2021 Dec 06

NLO #16

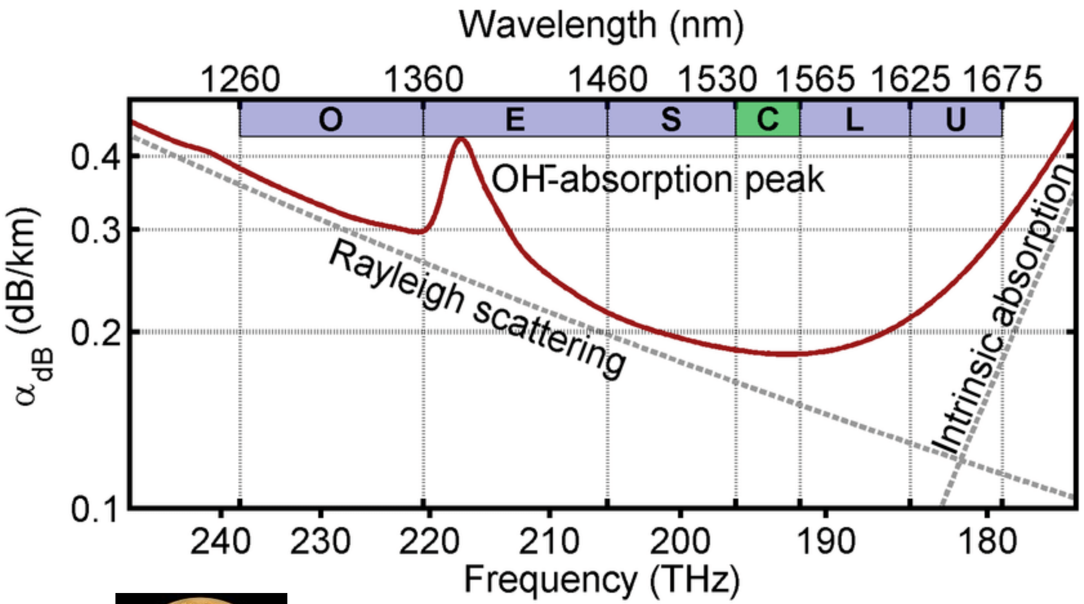
- Propagation of light through fibers and waveguides
  - Optical fibers
  - Planar Waveguide
  - Fiber modes
  - Higher order modes
  - Effective mode area
  - Finite element simulating
  - Dispersion and dispersion engineering in fiber
  - Integrated waveguides
  - Example: Supercontinuum in waveguides

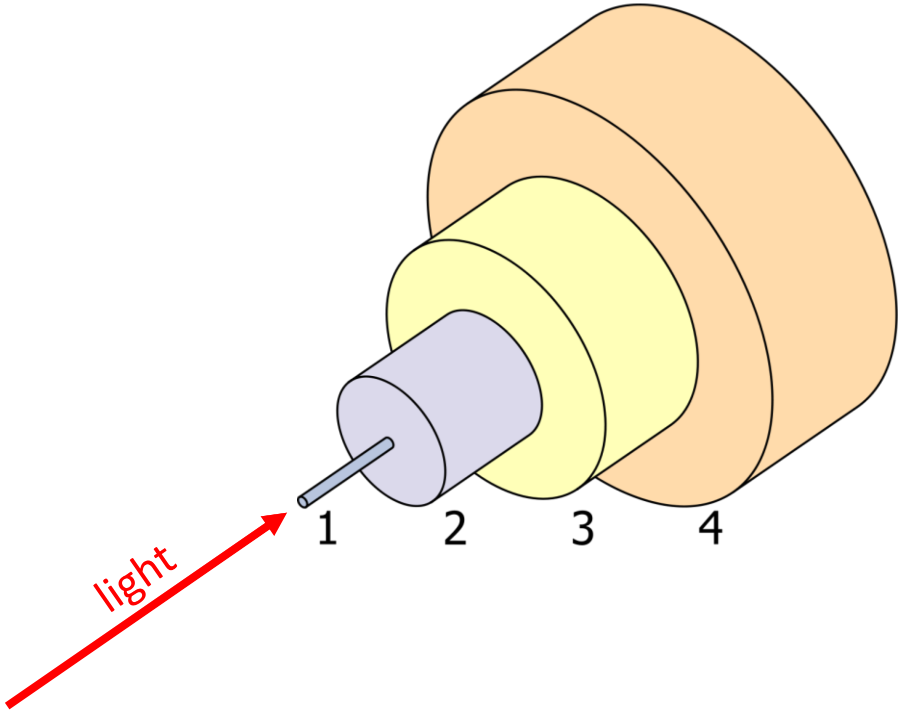
Optical fiber

Guiding through index contrast between core and cladding:

$$\Delta = \frac{n_{\text{core}} - n_{\text{clad}}}{n_{\text{clad}}}$$

Can maintain <u>small mode-cross section over long distance</u>.





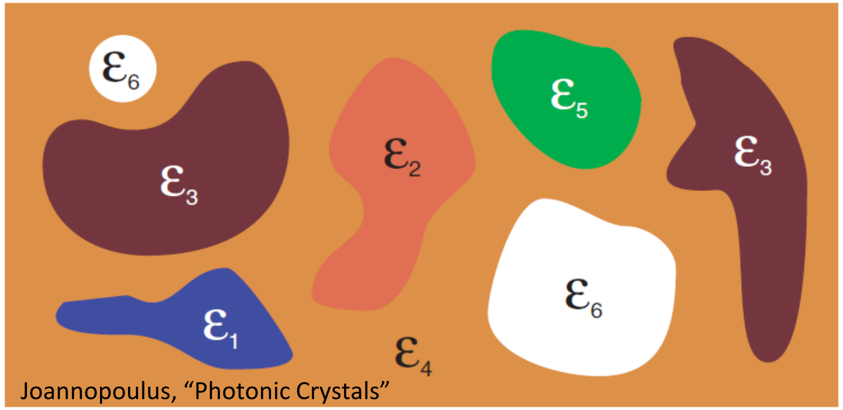
Optically relevant:
1 Core (typ. 5-10 µm diameter for single mode, up to hundreds of µm for multi-mode)
2 Cladding (125 µm diameter is often standard)

Mechanical protection:
3 Buffer (usually a polymer, 250 µm is standard)
4 Jacket optional additional protection, usually plastic.

Nobel Prize 2009
Charles Kao

0.2 dB/km : 50 % of the light is lost after 15 km (!) of propagation.

Light propagation in inhomogeneous media



Joannopoulus, “Photonic Crystals”

Eigenvalue equation

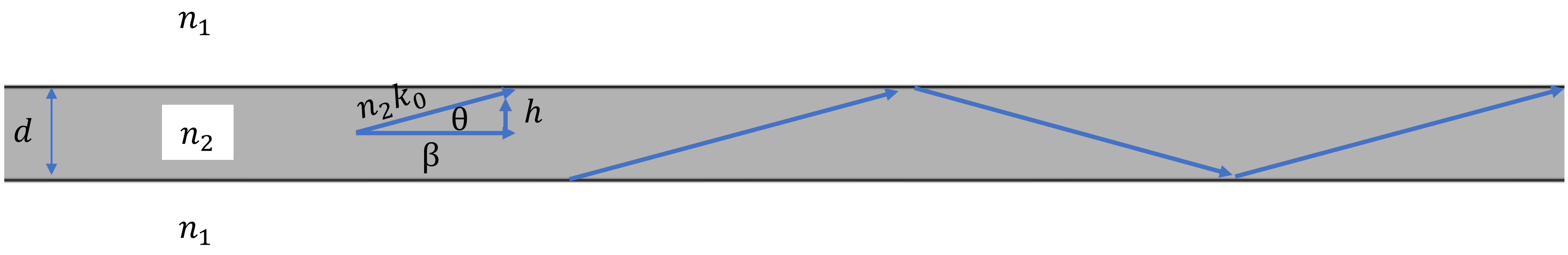
$$\underbrace{\nabla \times \left(\frac{1}{\varepsilon(\mathbf{r})} \nabla \times \right.}_{\text{linear hermetian operator}} \left. \mathbf{H}(\mathbf{r}) \right) = \underbrace{\left(\frac{\omega}{c} \right)^2}_{\text{Eigenvalue}} \mathbf{H}(\mathbf{r})$$

- Hermitian operator, real eigenvalues
- Eigenmodes orthogonal
- Confined structure have discrete modes

$$(\underbrace{\boldsymbol{H}_i}, \underbrace{\boldsymbol{H}_j}_{\text{Eigenmodes}}) = \int \boldsymbol{H}_i^*(\boldsymbol{r}) \boldsymbol{H}_j(\boldsymbol{r}) \, \mathrm{d}\boldsymbol{r} \propto \delta_{ij}$$

