UFS Lecture 10: Laser Dynamics

Laser CW operation continued

Stability and relaxation oscillations

Q-switching: active and passive,
first high energy short pulse generation

Steady state operation: output power vs small signal gain

Beyond the gain threshold, some time after the buildup phase, the laser reaches steady state. Neglecting the spontaneous emission, saturated gain and steady state power can be calculated:

$$\frac{d}{dt}g = -\frac{g-g_0}{\tau_L} - \frac{gP}{E_{sat}}$$

$$\frac{d}{dt}P = -\frac{1}{\tau_p}P + \frac{2g}{T_R}(P + P_{vac})$$

$$\frac{d}{dt}g = 0$$

$$\frac{d}{dt}P = 0$$

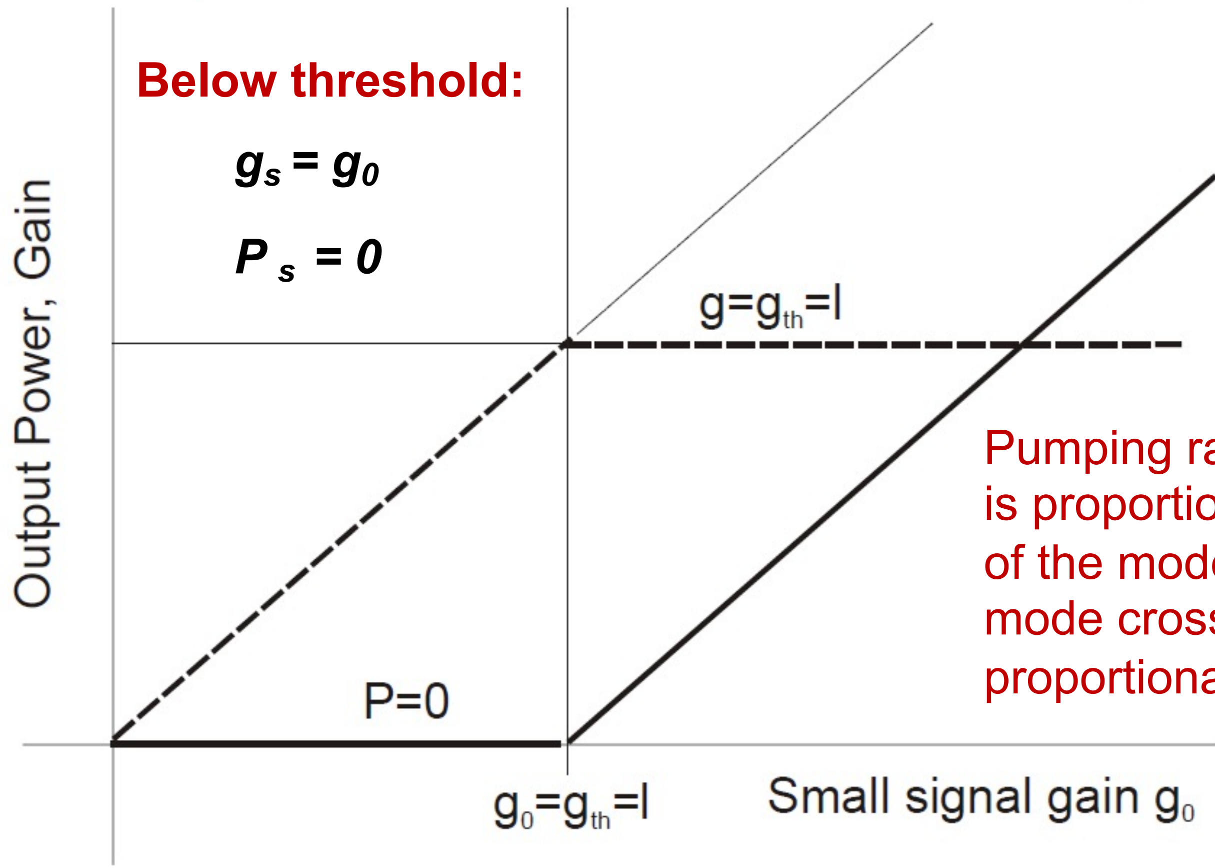
$$\rightarrow \quad g_s = \frac{g_0}{1+\frac{P_s}{P_{sat}}} = l$$

$$P_s = P_{sat}\left(\frac{g_0}{l} - 1\right)$$

Below threshold:

$\boldsymbol{g_s = g_0}$

$\boldsymbol{P_s = 0}$



$$g_{th} = l,$$

$$R_{p,th} = \frac{2l A_{eff}}{2^* \sigma \tau_L}$$

Pumping rate to reach threshold is proportional to the optical loss of the mode per roundtrip, the mode cross section, and inversely proportional to the $\sigma \cdot \tau_L$.

Spectroscopic parameters of selected laser materials

Laser Medium	Wave-length $\lambda_0 (nm)$	Cross Section σ (cm^2)	Upper-St. Lifetime τ_L (μs)	Linewidth Δf_{FWHM} $\frac{2}{T_2}$(THz)	Typ	Refr. index n
Nd^{3+}:YAG	1,064	$4.1 \cdot 10^{-19}$	1,200	0.210	H	1.82
Nd^{3+}:LSB	1,062	$1.3 \cdot 10^{-19}$	87	1.2	H	1.47
Nd^{3+}:YLF	1,047	$1.8 \cdot 10^{-19}$	450	0.390	H	1.82
Nd^{3+}:YVO_4	1,064	$2.5 \cdot 10^{-19}$	50	0.300	H	2.19
Nd^{3+}:glass	1,054	$4 \cdot 10^{-20}$	350	3	H/I	1.5
Er^{3+}:glass	1,55	$6 \cdot 10^{-21}$	10,000	4	H/I	1.46
Ruby	694.3	$2 \cdot 10^{-20}$	1,000	0.06	H	1.76
Ti^{3+}:Al_2O_3	660-1180	$3 \cdot 10^{-19}$	3	100	H	1.76
Cr^{3+}:LiSAF	760-960	$4.8 \cdot 10^{-20}$	67	80	H	1.4
Cr^{3+}:LiCAF	710-840	$1.3 \cdot 10^{-20}$	170	65	H	1.4
Cr^{3+}:LiSGAF	740-930	$3.3 \cdot 10^{-20}$	88	80	H	1.4
He-Ne	632.8	$1 \cdot 10^{-13}$	0.7	0.0015	I	$\sim$1
Ar^+	515	$3 \cdot 10^{-12}$	0.07	0.0035	I	$\sim$1
CO_2	10,600	$3 \cdot 10^{-18}$	2,900,000	0.000060	H	$\sim$1
Rhodamin-6G	560-640	$3 \cdot 10^{-16}$	0.0033	5	H	1.33
semiconductors	450-30,000	$\sim 10^{-14}$	$\sim$ 0.002	25	H/I	3 - 4

General description of laser operation

- Pumping begins when a laser is turned on. Then population inversion growth until it eventually reaches a steady-state value.

- This steady-state population inversion is determined by the pumping rate and the $\sigma\tau_L$ - product

- This steady-state population inversion corresponds to the small signal gain g_0.

- As the gain exceeds the cavity losses, the laser intra-cavity power begins to grow until it eventually reaches the saturation power and begins to extract energy from the medium.

- As the intra-cavity power grows, stimulated emission reduces the population inversion, and consequently the inversion reaches a new, lower steady-state value such that the reduced gain equals the losses in the cavity: $$g_s = \frac{g_0}{1+\frac{P_s}{P_{sat}}} = l$$

Stability and relaxation oscillations

How does the laser reach steady state, once a perturbation occurs?

$$\begin{aligned} P &= P_s + \Delta P \\ g &= g_s + \Delta g \end{aligned} \Rightarrow \begin{aligned} \frac{d\Delta P}{dt} &= +2\frac{P_s}{T_R}\Delta g \\ \frac{d\Delta g}{dt} &= -\frac{g_s}{E_{sat}}\Delta P - \frac{1}{\tau_{stim}}\Delta g \end{aligned}$$

Stimulated lifetime

$$\frac{1}{\tau_{stim}} = \frac{1}{\tau_L}\left(1 + \frac{P_s}{Psat}\right)$$

The perturbations decay or grow like

$$\begin{pmatrix} \Delta P \\ \Delta g \end{pmatrix} = \begin{pmatrix} \Delta P_0 \\ \Delta g_0 \end{pmatrix} e^{st} \Rightarrow A \begin{pmatrix} \Delta P_0 \\ \Delta g_0 \end{pmatrix} = \begin{pmatrix} -s & 2\frac{P_s}{T_R} \\ -\frac{T_R}{E_{sat} 2\tau_p} & -\frac{1}{\tau_{stim}} - s \end{pmatrix} \begin{pmatrix} \Delta P_0 \\ \Delta g_0 \end{pmatrix} = 0$$

Non-zero solution exists only if the determinant of the coefficient matrix is 0:

$$s\left(\frac{1}{\tau_{stim}} + s\right) + \frac{P_s}{E_{sat}\tau_p} = 0$$

$$\boxed{s_{1/2} = -\frac{1}{2\tau_{stim}} \pm \sqrt{\left(\frac{1}{2\tau_{stim}}\right)^2 - \frac{P_s}{E_{sat}\tau_p}}}$$

Stability and relaxation oscillations

Introducing the pump parameter $r = 1 + \frac{P_s}{P_{sat}} = \frac{g_0}{l}$, which tells us how much we pump the laser over threshold, the eigen frequencies can be rewritten as

$$\boxed{s_{1/2} = -\frac{1}{2\tau_{stim}}\left(1 \pm j\sqrt{\frac{4(r-1)}{r}\frac{\tau_{stim}}{\tau_p} - 1}\right) = -\frac{r}{2\tau_L} \pm j\sqrt{\frac{(r-1)}{\tau_L\tau_p} - \left(\frac{r}{2\tau_L}\right)^2}}$$

(i): The stationary state $(0, g_0)$ for $g_0 < l$ and (P_s, g_s) for $g_0 > l$ are always stable, i.e. $\mathrm{Re}\{s_i\} < 0$.

(ii): For lasers pumped above threshold, $r > 1$, and long upper state lifetimes, i.e. $\frac{r}{4\tau_L} < \frac{1}{\tau_p}$, the relaxation rate becomes complex, i.e. there are relaxation oscillations

$$s_{1/2} = -\frac{1}{2\tau_{stim}} \pm j\omega_R \qquad \omega_R = \sqrt{\frac{1}{\tau_{stim}\tau_p}} \qquad \tau_{stim} = \frac{\tau_L}{r}$$

If the laser can be pumped strong enough, i.e. r can be made large enough so that the stimulated lifetime becomes as short as the cavity decay time, relaxation oscillations vanish.

