Ultrafast Optical Physics II (SoSe 2021)
Lecture 22, June 22

(1) Pulse compression: general idea

(2) Dispersion compensation

Examples of ultrafast solid-state laser media

Broader gain bandwidth produces shorter laser pulses.

Laser Materials	Absorption Wavelength	Average Emission λ	Band Width	Pulse Width
Nd:YAG	808 nm	1064 nm	0.45 nm	$\sim$ 6 ps
Nd:YLF	797 nm	1047 nm	1.3 nm	$\sim$ 3 ps
Nd:LSB	808 nm	1062 nm	4 nm	$\sim$ 1.6 ps
$Nd:YVO_4$	808 nm	1064 nm	2 nm	$\sim$ 4.6 ps
Nd:fiber	804 nm	1053 nm	22-28 nm	$\sim$ 33 fs
Nd:glass	804 nm	1053 nm	22-28 nm	$\sim$ 60 fs
Yb:YAG	940, 968 nm	1030 nm	6 nm	$\sim$ 300 fs
Yb:glass	975 nm	1030 nm	30 nm	$\sim$ 90 fs
$Ti:Al_2O_3$	480-540 nm	796 nm	200 nm	$\sim$ 5 fs
$Cr^{4+}:Mg_2SiO_4$:	900-1100 nm	1260 nm	200 nm	$\sim$ 14 fs
Cr^{4+}:YAG	900-1100 nm	1430 nm	180 nm	$\sim$ 19 fs

Transform-limited pulse

$$\tilde{E}(\omega) = \int_{-\infty}^{\infty} E(t)\exp(-j\omega t)dt \qquad E(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \tilde{E}(\omega)\exp(j\omega t)d\omega$$

$\left|\tilde{E}(\omega)\right|^2$ has a spectrum bandwidth of $\Delta\nu$

$\left|E(t)\right|^2$ has a pulse duration of Δt

Both are measured at full-width at half-maximum (FWHM).

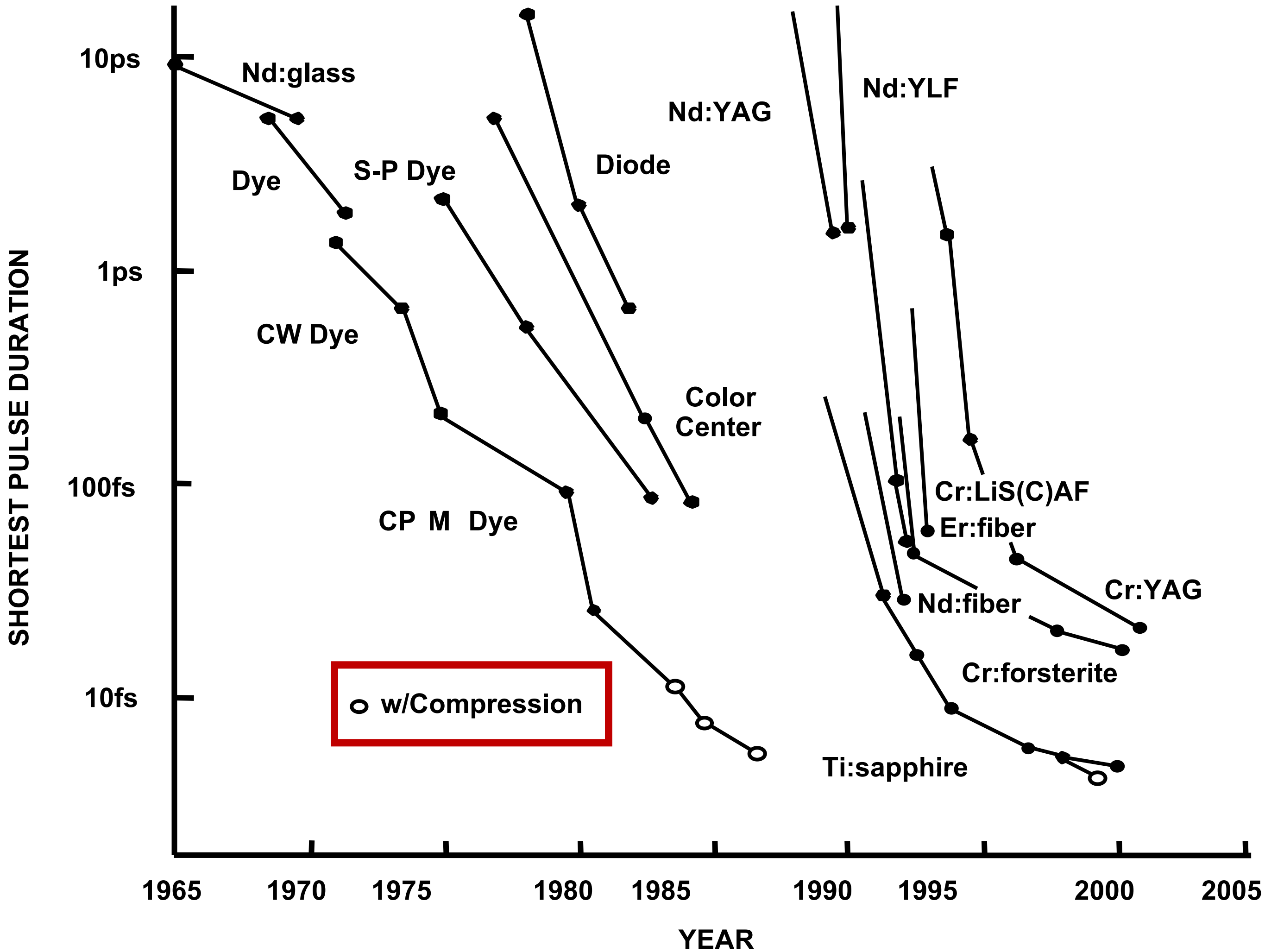
Uncertainty principle:

$$\underbrace{\Delta\nu\Delta t}_{\text{Time Bandwidth Product (TBP)}} \geq \underbrace{K}_{\text{A number depending only on pulse shape}}$$

For a given optical spectrum, there exist a lower limit for the pulse duration. If the equality is reached, we say the pulse is a transform-limited pulse.

<u>To get a shorter transform-limited pulse, one needs a broader optical spectrum.</u>

How to achieve ultrashort pulse?
To compress or not to compress



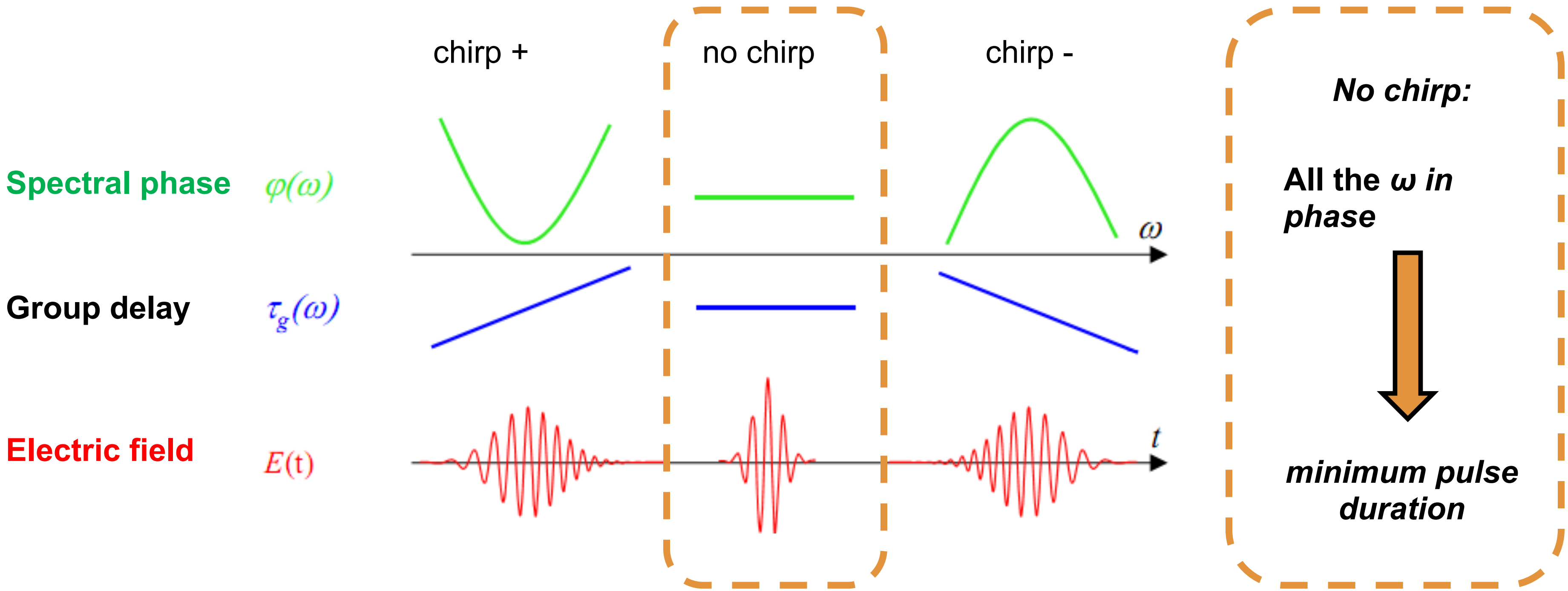
Spectral phase

Intensity Spectral phase

$$E(t) = \sqrt{I(t)}exp(i\omega_0 t - i\phi(t)) \xleftrightarrow{\text{Fourier Transform}} \tilde{E}(\omega) = \sqrt{I(\omega - \omega_0)}exp(-i\varphi(\omega - \omega_0))$$

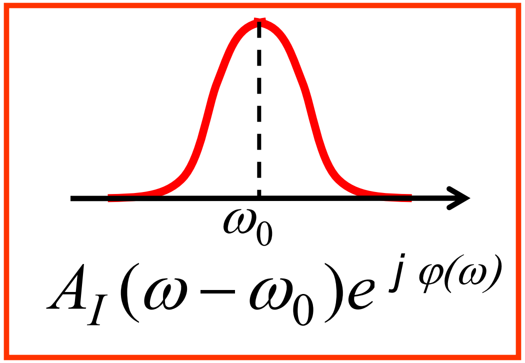
Instantaneous frequency: $\omega(t) = \frac{\mathrm{d}\phi_{tot}(t)}{\mathrm{d}t} = \omega_0 - \frac{\mathrm{d}\phi(t)}{\mathrm{d}t}$

