Ultrafast Sources 2021: Lecture 17, June 3, 2021

Lecture overview

- Historical Intro: frequency combs
- The comb in frequency and time domain, influence of noise
- Brief Introduction: Frequency metrology and clocks
- Application examples: Astro-combs and XUV combs

Lecture script chapters

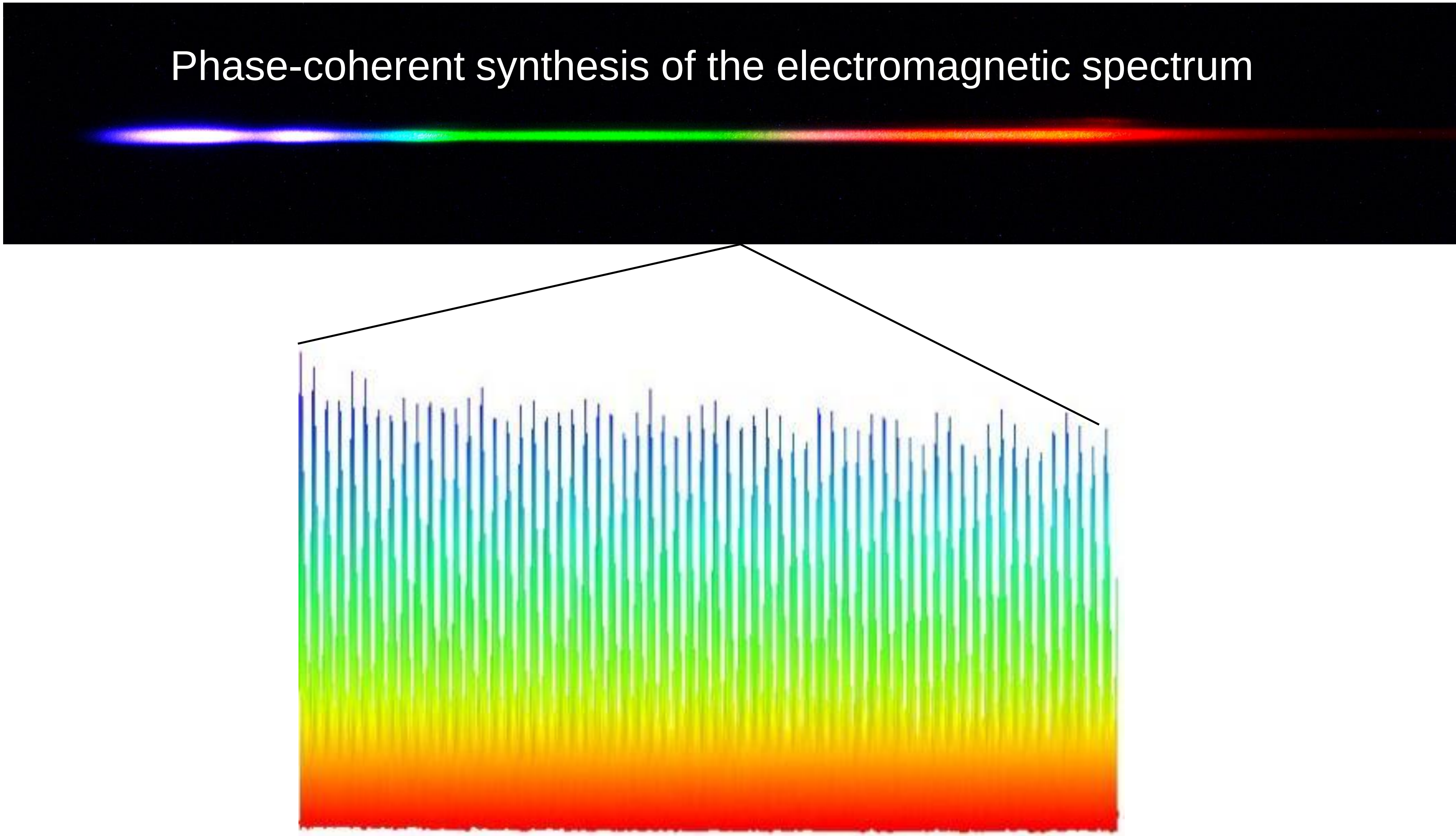
10 Femtosecond Laser Frequency Combs (Rulers for Light)

10.1 The Mode Comb

10.2 Group- and PhaseVelocity of Solitons

10.3 Femtosecond Laser Frequency Combs

The Frequency comb



Brief historical introduction

High precision lasers

- Continuous wave lasers
- Single frequency lasers
- Narrow spectral linewidth
- …

- Optical frequency measurements
- Testing fundamental theories
- Define time (clocks)
- …

Frequency combs

- Narrow linewidth
- Continuous wave like
- Broad spectra coverage
- Ultrashort pulse durations
- High peak power
- **Absolute frequency reference option!**

1960 2000 Time

High peak power lasers

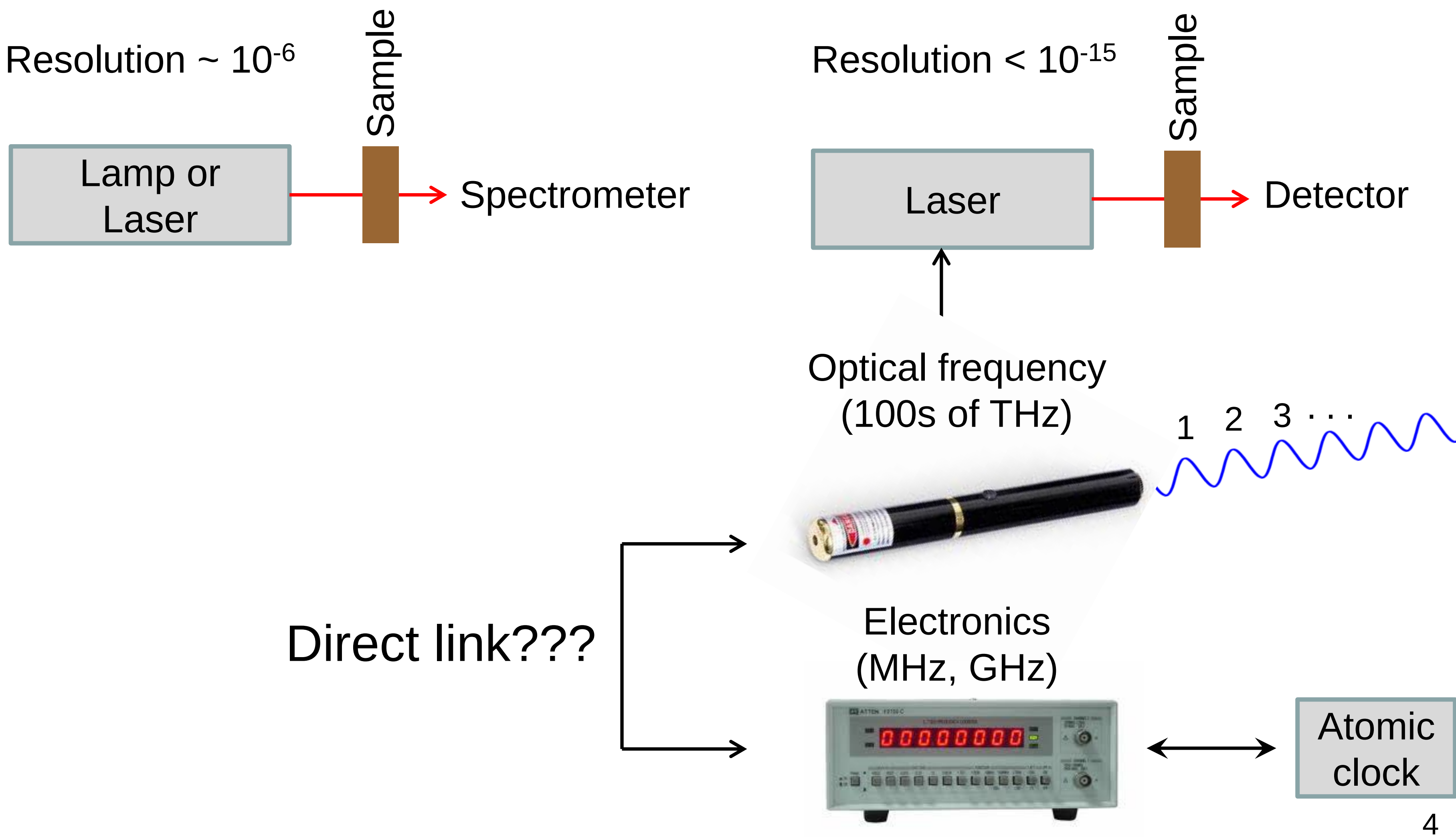
- Ultrashort pulse durations
- Broad bandwidth
- Highest pulse energies
- …

- Strong field physics
- Extreme nonlinear optics
- Material processing
- Nuclear fusion
- …

Brief historical introduction

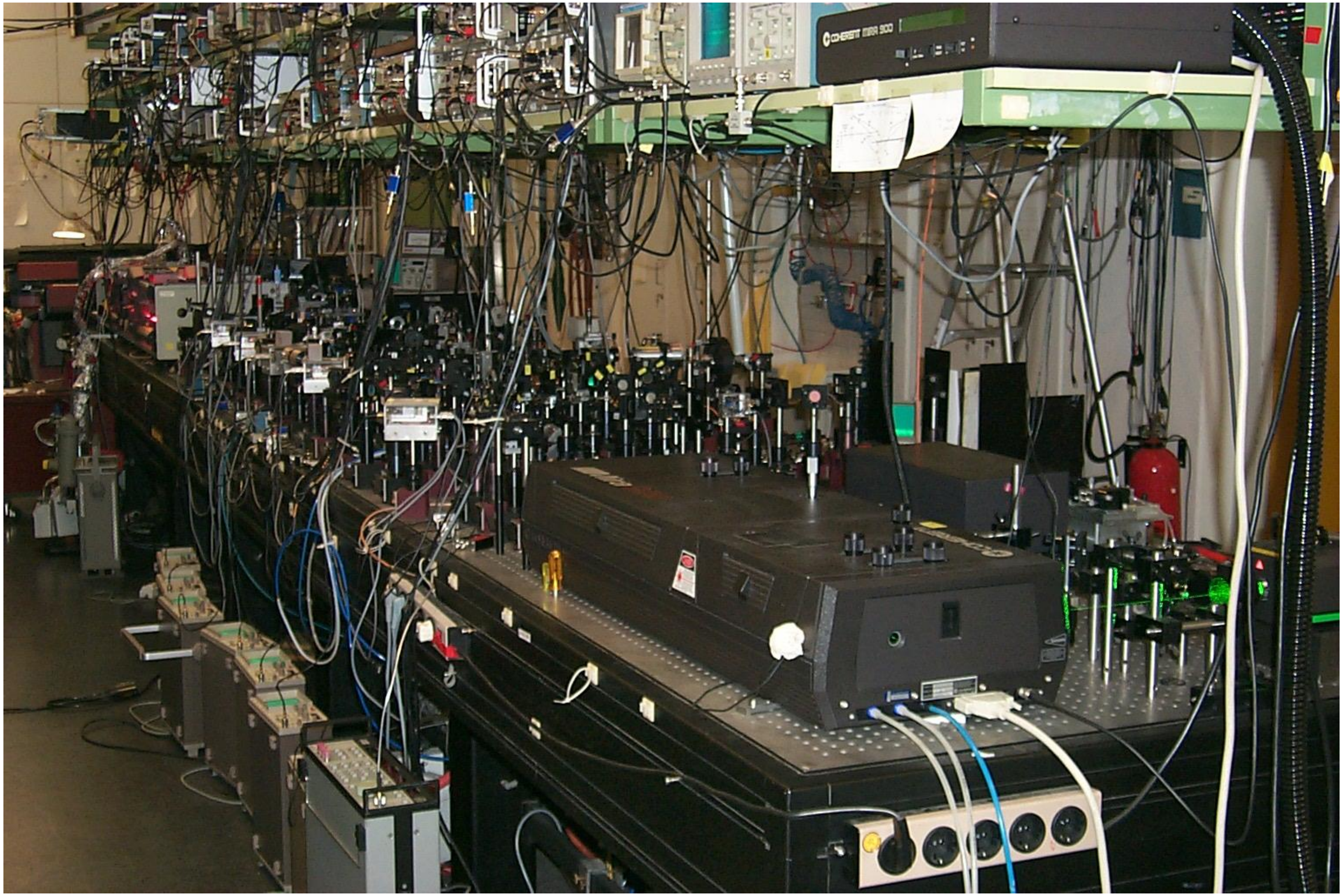
Traditional spectroscopy

Spectrocopy with direct frequency reference



Brief historical introduction

Group T. Hänsch: 7f – 8f self referencing in 1999:



[Phys. Rev. Lett. 84, 3232 (2000) & Phys. Rev. Lett. 84, 5496 (2000)]

The Nobel Prize in Physics 2005

"for his contribution to the quantum theory of optical coherence"

"for their contributions to the development of laser-based precision spectroscopy, including the optical frequency comb technique"

