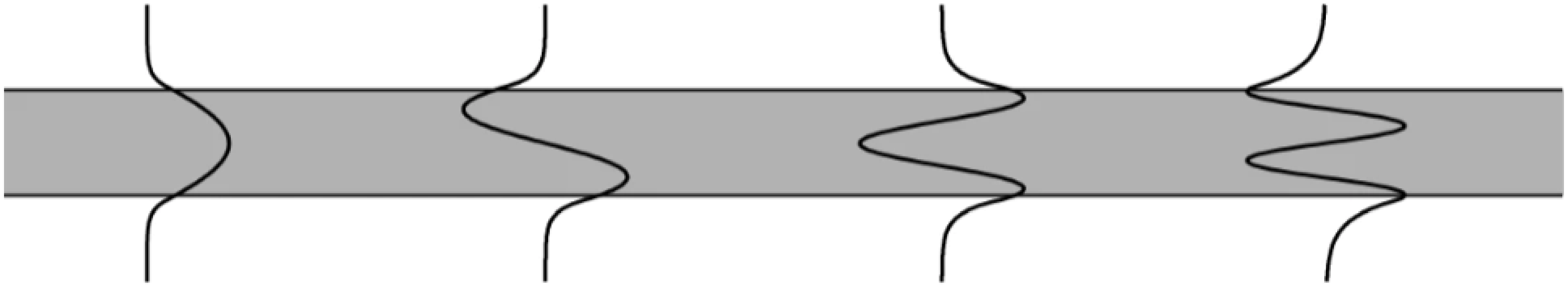
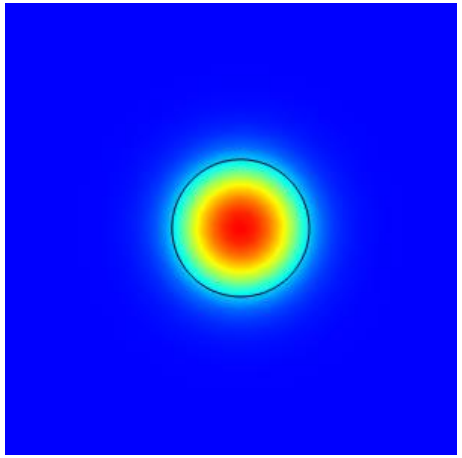
2021 Dec 08

NLO #17

- Dispersion engineering in fiber and wavegiudes (cont)
  - Photonic crystal fiber
  - Photonic crystal wavegiudes
  - Mode hybridization
- High-Q microresonators (linear regime)

Reminder Waveguides and fibers

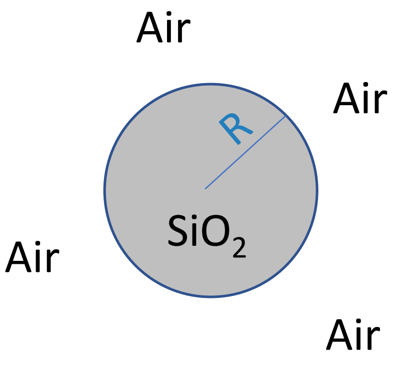
 

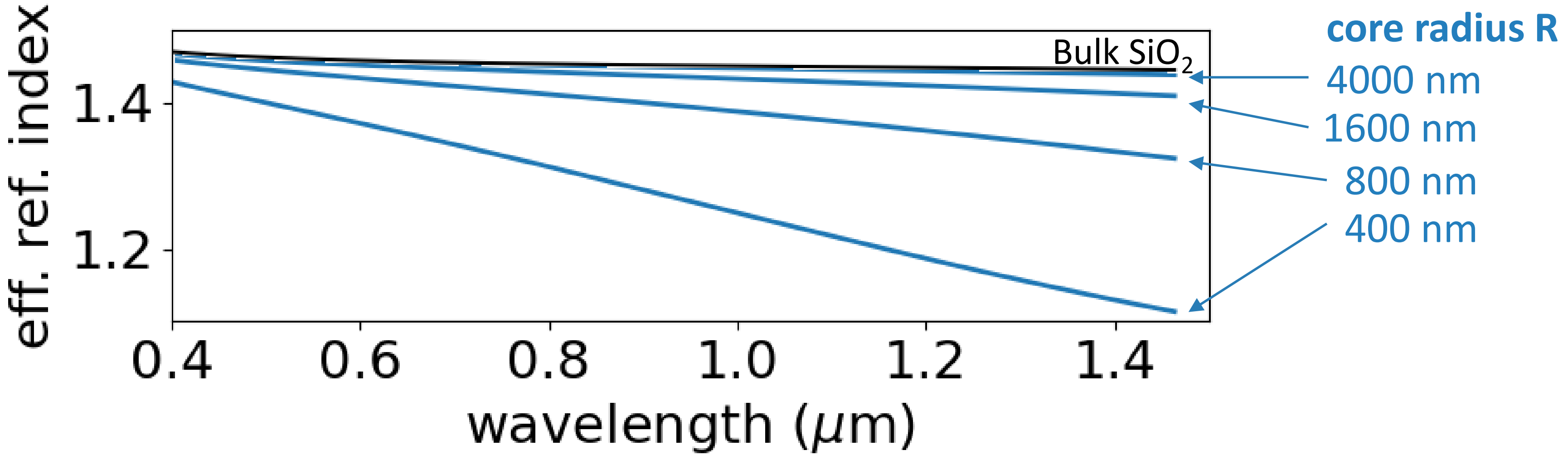
Guided mode a consequence of index contrast

High-index contrast and small core results in strong confinement

Group-velocity dispersion (GVD) can be modified e.g. by change of core size and index contrast

