UFS Lecture 11: Laser Dynamics

Finish Passive Q-Switching

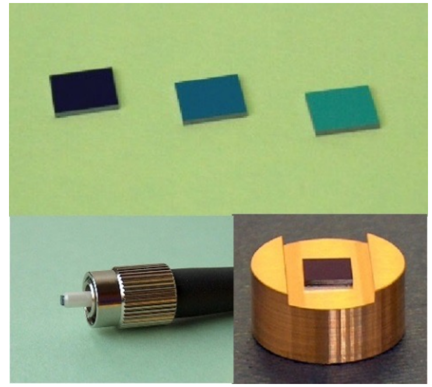
5 Active Mode Locking

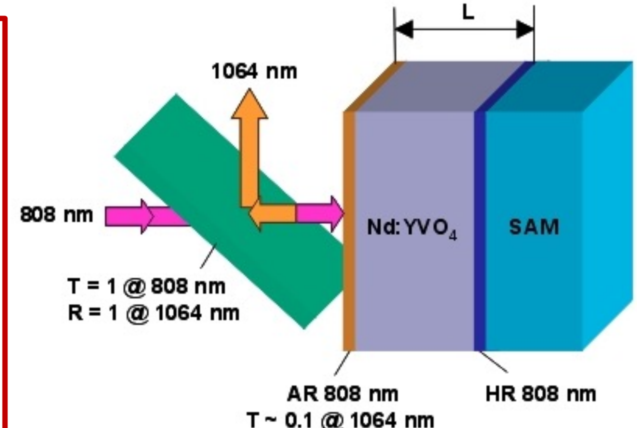
5.1 The Master Equation of Mode Locking
5.2 Active Mode Locking by Loss Modulation
5.3 Active Mode Locking by Phase Modulation

Passive Q-Switching: manage cavity loss using saturable absorber

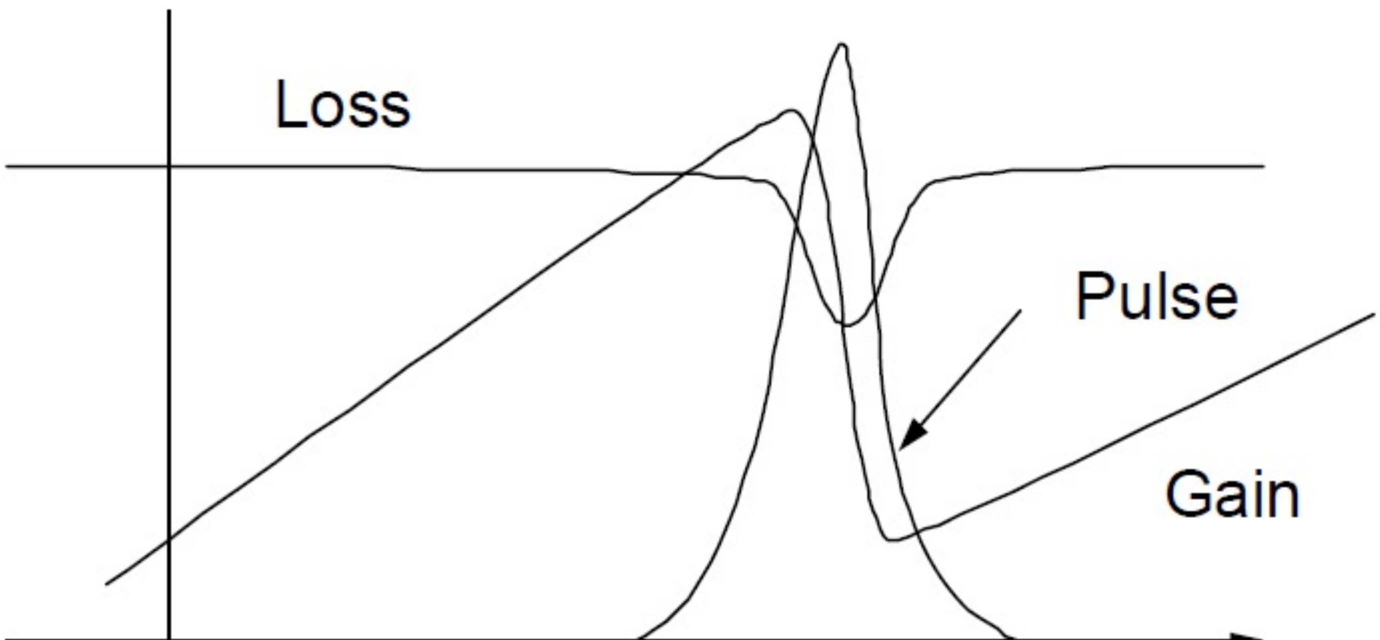
Saturable absorber: an optical passive device, which introduces large loss for low optical intensities and small loss at high optical intensities.

$$T_R \frac{dq}{dt} = -\frac{q - q_0}{T_A} - \frac{q T_R P}{E_A}$$

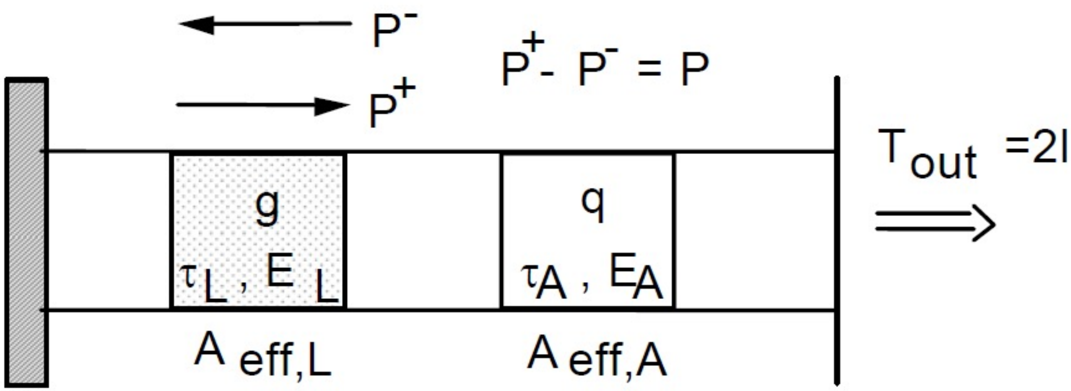




http://www.batop.de/products/products.html



Modeling of passively Q-switched lasers



We assume small output coupling so that the laser power within one roundtrip can be considered position independent.

Rate equations for a passively Q-switched laser

Assume that the changes in the laser intensity, gain and saturable absorption are small on a time scale on the order of the round-trip time T_R in the cavity, (i.e. less than 20%).

$$
\begin{aligned}
T_R \frac{dP}{dt} &= 2(g - l - q)P \\
T_R \frac{dg}{dt} &= -\frac{g - g_0}{T_L} - \frac{g T_R P}{E_L} \\
T_R \frac{dq}{dt} &= -\frac{q - q_0}{T_A} - \frac{q T_R P}{E_A}
\end{aligned}
$$

Normalized upper-state lifetime of the gain medium and the absorber recovery time

$$
\begin{aligned}
T_L &= \tau_L / T_R \\
T_A &= \tau_A / T_R
\end{aligned}
$$

Saturation energies of the gain and the absorber

$$
\begin{aligned}
E_L &= h\nu A_{eff,L} / 2^* \sigma_L \\
E_A &= h\nu A_{eff,A} / 2^* \sigma_A
\end{aligned}
$$

Passively Q-switched laser: fast saturable absorber

Typical solid-state lasers:

τ_L=100 μs T_R=10 ns τ_A=1-100 ps

$T_L \approx 10^4 \quad T_A \approx 10^{-4}$ to 10^{-2}

Fast Saturable Absorber: $T_A << T_L$, the absorber will follow the instantaneous laser power:

$$T_R \frac{dq}{dt} = -\frac{q-q_0}{T_A} - \frac{qT_RP}{E_A} \xrightarrow{\text{Adiabatic solution}} q = \frac{q_0}{1+P/P_A} \quad \text{with} \quad P_A = \frac{E_A}{\tau_A}$$

saturation power

New equations of motion:

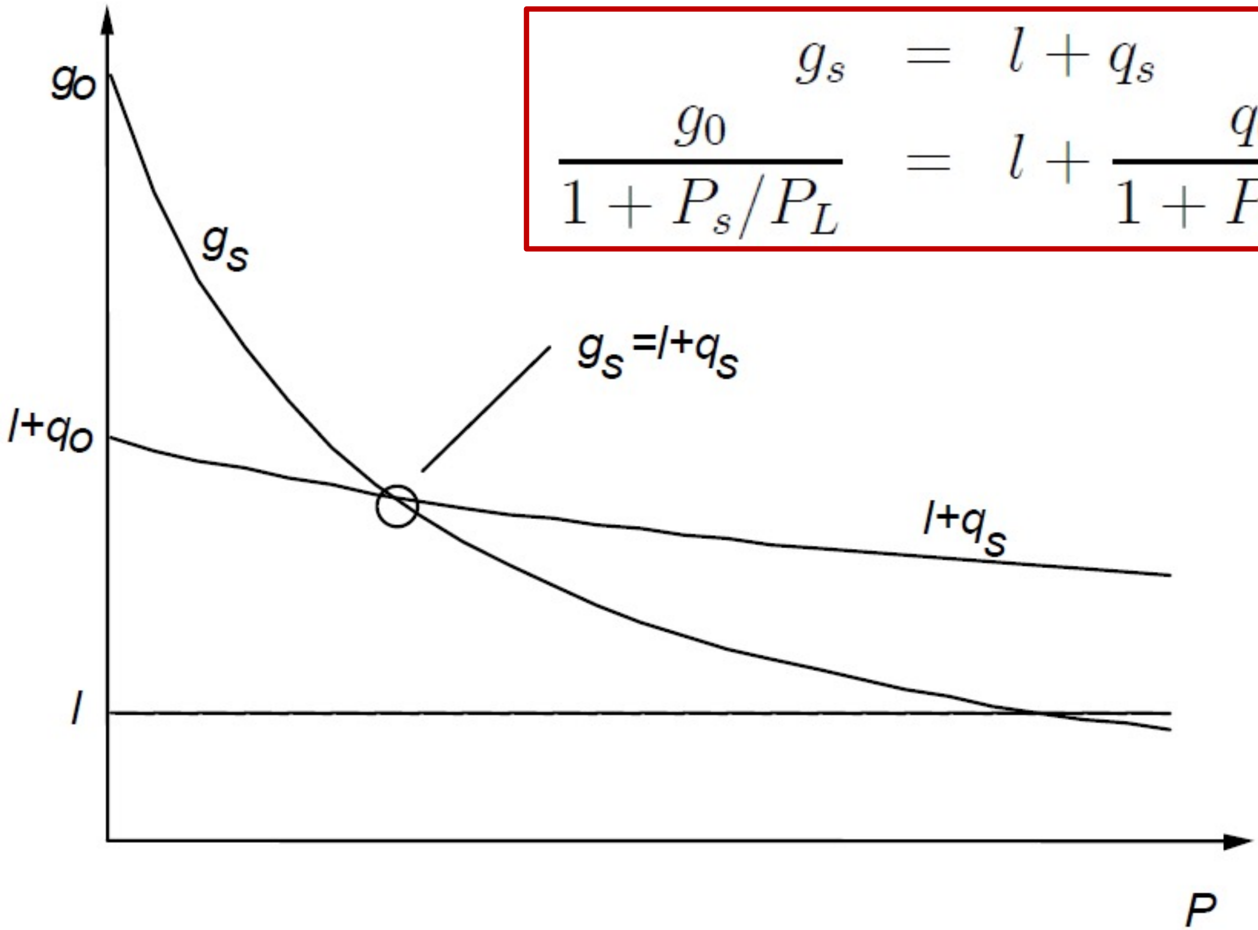
$$T_R \frac{dP}{dt} = 2(g-l-q(P))P$$

$$T_R \frac{dg}{dt} = -\frac{g-g_0}{T_L} - \frac{gT_RP}{E_L}$$

Passively Q-switched laser: stationary solution

As in the case for the cw-running laser the stationary operation point of the laser is determined by the point of zero net gain:

$$\boxed{\begin{aligned} g_s &= l + q_s \\ \frac{g_0}{1+P_s/P_L} &= l + \frac{q_0}{1+P_s/P_A} \end{aligned}}$$



Graphical solution of the stationary operating point

Stability of stationary operating point:

$$T_R \frac{d}{dt}\begin{pmatrix}\Delta P_0 \\ \Delta g_0\end{pmatrix} = A \begin{pmatrix}\Delta P_0 \\ \Delta g_0\end{pmatrix}, \quad \text{with } A = \begin{pmatrix} -2\left.\frac{dq}{dP}\right|_{cw} P_s & 2P_s \\ -\frac{g_s T_R}{E_L} & -\frac{T_R}{\tau_{stim}} \end{pmatrix}$$

Stationary operating point is stable if: Tr (A) < 0 and det (A) > 0

Tr (A) < 0: $$-2P\left.\frac{dq}{dP}\right|_{cw} < \frac{r}{T_L} \quad \text{with} \quad r = 1 + \frac{P}{P_L} \quad \text{and} \quad P_L = \frac{E_L}{\tau_L},$$

det (A) > 0: $$\left.\frac{dq}{dP}\right|_{cw}\frac{r}{T_L} + \cancel{2}g_s\frac{\cancel{r}-1}{T_L P_L} > 0.$$

$$\left|\left.\frac{dq}{dP}\right|_{cw}\right| < \left|\left.\frac{dg_s}{dP}\right|_{cw}\right|$$

Always fulfilled for self-starting laser

Stability condition:

$$\boxed{-2T_L P\left.\frac{dq}{dP}\right|_{cw} = 2T_L q_0 \left.\frac{\frac{P}{\chi P_L}}{\left(1+\frac{P}{\chi P_L}\right)^2}\right|_{cw} < r \quad \text{with} \quad \chi = \frac{P_A}{P_L}}$$

Stability of stationary operating point: Passive Q-switching

To find the stability criterion, we linearize the system just as we have done for laser CW operation:

$$T_R \frac{d}{dt}\begin{pmatrix} \Delta P_0 \\ \Delta g_0 \end{pmatrix} = A \begin{pmatrix} \Delta P_0 \\ \Delta g_0 \end{pmatrix}, \text{ with } A = \begin{pmatrix} -2\left.\frac{dq}{dP}\right|_{cw} P_s & 2P_s \\ -\frac{g_s T_R}{E_L} & -\frac{T_R}{\tau_{stim}} \end{pmatrix}$$

We look for the eigen solution:

$$\frac{d}{dt}\begin{pmatrix} \Delta P_0(t) \\ \Delta g_0(t) \end{pmatrix} = s \begin{pmatrix} \Delta P_0(t) \\ \Delta g_0(t) \end{pmatrix}$$

$$\boxed{s = \frac{1}{2}\left(\gamma_Q - \frac{1}{\tau_{stim}}\right) \pm j\sqrt{\gamma_Q \frac{r}{\tau_L} + \frac{r-1}{\tau_p \tau_L} - \left(\frac{\gamma_Q - \frac{1}{\tau_{stim}}}{2}\right)^2}}$$

