Universität Hamburg
Physics Department

Ultrafast Sources
SoSe 2021

Problem Set 7

Issued: June 1st, 2021. Due: June 8th, 2021

Problem 7.1: Noise and thermal equilibrium energies in electronic circuits (10 Points)

This problem shows that in thermal equilibrium RLC – Circuits store kT/2 thermal energy in an L or a C.

In class, we derived that a resistor including thermal or Johnson noise can be modelled as an ideal noise free resistor *R* or conductor *G = 1/R* with a noise voltage or current source in series or in parallel, respectively. Those noise source where characterized as white noise sources with correlation spectra as shown in the figure below.

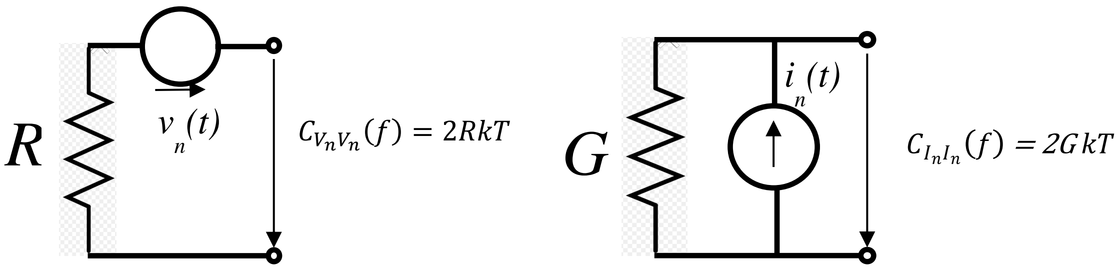


Figure 1: Equivalent circuit diagram of a noisy resistor *R* or conductor *G = 1/R*.

In class we derived, that if we put a capacitor in parallel to the noise resistor R, there builds up a steady state fluctuation voltage on the capacitor with correlation spectrum

$$C_{VV}(f) = \frac{2RkT}{1+(\omega RC)^2} .$$

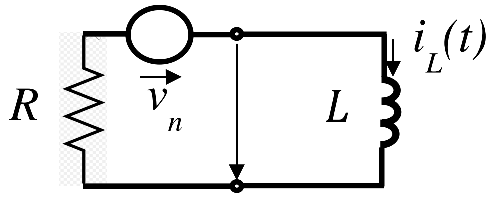
a.) What is the variance $\langle v^2 \rangle$ of the voltage at the capacitor? **(3 Points)**

Use: $\int_{-\infty}^{+\infty} \frac{1}{a^2+x^2} dx = \frac{\pi}{a}$.

b.) What is average thermal energy stored on the capacitor? **(1 Point)**

Now, we connect instead of a capacitor an inductor in parallel to the resistor.

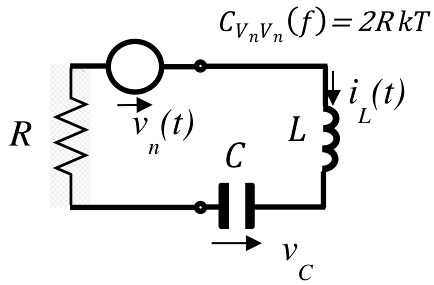
c.) What is the correlation spectrum of the current flowing through the inductor? **(2 Points)**



d.) What is the variance of the current $\langle {i_L}^2 \rangle$ through the inductor *L*? **(3 Points)**

e.) What is the average kinetic energy stored on the inductor in thermal equilibrium?**(1 Point)**

f.) Now, we connect both a capacitor and an inductor in series to the resistor, i.e. we form a LC-circuit, where the resistor models the loss in the LC-circuit.



We know that the average energy of an oscillator in thermal equilibrium is equal to kT. **Proof that this is also the case for our LC-circuit*.**

Proof as an intermediate step that the autocorrelation spectra of the current through inductor and the voltage at the capacitor are:

$$C_{I_LI_L}(f) = \frac{2RkT}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}; \quad C_{V_CV_C}(f) = \frac{(\omega C)^2 \cdot 2RkT}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

What is the variance of the voltage at the capacitor and the variance of the current in the inductor in thermal equilibrium?

This task is a bonus task, if you can proof or disproof the above statement * you will be granted full points on all problemsets.

Problem 7.2 Noise in modelocked Lasers (15 Points)

We consider the modelocked Er-doped fiber laser shown in Figure 1. We assume, that the HWHM gain bandwidth is 10 nm, the total losses due to the saturable aborber, output coupler and coupling losses are 26%. The center wavelength is 1550nm. The generated pulses are of 180 fs duration, the rep- etition rate of the laser is 400 MHz and the average output power is 12 mW. In addition we assume that the pumped Er-doped fiber has an excess noise factor of Θ = 2 and we estimate for the energy decay time into equilibrium to be $\tau_w = 50 \cdot T_R$.

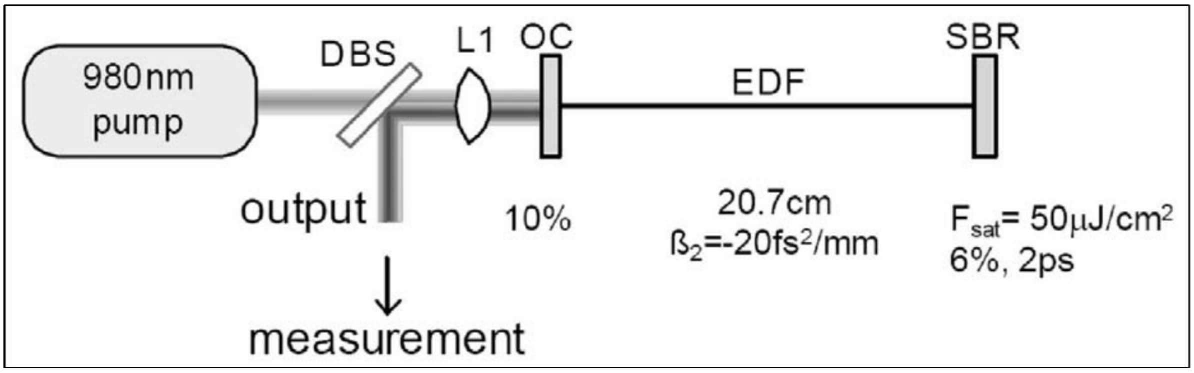


Figure 1: Schematic of soliton fiber laser modelocked with a semiconductor saturable Bragg reflector

a. What are the values of the saturated gain, the decay time for carrier frequency fluctuations, the average power and photon number in the laser? **(3 Points)**

b. What is the value of the soliton phase shift per roundtrip in the laser cavity? **(3 Points)**

c. Compute and represent graphically the timing jitter spectral density of the pulse train from the mode-locked laser due to the spontaneous emission noise of the fiber amplifier. **(3 Points)**

d. Compute and represent graphically the integrated timing jitter in the frequency range $[f_{min}\ ,\ \infty]$ as a function of f_{min}. **(3 Points)**

e. Compute and represent graphically the linewidth of each comb line as a function of mode number n. **(3 Points)**