Ultrafast Optical Physics II (SoSe 2021)
Lecture 20, June 15

1) Theory of Optical Parametric Amplification

2) Phase matching for OPA

[5] Largely follows the review paper of Cerullo et al., "Ultrafast Optical Parametric Amplifiers" Rev. Sci. Instr. 74, pp 1-17 (2003)

Theory of Optical Parametric Amplification

Undepleted pump approximation: $E_p = const.$

$$\frac{\partial E_s(z)}{\partial z} = -j\kappa_s \, E_p E_i^*(z) \, e^{-j\Delta k z},$$
$$\frac{\partial E_i(z)}{\partial z} = -j\kappa_i \, E_p E_s^*(z) \, e^{-j\Delta k z}.$$

with:

$$E_s(z=0) = E_s(0) \qquad E_i(z=0) = 0$$

$$E_s(z) \tilde{} E_s(0)\, e^{gz - j\Delta k z/2} \text{ and } E_i(z) \tilde{} E_i(0)\, e^{gz - j\Delta k z/2}$$

$$\rightarrow \quad \begin{vmatrix} g - j\frac{\Delta k}{2} & j\kappa_s \, E_p \\ j\kappa_i \, E_p^* & g + j\frac{\Delta k}{2} \end{vmatrix} = 0$$

$$g = \sqrt{\Gamma^2 - \left(\frac{\Delta k}{2}\right)^2}, \text{ with } \Gamma = \sqrt{\kappa_i \, \kappa_s \, |E_p|^2}.$$

Gain

Max. gain, when phase matched

Maximum Gain

$$\Gamma^2 = \frac{\omega_s\omega_i}{n_s n_i c_0^2} d_{eff}^2 \ |E_p|^2 = \frac{2Z_{F_0}\omega_s\omega_i}{n_p n_s n_i c_0^2} d_{eff}^2 \quad I_p$$

$$FOM = \frac{d_{eff}}{\sqrt{\lambda_s\lambda_i n_p n_s n_i}}$$

General solutions:

$$E_s(z) = \{E_s(0)\cosh gz + B\sinh gz\} e^{-j\Delta kz/2} \qquad B = -j\frac{\Delta k}{2g}E_s(0) - j\frac{\kappa_1}{g}E_p E_i^*(0)$$

$$E_i(z) = \{E_i(0)\cosh gz + D\sinh gz\} e^{-j\Delta kz/2} \qquad D = -j\frac{\Delta k}{2g}E_i(0) - j\frac{\kappa_2}{g}E_p^* E_s^*(0)$$

Here:

$$I_s(L) = I_s(0)\left[1 + \frac{\Gamma^2}{g^2}\sinh^2 gL\right]$$

$$I_i(L) = I_s(0)\frac{\omega_i}{\omega_s}\frac{\Gamma^2}{g^2}\sinh^2 gL.$$

For large gain: $\Gamma L >> 1$

$$I_s(L) = \frac{1}{4}I_s(0)\ e^{2\Gamma L},$$

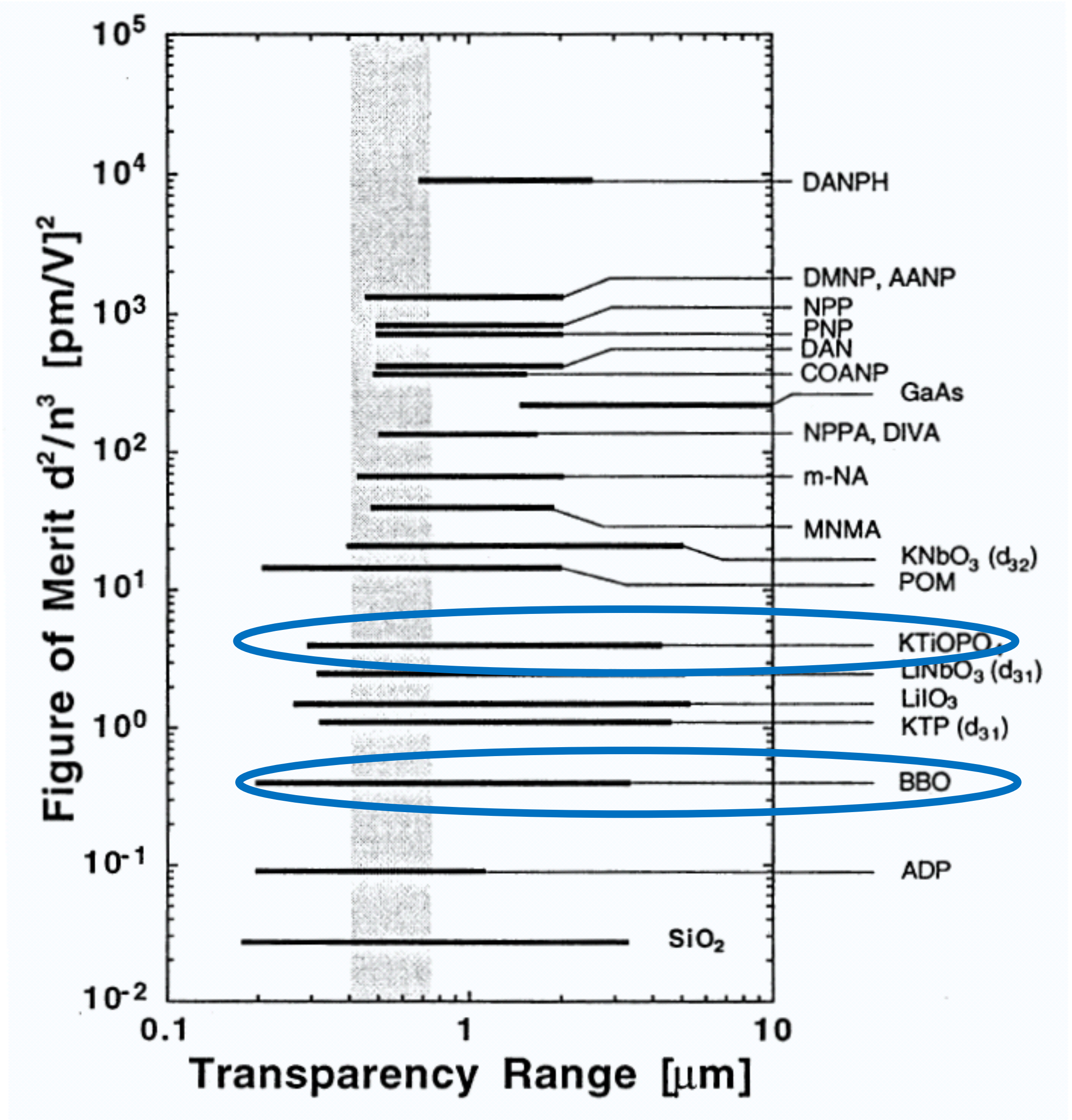
$$I_i(L) = \frac{1}{4}I_s(0)\ \frac{\omega_i}{\omega_s}\ e^{2\Gamma L}$$

→

$$G = \frac{I_s(L)}{I_s(0)} = \frac{1}{4}\ e^{2\Gamma L}$$

Figure of merit:

$$FOM = \frac{d_{eff}}{\sqrt{\lambda_s \lambda_i n_p n_s n_i}}$$



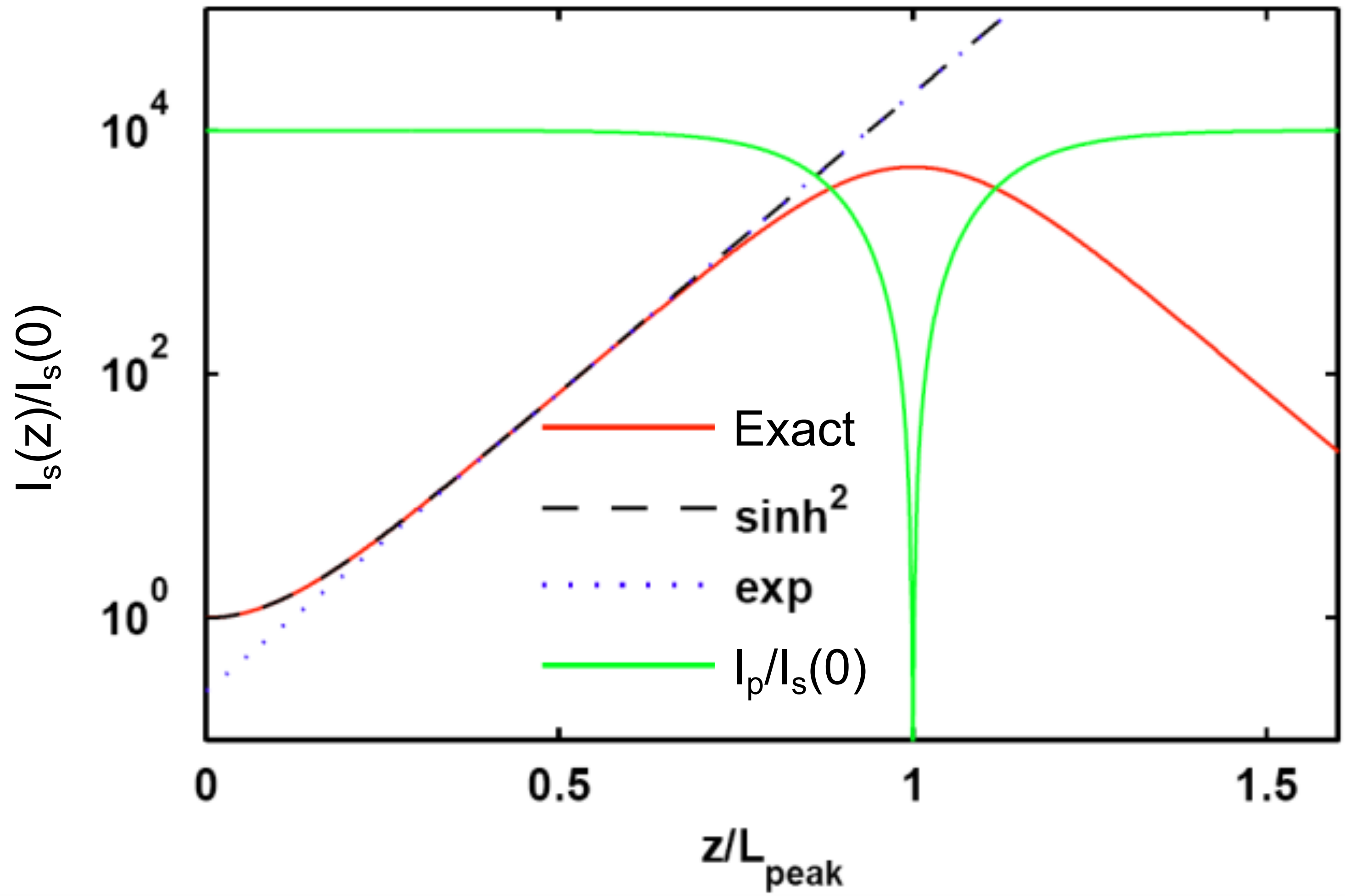


Fig. Exact solution for signal gain, plotted together with hyperbolic secant and exponential function solutions, approximate solutions derived by assuming the pump is undepleted. The exact solution for the pump intensity is also shown.