Growth rate introduced by the saturable absorber that destabilizes the laser relaxation oscillation:

$$\gamma_Q = -\frac{2}{T_R}\left.\frac{dq}{dP}\right|_{cw} P_s$$

Q-switching happens when $\gamma_Q > \frac{1}{\tau_{stim}}$

$$s = \frac{1}{2}\left(\gamma_Q - \frac{1}{\tau_{stim}}\right) \pm j\sqrt{\gamma_Q \frac{r}{\tau_L} + \frac{r-1}{\tau_p \tau_L} - \left(\frac{\gamma_Q - \frac{1}{\tau_{stim}}}{2}\right)^2}.$$

$$\gamma_Q = -\frac{2}{T_R} \left.\frac{dq}{dP}\right|_{cw} P_s$$

$$r = g_0/(l + q_s)$$

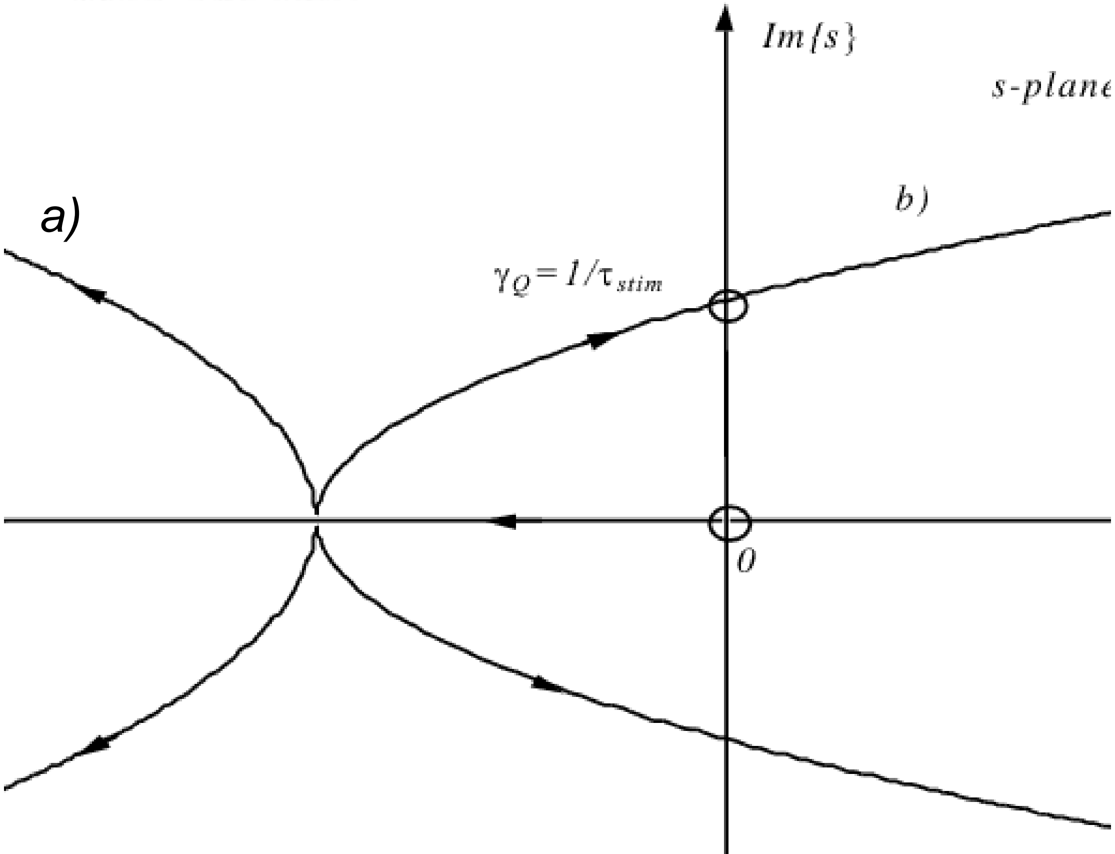


Fig. 4.26: Root locus plot as a function of destabilizing absorber. a) Without sat.absorber as a function of the pump parameter r; b) With sat. absorber as a function of γ_Q .

Passive Q-switching: a numerical example

τ_L=250μs, T_R=4ns, 2l=0.1, $2q_o$=0.005, $2g_o$=2, P_L/P_A=100.

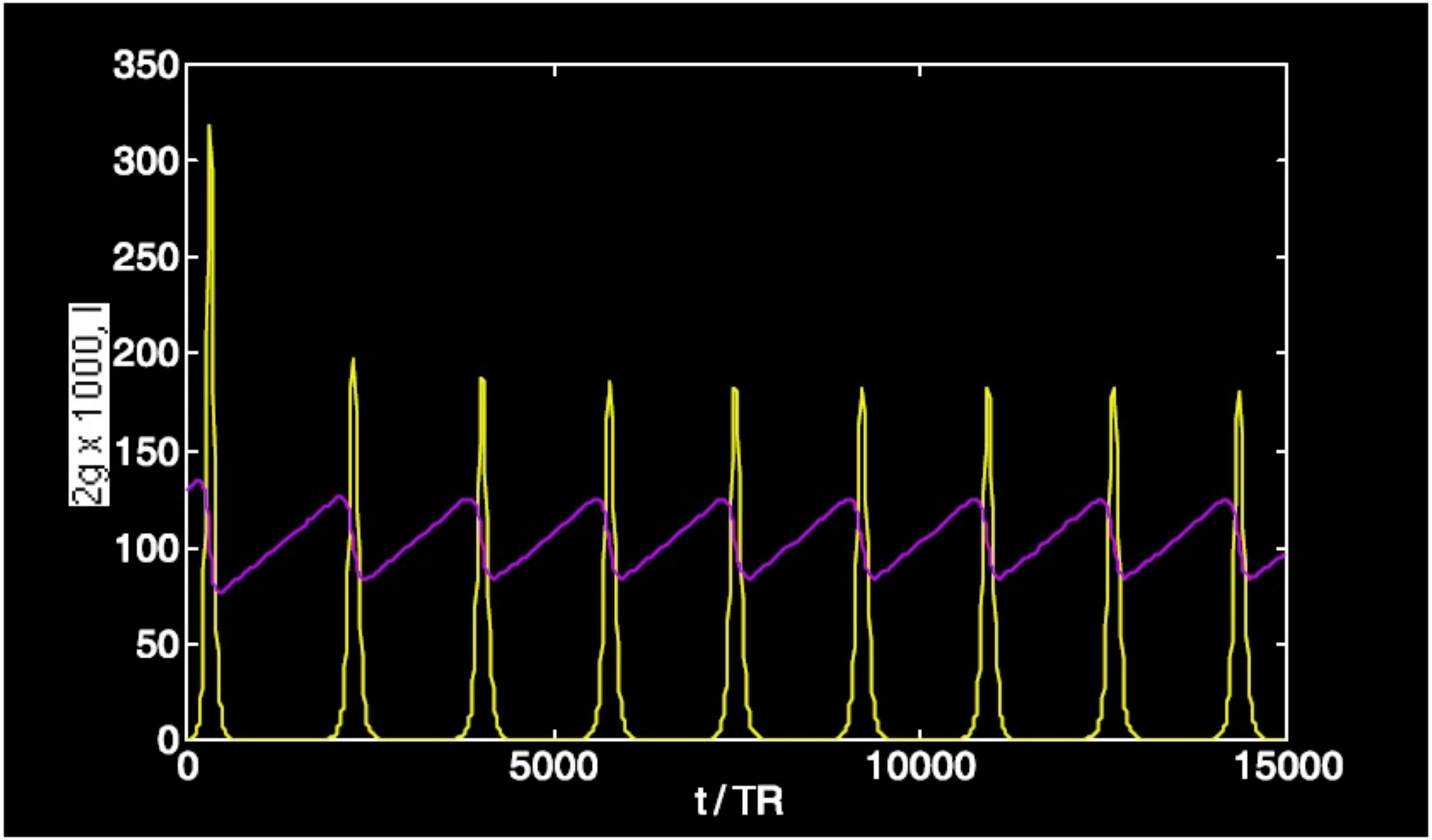
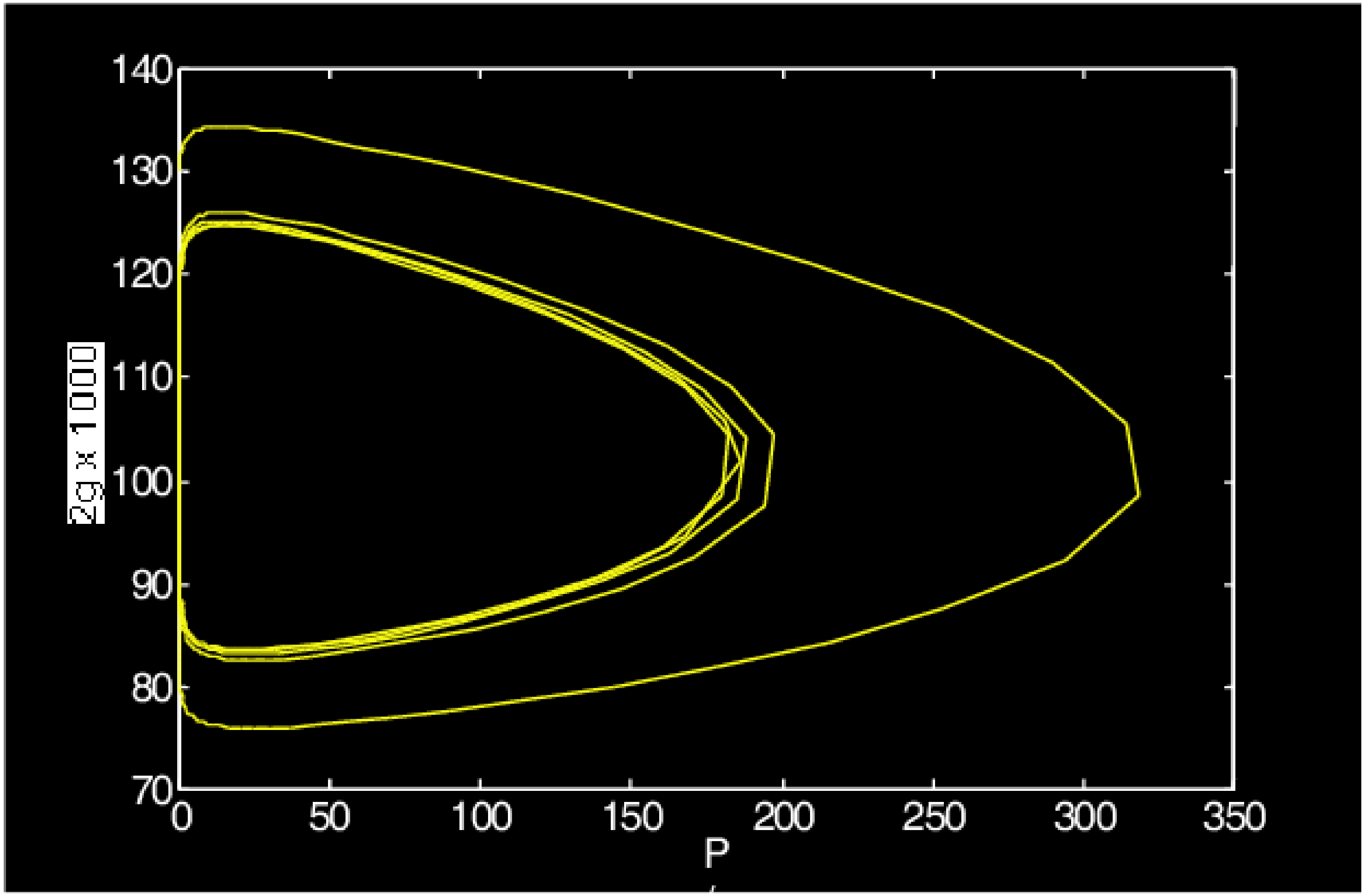


Fig. 4.28: Gain and output power as a function of time.



τ_L=250µs, T_R=4ns, 2l=0.1, $2q_o$=0.005, $2g_o$=2, P_L/P_A=100.

Fig. 4.27: Phase space solution for rate equations.

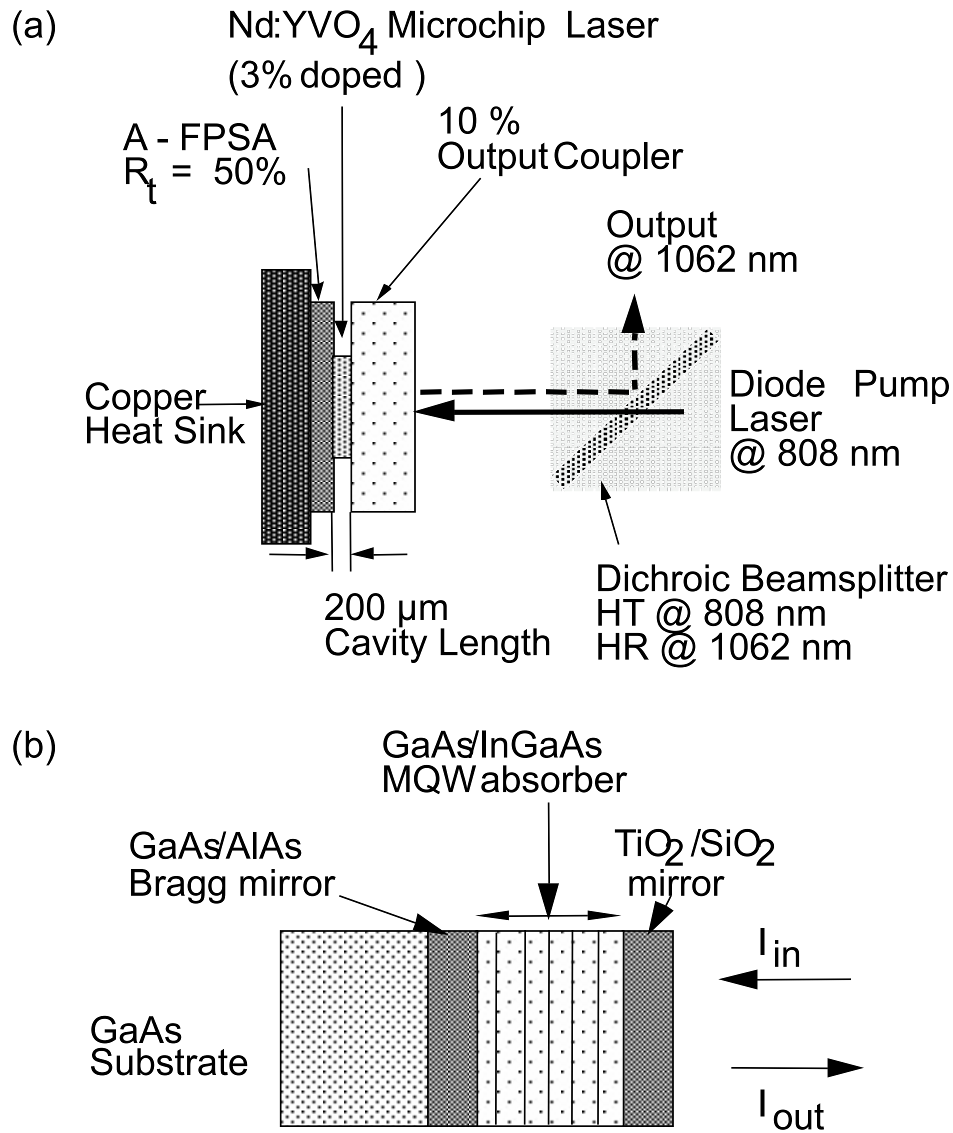


Fig. 4.29: Q-switched $Nd:YVO_4$-laser and absorber structure

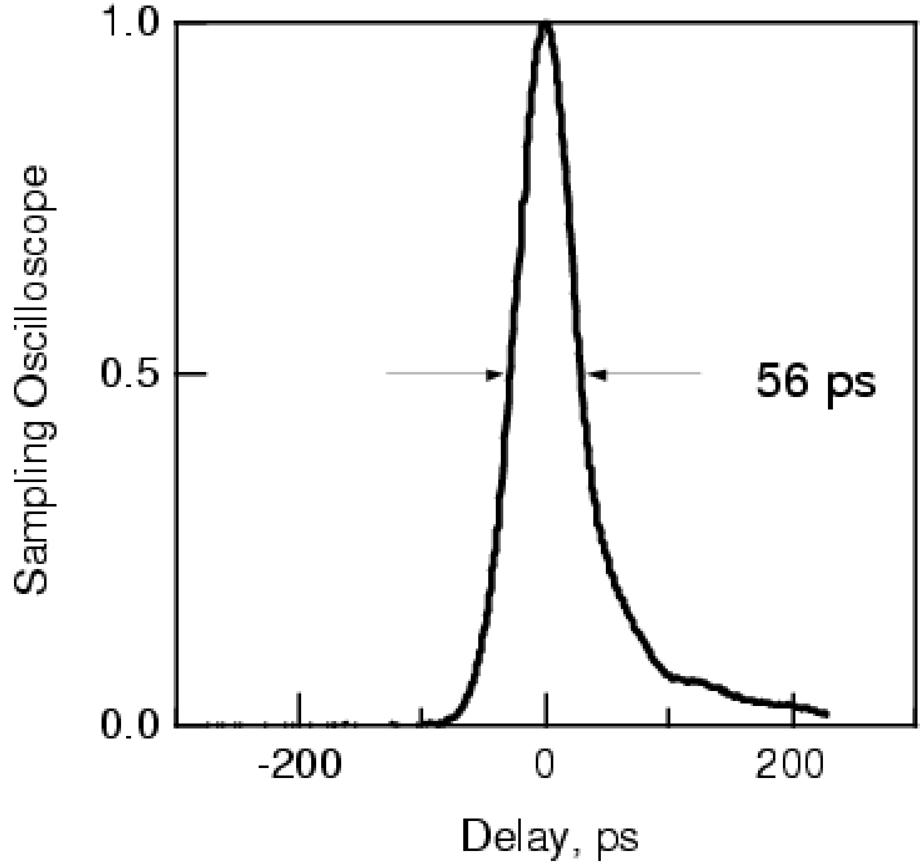


Fig. 4.30: Single-Mode Q-switched pulse from Nd:YVO_4 laser.

