Ultrafast Optical Physics II (SoSe 2017)
Lecture 8, June 2

Class schedule in following weeks:

June 9 (Friday): No class
June 16 (Friday): Lecture 9
June 23 (Friday): Lecture 10
June 30 (Friday): Lecture 11 shift to July 3?

Passive Mode Locking

Slow Saturable Absorber Mode Locking

Fast Saturable Absorber Mode Locking

Soliton Mode-Locking

Dispersion Managed Soliton Formation

Kerr-Lens Mode-Locking

Additive Pulse Mode-Locking

Semiconductor Saturable Absorber Mode-Locking

Principles of Passive Mode Locking

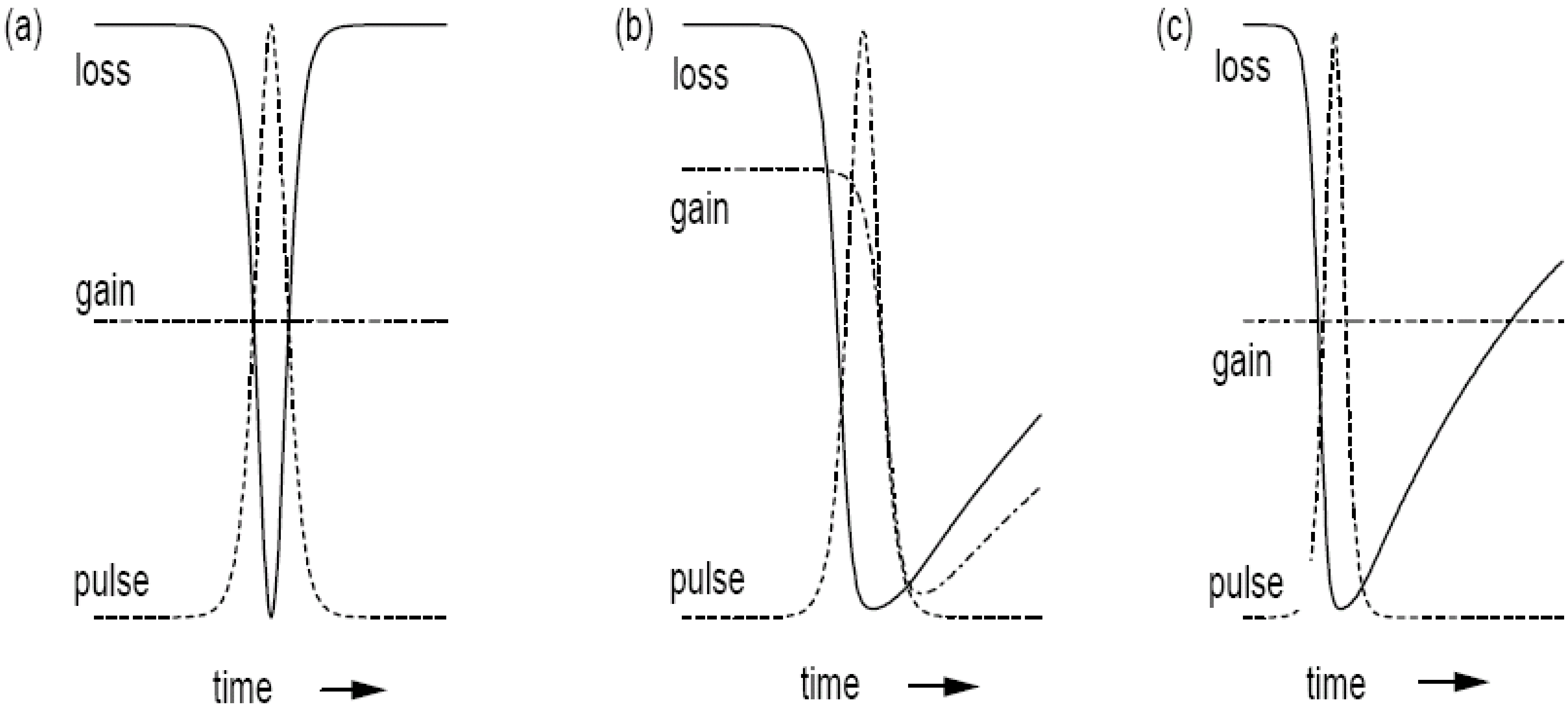
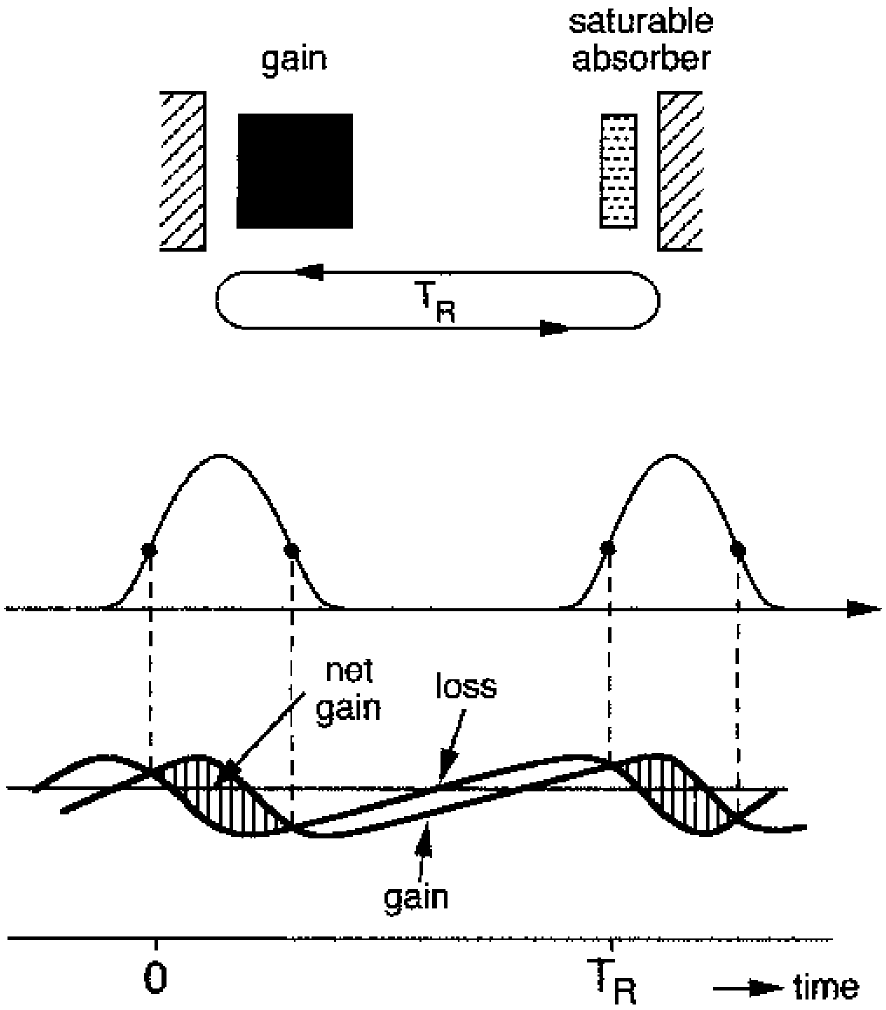