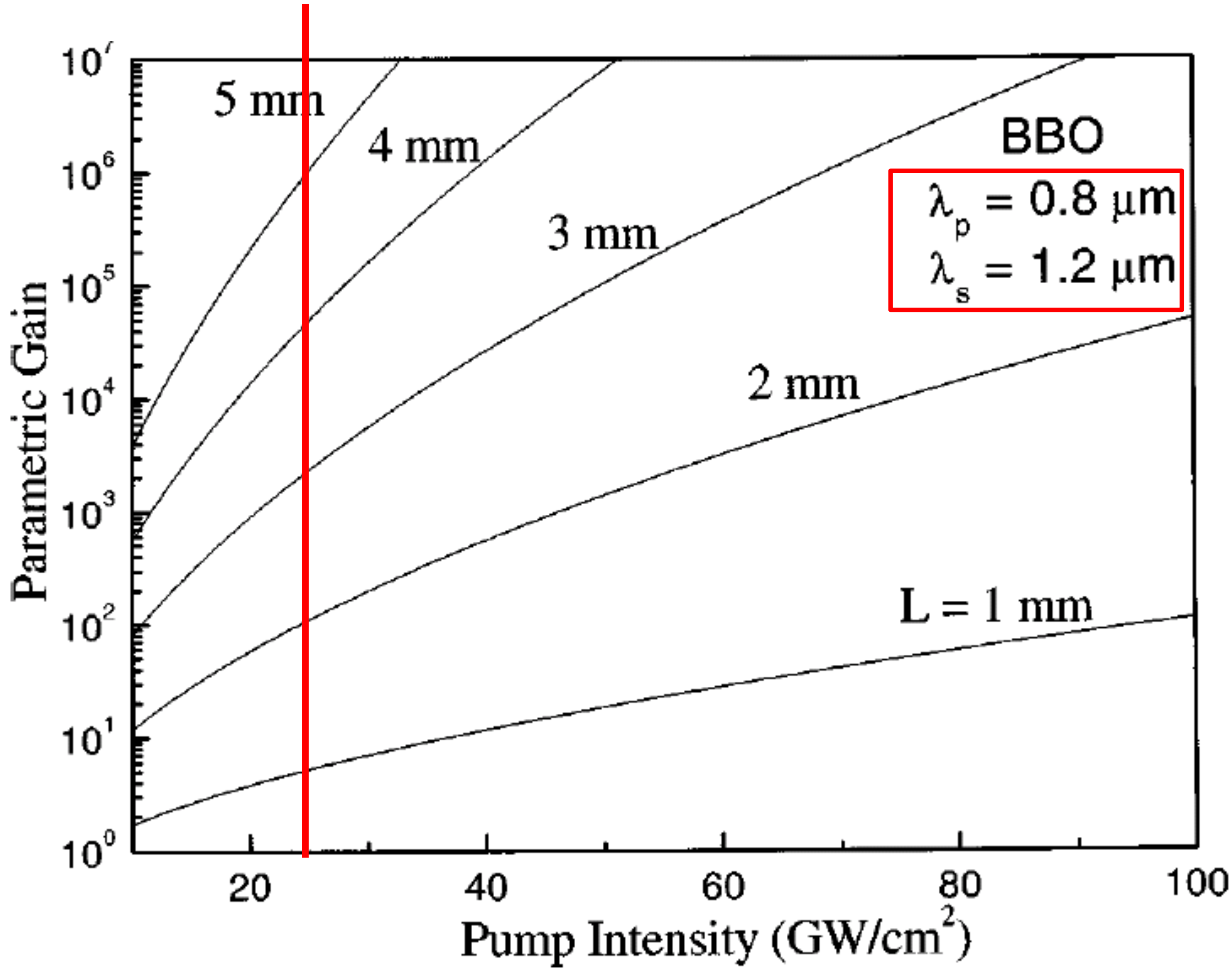


Fig. 12.24 Parametric gain for an OPA at the pump wavelength λ_p *= 0.8 μm* and the signal wavelength λ_s *= 1.2 μm, using type I phase matching in BBO* (d_{eff} *= 2 pm/V).*