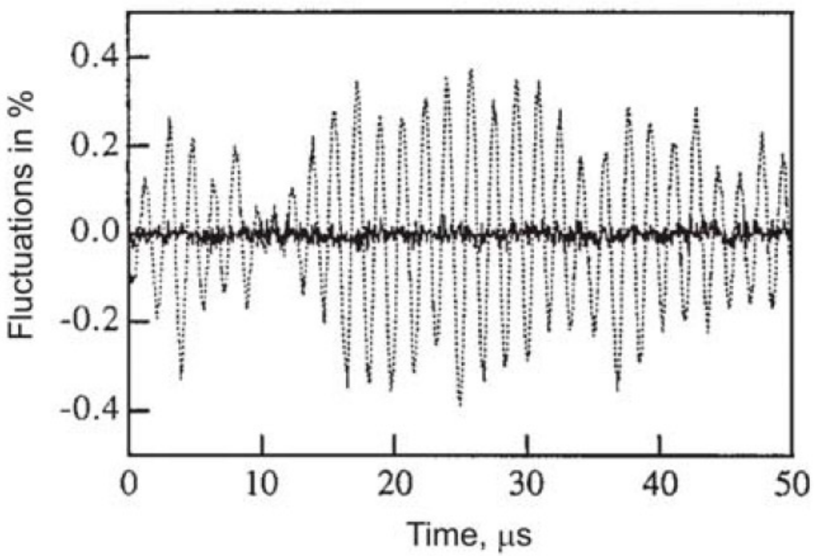
Relaxation oscillations: a case study

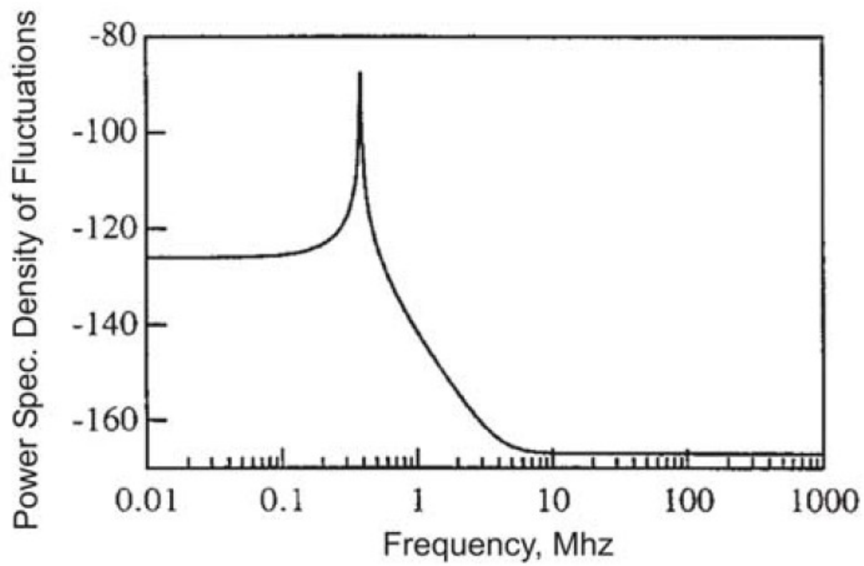
Diode-pumped Nd:YAG-Laser:

$$\lambda_0 = 1064\ \mathrm{nm},\ \sigma = 4 \cdot 10^{-20} cm^2,\ A_{eff} = \pi\,(100\mu m \times 150\mu m)$$
$$r = 50 \quad \tau_L = 1.2\ ms,\ l = 1\%,\ T_R = 10ns$$

$$I_{sat} = \frac{hf_L}{\sigma \tau_L} = 3.9\frac{kW}{cm^2},\ P_s = 91.5W$$
$$P_{sat} = I_{sat} A_{eff} = 1.8\ W,$$
$$\tau_{stim} = \frac{\tau_L}{r} = 24\mu s,\ \tau_p = 1\mu s$$
$$\omega_R = \sqrt{\frac{1}{\tau_{stim}\tau_p}} = 2 \cdot 10^5 s^{-1}$$

The physical reason for relaxation oscillations and later instabilities is, that the gain reacts too slow on the light field, i.e. the stimulated lifetime is long in comparison with the cavity decay time.





Typically observed relaxation oscillations in time and frequency domain.

Laser efficiency: how much pump power converted to laser output power

Steady-state intracavity power:

$$P_s = P_{sat}\left(\frac{2g_0}{2l} - 1\right)$$

$$2g_0 = 2^* \frac{R_p}{A_{eff}}\sigma\tau_L,$$

$$P_{sat} = \frac{hf_L}{2^*\sigma\tau_L}A_{eff}$$

Laser power losses include the internal losses $2l_{int}$ and the transmission T through the output coupling mirror: $2l = 2l_{int} + T$

Pump photon energy

Laser output power: $P_{out} = T \cdot P_{sat}\left(\frac{2g_0}{2l_{int}+T} - 1\right)$ Pump power: $P_p = R_p h f_P$

Efficiency: $\eta = \frac{P_{out}}{P_p}$ **Differential Efficiency:** $\eta_D = \frac{\partial P_{out}}{\partial P_p}$

If the laser is pumped many times over threshold: $r = 2g_0/2l \rightarrow \infty$

$$\eta_D = \eta = \frac{T}{2l_{int}+T}P_{sat}\frac{2^*}{A_{eff}hf_P}\sigma\tau_L = \frac{T}{2l_{int}+T}\cdot\frac{hf_L}{hf_P}$$

Laser efficiency is fundamentally limited by the ratio of output coupling to total losses and the quantum defect in pumping.

How to efficiently use the energy storage capability?

- Typical cavity length of a solid-state laser: 0.1-10 m
- Cavity round-trip time: ~1-100 ns
- Photon lifetime in the cavity $\tau_p = T_R / 2l$: ~0.1-1 us
- Upper level lifetime of solid-state gain materials: 10-1000 us

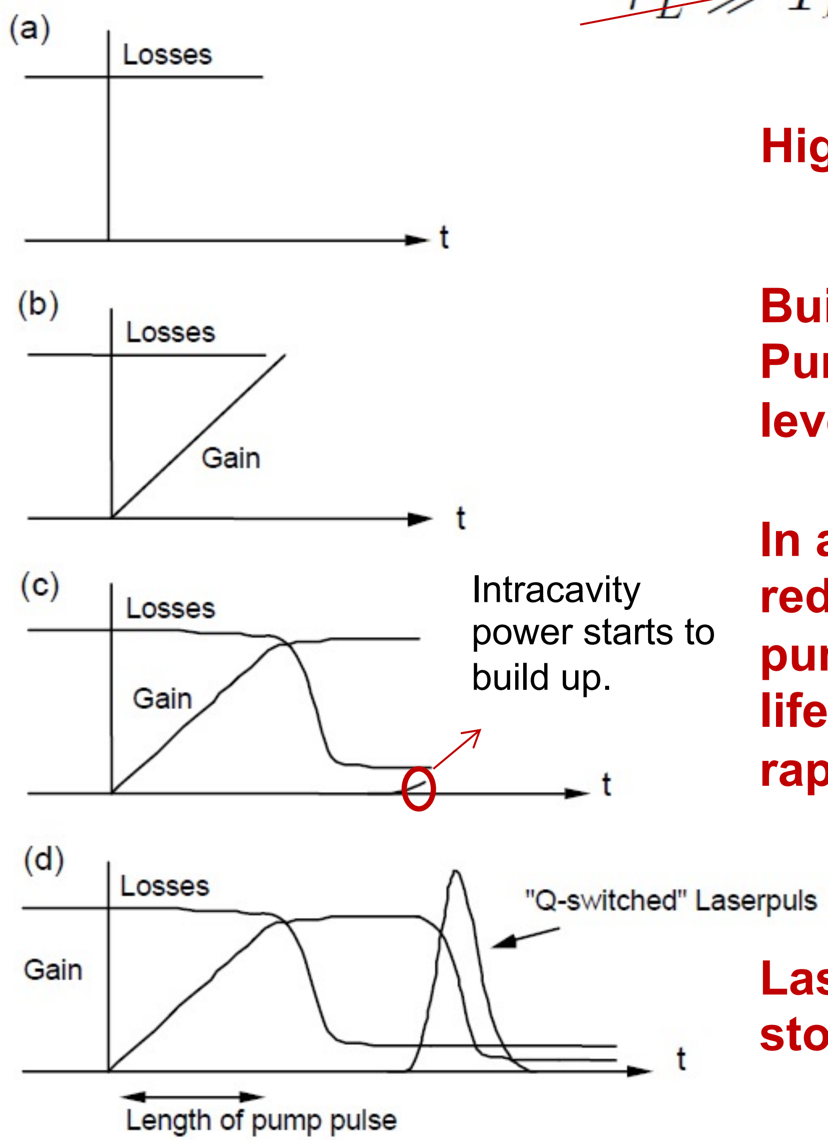
$$\tau_L >> \tau_p >> T_R$$

- Given pumping rate, τ_L time is needed such that the full energy storage capability is reached.
- However, the intra-cavity power starts to build up as the gain exceeds loss and it grows much faster such that it reaches the saturation power and starts to saturate population inversion. So the full energy storage capability cannot be reached.

How to solve this dilemma?

Q-Switching: manage cavity loss to obtain giant pulses

$$\cancel{\tau_L \gg T_R \gg \tau_p} \quad \tau_L >> \tau_p >> T_R \qquad (4.137)$$



High losses, laser is below threshold.

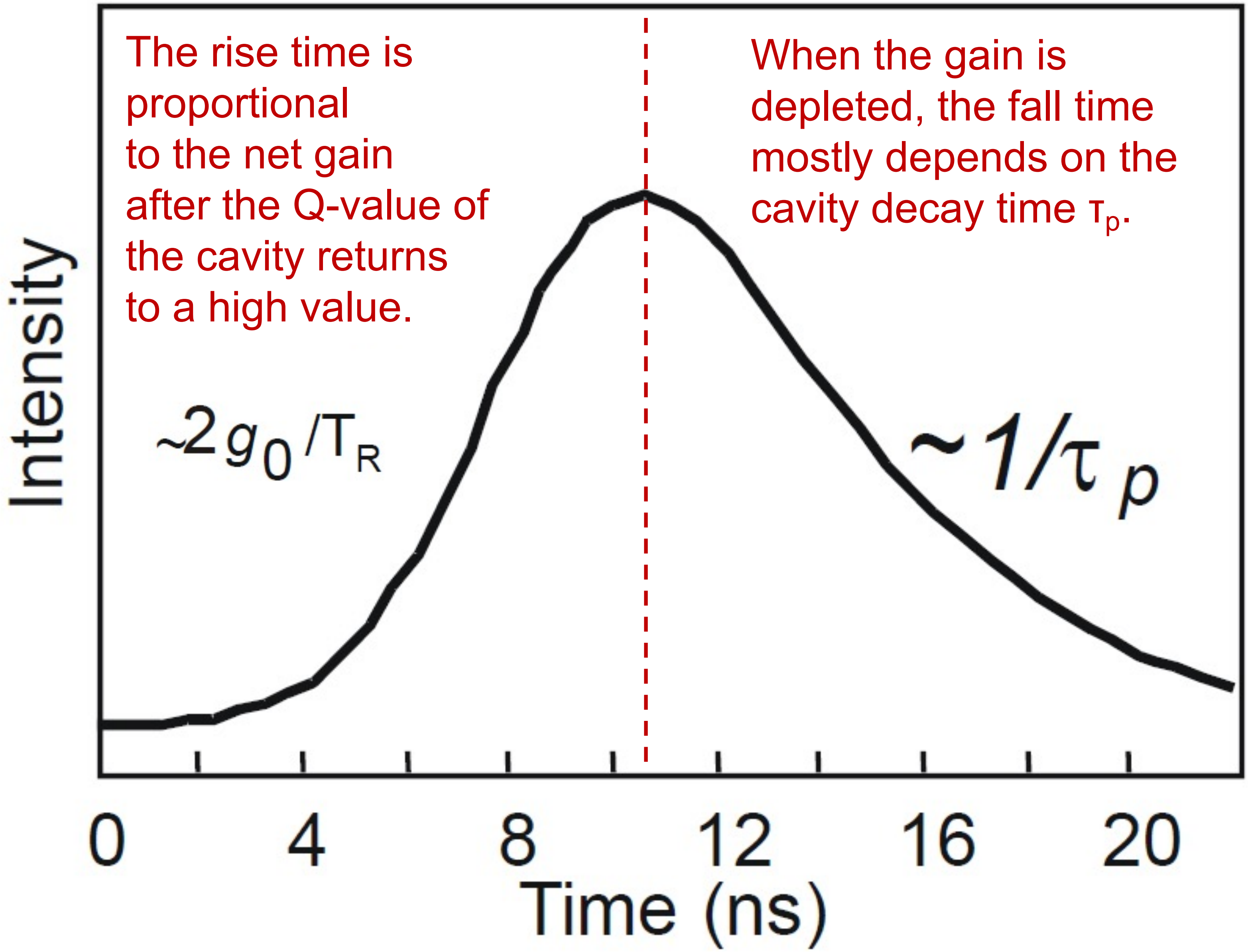
Build-up of inversion by pumping. Pump population to upper laser level until gain approaches loss.

