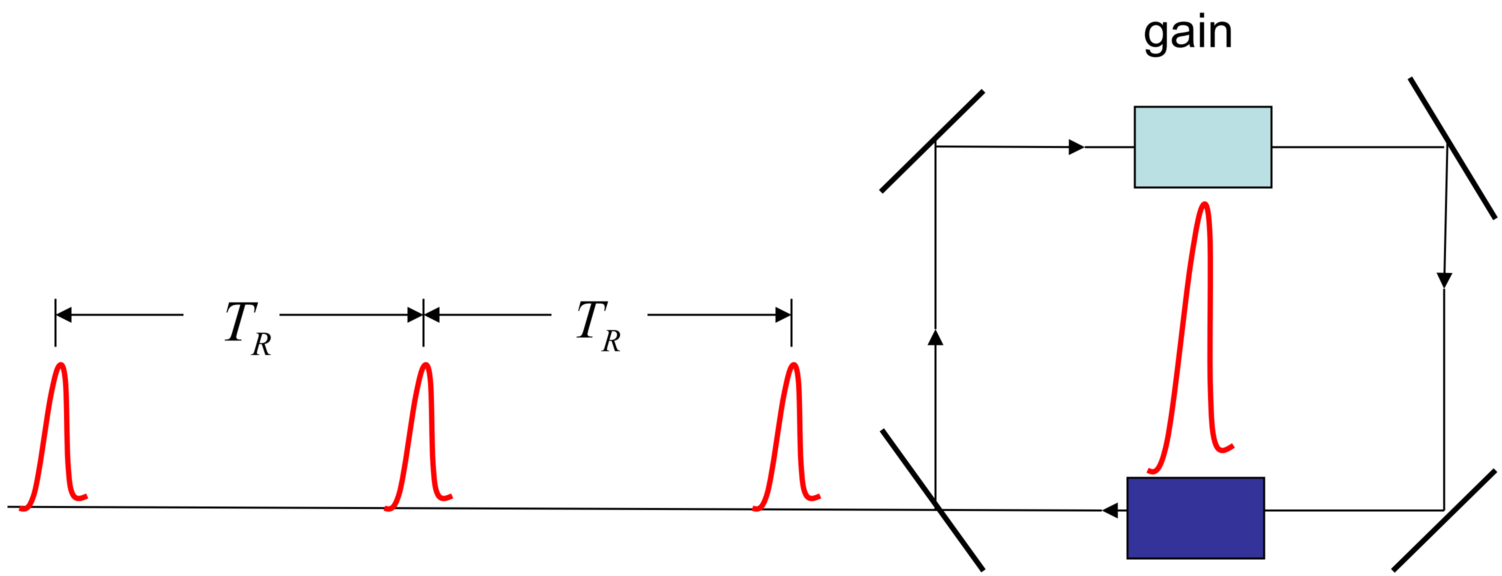
UFS Lecture 12: Passive Modelocking

5. Active mode-locking using amplitude modulator

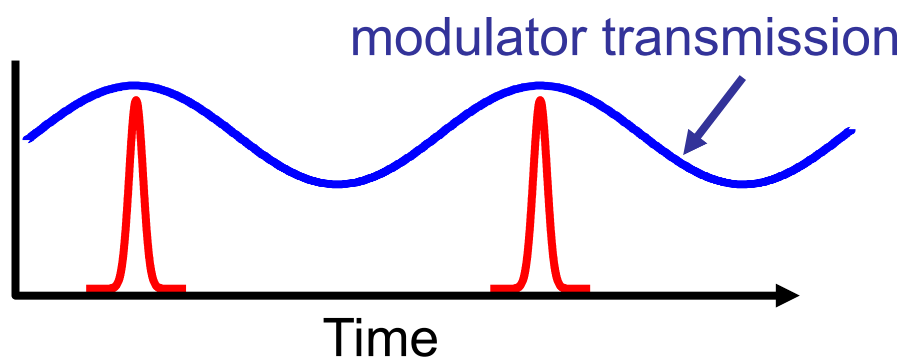


Loss coefficient $q(t) = M(1-\cos\omega_m t)$

Transmission of the modulator

$$T_m = e^{-M(1-\cos\omega_m t)}$$

$$T_m \approx 1 - M(1-\cos\omega_m t)$$

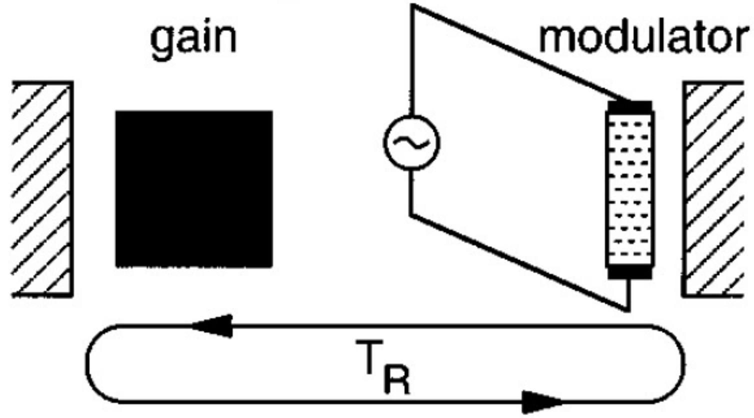


Active mode-locking using amplitude modulator

$$T_R \frac{\partial A}{\partial T} = \left[g(T) + D_g \frac{\partial^2}{\partial t^2} - l - M\left(1 - \cos(\omega_M t)\right) \right] A.$$

$$T_R \frac{\partial A}{\partial T} = \left[g - l + D_g \frac{\partial^2}{\partial t^2} - M_s t^2 \right] A.$$

$$D_g = \frac{g}{\Omega_g^2} \qquad M_s = \frac{M\omega_M^2}{2}$$



Hermite-Gaussian Solution

$$A_n(t) = \sqrt{\frac{W_n}{2^n \sqrt{\pi} n! \tau_a}} H_n(t/\tau_a) e^{-\frac{t^2}{2\tau_a^2}}$$

$$\tau_a = \sqrt[4]{D_g / M_s} \longrightarrow \boxed{\tau_a = \sqrt[4]{2} (g/M)^{1/4} / \sqrt{\Omega_g \omega_M}}$$

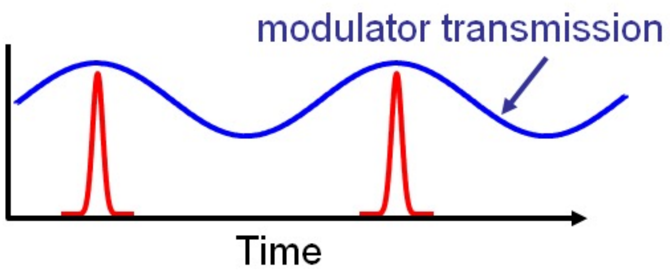
Comments on active mode-locking

Pulse duration: $\tau_a = \sqrt[4]{2}(g/M)^{1/4} / \sqrt{\Omega_g \omega_M}$

1) Larger modulation depth, M, and higher modulation frequency will give shorter pulses because the "low loss" window becomes narrower and shortens the pulse.
2) A broader gain bandwidth yields shorter pulses because the filtering effect of gain narrowing is lower and more modes are lasing.

Disadvantages of active mode-locking:

1) It requires an externally driven modulator. Its modulation frequency has to match precisely the cavity mode spacing.



2) The pulse width shortens only inversely proportional to the square root of the gain bandwidth, so it is hard to reach femtosecond pulses.

Principles of Passive Mode Locking

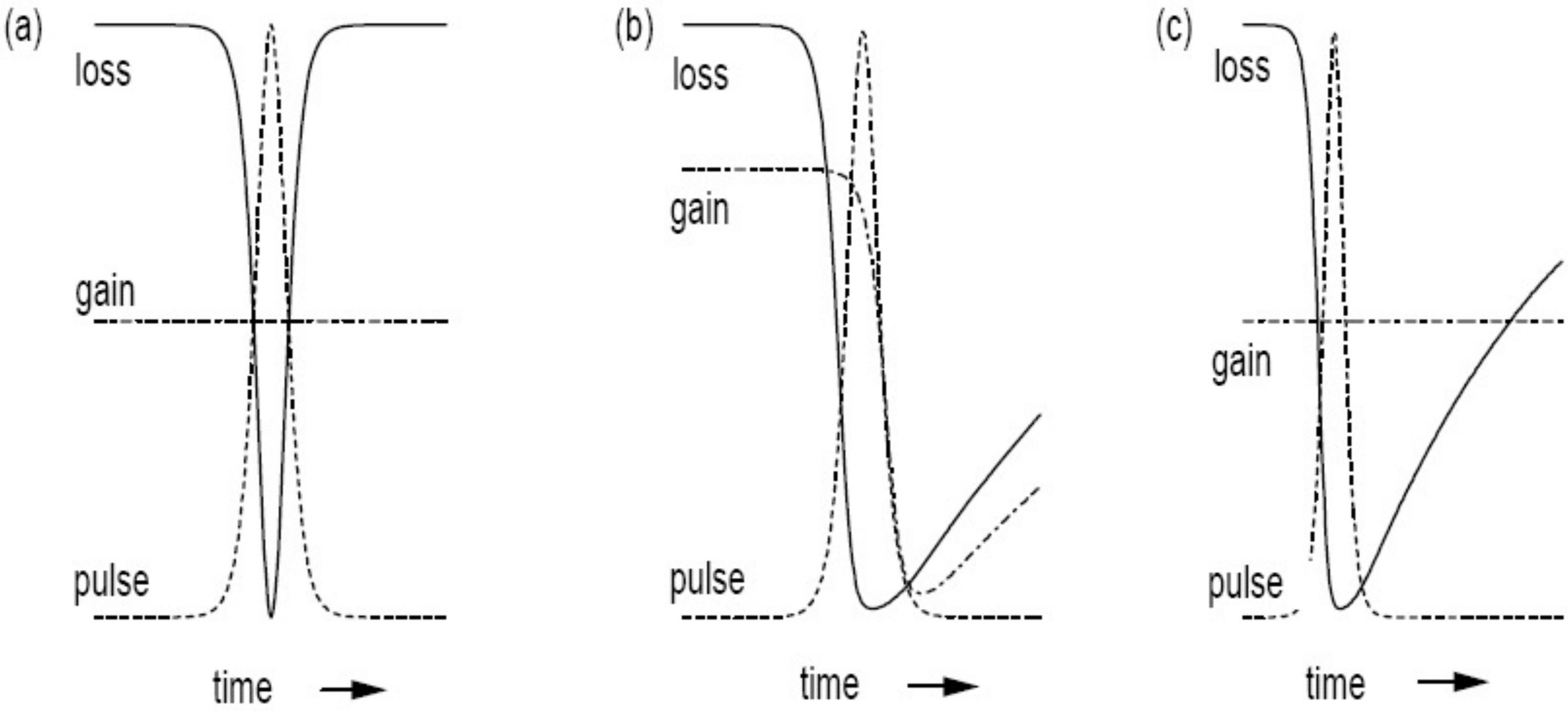


Fig. 6.1: Principles of mode locking

6.1 Slow Saturable Absorber Mode Locking

$$\frac{dg}{dt} = -g\frac{|A(t)|^2}{E_L}$$

Introduce pulse energy: $E(t) = \int_{-T_{R/2}}^{t} dt |A(t)|^2$

$$\rightarrow \quad g(t) = g_i \exp\left[-E(t)/E_L\right]$$

$$q(t) = q_0 \exp\left[-E(t)/E_A\right]$$

Master Equation:

$$T_R \frac{\partial}{\partial T} A = \left[g_i \left(\exp\left(-E(t)/E_L\right)\right) A - lA - q_0 \exp\left(-E(t)/E_A\right)\right] A + \frac{1}{\Omega_f^2}\frac{\partial^2}{\partial t^2} A$$