parameter	value
$2\, g_0$	0.7
$2\, q_0$	0.03
$2\, l$	0.14
T_R	2.7 ps
τ_L	87 μs
τ_A	24 ps
E_L	20 μJ
E_A	7.7 nJ

Table 4.2:

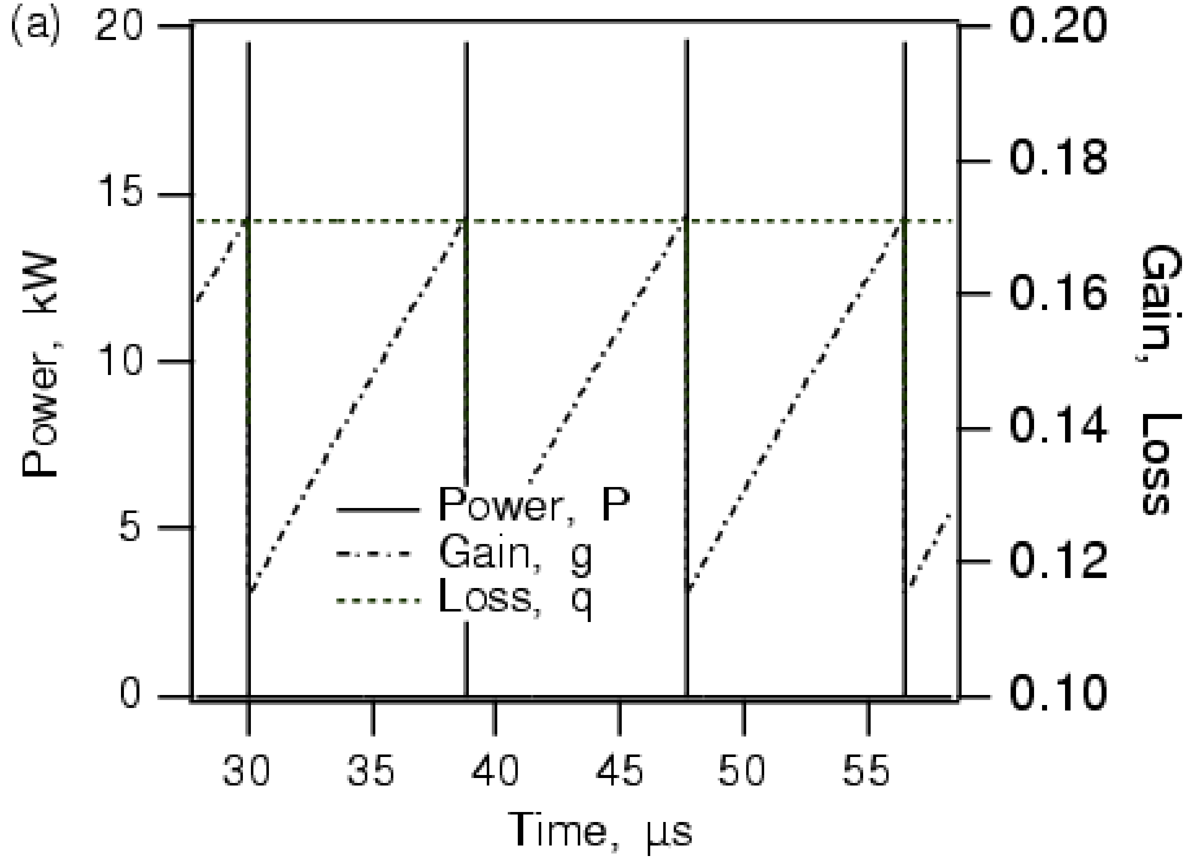


Fig. 4.31: Dynamics of the Q-switched microchip laser on a microsecond timescale

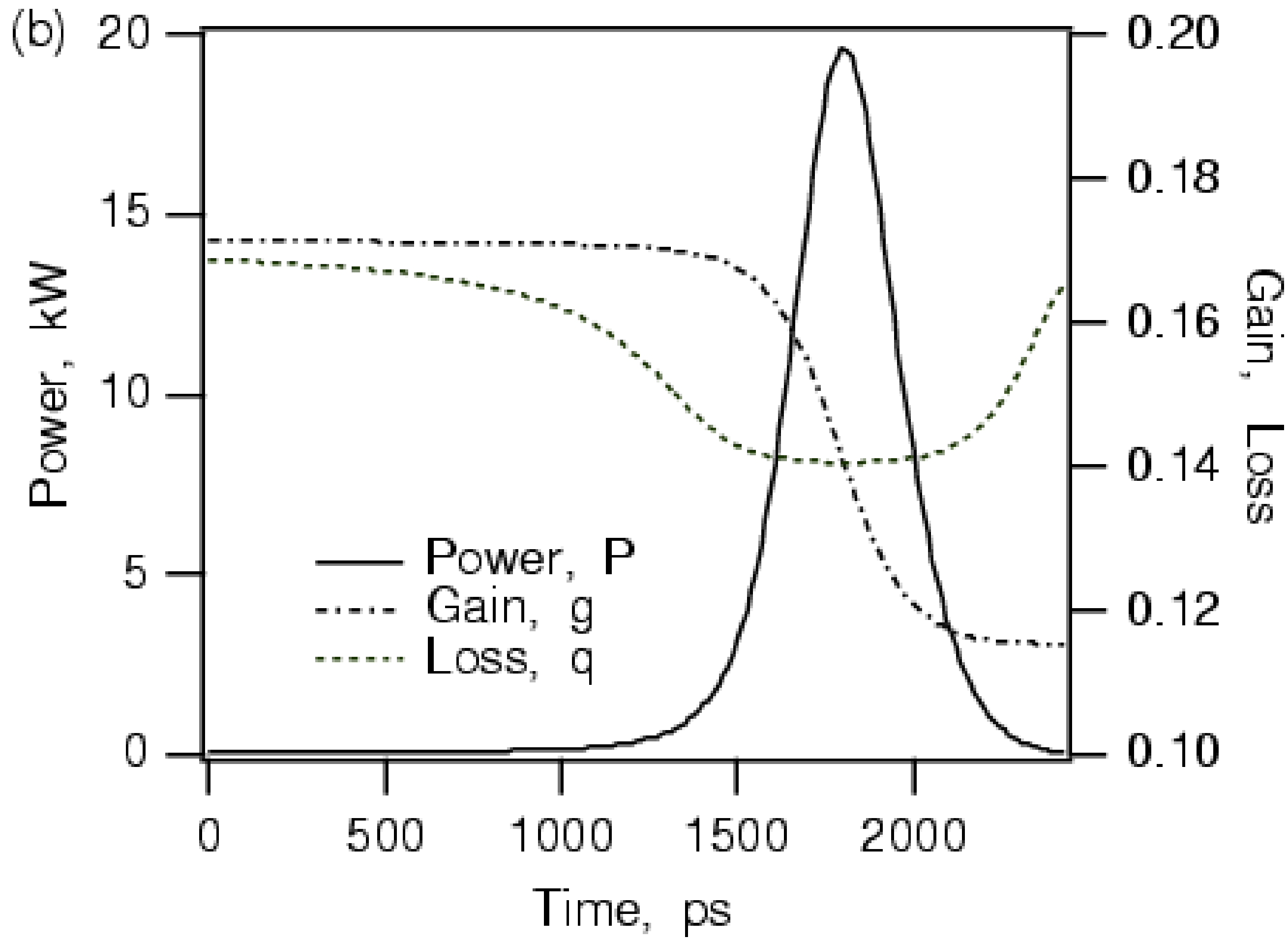


Fig. 4.32: On a picosecond time scale

5. Laser Mode Locking

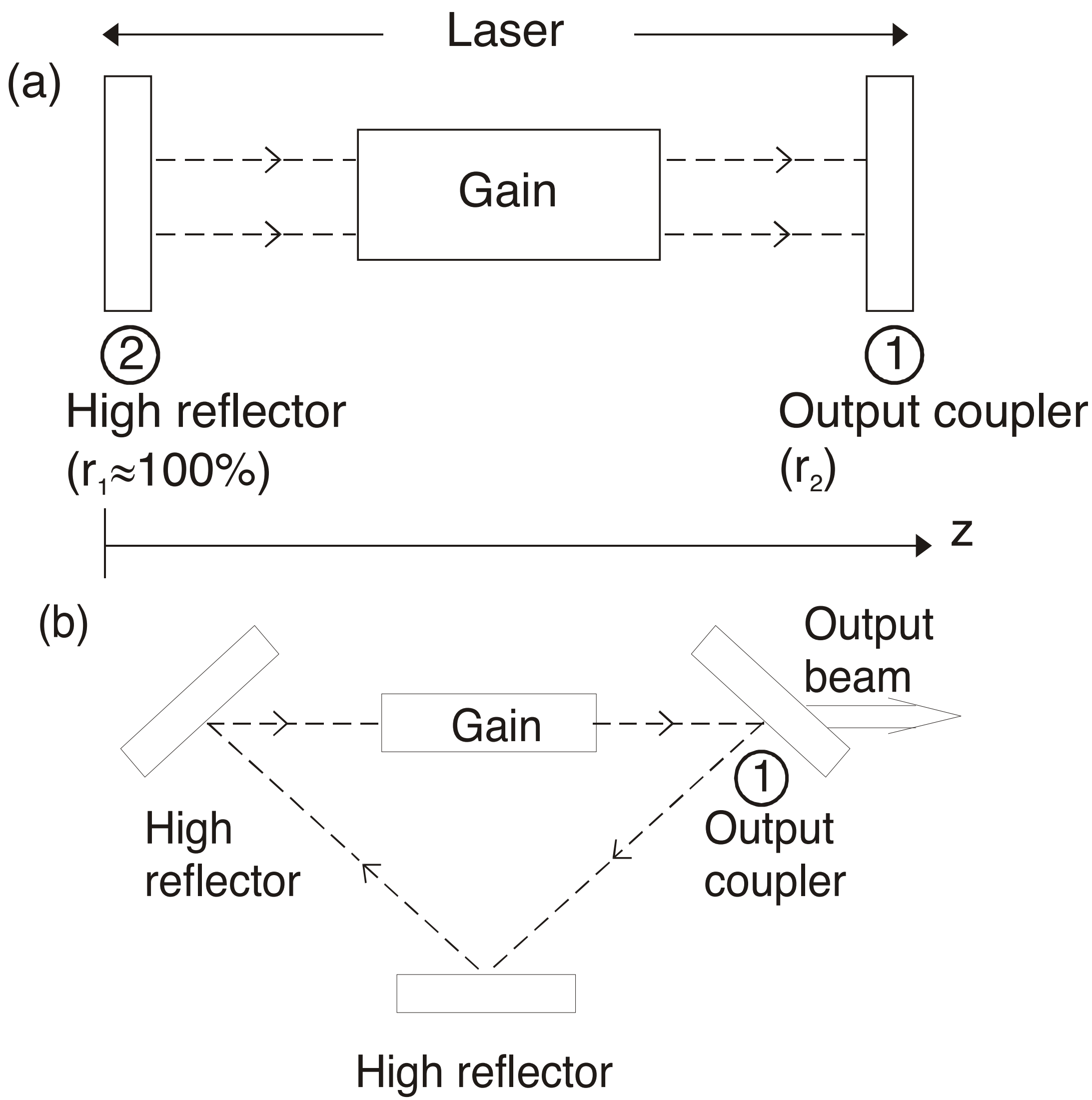


Figure 1.12: Possible cavity configurations

Steady State Lasing

$$E(z,t) = \Re\left\{E_0 e^{\mathrm{j}(\omega t - kz)}\right\}$$

Gain and loss: $n = n' + \mathrm{j}n''$

$$k = \frac{\omega}{c} n$$

After propagation through gain medium and air path: $\ell = n_g' \ell_g + \ell_a$

$$E = \Re\left\{E_0 e^{\frac{\omega}{c} n_g'' \ell_g} e^{\mathrm{j}\omega t} e^{-\mathrm{j}\frac{\omega}{c}(n_g' \ell_g + \ell_a)}\right\}$$

Steady-State Condition:

$$E = \Re\left\{r_1 r_2 e^{2\frac{\omega}{c} n_g'' \ell_g} E_0 e^{\mathrm{j}\omega t - \mathrm{j}2\frac{\omega}{c}\ell}\right\} \Rightarrow r_1 r_2 e^{2\frac{\omega}{c} n_g'' \ell_g} = 1$$

