

Problem Set 5

Issued: May 18, 2021.

Due: May 25, 2021.

*Instruction: Please submit your answers to the problem set in digital form (e.g. “ *.pdf ” or similar) to your tutor. If you collaborated on a problem, please note down their names as well.*

In the previous problem set we discussed the rate equations for a 4-level system and how that influences the output power of our laser cavity. Now we will continue to look at more applied properties of such a laser cavity. Specifically, we will look at how the output power reacts to fluctuations in the pump power and how you can achieve pulsed operation of the cavity via passive Q-switching.

Problem 5.1: Relaxation Oscillations (10 points)

We consider a continuously operating Ti:sapphire laser at a wavelength of $\lambda_0 = 0.8\mu\text{m}$, 100 MHz repetition rate, pumped with 5 W power. The laser threshold is 0.5 W. The upper state lifetime of Ti:sapphire is $2.5\ \mu\text{s}$ and a 10% output coupler is used.

- a) Is the relaxation oscillation in this laser over- or underdamped? **(2 Points)**
- b) What is the relaxation oscillation frequency and the damping rate of the relaxation oscillation as a function of the pump parameter r ? **(3 Points)**
- c) Plot the root locus of the complex relaxation oscillation frequency as a function of the pump parameter. What is the result for the pumping conditions described above? **(3 Points)**
- d) How much pump power is required to suppress relaxation oscillations? **(2 Points)**

Problem 5.2: Passive Q-Switching (20 points)

We consider a microchip laser with the following laser cavity parameters:

Parameter	Value
$2g_0$	0.7
$2l$	0.14
T_R	2.7 ps
τ_L	0.87 μ s
E_L	0.2 μ J

To achieve a pulsed output from the laser we use passive Q-Switching. For passive Q-Switching we need a passive modulation of our losses inside our laser cavity, therefore we will try two different saturable absorbers. (1) A fast saturable absorber with an absorption depth of $2q_0 = 0.03$, a saturation energy of $E_A^{FSA} = 0.77$ pJ, and an absorber recovery time of $\tau_A^{FSA} = 1$ ps; and (2) a slow saturable absorber with an absorption depth of $2q_0 = 0.03$, a saturation energy of $E_A^{SSA} = 77$ pJ, and an absorber recovery time of $\tau_A^{SSA} = 100$ ps.

Hint: The fast saturable absorber is so fast that we can assume that it reacts instantaneously to changes of power. See lecture 11 page 4 for more.

- Is the laser with the fast saturable absorber Q-switching? **(2 Points)**
- Formulate the rate equations for case (i) and (ii). **(3 Points)**
- Write a short MATLAB routine that numerically solves these rate equations using the routine ode45 (of course, also any other numerical ode-solver can be used), see example attached. It helps to add spontaneous emission for numerical stability: for example, add a term $10^{-8}g$ in your power rate equation. **(10 Points)**
- Integrate the rate equations over many cavity round-trip times, and plot the corresponding solutions versus time (similar to Fig. 4.28 in the lecture note L11) and in a phase space (similar to Fig. 4.27 in the lecture note 11). Normalize the corresponding quantities properly. **(3 Points)**
- For the slow saturable absorber case, extract the characteristic pulse properties of the Q-switched pulse trains (pulse energy, pulse width and repetition rate) from the numerical simulations. **(2 Points)**

Example code:

```
clear all;
close all;

[t,y]=ode45(@rateEq, [0 20], [0 1]);
% "@yp_pr6": Defining the differential equation system with that function
% "[0,20]" start and end condition in time
% "[0 1]" initial conditions

figure;
yyaxis left
plot(t,y(:,1))
ylabel('state 1 amplitude')
xlabel('time (arb. u.)')
yyaxis right
plot(t,y(:,2))

ylabel('state 2 amplitude')

function dy = rateEq(t, y)
% y(1) : state 1
% y(2) : state 2

tau1= 0.5; % Time constant
%tau2=1;
% system of equations defining our linear differential equation system
dy = zeros(2,1);
dy(1) = -tau1*y(1)+y(2);
dy(2) = -tau1*y(1);
end
```