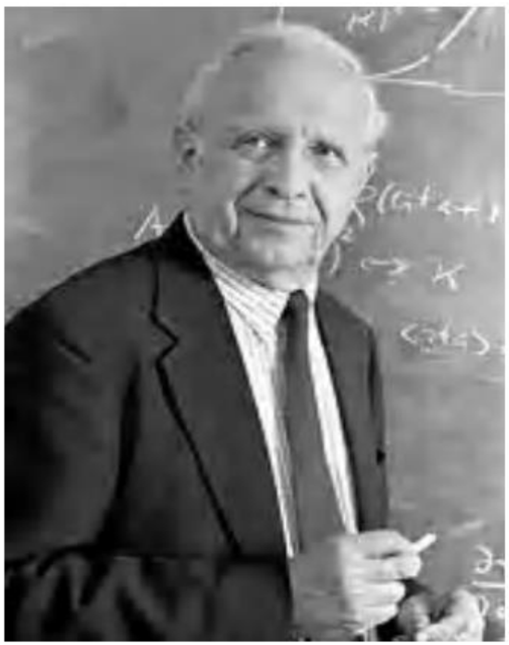


Photo: J.Reed

Roy J. Glauber

Photo: Sears.P.Studio

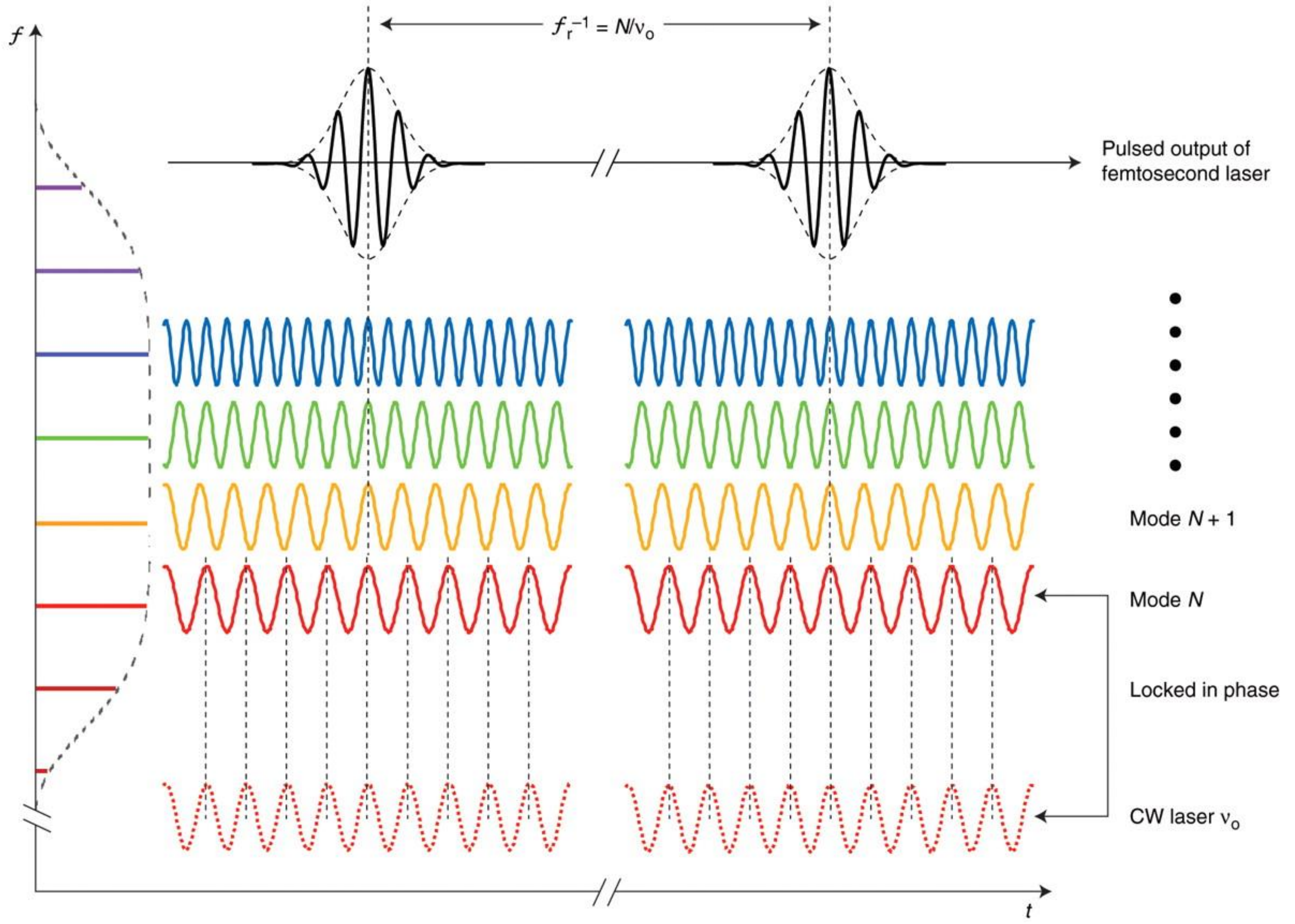
John L. Hall

Photo: F.M. Schmidt

Theodor W. Hänsch

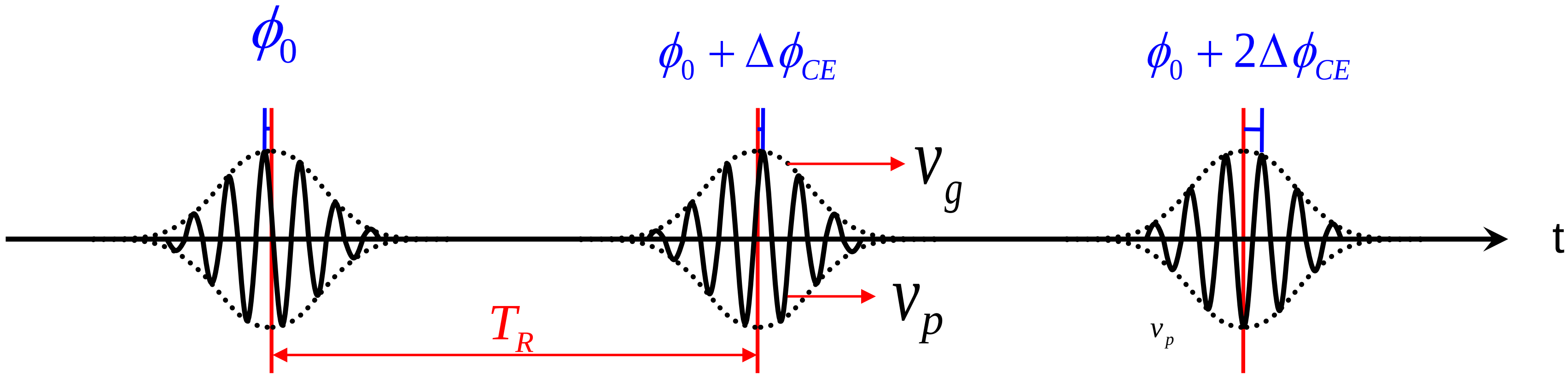
http://nobelprize.org/nobel_prizes/physics/laureates/2005/hall-lecture.html

Revisit Mode-Locking: Time-Domain



S. A. Diddams et al. Science 2004;306:1318-1324

Pulse Train from a Modelocked Laser



$$T_R = \frac{2L}{v_g}$$

$$A(t) = \sum_{m=-\infty}^{+\infty} a(T = mT_R, t) e^{j\left[\omega_c\left(t - \frac{1}{v_p} 2mL\right)\right]} \quad (10.1)$$

$$= \sum_{m=-\infty}^{+\infty} a(T = mT_R, t) e^{j\left[\omega_c\left(t - mT_R + \left(\frac{1}{v_g} - \frac{1}{v_p}\right) 2mL\right)\right]} \quad (10.2)$$

$$\Delta\phi_{CE} = 2\omega_c L\left(\frac{1}{v_g} - \frac{1}{v_p}\right)$$

Optical Spectrum

$$A(t) = \sum_{m=-\infty}^{+\infty} a_s(t - mT_R)\; e^{j[\omega_c(t-mT_R)+m\Delta\phi_{CE}]}$$

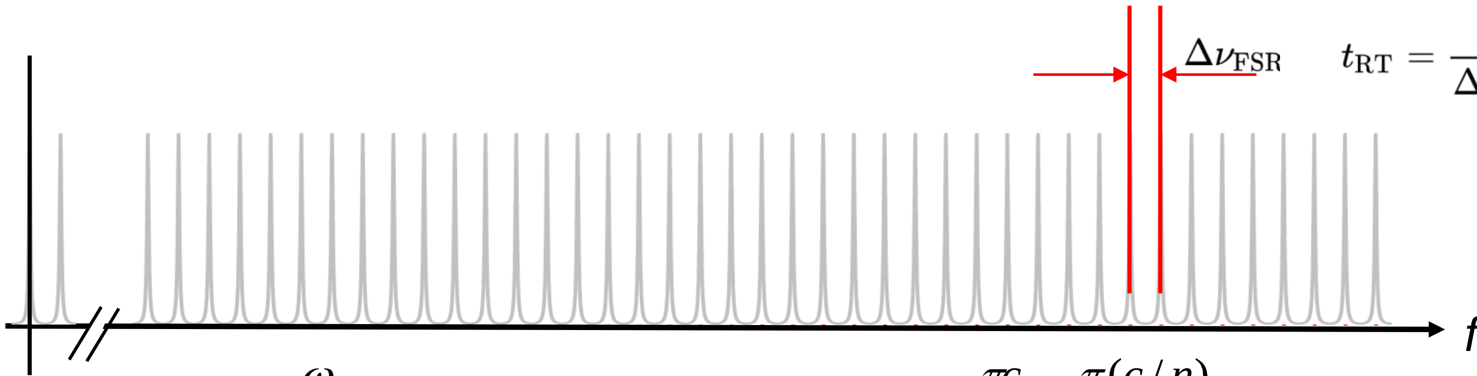
The Fourier transform of the unperturbed pulse train is

$$\begin{aligned} \hat{A}(\omega) &= \hat{a}_s(\omega - \omega_c) \sum_{m=-\infty}^{+\infty} e^{j(\Delta\phi_{CE} - \omega T_R)m} \\ &= \hat{a}_s(\omega - \omega_c) \sum_{m=-\infty}^{+\infty} e^{jmT_R\left(\frac{\Delta\phi_{CE}}{T_R} - \omega\right)} \\ &= \hat{a}_s(\omega - \omega_c) \frac{2\pi}{T_R} \sum_{n=-\infty}^{+\infty} \delta\left(\omega - \left(\frac{\Delta\phi_{CE}}{T_R} + n\omega_R\right)\right). \end{aligned}$$

Poisson's Summation Formula

$$\sum_{m=-\infty}^{+\infty} e^{jm2\pi x} = \sum_{n=-\infty}^{+\infty} \delta\left(x - 2\pi n\right)$$

Revisit Mode-Locking: The Mode Comb



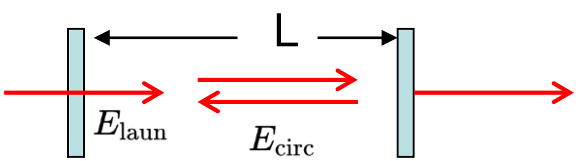
$$t_{\mathrm{RT}} = \frac{1}{\Delta\nu_{\mathrm{FSR}}} = \frac{2\ell}{c}.$$

Dispersionless cavity

0 $\omega_0 = 0$

$$\frac{\omega_N}{c} 2nL = 2N\pi \longrightarrow \omega_{N+1} - \omega_N = \frac{\pi c}{nL} = \frac{\pi(c/n)}{L}$$

Fabry–Pérot interferometer & Airy distributions



$$|r_i|^2 = R_i.$$

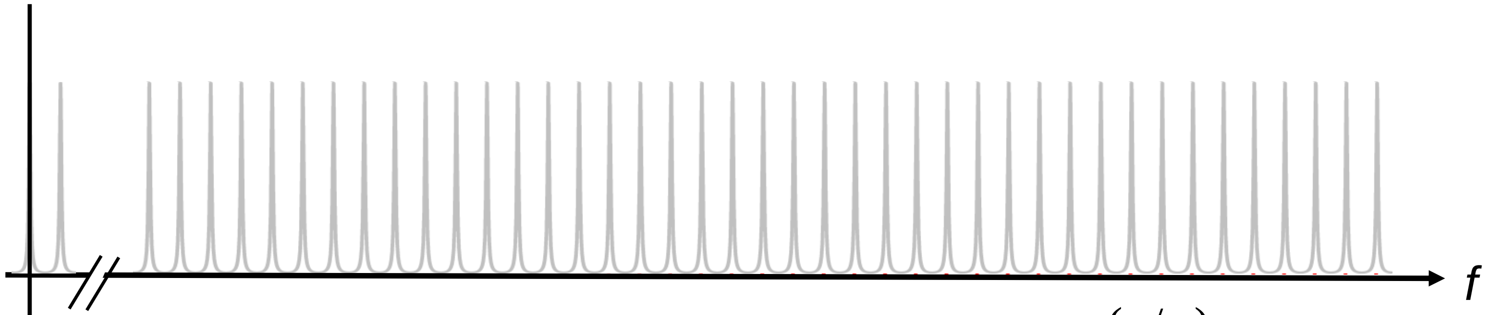
$$E_{\mathrm{circ}} = E_{\mathrm{laun}} + E_{\mathrm{RT}} = E_{\mathrm{laun}} + r_1 r_2 e^{-i2\phi} E_{\mathrm{circ}} \Rightarrow \frac{E_{\mathrm{circ}}}{E_{\mathrm{laun}}} = \frac{1}{1 - r_1 r_2 e^{-i2\phi}}.$$

$$2\phi(\nu) = 2\pi\nu t_{\mathrm{RT}}.$$

$$A_{\mathrm{circ}} = \frac{I_{\mathrm{circ}}}{I_{\mathrm{laun}}} = \frac{|E_{\mathrm{circ}}|^2}{|E_{\mathrm{laun}}|^2} = \frac{1}{|1 - r_1 r_2 e^{-i2\phi}|^2} = \frac{1}{\left(1 - \sqrt{R_1 R_2}\right)^2 + 4\sqrt{R_1 R_2}\sin^2(\phi)}.$$

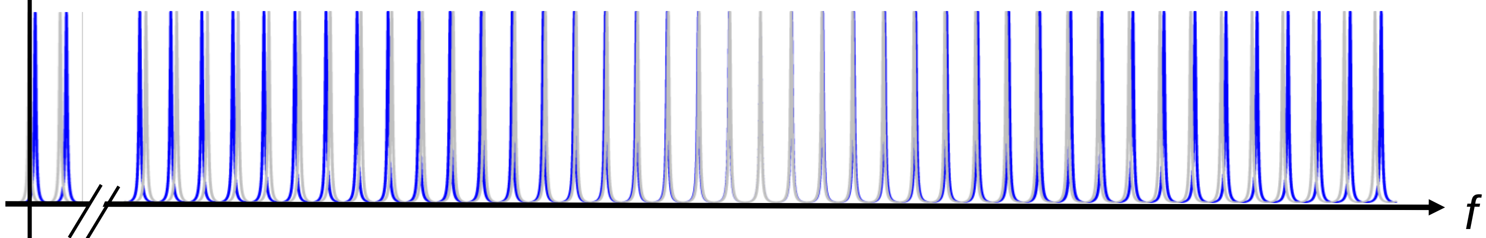
On resonance: $$A_{\mathrm{circ}}(\nu_q) = \frac{1}{\left(1 - \sqrt{R_1 R_2}\right)^2}.$$

Revisit Mode-Locking: The Mode Comb



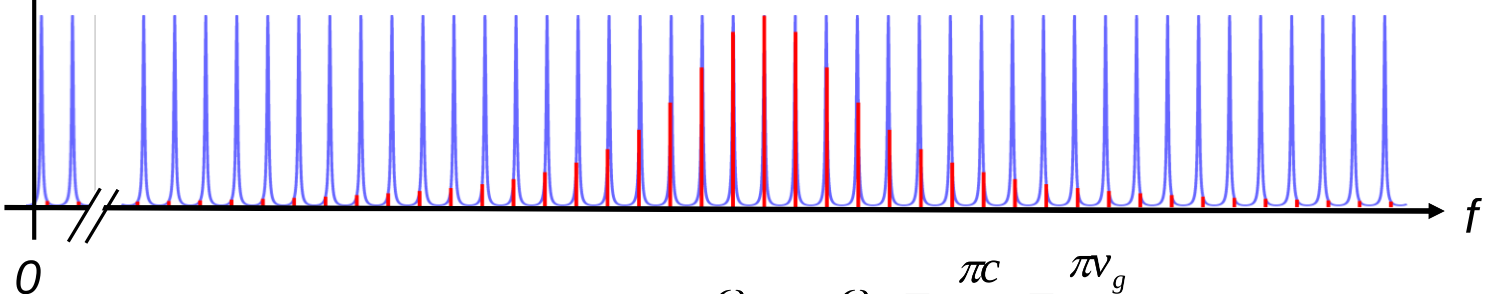
Dispersionless cavity

$$\omega_0 = 0 \qquad \frac{\omega_N}{c} 2nL = 2N\pi \longrightarrow \omega_{N+1} - \omega_N = \frac{\pi c}{nL} = \frac{\pi (c/n)}{L}$$