Fig. 6.1: Principles of mode locking

6.1 Slow Saturable Absorber Mode Locking



No fast element necessary:
Both absorber and gain
may recover on ns-time scale

Fig. 6.2: Slow saturable absorber modelocking

$$\frac{dg}{dt} = -g\frac{|A(t)|^2}{E_L}$$

Introduce pulse energy: $E(t) = \int_{-T_{R/2}}^{t} dt|A(t)|^2$

$$\longrightarrow \quad g(t) = g_i \exp\left[-E(t)/E_L\right]$$

$$q(t) = q_0 \exp\left[-E(t)/E_A\right]$$

Master Equation:

$$T_R\frac{\partial}{\partial T}A \;=\; \left[g_i\left(\exp\left(-E(t)/E_L\right)\right)A - lA - \right.$$

Fixed filtering / finite bandwidth

$$\left. q_0 \exp\left(-E(t)/E_A\right)\right]A + \tfrac{1}{\Omega_f^2}\tfrac{\partial^2}{\partial t^2}A$$

Approximate absorber response:

$$q_0 \exp\left(-E(t)/E_A\right) \;\approx\; q_0\left[1 - \left(E(t)/E_A\right) + \frac{1}{2}\left(E(t)/E_A\right)^2\right]$$

$$T_R \frac{\partial A(T,t)}{\partial T} = \left[g(t) - q(t) - l + D_f \frac{\partial^2}{\partial t^2} \right] A(T,t)$$

Ansatz: $A(t) = A_o \, \mathrm{sech}(t/\tau)$

Stationary solution: *A(T+T_R,t)* reproduces itself up to a timing shift?

$$A(t,T) \;=\; A_o \;\mathrm{sech}\left(\tfrac{t}{\tau} + \alpha \tfrac{T}{T_R}\right)$$

$$E(t) \;=\; \int_{-T_{R/2}}^{t} dt |A(t)|^2 = \frac{W}{2}\left(1 + \tanh\left(\frac{t}{\tau} + \alpha \frac{T}{T_R}\right)\right)$$

Shortest pulse width possible: $\tau = \frac{2\sqrt{2}}{\sqrt{q_0}\Omega_f}\frac{E_A}{W} > \frac{\sqrt{2}}{\sqrt{q_0}\Omega_f}$

6.2 Fast Saturable Absorber Mode Locking

Saturable absorption responds to instantaneous power: $q(A) = \dfrac{q_0}{1 + \frac{|A|^2}{P_A}}$

Approximately: $q(A) = q_0 - \gamma |A|^2$ with: $l_0 = l + q_0$ and $\gamma = q_0 / P_A$

$$T_R \frac{\partial A(T,t)}{\partial T} = \left[g - l_0 + D_f \frac{\partial^2}{\partial t^2} + \gamma |A|^2 + jD_2 \frac{\partial^2}{\partial t^2} - j\delta |A|^2 \right] A(T,t)$$

Dispersion + SPM

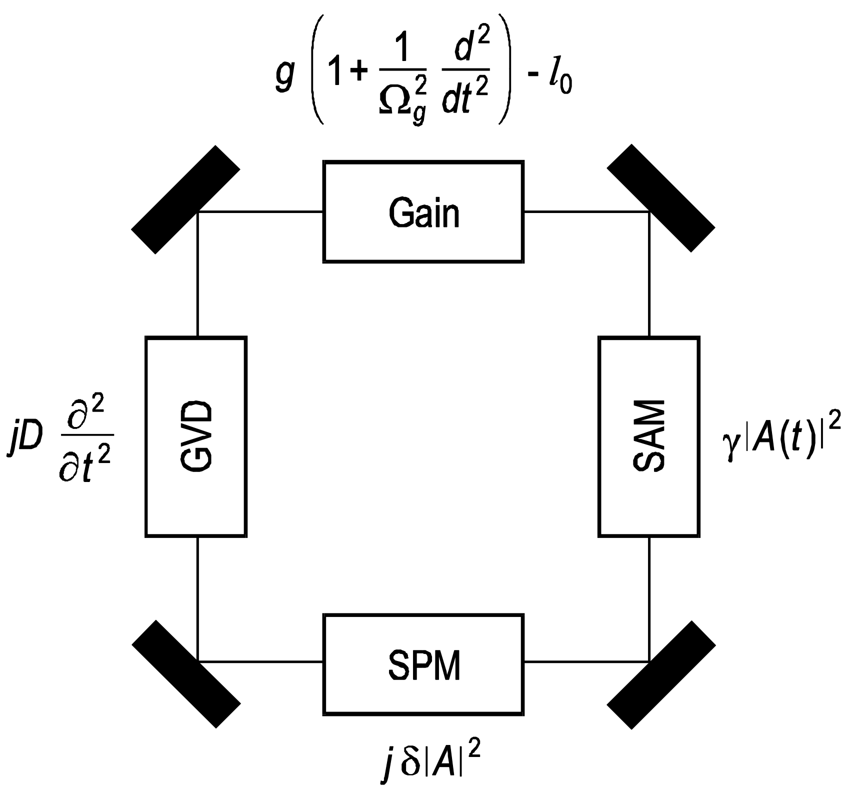


Fig. 6.3: Fast saturable absorber modelocking

Without GDD and SPM

$$T_R \frac{\partial A(T,t)}{\partial T} = \left[g - l_0 + D_f \frac{\partial^2}{\partial t^2} + \gamma |A|^2 \right] A(T,t)$$