Fixed filtering / finite bandwidth

Approximate absorber response:

$$q_0 \exp\left(-E(t)/E_A\right) \approx q_0 \left[1 - \left(E(t)/E_A\right) + \frac{1}{2}\left(E(t)/E_A\right)^2\right]$$

$$T_R \frac{\partial A(T,t)}{\partial T} = \left[g(t) - q(t) - l + D_f \frac{\partial^2}{\partial t^2} \right] A(T,t)$$

Ansatz: $A(t) = A_o \,\mathrm{sech}(t/\tau)$

Stationary solution: *A(T+T_R,t)* reproduces itself up to a timing shift?

$$A(t,T) = A_o \,\mathrm{sech}(\tfrac{t}{\tau} + \alpha \tfrac{T}{T_R})$$

$$E(t) = \int_{-T_{R/2}}^{t} dt |A(t)|^2 = \frac{W}{2}\left(1 + \tanh(\frac{t}{\tau} + \alpha \frac{T}{T_R})\right)$$

$$E(t)^2 = \left(\frac{W}{2}\right)^2 \left(2 + 2\tanh(\frac{t}{\tau} + \alpha\frac{T}{T_R}) - \mathrm{sech}^2(\frac{t}{\tau} + \alpha\frac{T}{T_R})\right)$$

$$T_R \frac{\partial}{\partial T} A(t,T) = -\alpha \tanh(\frac{t}{\tau} + \alpha \frac{T}{T_R}) A(t,T)$$

$$\frac{1}{\Omega_f^2}\frac{\partial^2}{\partial t^2} A(t,T) = \frac{1}{\Omega_f^2 \tau^2}\left(1 - 2\mathrm{sech}^2(\frac{t}{\tau} + \alpha\frac{T}{T_R})\right) A(t,T)$$

Substitution into master equation and comparison of coefficients:

Constants:

$$g_i\left[1-\frac{W}{2E_L}+\left(\frac{W}{2E_L}\right)^2\right]=l+q_0\left[1-\frac{W}{2E_A}+\left(\frac{W}{2E_A}\right)^2\right]-\frac{1}{\Omega_f^2\tau^2} \quad (6.12)$$

Tanh:

$$\alpha=\frac{\Delta t}{\tau}=g_i\left[\frac{W}{2E_L}-\left(\frac{W}{2E_L}\right)^2\right]-q_0\left[\frac{W}{2E_A}-\left(\frac{W}{2E_A}\right)^2\right]$$

sech²:

$$\frac{1}{\tau^2}=\frac{\Omega_f^2W^2}{8}\left(\frac{q_0}{E_A^2}-\frac{g_i}{E_L^2}\right) \quad (6.14) \qquad \longrightarrow \qquad q_0/E_A^2 > g_i/E_L^2.$$

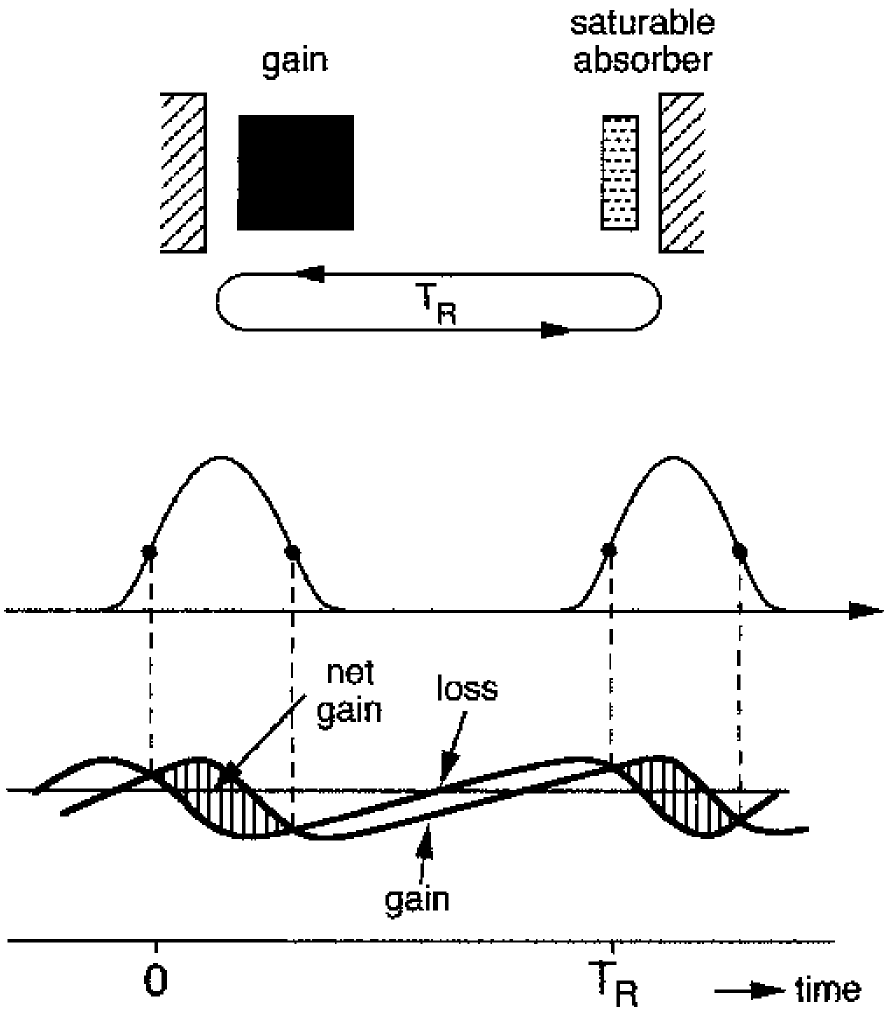
Stronger focusing into absorber lowers E_A!

From (6.12) and (6.14)

$$g_i-q_0-l \;=\; g_i\left[\frac{W}{2E_L}\right]-q_0\left[\frac{W}{2E_A}\right]+\frac{1}{\Omega_f^2\tau^2} \qquad (6.16)$$

Net gain before pulse (and also after pulse) must be < 0 for stability!

Shortest pulse width possible: $$\tau = \frac{2\sqrt{2}}{\sqrt{q_0}\Omega_f}\frac{E_A}{W} > \frac{\sqrt{2}}{\sqrt{q_0}\Omega_f}$$



No fast element necessary: Both absorber and gain may recover on ns-time scale

Fig. 6.2: Slow saturable absorber modelocking

6.2 Fast Saturable Absorber Mode Locking

Saturable absorption responds to instantaneous power: $q(A) = \dfrac{q_0}{1 + \frac{|A|^2}{P_A}}$

Approximately: $q(A) = q_0 - \gamma |A|^2$ with: $l_0 = l + q_0$ and $\gamma = q_0 / P_A$

$$T_R \frac{\partial A(T,t)}{\partial T} = \left[g - l_0 + D_f \frac{\partial^2}{\partial t^2} + \gamma |A|^2 + jD_2 \frac{\partial^2}{\partial t^2} - j\delta |A|^2 \right] A(T,t)$$

Dispersion + SPM

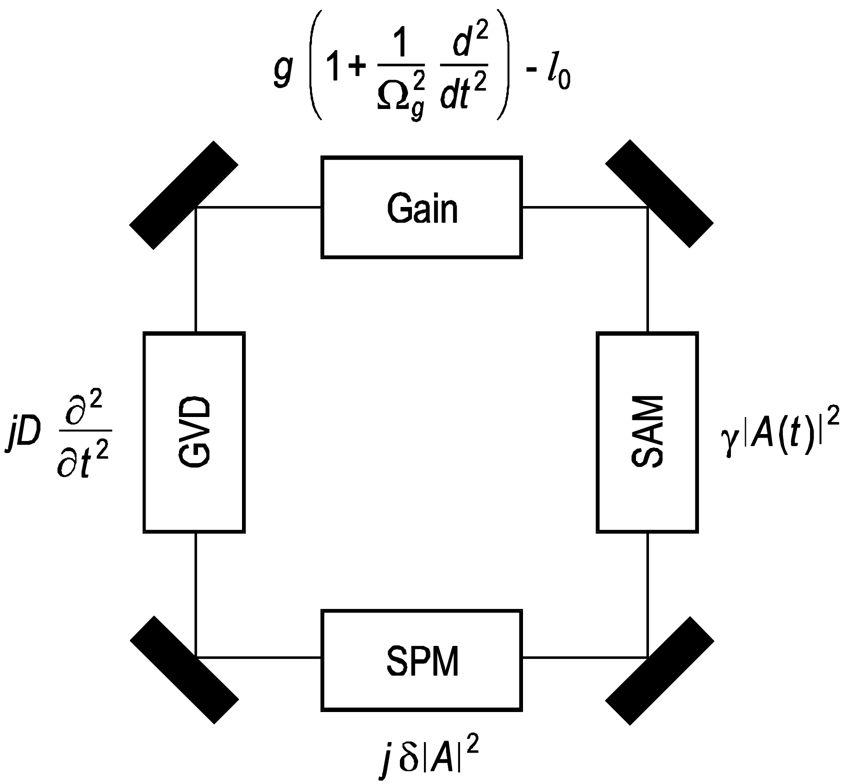


Fig. 6.3: Fast saturable absorber modelocking

Without GDD and SPM

$$T_R \frac{\partial A(T,t)}{\partial T} = \left[g - l_0 + D_f \frac{\partial^2}{\partial t^2} + \gamma |A|^2 \right] A(T,t)$$

$$T_R \frac{\partial A_s(T,t)}{\partial T} = 0. \longrightarrow A_s(T,t) = A_s(t) = A_0 \operatorname{sech}\left(\frac{t}{\tau}\right)$$

$$\begin{aligned} 0 \;=\; & \left[(g - l_0) + \frac{D_f}{\tau^2} \left[1 - 2\operatorname{sech}^2\left(\frac{t}{\tau}\right) \right] \right. \\ & \left. + \gamma |A_0|^2 \operatorname{sech}^2\left(\frac{t}{\tau}\right) \right] \cdot A_0 \operatorname{sech}\left(\frac{t}{\tau}\right) \end{aligned}$$

$$\begin{aligned} \frac{D_f}{\tau^2} &= \frac{1}{2} \gamma |A_0|^2 \\ g &= l_0 - \frac{D_f}{\tau^2} \end{aligned}$$