In active Q-switching, the losses are reduced, after the laser medium is pumped for as long as the upper state lifetime. Then the loss is reduced rapidly and laser oscillation starts.

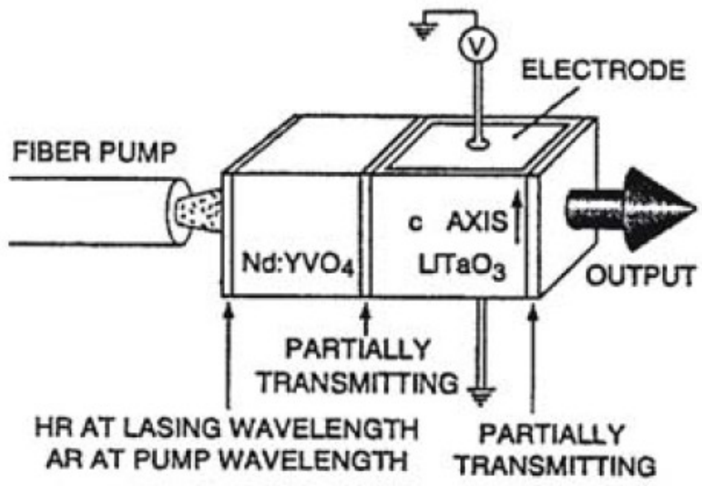
Laser emission stops after the energy stored in the gain medium is extracted.

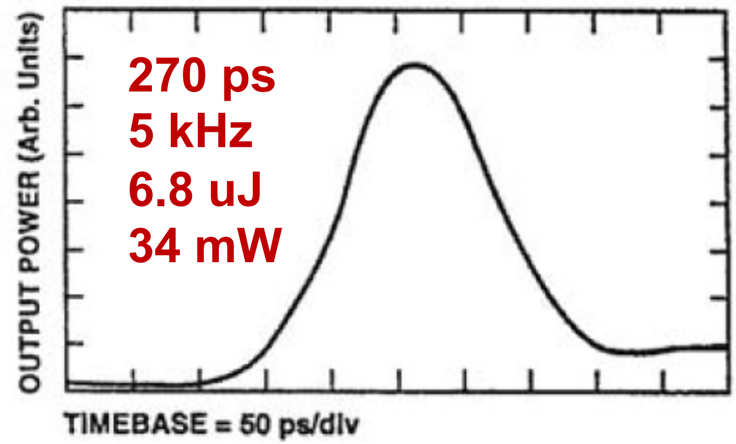
Gain and Loss Dynamics of Q-switched Laser

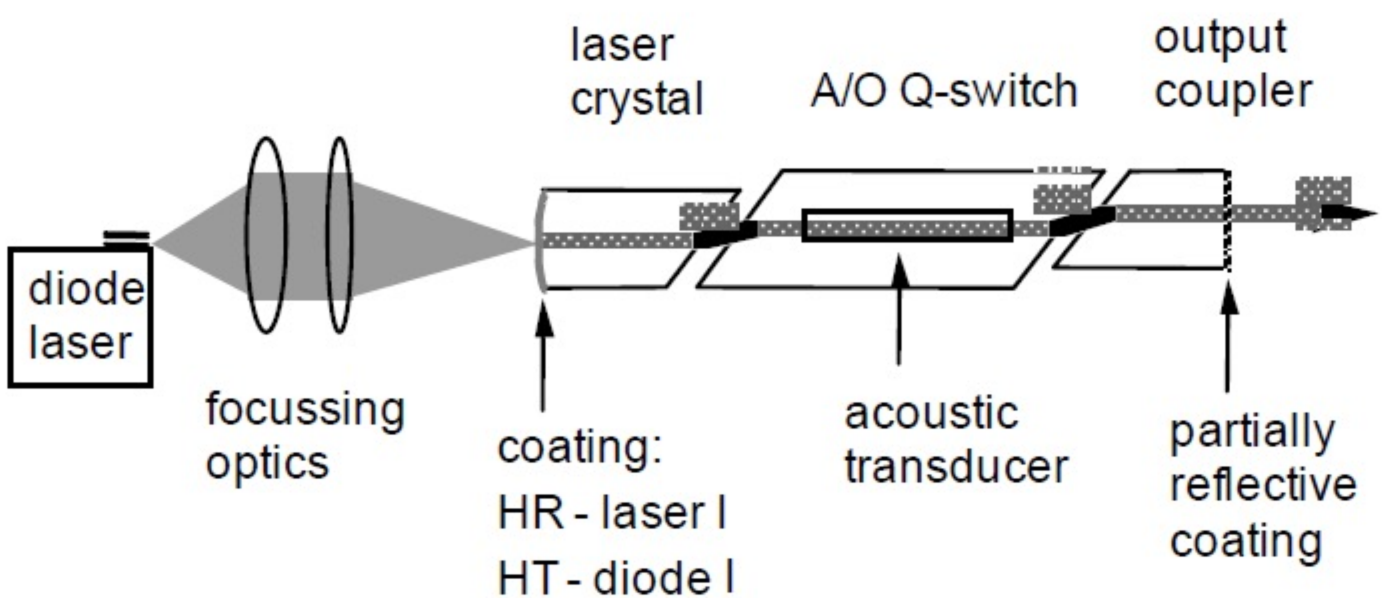
Asymmetric actively Q-switched pulse



Active Q-switching lasers: EO switch and AO switch







4.6.2 Single-Frequency Q-switched Laser

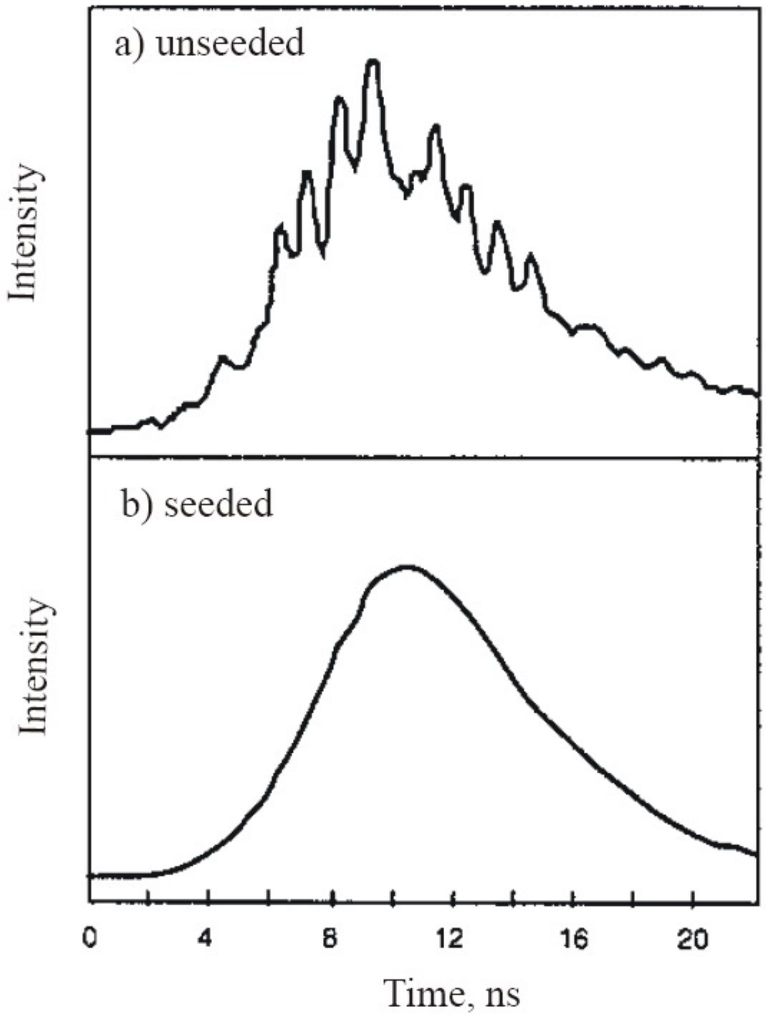


Figure 4.15: Output intensity of a Q-switched laser without a) and with seeding b).