Mode Condition: $$\frac{2\omega\ell}{c} = 2m\pi$$

Resonance Frequencies: $$\omega_m = \frac{m\pi c}{\ell} \qquad f_m = \frac{mc}{2\ell}$$

$$\Delta f = f_m - f_{m-1} = \frac{c}{2\ell}$$

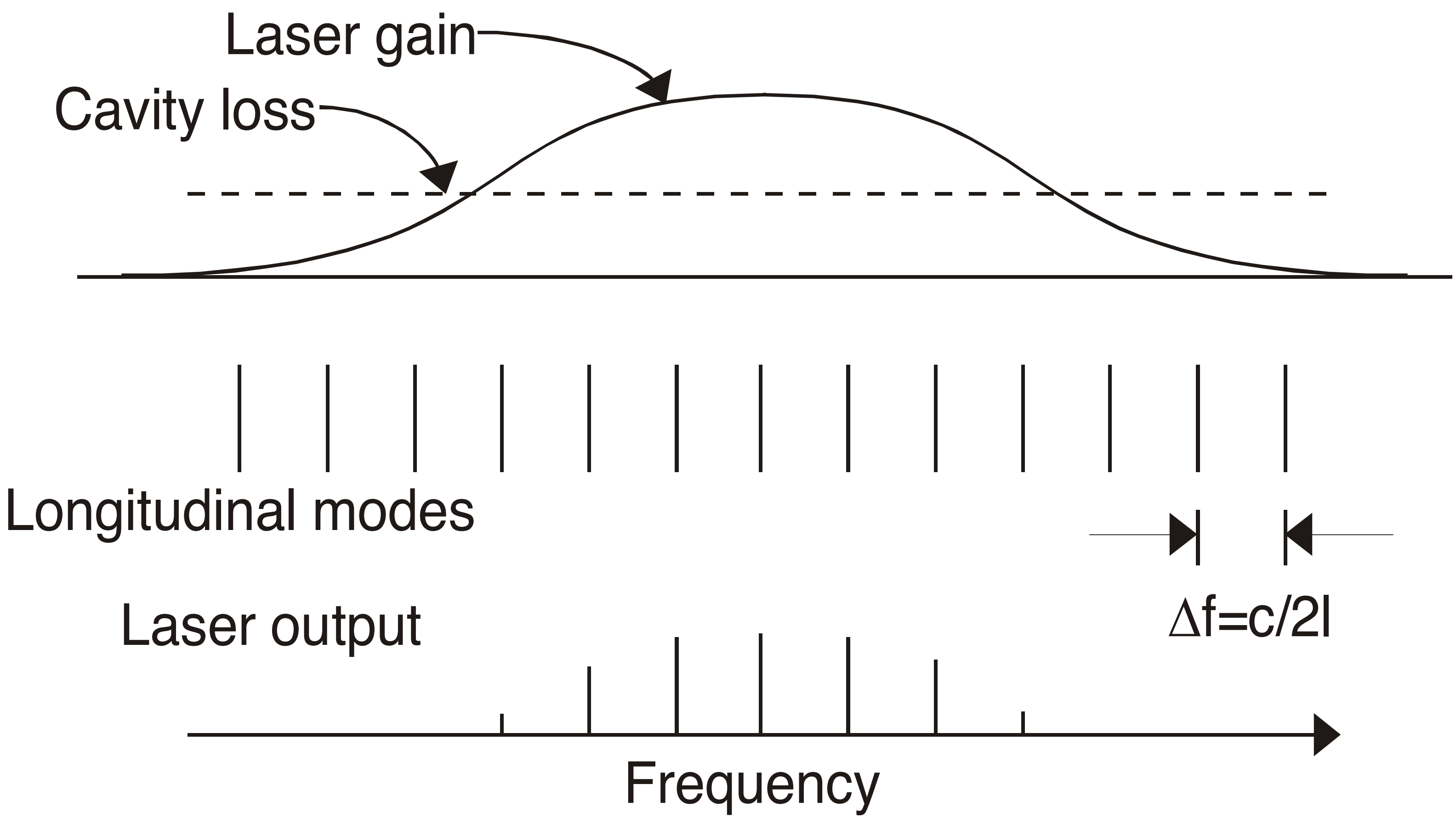


Figure 1.13: Laser gain and cavity loss spectrum

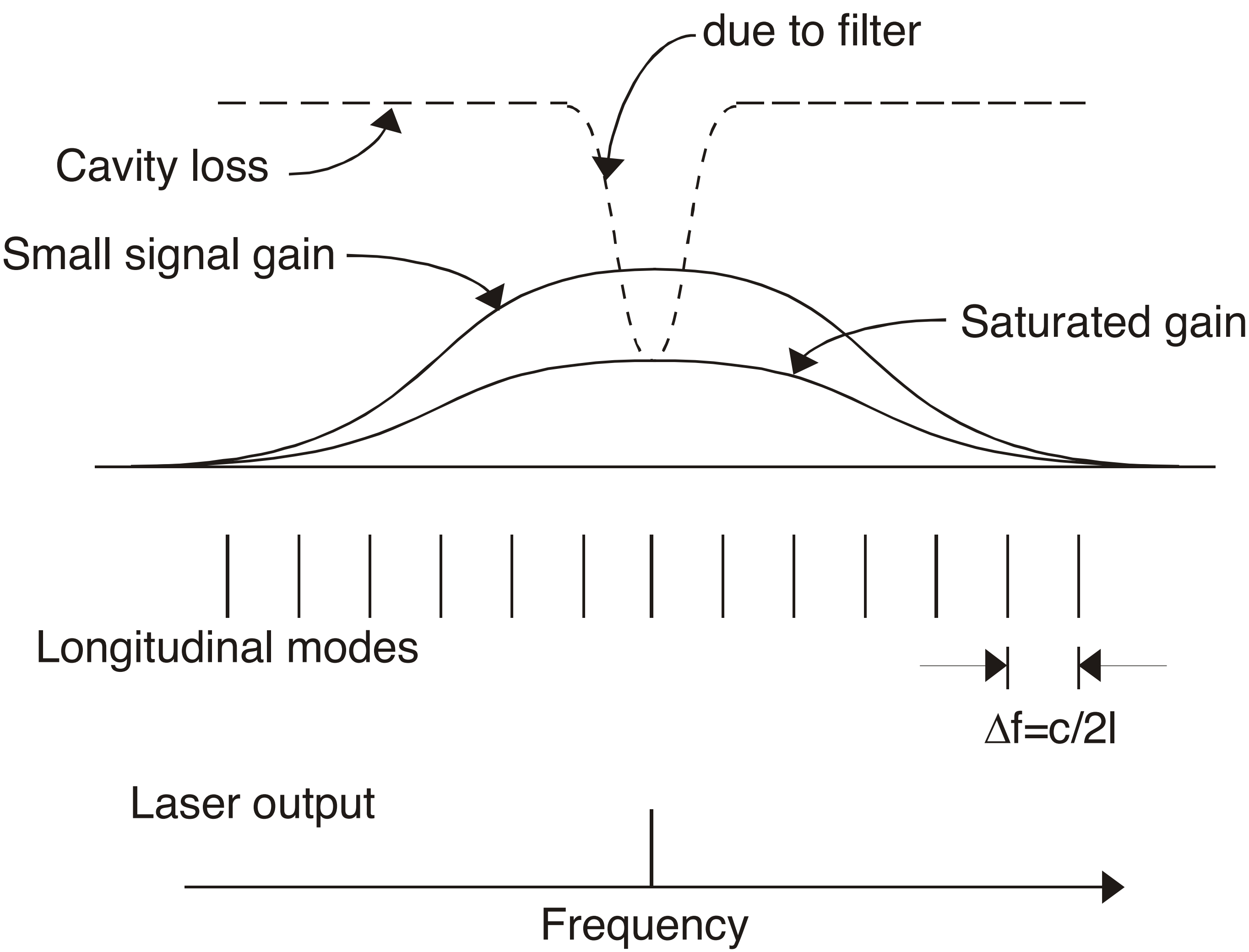


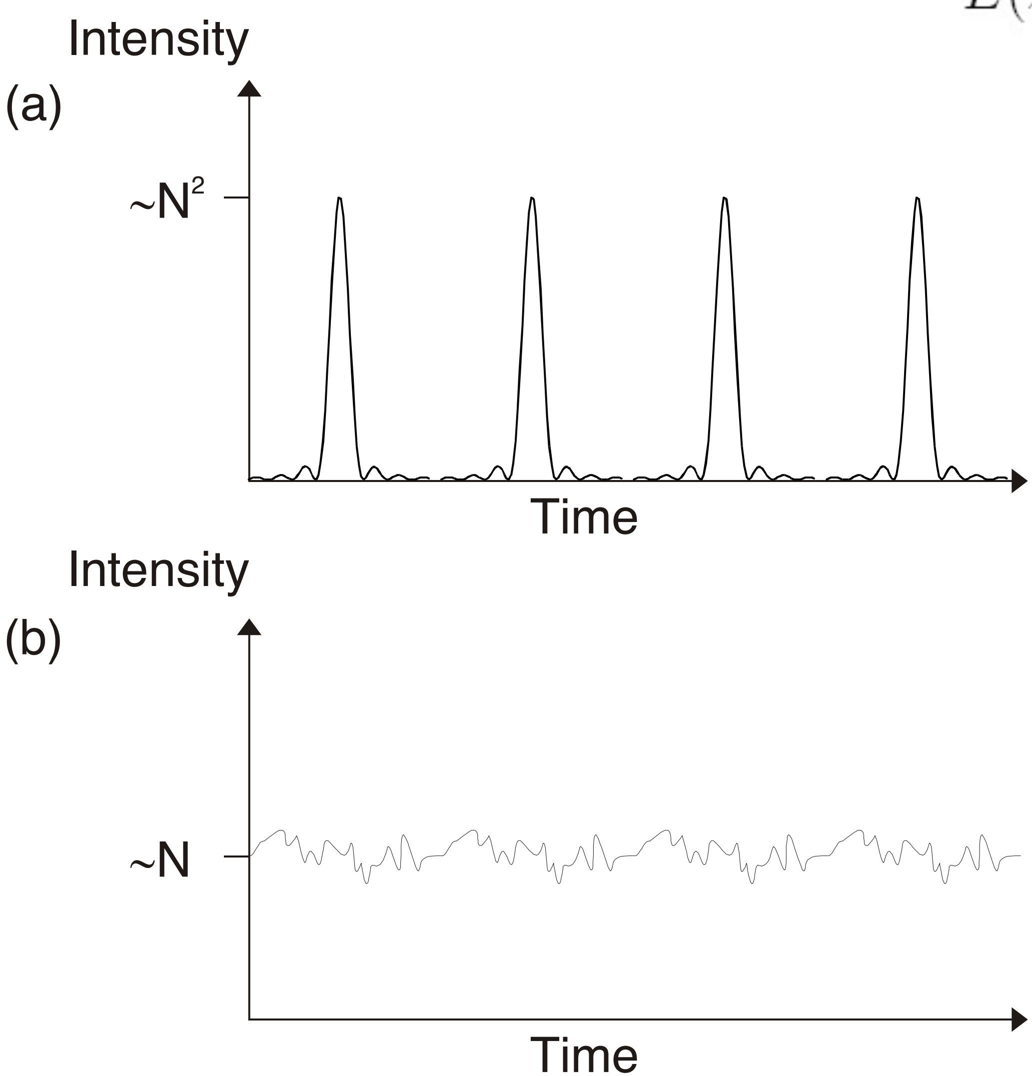
Figure 1.14: Gain and loss spectra.

Small signal gain

$$g = \frac{g_0}{1 + \frac{I}{I_{sat}}}$$

Saturation Intensity

Mode Locking:



$$
\begin{aligned}
E(z,t) &= \Re\left[\sum_m \tilde{E}_m e^{j(\omega_m t - k_m z + \phi_m)}\right] \\
\omega_m &= \omega_0 + m\Delta\omega = \omega_0 + \frac{m\pi c}{\ell}, \\
k_m &= \frac{\omega_m}{c}, \\
&= \Re\left[A(t - z/c) e^{j\omega_0 (t - z/c)}\right]
\end{aligned}
$$

Envelope Carrier

$$
\Delta t = \frac{2\pi}{N\Delta\omega} = \frac{1}{N\Delta f}
$$

Figure 1.15: (a) mode-locked laser output with constant pulse
(b): with random phase.

Self-consistent Model for Mode-Locked Laser

Medium modeled by pumped two level atoms
(Polarization and Inversion, at each point in space and time)

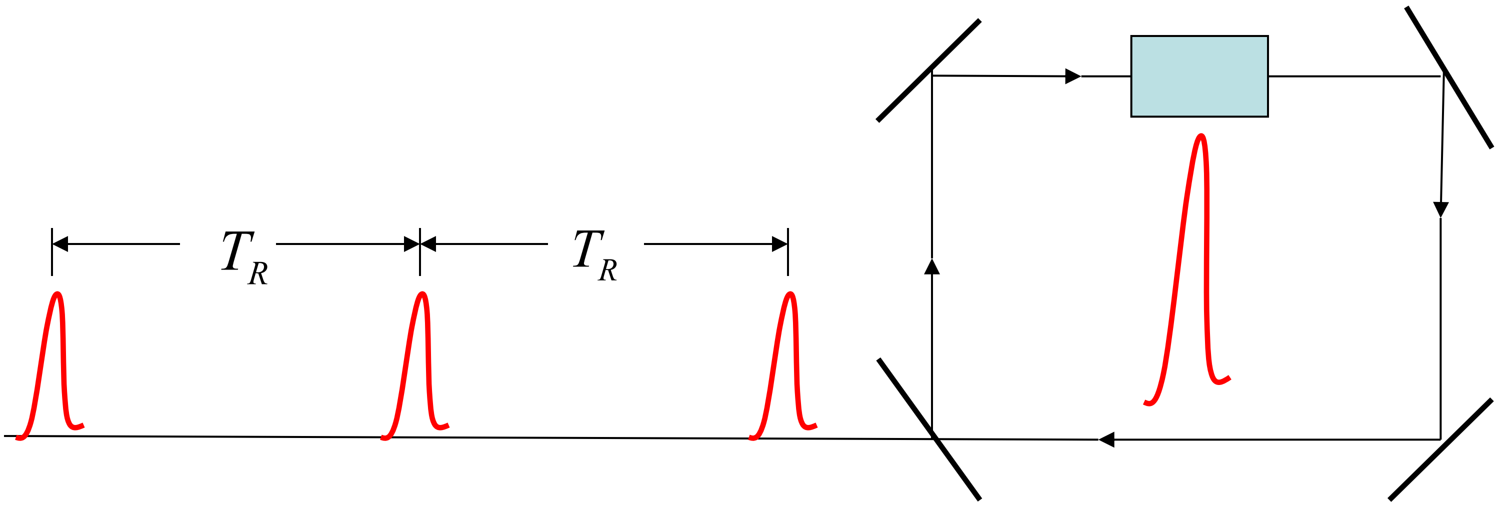
Medium Polarization is source in the Maxwell's Equations

Maxwell Equations describe Field in Resonator

Resonator included by proper boundary conditions for fields at the cavity mirrors.

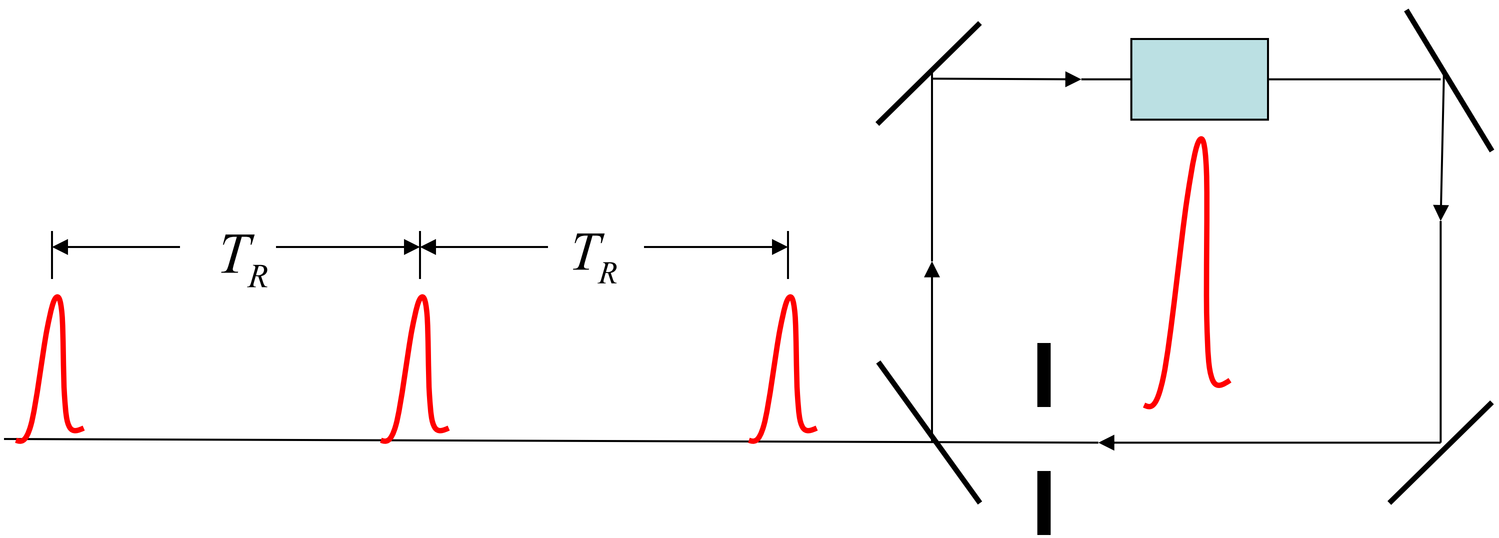
Directly write field as a sum of modes whose amplitudes change slowly with time due to coupling to the gain medium, dispersion, ….