Pulse Energy: $W = 2 A_0^2 \tau \longrightarrow \tau = \dfrac{4 D_f}{\gamma W}$.

Pulse Energy Evolution:

$$\begin{aligned} T_R \frac{\partial W(T)}{\partial T} &= T_R \frac{\partial}{\partial T} \int_{-\infty}^{\infty} |A(T,t)|^2 \, dt \\ &= T_R \int_{-\infty}^{\infty} \left[A(T,t)^* \frac{\partial}{\partial T} A(T,t) + c.c. \right] dt \\ &= 2G(g_s, W) W, \end{aligned}$$

$$\begin{aligned} \int_{-\infty}^{\infty} \left(\mathrm{sech}^2 x\right) dx &= 2, \\ \int_{-\infty}^{\infty} \left(\mathrm{sech}^4 x\right) dx &= \frac{4}{3}, \\ - \int_{-\infty}^{\infty} \mathrm{sech} x \frac{d^2}{dx^2} \left(\mathrm{sech} x\right) dx &= \int_{-\infty}^{\infty} \left(\frac{d}{dx} \mathrm{sech} x \right)^2 dx = \frac{2}{3} \end{aligned}$$

$$\begin{aligned} G(g_s, W) &= g_s - l_0 - \frac{D_f}{3\tau^2} + \frac{2}{3} \gamma |A_0|^2 \\ &= g_s - l_0 + \frac{1}{2} \gamma |A_0|^2 = g_s - l_0 + \frac{D_f}{\tau^2} = 0 \end{aligned}$$

$$g_s(W) = \frac{g_0}{1 + \frac{W}{P_L T_R}}$$

Steady State Pulse Energy:

$$g_s(W) = \frac{g_0}{1+\frac{W}{P_L T_R}} = l_0 - \frac{D_f}{\tau^2}$$

$$= l_0 - \frac{(\gamma W)^2}{16 D_{\cancel{g}}} \quad \text{Replace by } f$$

With $q_0 = \gamma A_0^2.$

$$\boxed{\frac{D_f}{\tau^2} = \frac{q_0}{2}, \qquad \tau = \sqrt{\frac{2}{q_0}}\frac{1}{\Omega_f}.}$$

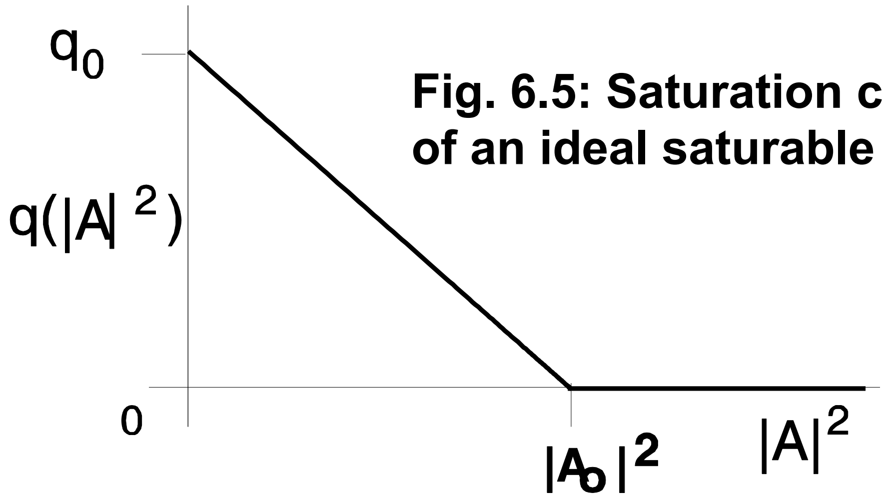


Fig. 6.5: Saturation characteristic of an ideal saturable absorber

Minimum Pulse Width:

$$g_s = l \overset{-}{\cancel{+}} \tfrac{1}{2} q_0 \qquad D_f = D_g = \tfrac{g}{\Omega_g^2}$$

$$\tau_{\min} = \frac{1}{\Omega_g} \longrightarrow \Delta f_{FWHM} = \frac{0.315}{1.76 \cdot \tau_{\min}} = \frac{\Omega_g}{1.76 \cdot \pi}$$

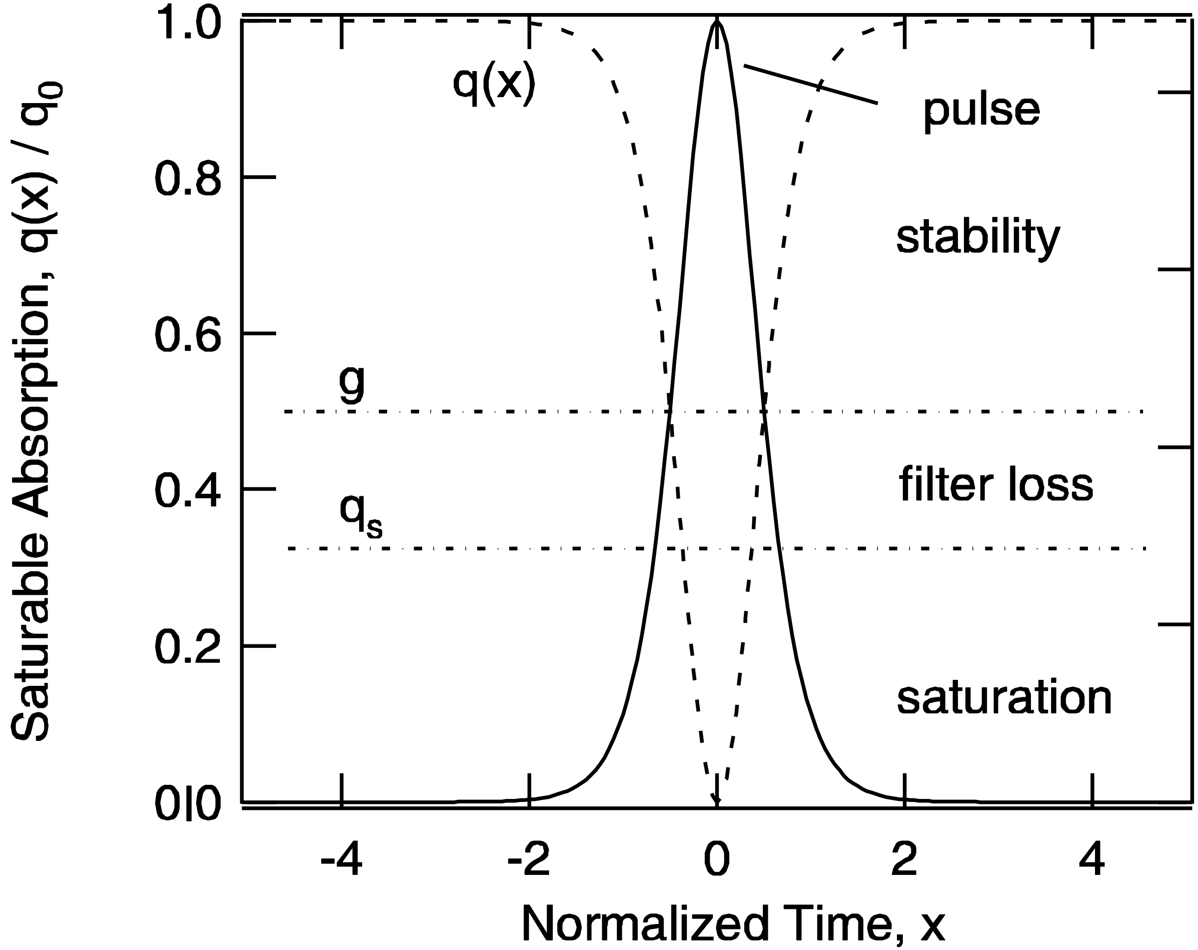


Fig. 6.4: Gain and loss in a fast saturable absorber (FSA) modelocked laser

6.2.2 Fast SA mode locking with GDD and SPM

$$T_R \frac{\partial A(T,t)}{\partial T} = \left[g - l_0 + D_f \frac{\partial^2}{\partial t^2} + \gamma |A|^2 + jD_2 \frac{\partial^2}{\partial t^2} - j\delta |A|^2 \right] A(T,t)$$

Steady-state solution is chirped sech-shaped pulse with 4 free parameters:

$$A_s(T,t) \;=\; A_0 \left(\operatorname{sech}\left(\frac{t}{\tau} \right) \right)^{(1+j\beta)} e^{j\psi T/T_R}$$