Planar waveguide

Intuitive ray optics picture



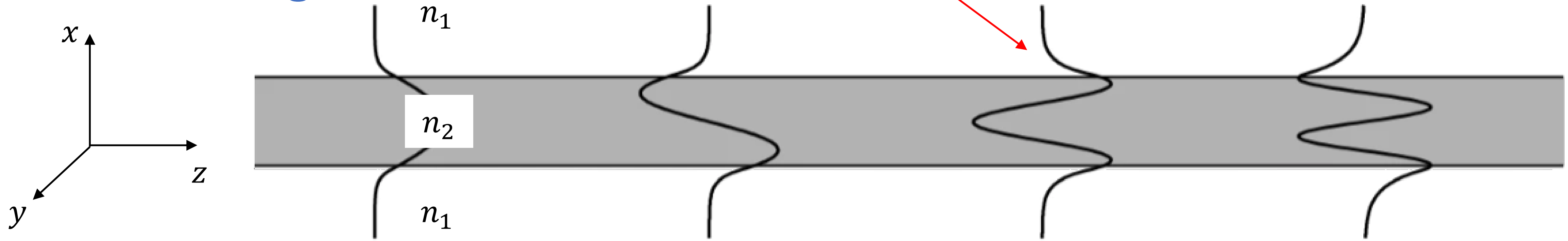
Guiding by total internal reflection if

$$\theta < \theta_c, \cos\theta_c = \frac{n_1}{n_2}$$

Transverse resonance condition (discrete modes)

$$2dn_2k_0\sin\theta + 2\phi_r = m\pi$$

Planar waveguide



$$\left.\begin{aligned}&\nabla^2 \boldsymbol{E}(\boldsymbol{r}) + k_0 n^2(\boldsymbol{r}) \boldsymbol{E}(\boldsymbol{r}) = 0 \\ &\boldsymbol{E}(\boldsymbol{r}, t) = \boldsymbol{E}(x, y) \exp[i(\omega t - \beta z)]\end{aligned}\right\} \xrightarrow{\frac{\partial}{\partial y} = 0}$$

$$\frac{\partial^2}{\partial x^2} E_i(x, y) + (k_0^2 n_1^2 - \beta^2) E_i(x, y) = 0 \quad \text{Cladding}$$

$$\frac{\partial^2}{\partial x^2} E_i(x, y) + (k_0^2 n_2^2 - \beta^2) E_i(x, y) = 0 \quad \text{Core}$$

Case	Core	Clad	Mode
$\beta > k_0 n_2 \, , \, k_0 n_1$	-	-	No physical solution
$k_0 n_1 < \beta < k_0 n_2$	"oscillating"	exponential decay	**Confined Guides mode**
$\beta < k_0 n_2 \, , \, k_0 n_1$	"oscillating"	"oscillating"	radiation mode

General characteristics of solution:

$$\frac{1}{E_i} \frac{\partial^2 E_i}{\partial x^2} = \text{const} > 0$$ **"exponential"** (decay / growth)

$$\frac{1}{E_i} \frac{\partial^2 E_i}{\partial x^2} = \text{const} < 0$$ **"oscillating"**

Fiber modes

Cylindrical symmetry implies modes 2π-periodic boundary condition in angular coordinate ϕ

$$E \propto F(r)\,\exp(il\phi)\,\exp(i\beta z)$$

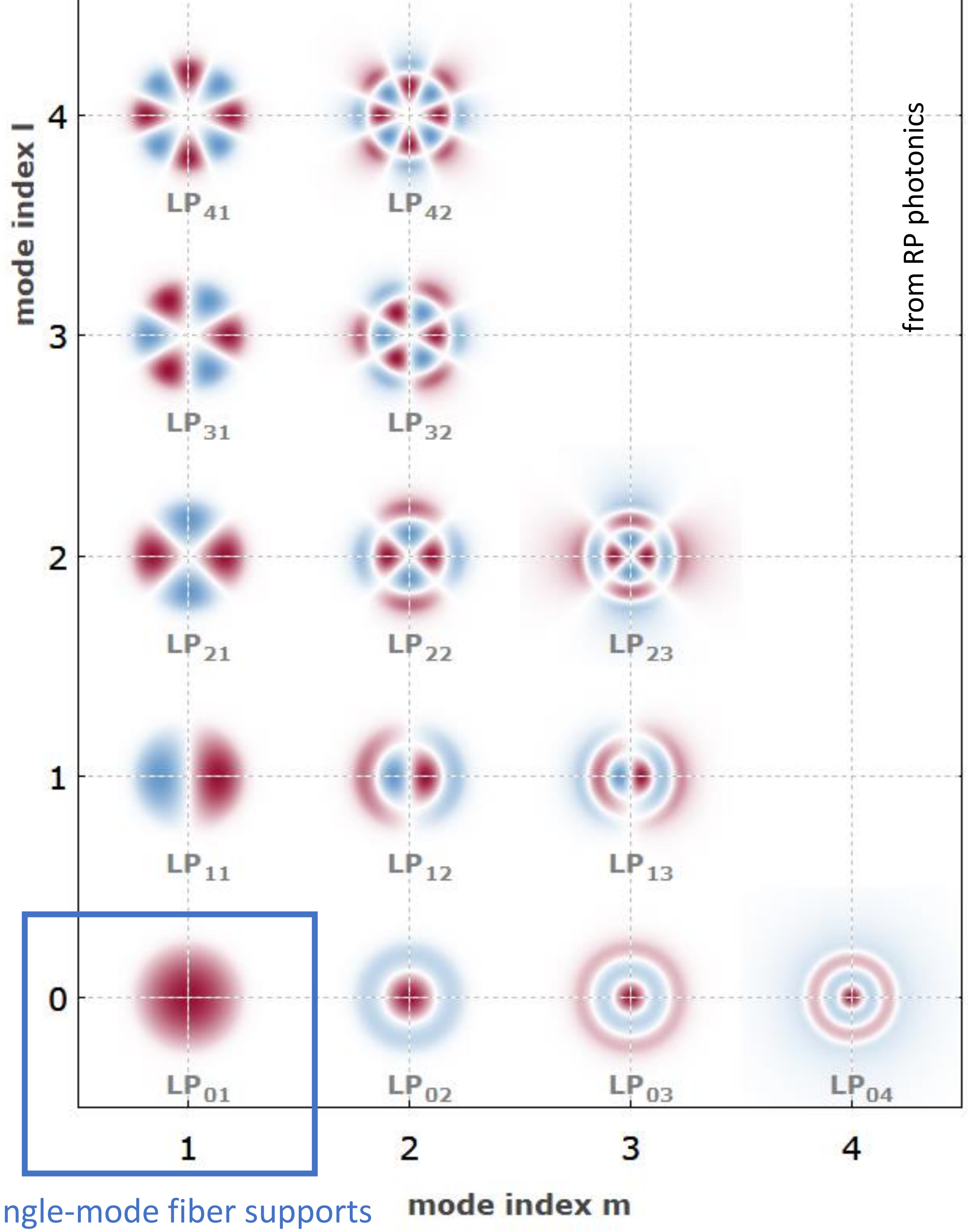
azimuthal mode index l

radial mode index m

Single mode condition

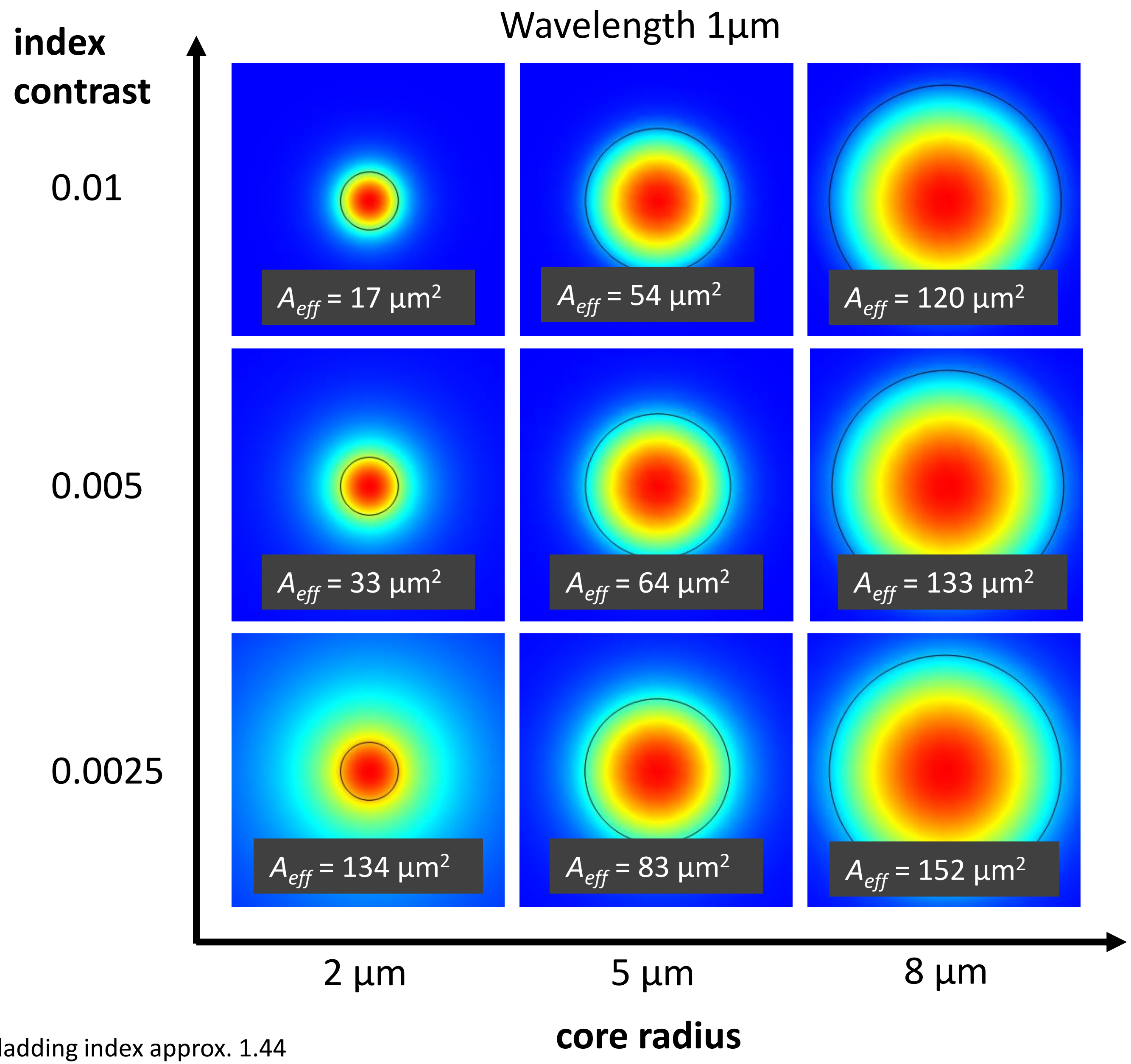
$$V = k_0 r_{\text{core}}\left(n_{\text{core}}^2 - n_{\text{clad}}^2\right)^{1/2}$$

