2021 Nov 08 - Chapter 5

NLO #9

Electro-optic effect

- How do *static* electric fields modify the permittivity?
- Linear electro-optic effect / Pockels effect
- Longitudinal and transverse electro-optic effect

Electro-optic modulators

- Phase and intensity modulation
- Waveguide-based modulators
- Experiment (PM/IM)

Reminder

Propagation of light through a linear medium (NLO #6)

$$\boldsymbol{D} = \epsilon_0 \boldsymbol{E} + \boldsymbol{P} \quad \xrightarrow[\epsilon = 1 + \chi^{(1)}]{\text{linear medium}} \quad D_i = \epsilon_0 \sum_j \epsilon_{ij} E_j$$

In matrix form:

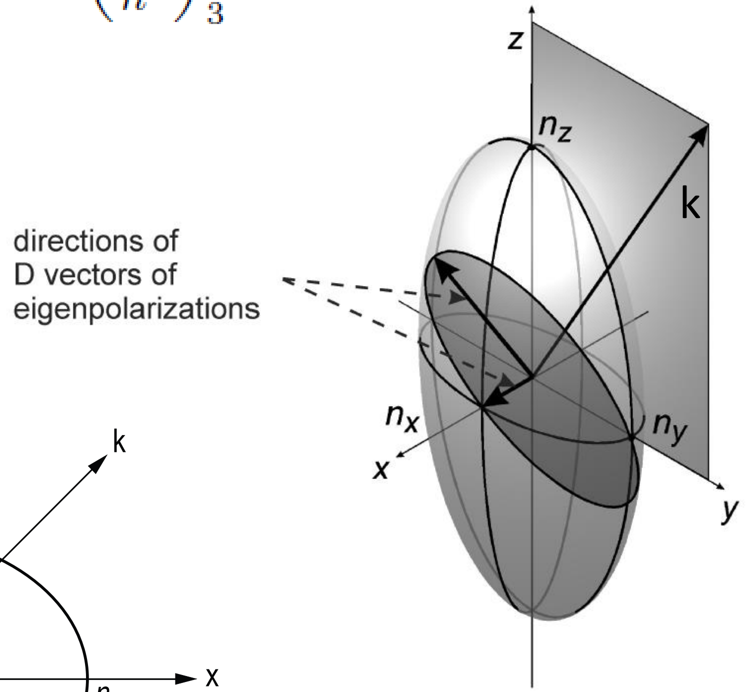
$$\begin{pmatrix} D_x \\ D_y \\ D_z \end{pmatrix} = \underbrace{\begin{pmatrix} \epsilon_{xx} & \epsilon_{xy} & \epsilon_{xz} \\ \epsilon_{yx} & \epsilon_{yy} & \epsilon_{yz} \\ \epsilon_{zx} & \epsilon_{zy} & \epsilon_{zz} \end{pmatrix}}_{\epsilon} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}$$

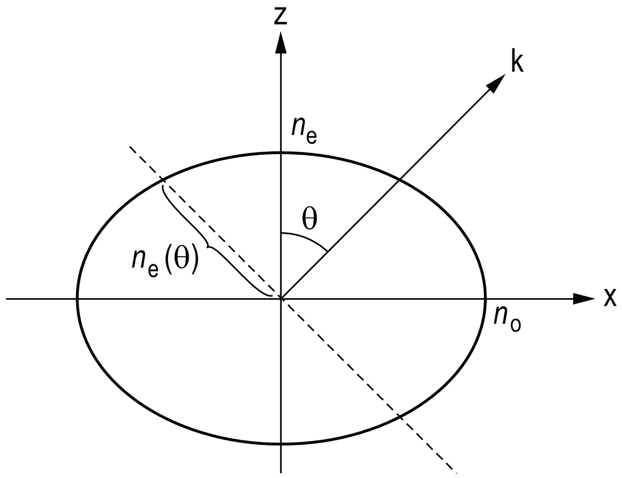
Can be diagonalized (for reciprocal media):

$$\begin{pmatrix} D_x \\ D_y \\ D_z \end{pmatrix} = \begin{pmatrix} \epsilon_{xx} & 0 & 0 \\ 0 & \epsilon_{yy} & 0 \\ 0 & 0 & \epsilon_{zz} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}$$

Index ellipsoid:

$$\left(\frac{1}{n^2}\right)_1 x^2 + \left(\frac{1}{n^2}\right)_2 y^2 + \left(\frac{1}{n^2}\right)_3 z^2 + = 1$$





Electro-optic effect

What happens in a *nonlinear* medium if we apply a static electric field?

$$\boldsymbol{E}_{\text{total}} = \boldsymbol{E} + \boldsymbol{E}^{\omega}$$

light wave

$$\boldsymbol{D} = \epsilon_0 \boldsymbol{E}_{\text{total}} + \boldsymbol{P}(\boldsymbol{E}_{\text{total}})$$

Only keep terms oscillating with ω (i.e. describing the linear propagation of the light wave under a static E-field):

$$D_i^{\omega} = \epsilon_0 \sum_j \epsilon_{ij} E_j^{\omega} + 2\epsilon_0 \sum_{jk} \chi_{ijk}^{(2)} E_j^{\omega} E_k + 3\epsilon_0 \sum_{jkl} \chi_{ijkl}^{(3)} E_j^{\omega} E_k E_l + \ldots$$

$$\boxed{\epsilon_{ij} \to \epsilon'_{ij} = \epsilon_{ij} + \underbrace{2\sum_k \chi_{ijk}^{(2)} E_k}_{\text{linear electro-optic effect}} + \underbrace{3\sum_{kl} \chi_{ijkl}^{(3)} E_k E_l}_{\text{quadratic electro-optic effect}}}$$

New ϵ-tensor may not be diagonal anymore (change of principal axes of the index ellipsoid)

Linear electro-optic effect (Pockels effect) and electro-optic tensor

In an arbitrary coordinate system {*x,y,z*} the index ellipsoid (**indicatrix**) can be expressed as

$$\left(\frac{1}{n^2}\right)_1 x^2+\left(\frac{1}{n^2}\right)_2 y^2+\left(\frac{1}{n^2}\right)_3 z^2+2\left(\frac{1}{n^2}\right)_4 yz+2\left(\frac{1}{n^2}\right)_5 xz+2\left(\frac{1}{n^2}\right)_6 xy=1$$

$$\left(\frac{1}{n^2}\right)_i=(\epsilon^{-1})_i$$

Definition of the electro-optic tensor:

Describes the change of the index ellipsoid's axes (zero in case of inversion symmetry)

$$\boxed{\Delta\left(\frac{1}{n^2}\right)_i=\sum_j r_{ij}E_j}$$

i=1..6 (contracted index; 1=*xx*, 2=*yy*, 3=zz, 4=yz, zy, 5=xz, zx, 6=*xy, yx*)

j=1..3 (1=*x*, 2=*y*, 3=*z*)

$$r_{ij}\sim 10^{-12}\ \mathrm{m/V}$$

$$\begin{pmatrix}\Delta(1/n^2)_1\\ \Delta(1/n^2)_2\\ \Delta(1/n^2)_3\\ \Delta(1/n^2)_4\\ \Delta(1/n^2)_5\\ \Delta(1/n^2)_6\end{pmatrix}=\begin{pmatrix}r_{11}&r_{12}&r_{13}\\ r_{21}&r_{22}&r_{23}\\ r_{31}&r_{32}&r_{33}\\ r_{41}&r_{42}&r_{43}\\ r_{51}&r_{52}&r_{53}\\ r_{61}&r_{62}&r_{63}\end{pmatrix}\begin{pmatrix}E_x\\ E_y\\ E_z\end{pmatrix}$$