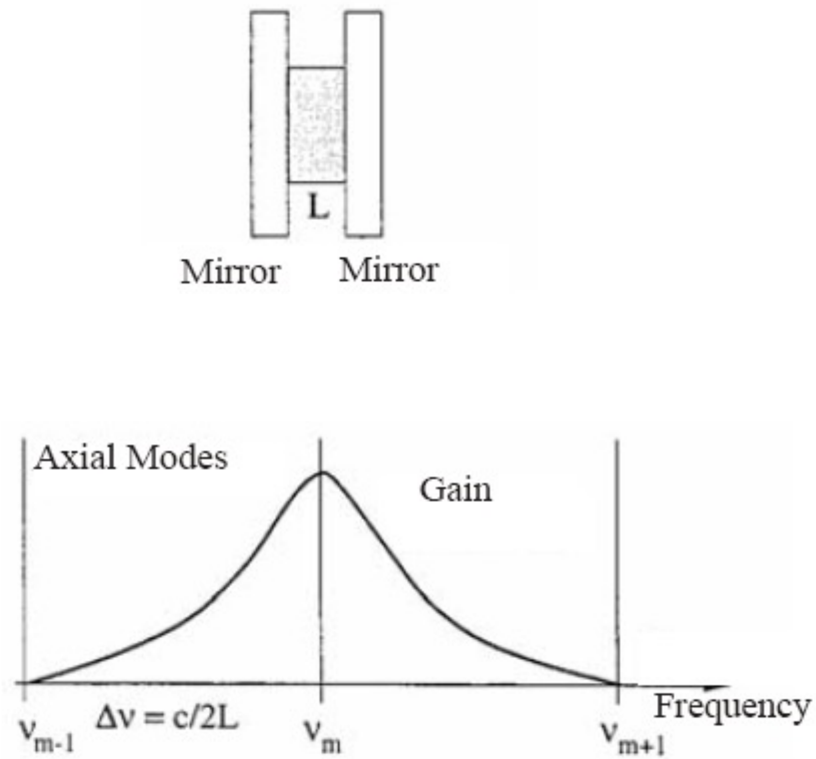
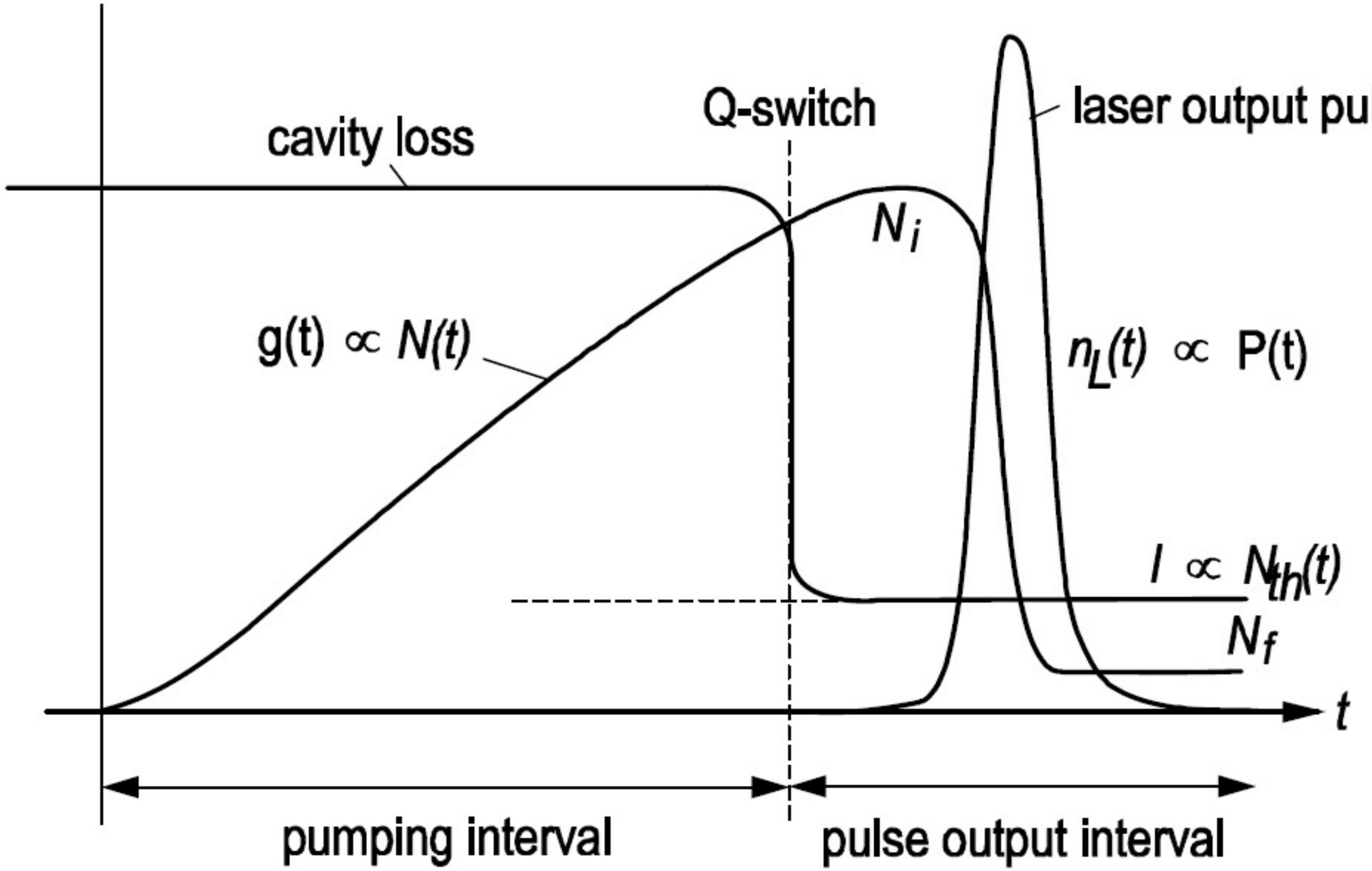


Figure 4.16: In a microchip laser the resonator can be so short, only one longitudinal mode within the gain bandwidth.

Theory on active Q-switching



Active Q-switching dynamics assuming an instantaneous switching

Theory on active Q-switching

Rate equations:

$$\frac{d}{dt}g = -\frac{g-g_0}{\tau_L} - \frac{gP}{E_{sat}} \qquad \frac{d}{dt}P = -\frac{1}{\tau_p}P + \frac{2g}{T_R}(P+P_{vac})$$

Pump interval with constant R_p:

$$\frac{d}{dt}g = -\frac{g-g_0}{\tau_L} \longrightarrow g(t) = g_0(1-e^{-t/\tau_L})$$

Pulse built-up phase:

Index is left away since there is only an upper state population.

From Eqn 4.101 and 4.105, we know the initial gain:

$$g_i = hf_L N_{2i}/(2E_{sat}) = hf_L N_i/(2E_{sat})$$

Assume that during pulse buildup, stimulated emission rate is the dominant term changing the inversion:

$$\frac{d}{dt}g = -\frac{gP}{E_{sat}}$$

$$\frac{d}{dt}P = \frac{2(g-l)}{T_R}P \longrightarrow \frac{dP}{dg} = \frac{2E_{sat}}{T_R}\left(\frac{l}{g}-1\right)$$

Theory on active Q-switching

Initial conditions: $g(t = 0) = g_i = r \cdot l$ $\quad P(t=0) = 0$

How many times the laser is pumped above the threshold after the Q-switch is operated.

Intra-cavity loss after the Q-switch is operated.

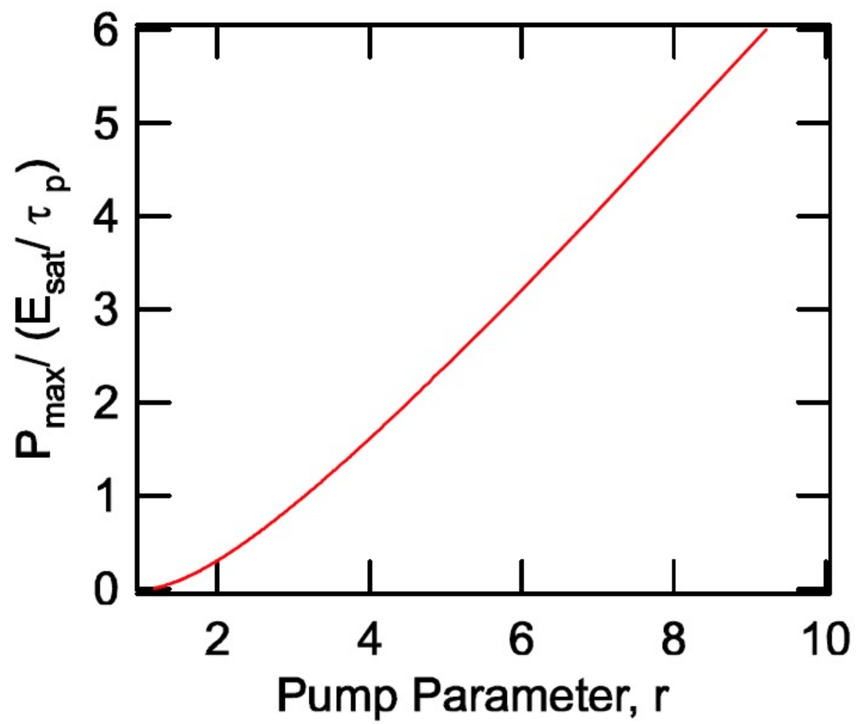
Intra-cavity power evolution:

$$P(t) = \frac{2E_{sat}}{T_R}\left(g_i - g(t) + l \ln \frac{g(t)}{g_i}\right)$$

Maximum power as gain equals loss:

$$P_{\max} = \frac{2lE_{sat}}{T_R}(r - 1 - \ln r)$$

$$= \frac{E_{sat}}{\tau_p}(r - 1 - \ln r)$$



Energy extraction

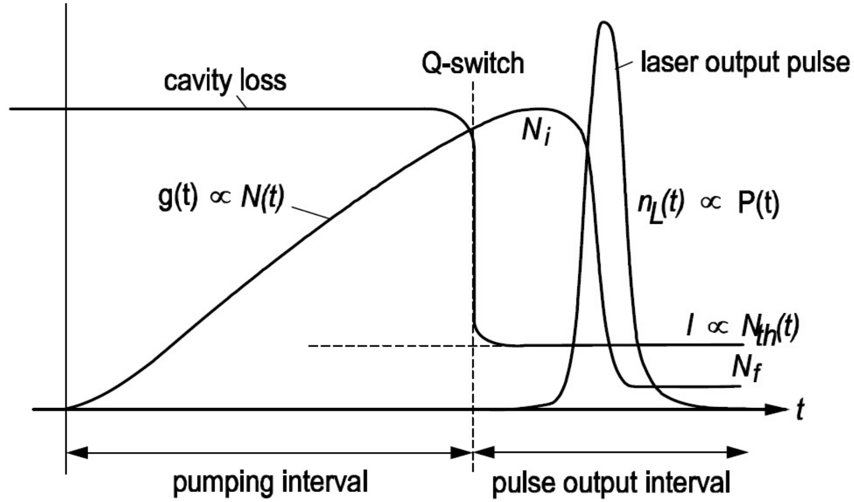
Final gain when power vanishes:

$$\left(g_i - g_f + l \ln\left(\frac{g_f}{g_i}\right)\right) = 0 \qquad r = g_i/l$$

$$\downarrow$$

$$1 - \frac{g_f}{g_i} + \frac{1}{r}\ln\left(\frac{g_f}{g_i}\right) = 0$$

$$1 - \frac{N_f}{N_i} + \frac{1}{r}\ln\left(\frac{N_f}{N_i}\right) = 0$$



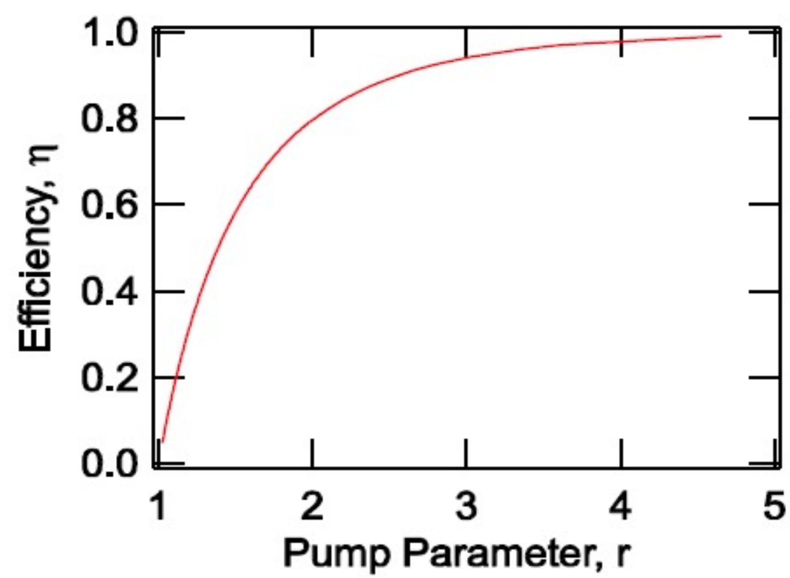
Assuming no internal losses, the pulse energy is: $E_P = (N_i - N_f)\,hf_L$

Energy extraction efficiency:

$$\eta = \frac{N_i - N_f}{N_i}$$

$$\eta + \frac{1}{r}\ln(1-\eta) = 0$$

Energy extraction efficiency only depends on the pump parameter r.