$$T_R \frac{\partial A_s(T,t)}{\partial T} = 0. \longrightarrow A_s(T,t) = A_s(t) = A_0 \text{sech}\left(\frac{t}{\tau}\right)$$

$$\begin{aligned} 0 \;=\; & \left[(g - l_0) + \frac{D_f}{\tau^2}\left[1 - 2\text{sech}^2\left(\frac{t}{\tau}\right)\right] \right. \\ & \left. + \gamma |A_0|^2 \text{sech}^2\left(\frac{t}{\tau}\right) \right] \cdot A_0 \text{sech}\left(\frac{t}{\tau}\right) \end{aligned}$$

$$\frac{D_f}{\tau^2} = \frac{1}{2}\gamma |A_0|^2 \qquad \text{Pulse Energy:} \quad W = 2A_0^2\tau \longrightarrow \tau = \frac{4D_f}{\gamma W}.$$

$$g = l_0 - \frac{D_f}{\tau^2}$$

Pulse Energy Evolution:

$$\begin{aligned} T_R \frac{\partial W(T)}{\partial T} &= T_R \frac{\partial}{\partial T} \int_{-\infty}^{\infty} |A(T,t)|^2 \, dt \\ &= T_R \int_{-\infty}^{\infty} \left[A(T,t)^* \frac{\partial}{\partial T} A(T,t) + c.c. \right] dt \\ &= 2G(g_s, W)W, \end{aligned}$$

$$\begin{aligned} \int_{-\infty}^{\infty} \left(\mathrm{sech}^2 x\right) dx &= 2, \\ \int_{-\infty}^{\infty} \left(\mathrm{sech}^4 x\right) dx &= \frac{4}{3}, \\ -\int_{-\infty}^{\infty} \mathrm{sech} x \frac{d^2}{dx^2} \left(\mathrm{sech} x\right) dx &= \int_{-\infty}^{\infty} \left(\frac{d}{dx} \mathrm{sech} x\right)^2 dx = \frac{2}{3} \end{aligned}$$

$$\begin{aligned} G(g_s, W) &= g_s - l_0 - \frac{D_f}{3\tau^2} + \frac{2}{3}\gamma |A_0|^2 \\ &= g_s - l_0 + \frac{1}{2}\gamma |A_0|^2 = g_s - l_0 + \frac{D_f}{\tau^2} = 0 \end{aligned}$$

$$g_s(W) = \frac{g_0}{1 + \frac{W}{P_L T_R}}$$

Steady State Pulse Energy:

$$g_s(W) = \frac{g_0}{1+\frac{W}{P_L T_R}} = l_0 - \frac{D_f}{\tau^2}$$

$$= l_0 - \frac{(\gamma W)^2}{16 D_{\cancel{g}}}$$ Replace by *f*

With $q_0 = \gamma A_0^2.$

$$\boxed{\frac{D_f}{\tau^2} = \frac{q_0}{2}, \qquad \tau = \sqrt{\frac{2}{q_0}} \frac{1}{\Omega_f}.}$$

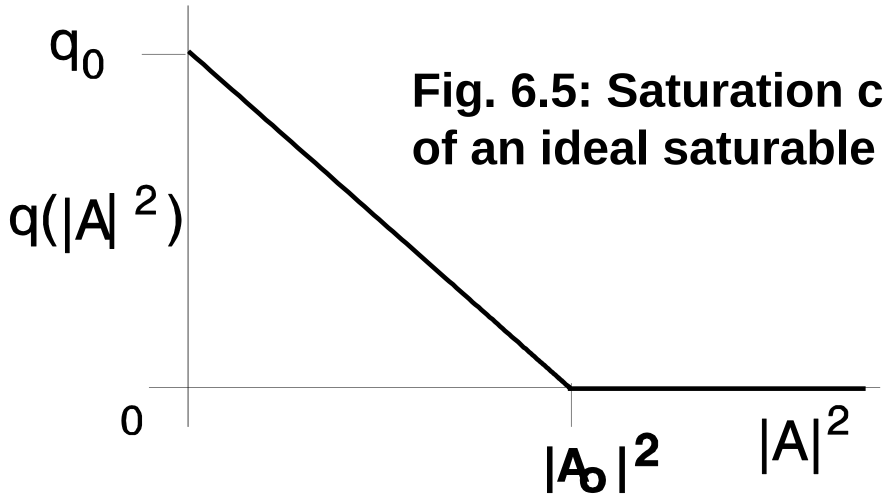


Fig. 6.5: Saturation characteristic of an ideal saturable absorber

Minimum Pulse Width: $g_s = l \overset{-}{\cancel{+}} \frac{1}{2} q_0$ $\qquad D_f = D_g = \frac{g}{\Omega_g^2}$

