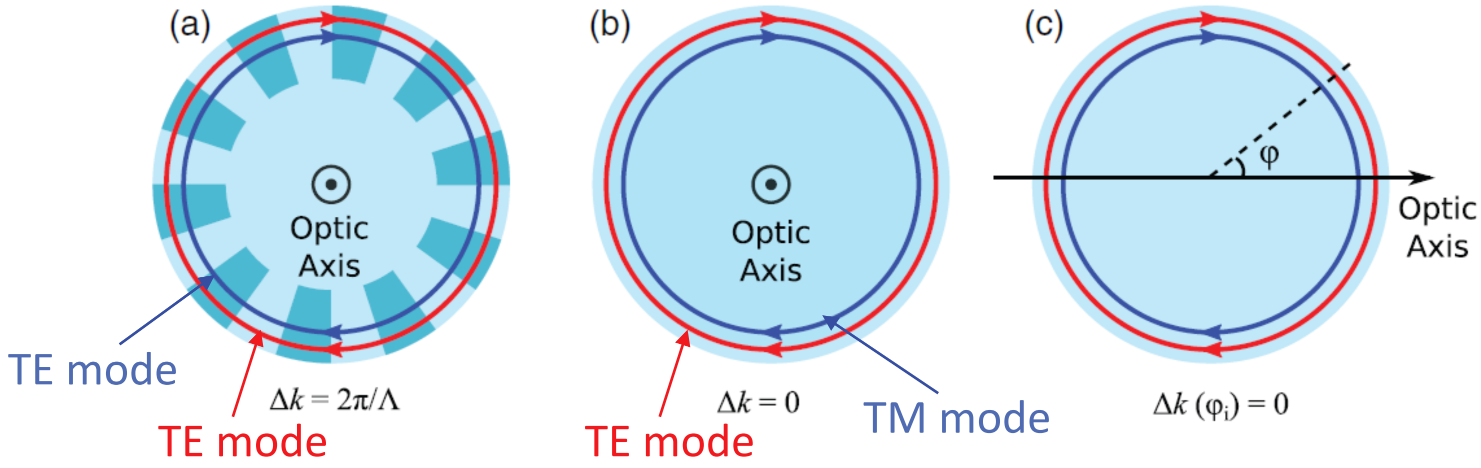


Figure from Lin/Chembo et al. Adv. In Opt. Phot. (2017)