Group delay: $\tau_g = \frac{\mathrm{d}\varphi}{\mathrm{d}\omega}$

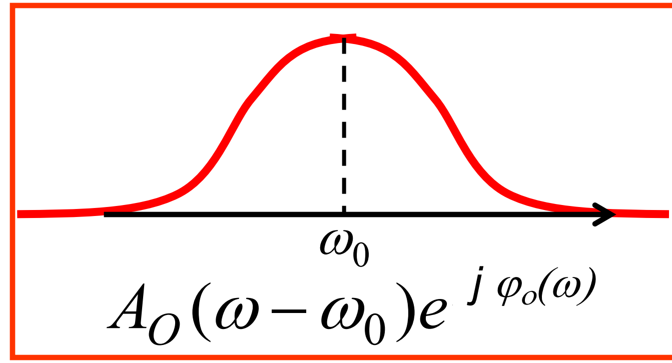


General idea of pulse compression

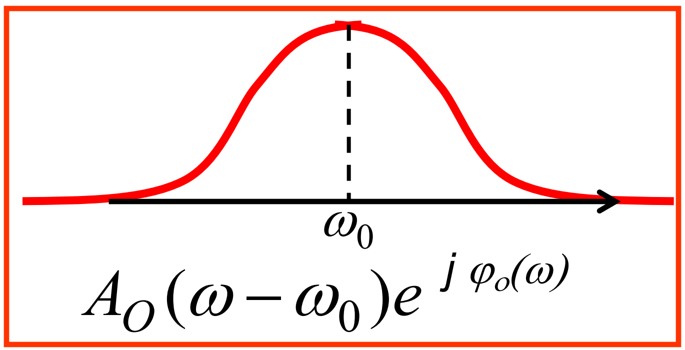
Step 1: nonlinear spectral broadening



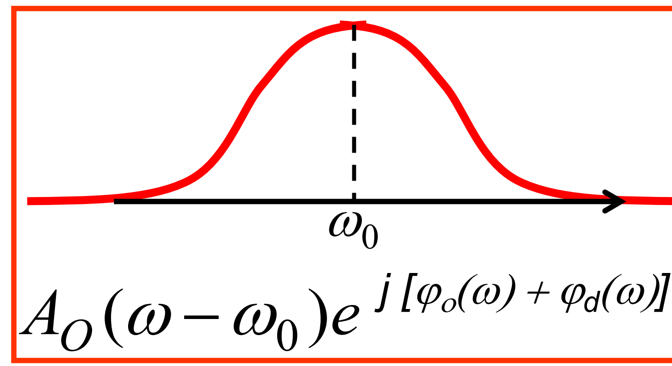
spectral broadening by light-matter nonlinear interaction



Step 2: pulse compression by a linear dispersive device



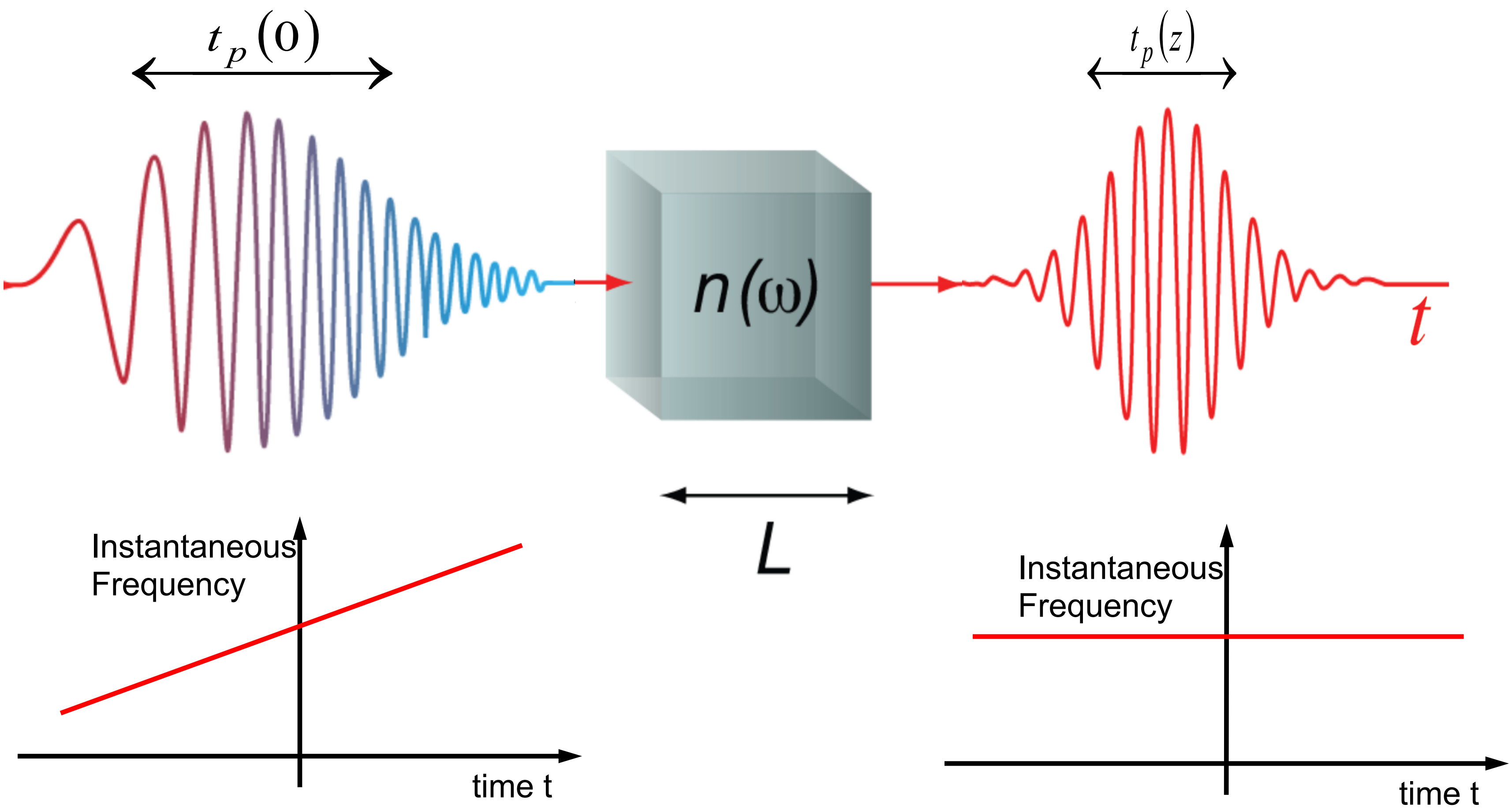
pick up an extra spectral phase



Ideal scenario: $\varphi_o(\omega)+\varphi_d(\omega)=\varphi_0+\varphi_1(\omega-\omega_0)$ $\quad \tau_g = d\varphi/d\omega = \varphi_1$

This condition guarantees a transform-limited pulse—the shortest pulse allowed by the spectrum.

Pulse travels through a dispersive bulk medium



Positive chirp

Transform-limited pulse

A dispersion compensating device can compensate for the spectral phase and then compress the stretched pulse to its transform-limited duration.

General idea of pulse compression

$$\varphi_o(\omega) = \varphi_{o,0} + \varphi_{o,1} \times (\omega - \omega_0) + \frac{1}{2}\varphi_{o,2} \times (\omega - \omega_0)^2 + \frac{1}{6}\varphi_{o,3} \times (\omega - \omega_0)^3 + \ldots$$

$$\varphi_d(\omega) = \varphi_{d,0} + \underline{\varphi_{d,1}} \times (\omega - \omega_0) + \frac{1}{2}\underline{\varphi_{d,2}} \times (\omega - \omega_0)^2 + \frac{1}{6}\underline{\varphi_{d,3}} \times (\omega - \omega_0)^3 + \ldots$$

Group delay | Group delay dispersion | 3rd-order dispersion

<u>Ideal scenario:</u>

$$\varphi_o(\omega) + \varphi_d(\omega) = \varphi_0 + \varphi_1 (\omega - \omega_0)$$

$$\varphi_{o,2} = -\varphi_{d,2}$$

$$\varphi_{o,3} = -\varphi_{d,3}$$

The broader the spectrum, the more higher-order dispersion should be matched.

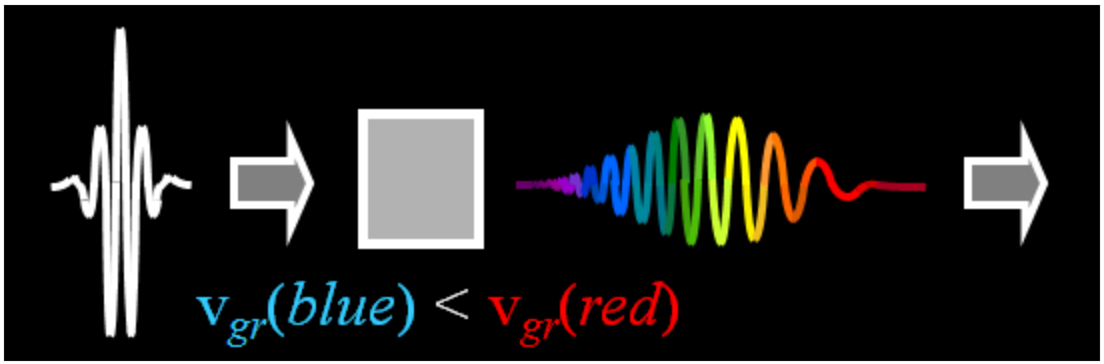
Dispersion parameters for various materials

material	λ [nm]	$n(\lambda)$	$\frac{dn}{d\lambda}\cdot 10^{-2}\left[\frac{1}{\mu m}\right]$	$\frac{d^2n}{d\lambda^2}\cdot 10^{-1}\left[\frac{1}{\mu m^2}\right]$	$\frac{dn^3}{d\lambda^3}\left[\frac{1}{\mu m^3}\right]$	$T_g\left[\frac{fs}{mm}\right]$	$GDD\left[\frac{fs^2}{mm}\right]$	$TOD\left[\frac{fs^3}{mm}\right]$
BK7	400	1,5308	-13,17	10,66	-12,21	5282	120,79	40,57
	500	1,5214	-6,58	3,92	-3,46	5185	86,87	32,34
	600	1,5163	-3,91	1,77	-1,29	5136	67,52	29,70
	800	1,5108	-1,97	0,48	-0,29	5092	43,96	31,90
	1000	1,5075	-1,40	0,15	-0,09	5075	26,93	42,88
	1200	1,5049	-1,23	0,03	-0,04	5069	10,43	66,12
SF10	400	1,7783	-52,02	59,44	-101,56	6626	673,68	548,50
	500	1,7432	-20,89	15,55	-16,81	6163	344,19	219,81
	600	1,7267	-11,00	6,12	-4,98	5980	233,91	140,82
	800	1,7112	-4,55	1,58	-0,91	5830	143,38	97,26
	1000	1,7038	-2,62	0,56	-0,27	5771	99,42	92,79
	1200	1,6992	-1,88	0,22	-0,10	5743	68,59	107,51
Sapphire	400	1,7866	-17,20	13,55	-15,05	6189	153,62	47,03
	500	1,7743	-8,72	5,10	-4,42	6064	112,98	39,98
	600	1,7676	-5,23	2,32	-1,68	6001	88,65	37,97
	800	1,7602	-2,68	0,64	-0,38	5943	58,00	42,19
	1000	1,7557	-1,92	0,20	-0,12	5921	35,33	57,22
	1200	1,7522	-1,70	0,04	-0,05	5913	13,40	87,30
Quartz	300	1,4878	-30,04	34,31	-54,66	5263	164,06	46,49
	400	1,4701	-11,70	9,20	-10,17	5060	104,31	31,49
	500	1,4623	-5,93	3,48	-3,00	4977	77,04	26,88
	600	1,4580	-3,55	1,59	-1,14	4934	60,66	25,59
	800	1,4533	-1,80	0,44	-0,26	4896	40,00	28,43
	1000	1,4504	-1,27	0,14	-0,08	4880	24,71	38,73
	1200	1,4481	-1,12	0,03	-0,03	4875	9,76	60,05

Negative GDD using angular dispersion

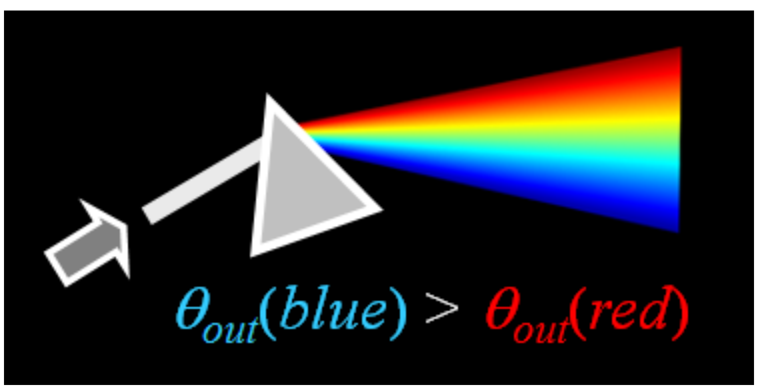
The dependence of the refractive index on wavelength has two effects on a pulse, one in time and the other in space.