Gray: Dispersionless
Blue: Dispersive cavity

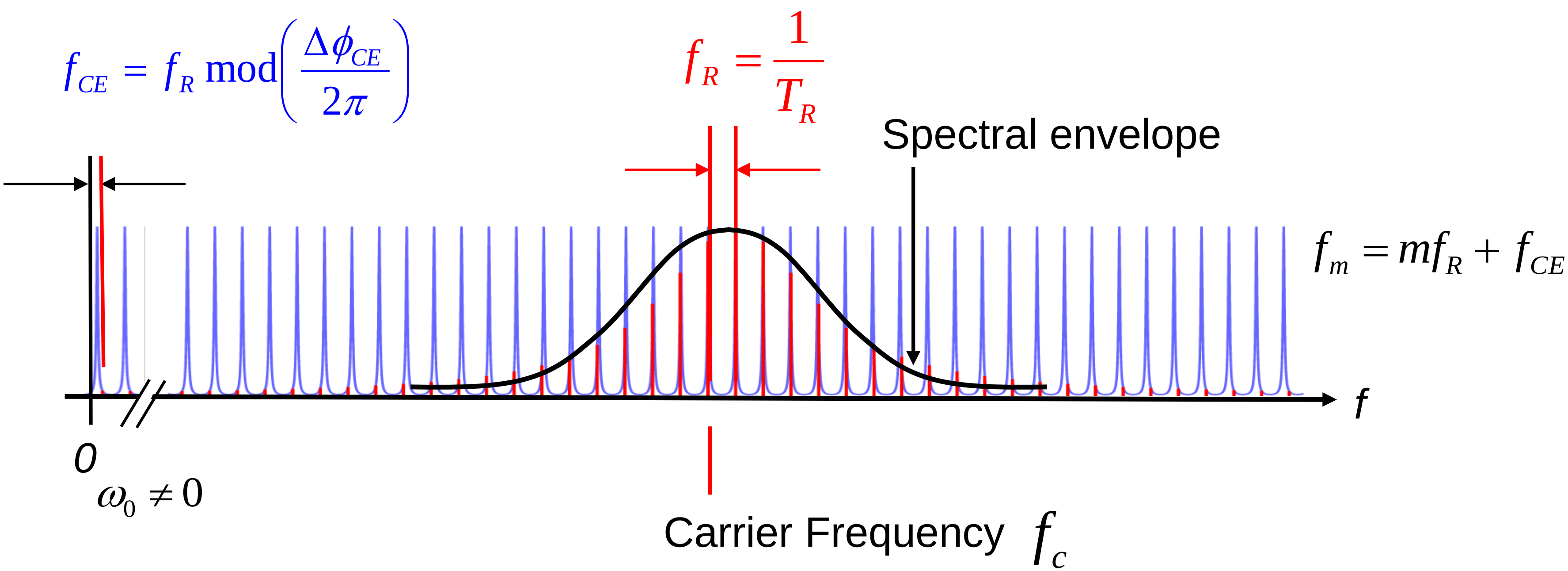
$$\omega_0 \neq 0 \qquad \frac{\omega_N}{c} 2n(\omega)L = 2N\pi \qquad \omega_{N+1} - \omega_N = \frac{\pi c}{n(\omega_N)L}$$



Dispersive cavity when modelocked

$$\omega_0 \neq 0 \qquad \omega_{N+1} - \omega_N = \frac{\pi c}{n_g L} = \frac{\pi v_g}{L}$$

The Comb with two degrees of freedom



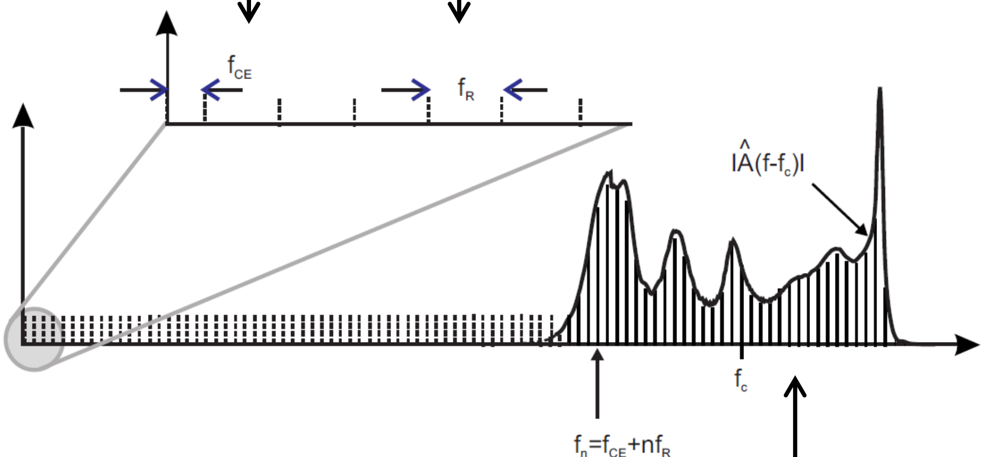
$$\hat{a}_s(\omega - \omega_c)\,\frac{2\pi}{T_R}\sum_{n=-\infty}^{+\infty}\delta\left(\omega - \left(\frac{\Delta\phi_{CE}}{T_R} + n\omega_R\right)\right)$$

It is straightforward to measure (using RF spectrum analyzer) and control (i.e. tuning cavity length) $f_{R.}$

1) How to measure f_{CE}?
2) How to control f_{CE} or what determines f_{CE}?
3) Does pump power only relate to f_{CE}?

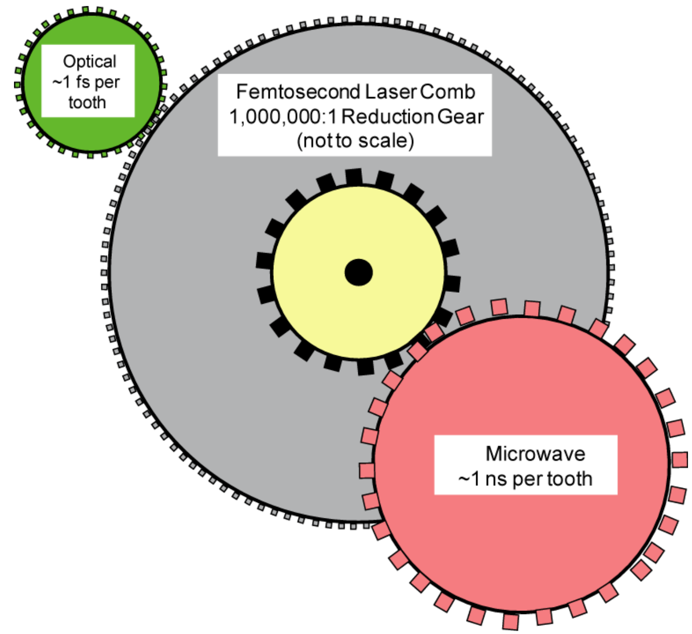
Linking optical and RF frequencies

Radio frequencies



Optical frequency

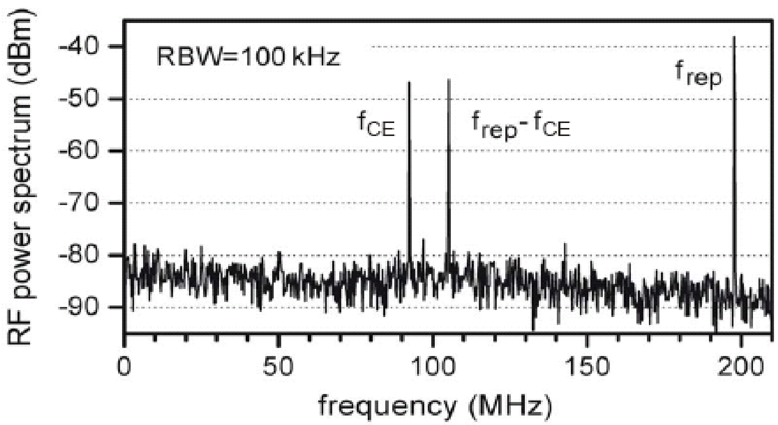
gear picture:
Ref. Scott Diddams

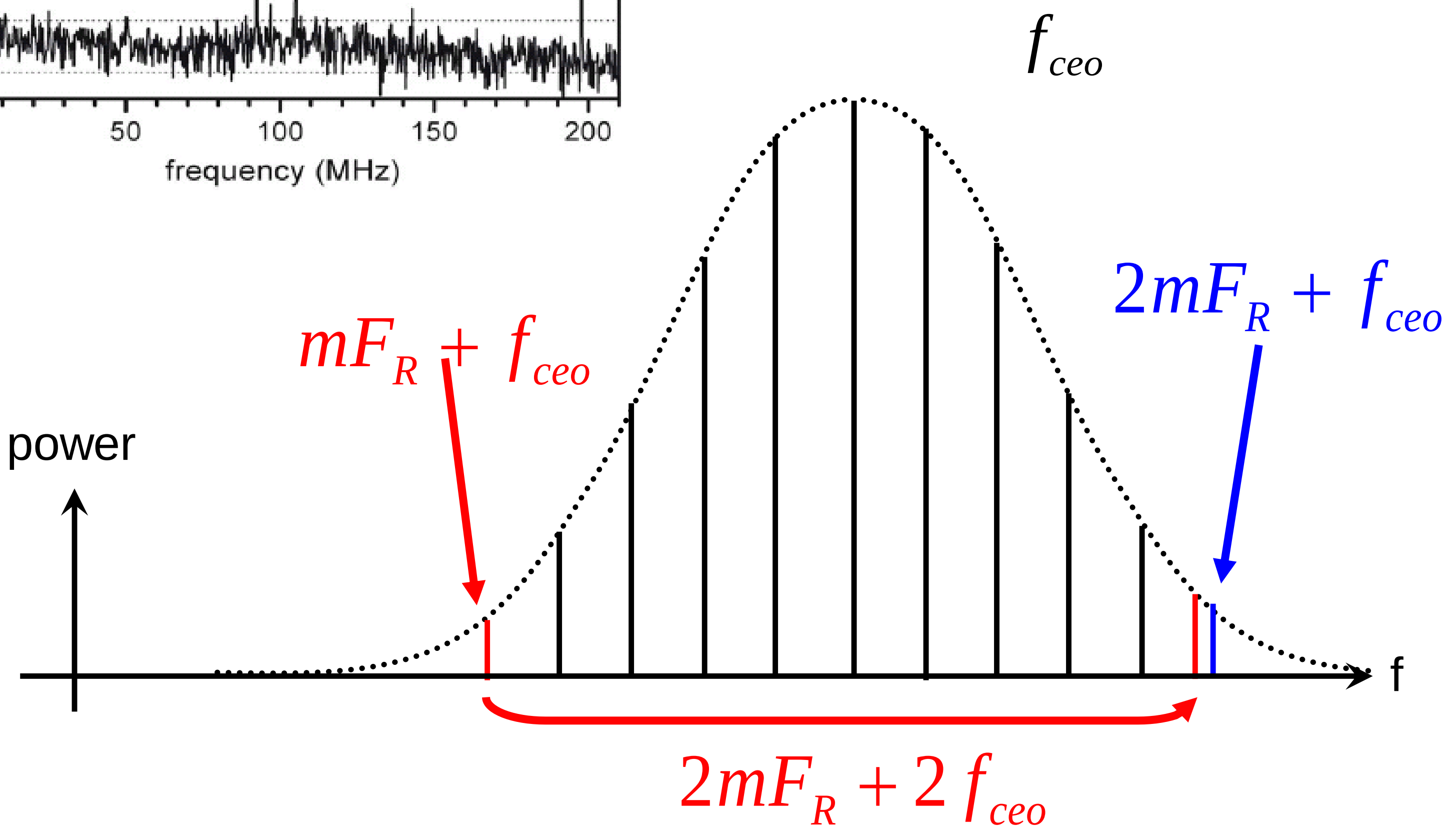


Frequency determination:

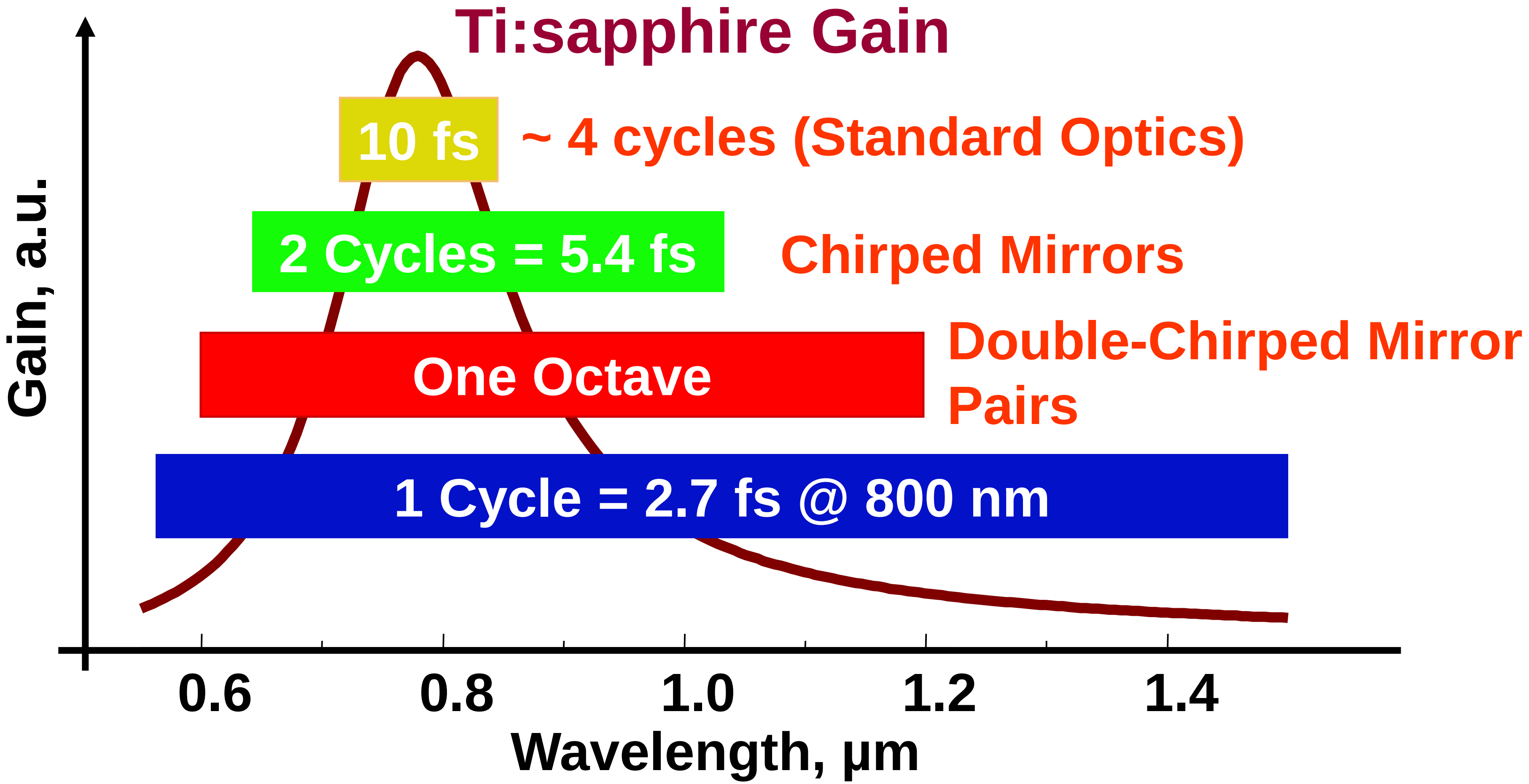
- Measure f_{CE}
- Measure f_R
- Determine n
- Claculate f_n