Time-domain picture of mode-locking



Each time the pulse hits the output coupler, a small fraction of the power is transmitted out of the cavity. The output is a pulse train with repetition rate $1/T_{R.}$

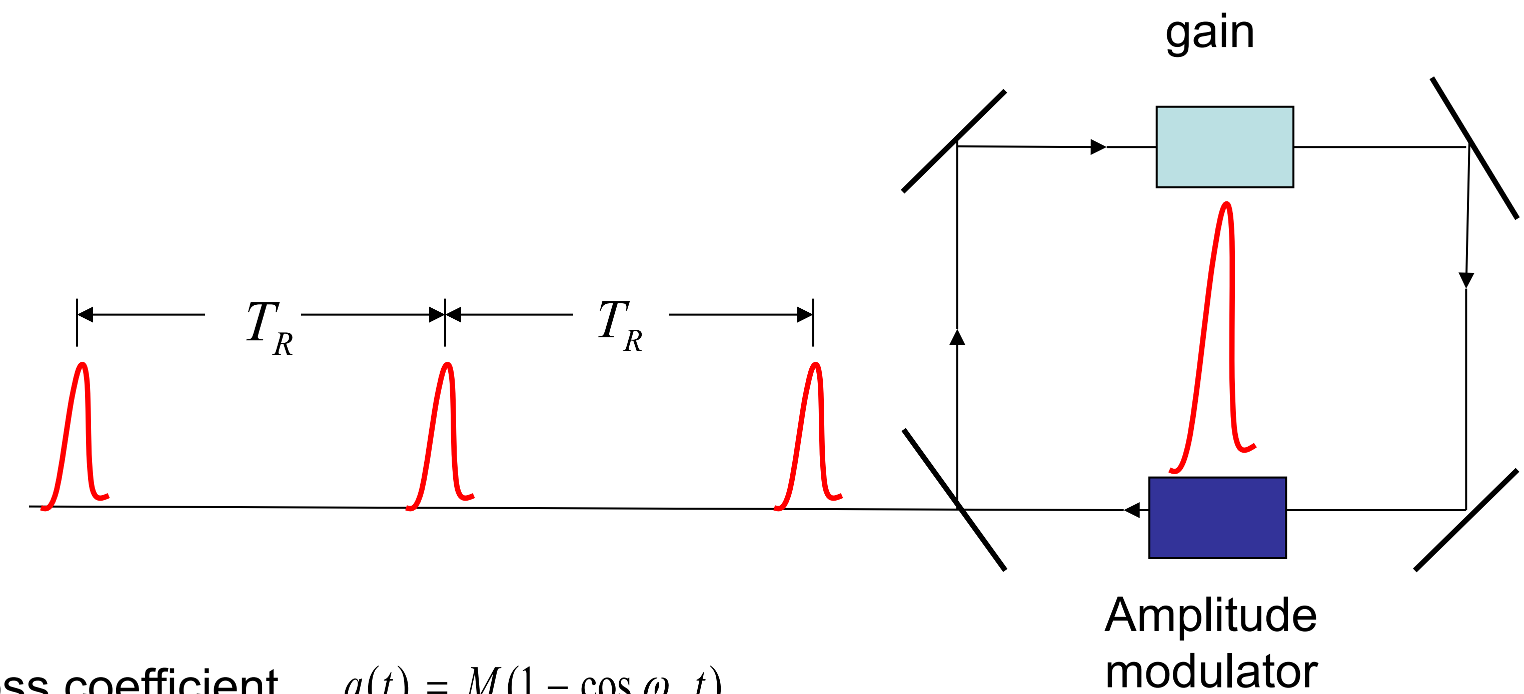
Force laser to generate short-pulse train using a "shutter" to modulate cavity loss



Transient process:

- Shutter is opened, loss is low → laser is above threshold → peak builds up
- Shutter is closed, loss is high → laser is blow threshold → wings developed

Active mode-locking

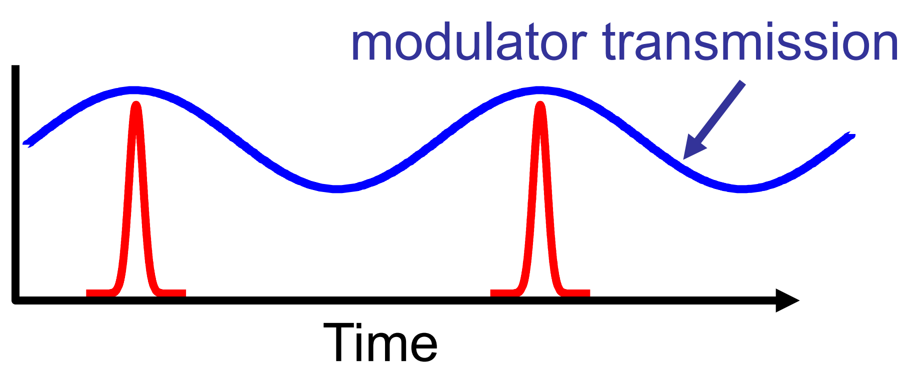


Loss coefficient $q(t) = M(1 - \cos\omega_m t)$

Transmission of the modulator

$$T_m = e^{-M(1-\cos\omega_m t)}$$

$$T_m \approx 1 - M(1-\cos\omega_m t)$$

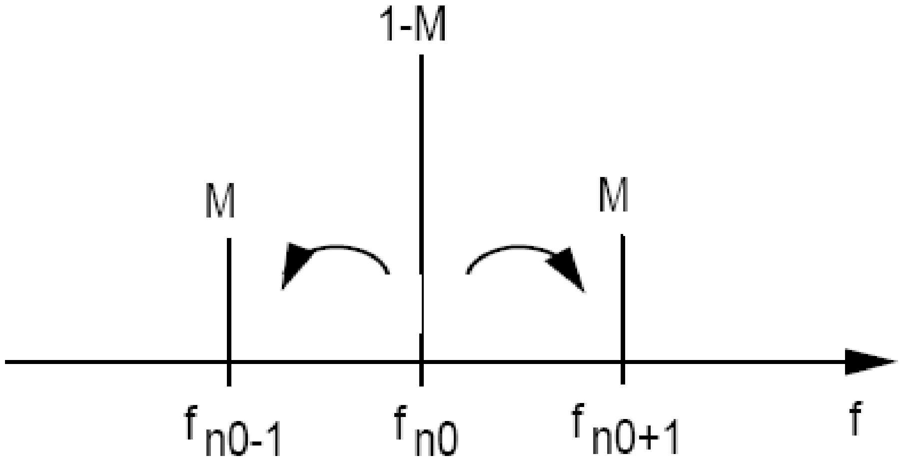


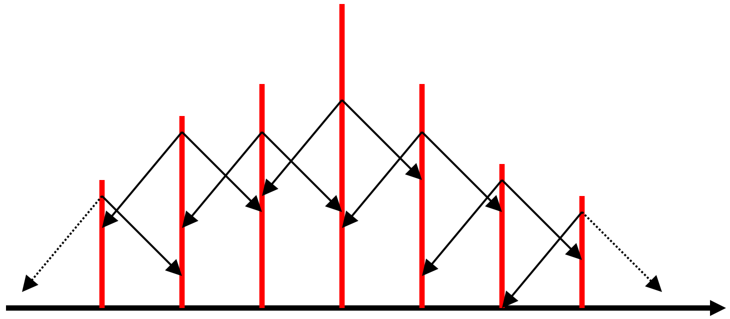
Talking among modes

$$e^{j\omega_{n0}t}T_m \approx e^{j\omega_{n0}t}(1-M+M\cos\omega_m t)$$

$$=(1-M)e^{j\omega_{n0}t}+\frac{M}{2}(e^{j\omega_m t}+e^{-j\omega_m t})e^{j\omega_{n0}t}$$

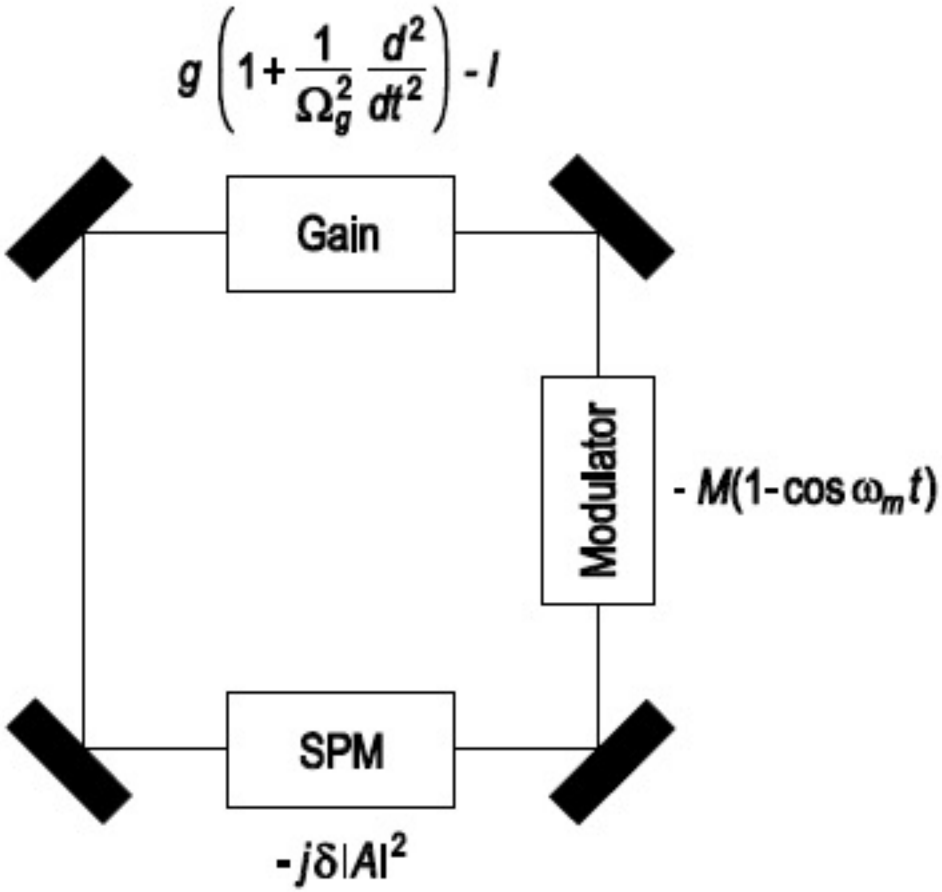
$$=(1-M)e^{j\omega_{n0}t}+\frac{M}{2}e^{j(\omega_{n0}+\omega_m)t}+\frac{M}{2}e^{j(\omega_{n0}-\omega_m)t}$$





The modulator actually takes power out of the central mode and redistributes it to the other modes. This is how mode-locking can occur in a homogeneously broadened laser.

5.1 Master equation of mode-locking



Assume in steady state, the change in the pulse caused by each element in the cavity small.

$$T_{\mathrm{R}} \frac{\partial A(T,t)}{\partial T} = \sum_i \Delta A_i = 0$$

A: the pulse envelope
T_{R}: the cavity round-trip time
T: the time that develops on a time scale of the order of T_R
t: the fast time of the order of the pulse duration
ΔA_i: the changes of the pulse envelope due to different elements in the cavity.

Loss: $$T_R \left.\frac{\partial A(T,t')}{\partial T}\right|_{(loss)} = -lA(T,t') \qquad (5.15)$$

Gain: $$T_R \left.\frac{\partial A(T,t')}{\partial T}\right|_{(gain)} = \left(g(T) + D_g \frac{\partial^2}{\partial t'^2}\right) A(T,t'), \qquad D_g = \frac{g(T)}{\Omega_g^2}$$

dispersion

loss

$$\boxed{\begin{aligned} T_R \frac{\partial A(T,t')}{\partial T} &= -lA(T,t') + j\sum_{n=2}^{\infty} D_n \left(j\frac{\partial}{\partial t'}\right)^n A(T,t') \\ &+ g(T)\left(1 + \frac{1}{\Omega_g^2}\frac{\partial^2}{\partial t'^2}\right) A(T,t') \\ &- q(T,t')A(T,t') - j\delta |A(T,t')|^2 A(T,t'). \end{aligned}} \qquad (5.21)$$

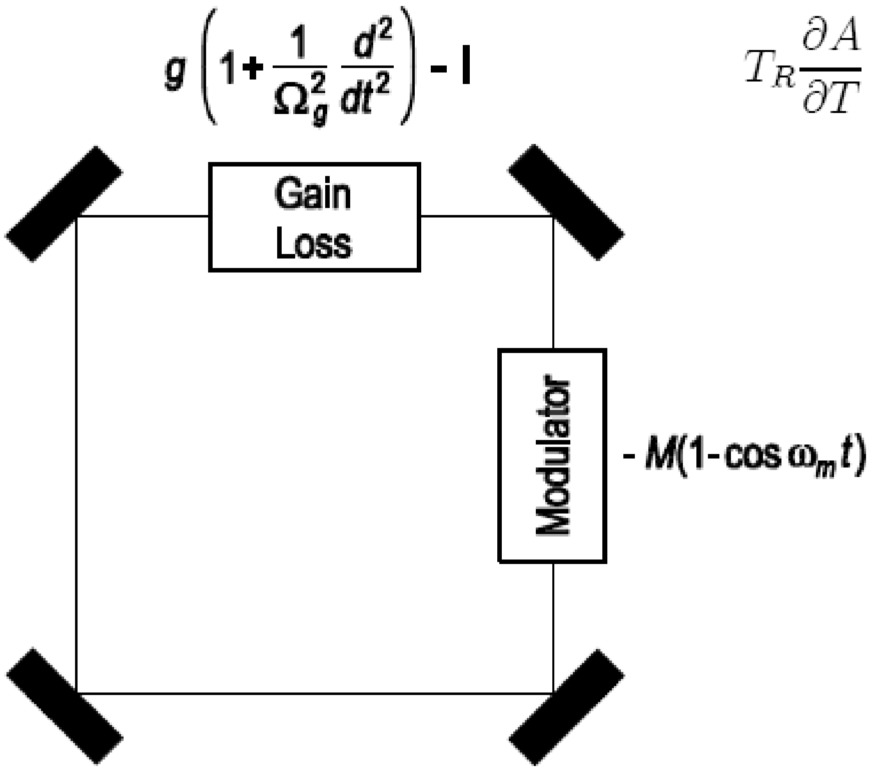
gain

Mode-locking element

Gain dispersion

Self-phase modulation

5.2 Active mode-locking by loss modulation



$$T_R\frac{\partial A}{\partial T}=\left[g(T)+D_g\frac{\partial^2}{\partial t^2}-l-M\left(1-\cos(\omega_M t)\right)\right]A.$$

$$\downarrow$$

$$T_R\frac{\partial A}{\partial T}=\left[g-l+D_g\frac{\partial^2}{\partial t^2}-M_s t^2\right]A.$$

$$D_g = \frac{g}{\Omega_g^2},$$

$$M_s = \frac{M\omega_M^2}{2}.$$

Gain tends to narrow the spectrum,
spectrum broadened by modulator.