Pulse amplitude: A_0 or Energy: $W = 2 A_0{}^2 \tau$

Pulse width: τ

Chirp parameter : β

Carrier-Envelope phase shift : ψ

Substitute above trial solution into the master equation and comparing the coefficients to the same functions leads to two complex equations:

$$\frac{1}{\tau^2} (D_f + jD_2) \left(2 + 3j\beta - \beta^2 \right) \;=\; (\gamma - j\delta) |A_0|^2 \qquad (6.49)$$

$$l_0 - \frac{(1+j\beta)^2}{\tau^2} (D_f + jD_2) \;=\; g - j\psi \qquad (6.50)$$

Fast SA mode locking with GDD and SPM

The real part and imaginary part of Eq.(6.49) give

$$\frac{1}{\tau^2}\left[D_f\left(2-\beta^2\right)-3\beta D_2\right] \;=\; \gamma|A_0|^2 \qquad (6.52)$$

$$\frac{1}{\tau^2}\left[D_2\left(2-\beta^2\right)+3\beta D_f\right] \;=\; -\delta|A_0|^2 \qquad (6.53)$$

Normalized parameters:

Normalized nonlinearity

$$\delta_n = \delta/\gamma$$

Normalized dispersion

$$D_n = D_2/D_f$$

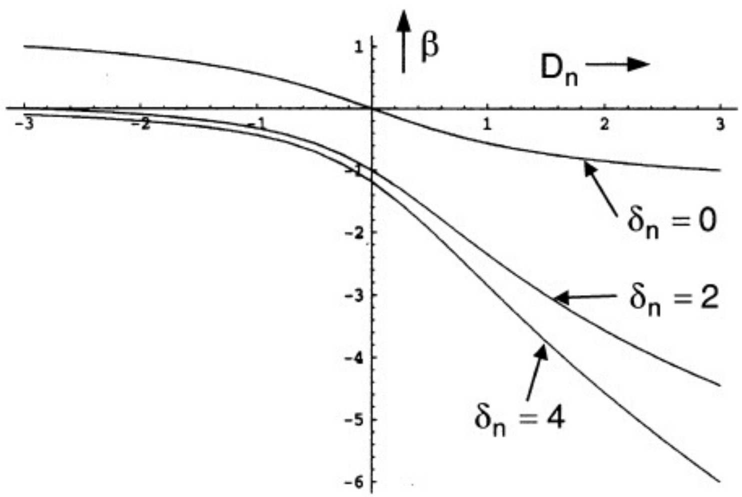
Dividing Eq.(6.53) by (6.52) leads to a quadratic equation for the chirp:

$$\frac{D_n\left(2-\beta^2\right)+3\beta}{\left(2-\beta^2\right)-3\beta D_n} = -\delta_n \longrightarrow \frac{3\beta}{2-\beta^2} = \frac{\delta_n + D_n}{-1+\delta_n D_n} \equiv \frac{1}{\chi} \qquad (6.54)$$

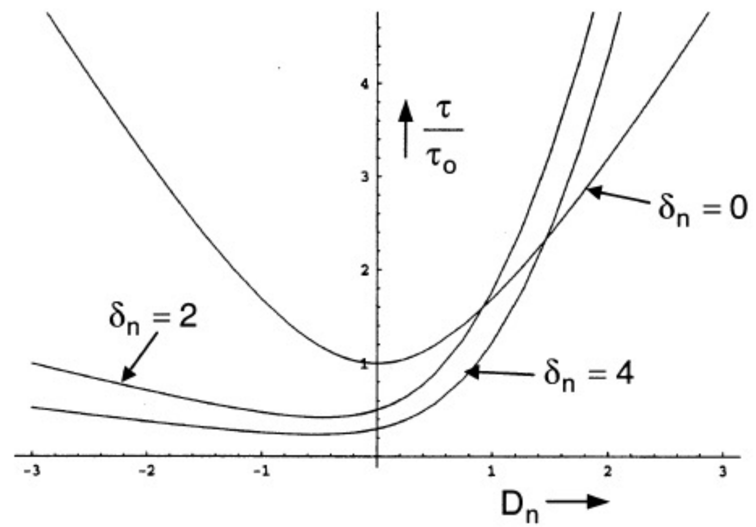
depends only on the system parameters

Fast SA mode locking with GDD and SPM

Chirp $\beta = -\frac{3}{2}\chi \pm \sqrt{\left(\frac{3}{2}\chi\right)^2 + 2}$



Pulse width $\tau = \frac{3\tau_0}{2}\beta\left(\chi - D_n\right)$



- strong soliton-like pulse shaping if $\delta_n \gg 1$ and $-D_n \gg 1$ the chirp is always much smaller than for positive dispersion and the pulses are solitonlike.
- pulses are even chirp free if $\delta_n = -D_n$, with the shortest with directly from the laser, which can be a factor 2-3 shorter than by pure SA modelocking.
- Without SPM and GDD, SA has to shape the pulse. When SPM and GDD included, they can shape the pulse via soliton formation; SA only has to stabilize the pulse.

Fast SA mode locking with GDD and SPM

$$l_0 - \frac{(1+j\beta)^2}{\tau^2}(D_f + jD_2) \;=\; g - j\psi \tag{6.50}$$

The real part of Eq.(6.50) gives the saturated gain:

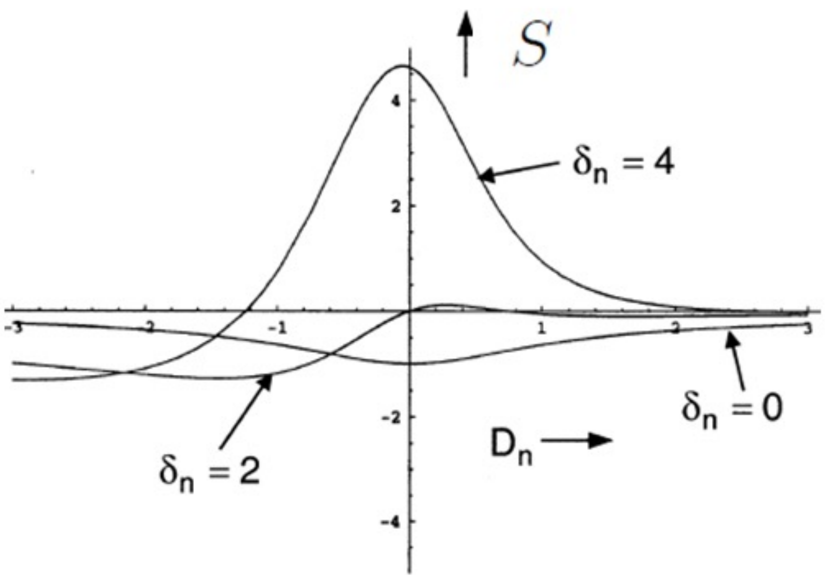
$$g = l_0 - \frac{1-\beta^2}{\tau^2}D_f + \frac{2\beta D_2}{\tau^2}$$

A necessary but not sufficient criterion for the pulse stability is that there must be net loss leading and following the pulse:

$$g - l_0 = -\frac{1-\beta^2}{\tau^2}D_f + \frac{2\beta D_2}{\tau^2} < 0$$

If we define the stability parameter S

$$S = 1 - \beta^2 - 2\beta D_n < 0$$



- Without SPM, the pulses are always stable.
- Excessive SPM can lead to instability near zero dispersion and for positive dispersion.