Measure carrier-envelope offset frequency using 1f-2f Interferometer





Bandwidth of Few-Cycle Optical Pulses

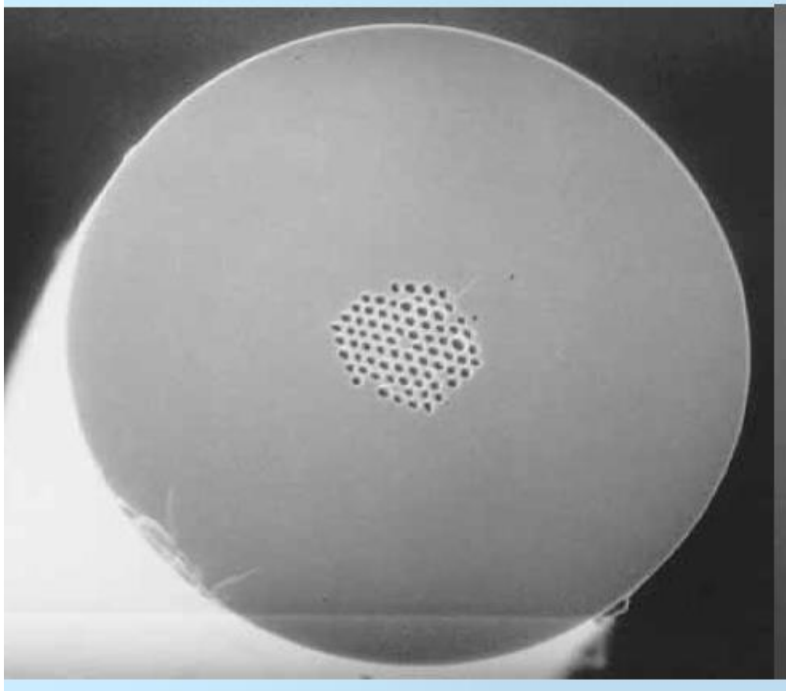


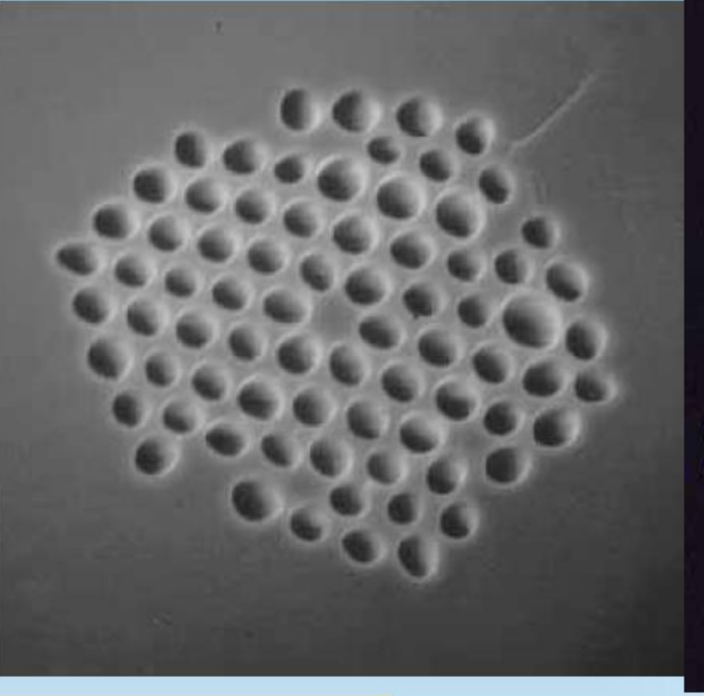
First demonstration of octave spanning Ti:Sapphire Laser:

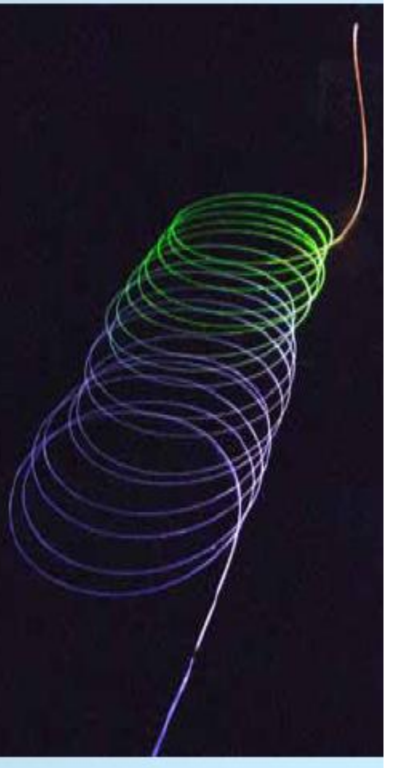
U. Morgner, et al., PRL 86, 5462-5465, 2001.

Honeycomb Microstructure Optical Fiber

CLEO,May, 1999







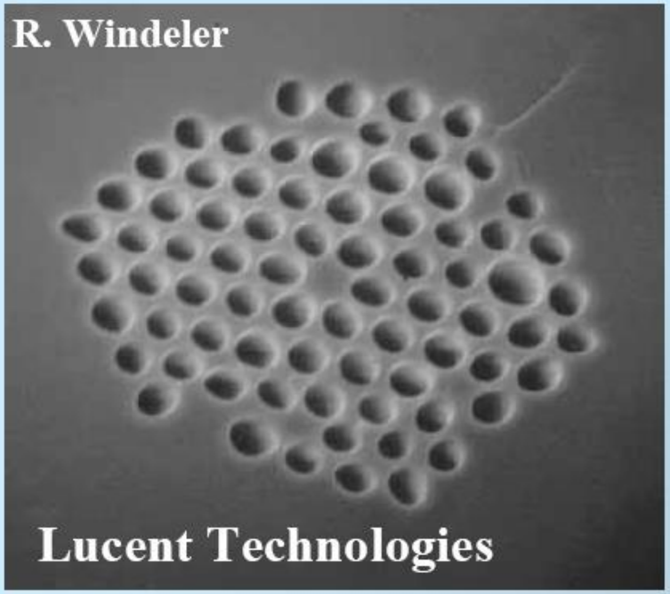
Dawn of a new Epoch !

Phillip Russel, University of Bath

courtesy of Jinendra Ranka

Lucent Technologies

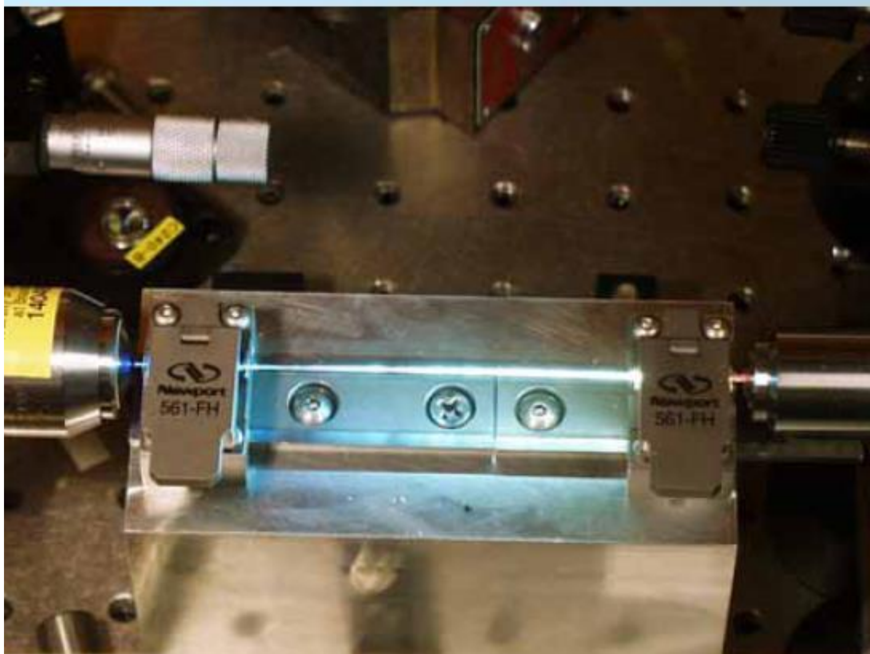
Bell Labs Innovations

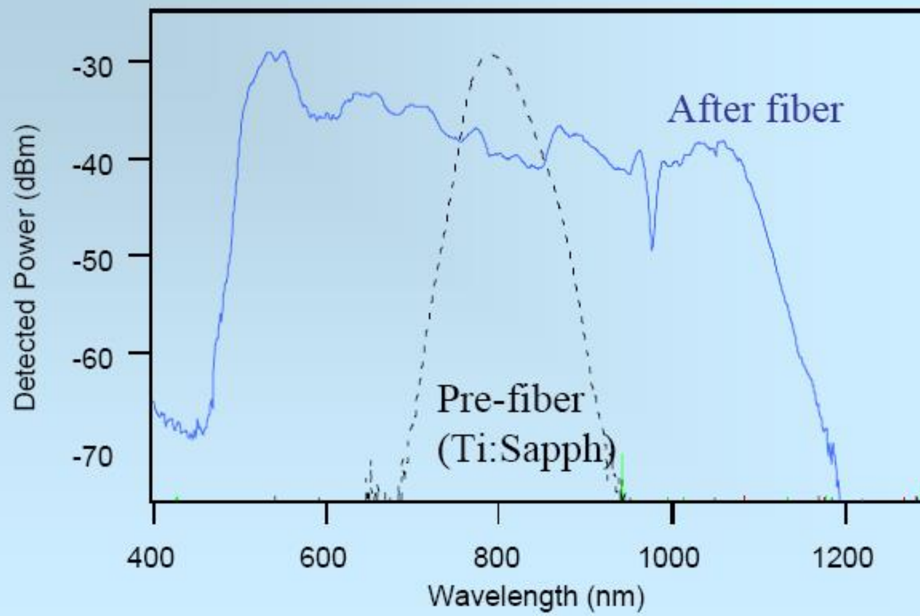


J.K Ranka, R. S. Windeler, A. Stenz, Opt. Lett. **25**, *25 (Jan. 2000)*

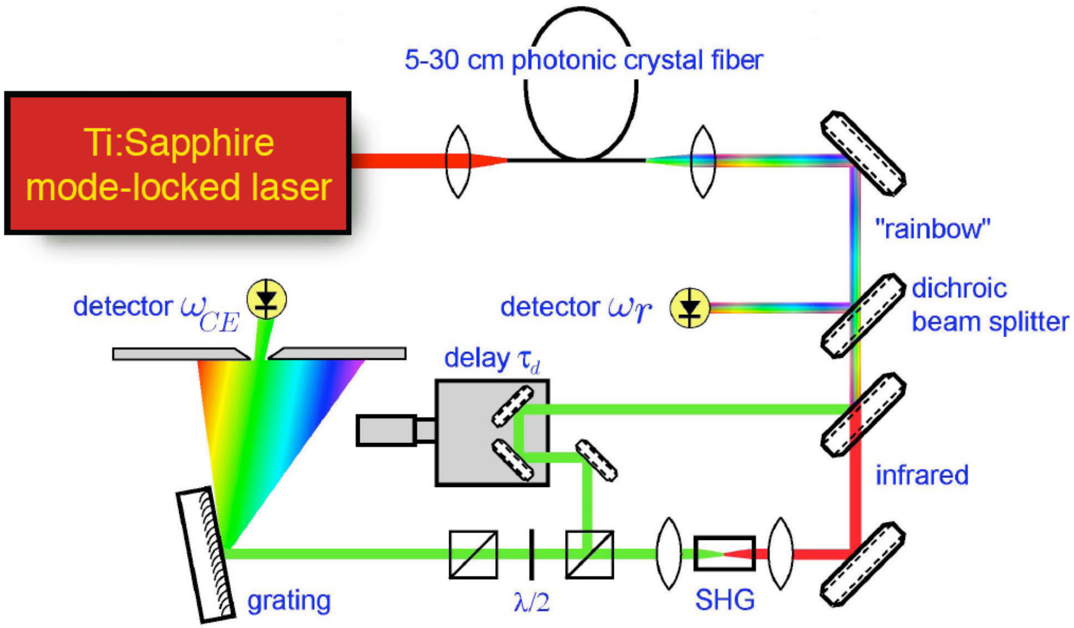
Microstructured fiber

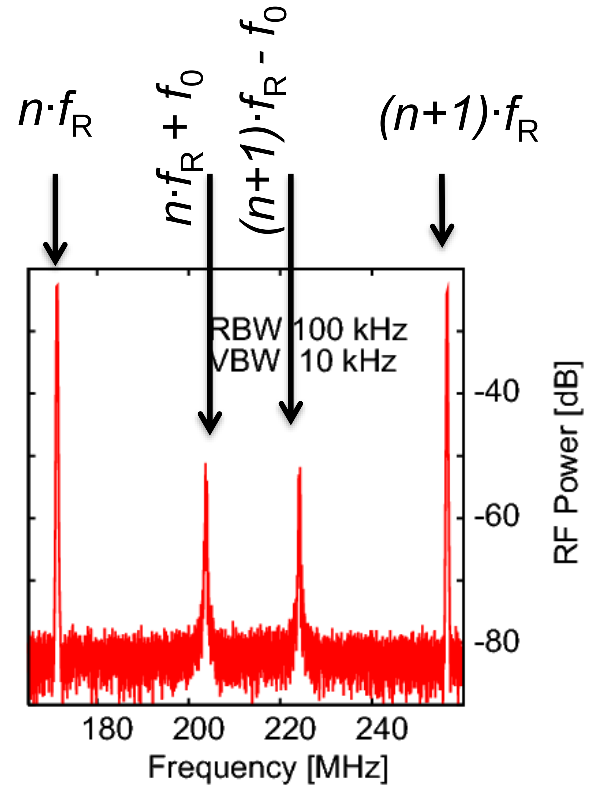
- dispersion zero at ~800 nm
- pulses do not spread
- continuum generation via self-phase modulation