Deformation of ellipsoid, principal axes' directions preserved

Deformation of ellipsoid, principal axes' directions modified

Modification of the index ellipsoid

Principle axes are aligned with coordinate system $\{x, y, z\}$:

$$\left(\frac{1}{n^2}\right)_1 x^2 + \left(\frac{1}{n^2}\right)_2 y^2 + \left(\frac{1}{n^2}\right)_3 z^2 = 1$$

Applying external field …

$$\left(\frac{1}{n^2}\right)_i \to \left(\frac{1}{n^2}\right)_i + \Delta\left(\frac{1}{n^2}\right)_i$$

… modifies the ellipsoid

$$\left[\left(\frac{1}{n^2}\right)_1 + \Delta\left(\frac{1}{n^2}\right)_1\right] x^2 + \left[\left(\frac{1}{n^2}\right)_2 + \Delta\left(\frac{1}{n^2}\right)_2\right] y^2 + \left[\left(\frac{1}{n^2}\right)_3 + \Delta\left(\frac{1}{n^2}\right)_3\right] z^2$$
$$+2\Delta\left(\frac{1}{n^2}\right)_4 yz + 2\Delta\left(\frac{1}{n^2}\right)_5 xz + 2\Delta\left(\frac{1}{n^2}\right)_6 xy = 1$$

Find **new semi-diameters** of ellipsoid and **new coordinate system** $\{\boldsymbol{x}', \boldsymbol{y}', \boldsymbol{z}'\}$ that aligns with new principal axes.

Find eigenvalues and eigenvectors of

$$\left(\frac{1}{n^2}\right)_i + \Delta\left(\frac{1}{n^2}\right)_i$$

Principle axes are aligned with ***new*** coordinate system $\{x', y', z'\}$:

$$\left(\frac{1}{n'^2}\right)_1 x'^2 + \left(\frac{1}{n'^2}\right)_2 y'^2 + \left(\frac{1}{n'^2}\right)_3 z'^2 = 1$$

Example: KDP

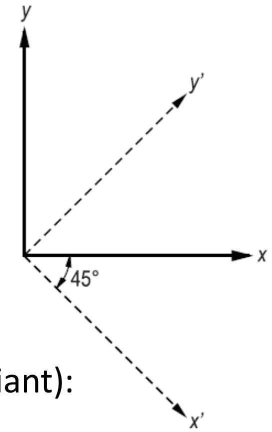
KDP (KH_2PO_4, potassium dihydrogen phosphate)
Crystal class $\bar{4}2m$

$$\mathbf{r} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & r_{41} & 0 \\ 0 & 0 & r_{63} \end{pmatrix}$$

$$\frac{1}{n_o^2}x^2 + \frac{1}{n_o^2}y^2 + \frac{1}{n_e^2}z^2 + 2r_{41}E_x yz + 2r_{41}E_y xz + 2r_{63}E_z xy = 1$$

Apply $\boldsymbol{E}$-field in z-direction (***will not change optical axis***):

$$\frac{1}{n_o^2}x^2 + \frac{1}{n_o^2}y^2 + \frac{1}{n_e^2}z^2 + 2r_{63}E_z xy = 1$$



New coordinate system (z invariant):

$$\begin{aligned} x &= x' \cos 45° + y' \sin 45°, \\ y &= -x' \sin 45° + y' \cos 45° \end{aligned}$$

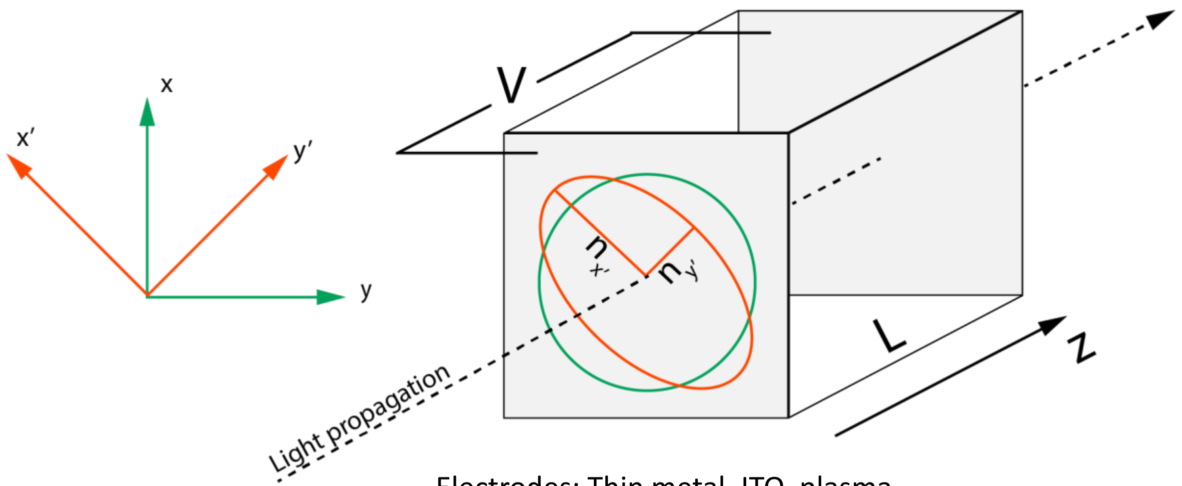
In coordinate system aligned with new principal axes:

$$\left(\frac{1}{n_0^2} - r_{63}E_z\right) x'^2 + \left(\frac{1}{n_0^2} + r_{63}E_z\right) y'^2 + \frac{z^2}{n_e^2} = 1$$

$$n_{x'} = n_0 \left(1 + \frac{1}{2}n_0^2 r_{63} E_z\right); \quad n_{y'} = n_0 \left(1 - \frac{1}{2}n_0^2 r_{63} E_z\right); \quad n_{z'} = n_e$$

approximation valid for
$n_0^2 r_{63} E_z \ll 1$

Longitudinal electro-optic effect
(Example: KDP, external field and propagation in z-direction)



