Universität Hamburg
Physics Department
Ultrafast Sources
SoSe 2021

Problem Set 1

Issued: April 13, 2021 Due: April 20, 2021

*Instruction: Please submit your answers to the problemset in digital form (e.g. " *.pdf " or similar) to your tutor. If you collaborated on a problem, please note down their names as well. When submitting your answer to a numerical problem, please attach your code as well. The code should be executable without errors and ideally on button press.*

Problem 1.1: Time-Bandwidth Product (10 points in total)

The time-bandwidth product links the duration of an optical pulse in the time domain to its corresponding spectral width in the frequency domain. The values are pulse-shape specific, and follow from the Fourier transform relation or the uncertainty principle, as the case may be. In ultrafast optical physics, it is common to specify the full width at half-maximum (FWHM) in both time and frequency domain. The following expression describes a parabolic pulse in the time domain, in complex notation:

$$E(t) = E_0 \cdot \left(1 - \frac{t^2}{\tau^2}\right) e^{i\omega_0 t} \qquad \text{for} \qquad |t| \leq \tau \qquad (1)$$

$$E(t) = 0 \qquad \text{for} \qquad |t| \geq \tau \qquad (2)$$

where ω_0 is the angular frequency of the light field of interest.

(a) Sketch the intensity function $|E(t)|^2$ and calculate the full width at half maximum (FWHM) Δt_{FWHM} of the intensity function. **(4 points)**

(b) Calculate the Fourier transform $\tilde{E}(\omega)$. Sketch the power spectrum $\left|\tilde{E}(\omega)\right|^2$ and identify the full width at half maximum $\Delta\nu_{FWHM} = \Delta\omega_{FWHM}/2\pi$. **(4 points)**

(Hint: Introduce the variable $x = (\omega - \omega_0)\tau$ and calculate Δx_{FWHM} numerically.)
(Hint: $\lim_{x \to 0} \frac{\sin x - x \cos x}{x^3} = \frac{1}{3}$)
(Hint: www.wolframalpha.com can be helpful for a numerical calculation.)

(c) Calculate the time-bandwidth product $\Delta\nu_{FWHM} \cdot \Delta t_{FWHM}$ for this pulse shape. **(2 points)**

Problem 1.2: Material dispersion, phase velocity and group velocity (30 points in total)

Two Gaussian pulses are launched into a piece of optical fiber. Their center wavelengths are located at $\lambda_1 = 1030nm$ and $\lambda_2 = 1550nm$ respectively. Assume that the pulses are narrowband compaired to their center wavelength (spectra do not overlap).

Dispersion of the fiber can lead to pulse broadening as well as time delay between pulses as they propagate through the fiber. You are asked to estimate the pulse broadening and time delay from the dispersion characteristics of the fiber.

(a) The refractive index of fused silica, the fiber is made up from, can be empirically calculated from the Sellmeier equation. The Sellmeier equation of an fused silica is written as follows:

$$n^2 - 1 = \frac{1.04 \cdot \lambda^2}{\lambda^2 - 0.006} + \frac{0.23 \cdot \lambda^2}{\lambda^2 - 0.02} + \frac{1.01 \cdot \lambda^2}{\lambda^2 - 103.56}$$

where n is the refractive index, λ is the wavelength of interest in vacuum with the unit in μm.

i) Plot the refractive index as a function of wavelength spanning $0.5\ \mu m$ to $2\ \mu m$ **(1 Point)**
ii) Find the refractive index at λ_1 and λ_2 **(2 points)**
iii) Calculate the phase velocity and group velocity at 1030 nm and 1550 nm. **(2 points)**
iv) Numerically calculate the group velocity dispersion (GVD) at 1030 and 1550 nm. Determine the type of dispersion at these two wavelengths (positive / negative). **(4 points)**
v) Find the Zero-Dispersion-Wavelength (ZDW) of fused silica. **(2 points)**

(Hint: it will make life easier if one can plot the GVD as a function of wavelength)

(b) We will show that a gaussian pulse with transform limited pulse duration τ_0 (FWHM) centered at wavelength λ_0 will change it's pulse duration to τ_1 after propagating through a material of thickness z that only shows group velocity dispersion $\frac{d^2}{d\omega^2}k(\omega) := k''$ and no higher order dispersion.

i) What can be said about the relationship (larger, smaller, the same) between τ_0 and τ_1 before actually calculating the relationship? Make an educated guess and explain why. **(1 Point)**
ii) Write down the time-domain $E_0(t)$ and frequency-domain expression $\tilde{E}_0(f)$ for the gaussian pulse by only using natural constants and τ_0, λ_0. **(1 Point)**
iii) Write down the frequency domain expression $\tilde{E}_1(f)$ for the gaussian pulse after propagating through the medium with group velocity dispersion of thickness z by only using natural constants and $\tau_0, \lambda_0, z, k''$ **(2 Point)**
iv) Apply the inverse Fourier-transform to the dispersed pulse in the frequency domain and derive $E_1(t)$. By comparing $E_1(t)$ to $E_0(t)$ express the new pulse duration τ_1 by only using natural constants and $\tau_0, \lambda_0, z, k''$. **(4 Points)**

(c) This sub-task requires you to use the solution for subtask (b) (iii), if you were not able to solve (b), please feel free to contact your tutor and he will give you the required formulas.

Here we want you use the **numerical tools,** we discussed earlier, to describe the propagation and dispersion of optical pulses. Our model pulse will be a sech-pulse $E(t) = A_0 \text{ sech}\left(\frac{t}{\tau_0}\right) e^{-i2\pi f_0 t}$ with $\tau_0 = 100\ fs$ and $f_0 = 194\ THz$.

i) Plot the normalized electric field of the pulse and it's intensity envelope as a function time. **(1 Point)**

ii) *fft()* your pulse and plot the frequency-domain amplitude and phase as a function of frequency. **(2 Points)**

iii) To describe propagation of a pulse the group velocity needs to be incorporated. Find out how long it takes to propagate the above pulse through 1 mm of fused silica. Plot the time domain of the initial pulse and compare it to the propagated pulse. Do the same for amplitude and phase in the frequency domain.
(Use https://refractiveindex.info/?shelf=glass&book=fused_silica&page=Malitson as a resource. Hint: Calculate by using the correct group index.) **(3 Points)**

iv) The last example is unphysical in the sense that in most real materials the pulse would not only propagate, but also disperse due to group velocity dispersion. Find out how much group delay dispersion 1mm propagation in fused silica (see above) causes. Plot the intensity profiles of the initial pulse and the dispersed pulse in the time domain with their peak intensities centered at $t = 0$. Plot the same in the frequency domain in amplitude and phase **(3 Points).**

v) What amount of fused silica is necessary to reduce the peak power of the initial pulse by half? State the number and plot your result in comparison with the initial pulse. **(2 Point).**