Dispersion also disperses a pulse in time:



Group delay dispersion or Chirp
$d^2n/d\lambda^2$

Dispersion disperses a pulse in space (angle):

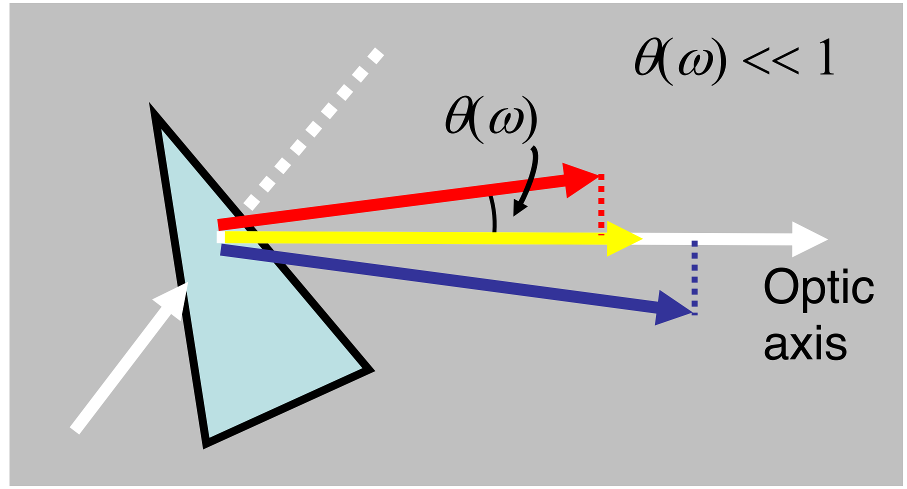


Angular dispersion
$dn/d\lambda$

<u>*Slide from Rick Trebino's Ultrafast Optics course*</u>

Negative GDD using angular dispersion

Taking the projection of $\vec{k}(\omega)$ onto the optic axis, a given frequency ω sees a phase delay of $\varphi(\omega)$:



$$\varphi(\omega) = \vec{k}(\omega) \cdot \overset{z}{\vec{r}}_{optic\ axis}$$

$$= k(\omega)\, z \cos[\theta(\omega)]$$

$$= (\omega / c)\, z \cos[\theta(\omega)]$$

We're considering only the GDD due to the angular dispersion $\theta(\omega)$ and not that of the prism material. Also $n = 1$ (that of the air after the prism).

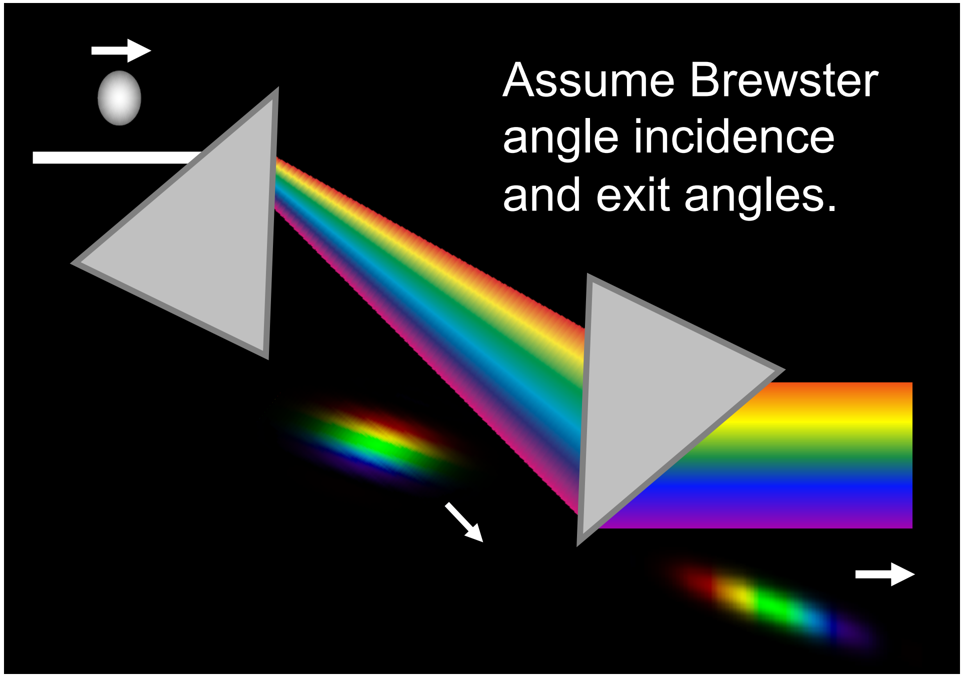
$$d\varphi / d\omega = (z / c) \cos(\theta) - (\omega / c)\, z \sin(\theta)\, d\theta / d\omega$$

$$\frac{d^2\varphi}{d\omega^2} = \cancel{-\frac{z}{c}\sin(\theta)\frac{d\theta}{d\omega}} \cancel{-\frac{z}{c}\sin(\theta)\frac{d\theta}{d\omega}} - \omega\frac{z}{c}\cos(\theta)\left(\frac{d\theta}{d\omega}\right)^2 \cancel{-\omega\frac{z}{c}\sin(\theta)\frac{d^2\theta}{d\omega^2}}$$

But $\theta << 1$, so the sine terms can be neglected, and $\cos(\theta) \sim 1$.

Slide from Rick Trebino's Ultrafast Optics course

A prism pair has negative GDD.



How can we use dispersion to introduce negative chirp conveniently?

Let L_{prism} be the path through each prism and L_{sep} ($z = L_{sep}$) be the prism separation.

$$\left.\frac{d^2\varphi}{d\omega^2}\right|_{\omega_0} \approx -4L_{sep}\,\frac{\lambda_0^3}{2\pi c^2}\left(\left.\frac{dn}{d\lambda}\right|_{\lambda_0}\right)^2 + L_{prism}\,\frac{\lambda_0^3}{2\pi c^2}\left.\frac{d^2 n}{d\lambda^2}\right|_{\lambda_0}$$

Always negative!

This term assumes that the beam grazes the tip of each prism

This term allows the beam to pass through an additional length, L_{prism}, of prism material.

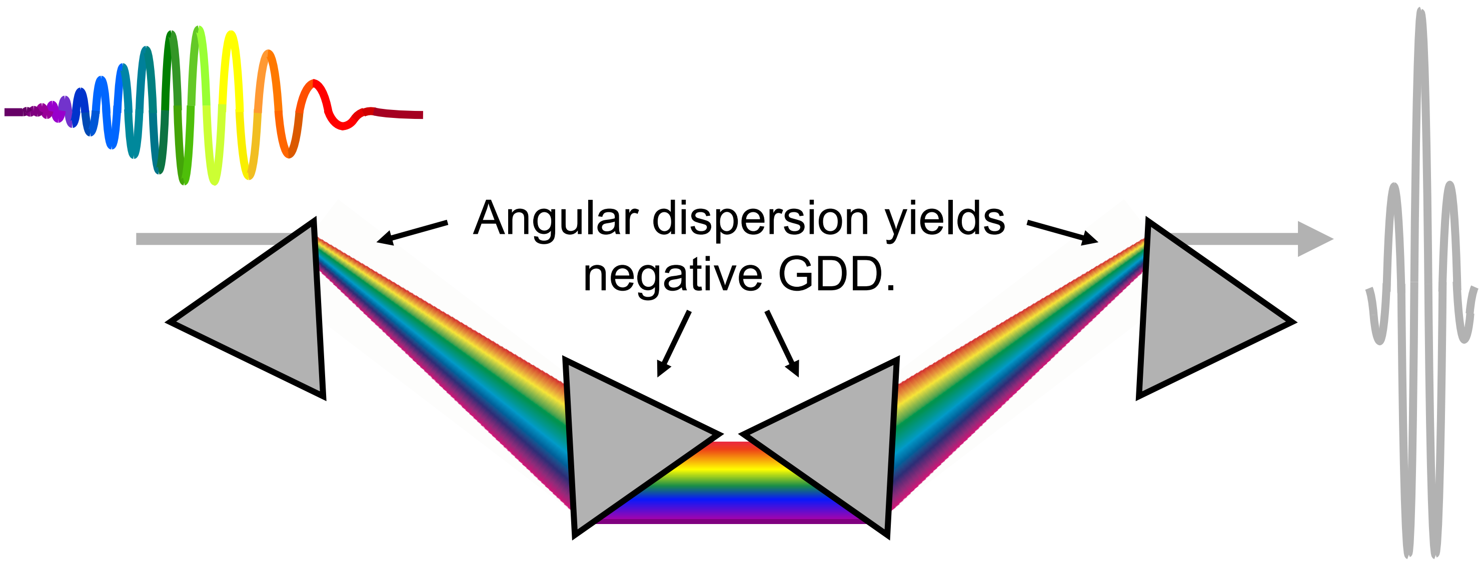
Always positive (in visible and near-IR)

Vary L_{sep} or L_{prism} to tune the GDD!

Slide from Rick Trebino’s Ultrafast Optics course

Pulse compressor using 4 prisms

This device, which also puts the pulse back together, has **negative** group-delay dispersion and hence can compensate for propagation through materials (i.e., for positive chirp).



It's routine to stretch and then compress ultrashort pulses by factors of >1000.