Electrodes: Thin metal, ITO, plasma, …

In KDP

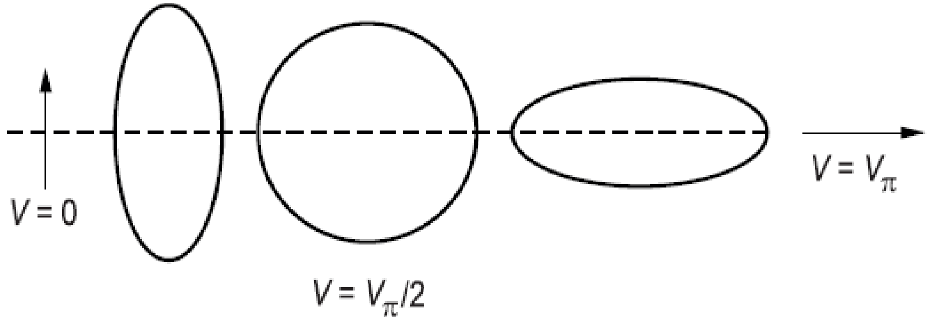
$$\Delta\phi = \frac{2\pi L}{\lambda_0}\left(n'_x - n'_y\right) = \frac{2\pi L n_0^3 r_{63} E_z}{\lambda_0}$$

Phase difference $\Delta\phi = \pi$ for $V_\pi = \dfrac{\lambda_0}{2n_0^3 r_{63}}$

$$r_{63} = -10.5\ \mathrm{pm/V},\ n_0 = 1.51$$

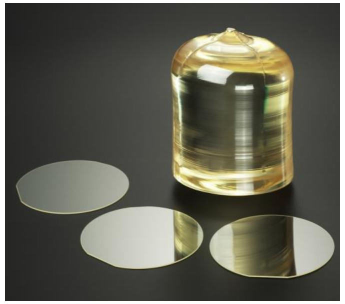
$632.8\ \mathrm{nm}$ $\quad \boxed{V_\pi = 8752\ \mathrm{V}}$

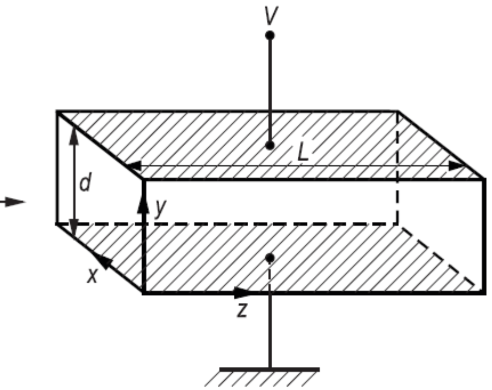
Voltage tunable waveplate



Transverse electro-optic effect (Example: Lithium Niobate, external field in y-direction)

Lithium niobate ($LiNbO_3$)
Crystal class 3m

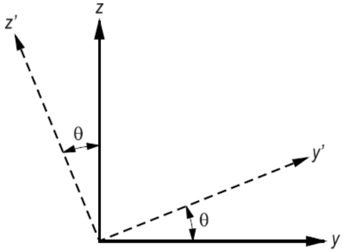




$$\left(\frac{1}{n_0^2} - r_{22}E_y\right)x^2 + \left[\left(\frac{1}{n_0^2} + r_{22}E_y\right)y^2 + \frac{z^2}{n_e^2} + 2r_{51}E_y yz\right] = 1.$$

New coordinates:

$$\begin{aligned} x &= x' \\ y &= y'\cos\theta - z'\sin\theta \\ z &= z'\cos\theta + y'\sin\theta \end{aligned}$$



$$\mathbf{r} = \begin{pmatrix} 0 & -r_{22} & r_{13} \\ 0 & r_{22} & r_{13} \\ 0 & 0 & r_{33} \\ 0 & r_{51} & 0 \\ r_{51} & 0 & 0 \\ -r_{22} & 0 & 0 \end{pmatrix}$$

$$\mathbf{r} = \begin{pmatrix} 0 & -3.4 & 8.6 \\ 0 & 3.4 & 8.6 \\ 0 & 0 & 30.8 \\ 0 & 28 & 0 \\ 28 & 0 & 0 \\ -3.4 & 0 & 0 \end{pmatrix} \times 10^{-13}\mathrm{mV}^{-1}$$

$$\left(\tfrac{1}{n_0^2} - r_{22}E_y\right)x'^2 + \left[\left(\tfrac{1}{n_0^2} + r_{22}E_y\right)\cos^2\theta + \tfrac{\sin^2\theta}{n_e^2} + r_{51}E_y\sin 2\theta\right]y'^2$$
$$+\left[\left(\tfrac{1}{n_0^2} + r_{22}E_y\right)\sin^2\theta + \tfrac{\cos^2\theta}{n_e^2} - r_{51}E_y\sin 2\theta\right]z'^2 = 1.$$

$\theta \sim 1$ mrad $\ll 1$

$$\left(\tfrac{1}{n_0^2} - r_{22}E_y\right)x'^2 + \left(\tfrac{1}{n_0^2} + r_{22}E_y + 2r_{51}E_y\theta + \tfrac{\theta^2}{n_e^2}\right)y'^2$$
$$+\left[\left(\tfrac{1}{n_0^2} + r_{22}E_y\right)\theta^2 - 2r_{51}E_y\theta + \tfrac{1}{n_e^2}\right]z'^2 = 1.$$

neglect terms including θ

$$\boxed{\begin{aligned} n_x &= \frac{n_0}{\sqrt{1 - n_0^2 r_{22}E_y}} = n_0\left(1 + \frac{1}{2}n_0^2 r_{22}E_y\right) \\ n_y &= \frac{n_0}{\sqrt{1 + n_0^2 r_{22}E_y}} = n_0\left(1 - \frac{1}{2}n_0^2 r_{22}E_y\right) \\ n_z &= n_e. \end{aligned}}$$

Intensity Modulation in bulk crystals

$$\Delta\phi = \frac{2\pi L}{\lambda_0}(n_x - n_y) = \frac{2\pi L n_o^3 r_{22} V}{\lambda_0 d}$$

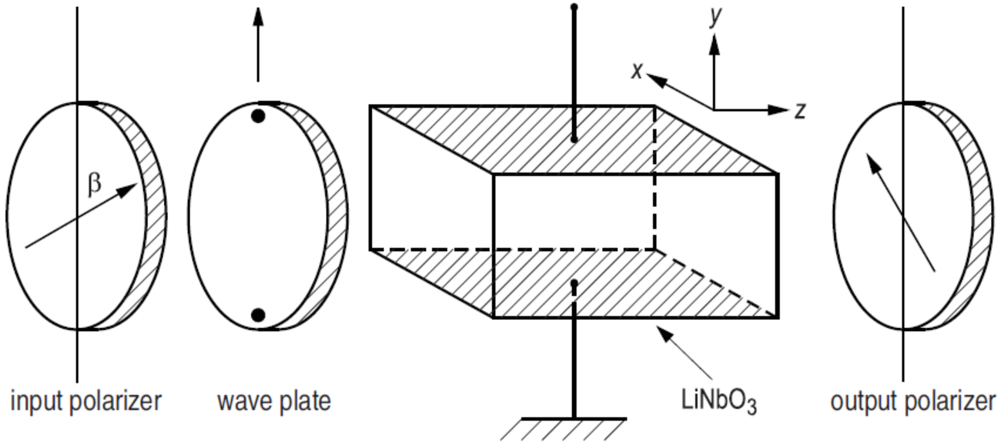
$$V_\pi = \frac{\lambda_0 d}{2 L n_0^3 r_{22}}$$