Modal phase matching

higher order mode sees lower effective refractive index

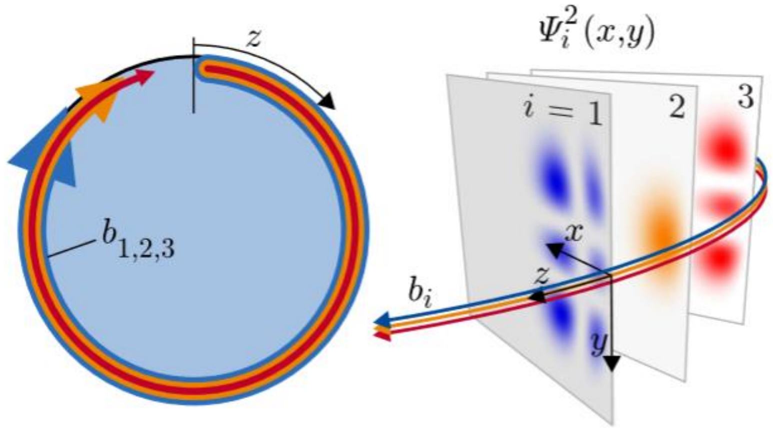
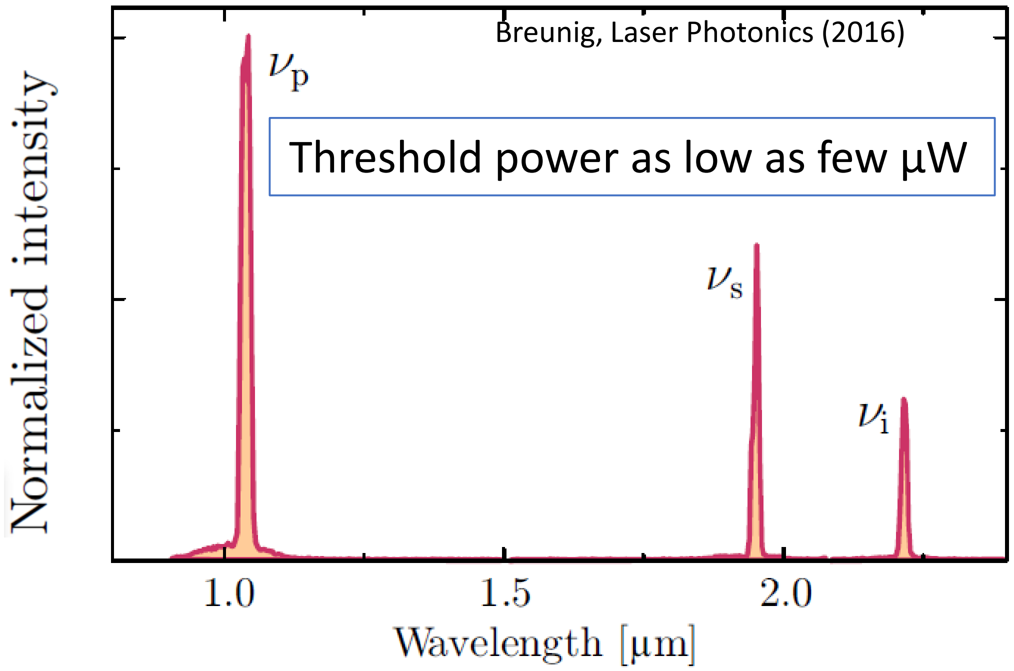
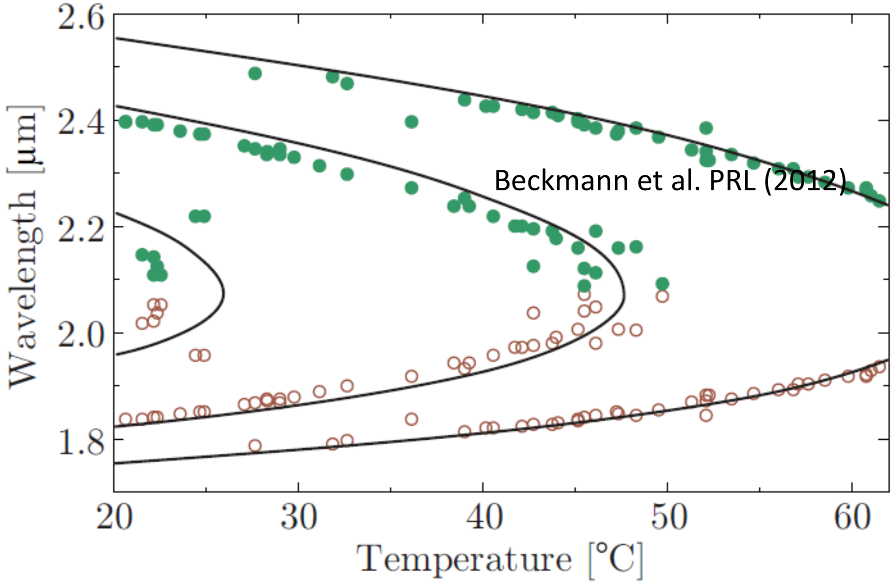