Slide from Rick Trebino's Ultrafast Optics course

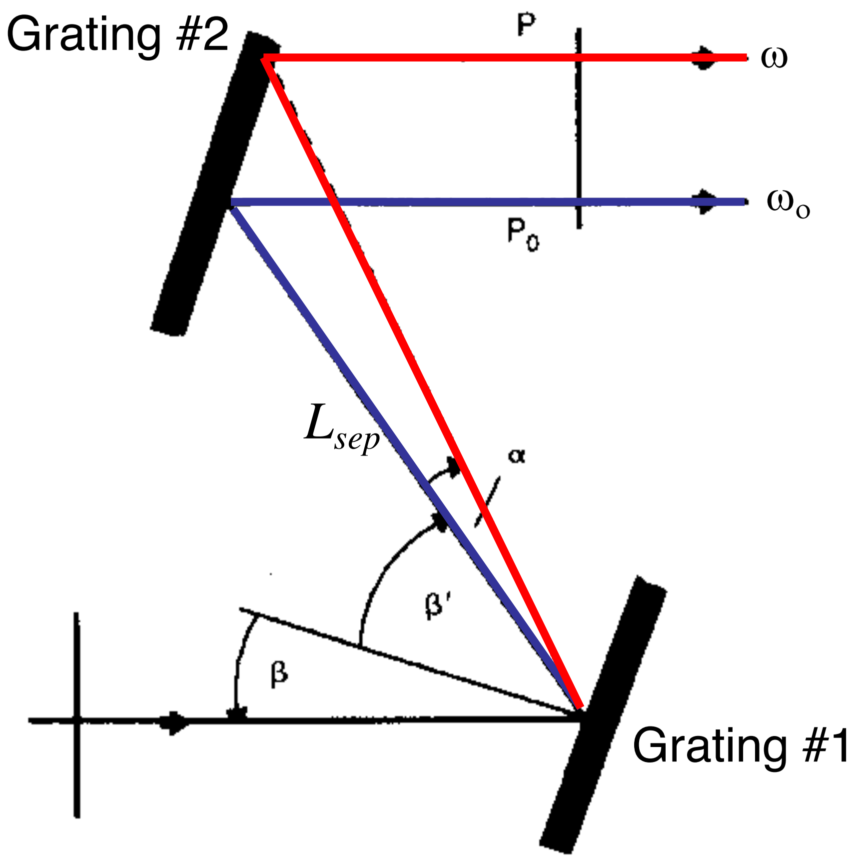
Pulse compressor using gratings

A grating pulse compressor also has negative second-order phase.

$$\left.\frac{d^2\varphi}{d\omega^2}\right|_{\omega_0} \approx -\frac{\lambda_0^3}{2\pi c^2 d^2}\frac{L_{sep}}{\cos^2(\beta')}$$

where d = grating period (same for both gratings)

Note that, as in the prism pulse compressor, the larger L_{sep}, the larger the negative GDD.



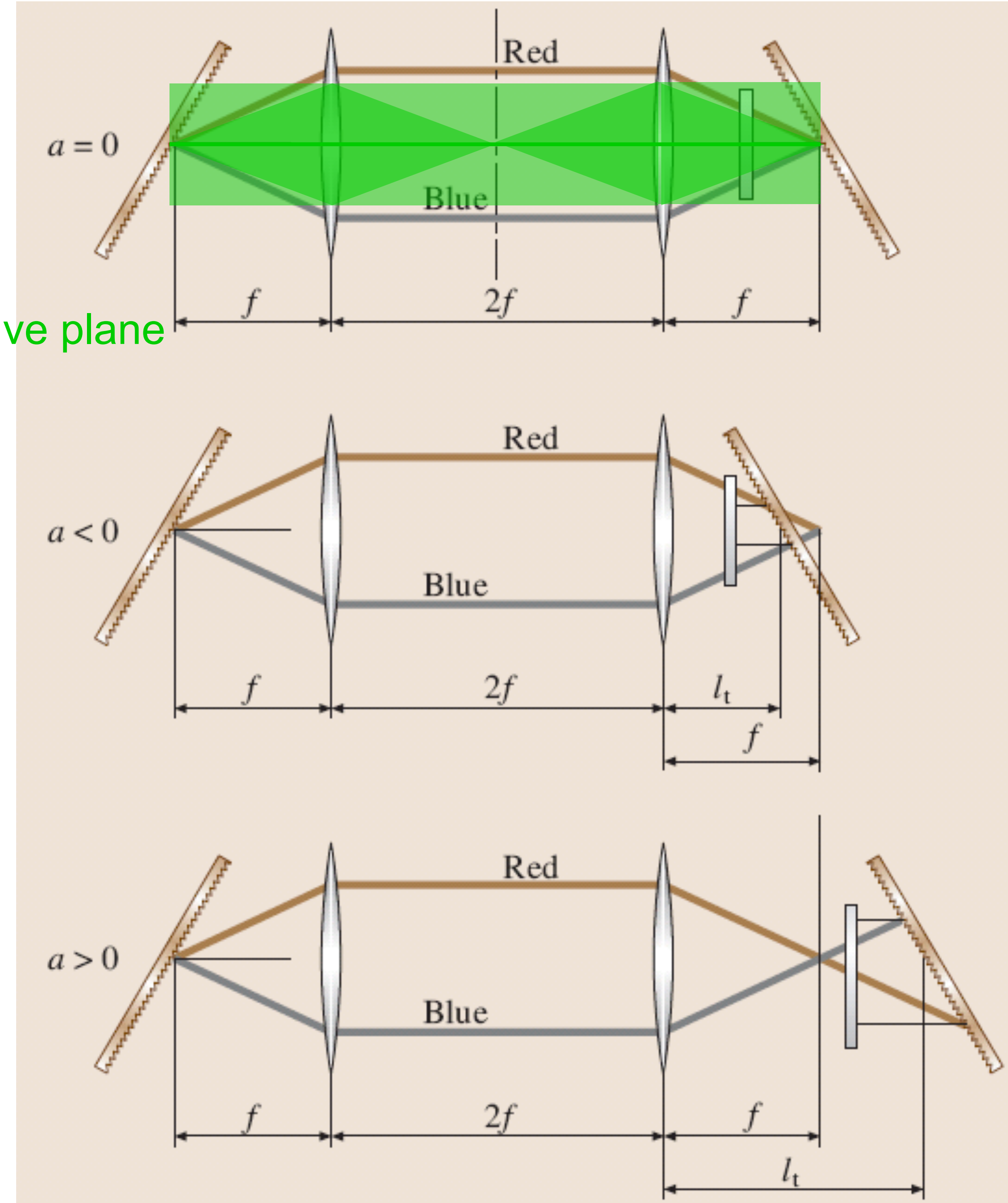
Types of grating pulse compressors

zero GDD

non-dispersive plane

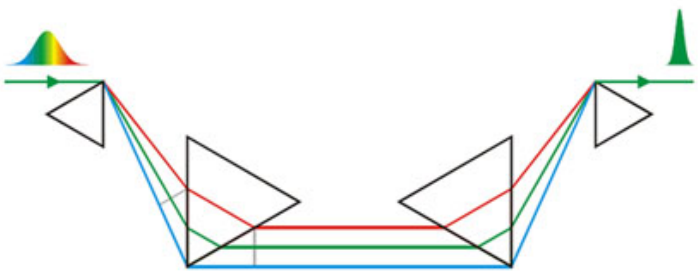
positive GDD *(stretcher)*

negative GDD *(compressor)*



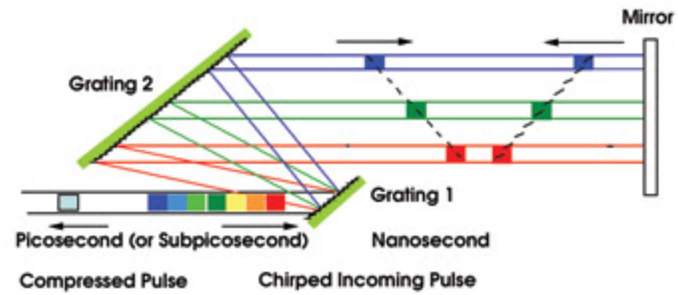
Dispersion compensation using angular dispersion

Prism pair



(1) Small dispersion
(2) Negligible loss at Brewster angle

Diffraction grating pair

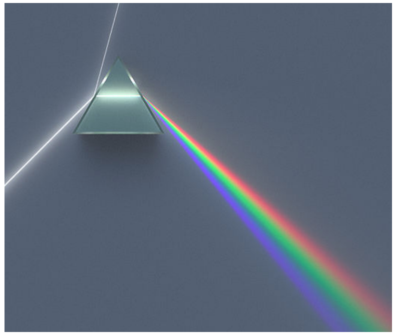


(1) Large dispersion
(2) Losses ~ 25%

Typical dispersion signs for material, grating pair, and prism pair

	Material	gratings	Prisms
2nd order dispersion	+	-	-
3rd order dispersion	+	+	-

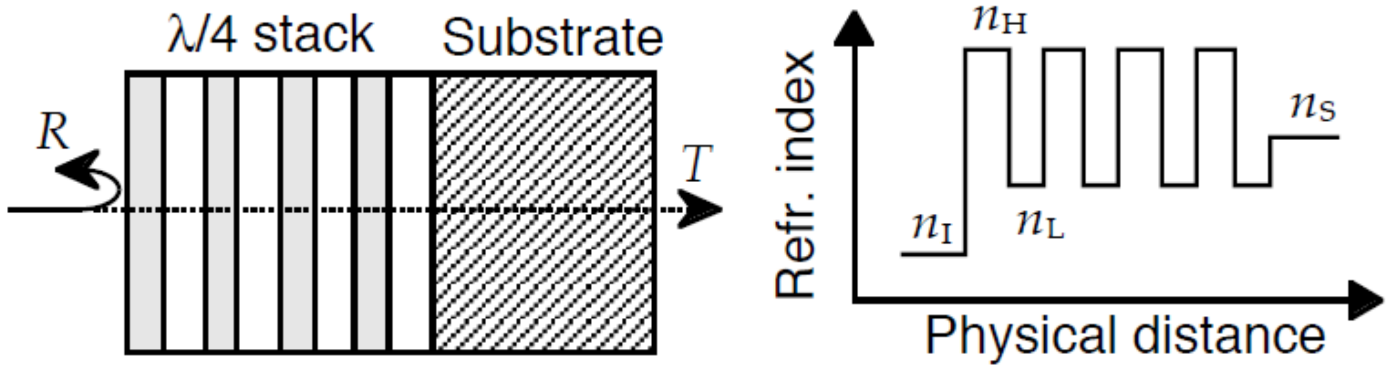
Grating pair versus prism pair



Device	λ_ℓ [nm]	φ" [fs^{+2}]	φ''' [fs^{+3}]
SQ1 ($L = 1$ cm) Piece of glass	620 800	550 362	240 280
Brewster prism pair, SQ1 $\ell = 50$ cm	620 800	-760 -523	-1300 -612
grating pair $b = 20$ cm; $\beta = 0°$ $d = 1.2$ μm	620 800	$-8.2\ 10^4$ $-3\ 10^6$	$1.1\ 10^5$ $6.8\ 10^6$

Dispersion of mirror structures: quarter-wave stack

High reflecting mirrors can be realized using a stack of thin dielectric films of different refractive indices.