$$V < V_c, \quad V_c \approx 2.405$$



A single-mode fiber supports only this mode profile (2 polarizations)

Effective mode area



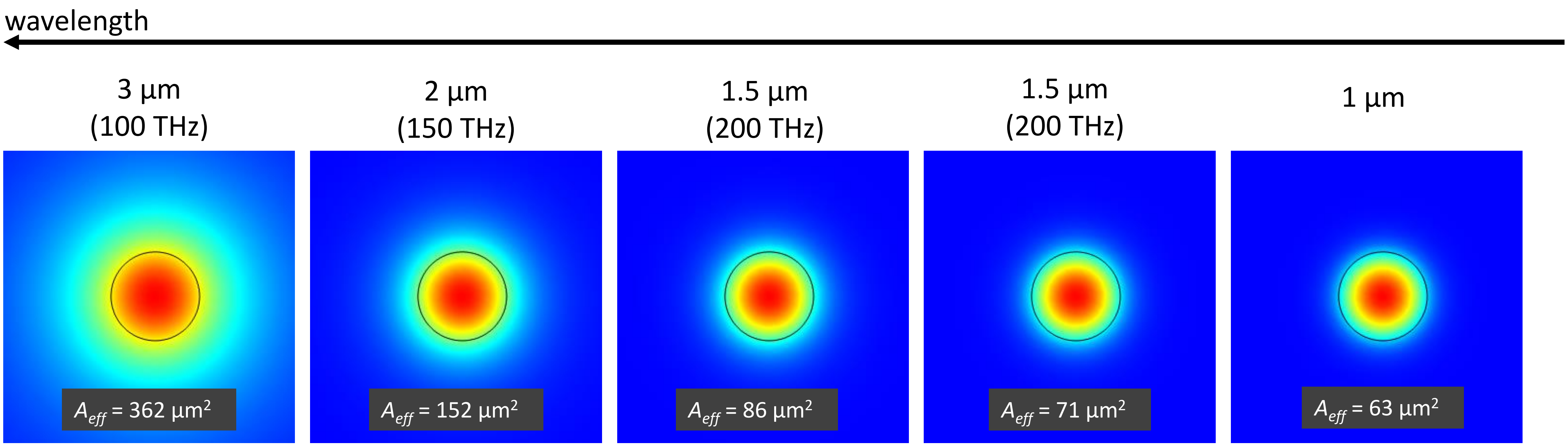
Effective mode area:

$$A_{\mathrm{eff}} = \frac{\left(\int |E|^2 \, \mathrm{d}A\right)^2}{\int |E|^4 \, \mathrm{d}A}$$

- Larger index contrast leads to stronger confinement
- Smaller core radius usually implies smaller effective area; however, when core radius and/or index contrast very small, the mode area can get larger

Cladding index approx. 1.44

Effective mode area and wavelength



- Larger mode area for larger wavelength
- Can lead to strong chromatic dispersion of nonlinearity, impacting τ_{shock}

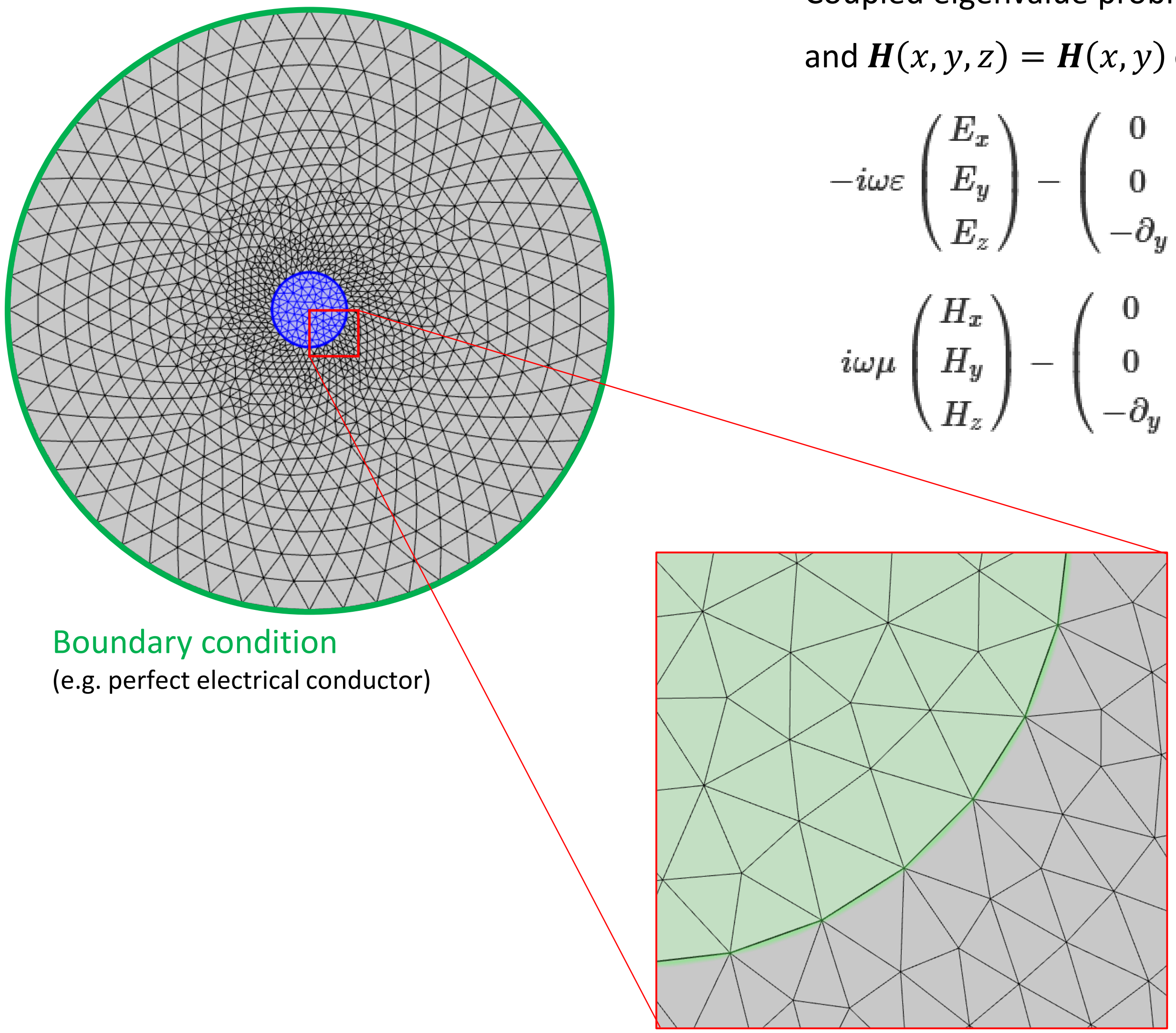
$$\left(\frac{\partial}{\partial z} + \frac{\alpha}{2} - iD\right) A = i\gamma \left(1 + i\tau_{\text{shock}} \frac{\partial}{\partial \tau}\right) A(z, \tau) \int R(\tau') |A(z, \tau - \tau')|^2 \, \mathrm{d}\tau'$$

Measures of dispersion

Often used measures of dispersion in fibers and waveguides:

Dispersion Measure	**Expression**	**Units**	**Sign of Anomalous Dispersion**
Dispersion Per Unit Length			
Group Velocity Dispersion (*GVD*) β_2 corresponds to k_2 in previous Taylor expanion	$\beta_2 = \frac{\partial^2 \beta}{\partial \omega^2} = \frac{1}{c}\left(2\frac{\partial n}{\partial \omega} + \omega \frac{\partial^2 n}{\partial \omega^2}\right) = -\frac{\lambda^2}{2\pi c} D$	s^2 / m (often as ps^2 / km)	-
Dispersion Parameter *D*	$D = -\frac{\lambda}{c}\frac{\partial^2 n}{\partial \lambda^2} = -\frac{2\pi c}{\lambda^2}\beta_2$	ps / (nm · km)	+

Finite element method (FEM)



Coupled eigenvalue problem with $\boldsymbol{E}(x, y, z) = \boldsymbol{E}(x, y)\exp[i\beta z]$

and $\boldsymbol{H}(x, y, z) = \boldsymbol{H}(x, y)\exp[i\beta z]$

$$-i\omega\varepsilon\begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} - \begin{pmatrix} 0 & 0 & \partial_y \\ 0 & 0 & -\partial_x \\ -\partial_y & \partial_x & 0 \end{pmatrix}\begin{pmatrix} H_x \\ H_y \\ H_z \end{pmatrix} = i\beta\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} H_x \\ H_y \\ H_z \end{pmatrix}$$

$$i\omega\mu\begin{pmatrix} H_x \\ H_y \\ H_z \end{pmatrix} - \begin{pmatrix} 0 & 0 & \partial_y \\ 0 & 0 & -\partial_x \\ -\partial_y & \partial_x & 0 \end{pmatrix}\begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = i\beta\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}$$

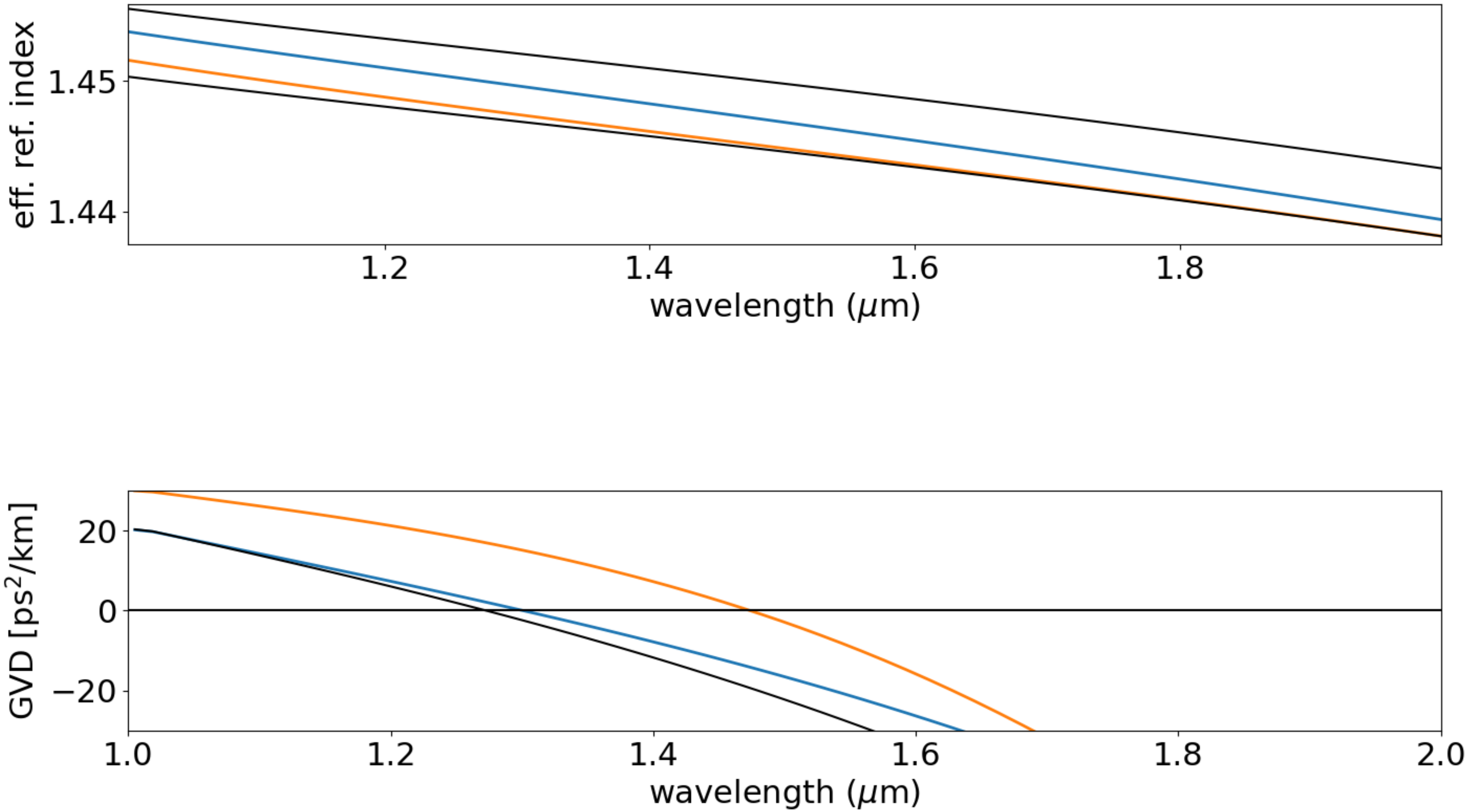
Solver finds Eigenvalue β for given ω

Other numeric techniques, e.g.

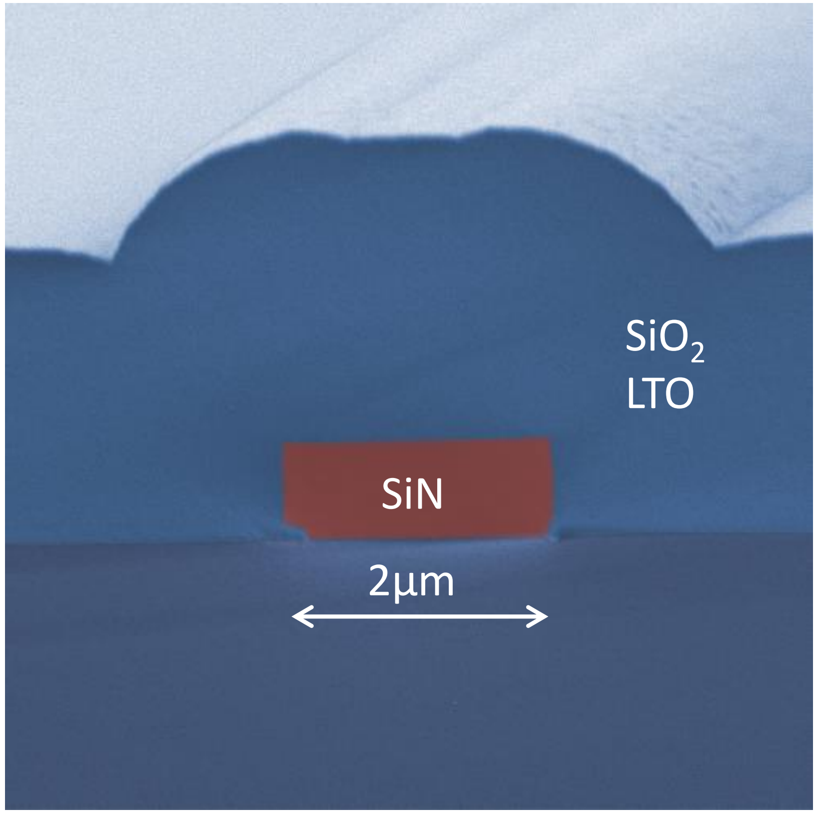
- **Finite difference method**
- **Beam propagation method**

and numerous variations exist.