Estimate of pulse width

We can estimate the pulse width of the emitted pulse by the ratio between pulse energy and peak power.

Emitted pulse energy can be written as $E_P = \eta(r) N_i h f_L$

Emitted pulse peak power: $P_{\max} = \frac{E_{sat}}{\tau_p}(r - 1 - \ln r)$

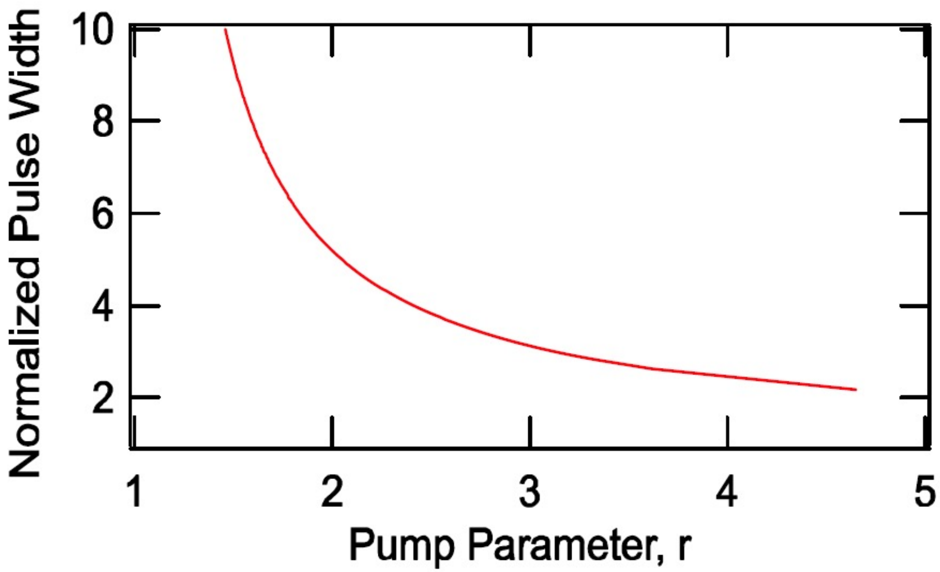
$$\tau_{Pulse} = \frac{E_P}{2l P_{peak}}$$

$$= \tau_p \frac{\eta(r)}{(r-1-\ln r)} \frac{N_i h f_L}{2l E_{sat}}$$

$$\boxed{g_i = h f_L N_i / (2E_{sat})}$$

$$= \tau_p \frac{\eta(r)}{(r-1-\ln r)} \frac{g_i}{l}$$

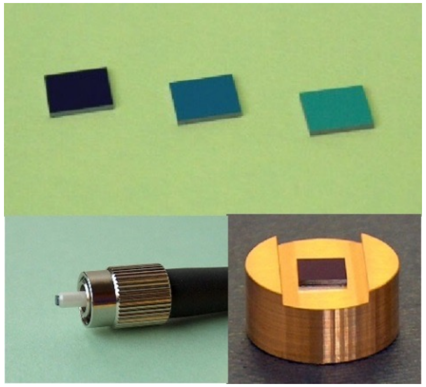
$$= \tau_p \frac{\eta(r) \cdot r}{(r-1-\ln r)}$$

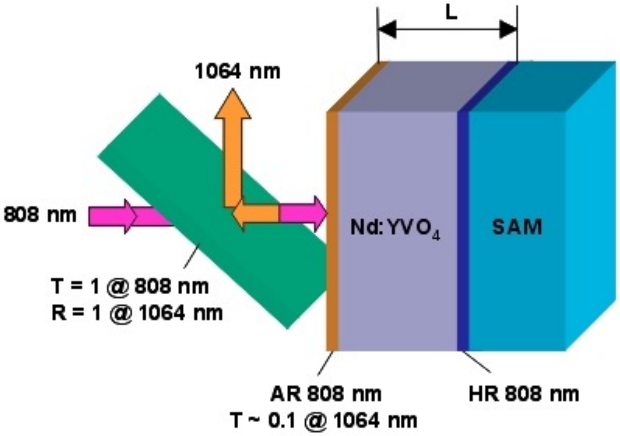


Passive Q-Switching: manage cavity loss using saturable absorber

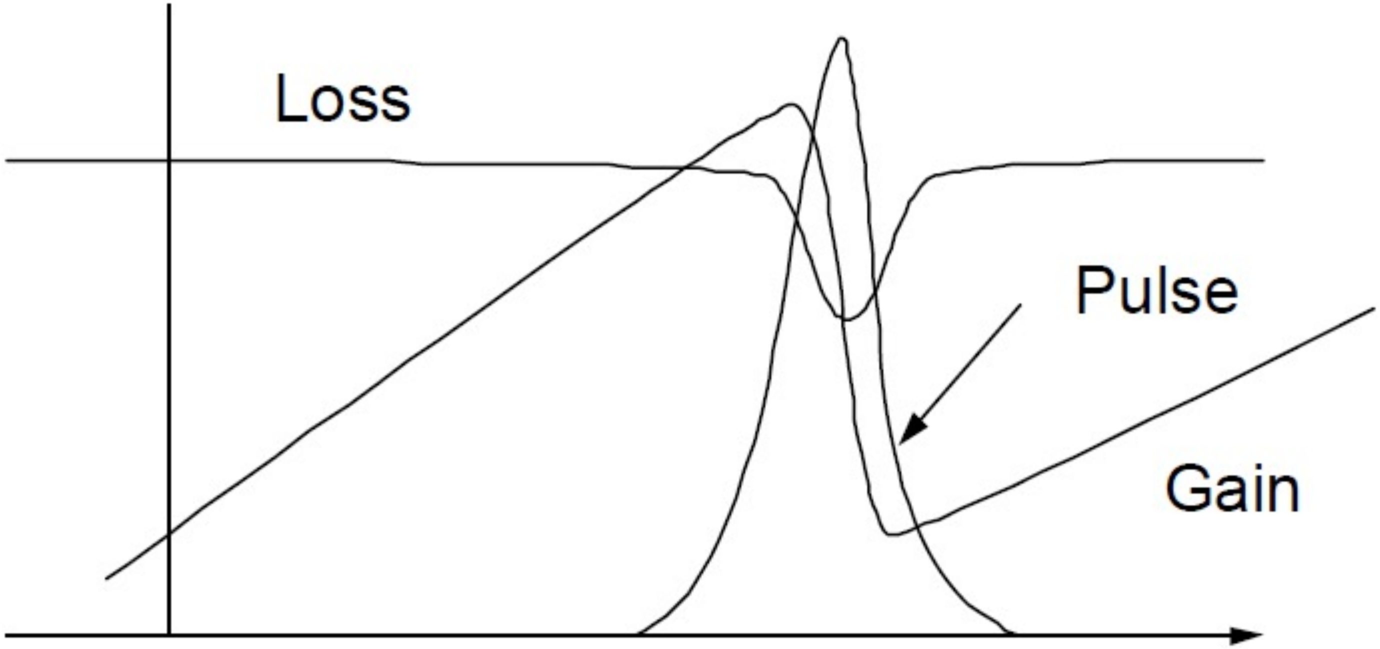
Saturable absorber: an optical passive device, which introduces large loss for low optical intensities and small loss at high optical intensities.

$$T_R \frac{dq}{dt} = -\frac{q - q_0}{T_A} - \frac{q T_R P}{E_A}$$

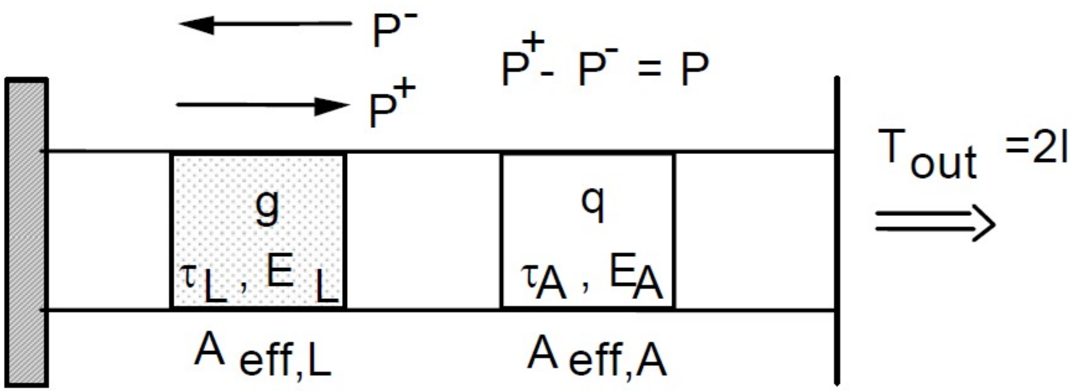




http://www.batop.de/products/products.html



Modeling of passively Q-switched lasers



We assume small output coupling so that the laser power within one roundtrip can be considered position independent.