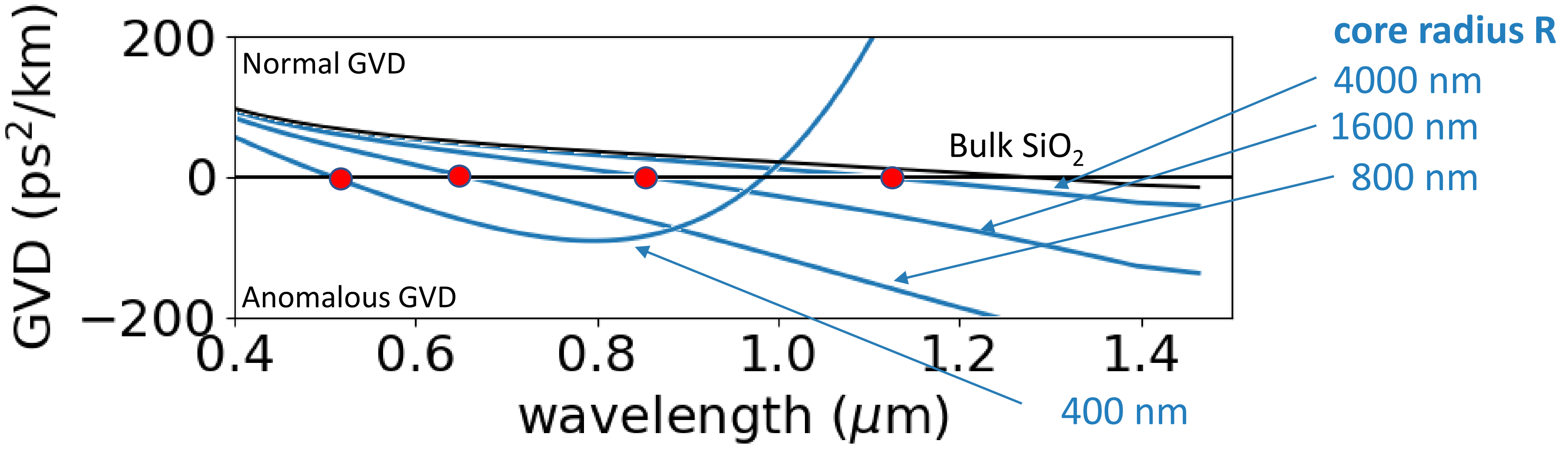
Air-clad dielectric rod

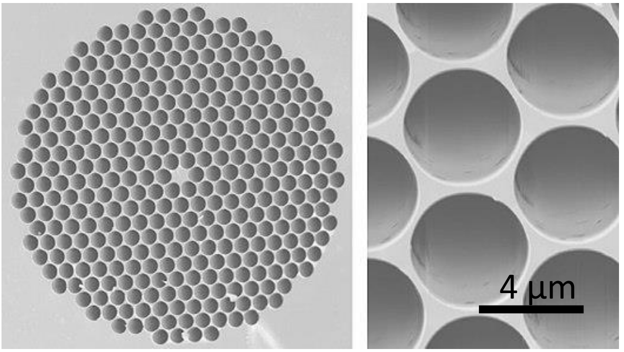




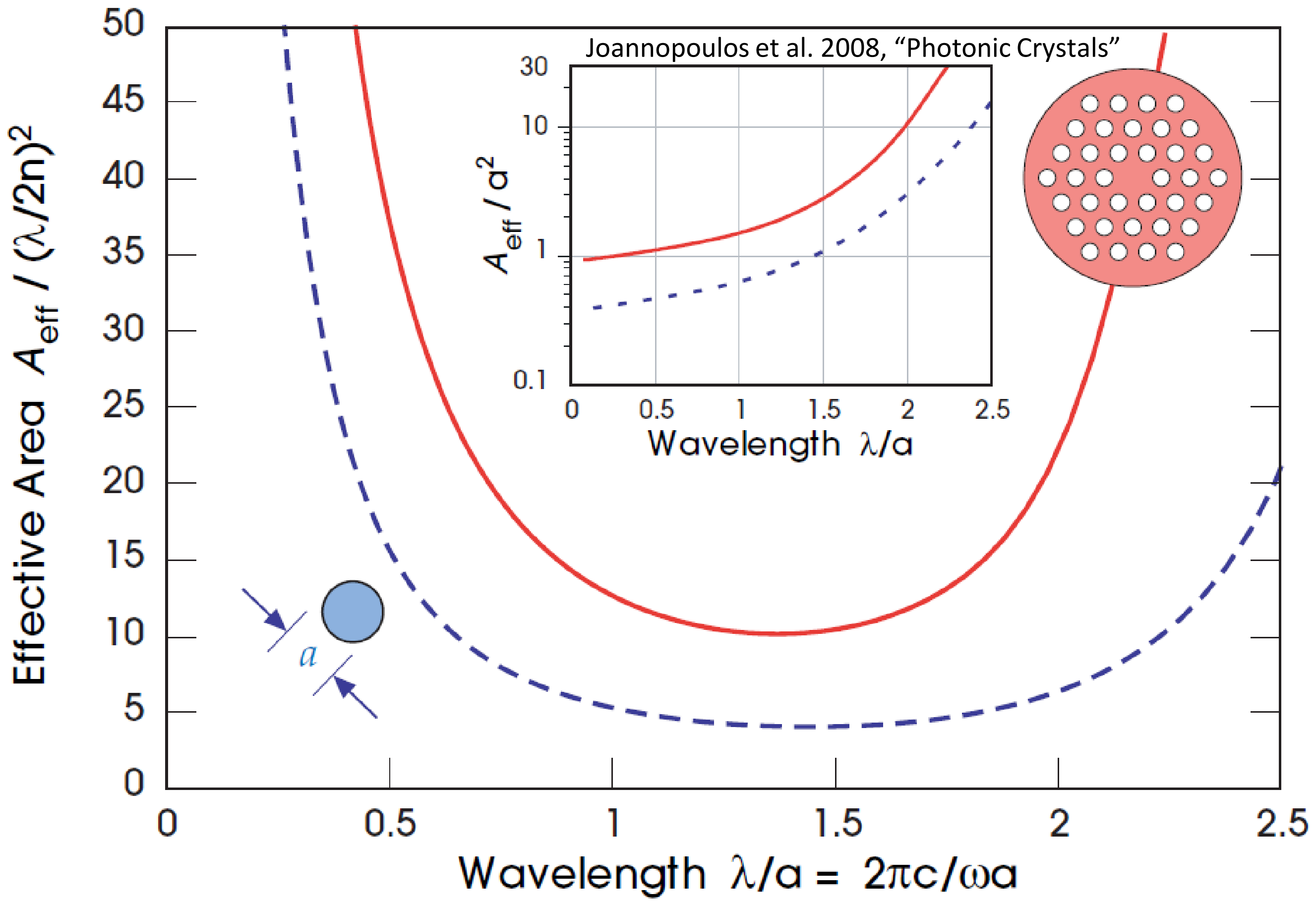
Need to suspend the fiber so that the mode is protected…

… **microstructured fiber**!

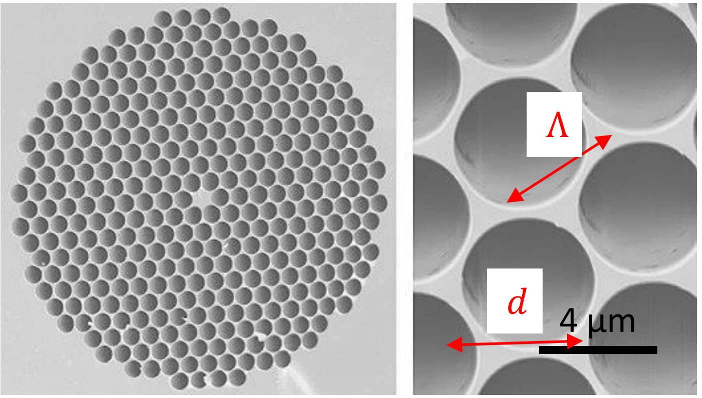




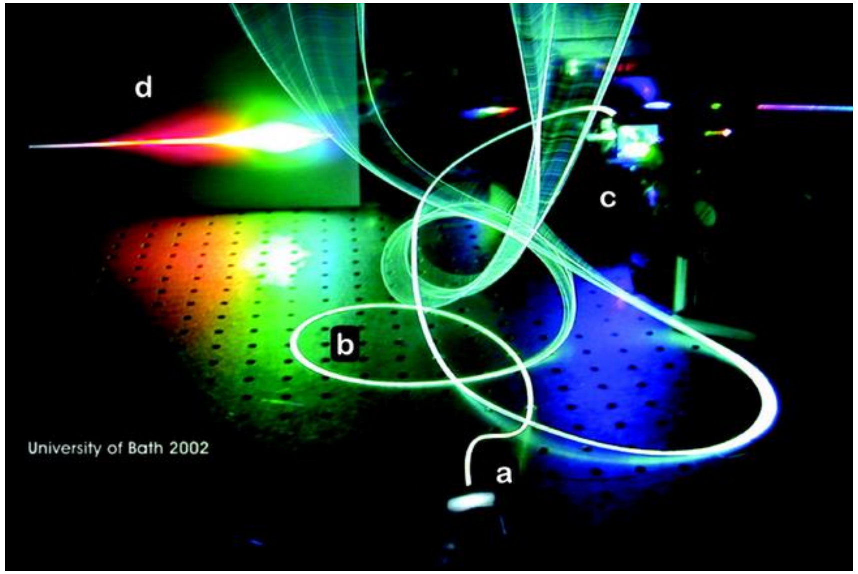
Comparison: “Dielectric Rod” and microstructured fiber



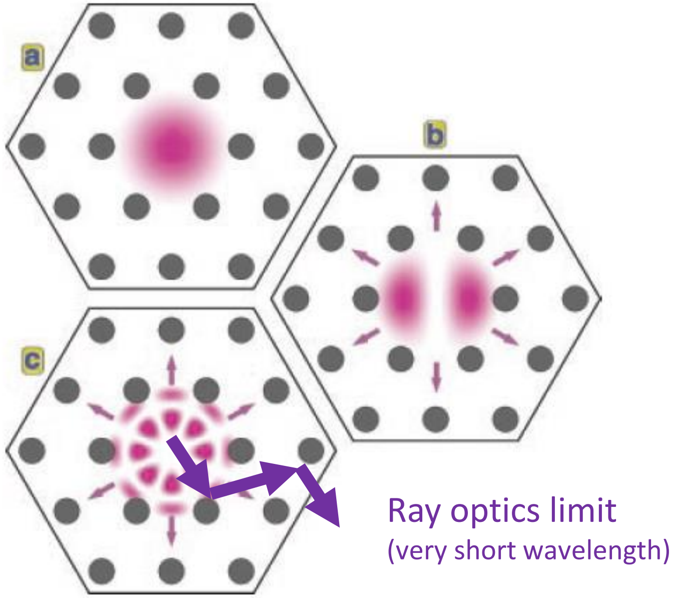
Microstructured photonic crystal fiber



- Large index contrast
- Modified internal refractive index guiding
- Can be made "endlessly single-mode"