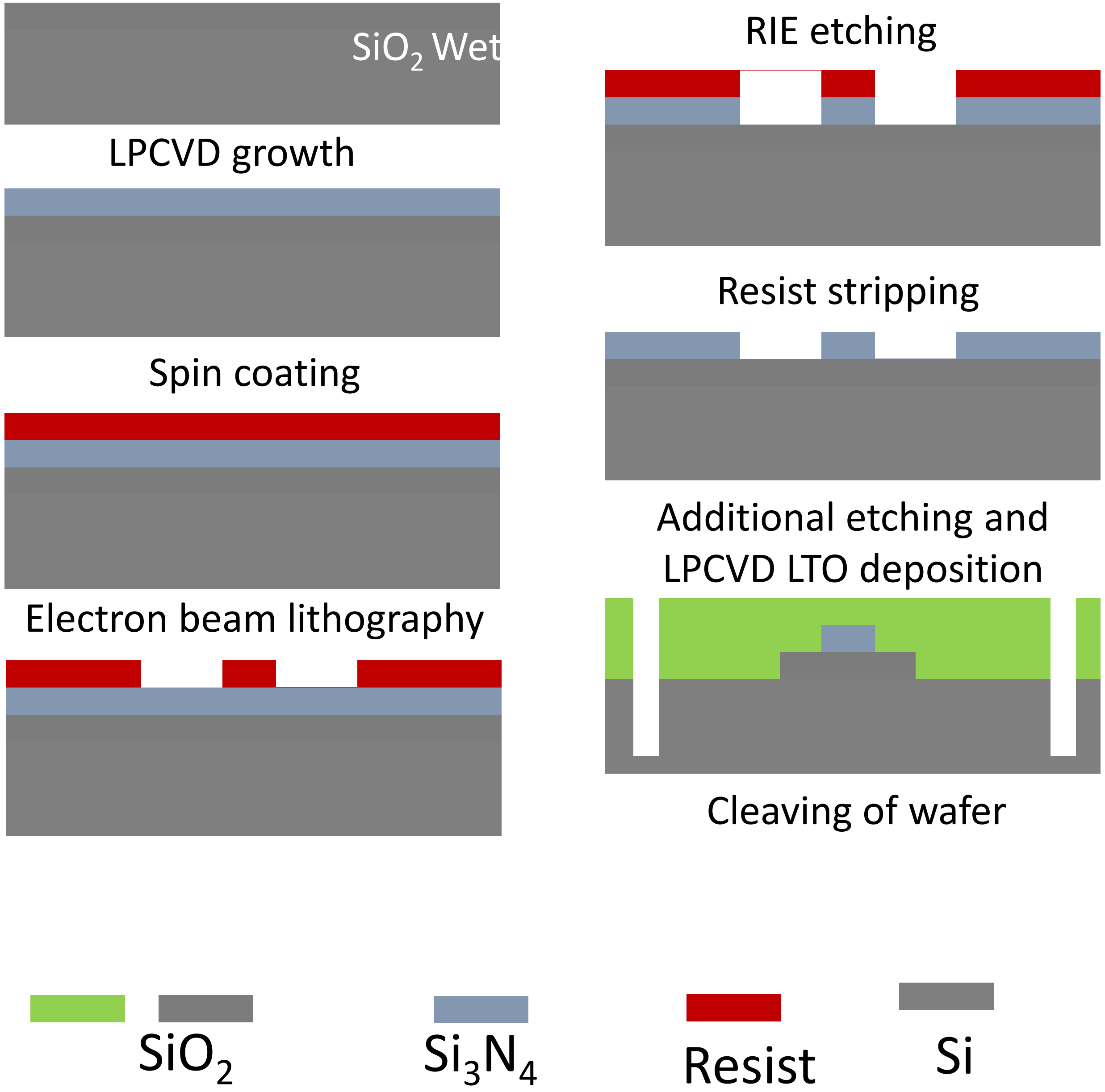
Dispersion Engineering in a fiber



Integrated waveguides

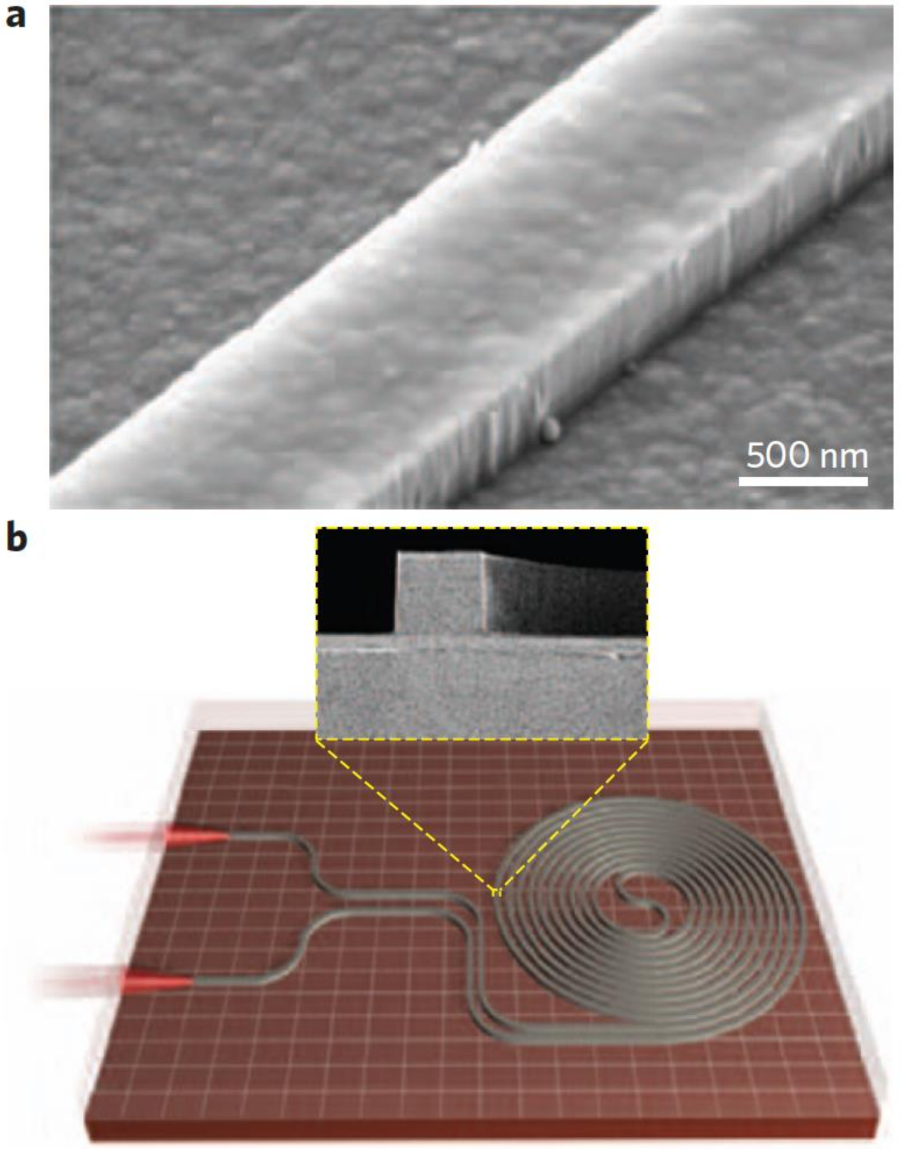


Mode area: ~ 1 µm
Loss: ~ 0.1 dB/cm



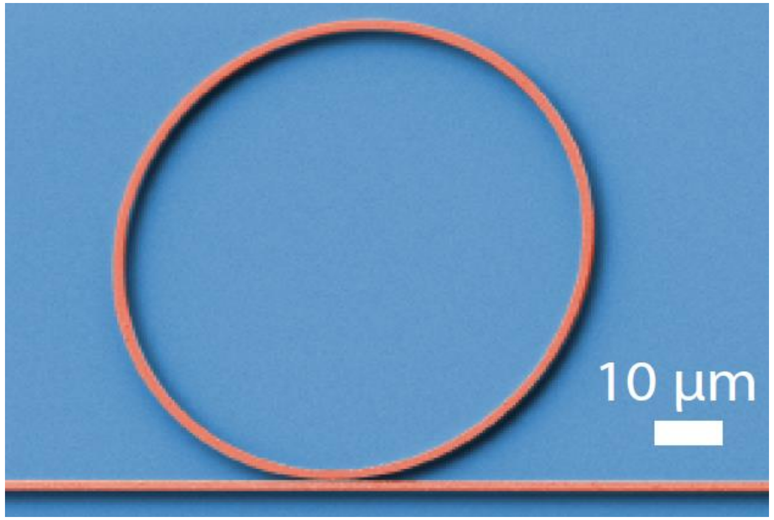
Examples

Long, high confinement waveguides

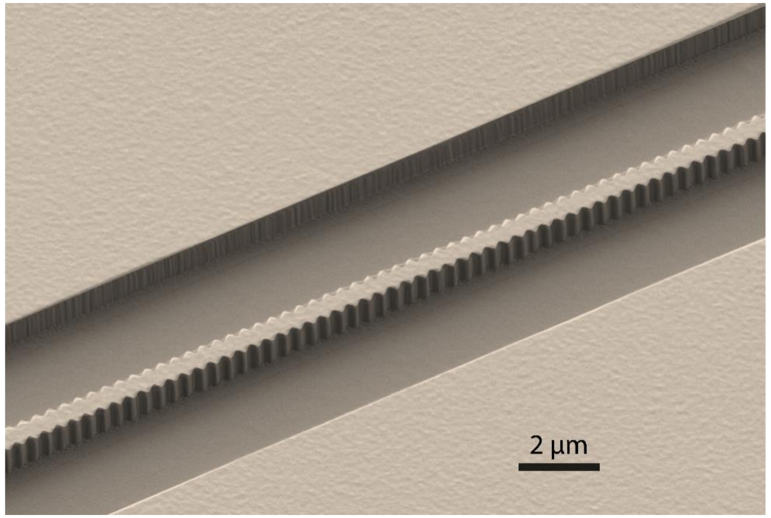


Moss et al., Nat. Phot. (2013)