Rate equations for a passively Q-switched laser

Assume that the changes in the laser intensity, gain and saturable absorption are small on a time scale on the order of the round-trip time T_R in the cavity, (i.e. less than 20%).

$$T_R \frac{dP}{dt} = 2(g - l - q)P$$

$$T_R \frac{dg}{dt} = -\frac{g - g_0}{T_L} - \frac{g T_R P}{E_L}$$

$$T_R \frac{dq}{dt} = -\frac{q - q_0}{T_A} - \frac{q T_R P}{E_A}$$

Normalized upper-state lifetime of the gain medium and the absorber recovery time

$$T_L = \tau_L / T_R$$
$$T_A = \tau_A / T_R$$

Saturation energies of the gain and the absorber

$$E_L = h\nu A_{eff,L} / 2^* \sigma_L$$
$$E_A = h\nu A_{eff,A} / 2^* \sigma_A$$

Passively Q-switched laser: fast saturable absorber

Typical solid-state lasers:	τ_L=100 μs	T_R=10 ns	τ_A=1-100 ps
	$T_L \approx 10^4$	$T_A \approx 10^{-4}$ to 10^{-2}	

Fast Saturable Absorber: $T_A << T_L$, the absorber will follow the instantaneous laser power:

$$T_R \frac{dq}{dt} = -\frac{q-q_0}{T_A} - \frac{qT_RP}{E_A} \xrightarrow{\text{Adiabatic solution}} q = \frac{q_0}{1+P/P_A} \quad \text{with} \quad P_A = \frac{E_A}{\tau_A}$$

saturation power

New equations of motion:

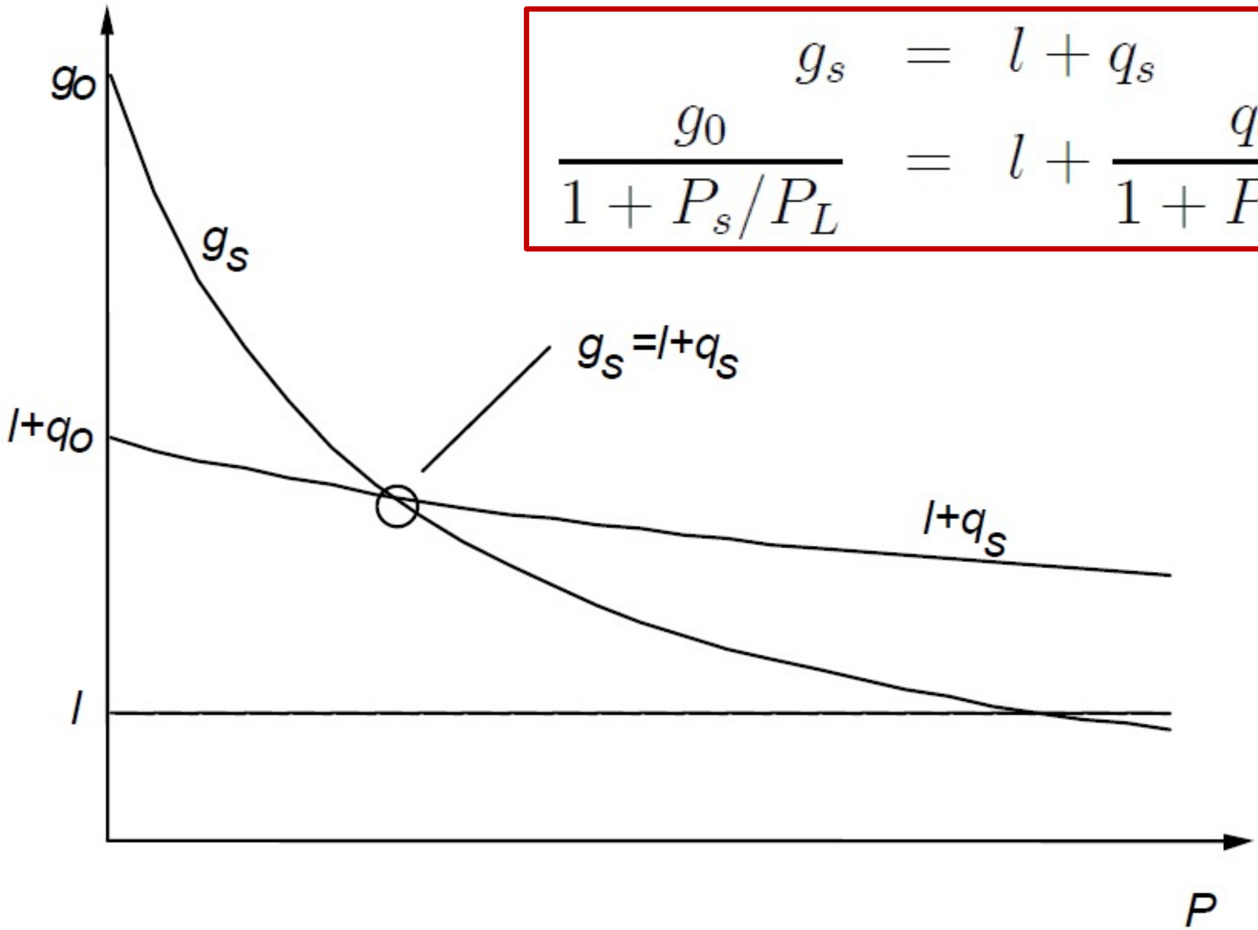
$$T_R \frac{dP}{dt} = 2(g-l-q(P))P$$

$$T_R \frac{dg}{dt} = -\frac{g-g_0}{T_L} - \frac{gT_RP}{E_L}$$

Passively Q-switched laser: stationary solution

As in the case for the cw-running laser the stationary operation point of the laser is determined by the point of zero net gain:

$$\begin{aligned} g_s &= l + q_s \\ \frac{g_0}{1 + P_s/P_L} &= l + \frac{q_0}{1 + P_s/P_A} \end{aligned}$$



Graphical solution of the stationary operating point

Stability of stationary operating point:

$$T_R \frac{d}{dt}\begin{pmatrix} \Delta P_0 \\ \Delta g_0 \end{pmatrix} = A \begin{pmatrix} \Delta P_0 \\ \Delta g_0 \end{pmatrix}, \quad \text{with } A = \begin{pmatrix} -2\left.\frac{dq}{dP}\right|_{cw} P_s & 2P_s \\ -\frac{g_s T_R}{E_L} & -\frac{T_R}{\tau_{stim}} \end{pmatrix}$$

Stationary operating point is stable if: Tr (A) < 0 and det (A) > 0

Tr (A) < 0: $$-2P\left.\frac{dq}{dP}\right|_{cw} < \frac{r}{T_L} \quad \text{with} \quad r = 1 + \frac{P}{P_L} \quad \text{and} \quad P_L = \frac{E_L}{\tau_L},$$

det (A) > 0: $$\left.\frac{dq}{dP}\right|_{cw} \frac{r}{T_L} + \cancel{2} g_s \frac{\cancel{r} - 1}{T_L P_L} > 0.$$

$$\left|\left.\frac{dq}{dP}\right|_{cw}\right| < \left|\left.\frac{dg_s}{dP}\right|_{cw}\right|$$

Always fulfilled for self-starting laser

Stability condition:

$$\boxed{-2T_L P\left.\frac{dq}{dP}\right|_{cw} = 2T_L q_0 \left.\frac{\frac{P}{\chi P_L}}{\left(1 + \frac{P}{\chi P_L}\right)^2}\right|_{cw} < r \quad \text{with} \quad \chi = \frac{P_A}{P_L}}$$

Stability of stationary operating point: Passive Q-switching

To find the stability criterion, we linearize the system just as we have done for laser CW operation:

$$T_R \frac{d}{dt} \begin{pmatrix} \Delta P_0 \\ \Delta g_0 \end{pmatrix} = A \begin{pmatrix} \Delta P_0 \\ \Delta g_0 \end{pmatrix}, \text{ with } A = \begin{pmatrix} -2 \left.\frac{dq}{dP}\right|_{cw} P_s & 2P_s \\ -\frac{g_s T_R}{E_L} & -\frac{T_R}{\tau_{stim}} \end{pmatrix}$$

We look for the eigen solution:

$$\frac{d}{dt} \begin{pmatrix} \Delta P_0(t) \\ \Delta g_0(t) \end{pmatrix} = s \begin{pmatrix} \Delta P_0(t) \\ \Delta g_0(t) \end{pmatrix}$$

$$\boxed{s = \frac{1}{2}\left(\gamma_Q - \frac{1}{\tau_{stim}}\right) \pm j\sqrt{\gamma_Q \frac{r}{\tau_L} + \frac{r-1}{\tau_p \tau_L} - \left(\frac{\gamma_Q - \frac{1}{\tau_{stim}}}{2}\right)^2}}$$

Growth rate introduced by the saturable absorber that destabilizes the laser relaxation oscillation:

$$\gamma_Q = -\frac{2}{T_R} \left.\frac{dq}{dP}\right|_{cw} P_s$$

Q-switching happens when

$$\gamma_Q > \frac{1}{\tau_{stim}}$$

$$s = \frac{1}{2}\left(\gamma_Q - \frac{1}{\tau_{stim}}\right) \pm j\sqrt{\gamma_Q \frac{r}{\tau_L} + \frac{r-1}{\tau_p \tau_L} - \left(\frac{\gamma_Q - \frac{1}{\tau_{stim}}}{2}\right)^2}.$$

$$\gamma_Q = -\frac{2}{T_R} \left.\frac{dq}{dP}\right|_{cw} P_s$$

$$r = g_0/(l + q_s)$$

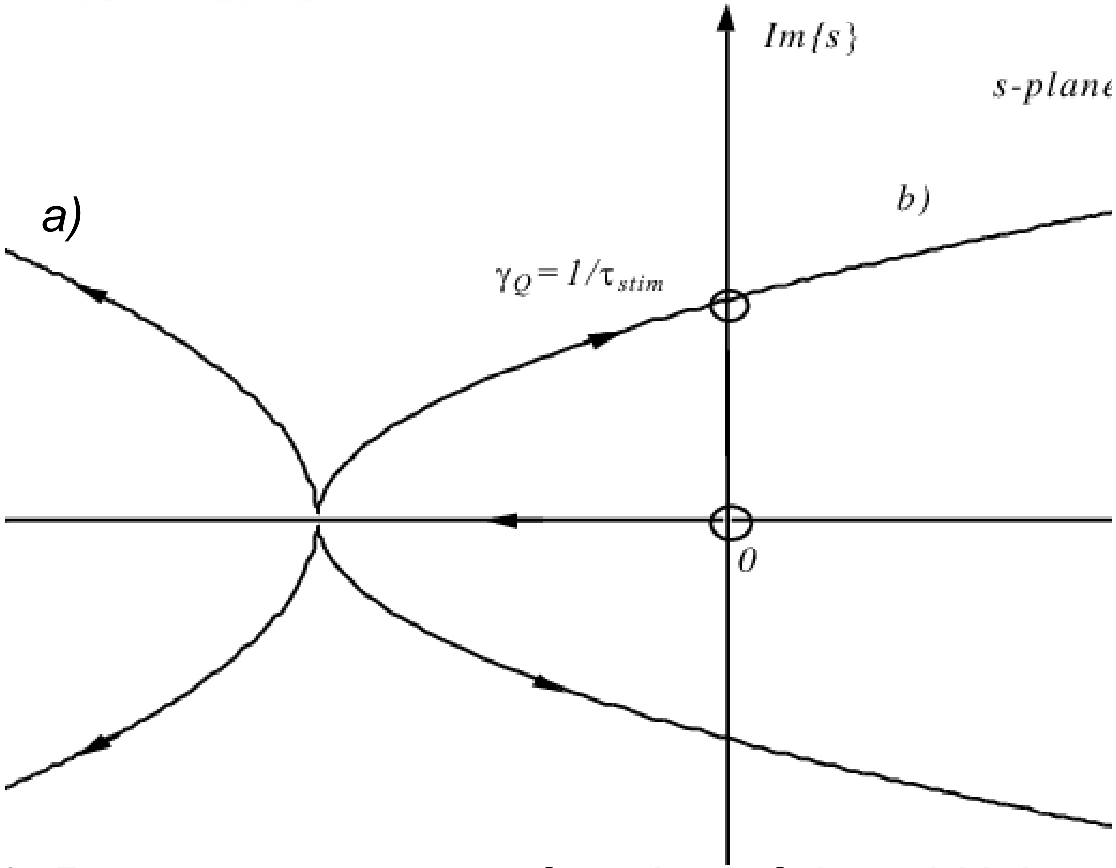


Fig. 4.26: Root locus plot as a function of destabilizing absorber. a) Without sat.absorber as a function of the pump parameter r; b) With sat. absorber as a function of γ_Q .