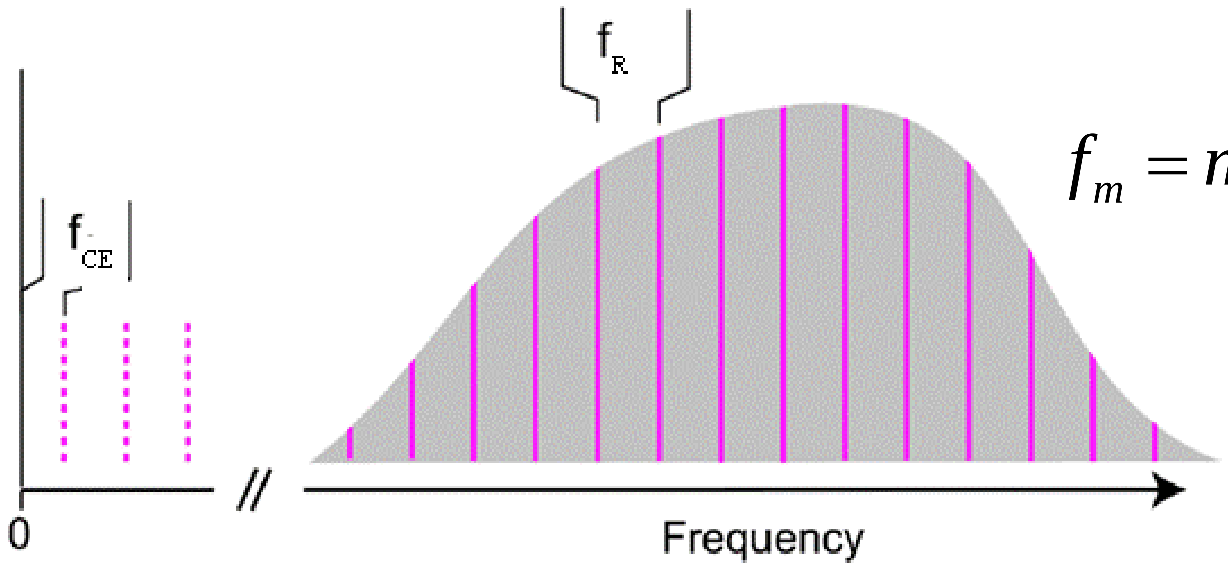
Measure both repetition rate and carrier-envelope offset frequency





http://nobelprize.org/nobel_prizes/physics/laureates/2005/hansch-lecture.html

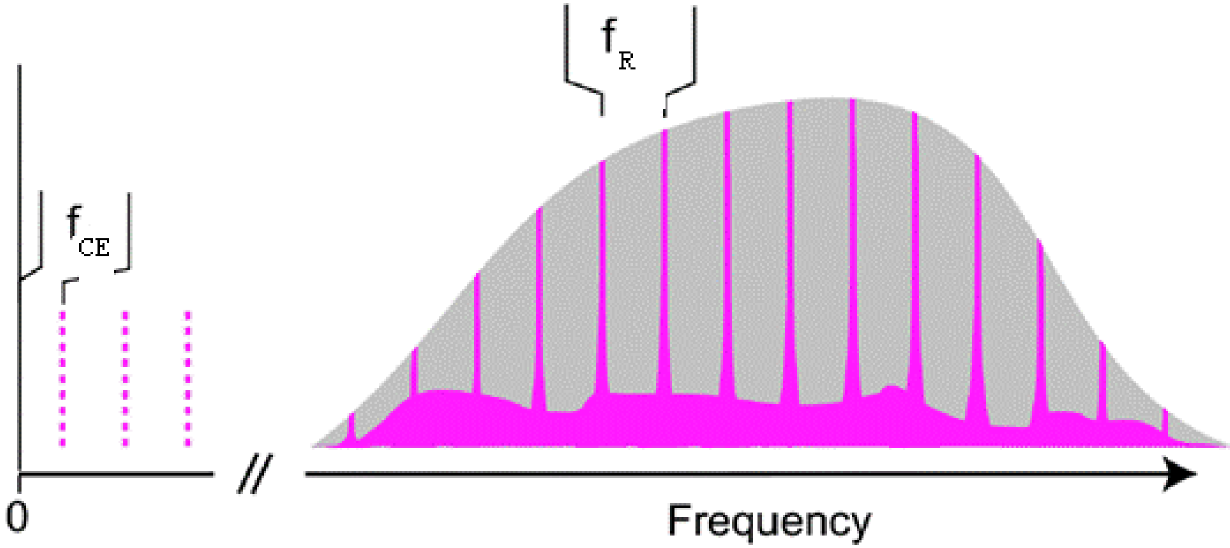
The noiseless comb



$$f_m = mf_R + f_{CE}$$

1) Each comb line is a noiseless CW laser with zero linewidth.
2) For a mode-locked laser with 100-fs pulse duration and 100 MHz repetition rate, the spectral bandwidth is about 1/(100 fs)=10 THz. So the number of comb lines within the bandwidth is (10 THz)/(100 MHz) = 100,000.
3) In other words, this frequency comb is equivalent to 100,000 CW lasers with their frequency fixed by $f_M = Mf_R + f_{CE}$

Effects of noise

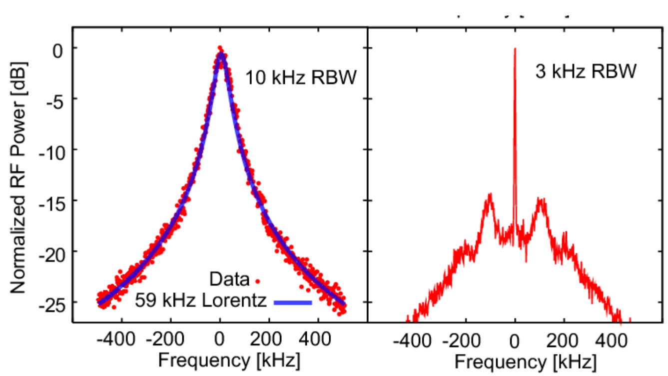


Newbury and Swann JOSA B **24**, pp. 1756-1770 (2007)
Benkler, et al. Opt. Express **13**, 5662–5668 (2005).

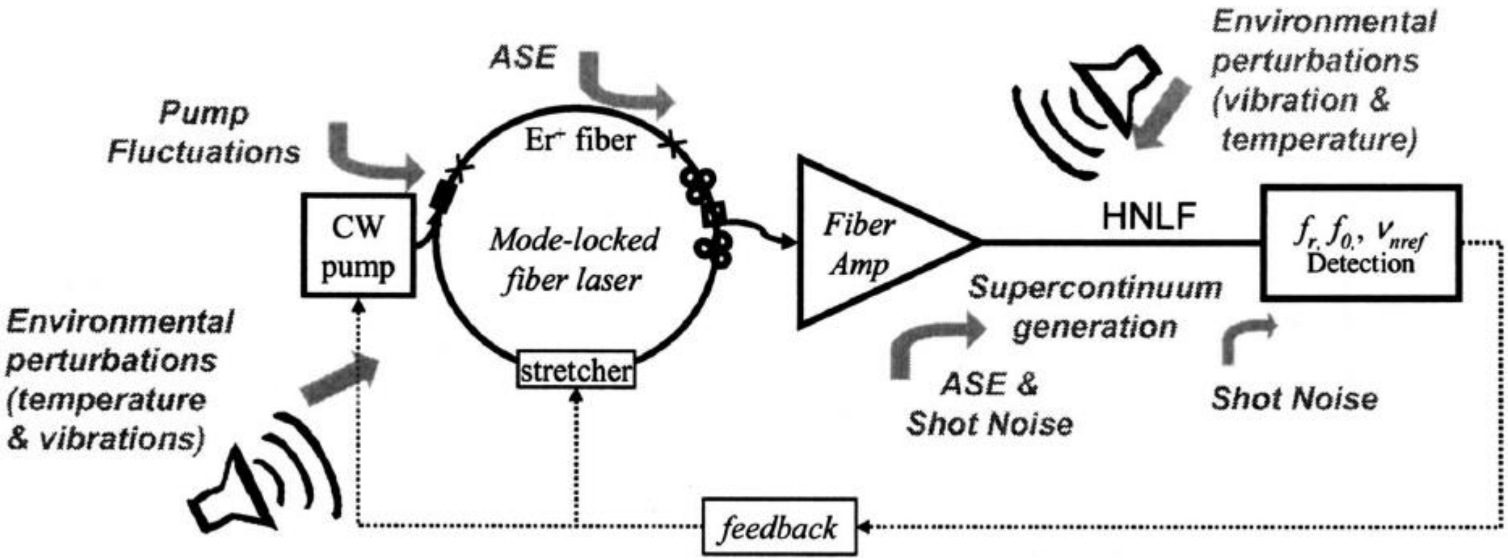
Intracavity noise: Broadens comb line
Cavity length (fixed point ~ 0)
Pump fluctuations (fixed point ~ comb center)

Extracavity noise: Reduces comb visibility
Continuum noise, amplifier ASE

Feedback: intracavity noise is reduced



Effects of noise & key noise quantities

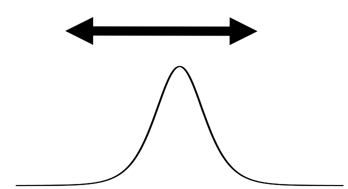


Intensity noise: fluctuations of the light field amplitude

Commonly used term: Relative intensity noise [dBc]

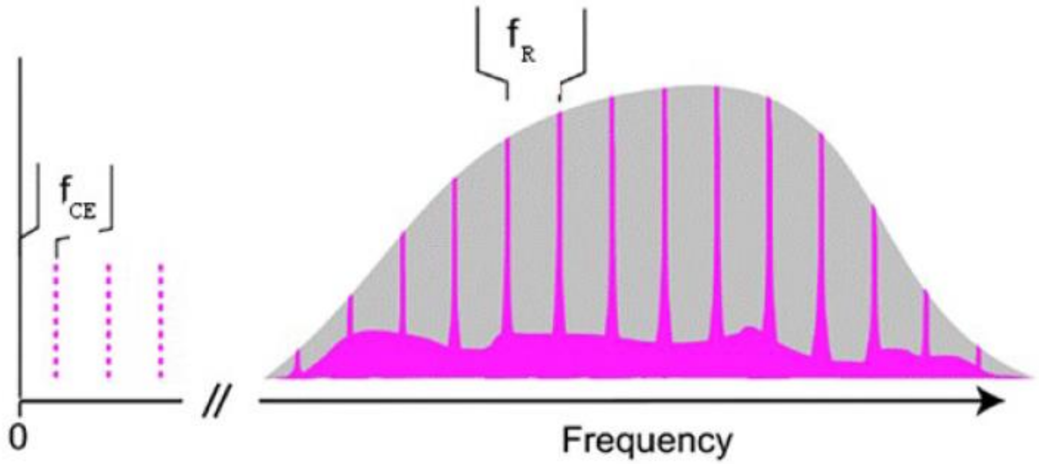
Frequency and phase noise: fluctuations of phase (and thus frequency) [rad²/Hz] [Hz²/Hz]

Timing jitter: Random walk of pulse position [fs]



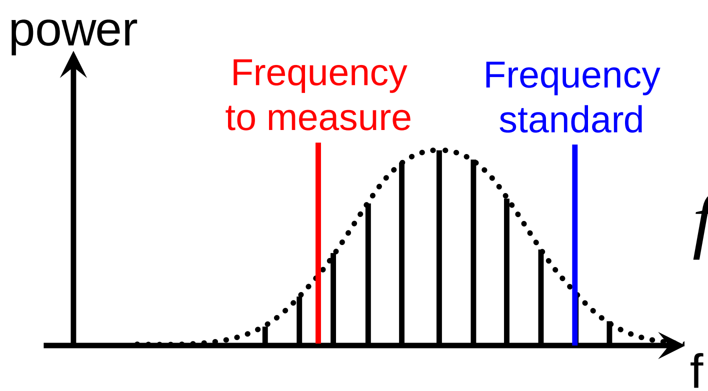
time

Construction Guidelines for Low Noise Lasers

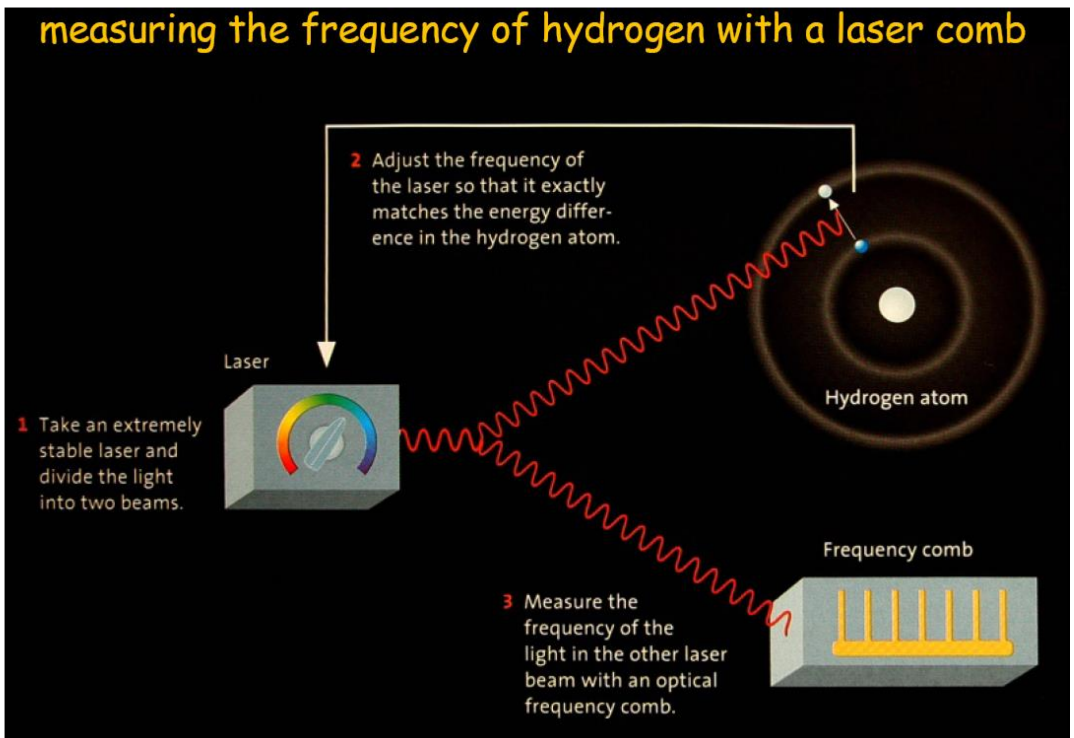


- Low loss: low gain and therefore less ASE
- Low net-cavity dispersion: minimizing Gordon-Haus effect (i.e., reducing conversion of center frequency noise to timing jitter)
- High intra-cavity power
- Shorter intra-cavity pulse: minimize the influence of ASE
- Long cavity (but it will pick up more environmental noise)