$$d = 5 \text{ mm}, \; L = 10 \text{ mm}, \; n_o = 2.3, \; r_{22} = 3.4 \cdot 10^{-12} \text{ m/V}$$

$$V_\pi = 1600 \text{ V}$$

Bulk crystal intensity modulator

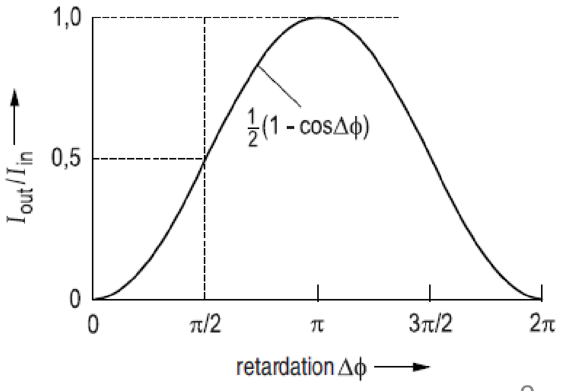
$\Delta\phi_{WP} = \pi/2$



$\beta = 45°$

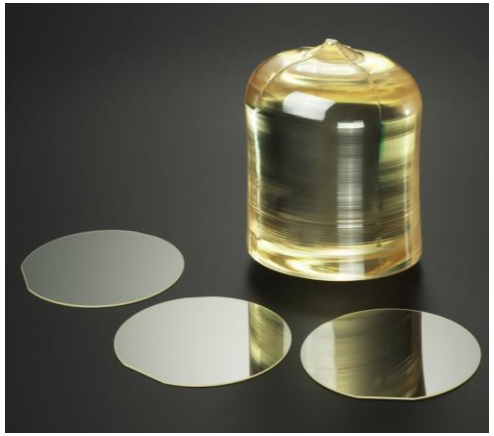
$$\frac{I_{out}}{I_{in}} = \frac{1}{2}\{1 - \cos[\Delta\phi_{WP} + aV(t)]\}$$
$$= \frac{1}{2}\{1 - \cos\Delta\phi_{WP}\cos[aV(t)] + \sin\Delta\phi_{WP}\sin[aV(t)]\}$$

$$\frac{I_{out}}{I_{in}} = \frac{1}{2}[1 + aV(t)]$$



Transverse electro-optic effect (Example: Lithium Niobate, external field in **z**-direction)

Lithium niobate ($LiNbO_3$)
Crystal class 3m



no cross-terms for E-field in z-direction

$$\mathbf{r} = \begin{pmatrix} 0 & -r_{22} & r_{13} \\ 0 & r_{22} & r_{13} \\ 0 & 0 & r_{33} \\ 0 & r_{51} & \boxed{0} \\ r_{51} & 0 & \boxed{0} \\ -r_{22} & 0 & \boxed{0} \end{pmatrix}$$

Apply $\boldsymbol{E}$-field in z-direction:

- **Principal axes' orientation unchanged**
- **Crystal is still uniaxial**

$$\frac{x^2}{n_0^2} + \frac{y^2}{n_0^2} + \frac{z^2}{n_e^2} = 1 \longrightarrow \left(\frac{1}{n_0^2} + r_{13}E_z\right) x^2 + \left(\frac{1}{n_0^2} + r_{13}E_z\right) y^2 + \left(\frac{1}{n_e^2} + r_{33}E_z\right) z^2 = 1$$

$$\frac{1}{n_0'^2} = \left(\frac{1}{n_0^2} + r_{13}E_z\right)$$

$$\frac{1}{n_e'^2} = \left(\frac{1}{n_e^2} + r_{33}E_z\right)$$

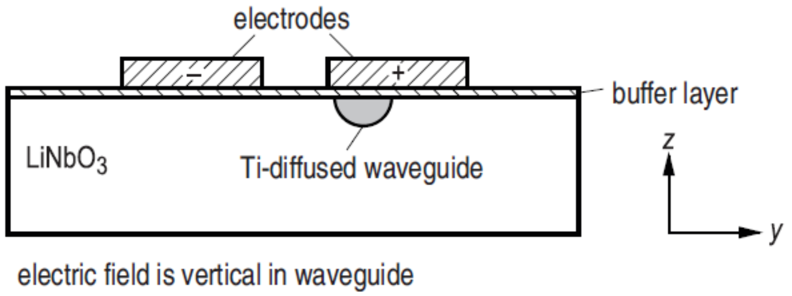
$$n_0' = \frac{n_0}{\sqrt{1 + r_{13}n_0^2 E_z}} = n_0\left(1 - \frac{1}{2} r_{13} n_0^2 E_z\right)$$

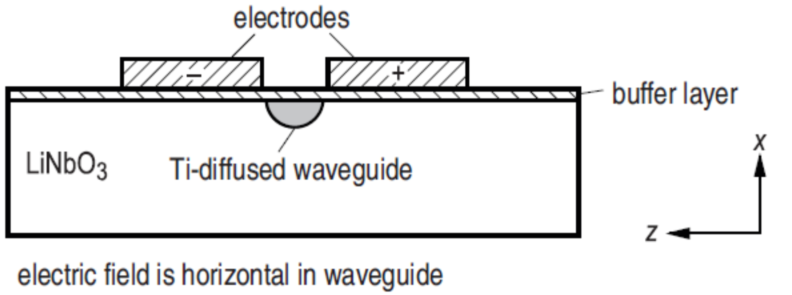
$$\boxed{n_e' = \frac{n_e}{\sqrt{1 + r_{33}n_e^2 E_z}} = n_e\left(1 - \frac{1}{2} r_{33} n_e^2 E_z\right)}$$

$r_{33} > r_{13}$

Interesting for integrated waveguide modulators!

Lithium niobate – Integrated phase modulators



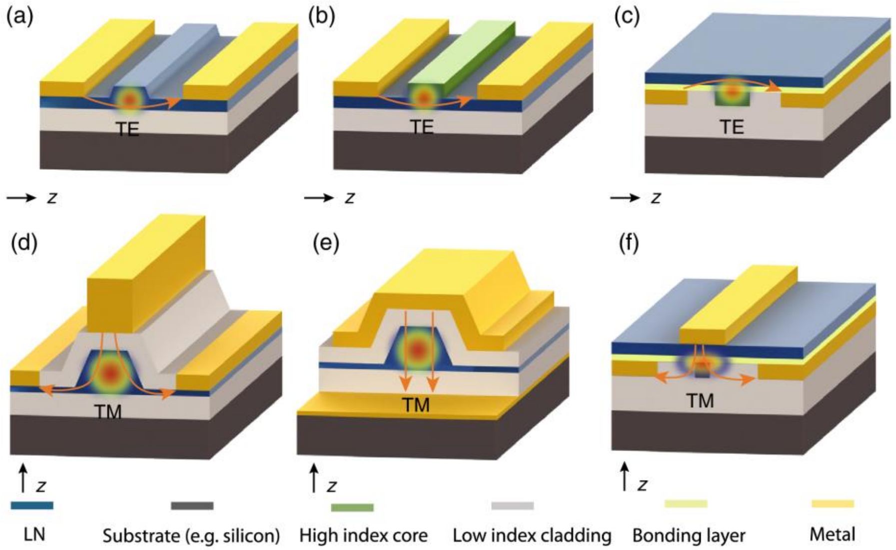


Phase retardation of extraordinary wave:

$$V_\pi \approx \frac{\lambda d}{L n_e^3 r_{33}}$$ (typ. 3-7 V)

d: effective distance between electrodes
r_{33}: 30 pm/V

Microphotonic waveguide phase modulators
Polarization of light-wave and external field in z-direction (extraordinary propagation)