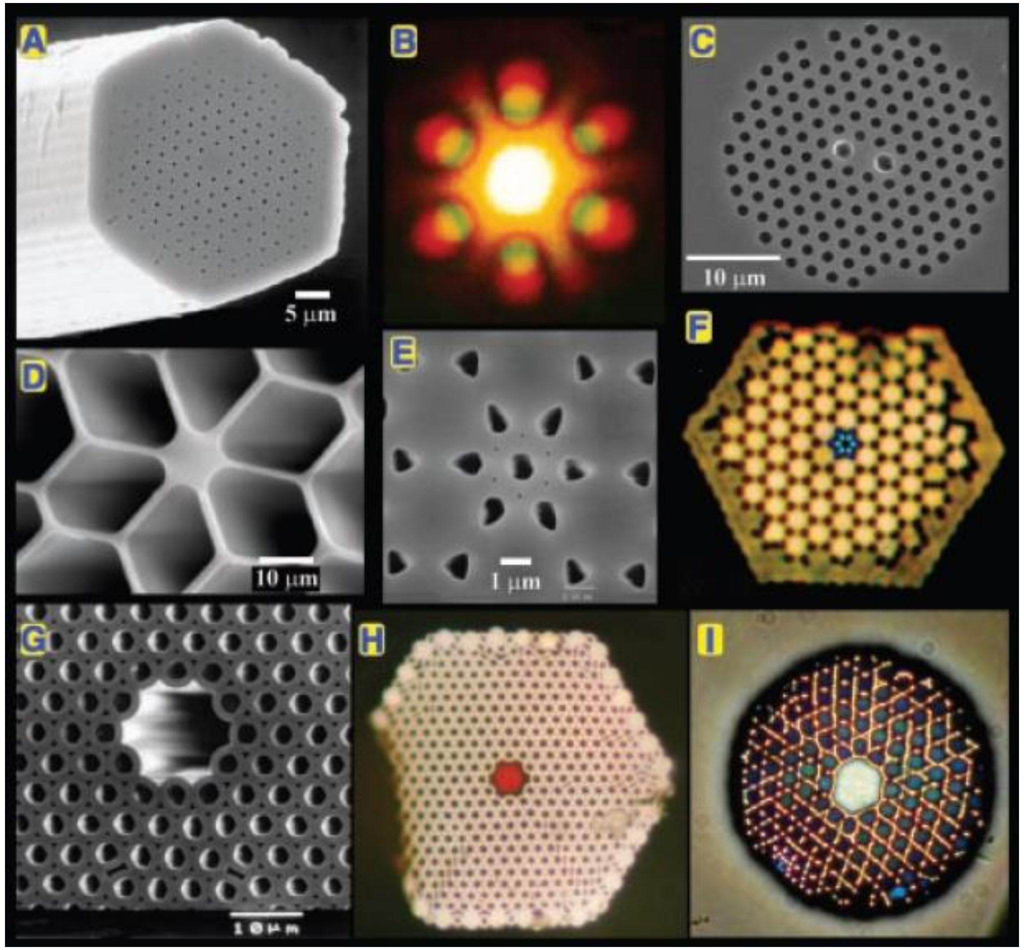
Russel, Science, 2003



Russel, Science, 2003

"Smaller features can escape"
Only the fundamental mode is guided.
(effective index contrast between core and air-hole cladding decreases for higher order modes, no more guiding)

Photonic crystal fiber



Russel, Science, 2003

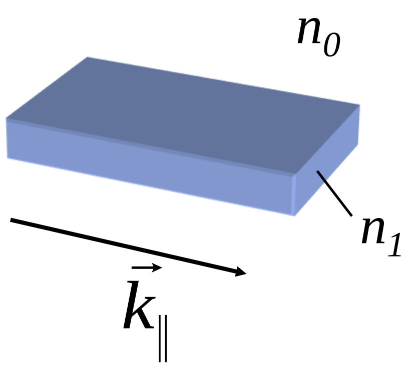
University of Southampton

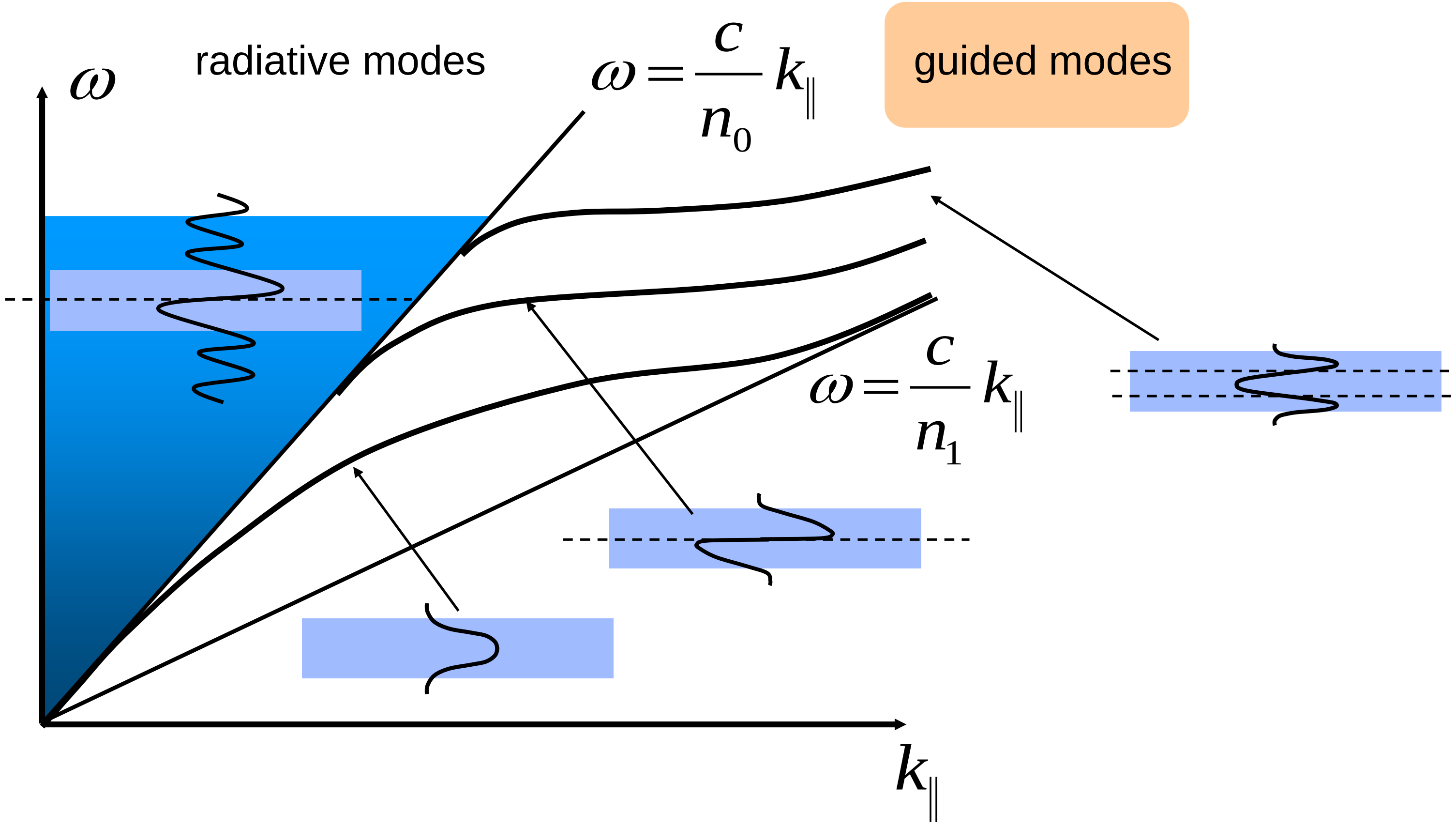
Photonic Crystal fiber (PCF)