Hermite-Gaussian Solutions

$$A_n(T,t) = A_n(t)e^{\lambda_n T/T_R} \qquad A_n(t) = \sqrt{\frac{W_n}{2^n\sqrt{\pi}n!\tau_a}}H_n(t/\tau_a)e^{-\frac{t^2}{2\tau_a^2}}$$

$$\tau_a = \sqrt[4]{D_g/M_s}. \longrightarrow \tau_a = \sqrt[4]{2}\left(\frac{g}{M}\right)^{\frac{1}{4}}\frac{1}{\sqrt{\Omega_g\omega_M}}$$

1) Larger modulation depth, M, and higher modulation frequency will give shorter pulses because the "low loss" window becomes narrower, thus shortening the pulses.
2) A broader gain bandwidth yields shorter pulses because the filtering effect of gain narrowing is lower and more modes are lasing.

$$\lambda_n = g_n - l - 2M_s\tau_a^2\left(n+\frac{1}{2}\right).$$

For given g, the eigen solution with n=0 has the largest gain per round-trip and saturate the gain to

$$g_s = l + M_s\tau_a^2$$

All other modes will decay.

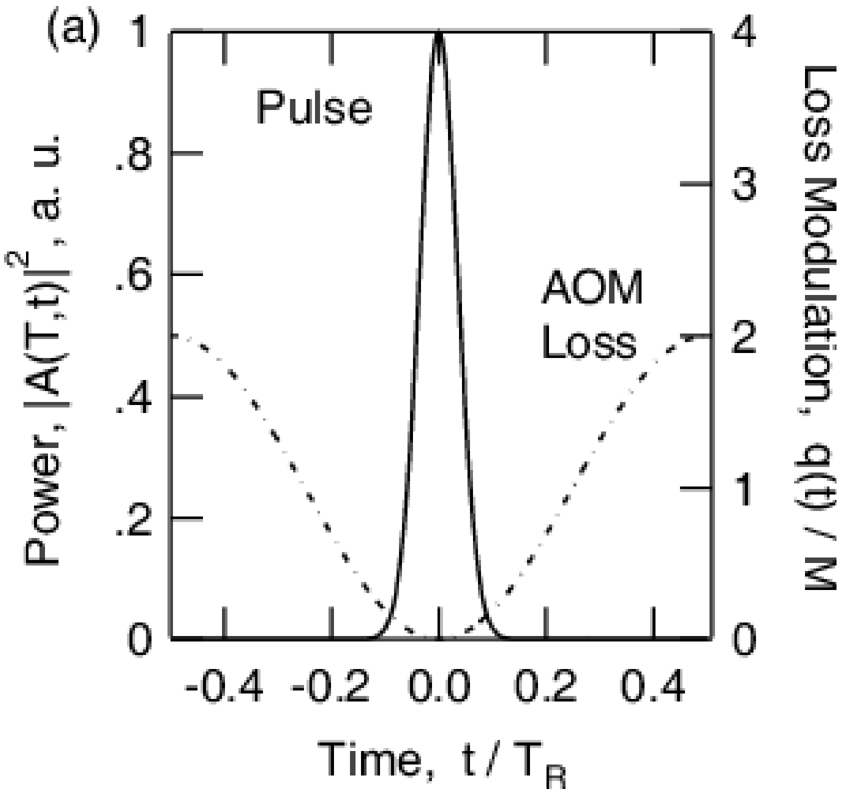


Fig. 5.4: Loss modulation results
in pulse shortening
in each roudntrip

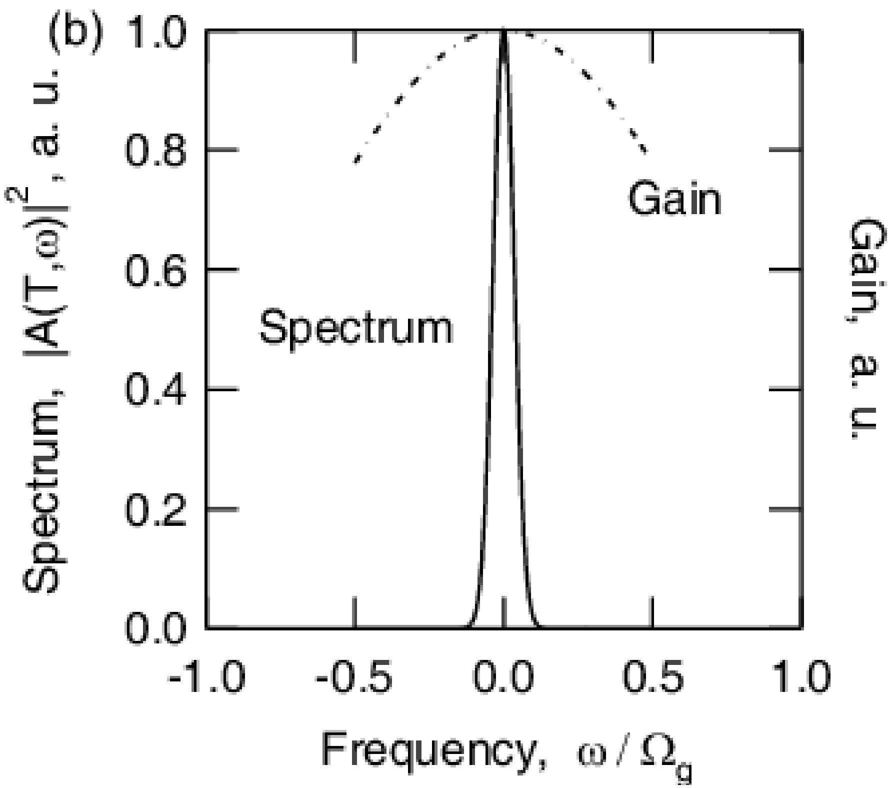


Fig. 5.5: Gain filtering broadens
the pulse in each roundtrip

Example:

Nd:YAG; 2l=2g=10%,

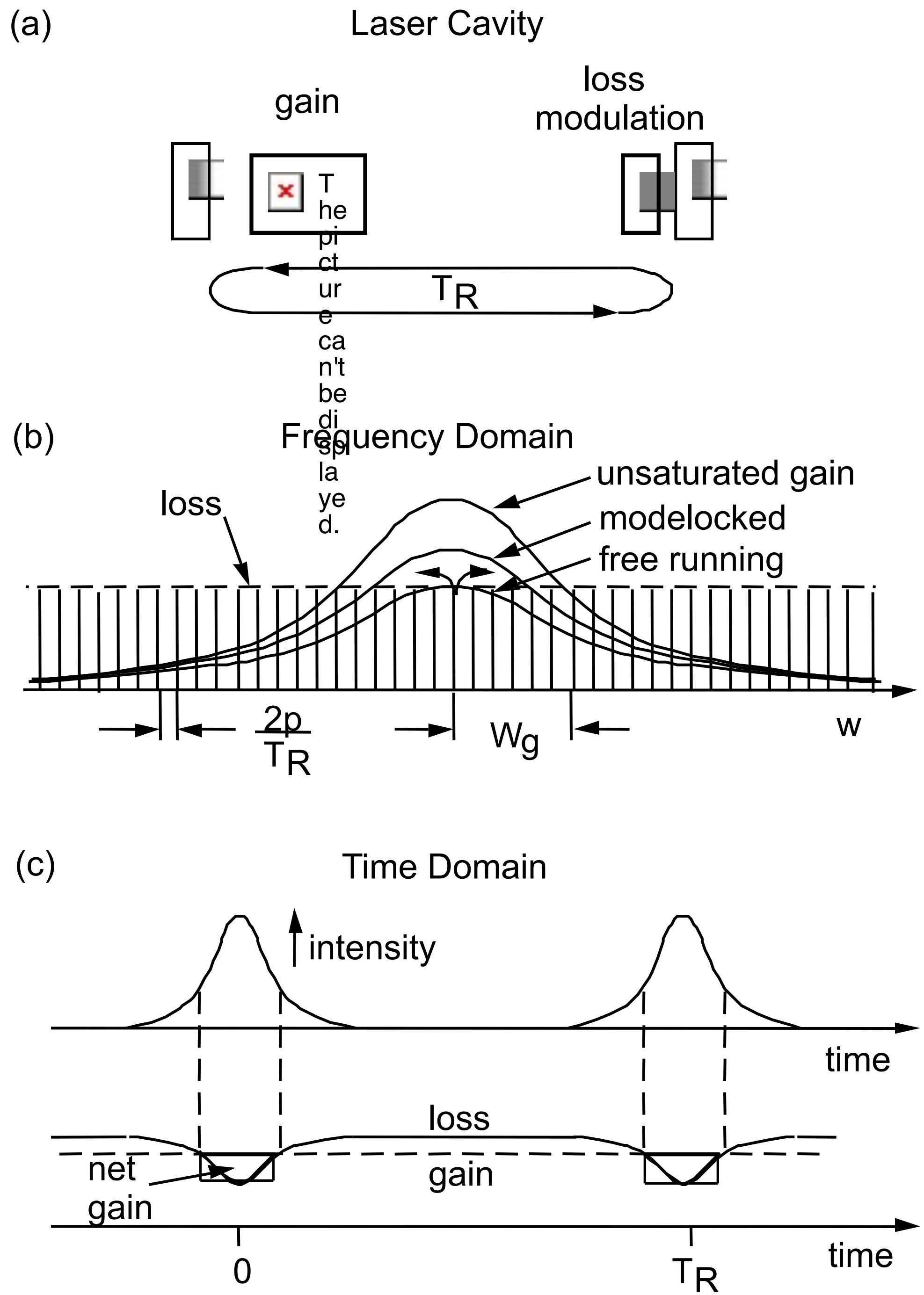
$\Omega_g = \pi\, \Delta f_{FWHM}$=0.65 THz,

M = 0.2, f_R =100 MHz,

D_g = 0.24 ps²,

M_s =4e16 /s².

$\tau \;=$ 99 ps.


(a)
Laser Cavity
gain
loss
modulation
The picture can't be displayed.
TR
(b)
Frequency Domain
unsaturated gain
modelocked
free running
loss
2p
TR
Wg
w
(c)
Time Domain
intensity
time
loss
gain
net
gain
0
TR
time

5.2 Active mode-locking by phase modulation

It can be modeled using master equation by replacing *M* by *jM*

$$T_R \frac{\partial A}{\partial T} = \left[g(T) + D_g \frac{\partial^2}{\partial t^2} - l - jM\left(1 - \cos(\omega_M t)\right)\right] A$$

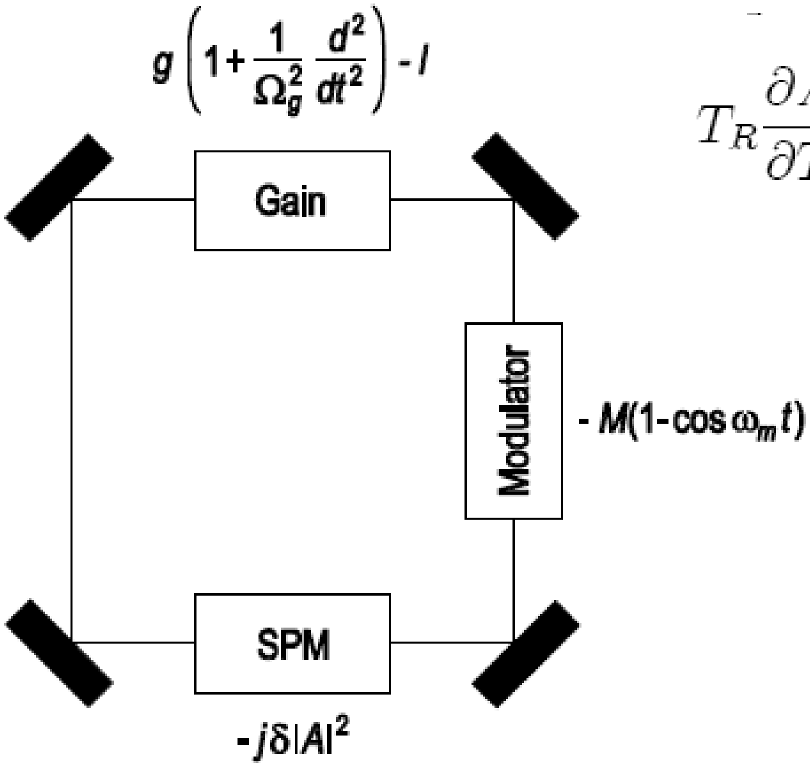
$$\tau_a' = \sqrt[4]{-j}\,\sqrt[4]{D_g/M_s}.$$

$$A_0(t) = \sqrt{\frac{W_s}{2^n \sqrt{\pi}\, n!\, \tau_a'}}\, e^{-\frac{t^2}{2\tau_a^2}\frac{1}{\sqrt{2}}(1+j)}$$

Chirp pulse

Pulse duration $\tau_a = \sqrt[4]{D_g/M_s}$

5.4 Active mode-locking with additional SPM



$$T_R\frac{\partial A}{\partial T} = \left[g(T) + D_g\frac{\partial^2}{\partial t^2} - l - M_s t^2 - j\delta|A|^2\right]A.$$

$$A_0(t) = Ae^{-\frac{t^2}{2\tau_a^2}(1+j\beta)+j\Psi T/T_R}$$

$$\begin{aligned} j\Psi A_0(t) \;&=\; \left\{g - l + D_g\left[\frac{t^2}{\tau_a^4}(1+j\beta)^2 - \frac{1}{\tau_a^2}(1+j\beta)\right]\right.\\ &\left. -M_s t^2 - j\delta\,|A|^2\, e^{-\frac{t^2}{\tau_a^2}}\right\} A_0(t). \end{aligned}$$

5.5 Active Modelocking with Soliton Formation

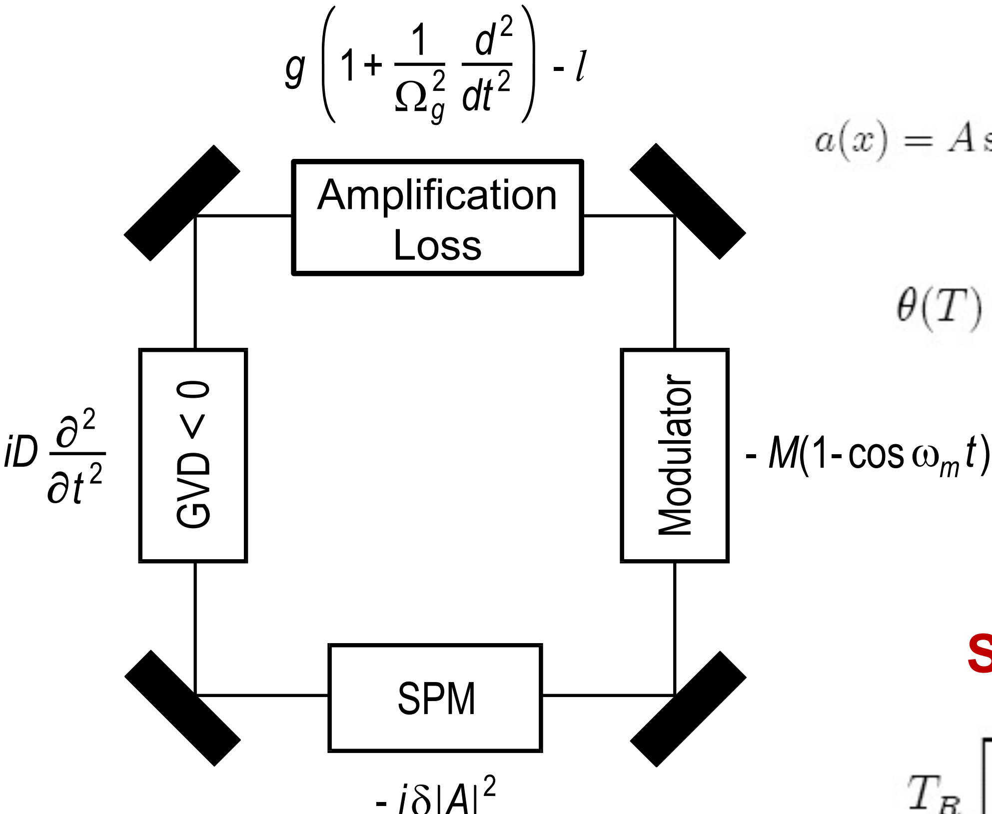


Fig. 5.8: Active Modelocking with Soliton Formation

$$A(T,t) = \left(a(x)e^{jpt} + a_c(T,t)\right)e^{-j\theta}$$

$$a(x) = A\,\mathrm{sech}(x), \quad \text{and} \quad x = \frac{1}{\tau}\left(t + 2D\int_0^T p(T')dT' - t_0\right)$$

$$\theta(T) = \theta_0(T) - \frac{D}{T_R}\int_0^T \left(\frac{1}{\tau(T')^2} - p(T')^2\right) dT'$$

$$\frac{|D|}{\tau(T)^2} = \frac{\delta A(T)^2}{2}.$$

Soliton Perturbation Theory

$$\begin{aligned} & T_R\left[\frac{\partial a_c}{\partial T} + \frac{\partial W}{\partial T}\mathbf{f}_w + \frac{\partial \Delta\theta}{\partial T}\mathbf{f}_\theta + \frac{\partial \Delta p}{\partial T}\mathbf{f}_p + \frac{\partial \Delta t}{\partial T}\mathbf{f}_t\right] \\ = \; & \phi_0 \mathbf{L}\left(\mathbf{a}_c + \Delta p \mathbf{f}_p\right) + \mathbf{R}(\mathbf{a} + \Delta p \mathbf{f}_p + \mathbf{a}_c) \\ & -M\omega_M \sin(\omega_M \tau x)\Delta t\, \mathbf{a}(x) \end{aligned}$$

$$\mathbf{R} = g\left(1 + \frac{1}{\Omega_g^2\tau^2}\frac{\partial^2}{\partial x^2}\right) - l - M\left(1 - \cos(\omega_M \tau x)\right),$$