$\tau_0 = 4D_f/(\gamma W)$

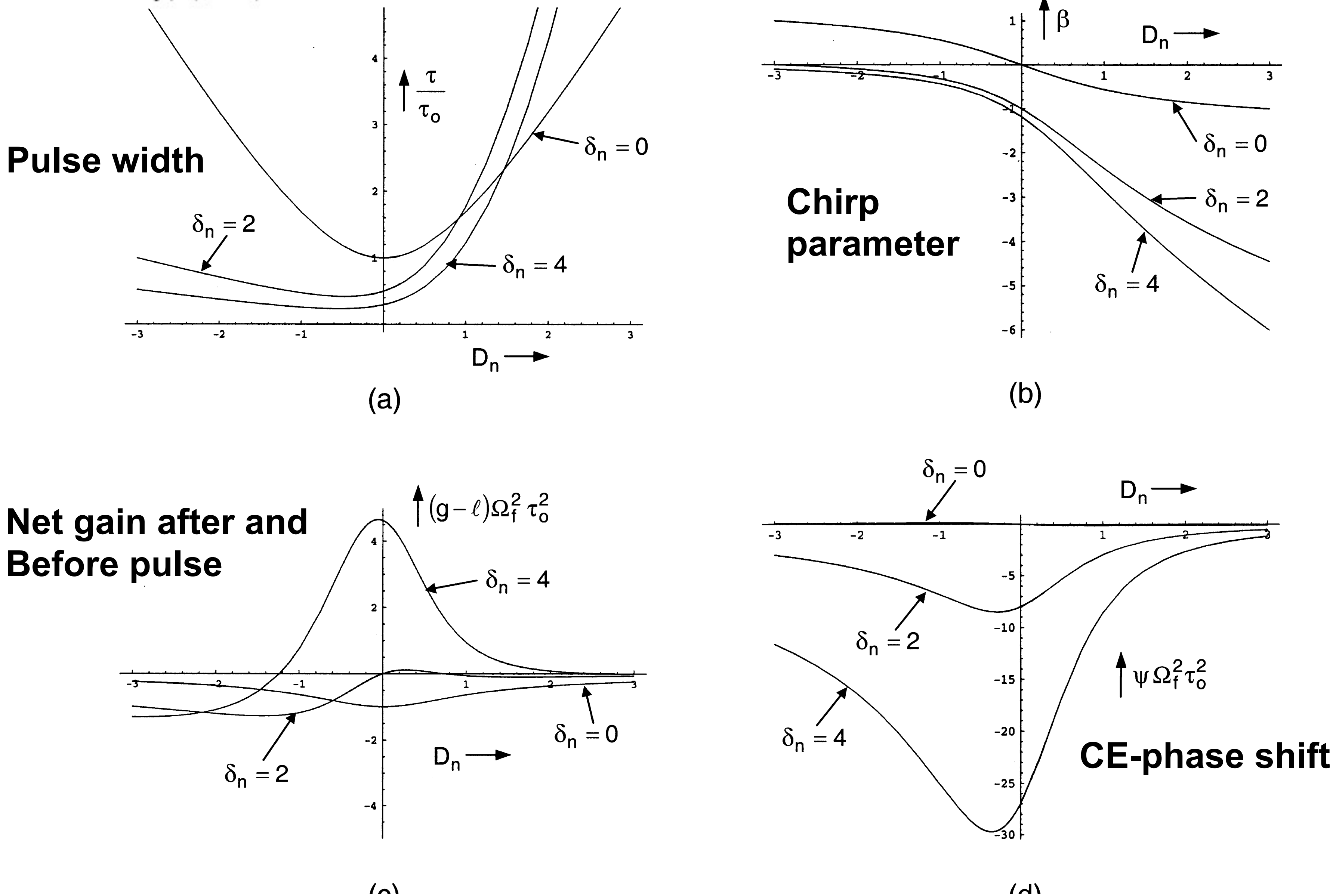
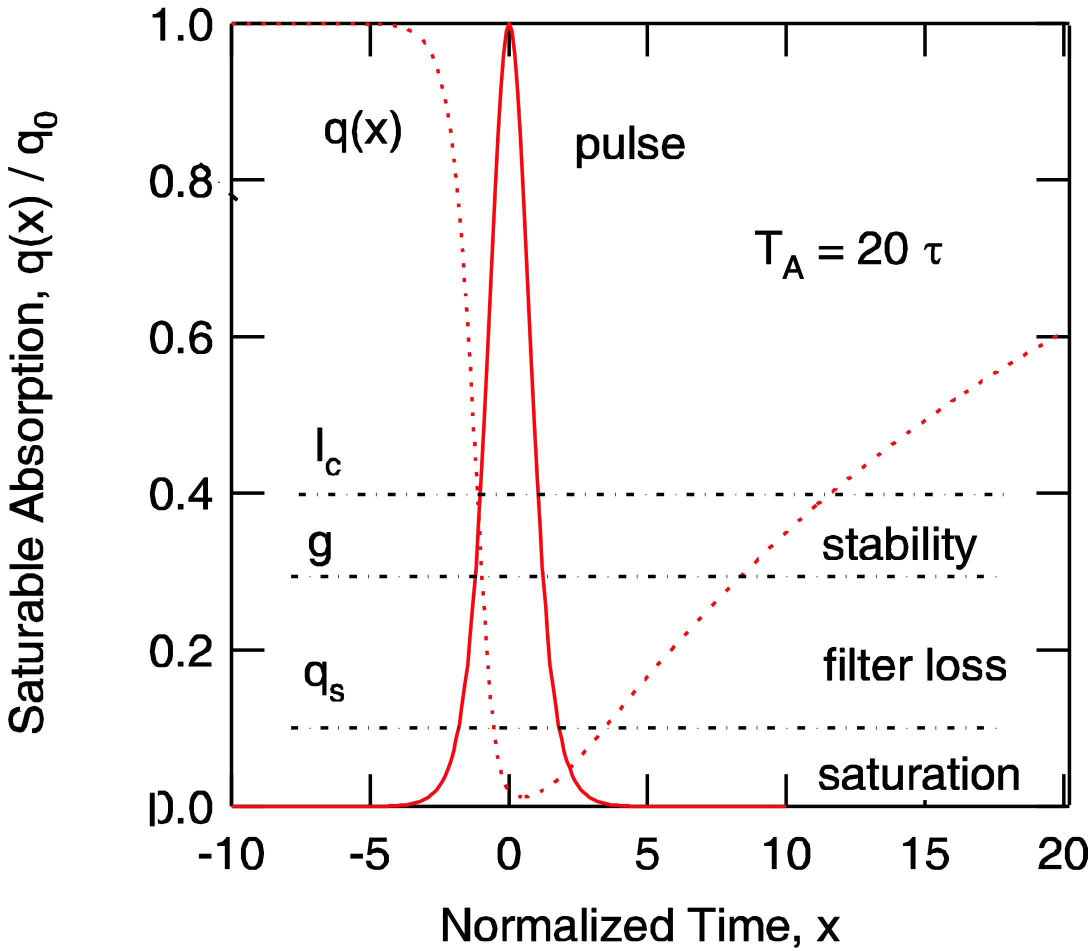


Figure 6.6: (a) Pulsewidth, (b) Chirp parameter, (c) Net gain following the pulse, which is related to stability. (d) Phase shift per pass. [4]

6.3 Soliton Mode Locking

$$T_R \frac{\partial A(T,t)}{\partial T} = \left[g - l + (D_f + jD) \frac{\partial^2}{\partial t^2} - j\delta |A(T,t)|^2 - q(T,t) \right] A(T,t).$$



$$\frac{\partial q(T,t)}{\partial t} = -\frac{q - q_0}{\tau_A} - \frac{|A(T,t)|^2}{E_A}.$$

Fig. 6.7: Soliton modelocking

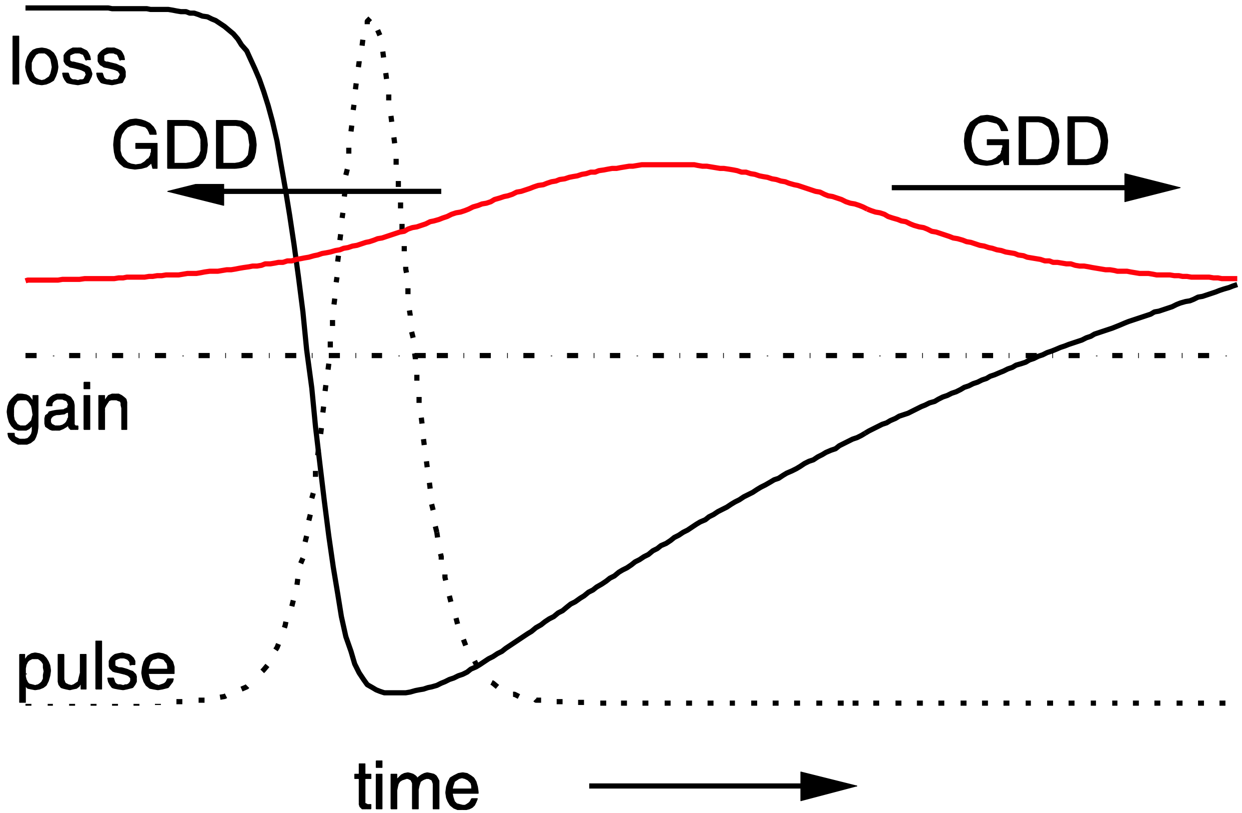


Fig. 6.8: Soliton Stability

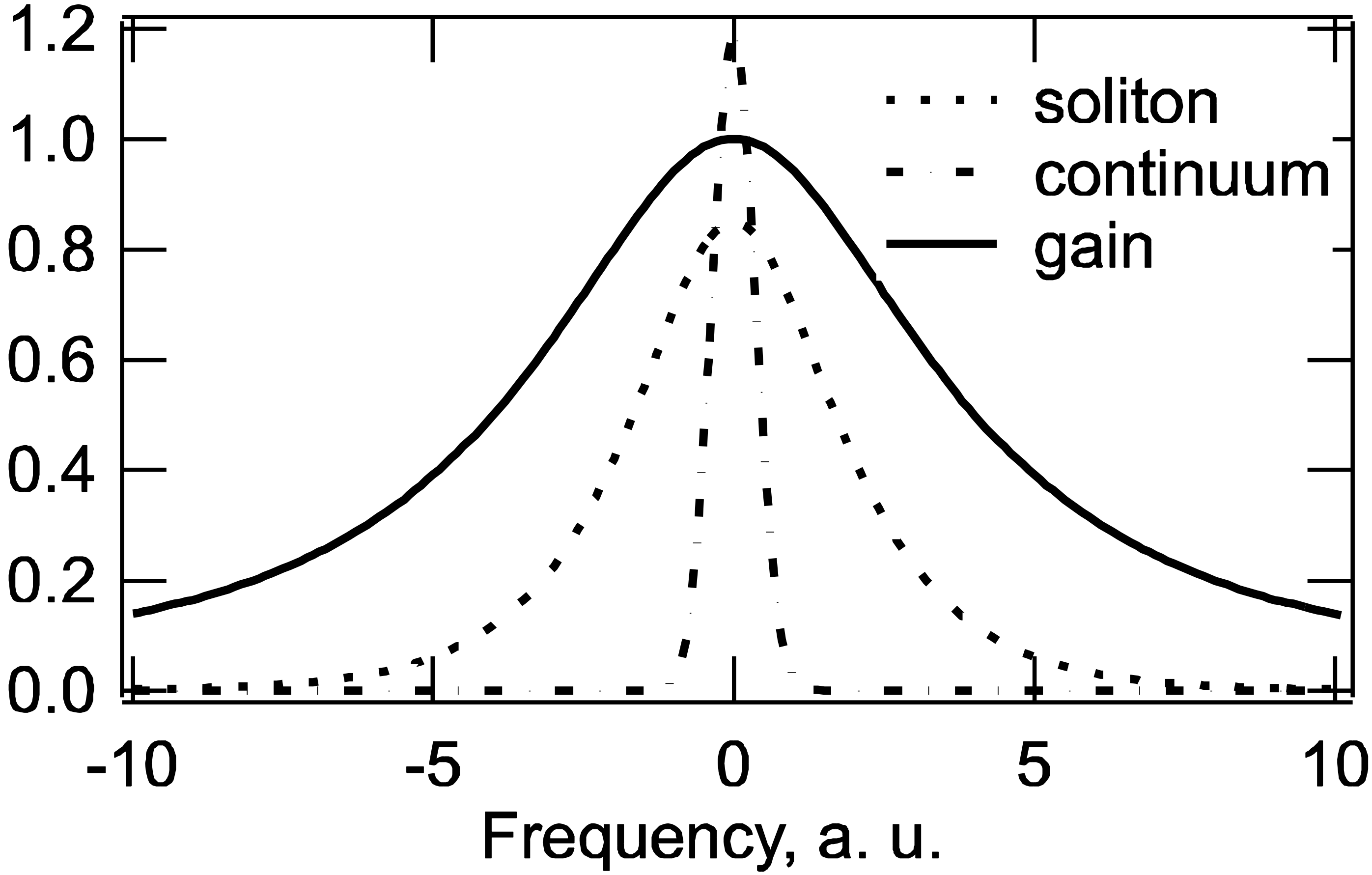


Fig. 6.9: Normalized gain, soliton and continuum.
The continuum is a long pulse exploiting the peak of the gain.