$$\tau_{\min} = \frac{1}{\Omega_g} \longrightarrow \Delta f_{FWHM} = \frac{0.315}{1.76 \cdot \tau_{\min}} = \frac{\Omega_g}{1.76 \cdot \pi}$$

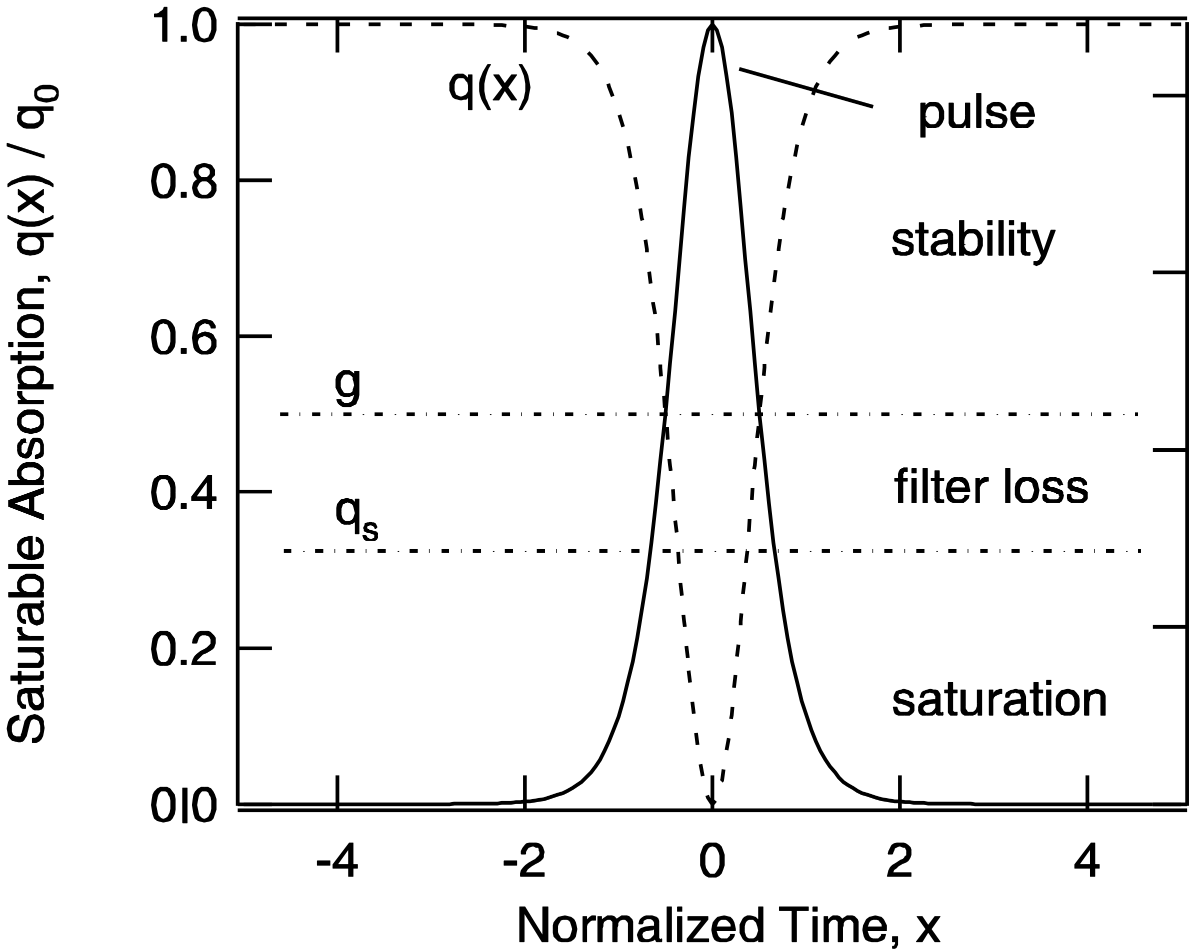
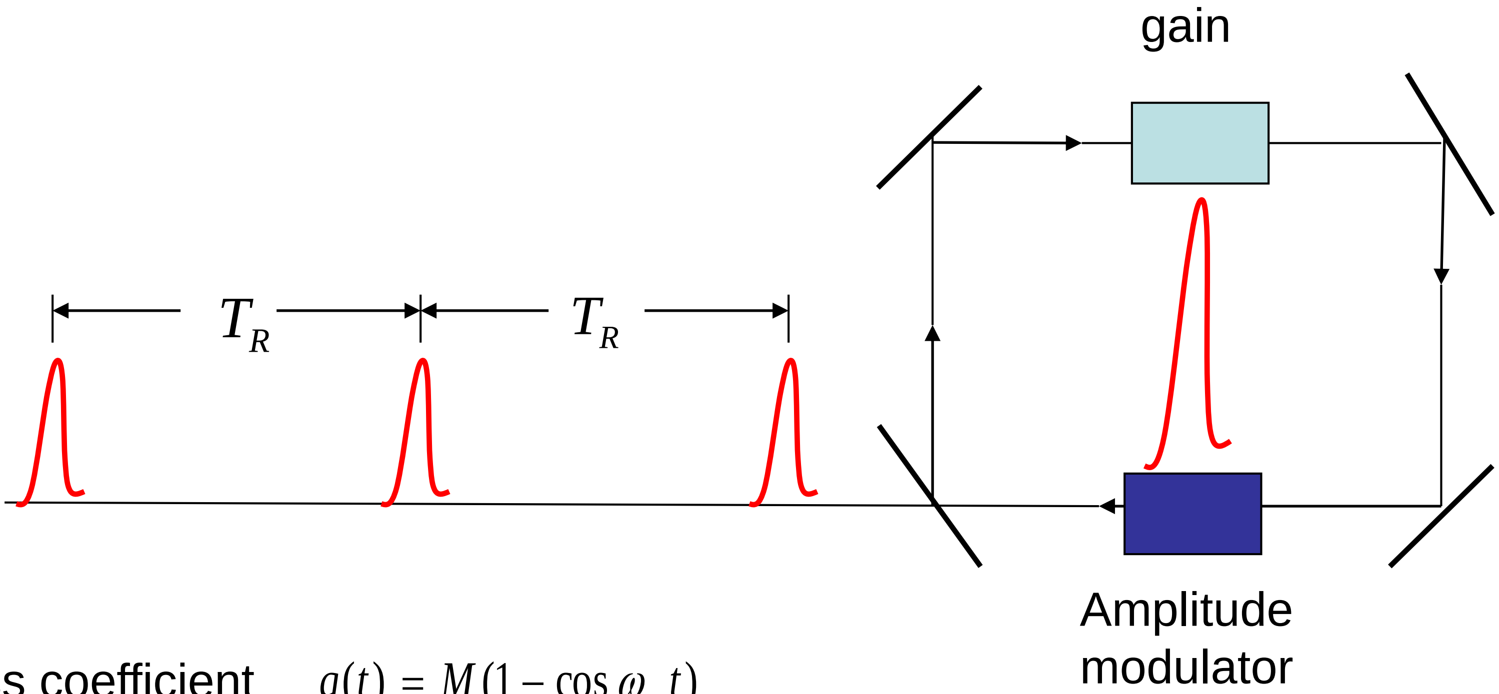