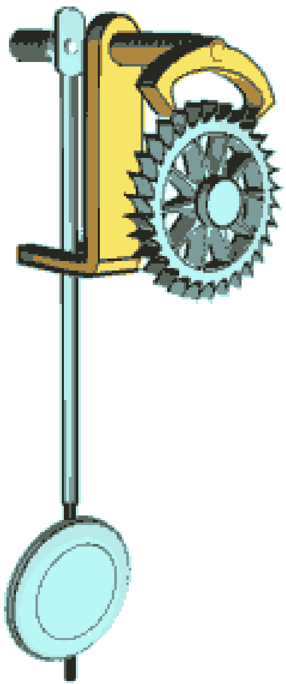
Frequency Metrology

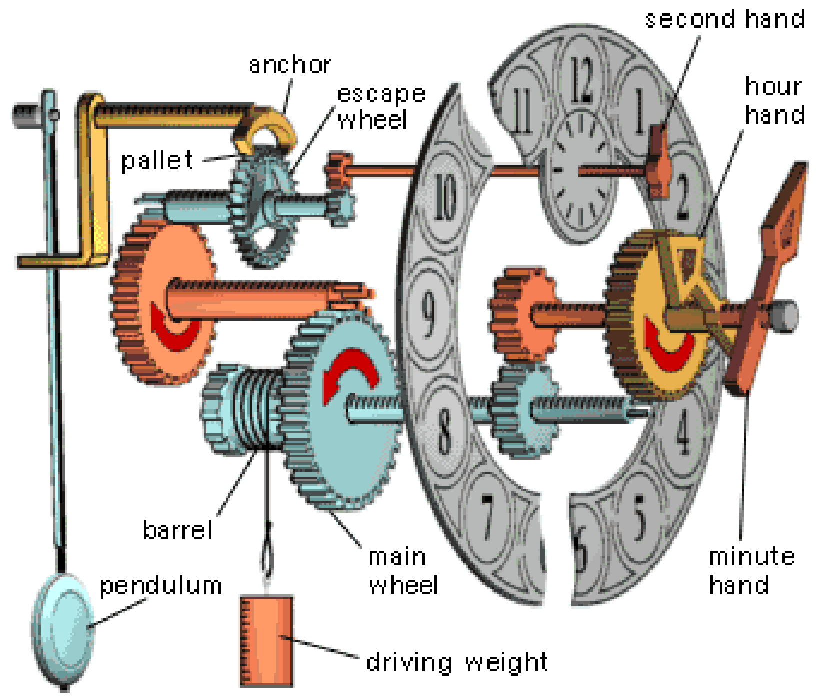


$$f_m = m f_R + f_{CE}$$



What is a clock?





an oscillator and a clockwork

Pendulum - Christiaan Huygens 1656

Chronometer - John Harrison (H4) 1761 (10^{-6} ~ 1 sec/ 9 days)

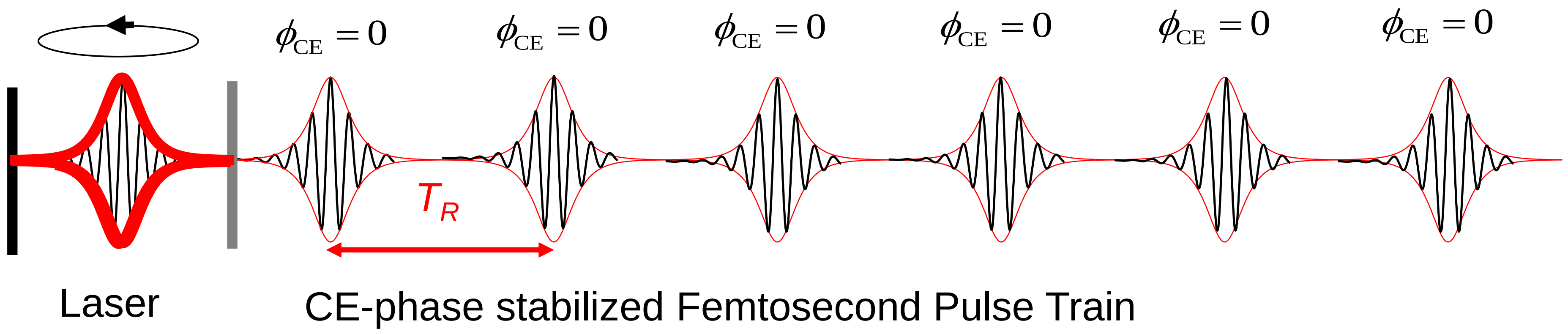
Quartz - W. Marrison, Bell Labs, 1928 (10^{-8} ~ 1 sec/3yrs)

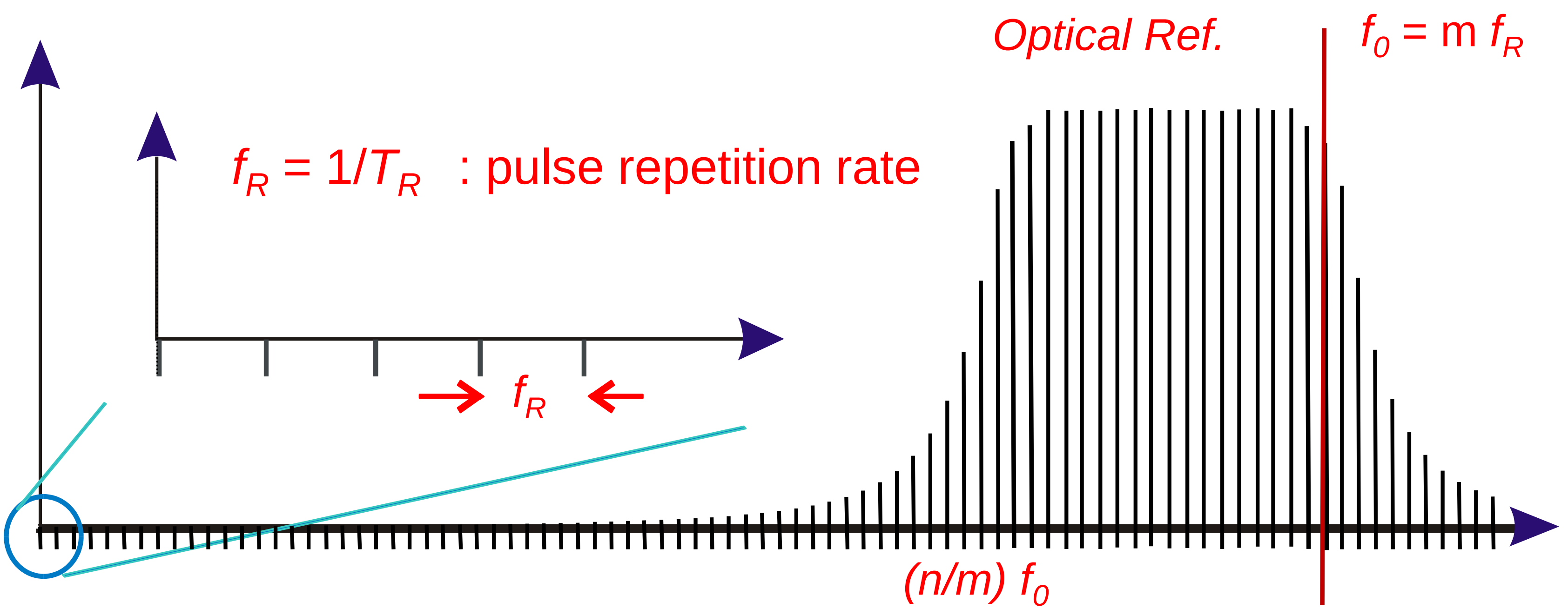
Cesium atom - 1955 (10^{-10} ~ 1sec/300yrs)

Atomic fountain - NIST-F1 ($1.7x10^{-15}$ ~ 1sec/20Myrs)

Hg ion – $5x10^{-18}$ ~ 10sec since big bang

Optical Clock





Each microwave cycle = M Optical Cycles

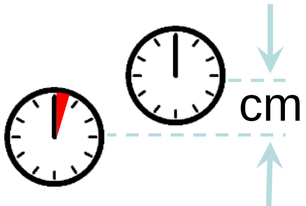
Low noise optical pulse train or microwaves

Defining time…

Current definition of 1 s: 9,192,631,770 caesium spin flips…

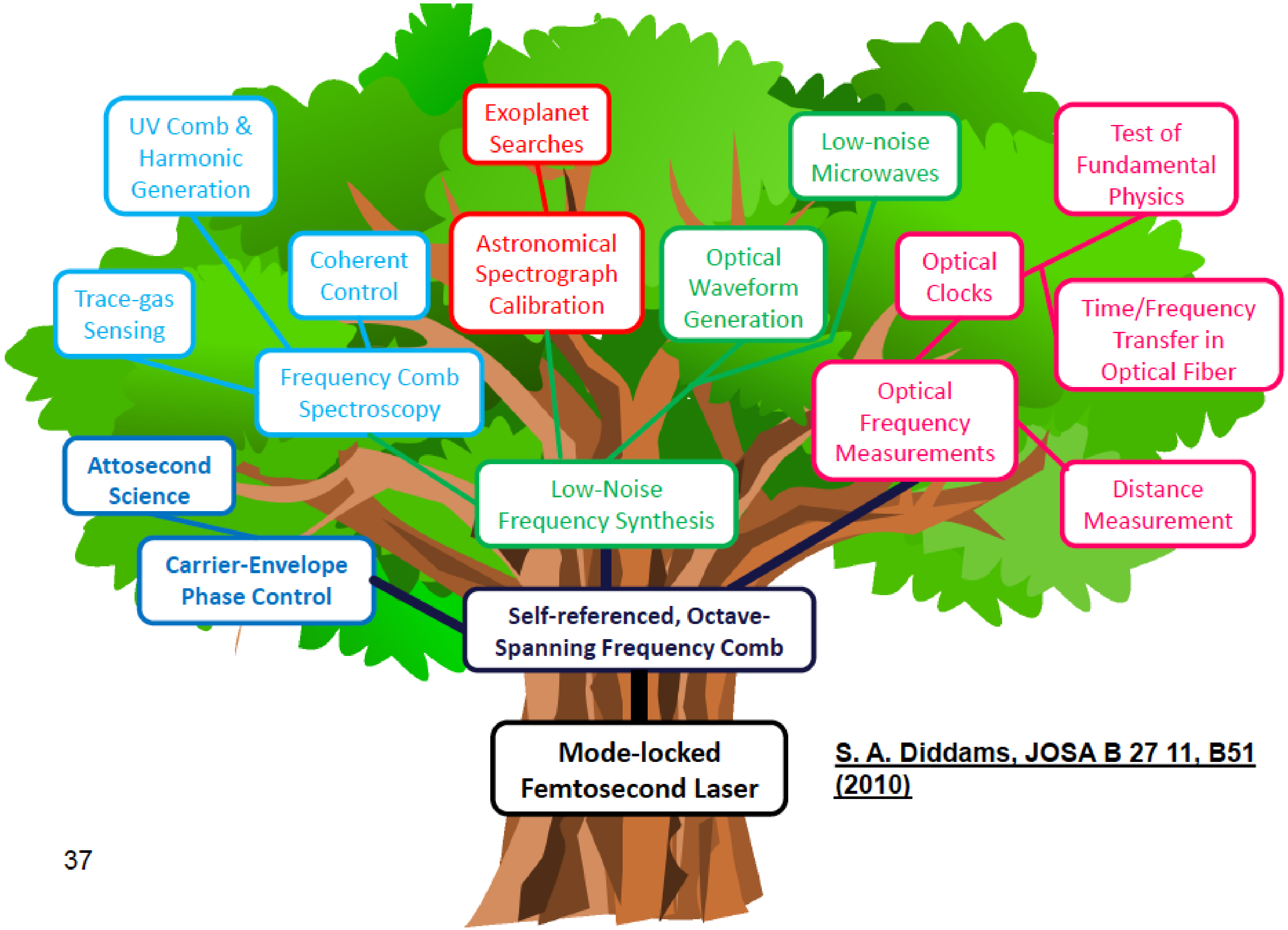
Reasons to define time using optical clocks:

- The higher the reference frequency, the better is the reference!
- Current standard lies in the microwave range
- Optical clocks with superior stability exist, using references at 1 million times higher frequency
- Great applications are waiting for better frequency references:
  - Clock-based geodesy
  - Clock-based gravitational wave detection
  - Searches for Physics beyond the standard model
  - …