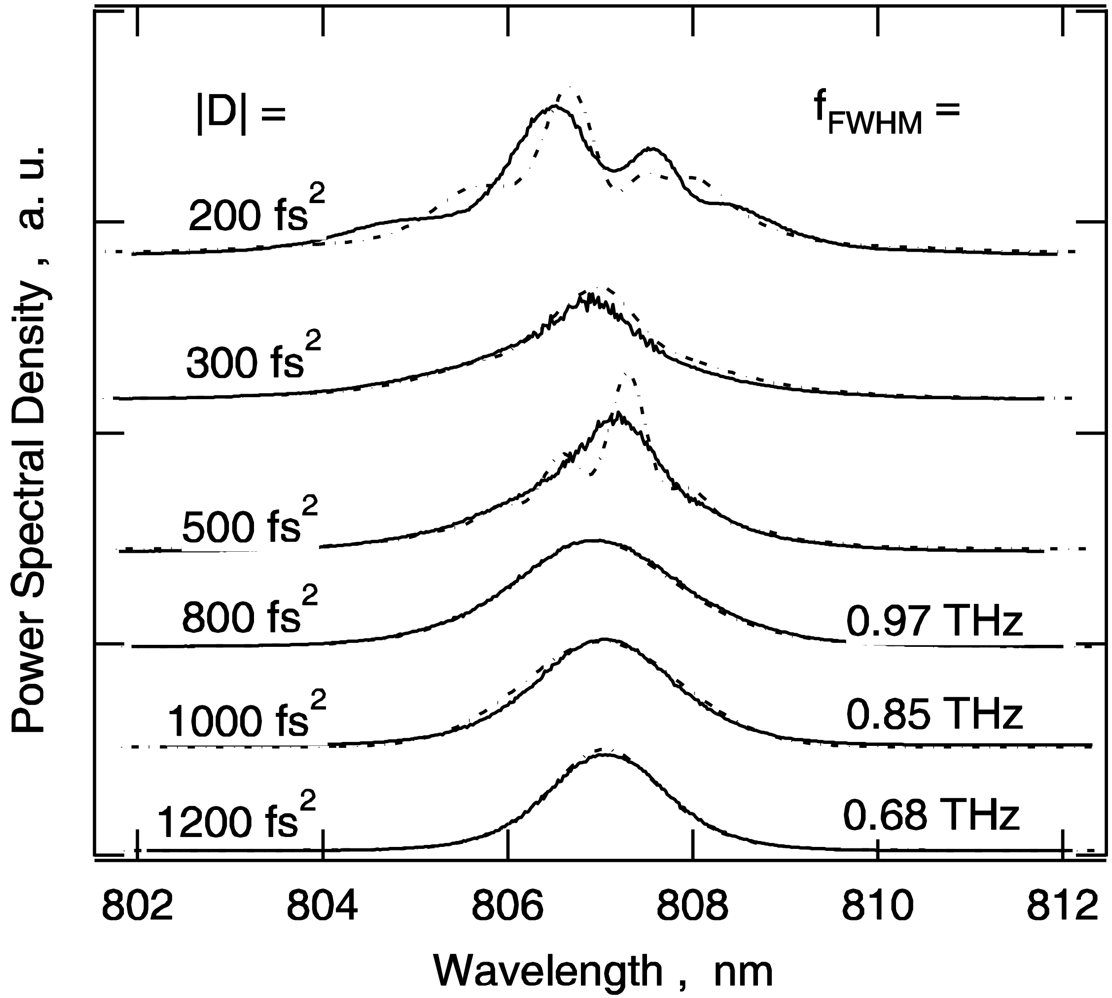


Fig. 6.10: Measured (---) and simulated (- - -) spectra from a semiconductor saturable absorber modelocked Ti:sapphire laser for different net intracavity dispersion.

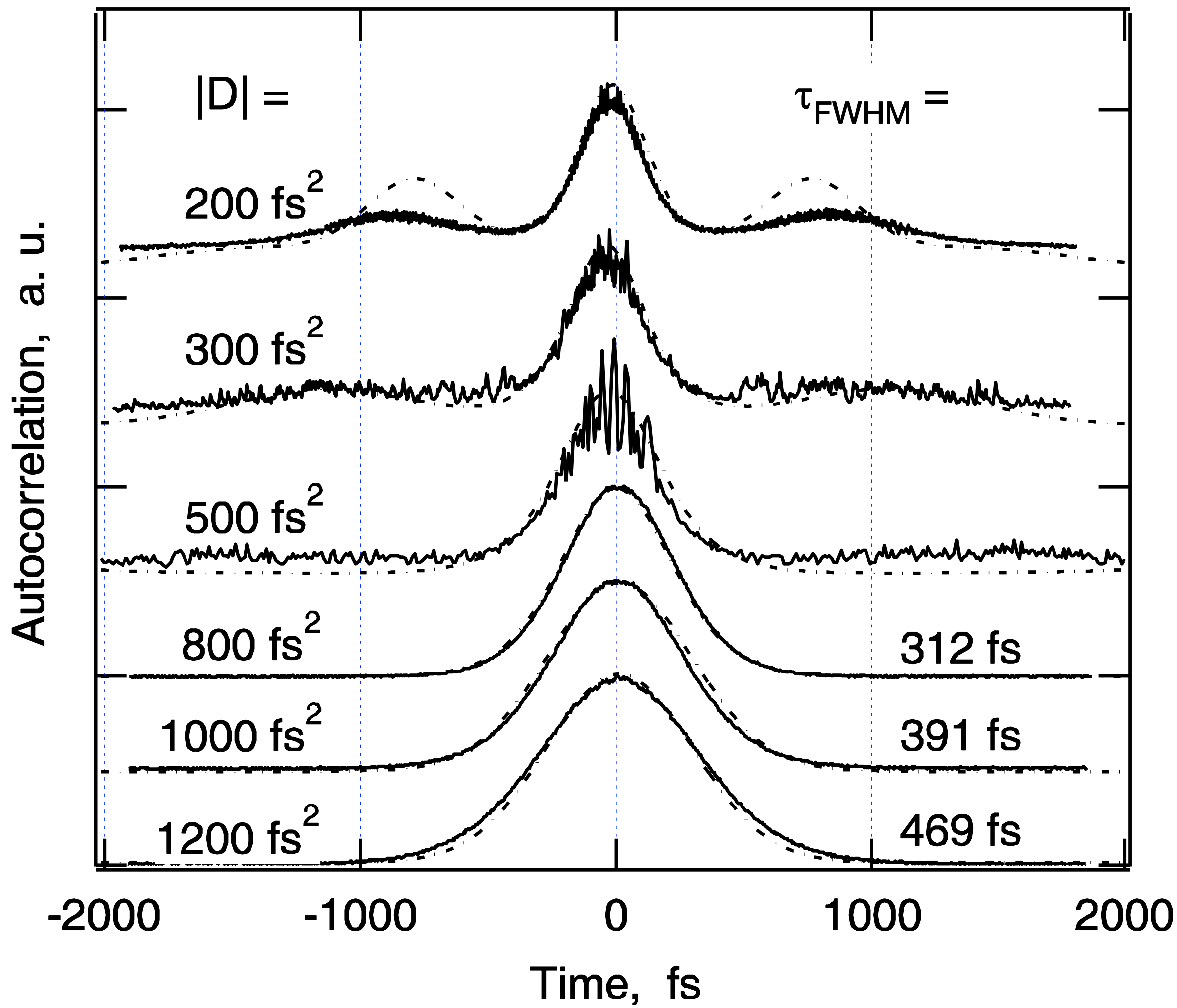


Fig. 6.11: Measured (----) and simulated (- - -) autocorrelations corresponding to the spectra shown in Figure 6.11