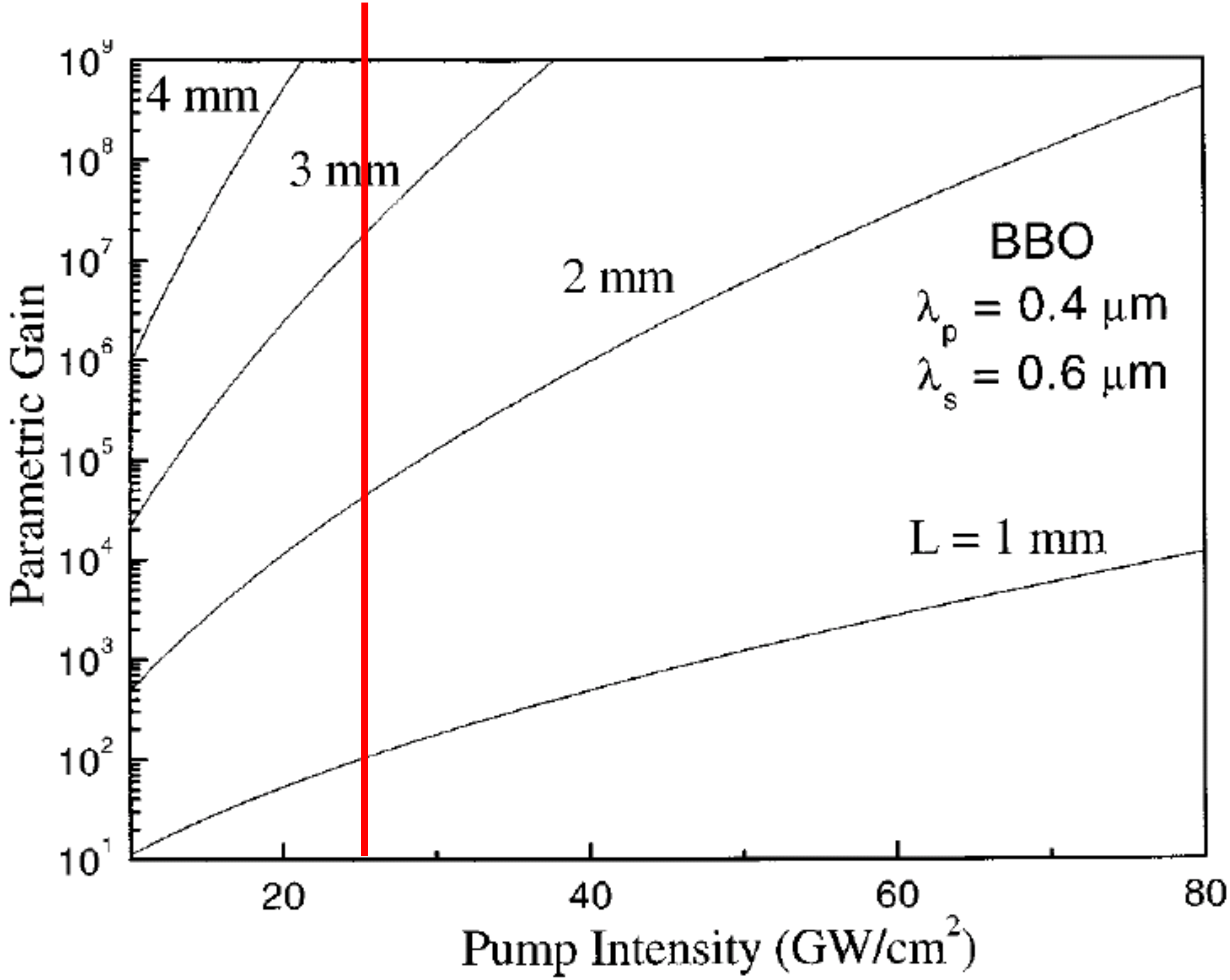


Fig. 12.25 Parametric gain for an OPA at the pump wavelength λ_p = *0.4 μm* and the signal wavelength λ_s *= 0.6 μm, using type I phase matching in BBO* (d_{eff} *= 2 pm/V).*

Phase Matching

$$\Delta k = 0 \longrightarrow n_p = \frac{n_s \omega_s + n_i \omega_i}{\omega_p}$$

Uniaxial Crystal: $n_e < n_o$

Type I: noncritical

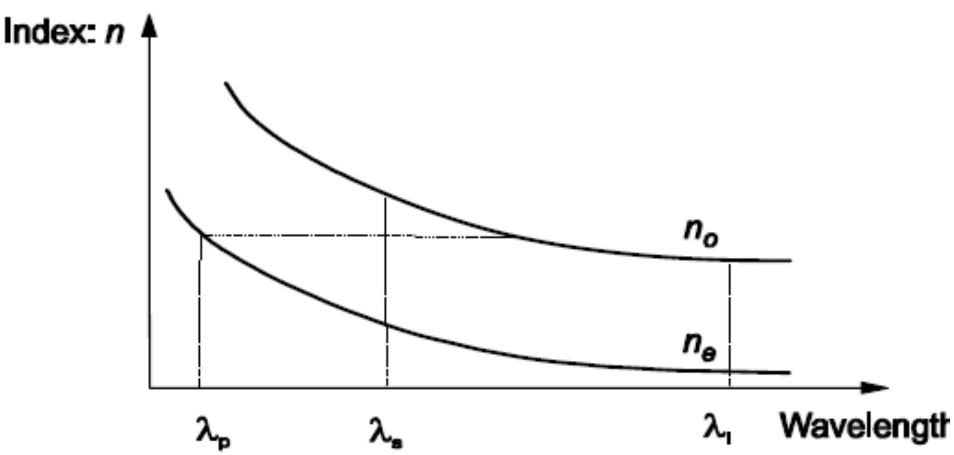


Fig. 12.26 Type I noncritical phase matching.

