Universität Hamburg
Physics Department
Ultrafast Optical Physics II
SoSe 2020

Problem Set 1

Issued: May 8, 2020 Due: May 15, 2020

Instruction: Please write your answer to each problem on separate paper sheet. If you are using programming language to do numerical calculations, attach the original code with your answers.

Problem 1.1: Time-Bandwidth Product (10 points in total)

The time-bandwidth product links the duration of an optical pulse in the time domain to its corresponding spectral width in the frequency domain. The values are pulse-shape specific, and follow from the Fourier transform relation or the uncertainty principle, as the case may be. In ultrafast optical physics, it is common to specify the full width at half-maximum (FWHM) in both time and frequency domain. The following expression describes a parabolic pulse in the time domain, in complex notation:

$$E(t) = E_0 \cdot \left(1 - \frac{t^2}{\tau^2}\right) e^{i\omega_0 t} \qquad \text{for} \qquad |t| \leq \tau \tag{1}$$

$$E(t) = 0 \qquad \text{for} \qquad |t| \geq \tau \tag{2}$$

where ω_0 is the angular frequency of the light field of interest.

(a) Sketch the intensity function $|E(t)|^2$ and calculate the full width at half maximum (FWHM) Δt_{FWHM} of the intensity function. **(4 points)**

(b) Calculate the Fourier transform $\tilde{E}(\omega)$. Sketch the power spectrum $\left|\tilde{E}(\omega)\right|^2$ and identify the full width at half maximum $\Delta\nu_{FWHM} = \Delta\omega_{FWHM}/2\pi$. **(4 points)**

(Hint: Introduce the variable $x = (\omega - \omega_0)\tau$ and calculate Δx_{FWHM} numerically.)
(Hint: $\lim_{x\to 0} \frac{\sin x - x\cos x}{x^3} = \frac{1}{3}$)
(Hint: www.wolframalpha.com can be helpful for a numerical calculation.)

(c) Calculate the time-bandwidth product $\Delta\nu_{FWHM} \cdot \Delta t_{FWHM}$ for this pulse shape. **(2 points)**

Problem 1.2: Material dispersion, phase velocity and group velocity (20 points in total)

Two Gaussian pulses are launched into a piece of optical fiber. Their center frequencies are located at $f_1 = 205\,THz$ and $f_2 = 210\,THz$ ($1\,THz = 10^{12}\,Hz$) respectively. Assume these two pulses' duration is relatively long so that their spectra are centered aroundf_1 and f_2 (they do not overlap).

Dispersion of the fiber can lead to pulse broadening as well as time delay between pulses as they propagate through the fiber. You are asked to estimate the pulse broadening and time delay from the dispersion characteristics of the fiber.

(a) The dependence of wave number k on frequency is linear as shown in Fig. 1 (a).

i) Which pulse arrives at the output first? Give a brief justification. **(2 points)**
ii) Which pulse is broadened by fiber dispersion more? Give a brief justification. **(2 points)**

(b) The dependence of wave number k on frequency is composed of linear segments as shown in Fig. 1 (b).

i) Which pulse arrives at the output first? Give a brief justification. **(2 points)**
ii) Calculate an *approximate* value for the time delay between the two pulses at the output of the fiber given the fiber length is $25\ km$. **(2 points)**
iii) Which pulse is broadened by fiber dispersion more? Give a brief justification. **(2 points)**

(c) The refractive index of transparent glass can be empirically calculated by Sellmeier equation. The Sellmeier equation of an unknown glass is written as follows:

$$n^2 - 1 = \frac{1.04 \cdot \lambda^2}{\lambda^2 - 0.006} + \frac{0.23 \cdot \lambda^2}{\lambda^2 - 0.02} + \frac{1.01 \cdot \lambda^2}{\lambda^2 - 103.56}$$

where n is the refractive index, λ is the wavelength of interest in vacuum with the unit in μm. The $n(\lambda)$ relationship is shown in Fig. 1 (c).

i) Find the refractive index at f_1 and f_2. **(2 points)**
ii) Calculate the phase velocity and group velocity at 1330 nm and 1550 nm. **(2 points)**
iii) Numerically calculate the GVD at 1030 and 1550 nm. Determine the type of dispersion at these two wavelengths (positive / negative). **(4 points)**
iv) Find the Zero-Dispersion-Wavelength (ZDW) of such glass. **(2 points)**

(Hint: it will make life easier if one can plot the GVD as a function of wavelength)