6.4 Dispersion Managed Soliton Formation

in Fiber Lasers

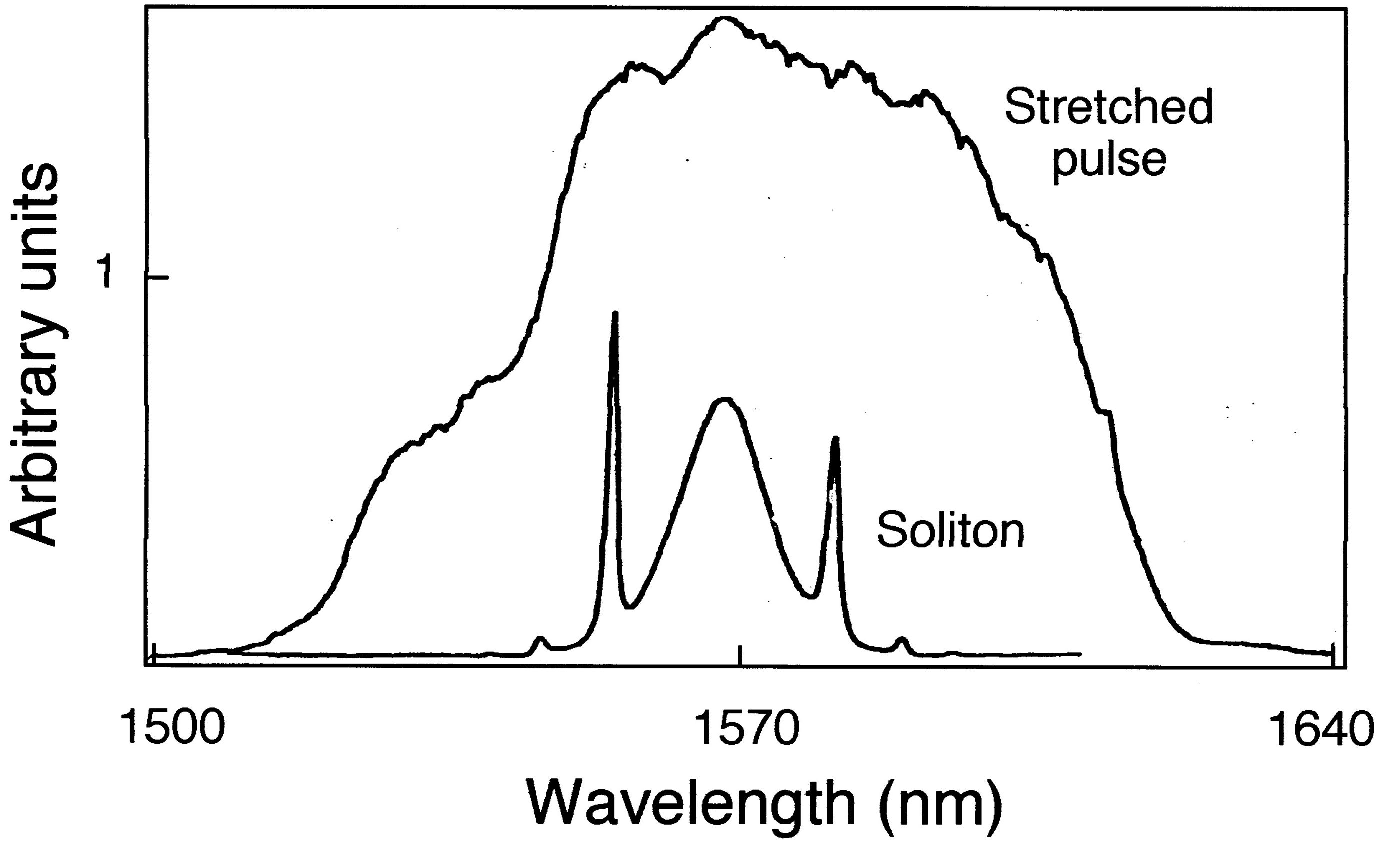


Fig. 6.12: Stretched pulse or dispersion managed soliton mode locking

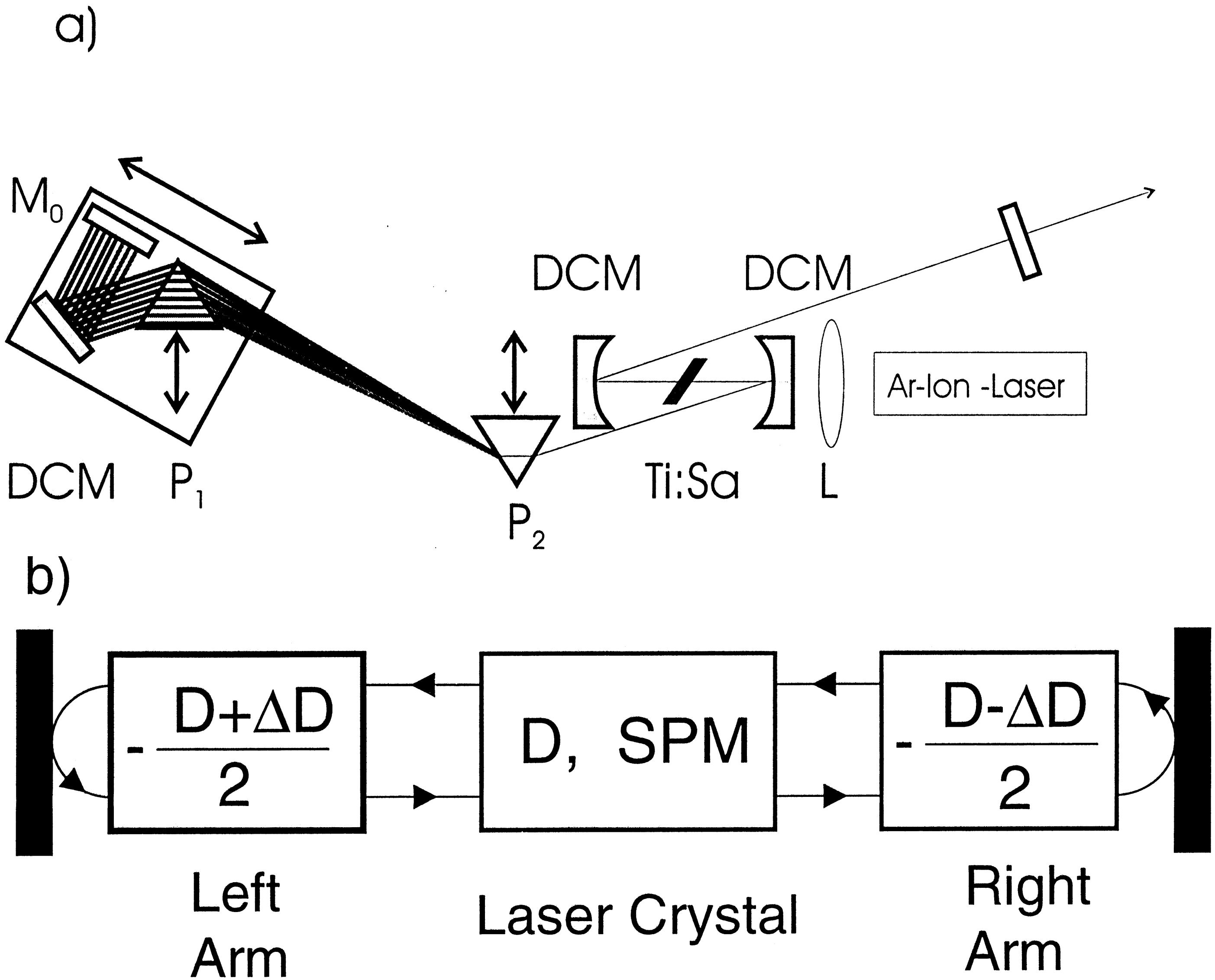
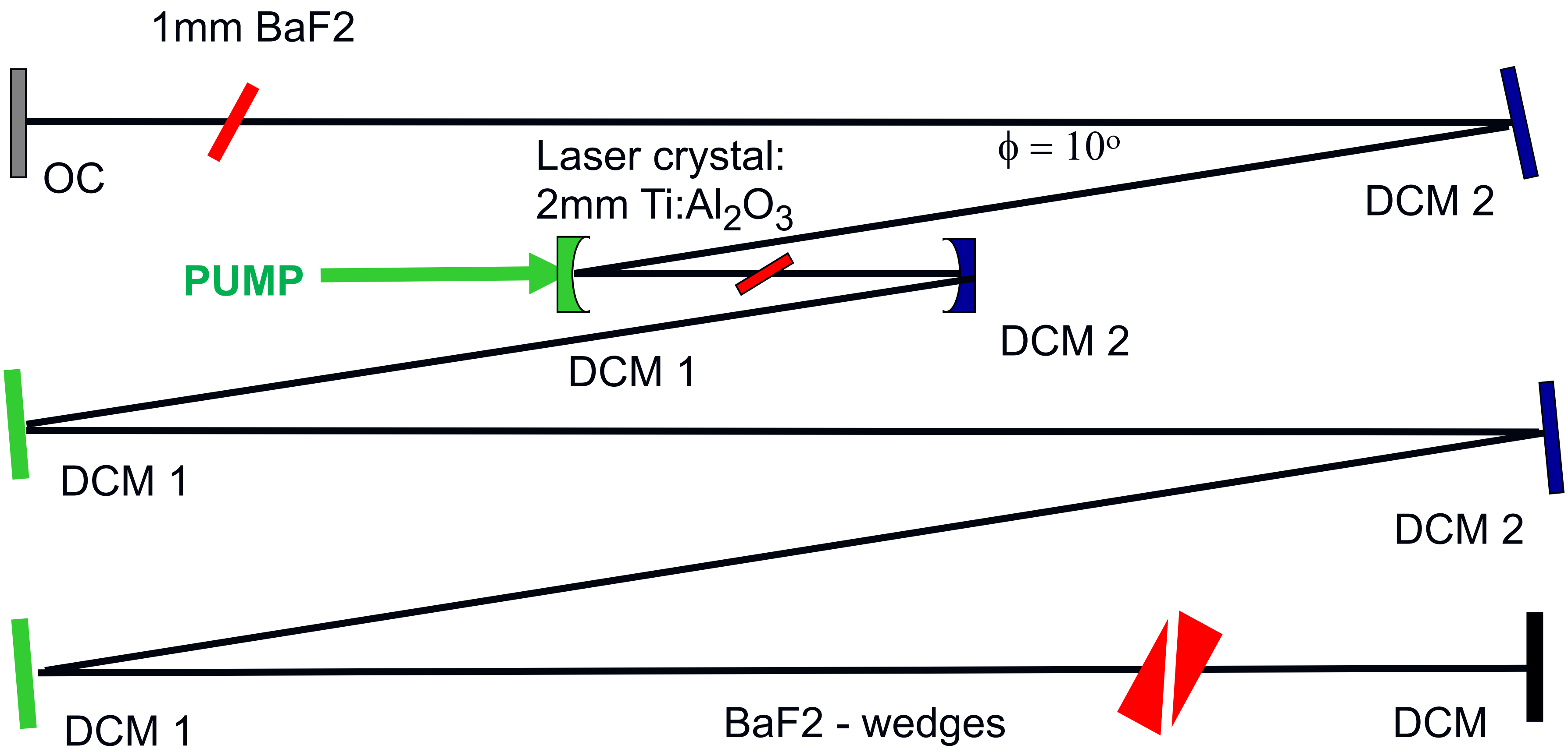


Fig. 6.13: (a) Kerr-lens mode-locked Ti:sapphire laser. (b) Correspondence with dispersion-managed fiber transmission.