Type I: critical

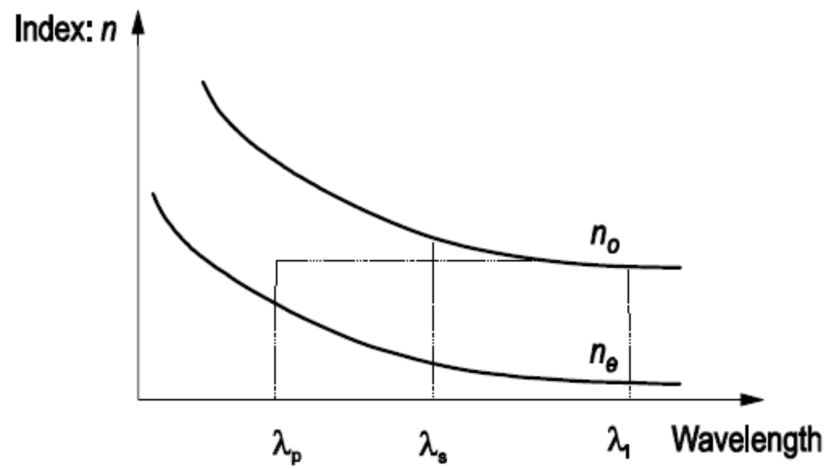
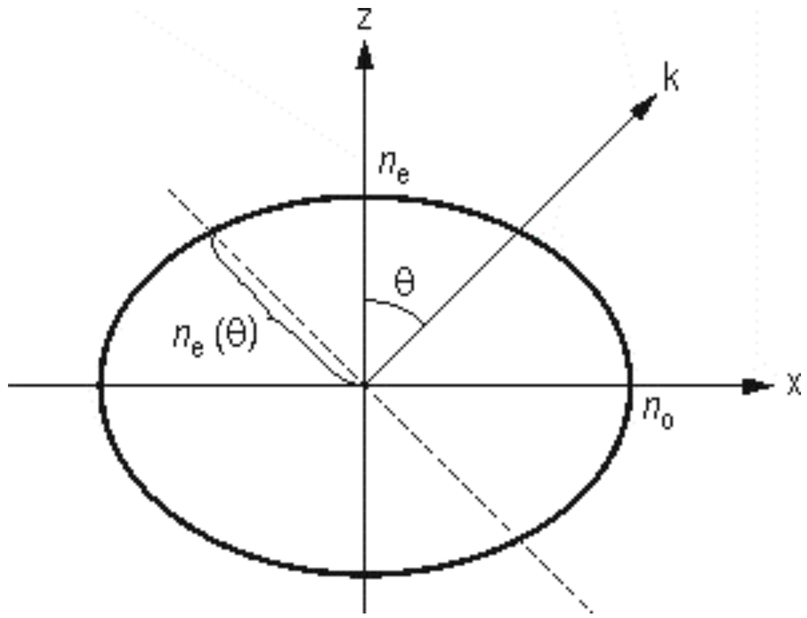


Fig. 12.27 Type I critical phase matching by adjusting the angle θ between wave vector of the propagating beam and the optical axis.

Phase Matching



Critical Phase Matching

$$n_{ep}(\theta)\omega_p = n_{os}\omega_s + n_{oi}\omega_i$$

$$\frac{1}{n_{ep}(\theta)^2} = \frac{\sin^2\theta}{n_{ep}^2} + \frac{\cos^2\theta}{n_{op}^2}$$

$$\theta = \arcsin\left[\frac{n_{ep}}{n_{ep}(\theta)}\sqrt{\frac{n_{op}^2 - n_{ep}^2(\theta)}{n_{op}^2 - n_{ep}^2}}\right]$$