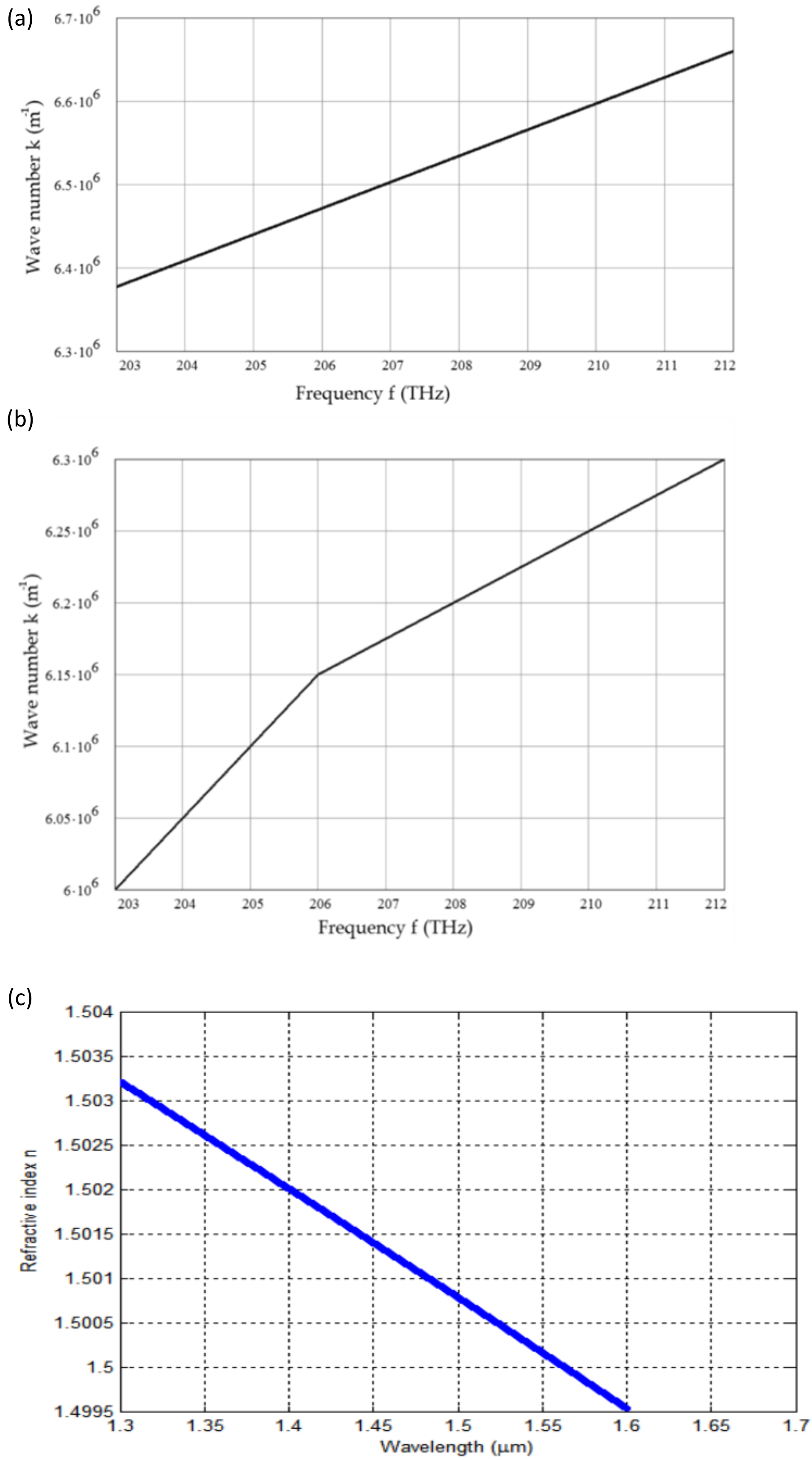
(a)
Wave number k (m-1)
6.3·10^6
6.4·10^6
6.5·10^6
6.6·10^6
6.7·10^6
203 204 205 206 207 208 209 210 211 212
Frequency f (THz)
(b)
Wave number k (m-1)
6·10^6
6.05·10^6
6.1·10^6
6.15·10^6
6.2·10^6
6.25·10^6
6.3·10^6
203 204 205 206 207 208 209 210 211 212
Frequency f (THz)
(c)
Refractive index n
1.4995
1.5
1.5005
1.501
1.5015
1.502
1.5025
1.503
1.5035
1.504
1.3 1.35 1.4 1.45 1.5 1.55 1.6 1.65 1.7
Wavelength (μm)

Figure 1

Problem 1.3: Gires-Tournois Interferometer (20 points in total)

Gires-Tournois Interferometer (GTI) is essentially a Fabry-Perot resonator with a 100% reflector. As with an ideal high-reflectivity mirror, the whole reflectivity of the device stays 100%. In contrast, the phase-delay is, as with a Fabry-Perot, frequency-dependent. Thus the GTI can be used in a laser resonator for dispersion compensation.