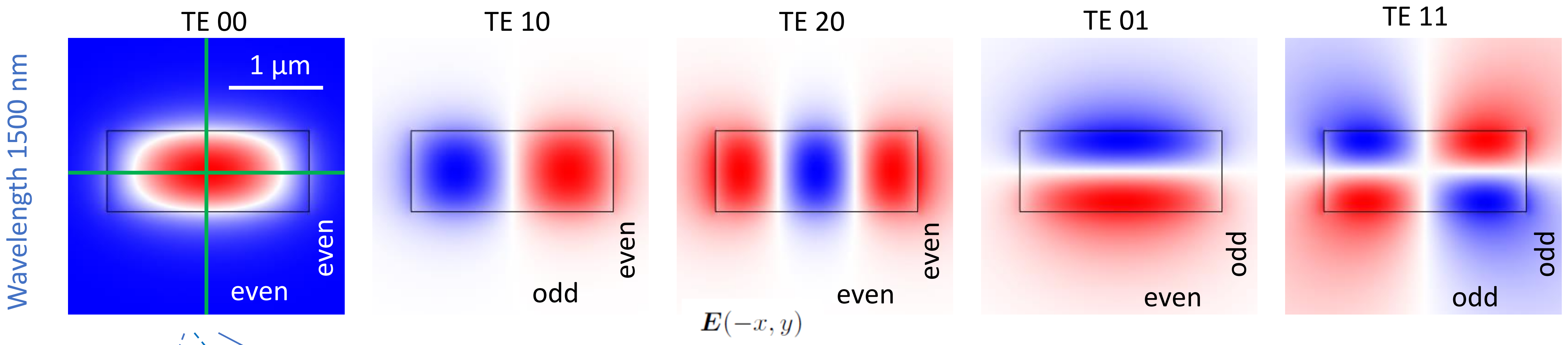
Resonators



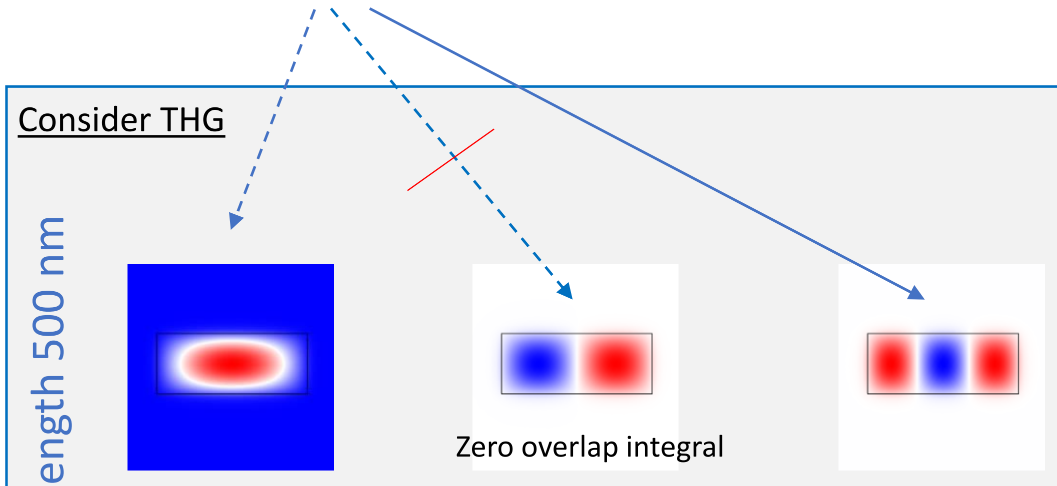
Change of waveguide parameter along length



Higher order modes



Consider THG



Zero overlap integral

Non-zero overlap integral but not generally phase matched

Non-zero overlap integral potentially phase matched

Odd and even modes arising from symmetry

Assume Eigenmode: $\boldsymbol{E}(x, y)$

If $\varepsilon(x, y) = \varepsilon(-x, y)$ then also $\boldsymbol{E}(-x, y)$ is a solution

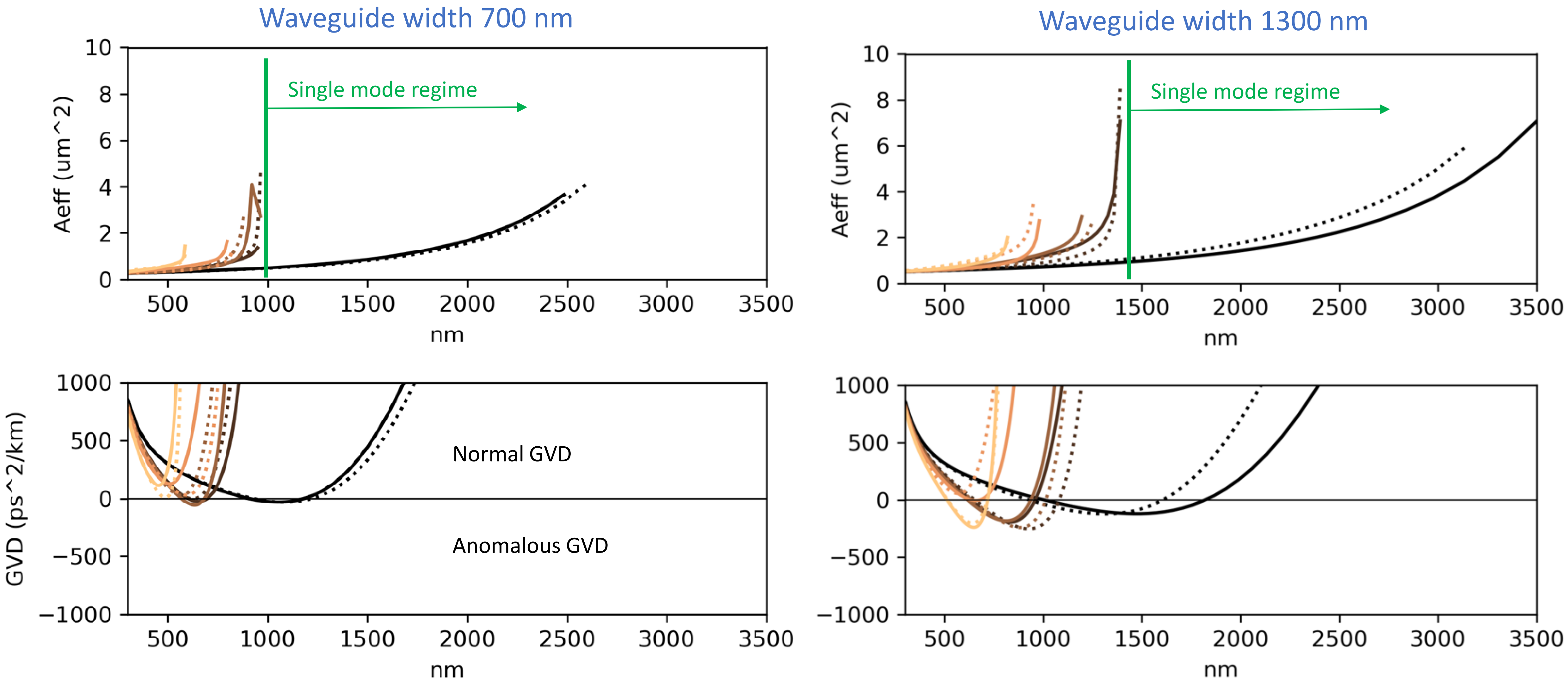
(same frequency, i.e. same mode)