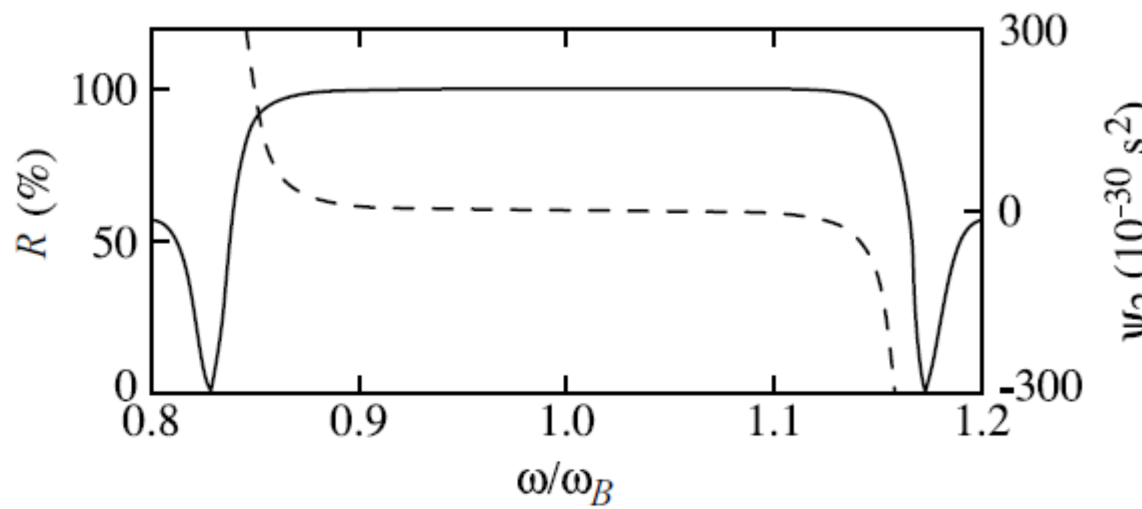
Bragg wavelength:

$$\lambda_B = 2(n_H d_H + n_L d_L)$$

$$n_H d_H = n_L d_L$$

Bandwidth of Bragg mirror:

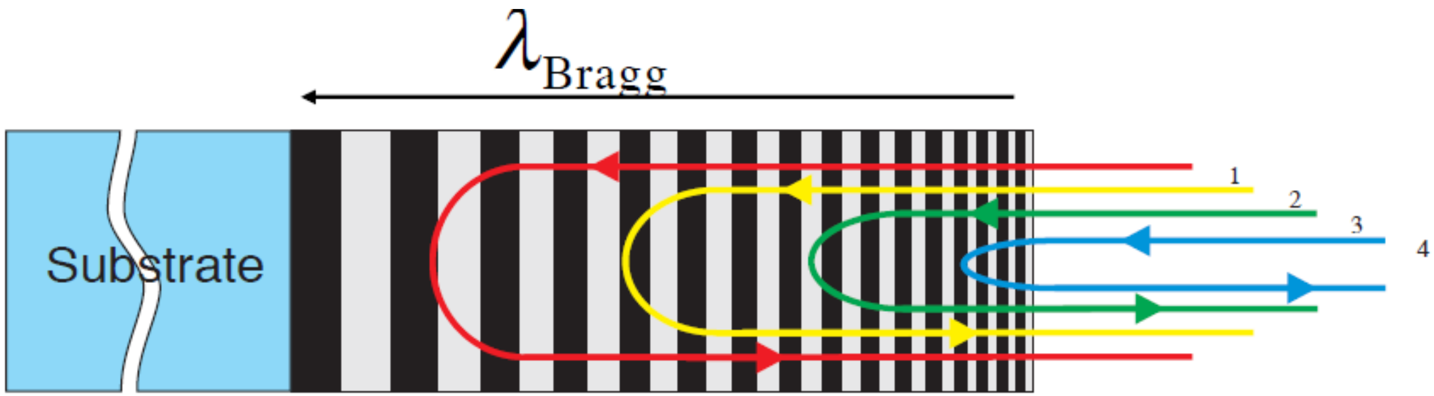
$$r_B = \frac{\Delta f}{f_c} = \frac{n_H - n_L}{n_H + n_L}$$



Typical coating example:

$$n_{SiO_2} = 1.48 \qquad n_{TiO_2} = 2.4 \longrightarrow \Delta f / f_c = 0.23$$

Chirped mirror by chirping the Bragg wavelength



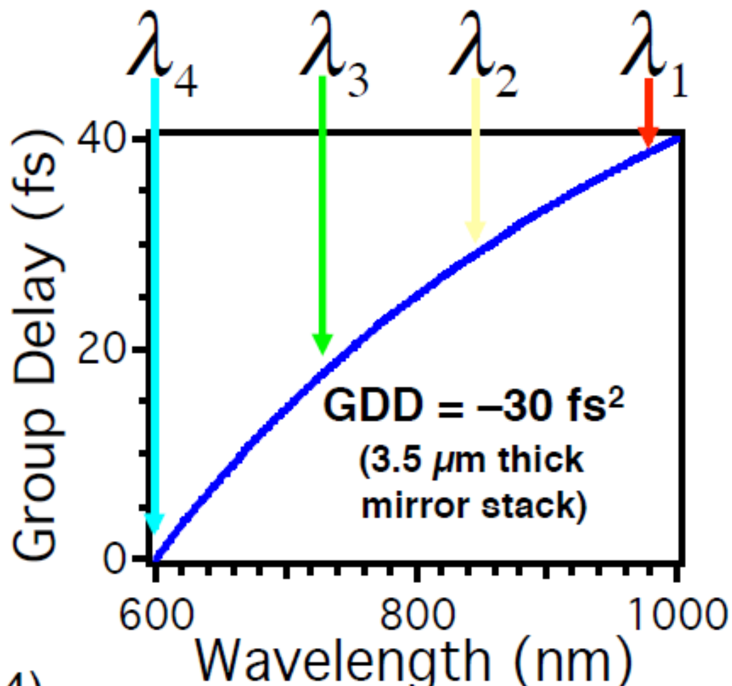
Chirp Bragg wavelength

⇒ wavelength-dependent penetration depth

⇒ engineerable dispersion

⇒ compensation of arbitrary material dispersion

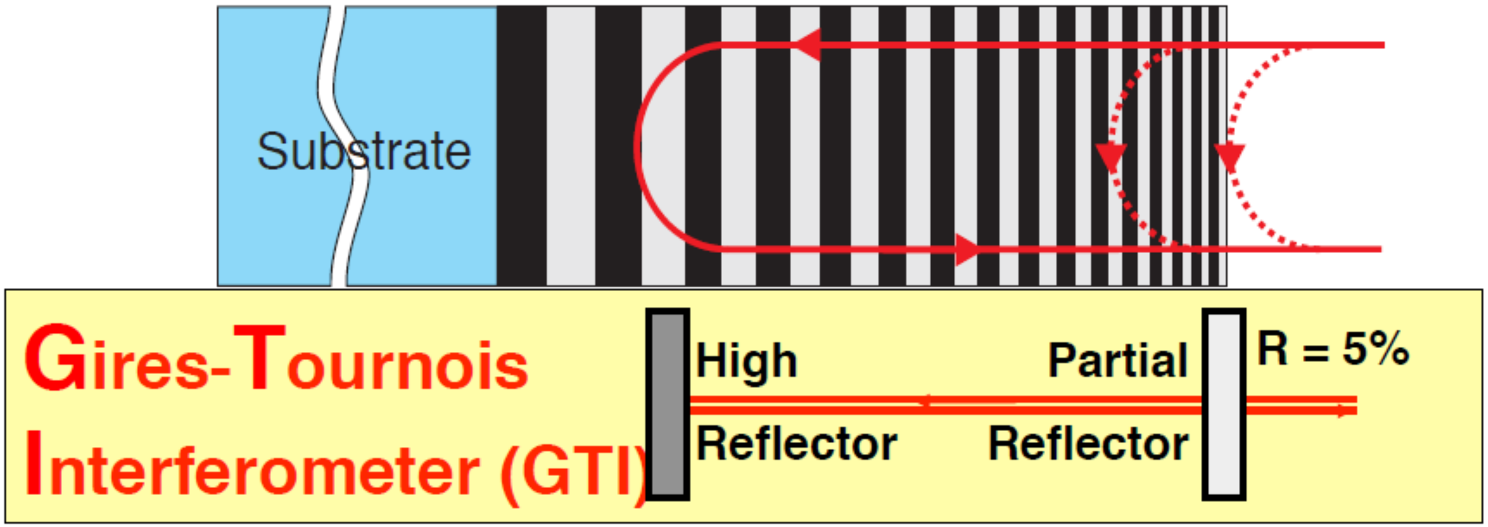
⇒ increased high-reflection bandwidth



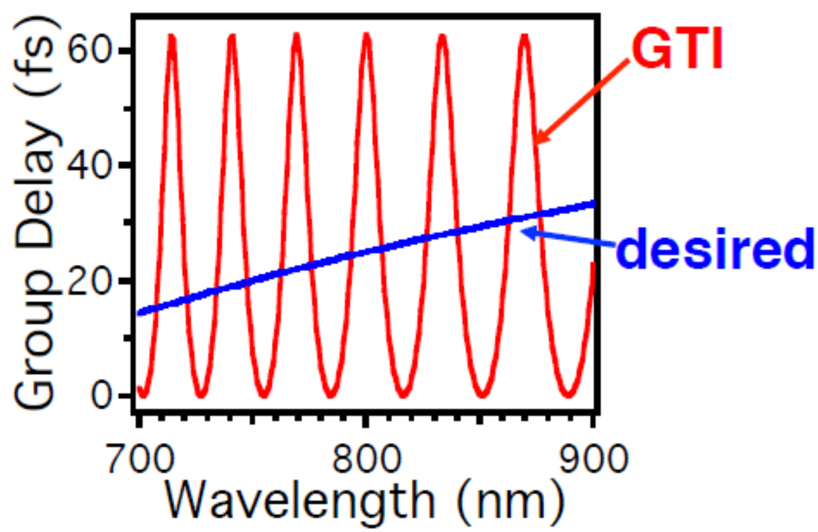
Szipöcs et al., *Opt. Lett.* **19**, 201 (1994)

Adapted from U. Keller's Ultrafast Laser Physics course

Interference causes ripples on group delay

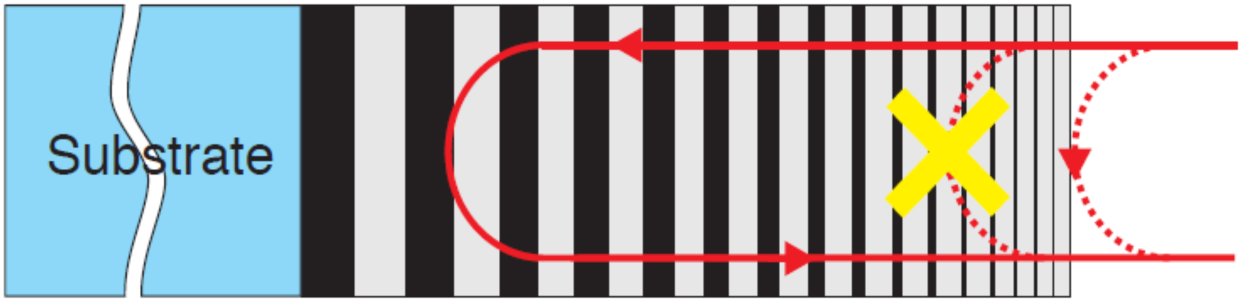


- front interface + high reflector form an *effective GTI*
- *dispersion oscillations*
- magnitude comparable to net mirror dispersion