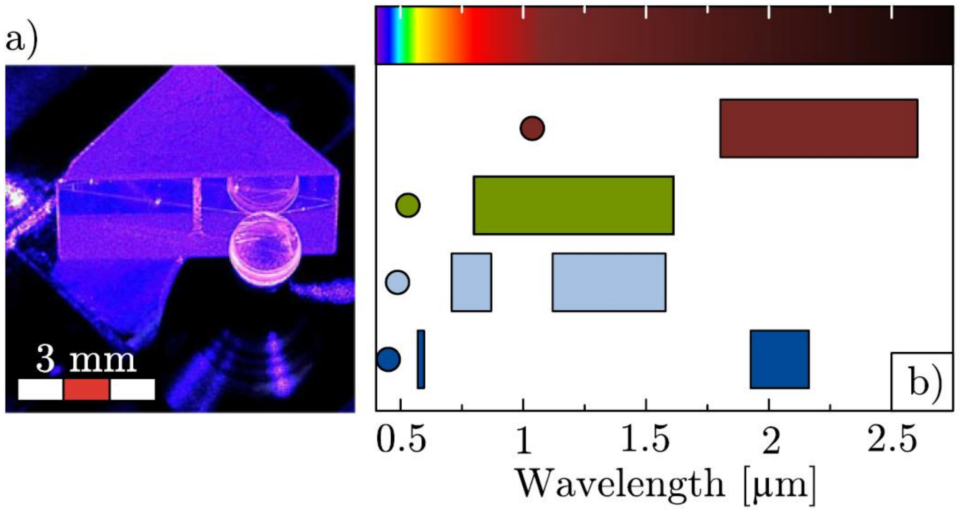
Figure from Breunig Laser Phot. Rev. (2016)