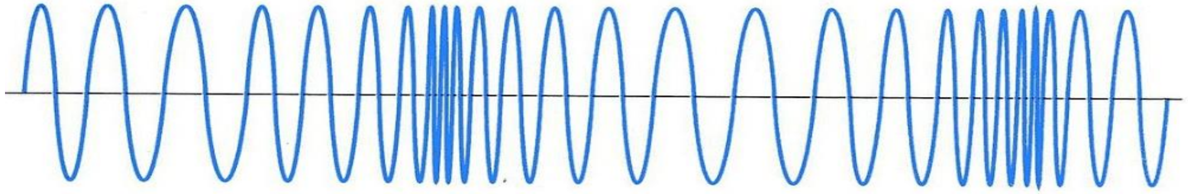


Mian Zhang et al., "Integrated lithium niobate electro-optic modulators: when performance meets scalability," Optica **8,** 652-667 (2021)

Phase modulation

When only ordinary or extraordinary wave is excited no polarization rotation occurs; only phase retardation.

$$E^{\omega}(t) = E_0 \cos\left[\omega t + \phi(t)\right]$$



$$\begin{aligned} E^{\omega}(t) &= E_0 \cos\left(\omega t + m \sin \omega_m t\right) \\ &= E_0 \left[\cos \omega t \cos\left(m \sin \omega_m t\right) - \sin \omega t \sin\left(m \sin \omega_m t\right)\right] \end{aligned}$$

$$\begin{aligned} E^{\omega}(t) = \; & E_0 \left[J_0(m) \cos \omega t \right. \\ & + J_1(m) \cos\left(\omega + \omega_m\right) t - J_1(m) \cos\left(\omega - \omega_m\right) t \\ & + J_2(m) \cos\left(\omega + 2\omega_m\right) t + J_2(m) \cos\left(\omega - 2\omega_m\right) t \\ & + J_3(m) \cos\left(\omega + 3\omega_m\right) t - J_3(m) \cos\left(\omega - 3\omega_m\right) t \\ & \left. + \ldots \right]. \end{aligned}$$

Some useful mathematics:

$$2 \sin A \sin B = \cos(A - B) - \cos(A + B),$$

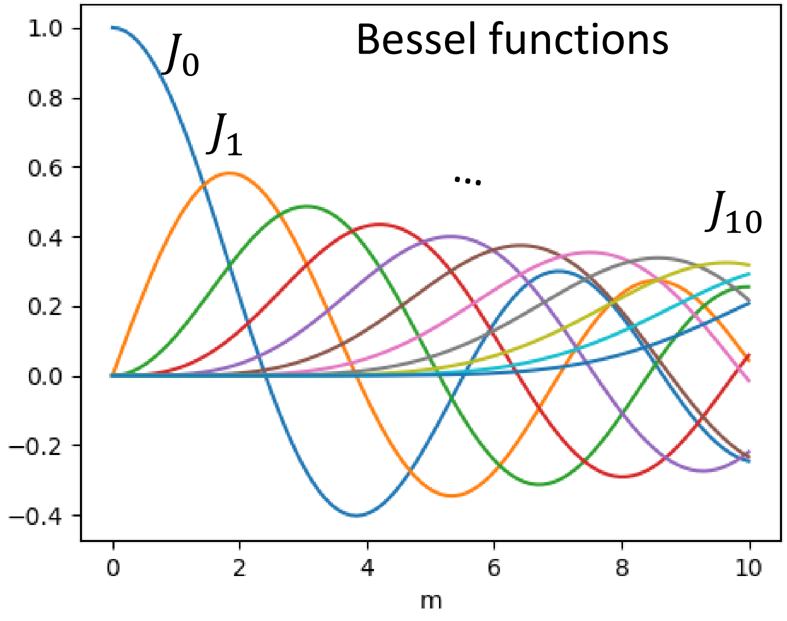
$$2 \cos A \cos B = \cos(A - B) + \cos(A + B),$$

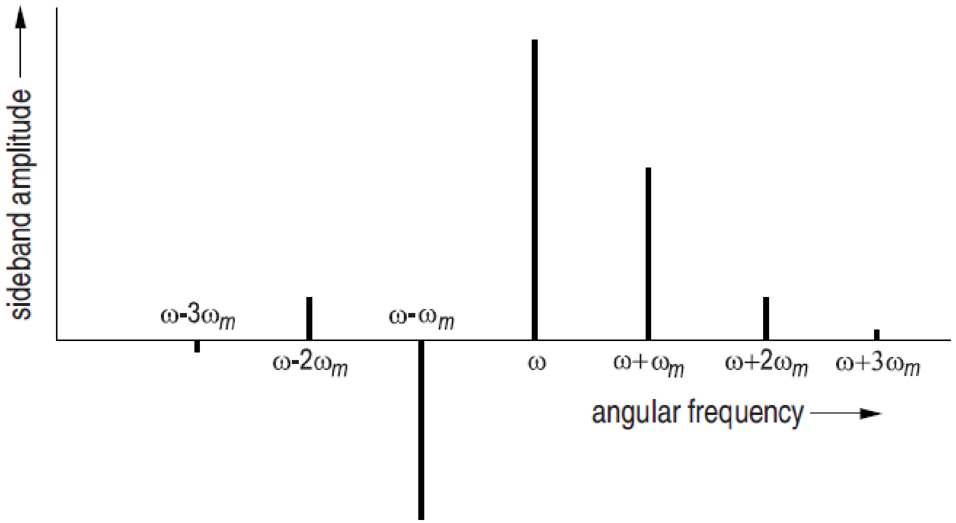
$$\cos\left(m \sin \omega_m t\right) = \boxed{J_0}(m) + 2 \sum_{k=1}^{\infty} \boxed{J_{2k}}(m) \cos\left(2k\omega_m t\right),$$