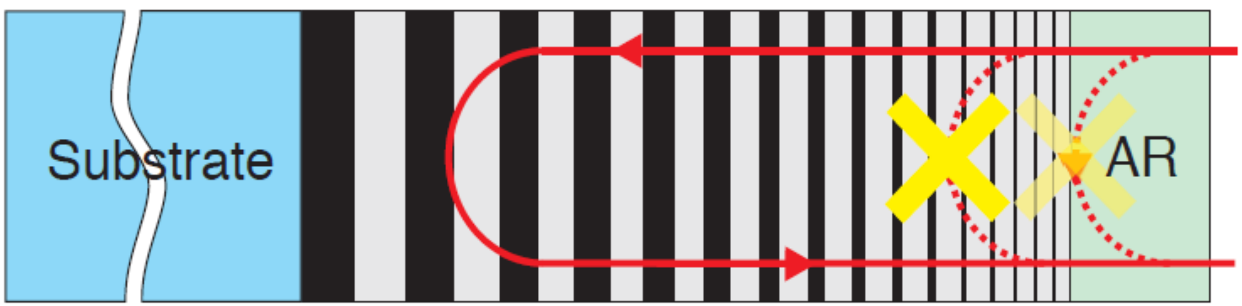


Adapted from U. Keller's Ultrafast Laser Physics course

Double chirped mirrors: eliminate dispersion oscillation



Additional chirp in the coupling between incident and reflected wave
⇔ chirp in the duty-cycle of high and low refractive index material
⇔ apodization of mirror impedance to first-layer impedance



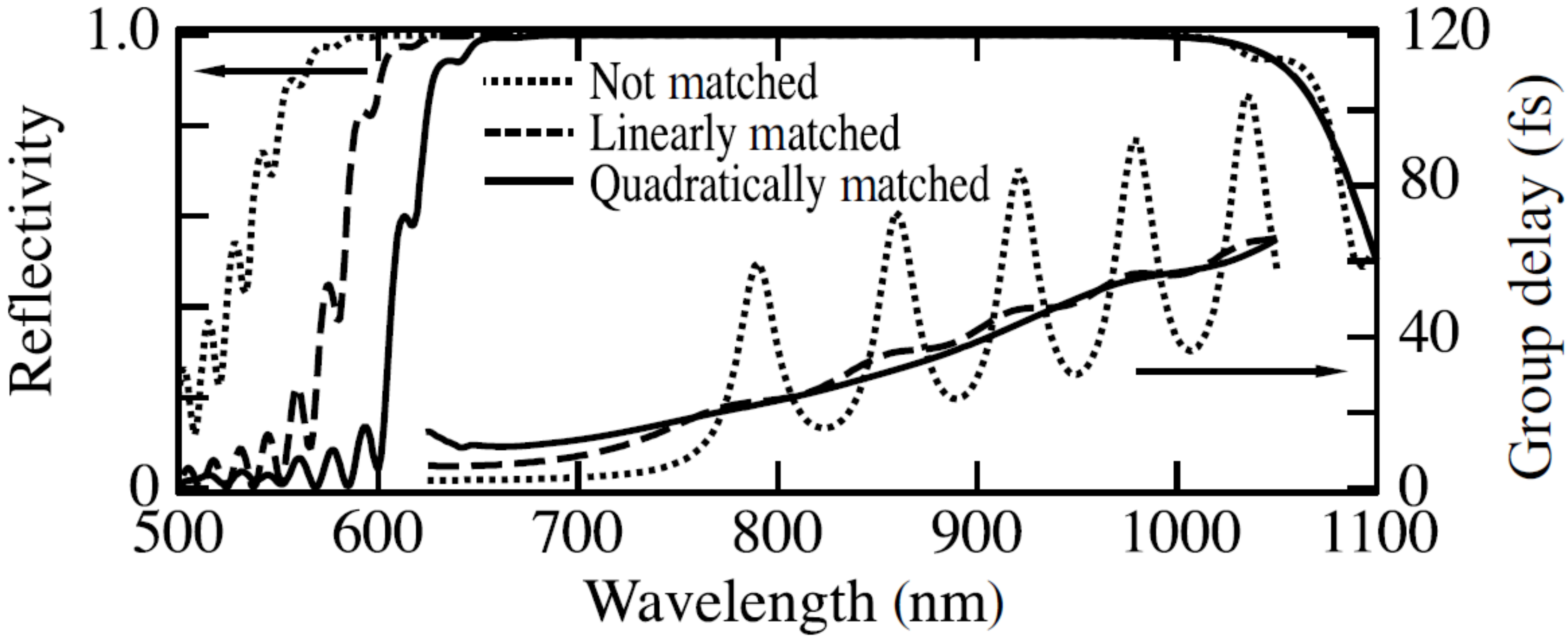
Anti-reflection coating matches first-layer impedance to air

Kärtner et al., Opt. Lett. **22**, 831 (1997)

Matuschek et al., IEEE J. Sel. Top. Quantum Electron. **4**, 197 (1998)

Adapted from U. Keller's Ultrafast Laser Physics course

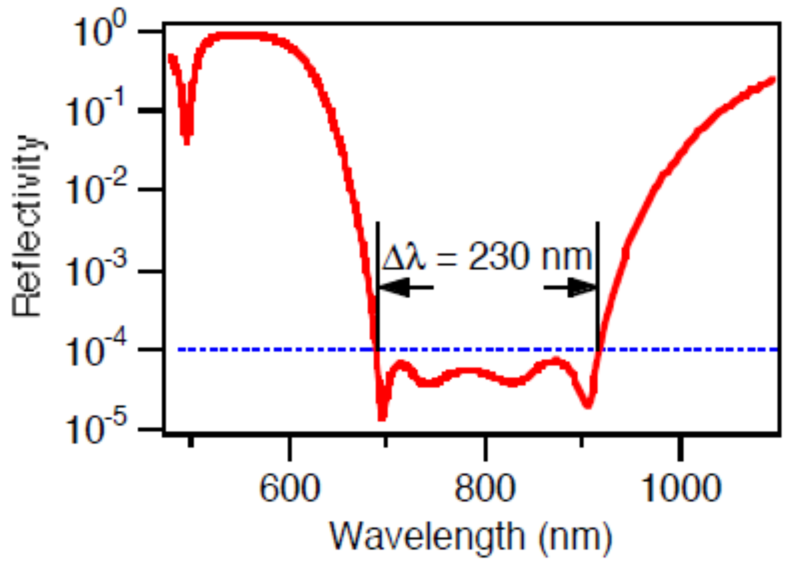
Comparison for different chirping of the high-index layer

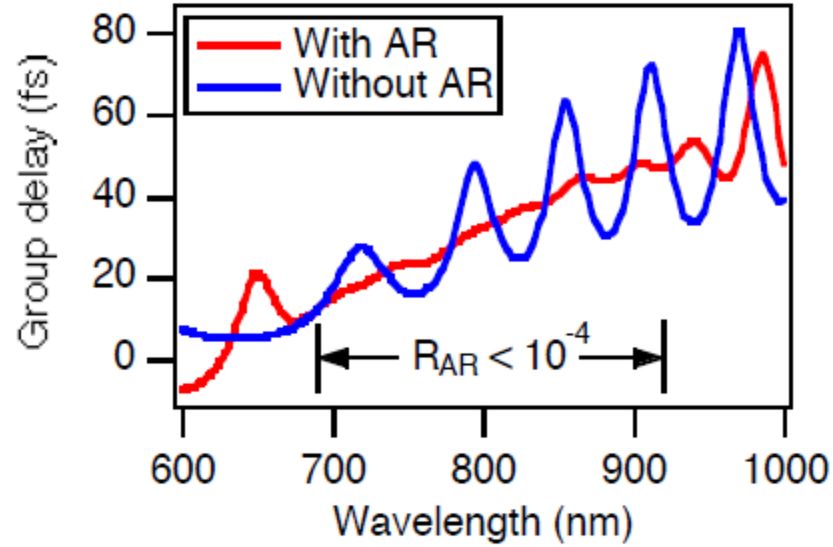


Calculated frequency-dependent reflectivity and group delay for 25-layer pair chirped mirrors with n_H = 2.5 and n_L = 1.5. The Bragg wavenumber $2\pi/\lambda_B$ is linearly chirped from $2\pi/(600\ \text{nm})$ to $2\pi/(900\ \text{nm})$ over the first 20 layer pairs, then held constant.

-- Dotted curve: a standard chirped mirror with d_H equal to a quarter-wave for all layers
-- Dashed curve: DCM with d_H linearly chirped over the first six layer pairs
-- Solid curve: DCM with d_H quadratically chirped over the first six layer pairs

Limitations of conventional DCMs





reduction of dispersion oscillations requires $R_{AR} < 10^{-4}$

⇒ limits bandwidth of direct approach to ≈ 250 nm

larger bandwidths by computer optimization of whole mirror structure, but...

⇒ dispersion oscillations strongly increase with bandwidth

⇒ increasing number of layers does not solve problem

"<u>cannot make arbitrarily low reflectivity and arbitrarily broad bandwidth at the same time</u>" J.A. Dobrowolski et al., *Appl. Opt.* **35**, 644 (1996)

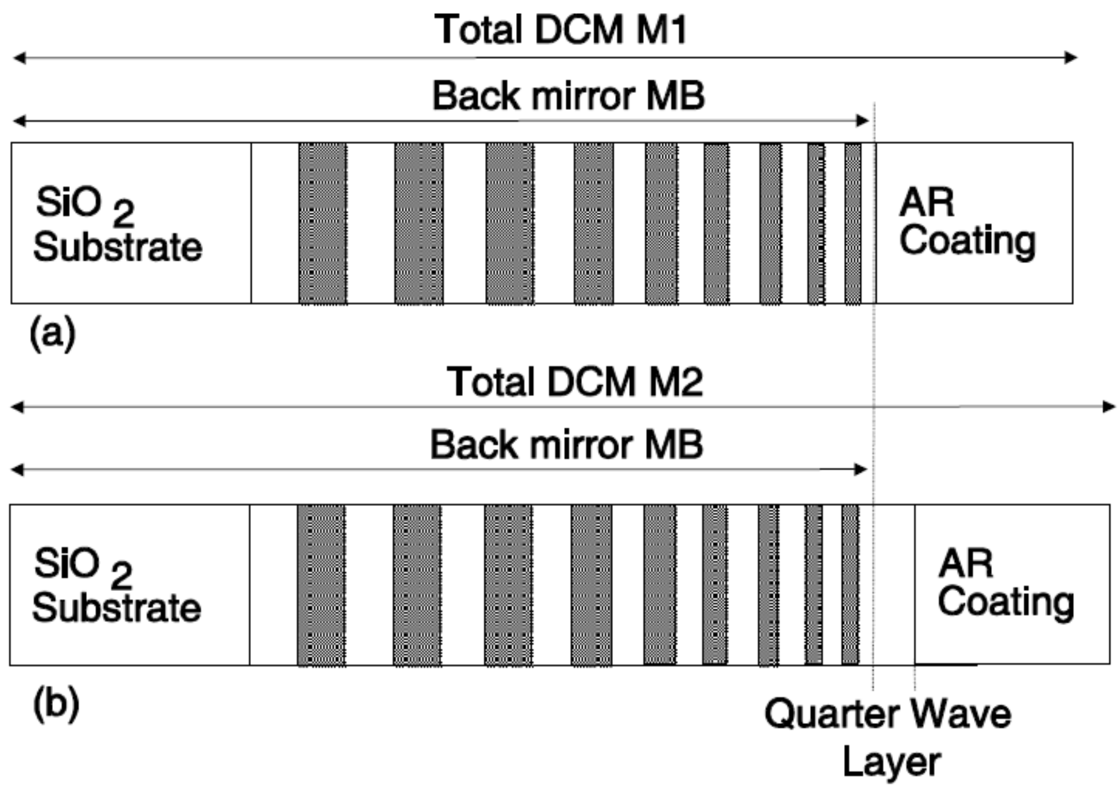
<u>Adapted from U. Keller's Ultrafast Laser Physics course</u>

Broadband DCM: DCM pair

The DCM M1 can be decomposed in a double-chirped back-mirror matched to a medium with the index of the top most layer.

