Problemset 6

Universität Hamburg
Physics Department
Ultrafast Sources
SoSe 2021

Issued: May 25th, 2021

Due: June 1st, 2021

Problem 6.1: Active Mode-Locking and Gaussian Pulse Analysis (30 points in total)

After having considered passive Q-switching during the last problemset, we now interest ourselves in active mode-locking. The objective of this exercise is to study analytically the steady-state operation of an actively mode-locked laser using Gaussian pulse analysis. In the later part, numerical simulation of its operation will be implemented using a similar technique as previously proposed for nonlinear pulse propagation in problemset 3.

The master equation for active mode-locking by pure loss modulation is given by the following expression

$$T_R \frac{\partial A}{\partial T} = \left[g - l + D_g \frac{\partial^2}{\partial t^2} - M_s t^2 \right] A, \tag{1}$$

where the loss due to the modulator has already been written as a parabolic approximation, and the other terms are explained in the lectures. As was the case for the Nonlinear Schrödinger equation (see problemset 3), this equation admits a formal solution which can be expanded to any desired order. We propose here to use a first order propagation scheme, to be compared to the order 3 propagation scheme proposed in problemset 3.

We can class the different operators involved in the propagation of the pulse in two distinct categories, depending on wether it is easier to apply them in the frequency or the time domain. As g and l are time and frequency independent on the timescales considered, they could be applied interchangeably in the time and the frequency domain. However, these terms result from an expansion in the frequency domain, and thus should be applied along the third operator, which is easily applied in the frequency domain, where the partial time derivatives are replaced with the angular frequency. The operator $M_s t^2$ is easier to apply in the time-domain.

We start the analysis of the system with a Gaussian pulse

$$A_0(t) = a_0 e^{-\Gamma_0 t^2/2}, \tag{2}$$

with complex Γ_0 parameter. After propagation through the gain and loss part of the system within one round trip, the amplitude field can be described as

$$\hat{A}_1(\omega) = e^{g-l-D_g\omega^2}\hat{A}_1(\omega). \tag{3}$$

Propagation through the modulator can then be written as

$$A_2(t) = e^{-M_s t^2}A_2(t). \tag{4}$$

a) Defining

$$A_i(t) = a_i e^{-\Gamma_i t^2/2}, \tag{5}$$

derive a_1, a_2, Γ_1, Γ_2 as functions of A_0, D_g, Γ_0, M_s, g and l. (5 points)

b) Define the equations for steady-state operation in terms of a_i and Γ_i. Using these equations and the results of the previous point, derive the equations linking A_0, D_g, Γ_0, M_s, g and l without solving them. (5 points)

c) Solve the equations derived in the last point under the assumption $D_g \ll 1/\Gamma_0$, keeping terms up to the first non-trivial order. Write the expression of the resulting steady-state pulse. What is the pulse duration in terms of the cavity parameters? (5 points)

d) Using the equations derived in point b), and without making the approximation $D_g \ll 1/\Gamma_0$, rewrite the expressions for $g - l$ and the steady-state parameter Γ in terms of cavity parameters. These non-approximated forms are needed for a proper implementation of the pulse propagation. (5 points)

e) Using the equations given above, the expressions derived in the last point and inspiration from problemset 3, implement the numerical propagation of a pulse in a mode-locked laser in steady state. Use $D_g = 0.24$ ps^2 and $M_s = 4 \cdot 10^{16}$ s^{-2}. Use Gaussian pulses as initial fields and propagate them for 10'000 roundtrips. Plot the frequency and time domain evolution of the pulses as a function of the number of roundtrips. What is the final pulse duration of a pulse with 50 ps FWHM as input? And for 100 ps? (10 points)