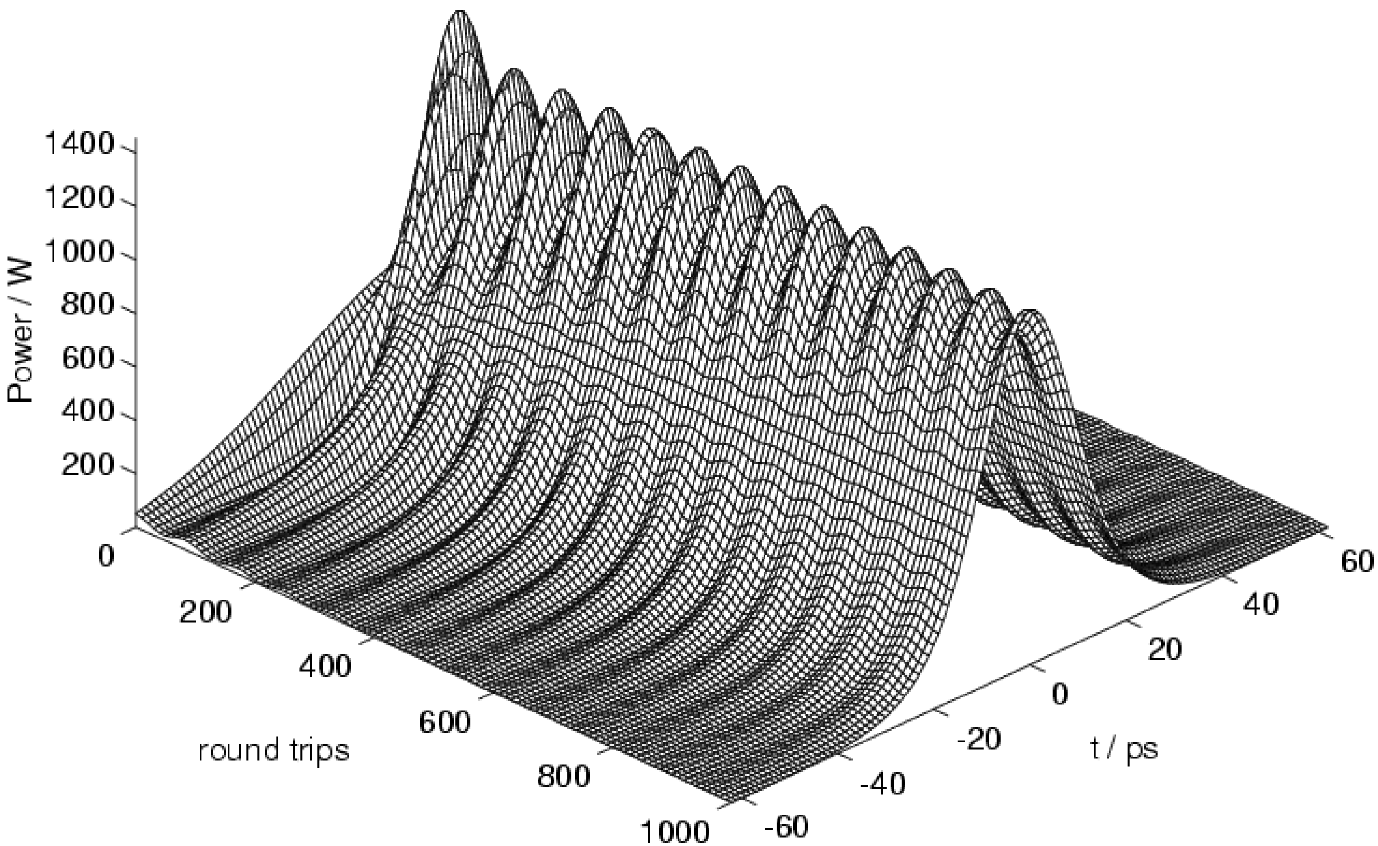


Fig. 5.10: Time evolution in a Nd:YAG laser D = -17ps^2 with initial pulsewidth 68 ps.