Index contrast guiding

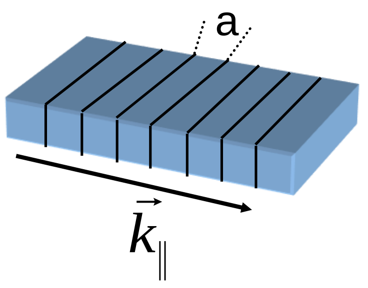
Photonic Bandgap guiding

Dispersion diagram





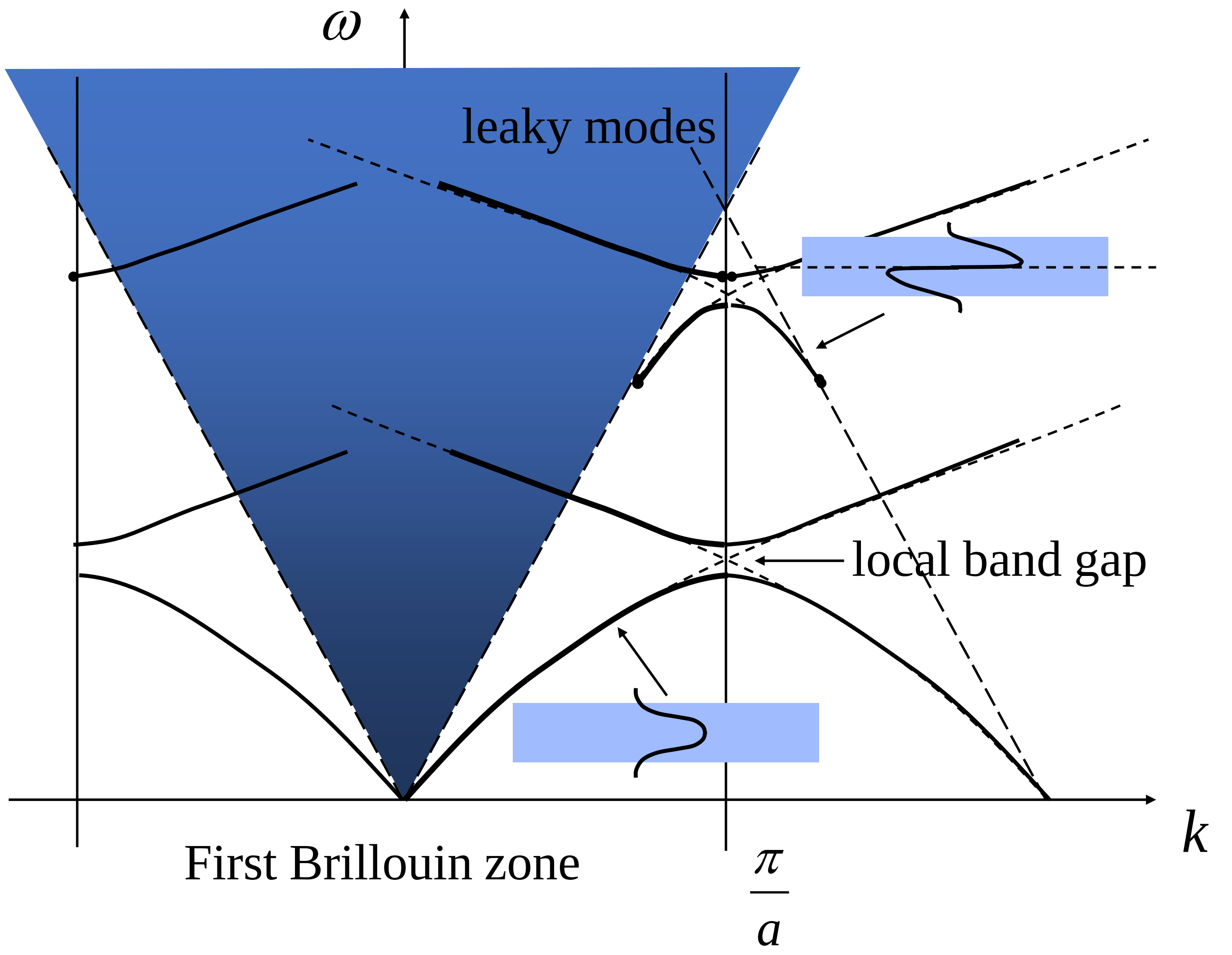
Photonic bandgap



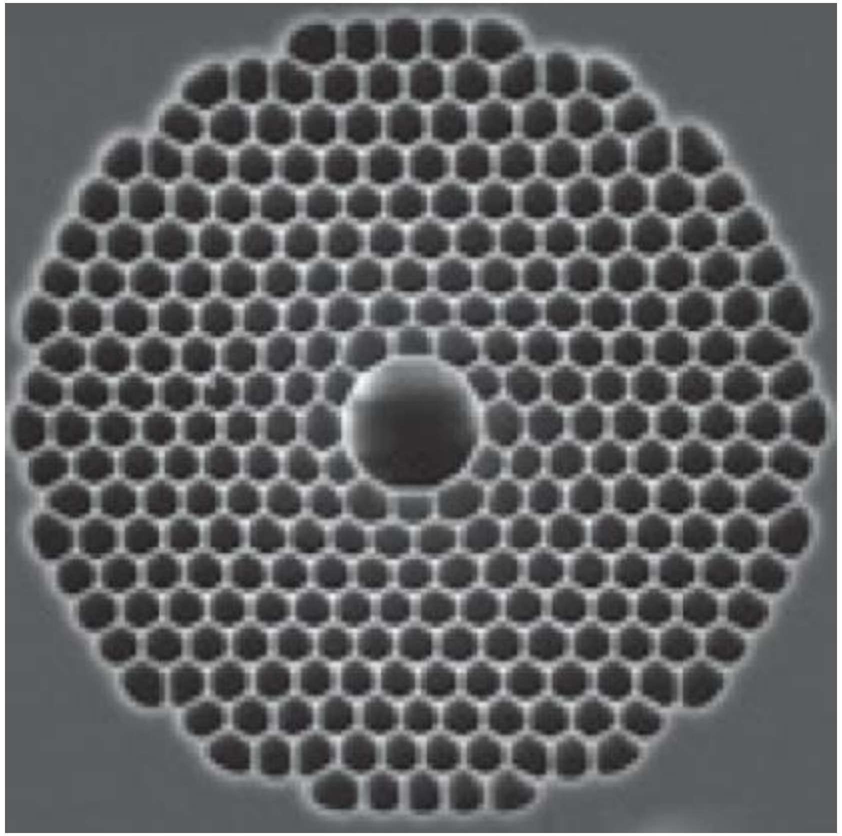
Bloch modes:

$$E(z) = u(z)\exp(ikz)$$

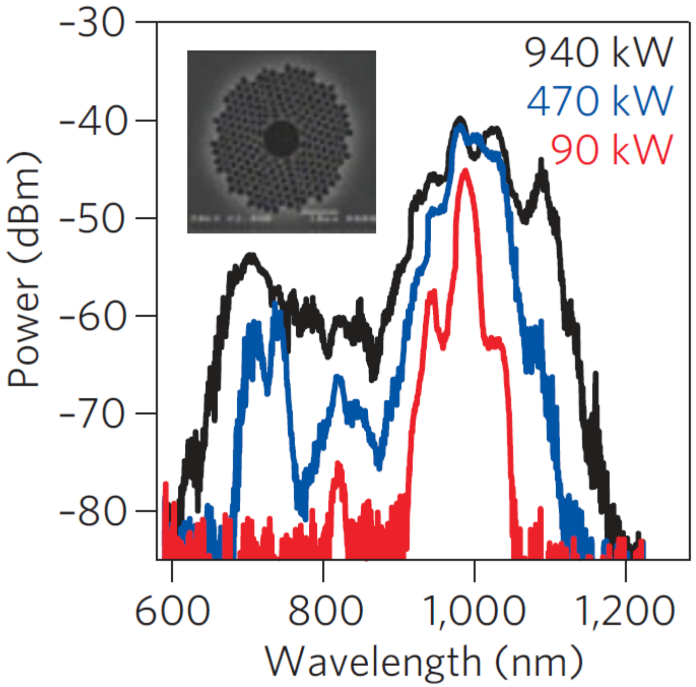
$$u(z+a) = u(z)$$



Photonic bandgap guidance



- Hollow-core and large mode area fibers can guide high power
- <u>Low</u> nonlinearity and low dispersion
- Gas or liquid filled for nonlinear and spectroscopic application

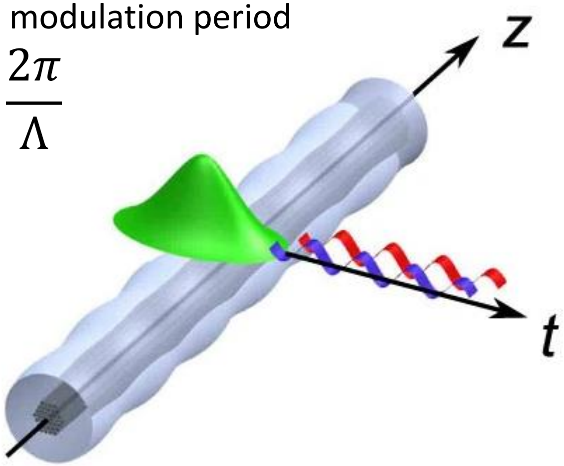


Bozolan et al. Optics Letters (2008)