M. Takamoto, *Nat. Photonics* **14,** 411–415 (2020).

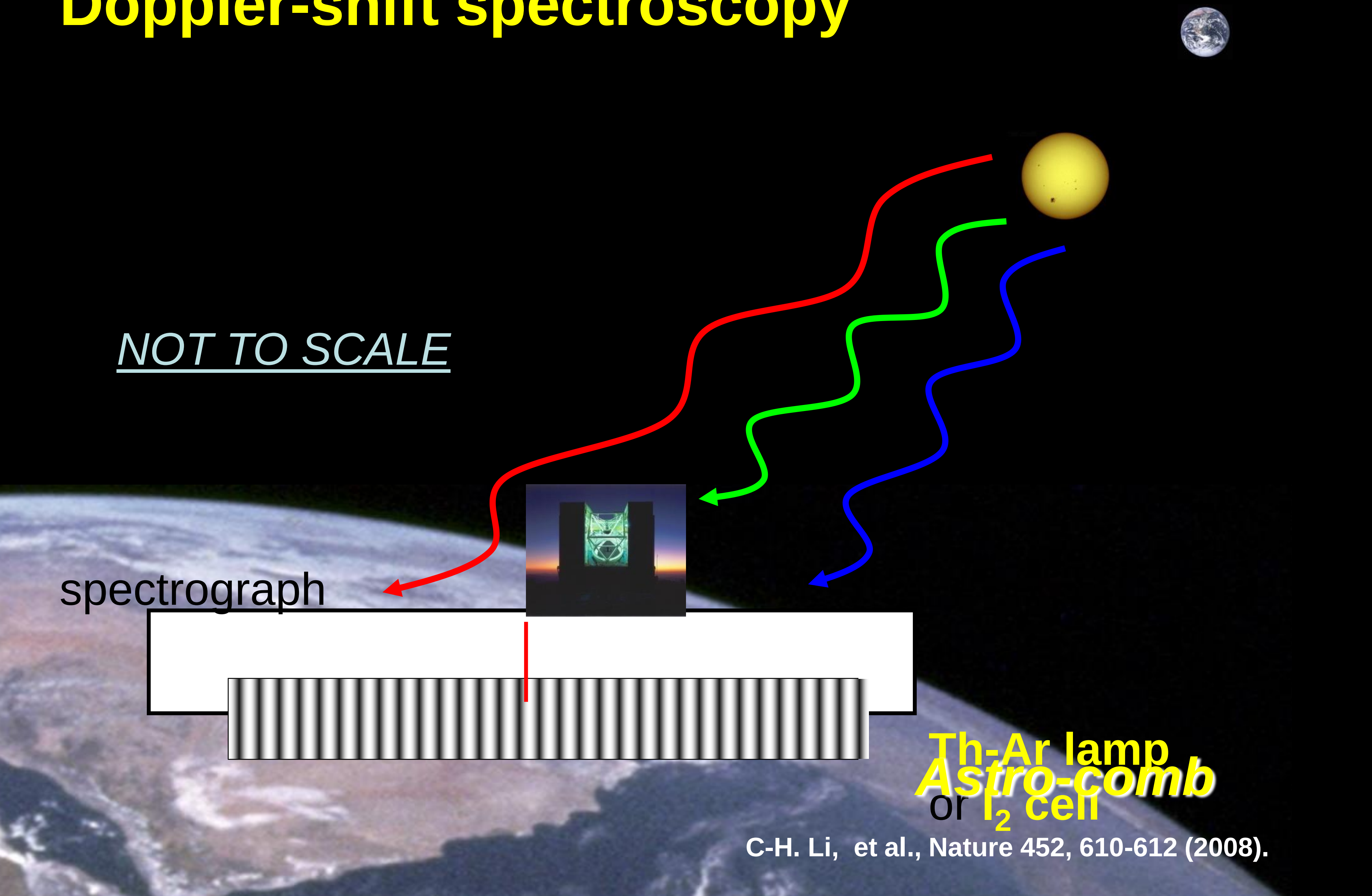
Frequency comb evolution & applications



S. A. Diddams, JOSA B 27 11, B51 (2010)

Example application 1
The Astrocomb

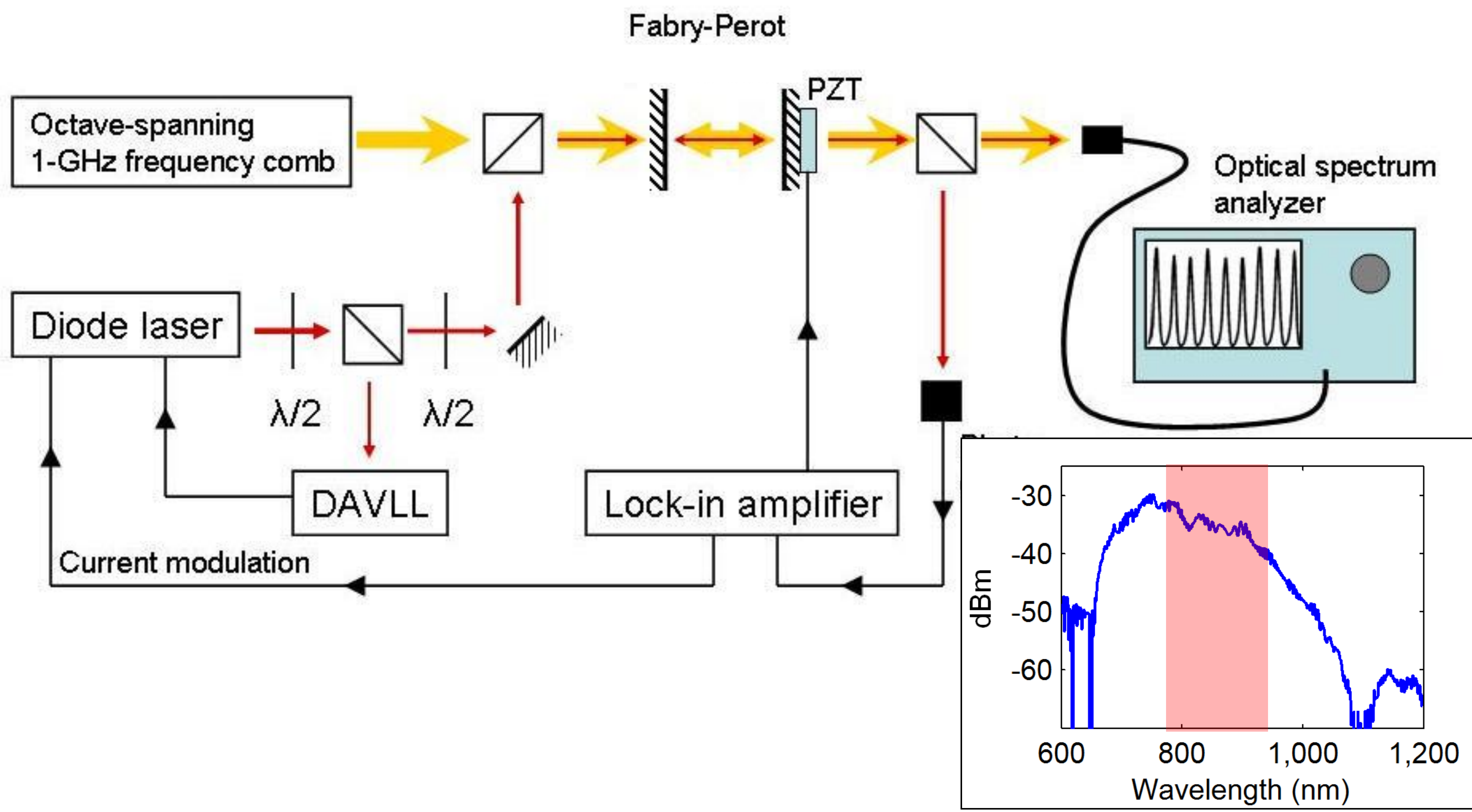
Doppler-shift spectroscopy



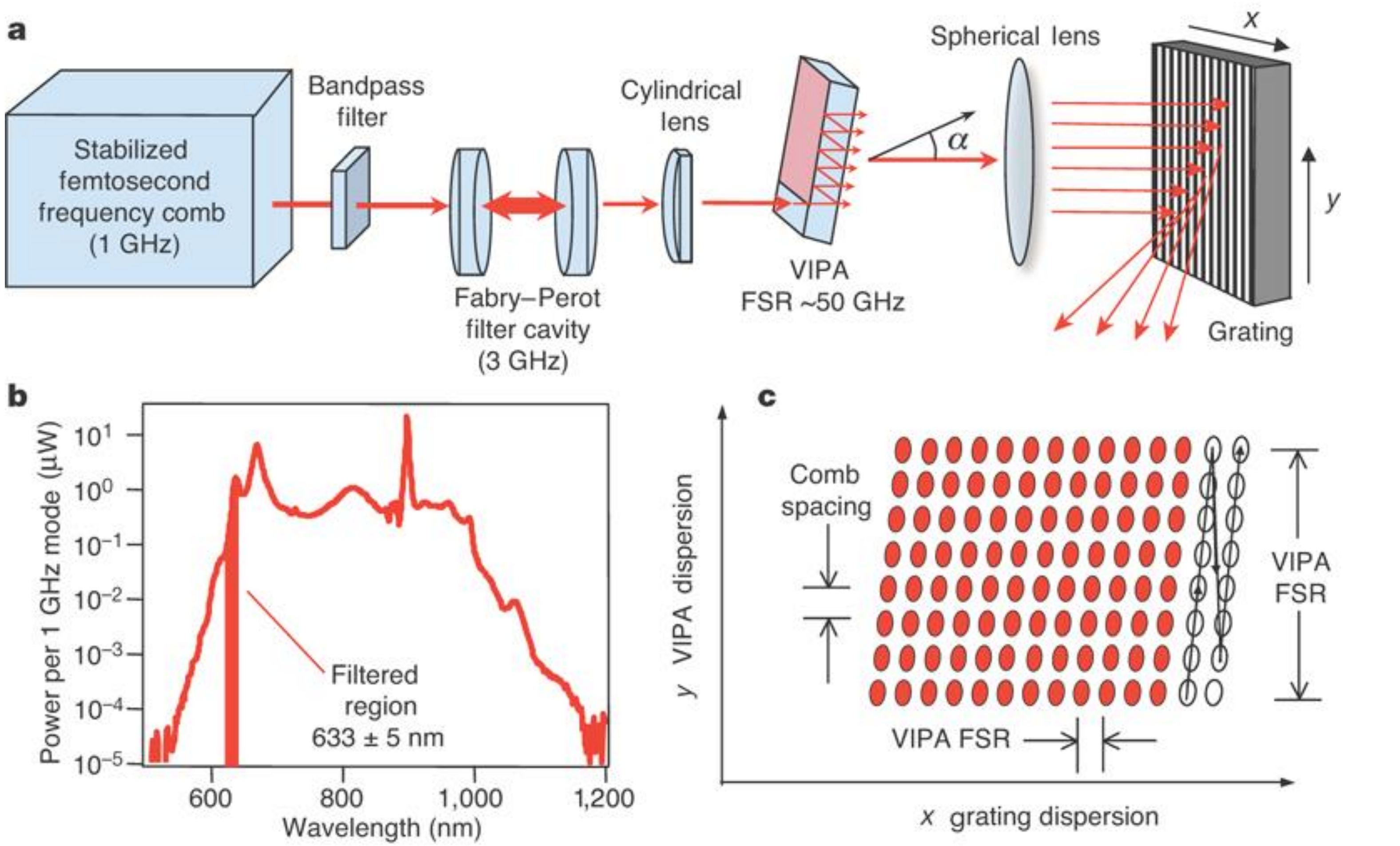
C-H. Li, et al., Nature 452, 610-612 (2008).

Astro comb: high repetition rates required

Remove comb lines with a stabilized Fabry-Perot cavity.

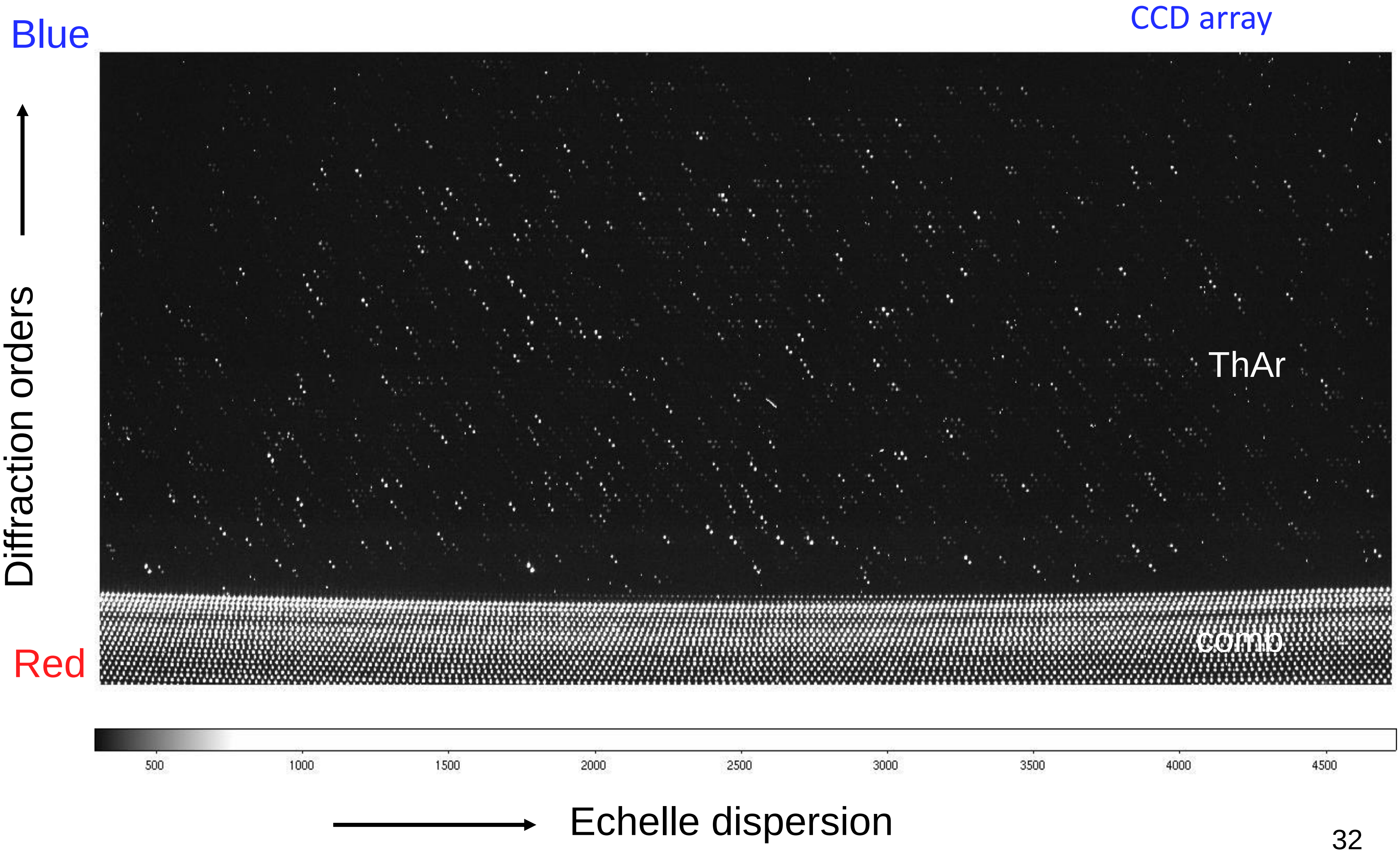


Resolving comb modes: The virtually imaged phased array (VIPA)

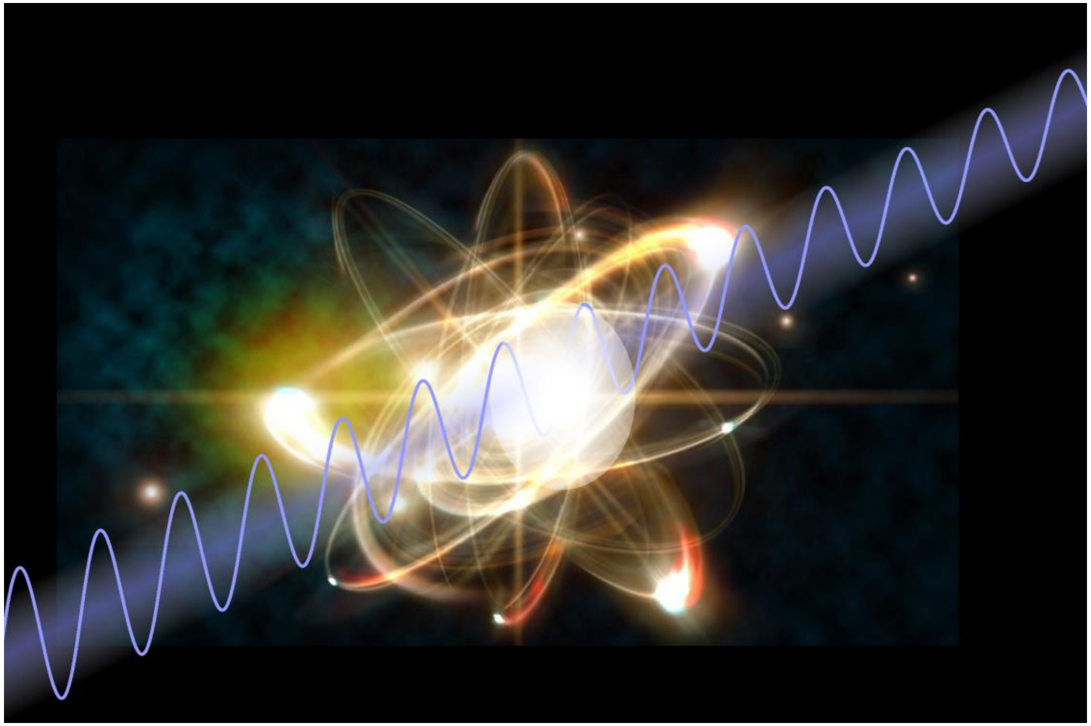


Diddams et al. , Nature, 2007

Th-Ar lamp versus Astro-Comb
(Whipple Obseratory, Arizona)