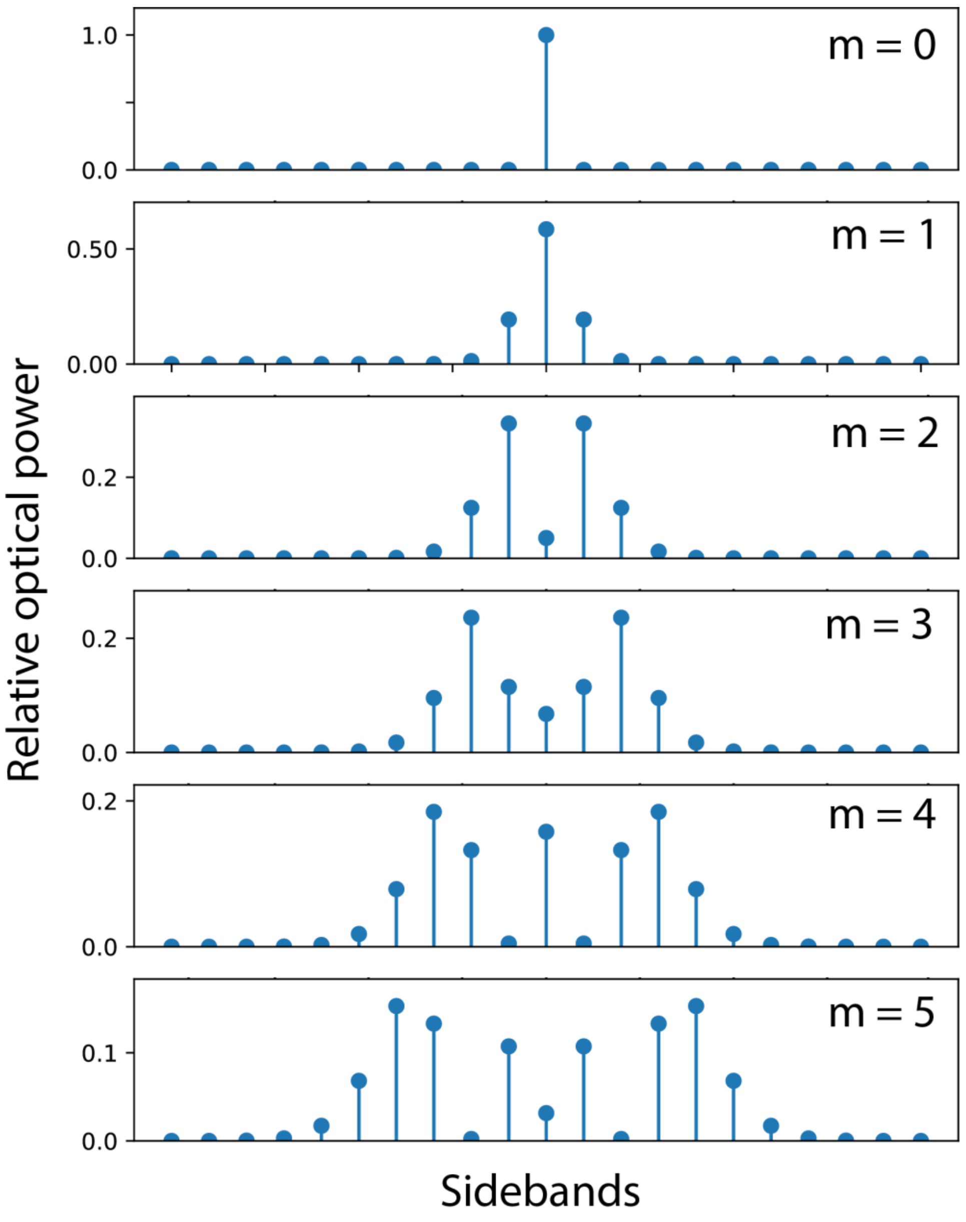
Bessel function
of the first kind

$$\sin\left(m \sin \omega_m t\right) = 2 \sum_{k=0}^{\infty} \boxed{J_{2k+1}}(m) \sin\left[\left(2k+1\right) \omega_m t\right],$$

Phase modulation



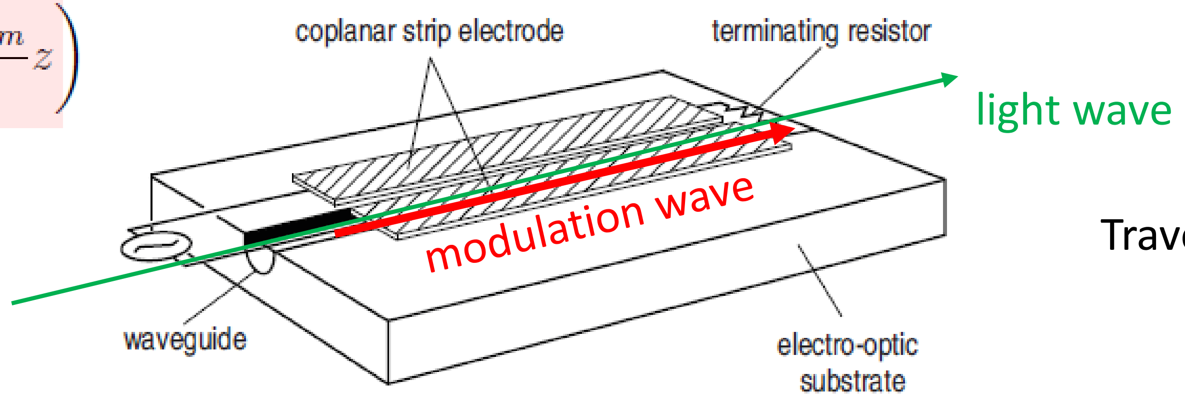




Microwave modulation

modulation:

$$V(z,t) = V_0 \cos\left(\omega_m t - \frac{\omega_m n_m}{c_0} z\right)$$

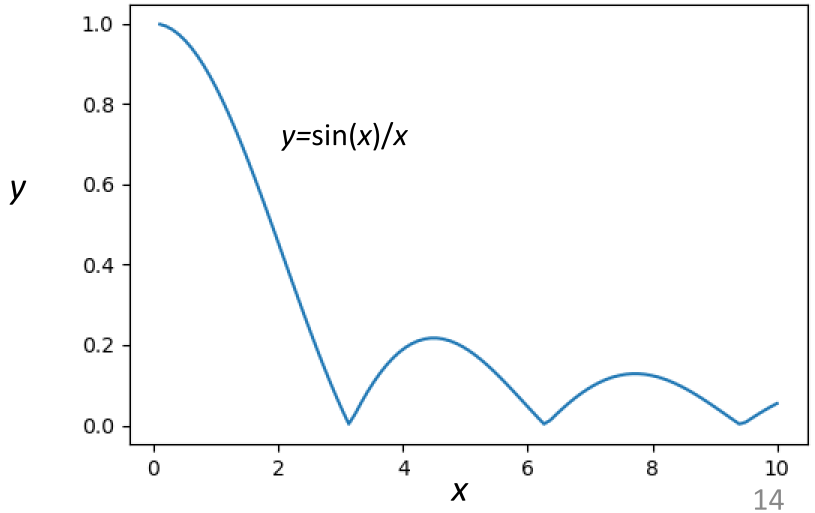


Travelling wave modulator

Experienced Voltage in frame co-moving with **light wave**

$$V(z) = V_0 \cos\left[\frac{\omega_m z}{c_0}(n - n_m)\right] \qquad \Delta n(z) = aV(z)$$