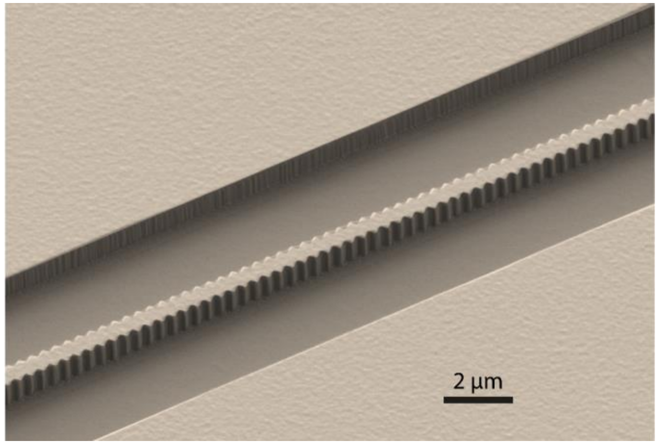
Modulated fibers/waveguides

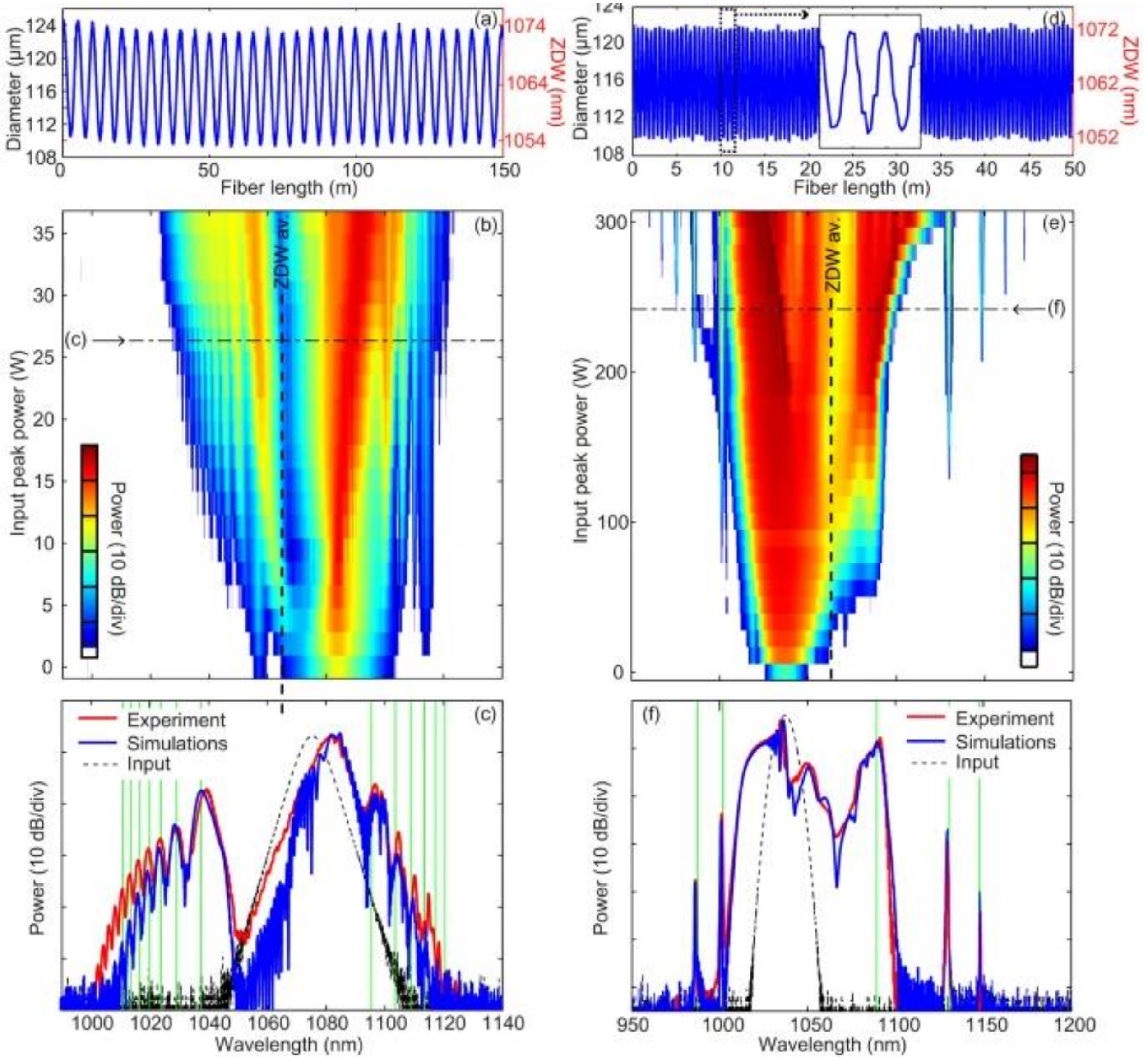
Λ spatial modulation period

$$\Delta k = \frac{2\pi}{\Lambda}$$



Quasi-phase matching through modulation of a fiber / waveguide parameter

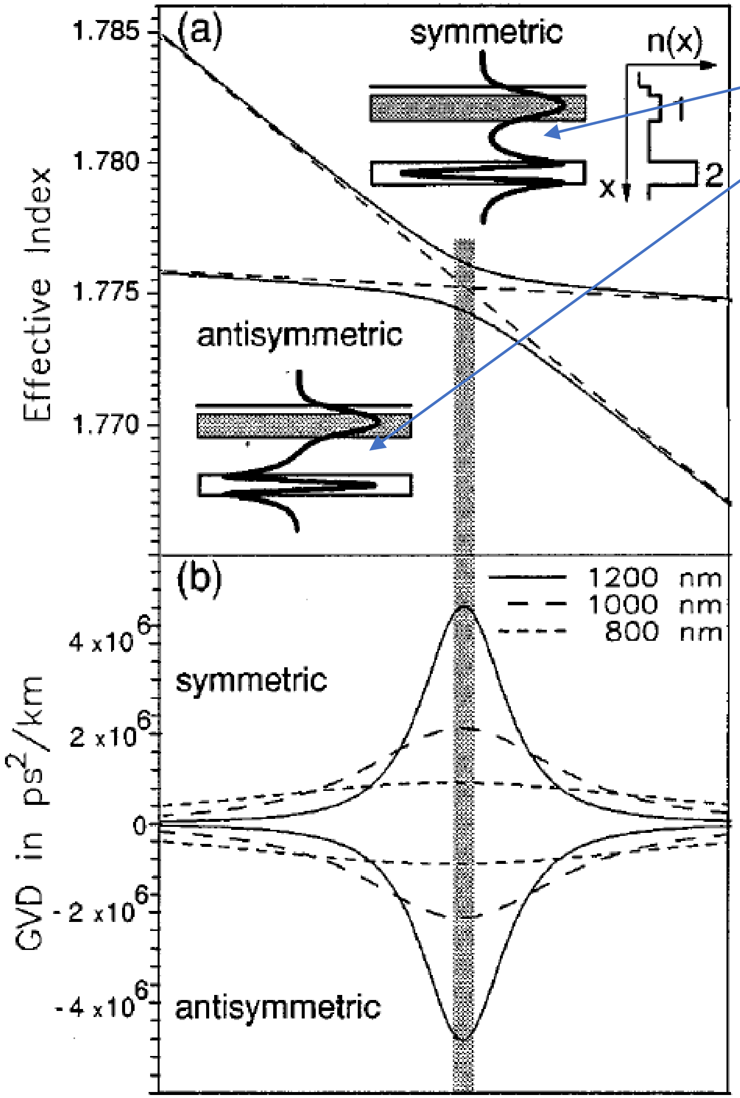




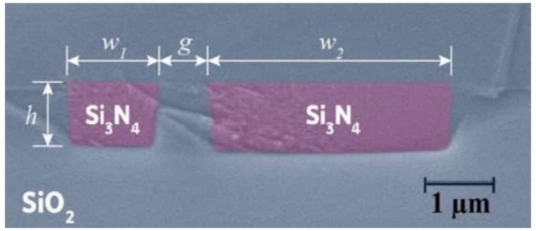
Conforti et al. Sci. Rep (2015)