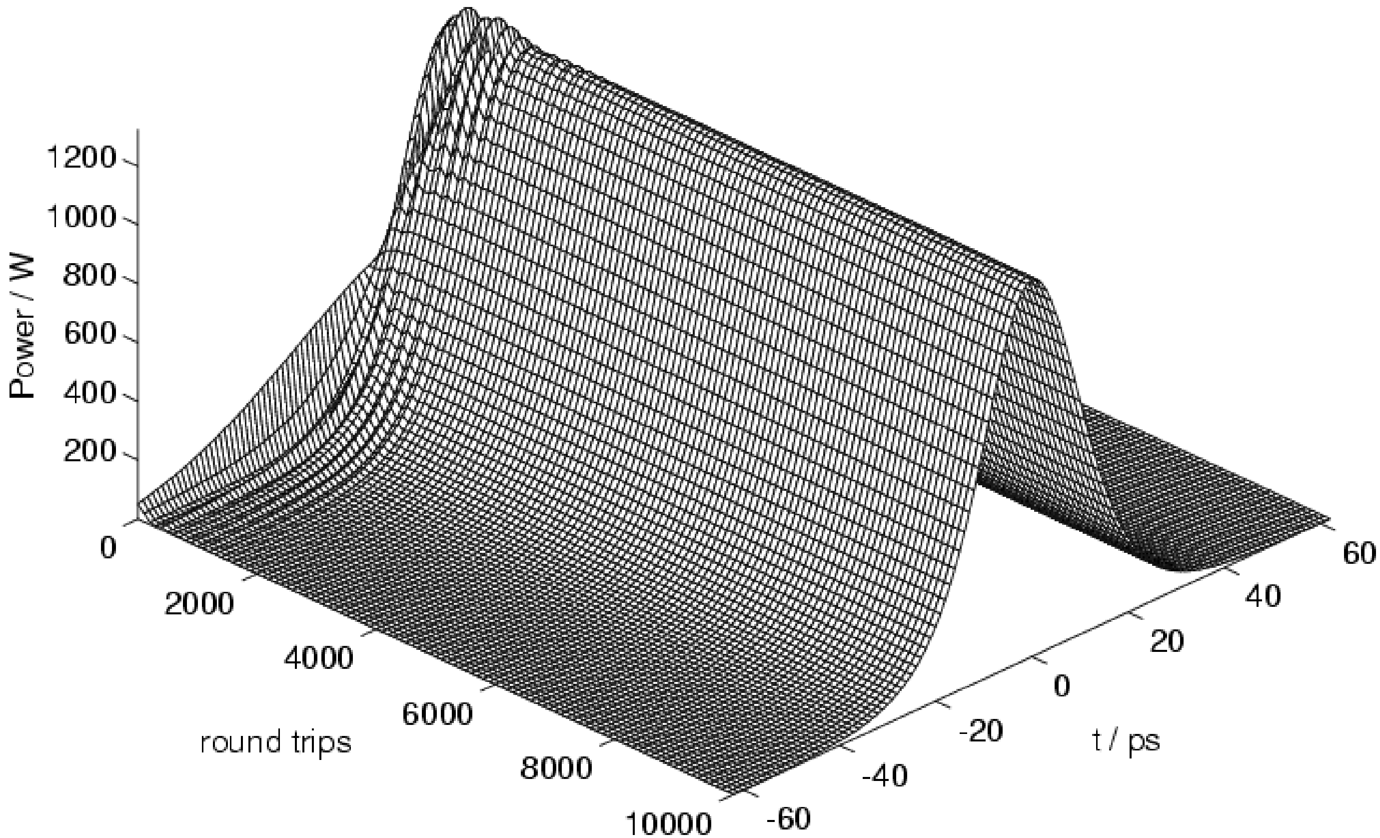


Fig. 5.11: 10000 roundtrips

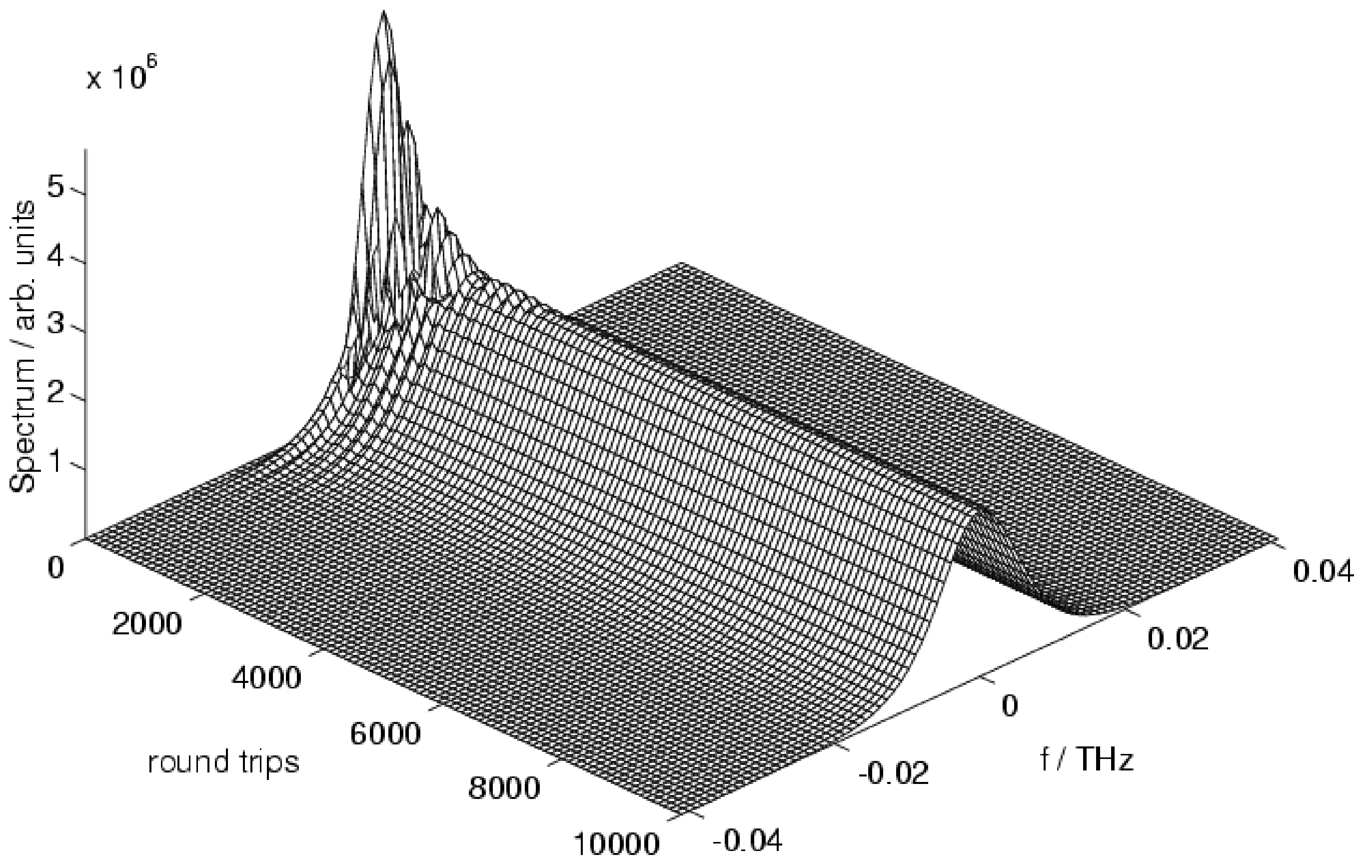


Fig. 5.11: 10000 roundtrips

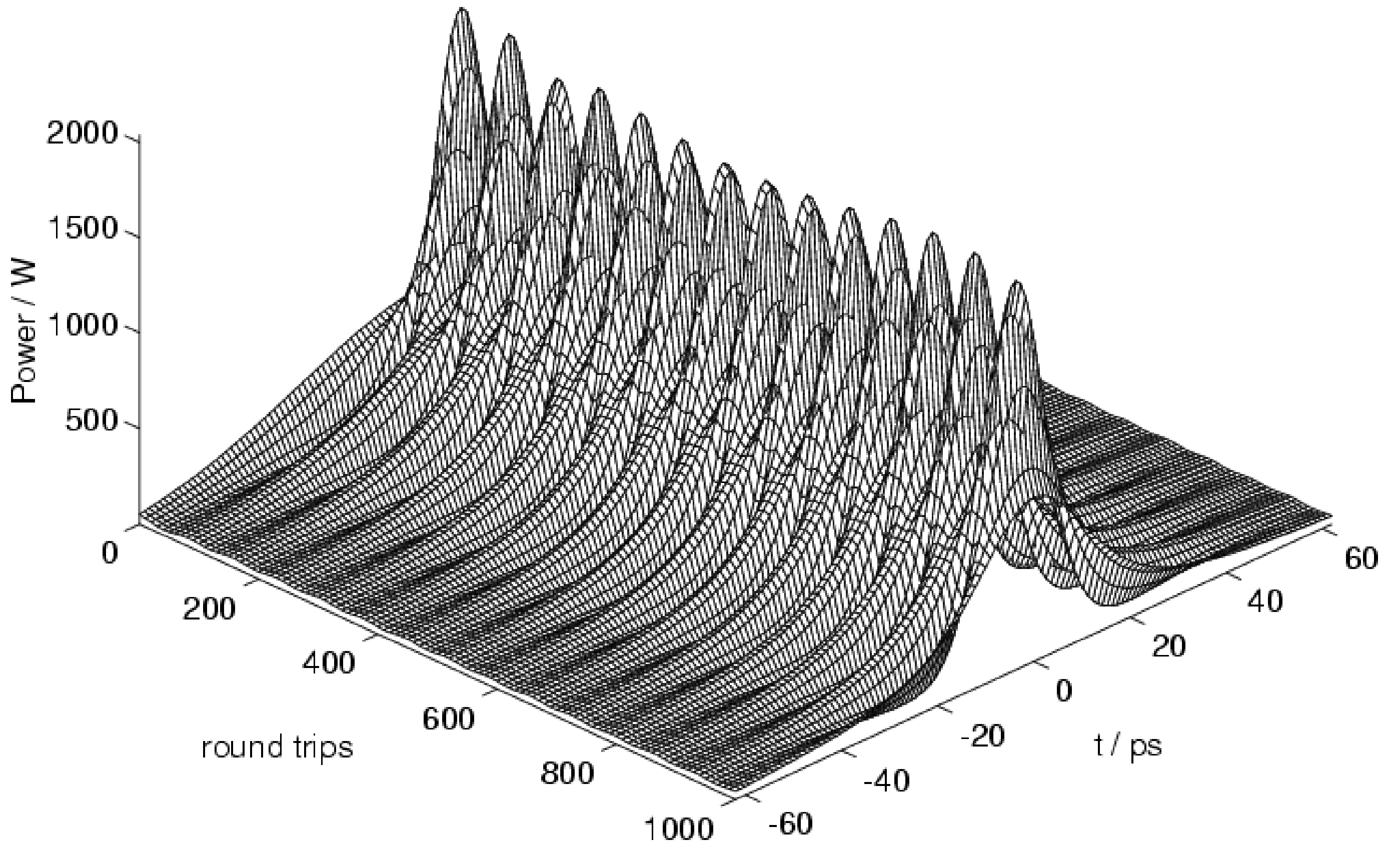


Fig. 5.12a: D = -10ps^2

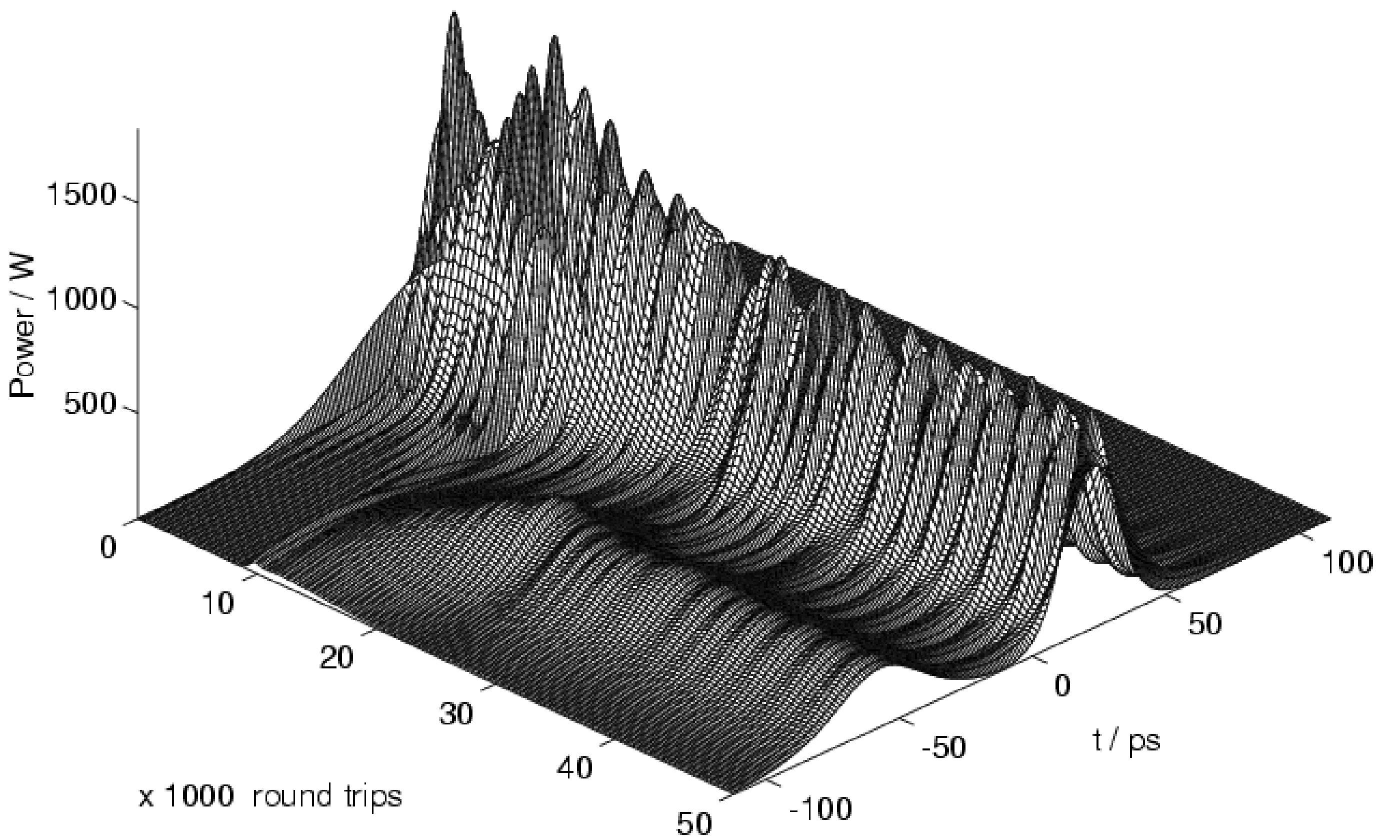


Fig. 5.12 b: 50000 roundtrips

5.6 Active Modelocking with Detuning

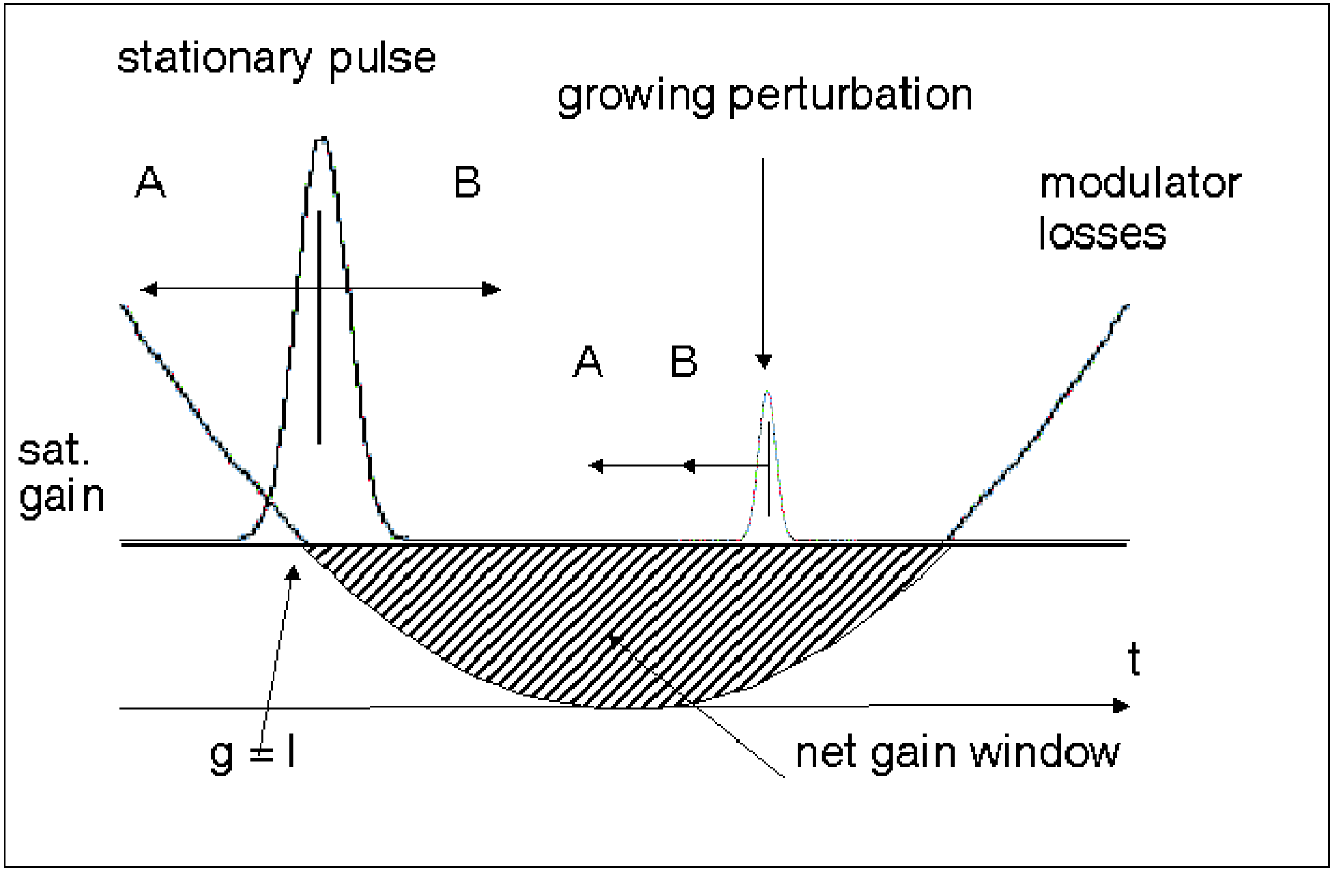


Fig. 5.15: Drifting pulse dynamics in a detuned laser

Normalized detuning: $$\Delta = \frac{1}{2\sqrt{2D_f M_s}}\frac{T_d}{\tau_a}.$$

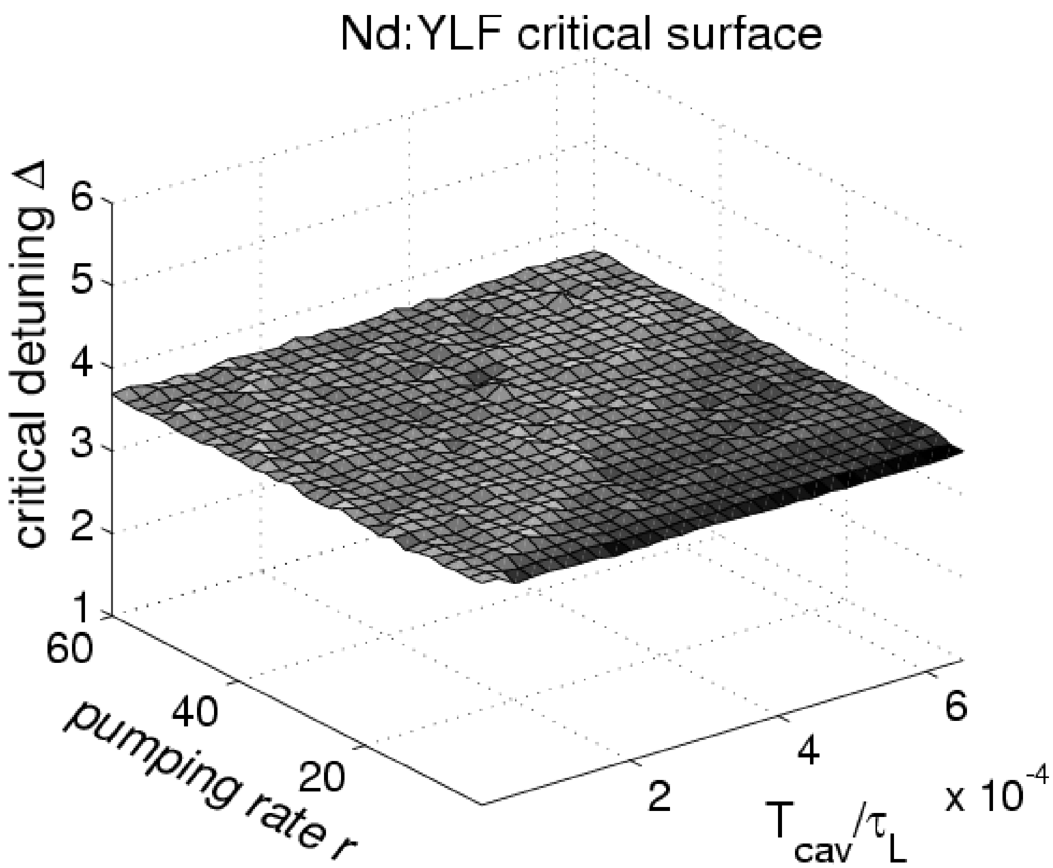


Fig. 5.25: Critical detuning ~ 3.65

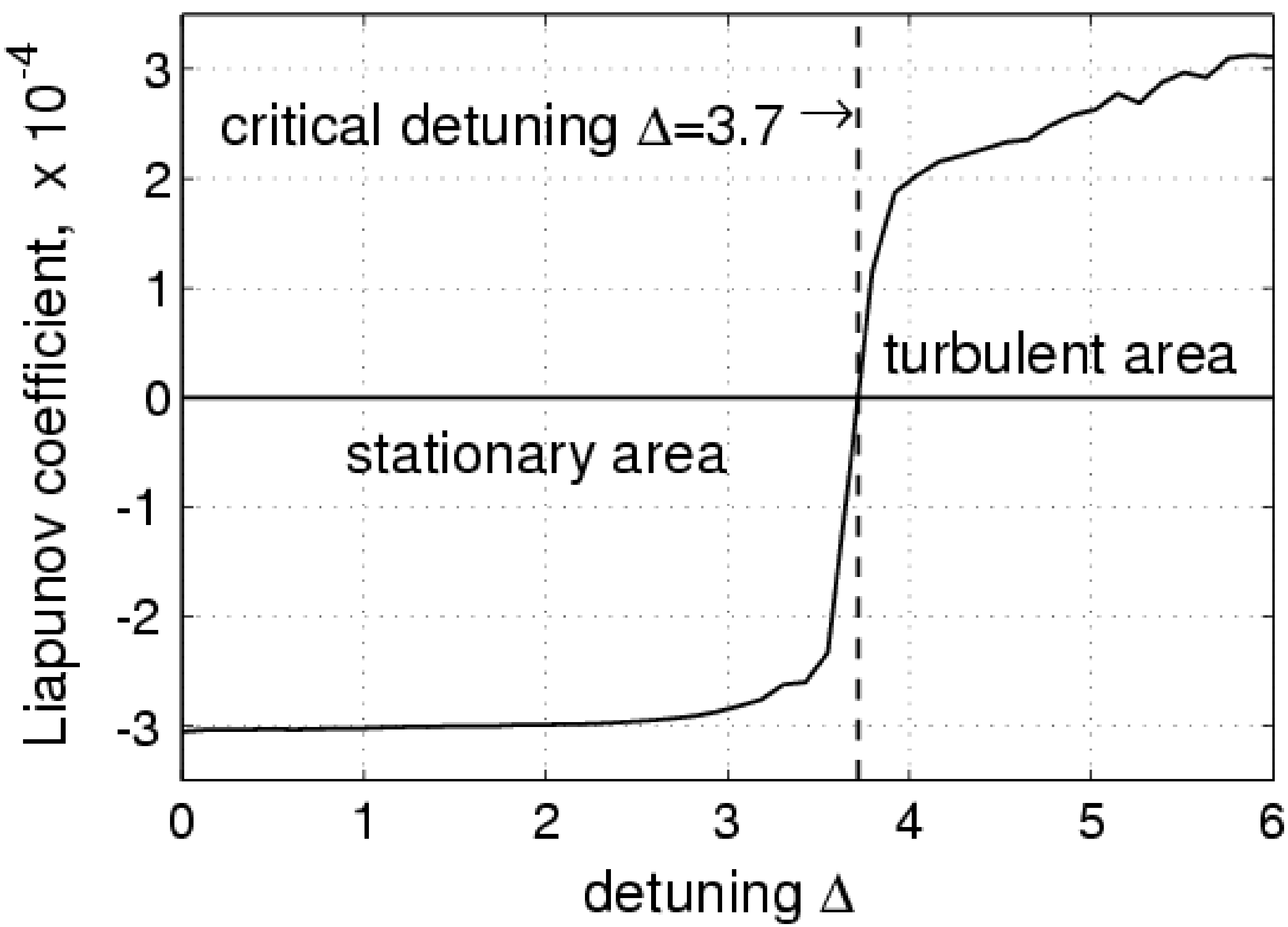


Fig. 5.26: Liapunov coefficient over normalized detuning