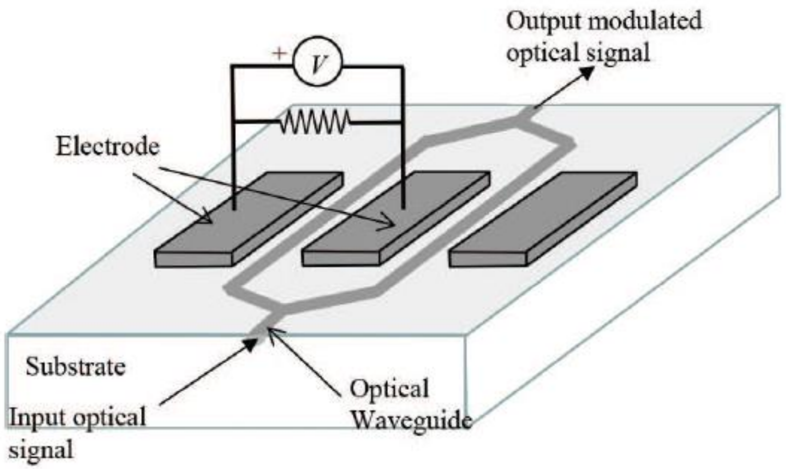
$$\Delta\phi = \int_0^\ell \frac{\omega \Delta n(z)}{c_0} dz = \frac{\omega a V_0}{\omega_m (n - n_m)} \sin\left[\frac{\omega_m \ell}{c_0}(n - n_m)\right]$$

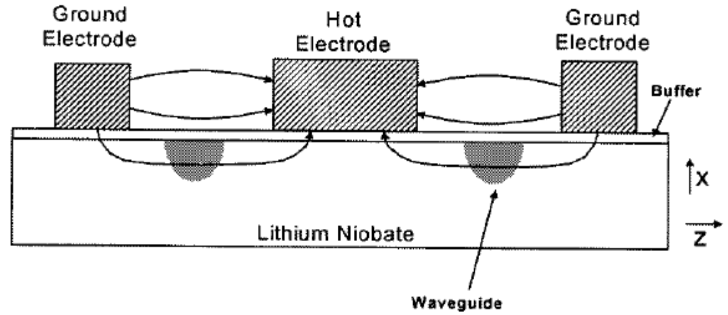


max. useful modulation frequency in commercial devices (l = 10 mm): $\omega_{m,max} \approx$ 20 GHz

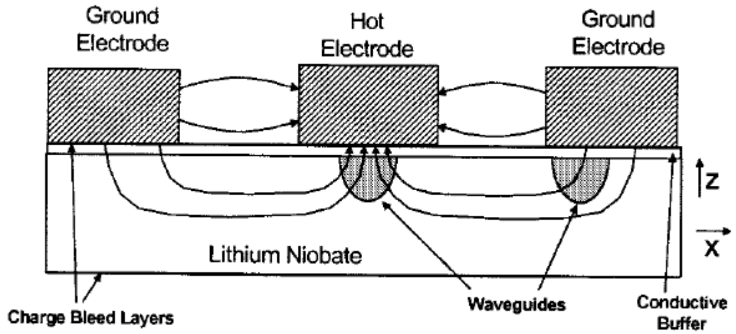
Intensity modulation in waveguides

Integrated intensity modulator

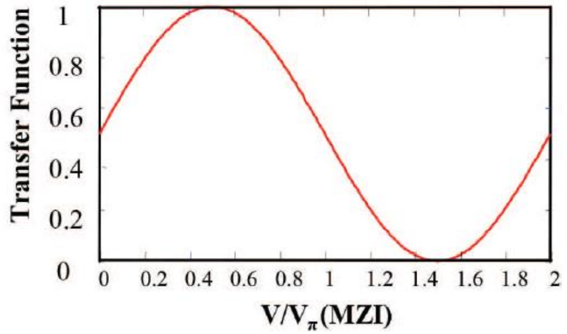


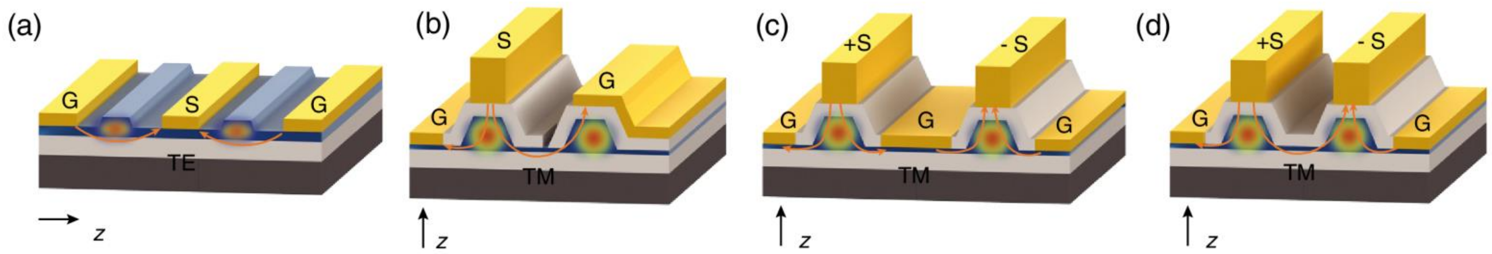


(b)



$$T_{\mathrm{MZI}}(V) = \frac{1}{2}\left[1 + \cos\left(\pi \frac{V}{V_\pi} + \theta\right)\right]$$





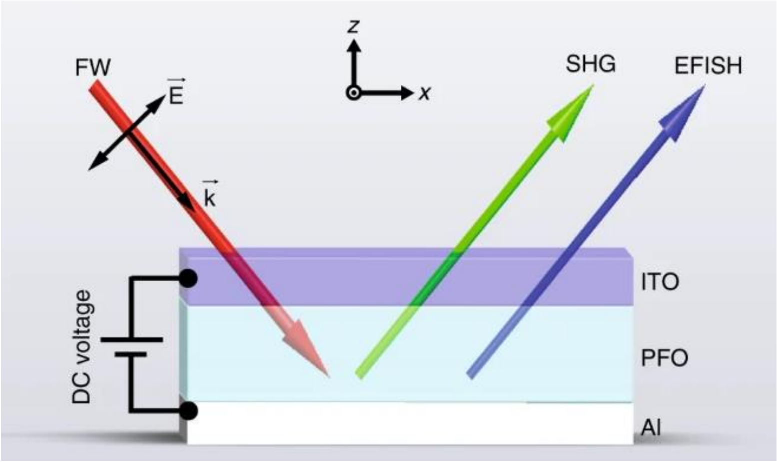
Electric-field induced quadratic nonlinearity

$$\chi^{(2)} = 0$$

$$\chi^{(3)} \neq 0 \longrightarrow \boxed{\text{“}\chi^{(2)}_{\text{ind}} = \chi^{(3)} E\text{”}}$$

$$\text{“}D_i^{2\omega} \sim \underbrace{\chi^{(3)}_{ijkl} E_j}_{\chi^{(2)}_{\text{ind},ikl}} E_k^{\omega} E_l^{\omega}\text{”}$$

Electric-field induced second harmonic generation (EFISH)