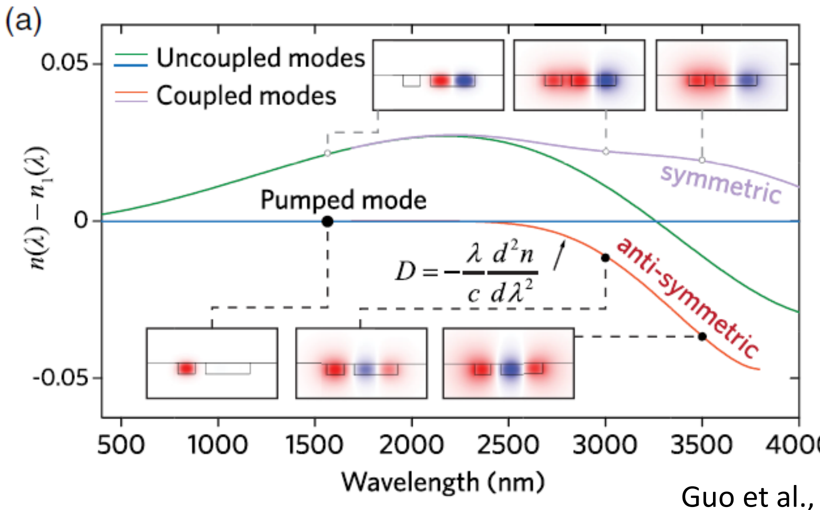
Mode-hybridization / Avoided-mode crossing

Supermodes

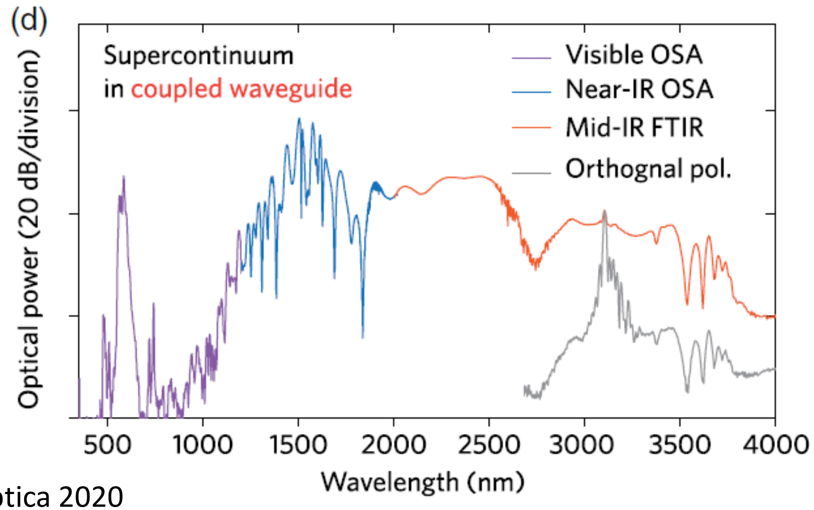


$$\beta^{(\pm)} = \frac{1}{2}(\beta_a + \beta_b) \pm \sqrt{\frac{1}{4}(\beta_a - \beta_b)^2 + \kappa^2}$$

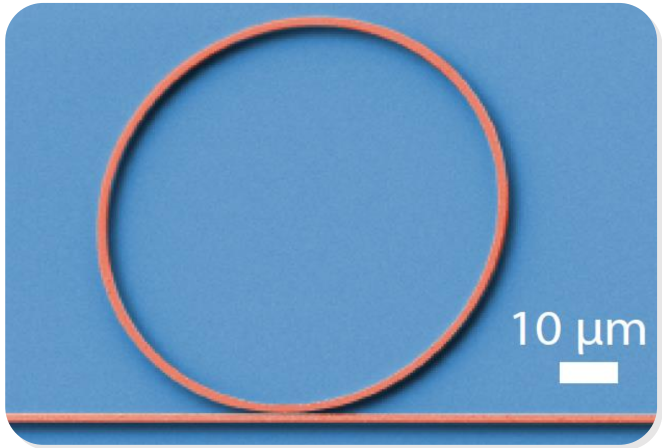




Guo et al., Optica 2020



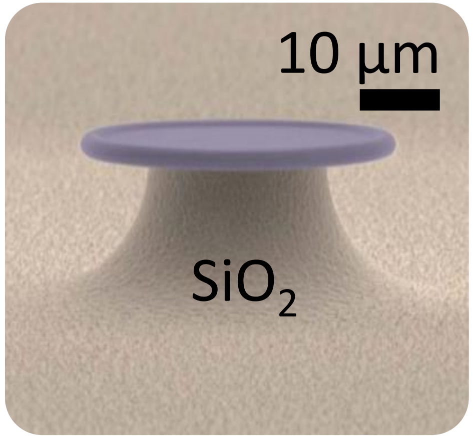
High-Q Microresonators

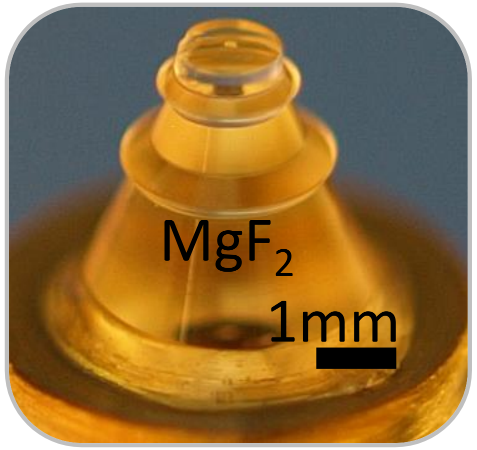


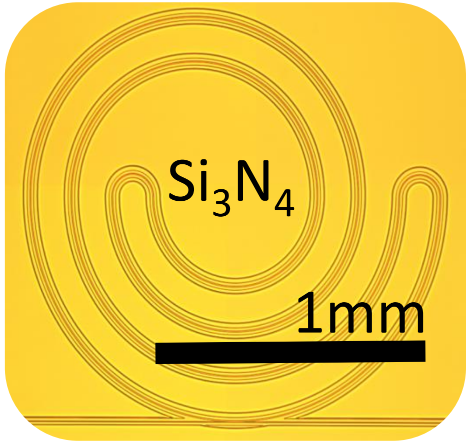
Free-spectral range (separation of 2 resonance frequencies)**:** 1 GHz to 1 THz

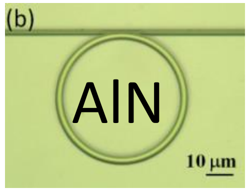
$$\mathrm{FSR} = \frac{c}{n_g L} = \frac{1}{T_R}$$