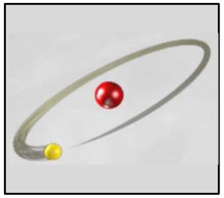


Example application 2
Thorium nuclear spectroscopy



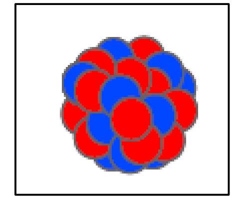
Th spectroscopy: motivation

Laser + atom



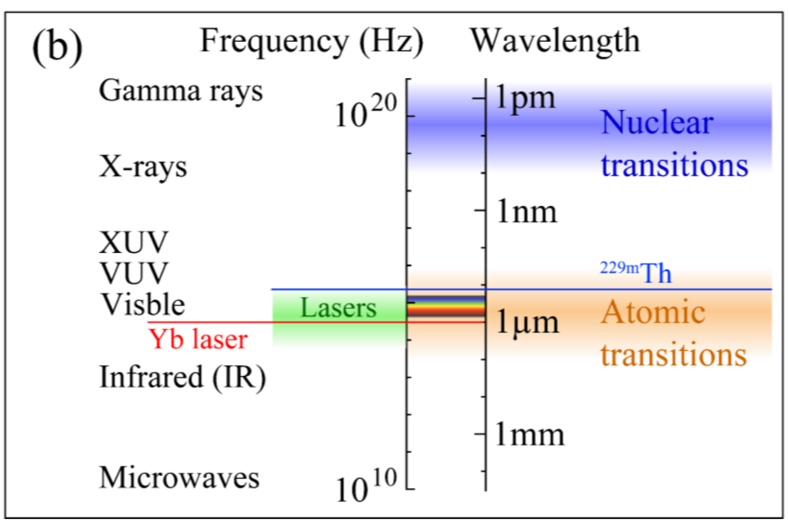
- Modern atomic physics!
- Test of fundamental theories
- Laser cooling - ultracold matter
- GPS, Radioastronomy
- Femtochemistry
- Attosecond physics
- …

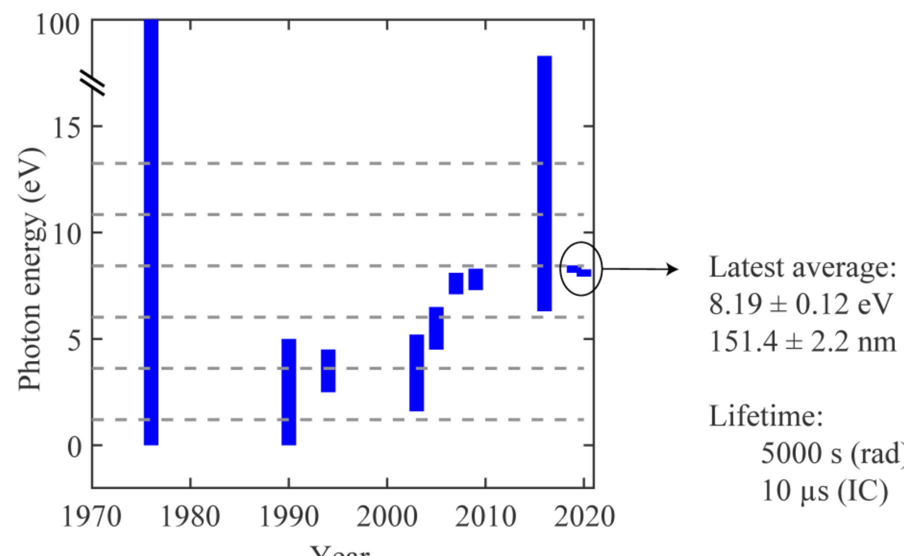
Laser + nucleus



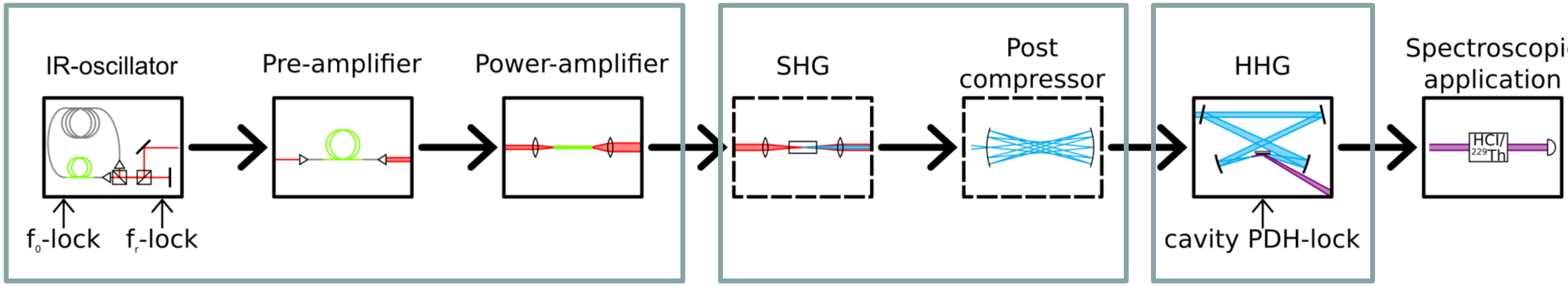
Envisioned applications:

- Precision tool for nuclear physics!
- Measure gravitational waves
- Physics beyond the standard model:
    - Fundamental constant variation?
    - Search for dark matter
    - …





Th spectroscopy: Frequency comb approach



Ytterbium laser:

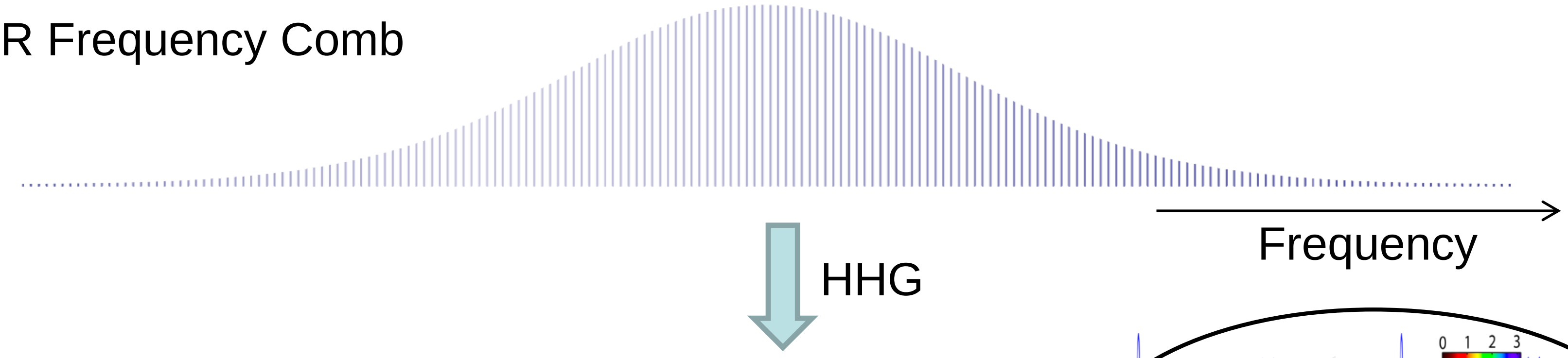
- high power (100 W)
- High repetition rate (65 MHz)
- Moderate pulse duration (200 fs)

Post-compression
< 50 fs

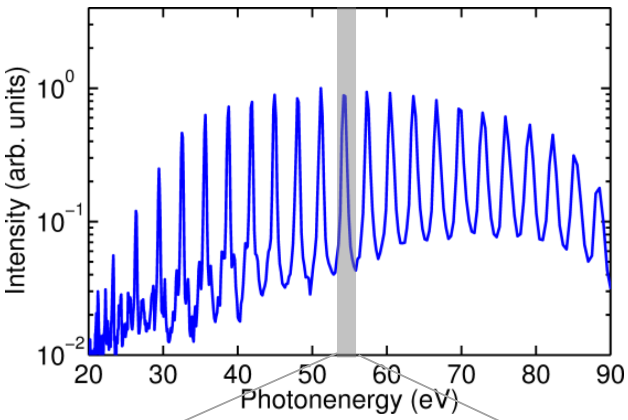
Passive power enhancement and
Coherent IR-XUV conversion!

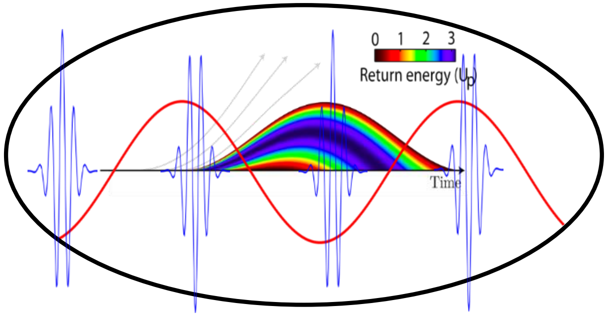
Phase-coherent conversion of combs

IR Frequency Comb

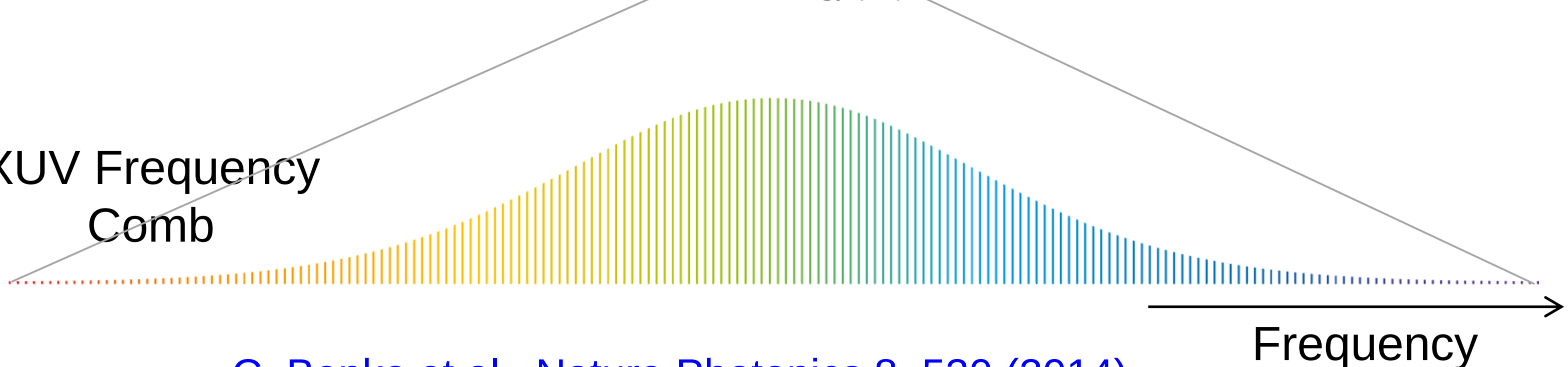


High-order Harmonic Generation





XUV Frequency Comb



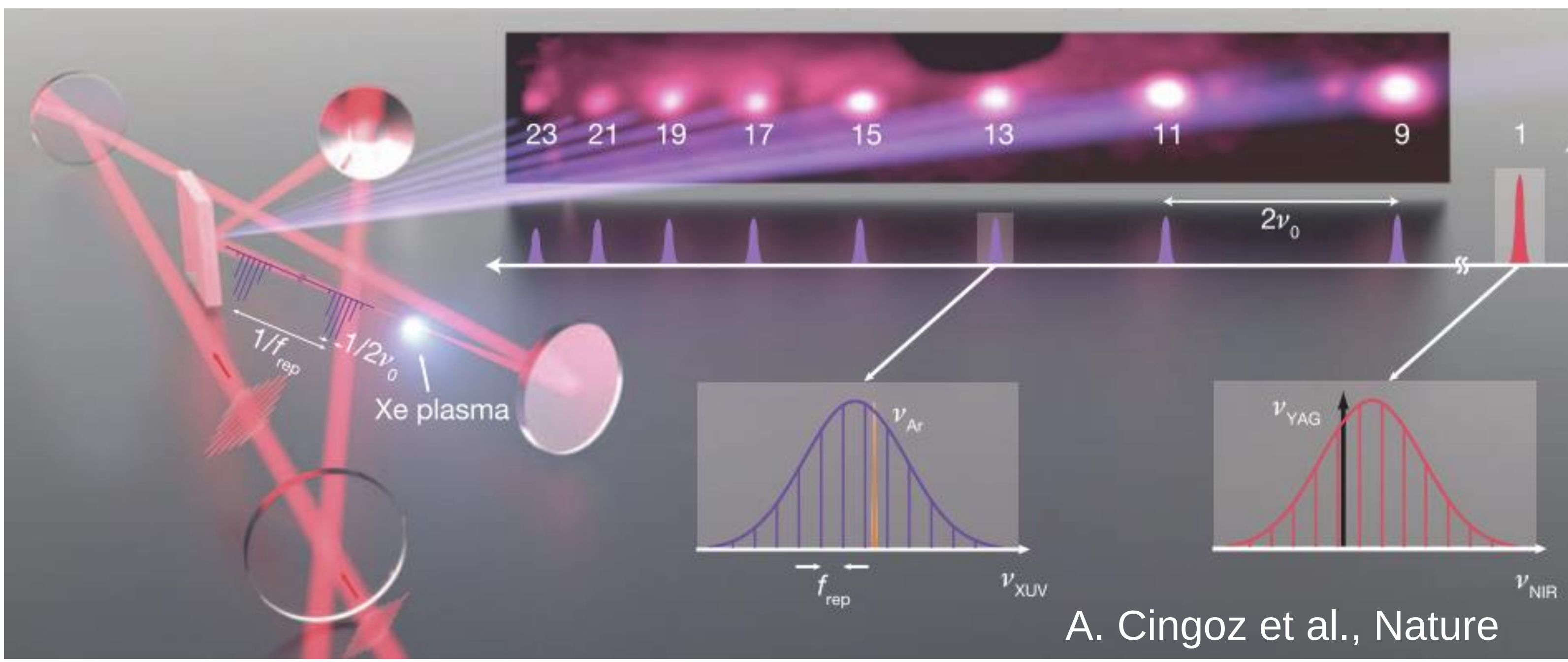
C. Benko et al., Nature Photonics 8, 530 (2014)

Femtosecond enhancement cavities

Frequency comb:
High repetition rate
(65 MHz)

HHG:
High intensity
(10^{13} – 10^{15} W/cm^2)

fs laser pulses with
~ 10 kW of average power!



A. Cingoz et al., Nature

End