In M2 a layer with a quarter wave thickness at the center frequency of the mirror and an index equivalent to the top most layer of the back-mirror MB is inserted between the back-mirror and the AR-coating.

The new back-mirror comprising the quarter wave layer can be re-optimized to achieve the same phase as MB with an additional π-phase shift over the whole octave of bandwidth.



DCM pair designed for Ti:Sapphire oscillator

Thick dash-dotted line with scale to the right: group delay design goal for perfect dispersion compensation of a prismless Ti:sapphire laser.

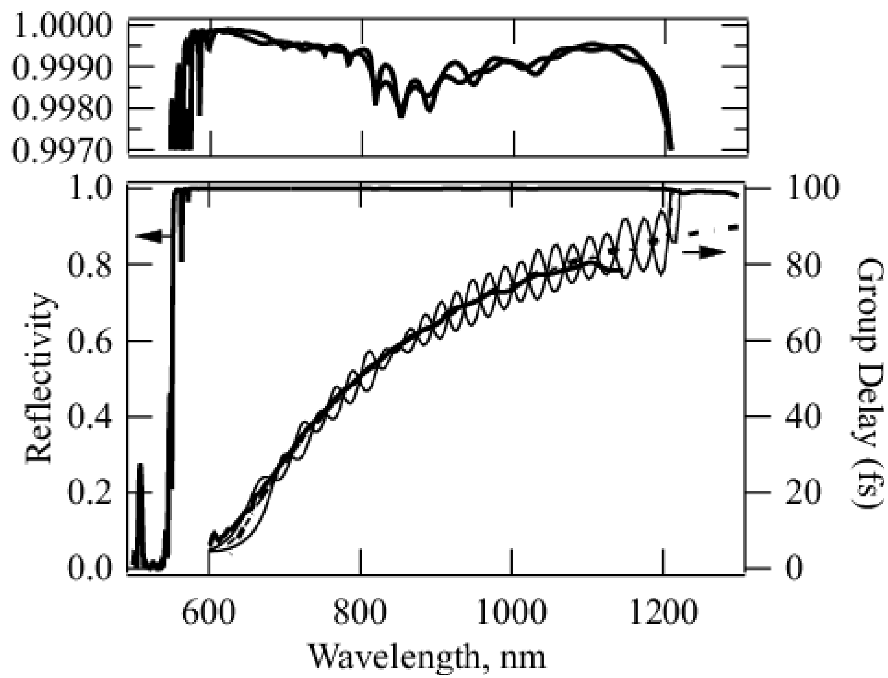
Thin line: individual group delay of the designed mirrors

Dashed line: average group delay of the two DCMs

Thick line: measured group delay from 600-1100 nm using white light interferometry

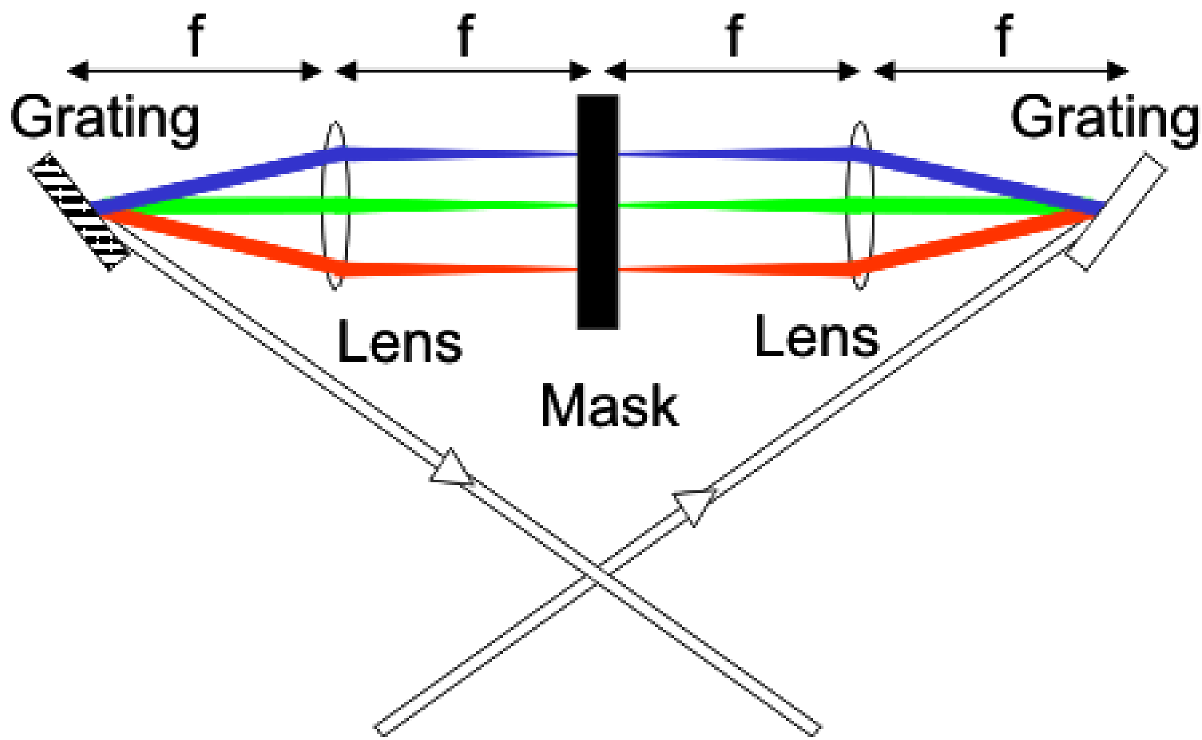
The average is almost identical with the design goal over the wavelength range form 650-1200 nm.

Beyond 1100nm the sensitivity of Si detector used prevented further measurements.



F. X. Kärtner et. al., "Ultrabroadband double-chirped mirror pairs for generation of octave spectra"*J. of the Opt. Soc. of Am. 18, 882-885 (2001).*

Active dispersion compensation: spatial light modulator

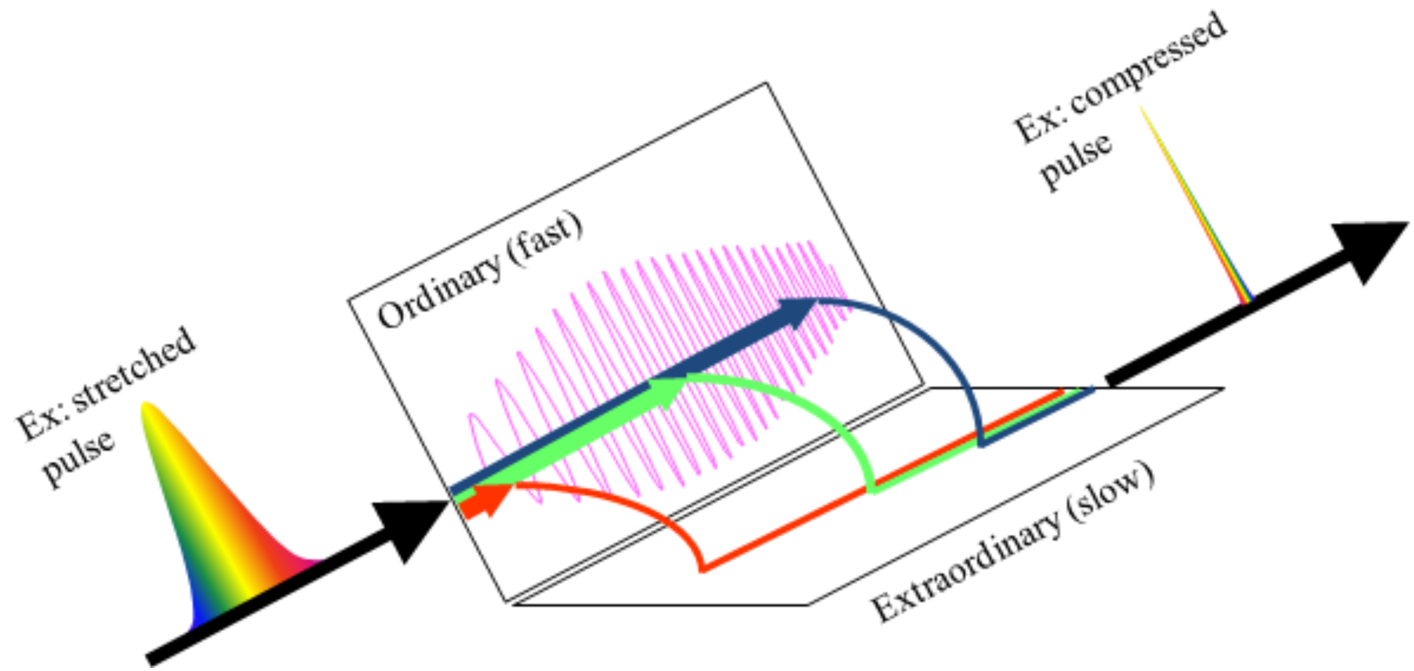


Dispersion Compensation with 4f-Pulse Shaper

Liquid crystal spatial light modulator (LCSLM) can be electronically controlled allowing programmable shaping of the pulse on a millisecond time scale.

A. M. Weiner, "Femtosecond pulse shaping using spatial light modulators" Rev. Sci. Instrum. 71, 1929 (2000).

Active dispersion compensation: AOPDF



Acousto-Optic Programmable Dispersive Filter (AOPDF), also known as Dazzler.

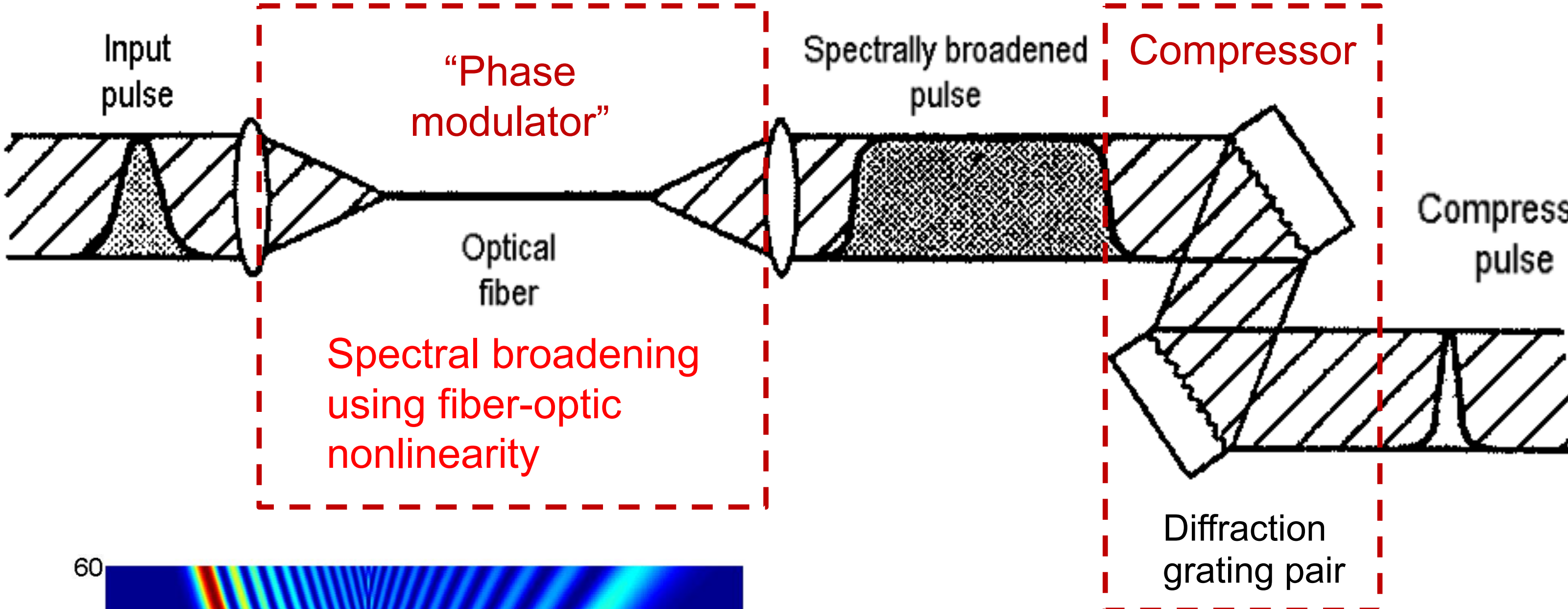
In an AOPDF, travelling acoustic wave induces variations in optical properties thus forming a dynamic volume grating.