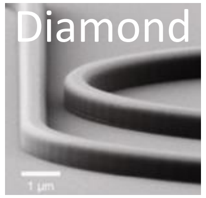
Power enhancement: 100x - 1’000’000x





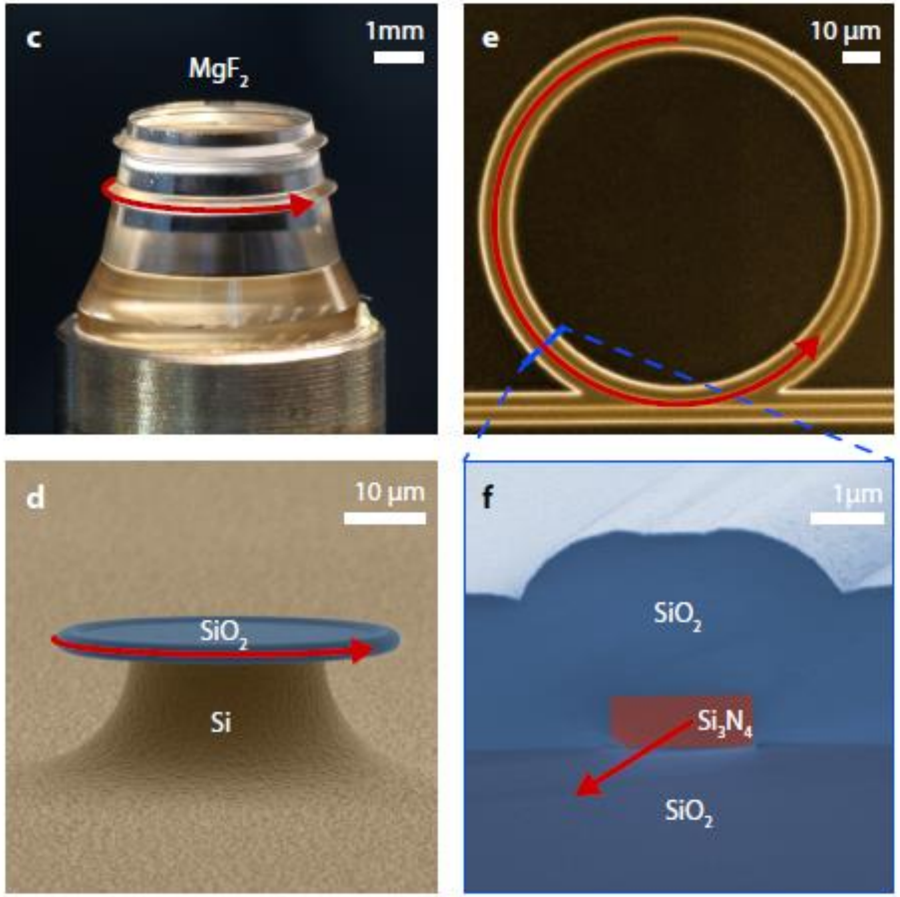




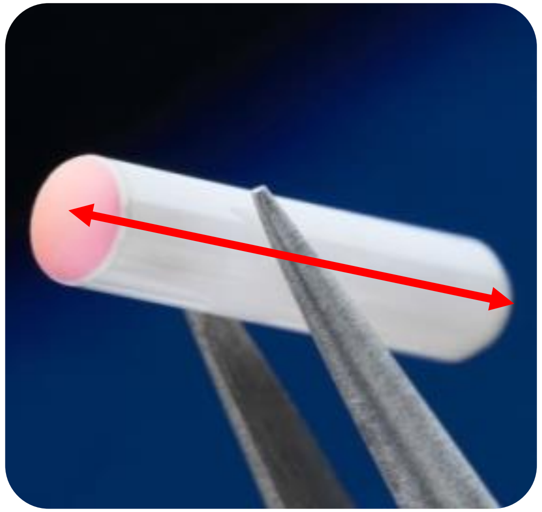


Travelling and Standing wave resonators

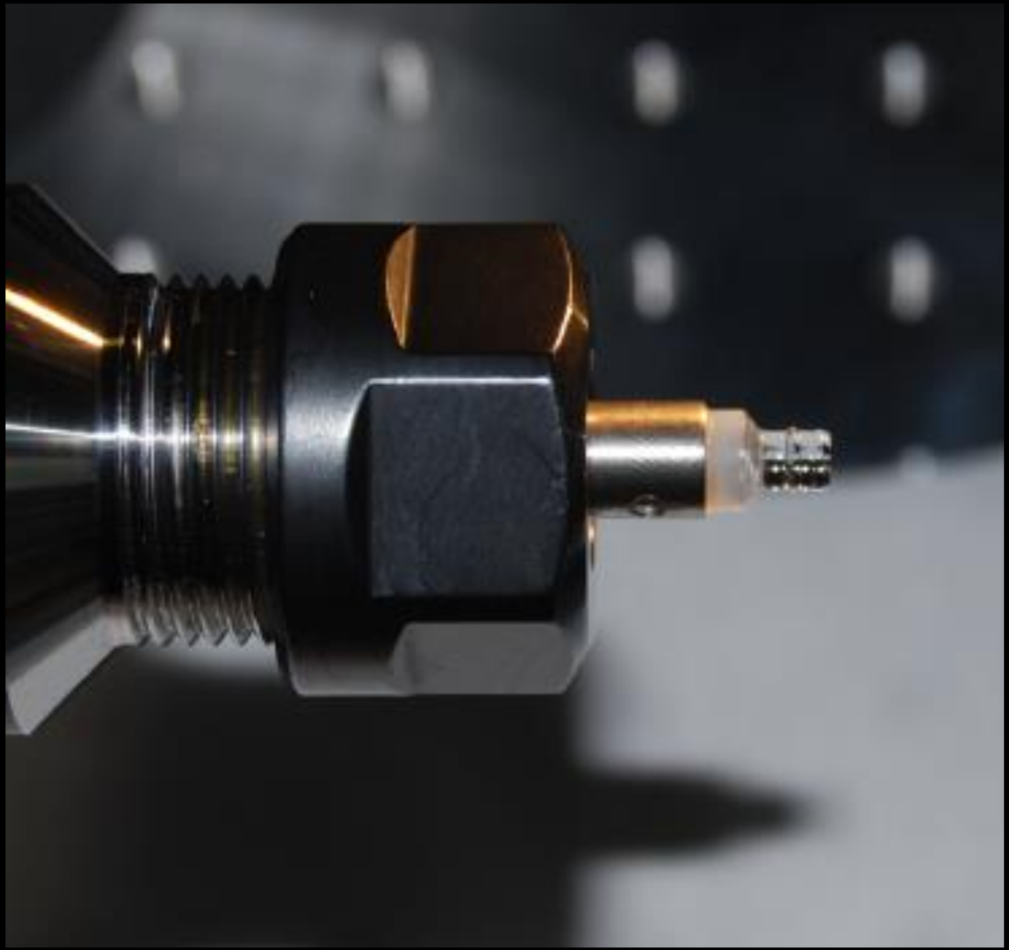
Travelling wave

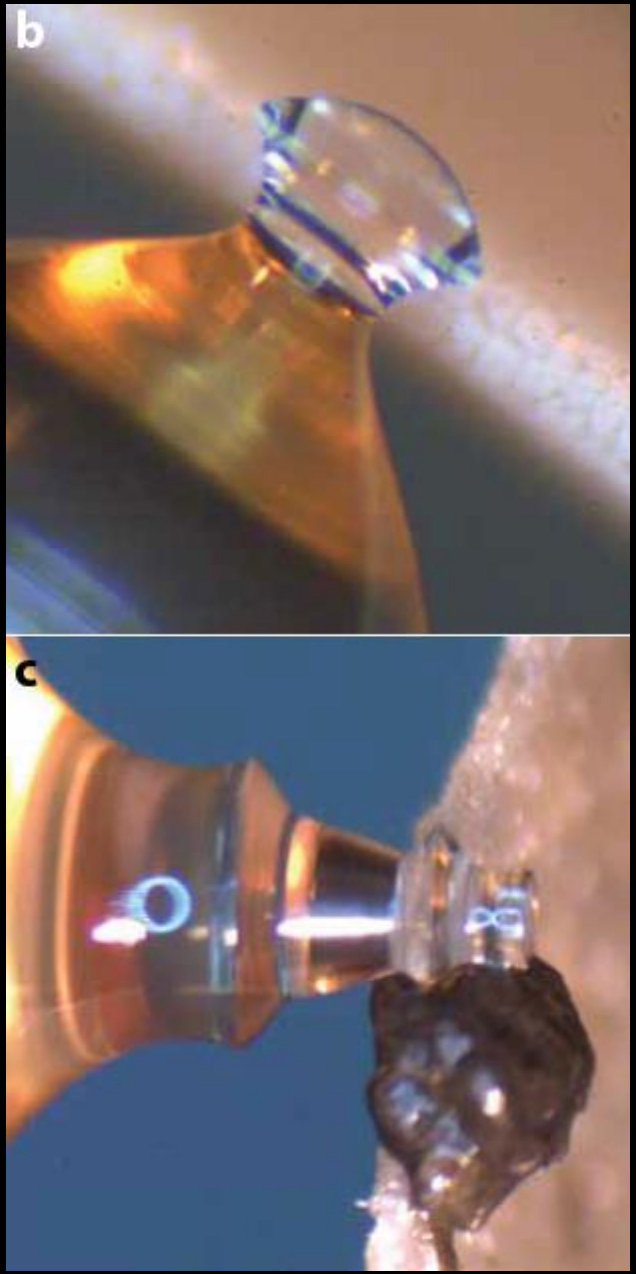


Standing wave



Achieving low loss





Can reach photon lifetime of > 10 µs

Coupling light to a resonator

$$\tilde{A} = A(t)\,\mathrm{e}^{-i\omega_0 t}$$ ($|A|^2$ is number of photons in cavity)

Time evolution:

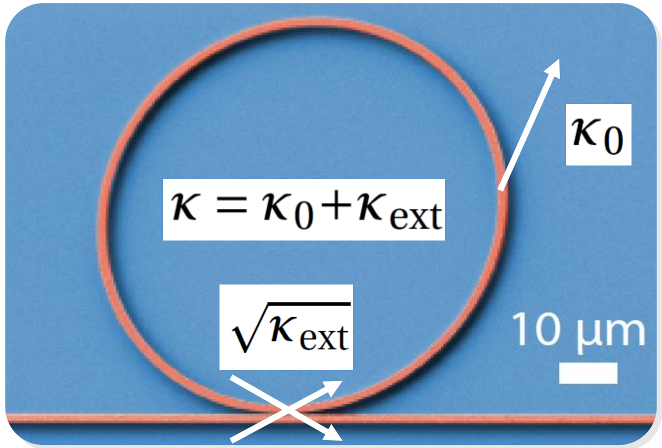
$$\frac{\mathrm{d}A(t)}{\mathrm{d}t} = -\frac{\kappa}{2}A(t) + \sqrt{\kappa_{\mathrm{ext}}}\,s_{\mathrm{in}}(t)\,\mathrm{e}^{-i(\omega_{\mathrm{p}}-\omega_0)t}$$