Passive Q-switching: a numerical example

τ_L=250μs, T_R=4ns, 2l=0.1, $2q_o$=0.005, $2g_o$=2, P_L/P_A=100.

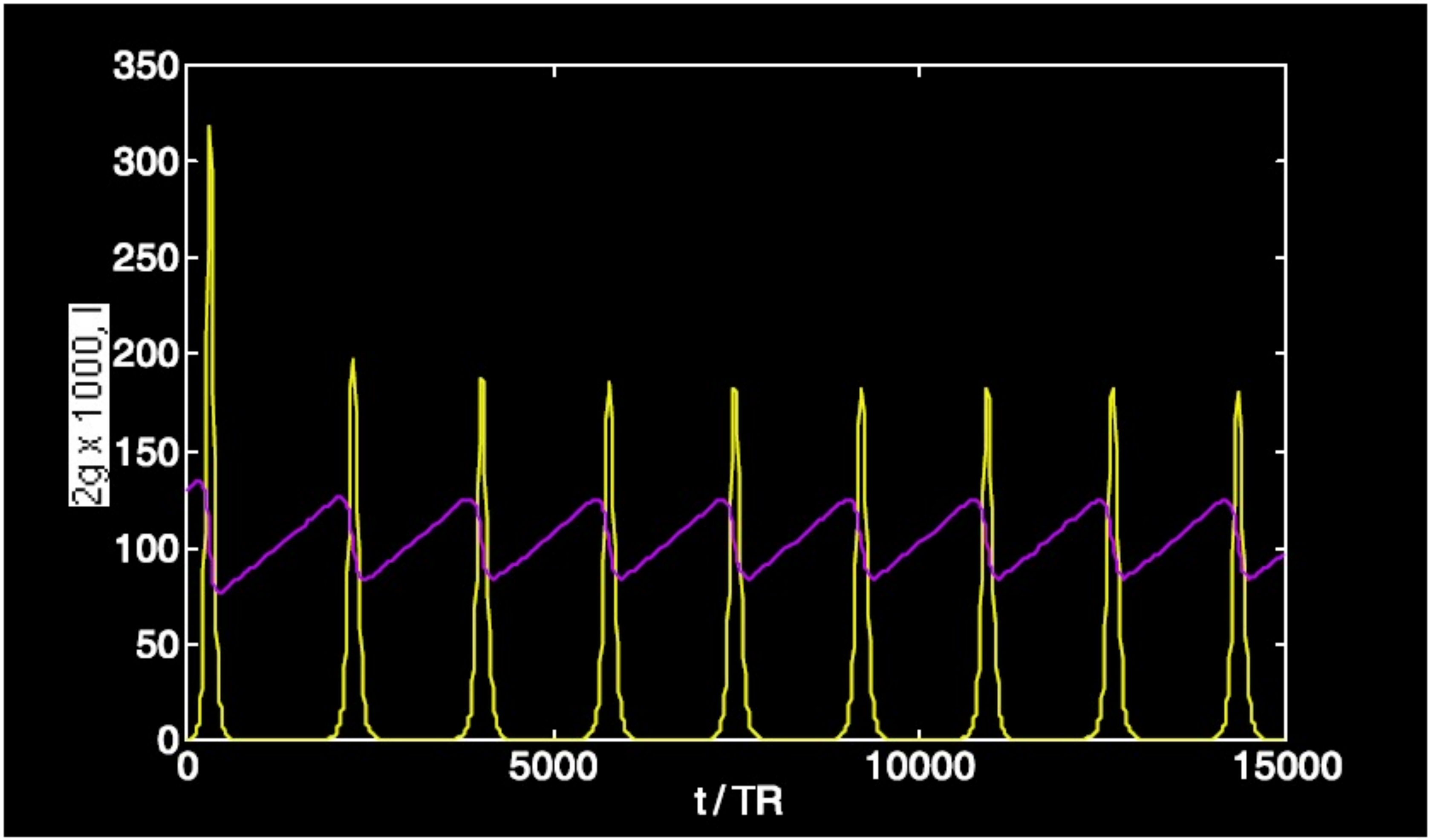
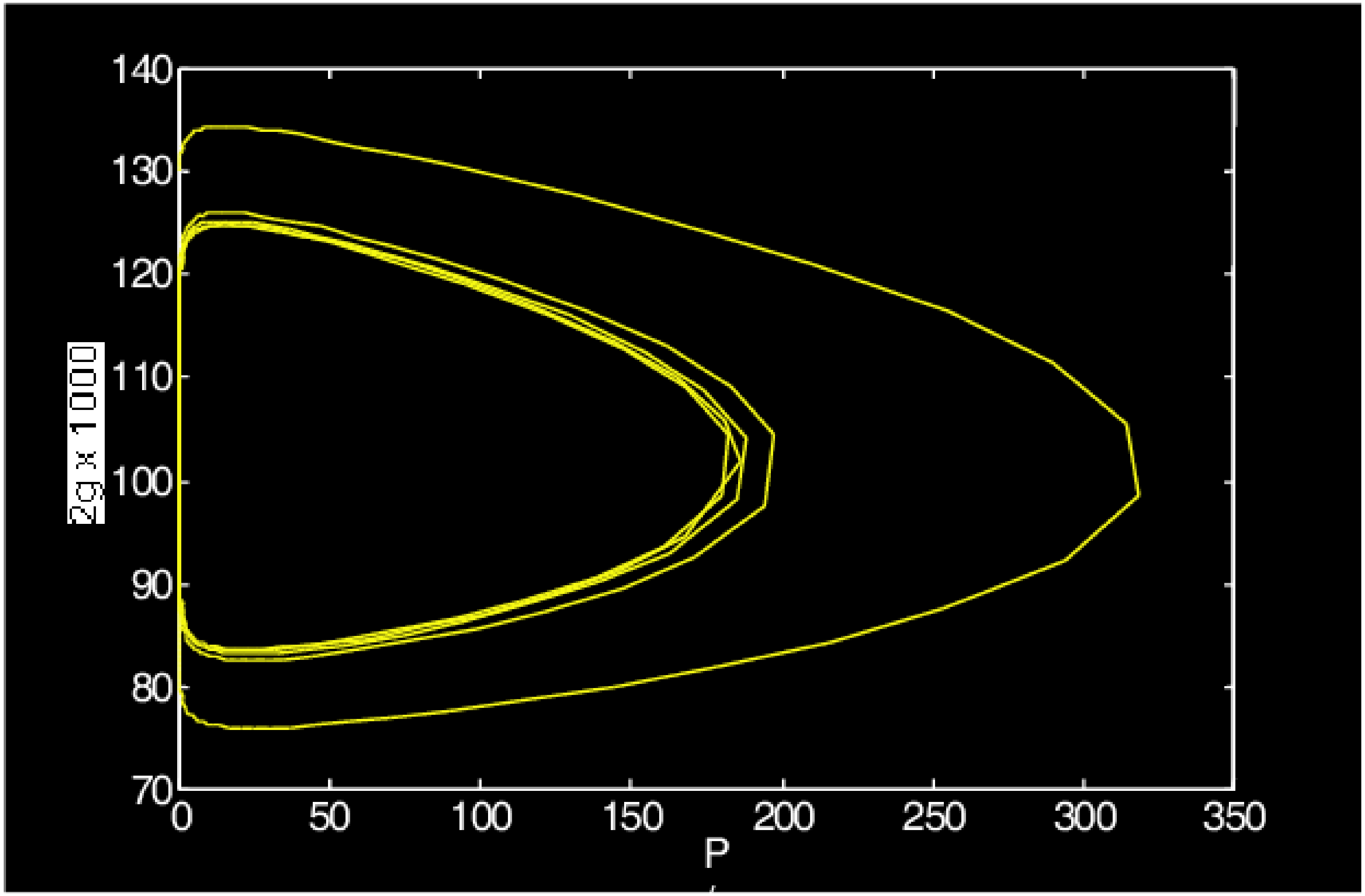


Fig. 4.28: Gain and output power as a function of time.



τ_L=250μs, T_R=4ns, 2l=0.1, $2q_o$=0.005, $2g_o$=2, P_L/P_A=100.

Fig. 4.27: Phase space solution for rate equations.