$$\boldsymbol{E}(-x, y) = a\boldsymbol{E}(+x, y)$$

$$\boldsymbol{E}(x, y) = a^2\boldsymbol{E}(+x, y)$$

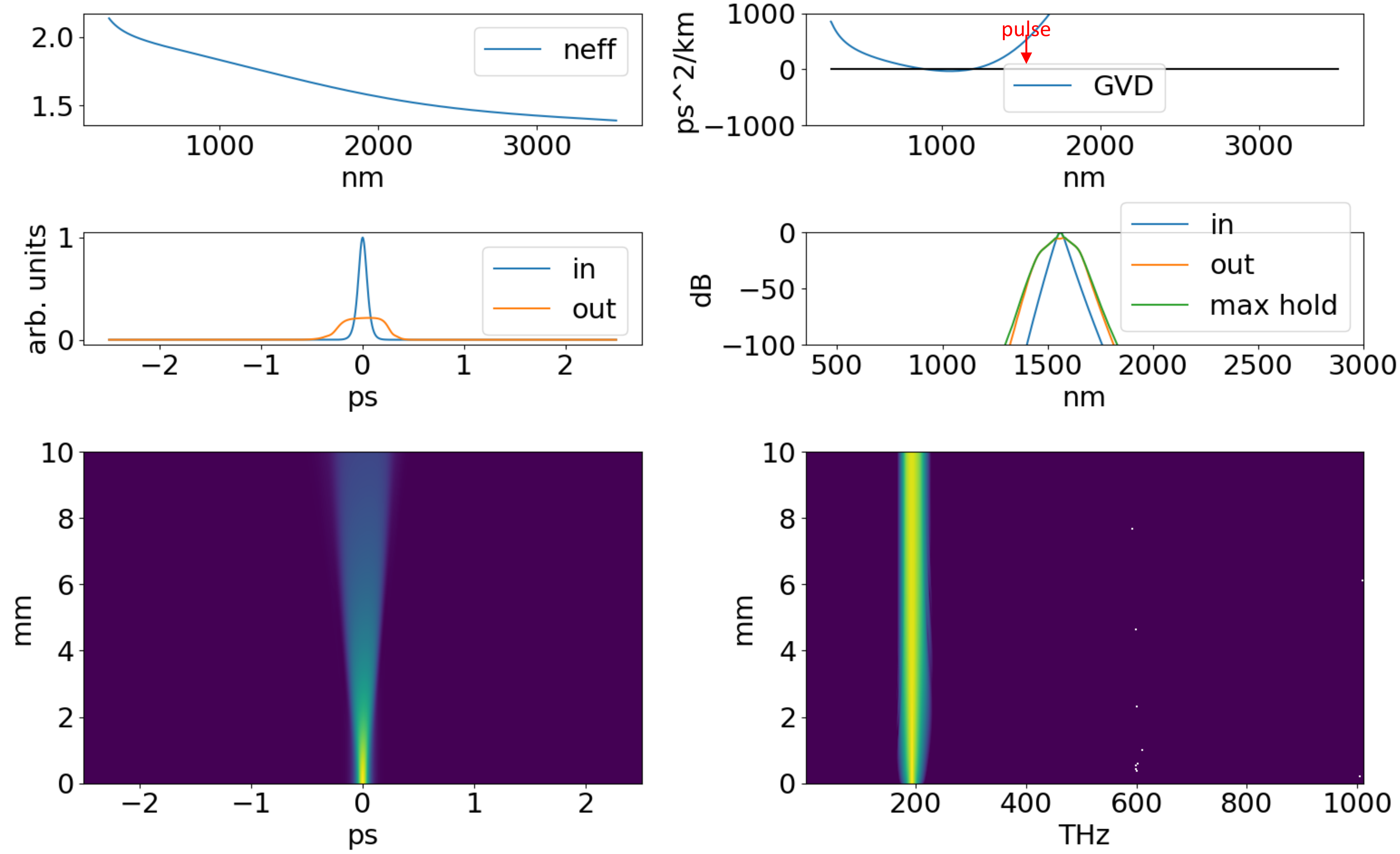
$a = \pm 1 \longrightarrow$ Odd and even modes

Dispersion integrated waveguides

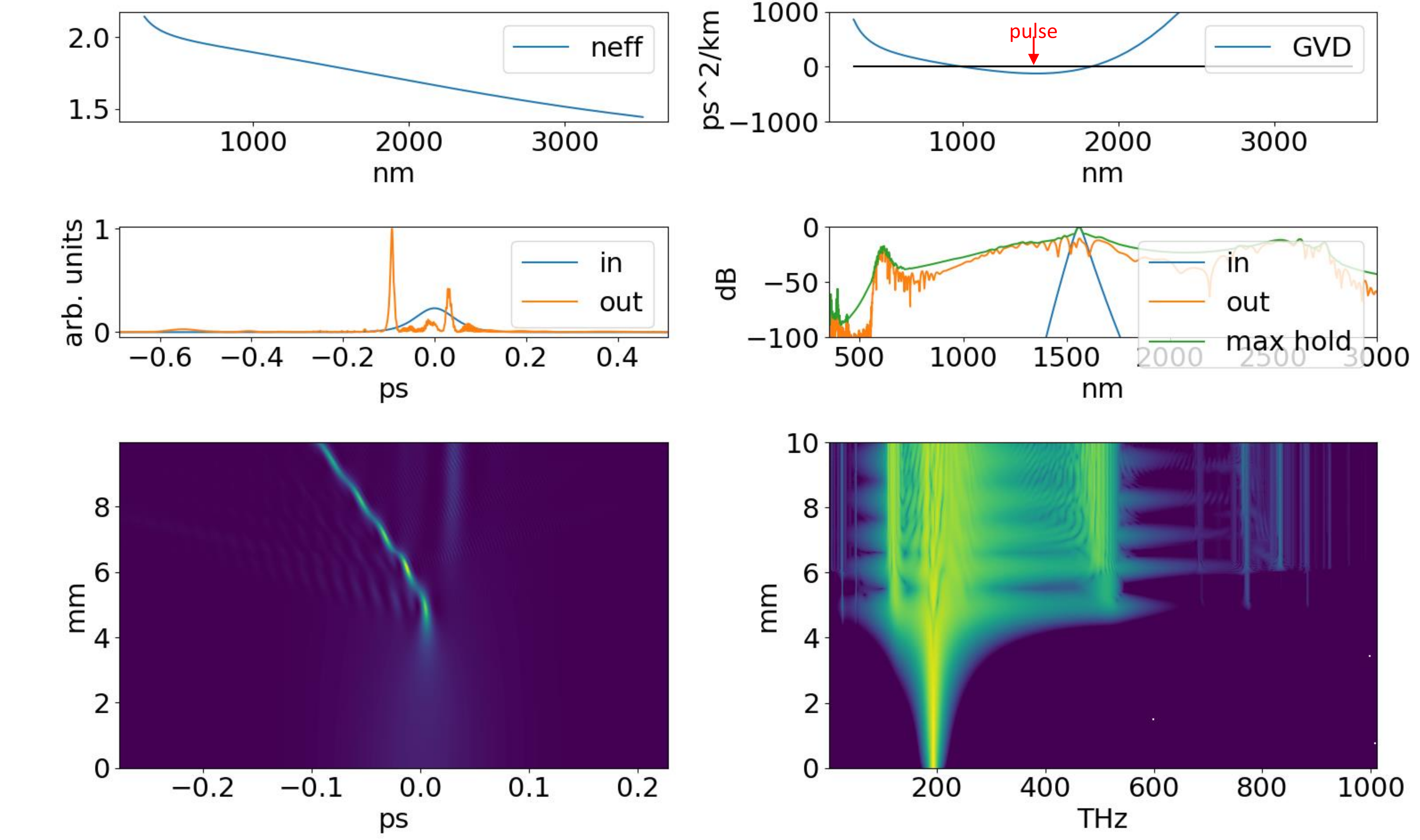


Silicon nitride wave guide of height 800 nm embedded in SiO_2
Solid line: TE, Dotted line: TM, black: fundamental mode, color: higher order (all modes same symmetry as fundamental mode)

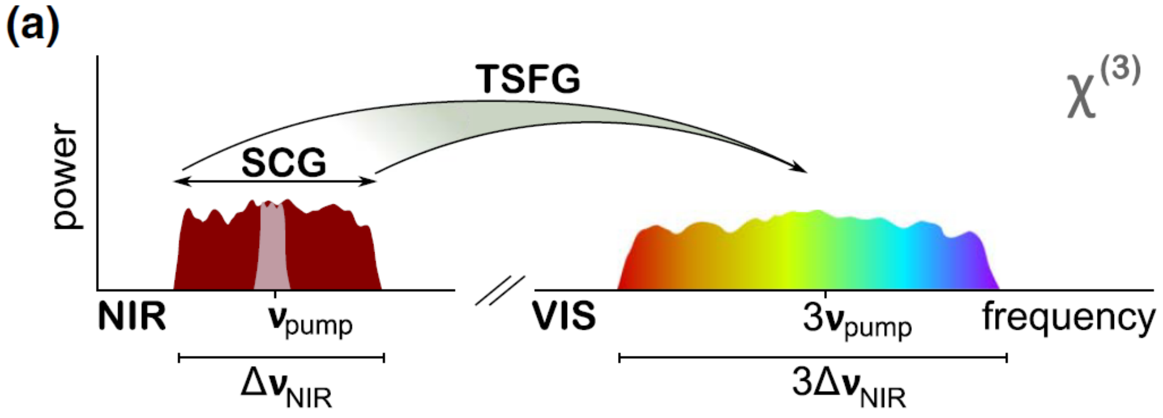
Example supercontinuum – waveguide width 700 mn