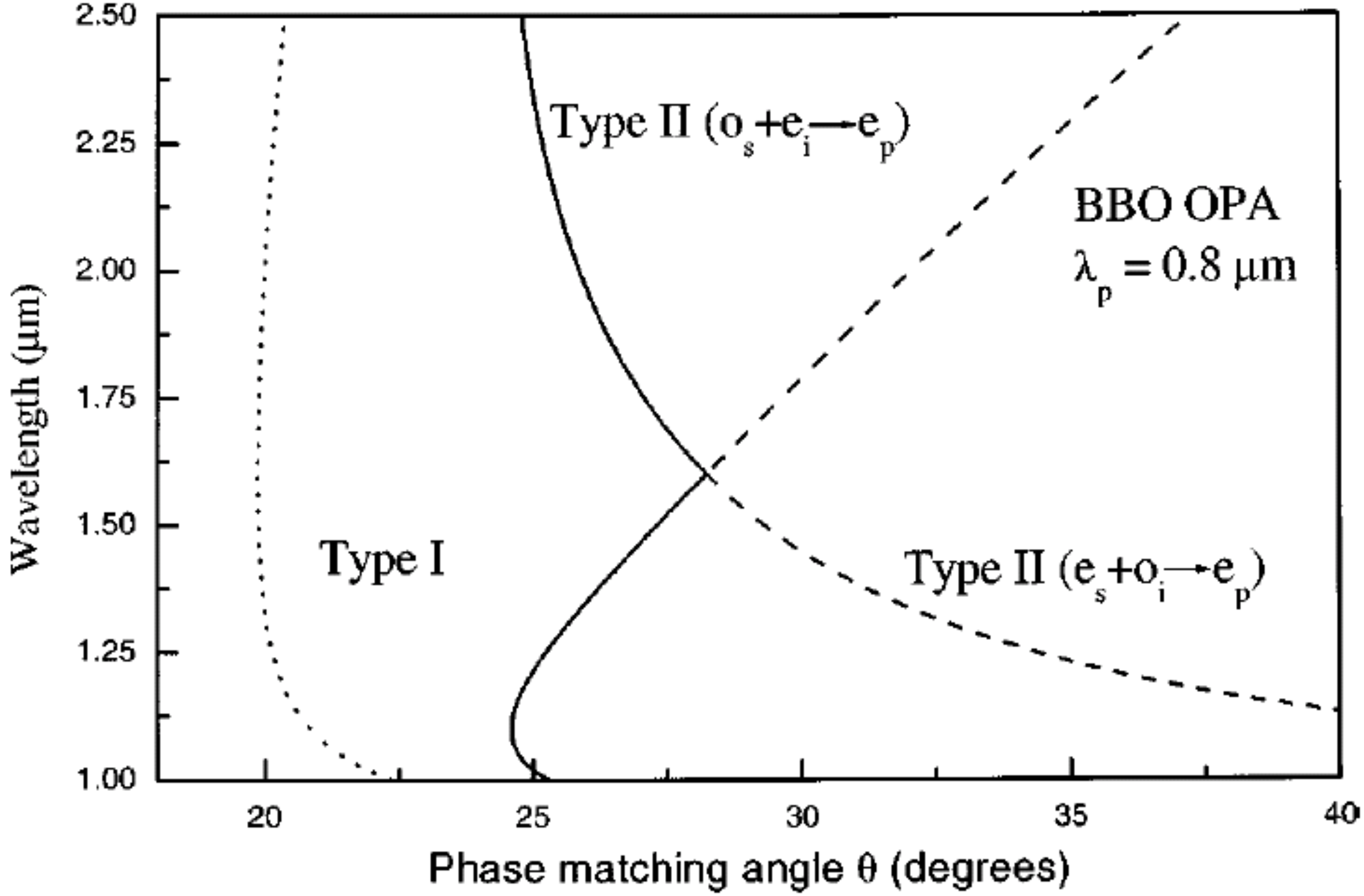


Fig. 12.28 Angle tuning curves for a BBO OPA at the pump wavelength λ_p=*0.8* μm for type I phase matching (dotted line), type II ($o_s + e_i \rightarrow e_p$) phase matching (solid line), and type II ($e_s + o_i \rightarrow e_p$) phase matching (dashed line).

