Universität Hamburg
Physics Department
Ultrafast Optical Physics II
SoSe 2020

Problem Set 2

Issued: May 15, 2020 Due: May 29, 2020

Instruction: Please return your answer electronically to your TA via slack. For this problem set please send to Yi-Kai (Group B) or Felix (Group A).

Problem 2.1: The Nonlinear Schrödinger Equation (NSE) and optical soliton (15 points in total)

The Nonlinear Schrödinger Equation is written as follows, (here we assume $D_2 < 0$)

$$\frac{\partial A(z,t)}{\partial z} = jD_2 \frac{\partial^2 A(z,t)}{\partial t^2} - j\delta |A(z,t)|^2 A(z,t)$$

(a) Show by using the following transform

$$\xi = \frac{z}{L_D}, \tau = \frac{t}{\tau_0},$$

$$L_D = \frac{\tau_0^2}{|D_2|}, u = \sqrt{\frac{\delta}{|D_2|}}\ \tau_0 A$$

the NSE can be rewritten in to the normalized form

$$\frac{\partial u(\xi,\tau)}{\partial \xi} = -j\frac{\partial^2 u(\xi,\tau)}{\partial \tau^2} - j|u(\xi,\tau)|^2 u(\xi,\tau).$$ **(5 points)**

(b) Prove that $u(\xi,\tau) = \mathrm{sech}\left(\frac{\tau}{\sqrt{2}}\right) \cdot \exp\left(-\mathrm{j}\frac{\xi}{2}\right)$ is the solution to the normalized NSE. **(5 points)**

Hint: See section 3.9, especially $\frac{d}{dx} sech(\mathrm{x}) = -tanh(\mathrm{x})\ sech(\mathrm{x})\ and\ \frac{d}{dx} tanh(x) = sech^2(\mathrm{x})$

(c) Ultrafast optical solitons have been generated in optical fiber SMF-28 at $\lambda = 1.55\ \mu\mathrm{m}$. Using pulses from mode-locked lasers, hyperbolic-secant pulses with $\tau_0 = 4\ \mathrm{ps}$ were obtained after 700 m propagation in the optical fiber. The nonlinear index coefficient of the fiber is $\mathrm{n}_2 = 3 \times 10^{-20} \mathrm{m}^2/\mathrm{W}$. Calculate the corresponding peak intensity and the corresponding peak power of the soliton. **(5 points)**

Hint: the relationship between fiber dispersion $D(\lambda)$ characterized by fiber communication community and group velocity dispersion β_2 or D_2 can be found at RP-Photonics website, http://www.rp-photonics.com/group_velocity_dispersion.html .

Problem 2.2: The Split-Step Fourier method (15 points in total)

The normalized Nonlinear Schrödinger Equation (NSE) can be numerically solved using the Split-Step Fourier transform. Firstly the NSE can be understood in the following way

$$\frac{\partial u(\xi,\tau)}{\partial \xi} = \left(\hat{D} + \hat{N}\right)u(\xi,\tau)$$

as the simultaneous action of a dispersion operator $\hat{D} = -j\frac{\partial^2 u(\xi,\tau)}{\partial \tau^2}$ (assume the dispersion is negative), and a nonlinear operator $\hat{N} = -j|u(\xi,\tau)|^2$. If the linear and nonlinear changes for the pulse evolution are small within a short distance of propagation $\Delta\xi$, the solution of the NSE can be symbolically written as

$$u(\Delta\xi,\tau) = e^{(\hat{D}+\hat{N})\Delta\xi}u(0,\tau)$$

and approximated by

$$u(\Delta\xi,\tau) = e^{\frac{1}{2}\hat{D}\Delta\xi}e^{\hat{N}\Delta\xi}e^{\frac{1}{2}\hat{D}\Delta\xi}u(0,\tau).$$

One can show that iterative application of this propagation step only leads to an error of order $\Delta\xi^3$. Since the linear operator can be easily applied in the Fourier domain and the nonlinear operator (self-phase modulation only) in the time domain, one can simulate the NSE over one propagation step $\Delta\xi$ by the following algorithm

$$u(\xi+\Delta\xi,\tau) = F^{-1}[e^{\frac{1}{2}j\Omega^2\Delta\xi}F[e^{-j|u(\xi,\tau)|^2\Delta\xi}F^{-1}[e^{\frac{1}{2}j\Omega^2\Delta\xi}F[u(\xi,\tau)]]]].$$

where $\Omega = \omega - \omega_0$ is the difference between the real frequency and the carrier frequency.

(a) Dispersion effect on pulse evolution. The electric field of an unchirped Gaussian pulse is written as $E(0,t) = \mathrm{A}_0 \cdot \exp(-t^2/2\tau_0^2) \cdot \exp(i\omega_0 t)$, where $\tau_0 = 100\ fs$ and the center wavelength of such pulse locates at 1550 nm. We let the pulse propagate inside optical fiber SMF-28 and we do NOT consider the nonlinear effect. Moreover, we ONLY consider second order dispersion effect. What is the characteristic dispersion length L_D (based on the definition in Problem 2.1)? Plot the intensity distribution of such pulse at 0, $1L_D$, $2L_D$, $3L_D$ and $4L_D$ (You can use normalized scale. Indicate proper axis label, i.e. t/τ_0). Is the pulse positively chirped or negatively chirped after propagation? **(5 points)**

(b) Nonlinear effect on the evolution of the pulse spectrum. Ignore the dispersion effect, the nonlinear pulse evolution can be written as $u(\Delta\xi,\tau) = e^{\hat{N}\Delta\xi}u(0,\tau)$, where $\hat{N} = -j|u(\xi,\tau)|^2$ and we have changed the NSE to its normalized form. Plot the spectrum $S(\Omega) = |\tilde{u}(\Omega)|^2$

as the pulse accumulates different nonlinear phase $\phi_{NL} = |u(\xi, 0)|^2 = 0, \frac{\pi}{2}, \pi, \frac{3}{2}\pi$ and 2π assuming $u(0, \tau) = \exp(-\tau^2)$. **(5 points)**

(c) Write a program and simulate the normalized NSE for the following initial pulse

$$u(0, \tau) = N \cdot \text{sech}\,(\tau/\sqrt{2})$$

for N = 1,2 and 3. Make use of the Fast Fourier Transform (FFT) and use at least 1024 points. Plot the pulse shape (in the time domain) and corresponding amplitude spectra (in the frequency domain) as a function of propagation distance. **(5 points)**