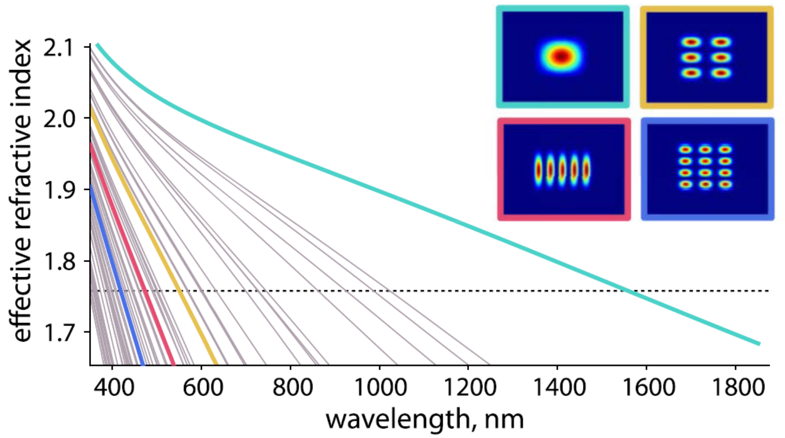


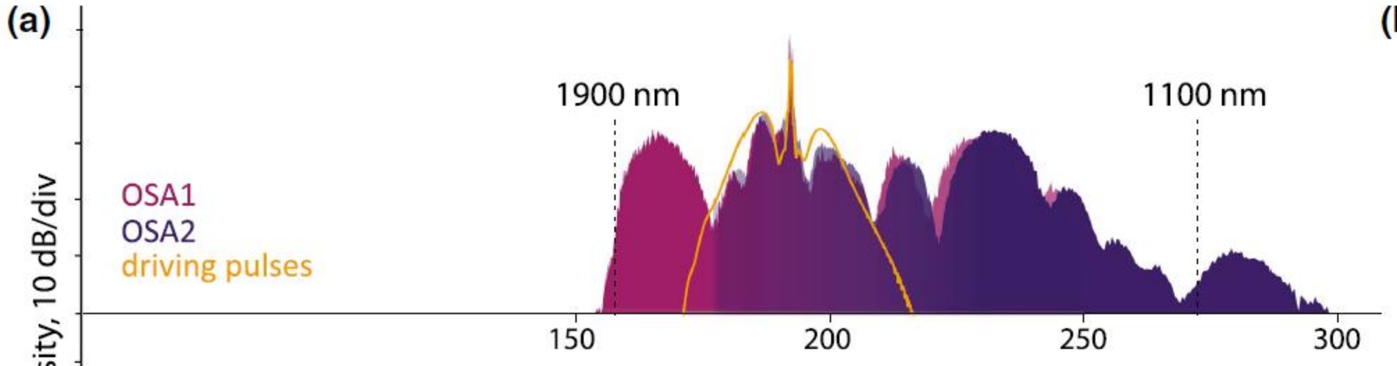
Example supercontinuum – waveguide width 1300 mn

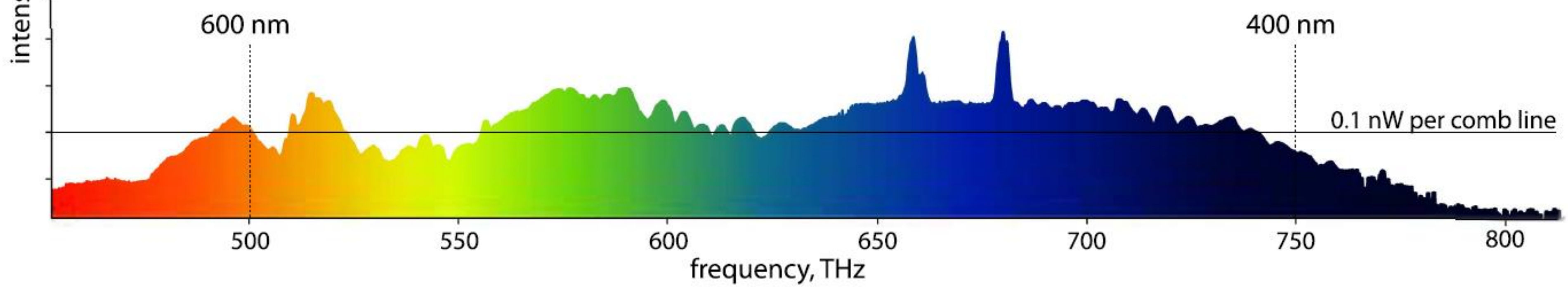


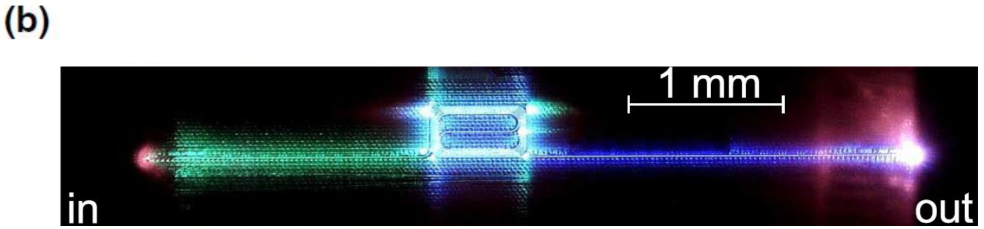
Multi-mode phase matching



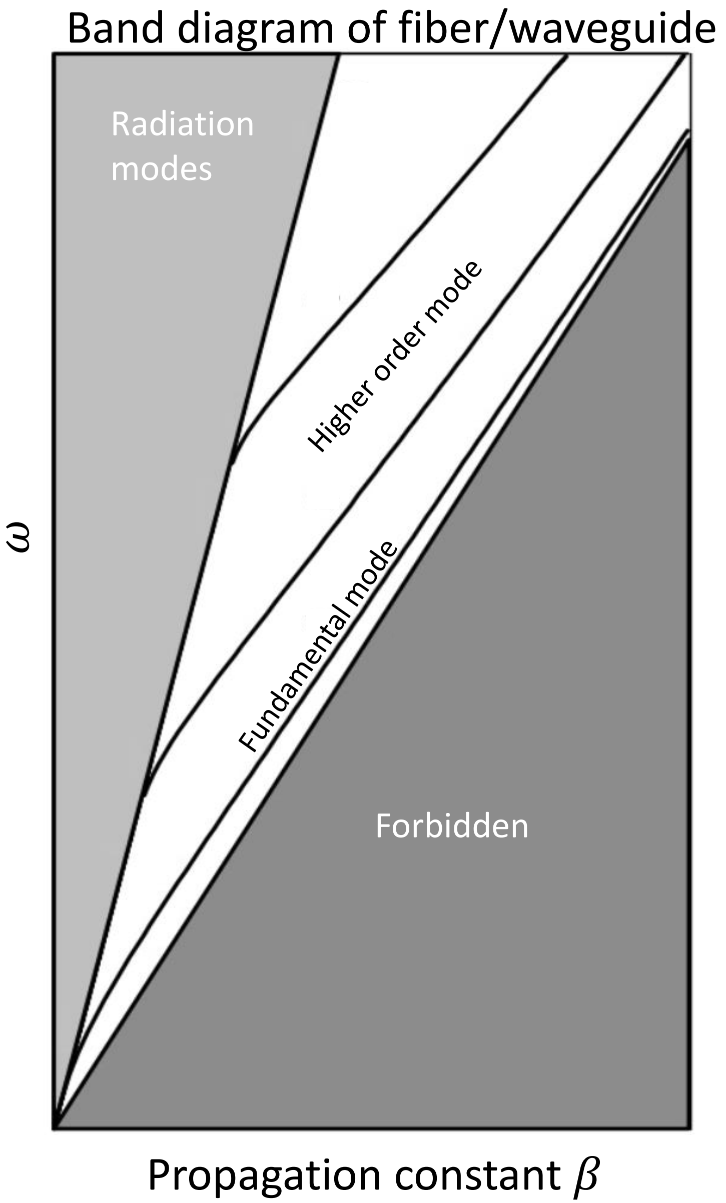




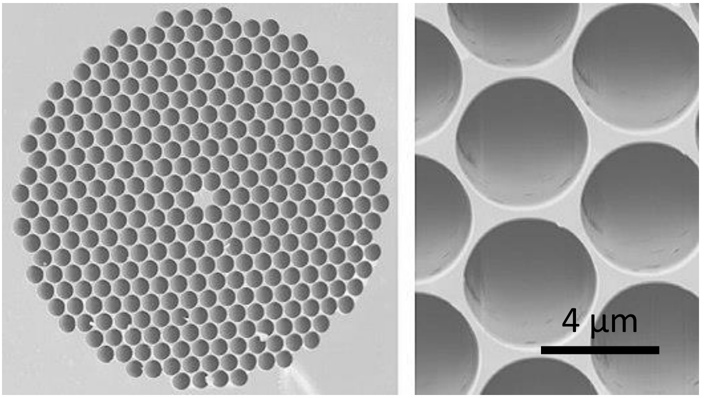


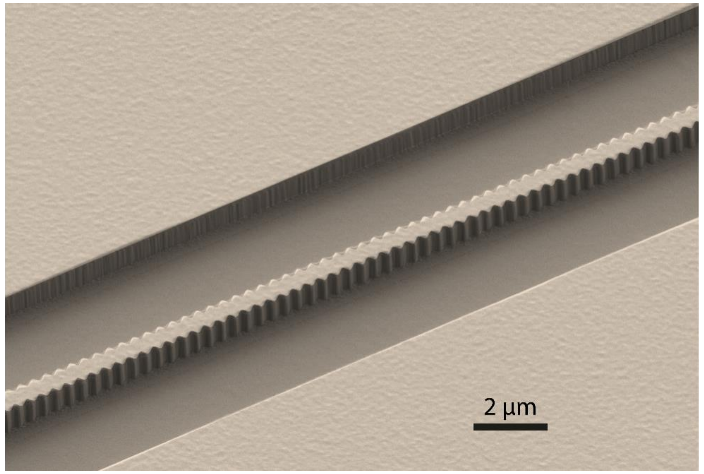


Photonic crystal fiber (PCF) and photonic crystal waveguides



Periodic structures can significantly change dispersion





Experiment: Fiber splicing

After lecture