Conversion efficiency of > 1000%/W is achievable

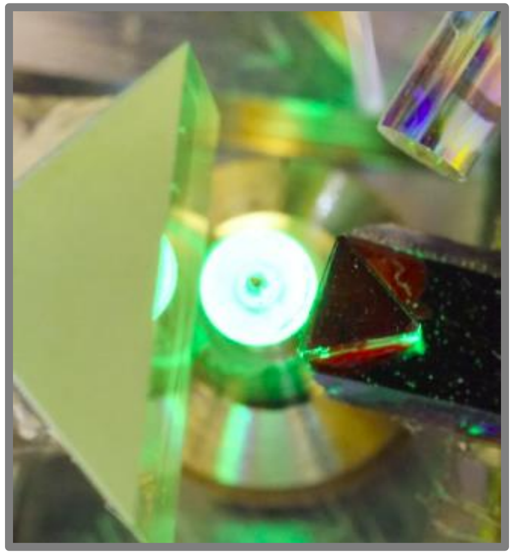
Optical parametric oscillators

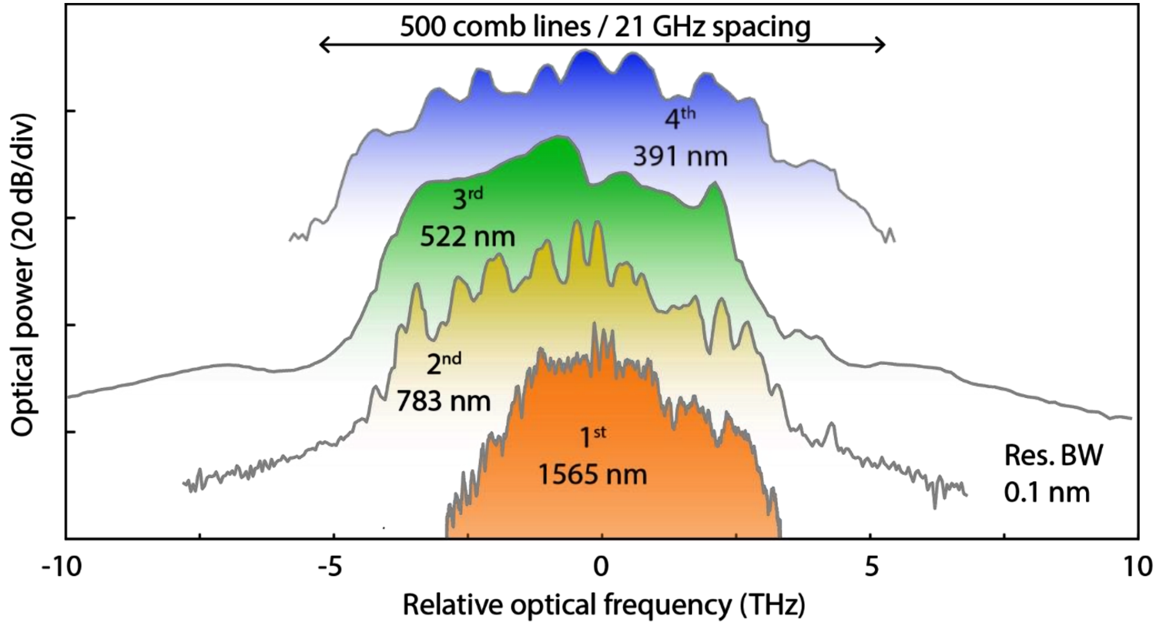


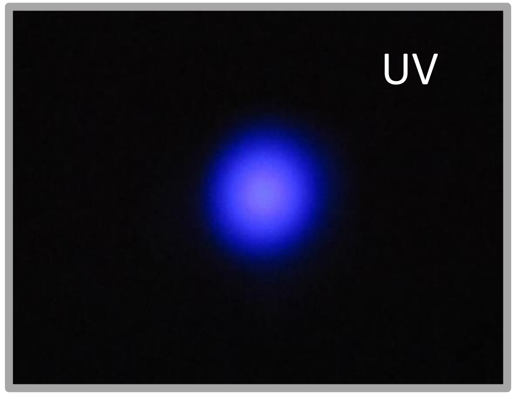