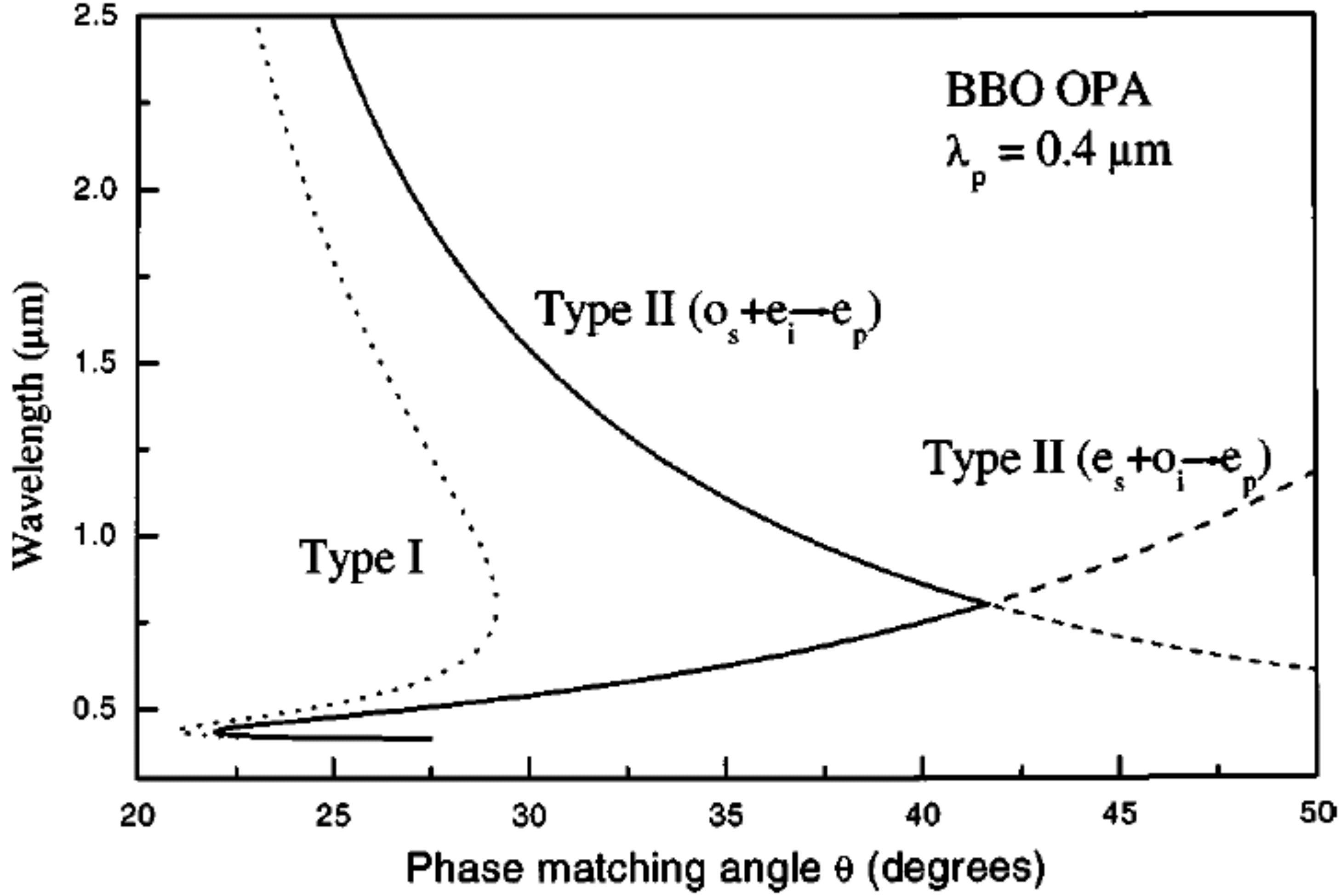
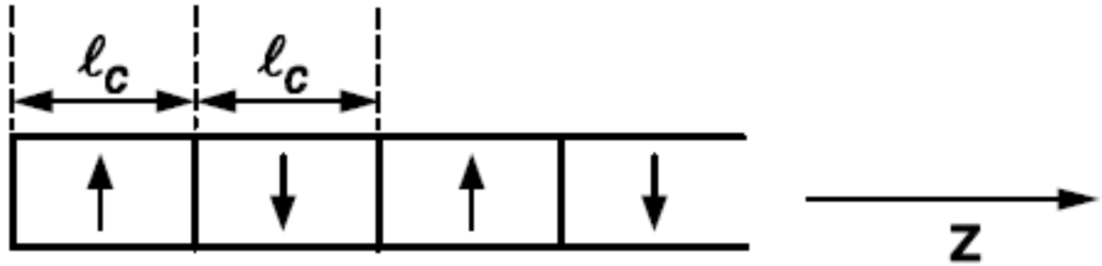


Fig. 12.28 Angle tuning curves for a BBO OPA at the pump wavelength $\lambda_p=0.4$ µm for type I phase matching (dotted line), type II ($o_s + e_i \rightarrow e_p$) phase matching (solid line), and type II ($e_s + o_i \rightarrow e_p$) phase matching (dashed line).

Quasi Phase Matching



Periodically poled crystal

Fig.12.30: Variation of def f in a quasi phase matched material as a function of propagation distance

$$d_{eff}(z) = \sum_{m=-\infty}^{+\infty} d_m e^{jm\kappa z}$$

$$\frac{\partial E_p(z)}{\partial z} = -j\kappa_p \, E_s(z)E_i(z) \, e^{j\Delta k z}$$