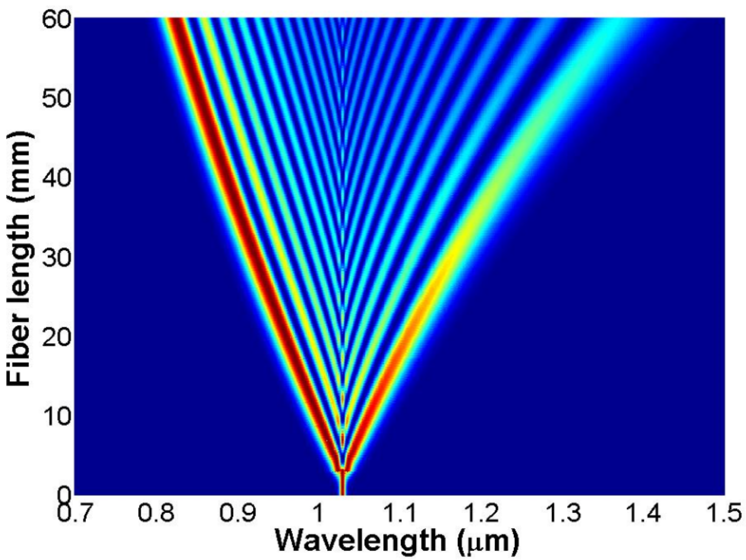
It is a programmable spectral filter, which can shape both the spectral phase and amplitude of ultrashort laser pulses.

Pierre Tournois, "Acousto-optic programmable dispersive filter for adaptive compensation of group delay time dispersion in laser systems," Optics Communications ***140****, 245 (1997).*

Pulse compression: general idea

Spectral broadening followed by dispersion compensation to compress (de-chirp) the pulse





$$T_g(\omega) = \phi'(\omega_0) + \phi''(\omega_0)\Delta\omega + \frac{1}{2}\phi'''(\omega_0)\Delta\omega^2 + \frac{1}{3!}\phi''''(\omega_0)\Delta\omega^3 + \cdots$$

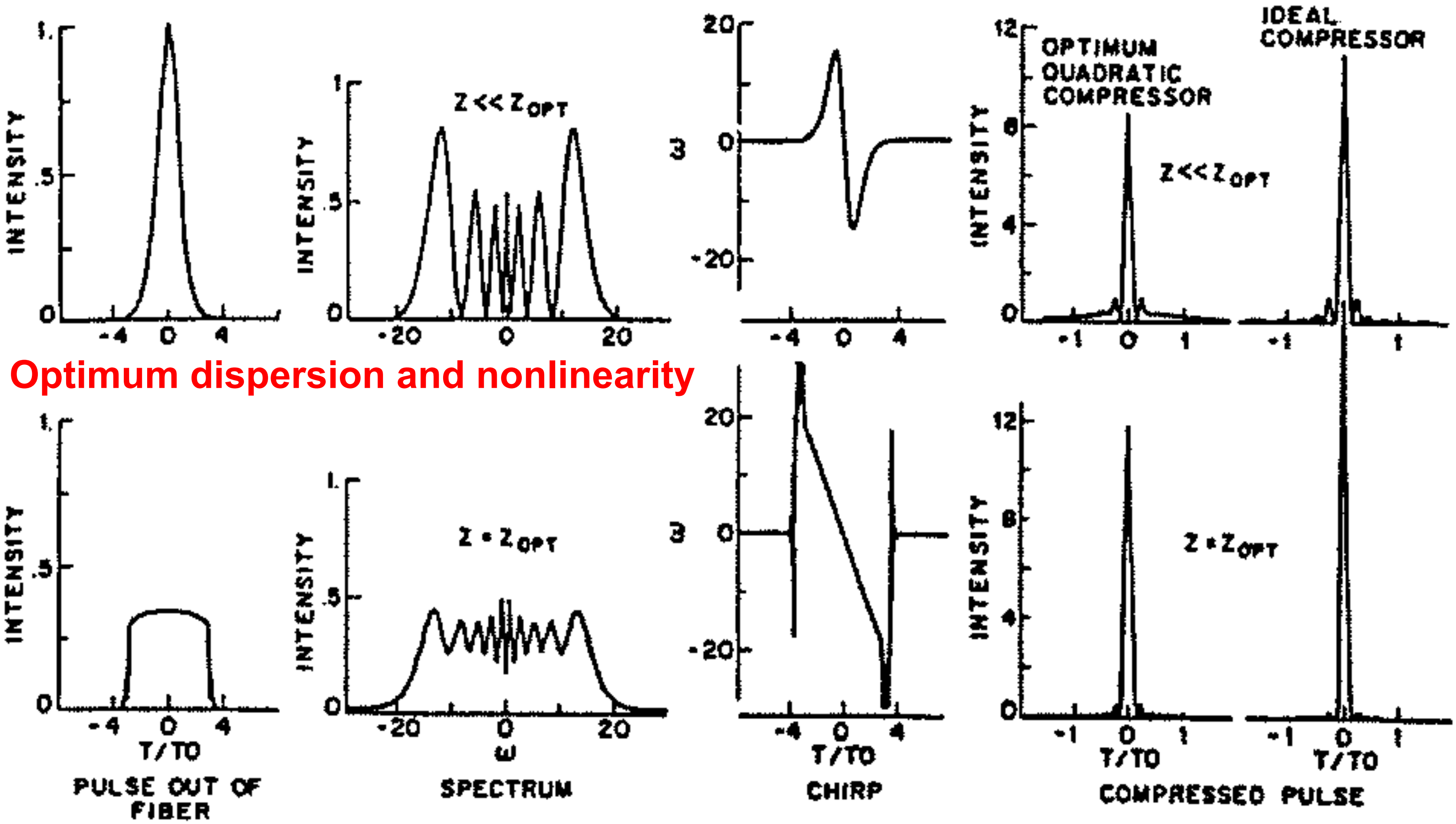
$$\begin{aligned} \phi''(\omega_0) &= \phi''_{modulator} + \phi''_{compresssor} = 0 \\ \phi'''(\omega_0) &= \phi'''_{modulator} + \phi'''_{compresssor} = 0 \end{aligned}$$

Variable dispersion by the grating pair.

Dispersion matters in spectral broadening

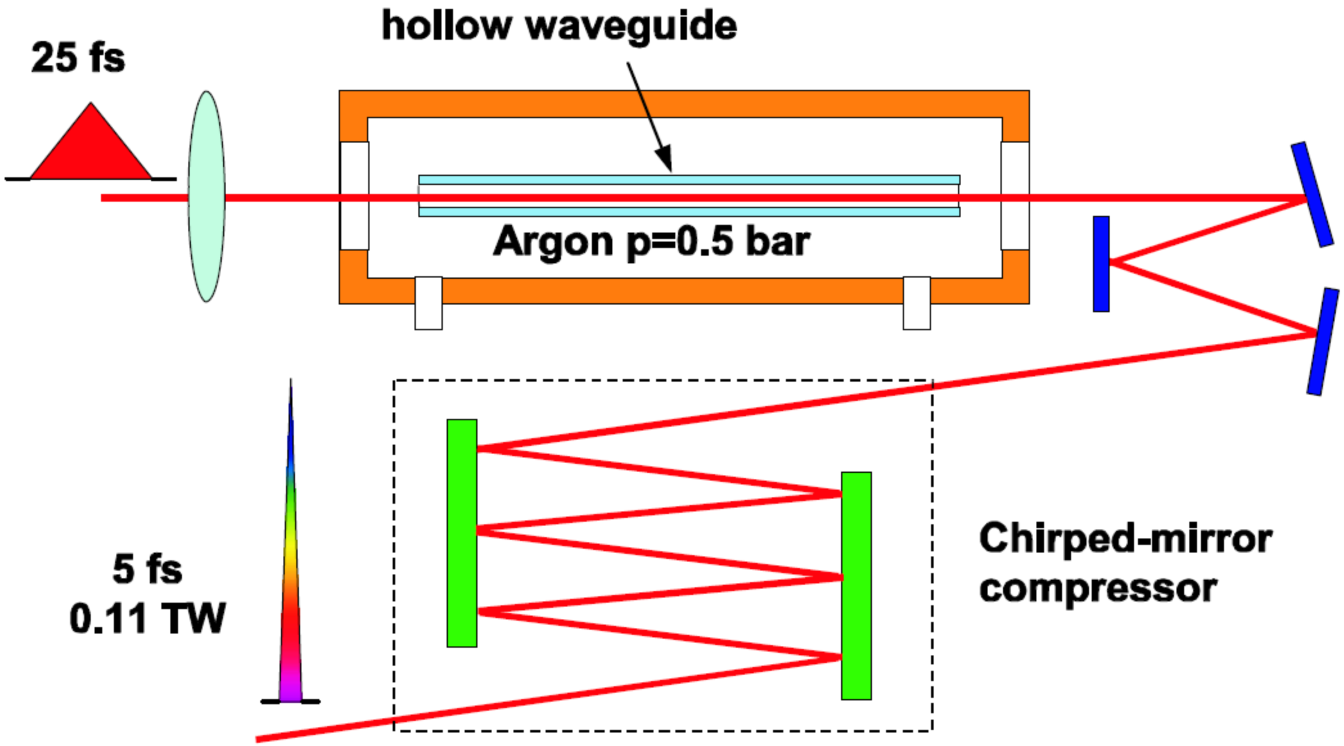
Dispersion negligible using short fiber, SPM dominates

Optimum dispersion and nonlinearity



Hollow fiber compression of mili-joule pulses

Self focusing threshold in fused silica is 4 MW. For ~100 fs pulse, the pulse energy allowed in a fused silica fiber is ~400 nJ before fiber breakdown.



The modes of the hollow fiber are leaky modes, i.e. they experience radiation loss. However, the EH_{11}mode has considerably less loss than the higher order modes and is used for pulse compression. The nonlinear index in the fiber can be controlled with the gas pressure. Typical fiber diameters are 100-500 μm and typical gas pressures are in the range of 0.1-3 bar.