Today's Broadband, Prismless Ti:sapphire Lasers



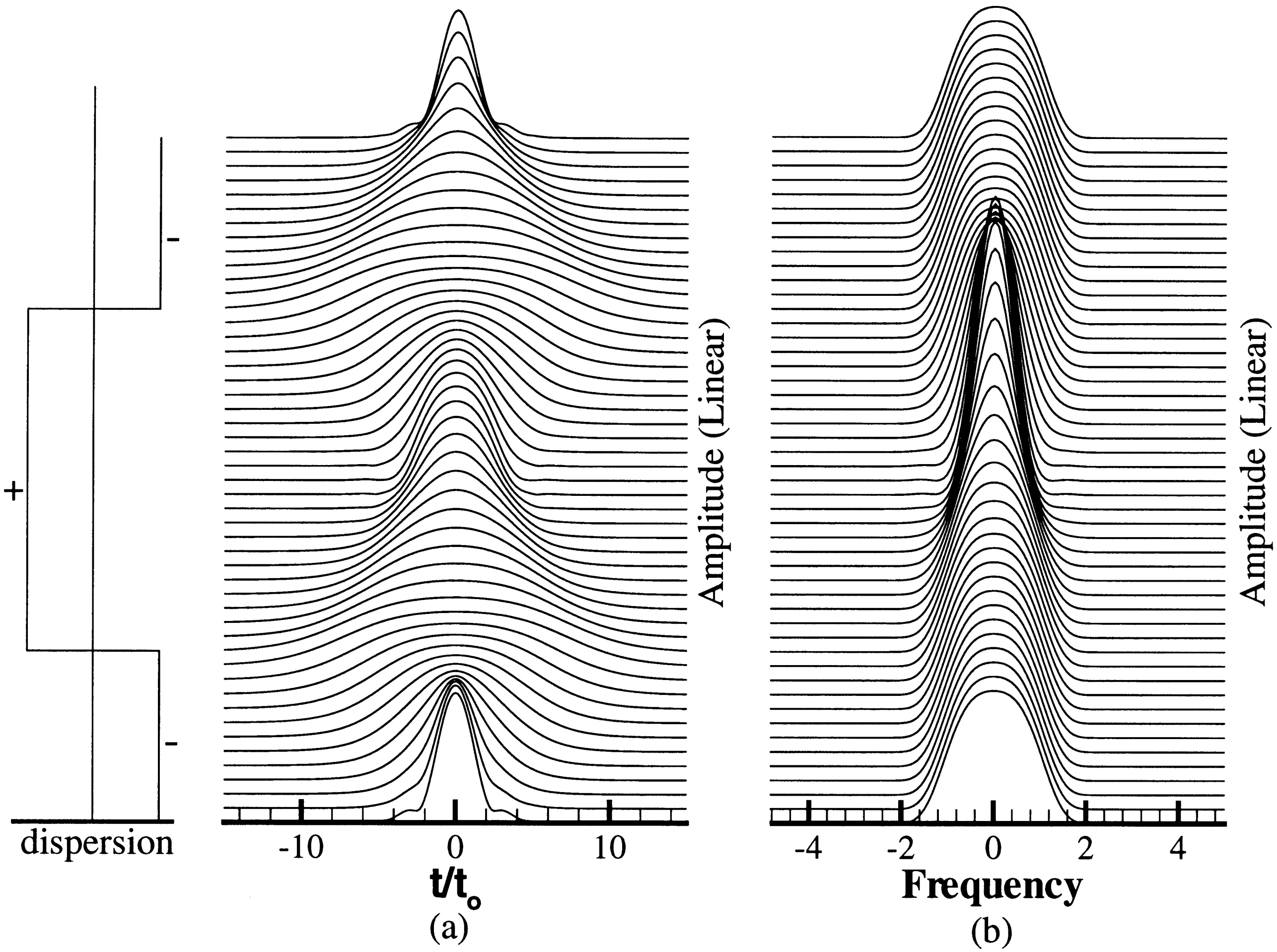


Fig. 6.14: Dispersion managed soliton including saturable absorption and gain filtering

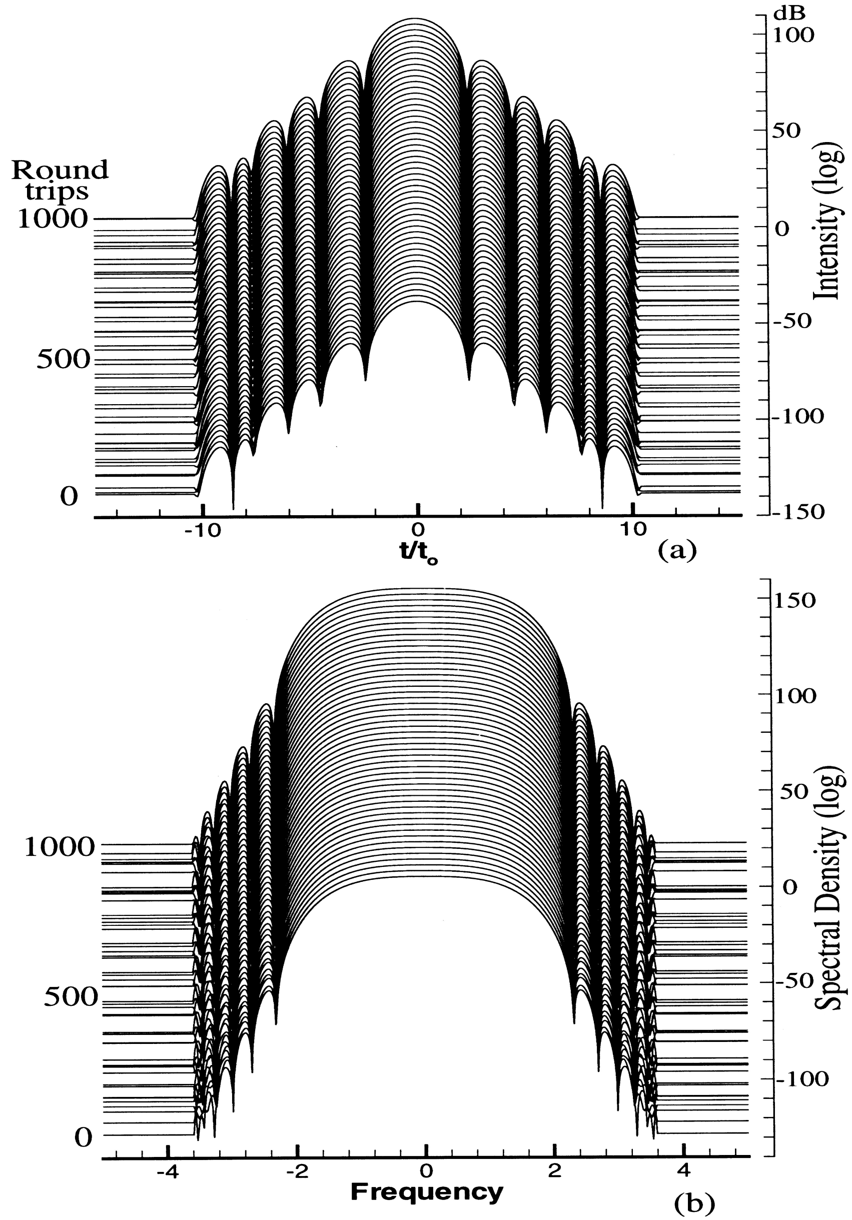


Fig. 6.15: Steady state profile if only dispersion and GDD is involved: Dispersion Managed Soliton

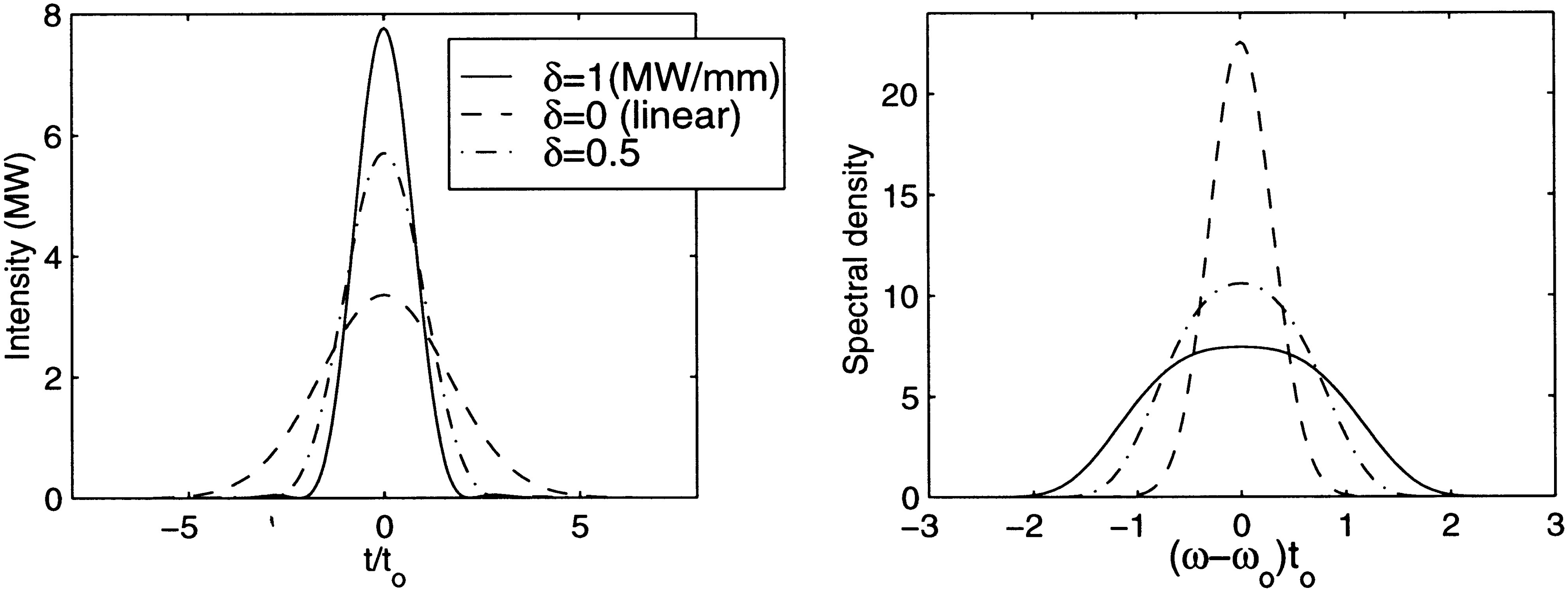


Fig. 6.16: Pulse shortening due to dispersion managed soliton formation