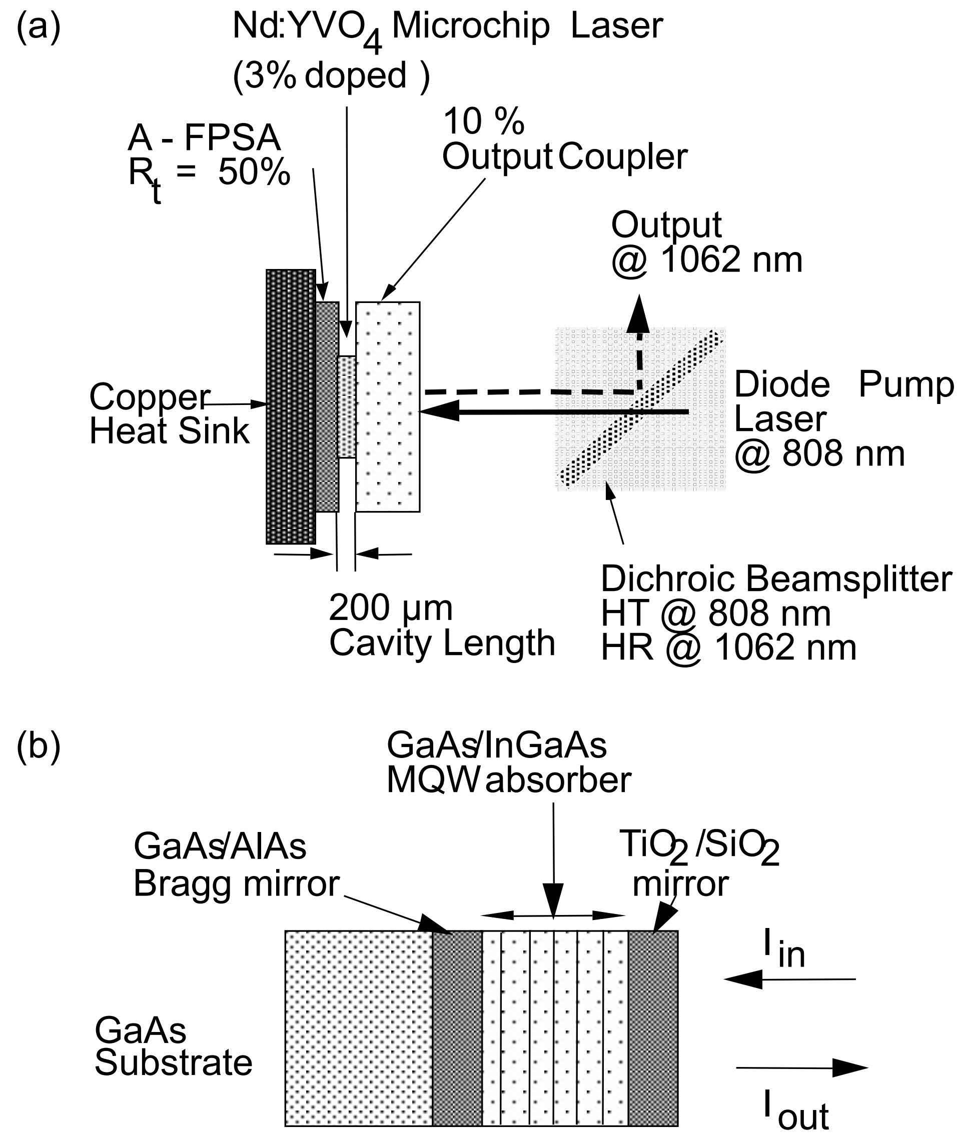


Fig. 4.29: Q-switched Nd:YVO_4-laser and absorber structure

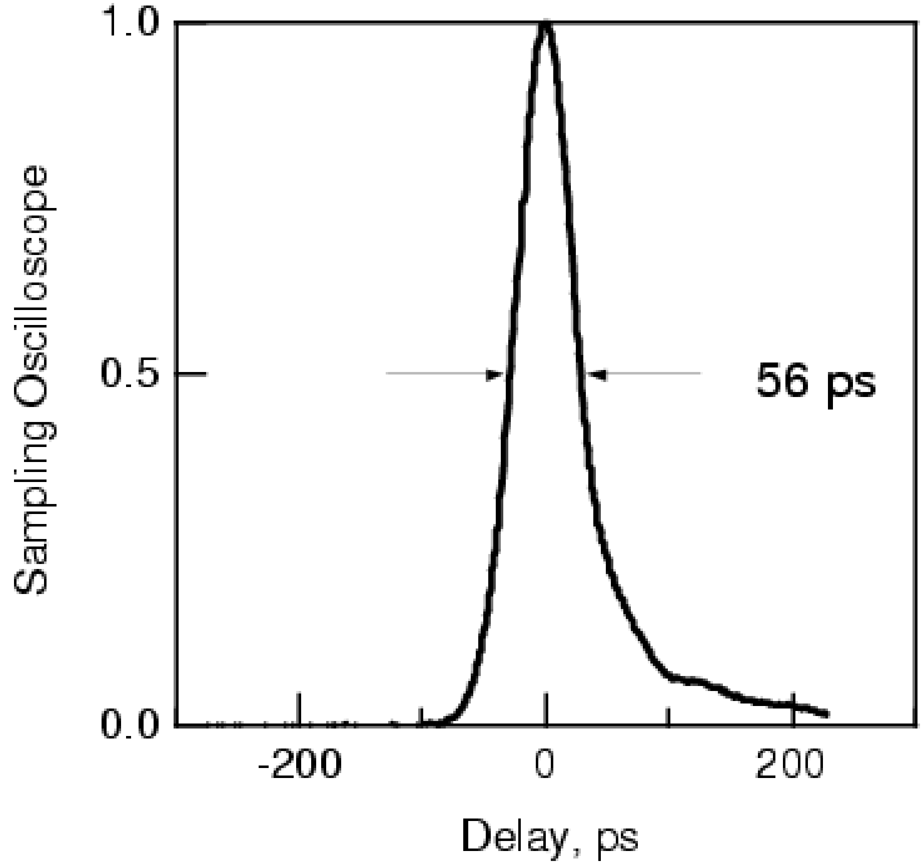


Fig. 4.30: Single-Mode Q-switched pulse from Nd:YVO_4 laser.

parameter	value
$2\ g_0$	0.7
$2\ q_0$	0.03
$2\ l$	0.14
T_R	2.7 ps
τ_L	87 μs
τ_A	24 ps
E_L	20 μJ
E_A	7.7 nJ

Table 4.2:

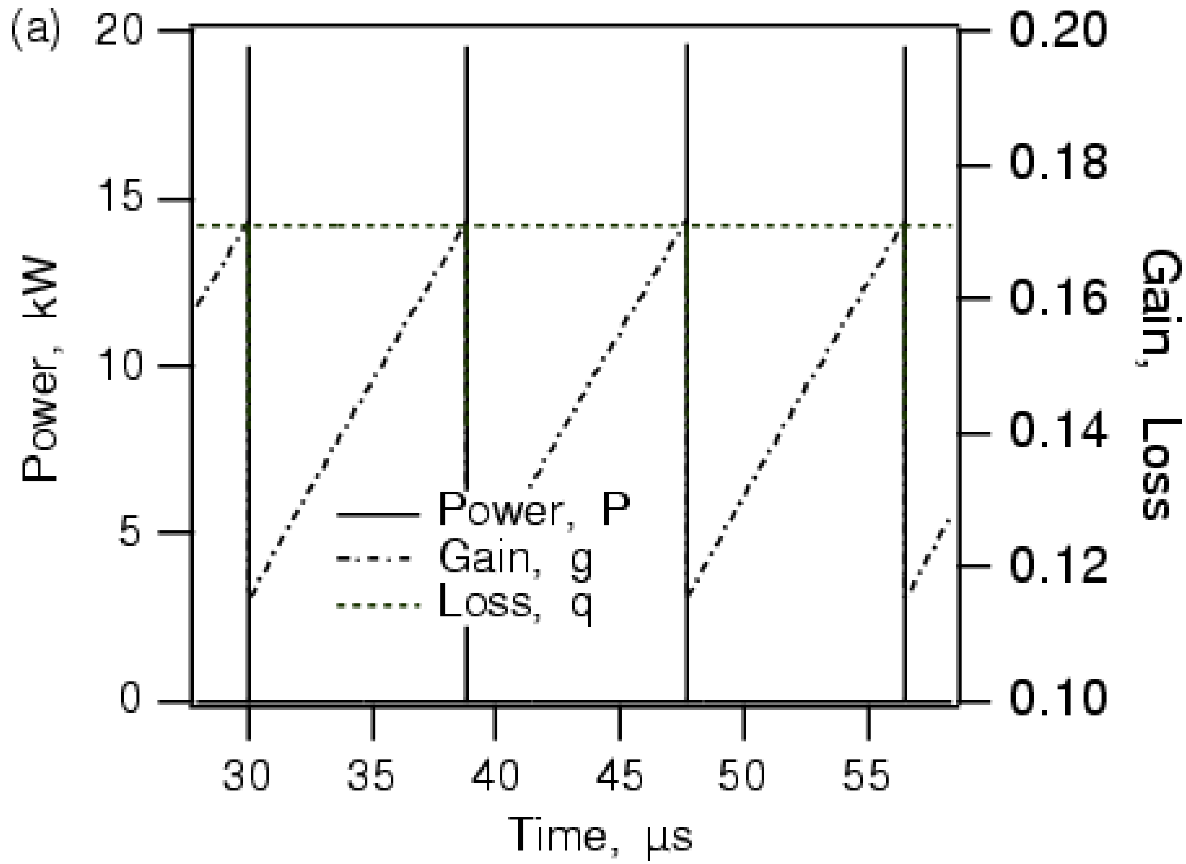


Fig. 4.31: Dynamics of the Q-switched microchip laser on a microsecond timescale

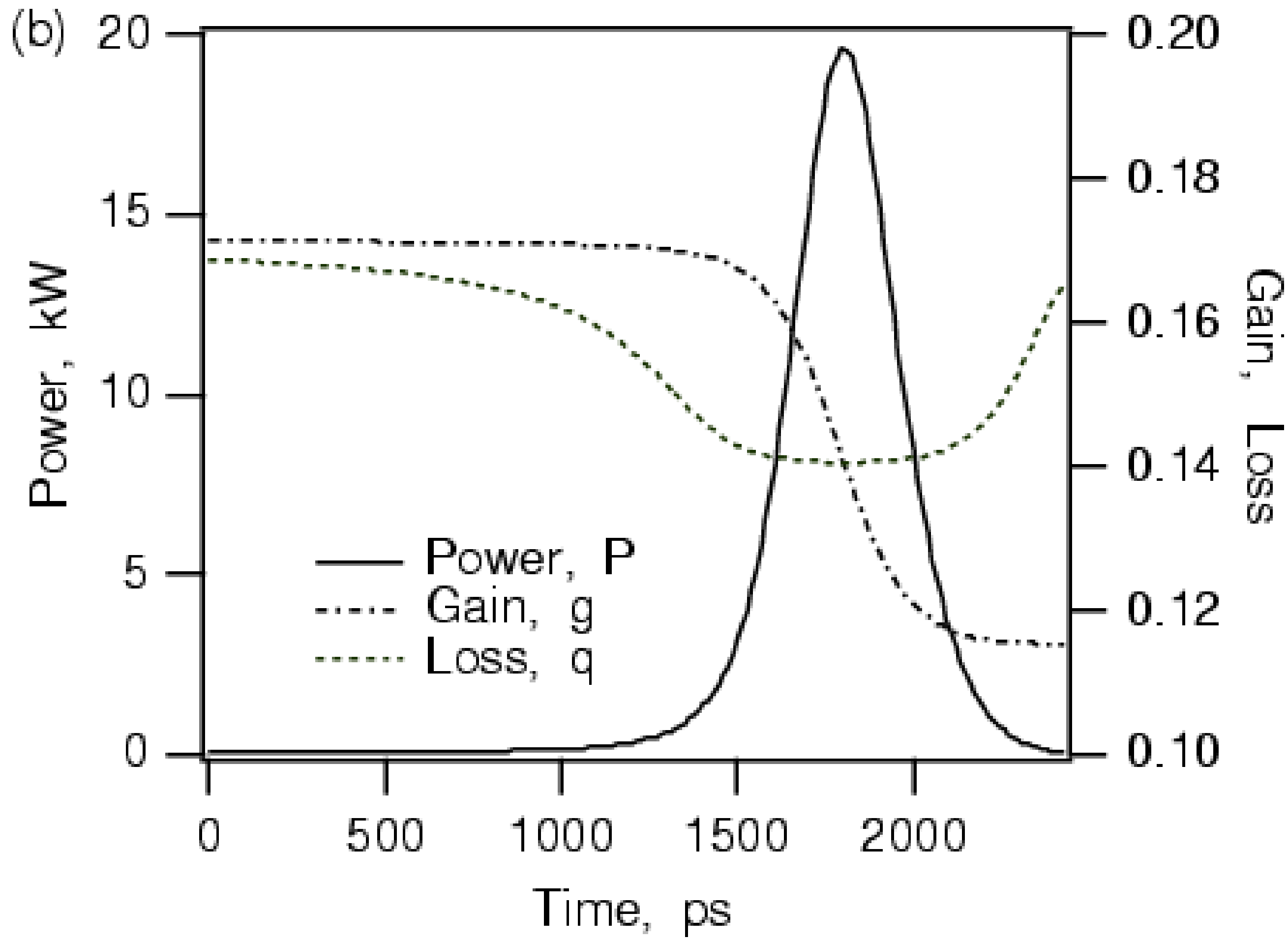


Fig. 4.32: On a picosecond time scale