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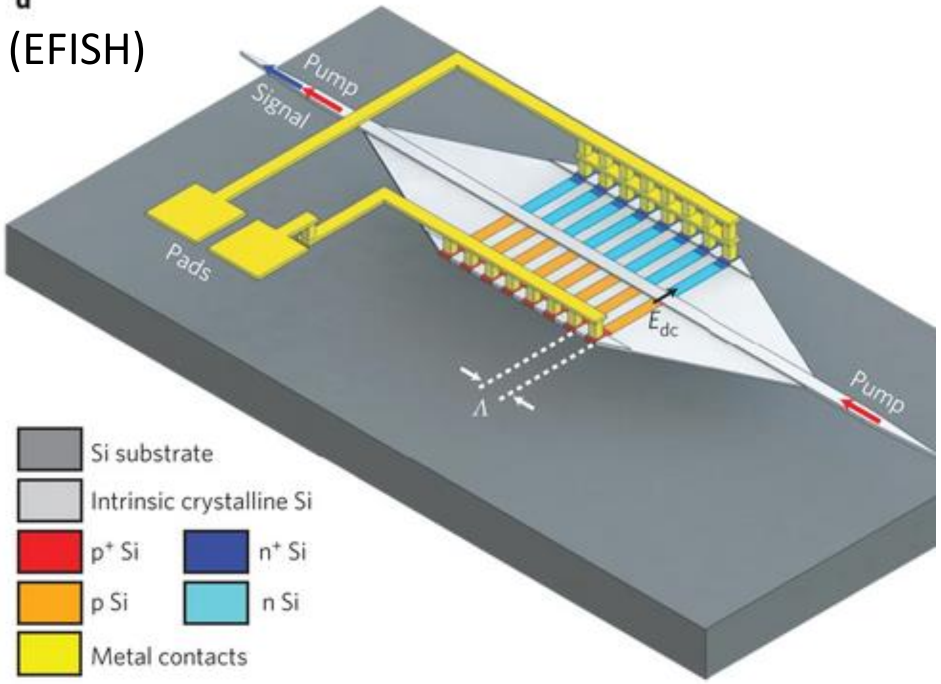
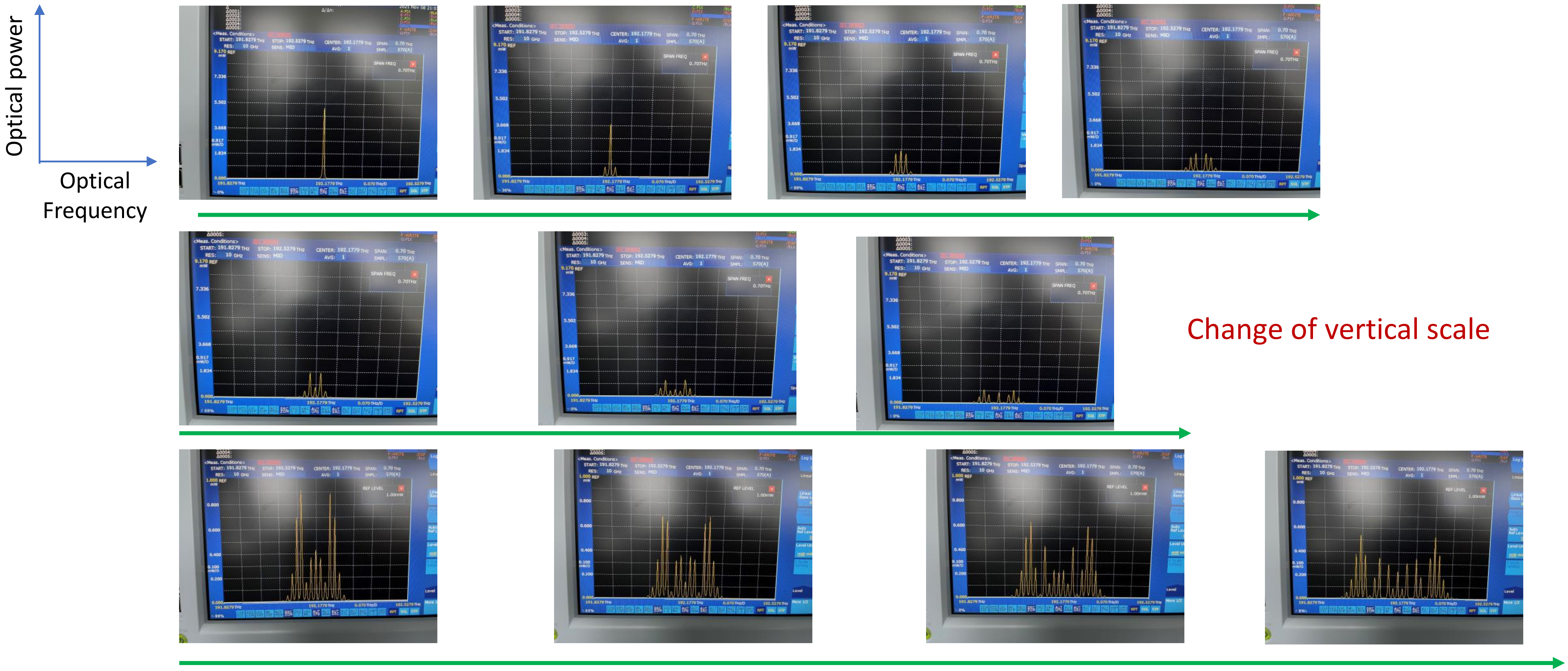


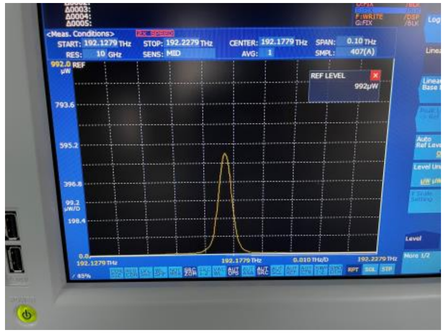
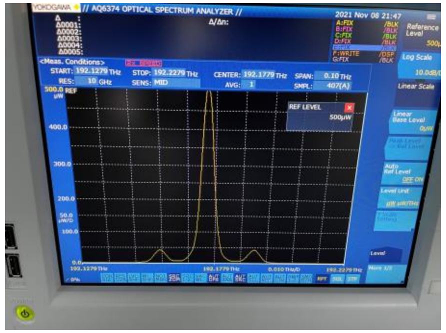
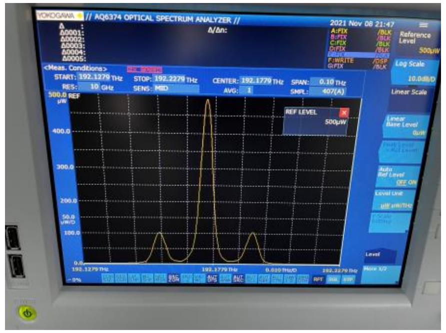
Experiment after lecture: Phase modulation



Increasing modulation amplitude

Modulation frequency 18 GHz

Experiment after lecture: Intensity modulation


Increasing modulation amplitude




Using phase bias in the interferometer to suppress central frequency component

Modulation frequency 18 GHz