Quality-factor

$$\boxed{Q = \omega/\kappa = \omega\tau}$$

($|s_{\mathrm{in}}|^2$ is number of photons per second)

$s_{\mathrm{in}}(t)$



Transform into rotating frame of pump:

$$a = A\,e^{i(\omega_{\mathrm{p}}-\omega_0)t} \longrightarrow \frac{\mathrm{d}a(t)}{\mathrm{d}t} = -\left(i(\omega_0-\omega_{\mathrm{p}}) + \frac{\kappa}{2}\right)a(t) + \sqrt{\kappa_{\mathrm{ext}}}\,s_{\mathrm{in}}(t)$$

Assume continuous wave laser:

$$s_{\mathrm{in}}(t) = s_{\mathrm{in}} \xrightarrow[\text{Steady-state}]{} a = \frac{\sqrt{\kappa_{\mathrm{ext}}}}{i(\omega_0-\omega_{\mathrm{p}}) + (\frac{\kappa}{2})}\cdot s_{\mathrm{in}} \longrightarrow$$

Steady-state number of photons in cavity

$$\boxed{|a|^2 = \frac{\kappa_{\mathrm{ext}}}{(\omega_0-\omega_{\mathrm{p}})^2 + (\frac{\kappa}{2})^2}\cdot |s_{\mathrm{in}}|^2}$$

Coupling light to a resonator

from previous page

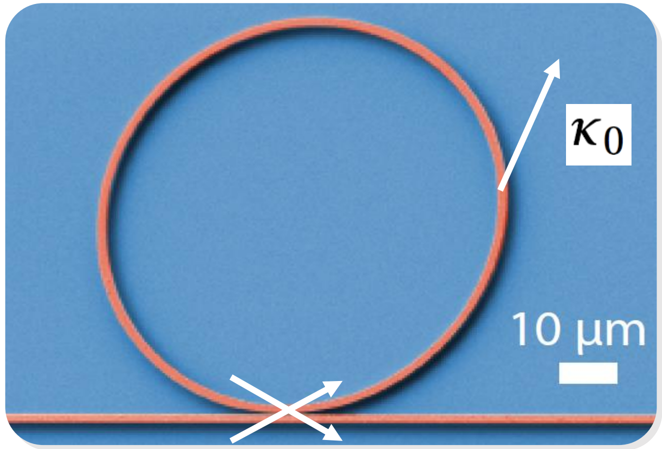
$$|a|^2 = \frac{\kappa_{\text{ext}}}{(\omega_0 - \omega_\text{p})^2 + (\frac{\kappa}{2})^2} \cdot |s_\text{in}|^2$$

Finesse

$$\boxed{F = \frac{\text{FSR}}{\kappa/2\pi}}$$

($|s_\text{in}|^2$ is number of photons per second)

$s_\text{in}(t)$



$$P_\text{cav} = \text{FSR} \cdot \frac{\kappa_\text{ext}}{(\omega_0 - \omega_\text{p})^2 + (\frac{\kappa}{2})^2} \cdot P_\text{in} = 2\eta \cdot \frac{F}{\pi} \cdot \frac{1}{\left(\frac{4(\omega_0-\omega_\text{p})^2}{\kappa^2}\right) + 1} \cdot P_\text{in}$$

Coupling ratio

$$\boxed{\eta = \kappa_\text{ext}/\kappa}$$

$$\boxed{P_\text{cav} = \frac{F}{\pi} \cdot P_\text{in}}$$

Power enhancement is proportional to Finesse

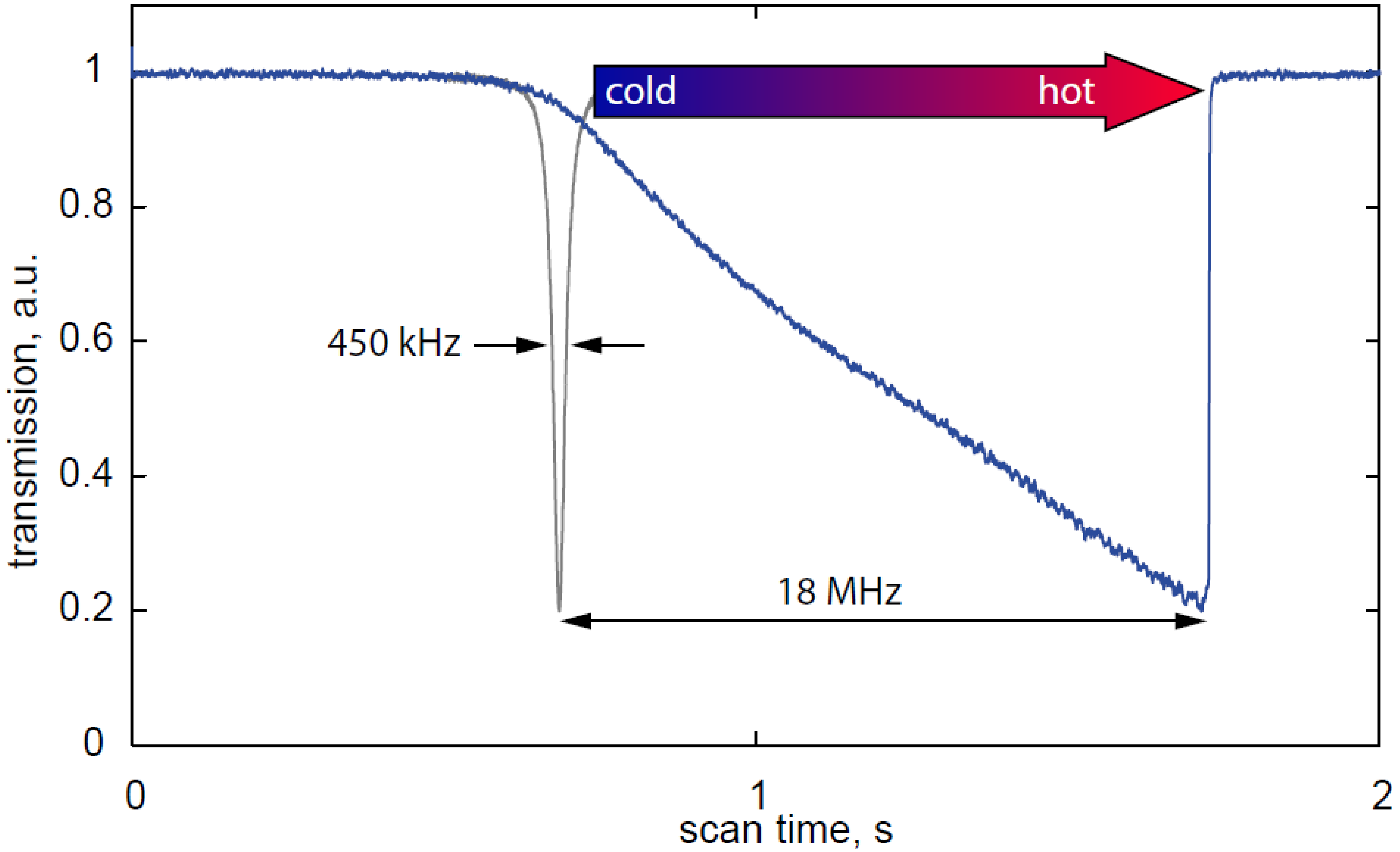
Coupling regimes

$\kappa_0 > \kappa_\text{ext}$	undercoupled
$\kappa_0 = \kappa_\text{ext}$	critically coupled
$\kappa_0 < \kappa_\text{ext}$	overcoupled

Thermal nonlinearity

$$-\frac{d\omega}{dT} = \frac{1}{n}\frac{dn}{dT} + \frac{1}{L}\frac{dL}{dT}$$

Absorption and heating leads to a intensity dependent refractive index and thermal expansion. This causes a **shift of the resonance frequency**



Nonlinear Microresonators

Polarization of the medium:

$$\tilde{P}(t) = \epsilon_0 \left[\chi^{(1)} \tilde{E}(t) + \chi^{(2)} \tilde{E}^2(t) + \chi^{(3)} \tilde{E}^3(t) + \cdots \right]$$