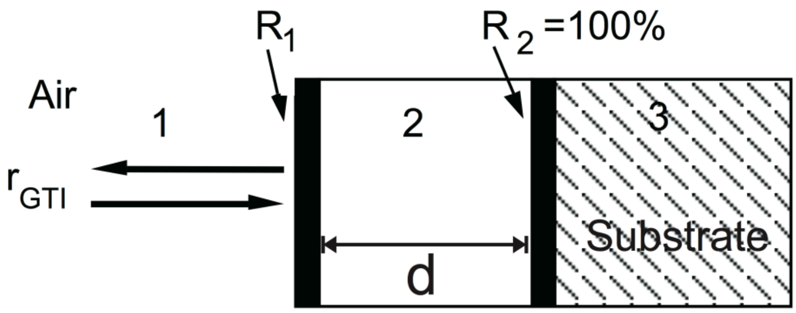

Figure 2: Gires-Tournois Interferometer. Intensity reflectivity for the corresponding surface is given with $\boldsymbol{R}$ and amplitude reflectivity with $\boldsymbol{r}$

Using $r_1 = -\sqrt{R_1}$, $r_2 = -\sqrt{R_2} = -1$ and assuming that medium 2 has a refractive index 1, the following expression for the amplitude reflectivity can be found, $\tilde{r}_{GTI}$:

$$\tilde{r}_{GTI} = \frac{-\sqrt{R_1} + e^{-i2kd}}{1 - \sqrt{R_1}e^{-i2kd}} \tag{5}$$

where $k = 2\pi/\lambda$ and d is the thickness of Medium 2.

(a) The relationship between intensity reflectivity R and amplitude reflectivity r is $R = |r|^2$. Show that the intensity reflectivity R_{GTI} is 100%, as long as there is no absorption or other loss in Medium 2. **(2 points)**

(b) Using the relation $\tilde{r}_{GTI} \stackrel{\text{def}}{=} |\tilde{r}_{GTI}| \cdot e^{-i\Phi_{GTI}}$ to calculate the phase Φ_{GTI} from the amplitude reflectivity. Give your answer in term of ωt_0 using the relation $\omega t_0 = 2kd$ in your final answer. **(2 points)**

(c) Calculate the group delay $T_g = -\frac{\partial \Phi_{GTI}}{\partial \omega}$ and the group delay dispersion $D_g = \frac{\partial T_g}{\partial \omega}$. **(2 points)**

From problem (d) to problem (h), suppose the thickness of medium 2 is $d = 150\ \mu m$ and the reflectivity at the interface between medium 1 and medium 2 is $R_1 = 4\%$.

(d) Plot T_g and D_g as functions of wavelength λ in the band from $798\,nm$ to $803\,nm$. Indicate proper units. **(2 points)**

(e) From the answer of (d), in which wavelength range can the GTI be used for dispersion compensation inside a laser resonator? Note that the laser crystals and air have positive group velocity dispersions. **(3 points)**

(f) Suppose a $100\,fs$ long Gaussian-shaped optical pulse (peak intensity is normalized to 1) centered at $\lambda = 800\,nm$ is reflected from the interface between medium 1 and 2 at $t = 0$. At $t = 10\,ps$, how will the reflected pulses look like? Sketch the pulses at this point of time and specify as many numeric values (intensity) as possible. **(3 points)**

(g) Now suppose a $10\,ps$ long Gaussian-shaped optical pulse (peak intensity is normalized to 1) centered at $\lambda = 800\,nm$ is reflected from the interface between medium 1 and 2 at $t = 0$. At $t = 20\,ps$, how will the reflected pulse look like? Sketch the pulse at this point in time and specify as many numeric values as possible. **(3 points)**

(h) The answers for problems (f) and (g) will look quite different. Briefly explain the reason in the frequency and/or time domains. **(3 points)**