Universität Hamburg
Physics Department
Ultrafast Sources
SoSe 2021

Problem Set 2

Issued: April 20, 2021. Due: April 27, 2021.

*Instruction: Please submit your answers to the problem set in digital form (e.g. " *.pdf " or similar) to your tutor. If you collaborated on a problem, please note down their names as well.*

The previous problem set addressed linear properties of femtosecond laser pulses. Here, we will extend to the regime where the response of materials interacting with such intense laser pulses is **not** linear as a function of the electric field. The material response, i.e. the polarization, is given by:

$$P(t) = P^{(1)}(t) + P^{(2)}(t) + P^{(3)}(t) + \dots$$

$$= \epsilon_0 \left[\chi^{(1)} E(t) + \chi^{(2)} E^{(2)}(t) + \chi^{(3)} E^{(3)}(t) + \dots\right]$$

Eq. 2 shows that for an incident field *E(t)*, the generation of new frequencies via the polarization *P(t)* is possible given sufficient intensity. The $\chi^{(i>1)}$ coefficients are the dielectric tensors that weight the nonlinear polarization orders and are characteristic to the material.

Problem 2.1: Second-order effects (10 points)

Consider an electric field composed of two frequencies ω_1 and ω_2 interacting with a nonlinear medium:

$$E(t) = E_1 \exp(i\omega_1 t) + c.c. + E_2 \exp(i\omega_2 t) + c.c.$$

(a) Write the electric field component contributing to the second-order polarization.

(b) Identify the new frequencies that arise.

(c) Name each of the nonlinear processes leading to the generation of these frequencies.

Problem 2.2: Electro-optic effect (10 points)

An interesting application exploiting nonlinear processes is the electro-optic modulator in which an external voltage applied to a nonlinear crystal modifies the refractive index. Consider an electric field $E(t) = E_0 \cos(\omega t)$ passing through such crystal that is modulated by an external sinusoidal voltage with amplitude *m* and frequency ω_m. An additional phase $\varphi(t) = m \sin \omega_m t$ is induced such that:

$$E(t) = E_0 \cos(\omega t + m \sin \omega_m t)$$

(a) Develop the resulting electric field and show that the phase modulator leads to the creation of sidebands.

Hint: Consider using trigonometric identities, in particular

$$\cos(\mathrm{A} + \sin(\mathrm{B})) = \cos(\mathrm{A})\cos(\sin(\mathrm{B})) - \sin(\mathrm{A})\sin(\sin(\mathrm{B})),$$

$$2\sin A \sin B = \cos(A - B) - \cos(A + B),$$

$$2\cos A \cos B = \cos(A - B) + \cos(A + B),$$

and

$$\cos(\mathrm{m}\sin\omega_{\mathrm{m}}\mathrm{t}) = \mathrm{J}_0(\mathrm{m}) + 2\sum_{\mathrm{k}=1}^{\infty}\mathrm{J}_{2\mathrm{k}}(\mathrm{m})\cos(2\mathrm{k}\omega_{\mathrm{m}}\mathrm{t}),$$

$$\sin(\mathrm{m}\sin\omega_{\mathrm{m}}\mathrm{t}) = 2\sum_{\mathrm{k}=0}^{\infty}\mathrm{J}_{2\mathrm{k}+1}(\mathrm{m})\sin[(2\mathrm{k}+1)\omega_{\mathrm{m}}\mathrm{t}]$$

with the Bessel functions J_i.

(b) Identify the new frequencies that arise.

(c) Sketch a simple optical setup that could take advantage of an electro-optic modulator to 'select' periodically some pulses from a laser pulse train. Explain qualitatively.

Problem 2.3: Nonlinear propagation in optical fiber (10 points)]

Nonlinear effect on the evolution of the pulse spectrum: The normalized nonlinear Schrödinger equation can be written as:

$$\frac{\partial u(\xi,\tau)}{\partial \xi} = \left(\widehat{D} + \widehat{N}\right)u(\xi,\tau)$$

With the dispersion operator $\widehat{D} = -j\frac{\partial^2 u(\xi,\tau)}{\partial \tau^2}$ and the nonlinear operator $\widehat{N} = -j|u(\xi,\tau)|^2$.

(a) Ignoring the dispersion effect, the nonlinear pulse evolution can be written as:

$$u(\Delta\xi,\tau) = e^{\widehat{N}\Delta\xi}u(0,\tau)$$

Using MATLAB, plot the spectrum $S(\Omega) = |\tilde{u}(\Omega)|^2$ as the pulse accumulates different nonlinear phase $\phi_{NL} = |u(\xi,0)|^2 = 0, \frac{\pi}{2}, \pi, \frac{3}{2}\pi$ and 2π, assuming $u(0,\tau) = \exp(-\tau^2)$.

(b) What nonlinear phenomenon can be responsible for what you observe? Explain briefly.