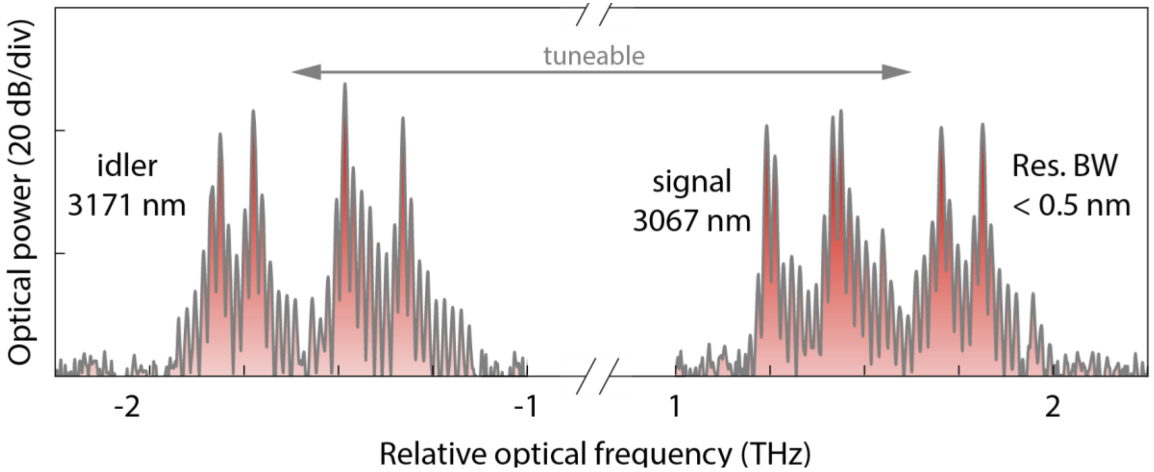


Up- and down conversion of a frequency comb









Summary: Nonlinear processes in microresonators

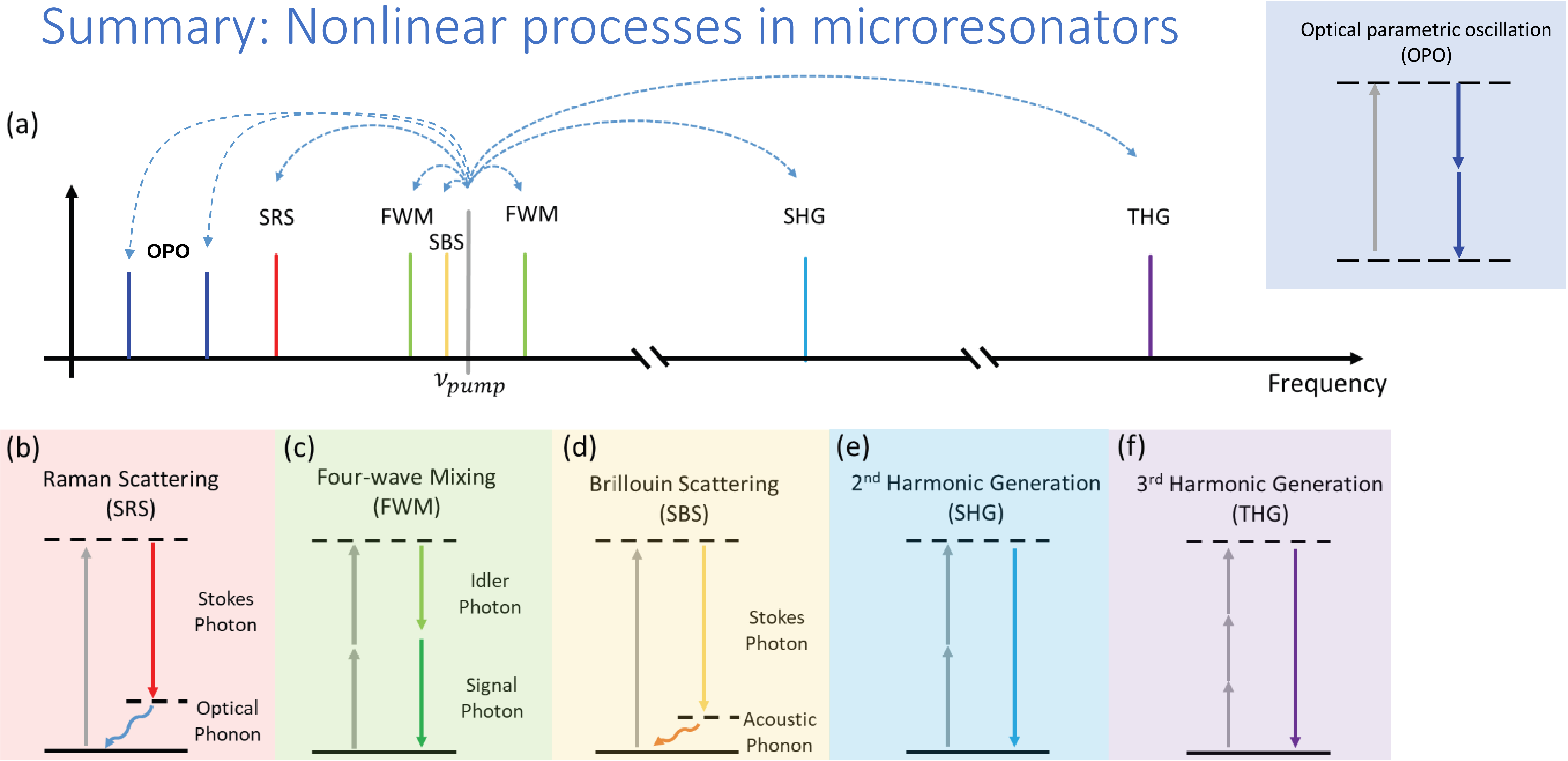


Figure from Yang et al. https://arxiv.org/abs/1809.04878