Fig. 6.4: Gain and loss in a fast saturable absorber (FSA) modelocked laser

Active mode-locking using amplitude modulator

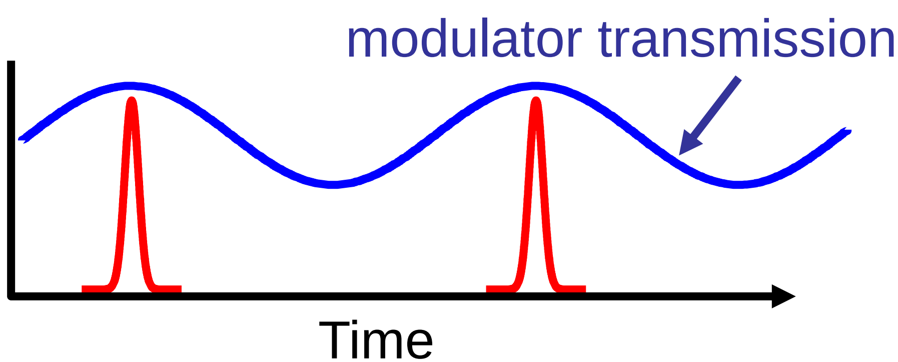


Loss coefficient $q(t) = M(1 - \cos\omega_m t)$

Transmission of the modulator

$$T_m = e^{-M(1-\cos\omega_m t)}$$

$$T_m \approx 1 - M(1-\cos\omega_m t)$$



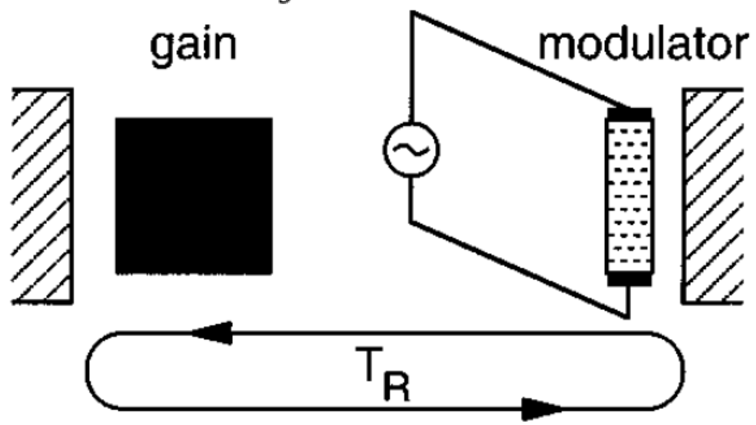
Active mode-locking using amplitude modulator

$$T_R \frac{\partial A}{\partial T} = \left[g(T) + D_g \frac{\partial^2}{\partial t^2} - l - M\left(1 - \cos(\omega_M t)\right) \right] A.$$

$$\downarrow$$

$$T_R \frac{\partial A}{\partial T} = \left[g - l + D_g \frac{\partial^2}{\partial t^2} - M_s t^2 \right] A.$$

$$D_g = \frac{g}{\Omega_g^2} \qquad M_s = \frac{M\omega_M^2}{2}$$



Hermite-Gaussian Solution

$$A_n(t) = \sqrt{\frac{W_n}{2^n \sqrt{\pi}\, n!\, \tau_a}} H_n(t/\tau_a) e^{-\frac{t^2}{2\tau_a^2}}$$

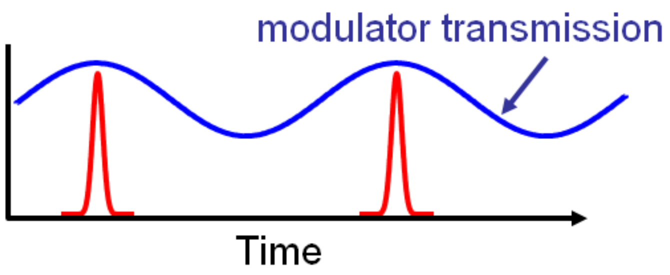
$$\tau_a = \sqrt[4]{D_g/M_s} \longrightarrow \boxed{\tau_a = \sqrt[4]{2}\,(g/M)^{1/4} / \sqrt{\Omega_g \omega_M}}$$

Comments on active mode-locking

Pulse duration: $\tau_a = \sqrt[4]{2}(g/M)^{1/4}/\sqrt{\Omega_g \omega_M}$

1) Larger modulation depth, M, and higher modulation frequency will give shorter pulses because the "low loss" window becomes narrower and shortens the pulse.
2) A broader gain bandwidth yields shorter pulses because the filtering effect of gain narrowing is lower and more modes are lasing.

Disadvantages of active mode-locking:



1) It requires an externally driven modulator. Its modulation frequency has to match precisely the cavity mode spacing.

1) A broader gain bandwidth yields shorter pulses because the filtering effect of gain narrowing is lower and more modes are lasing.

Principles of Passive Mode Locking

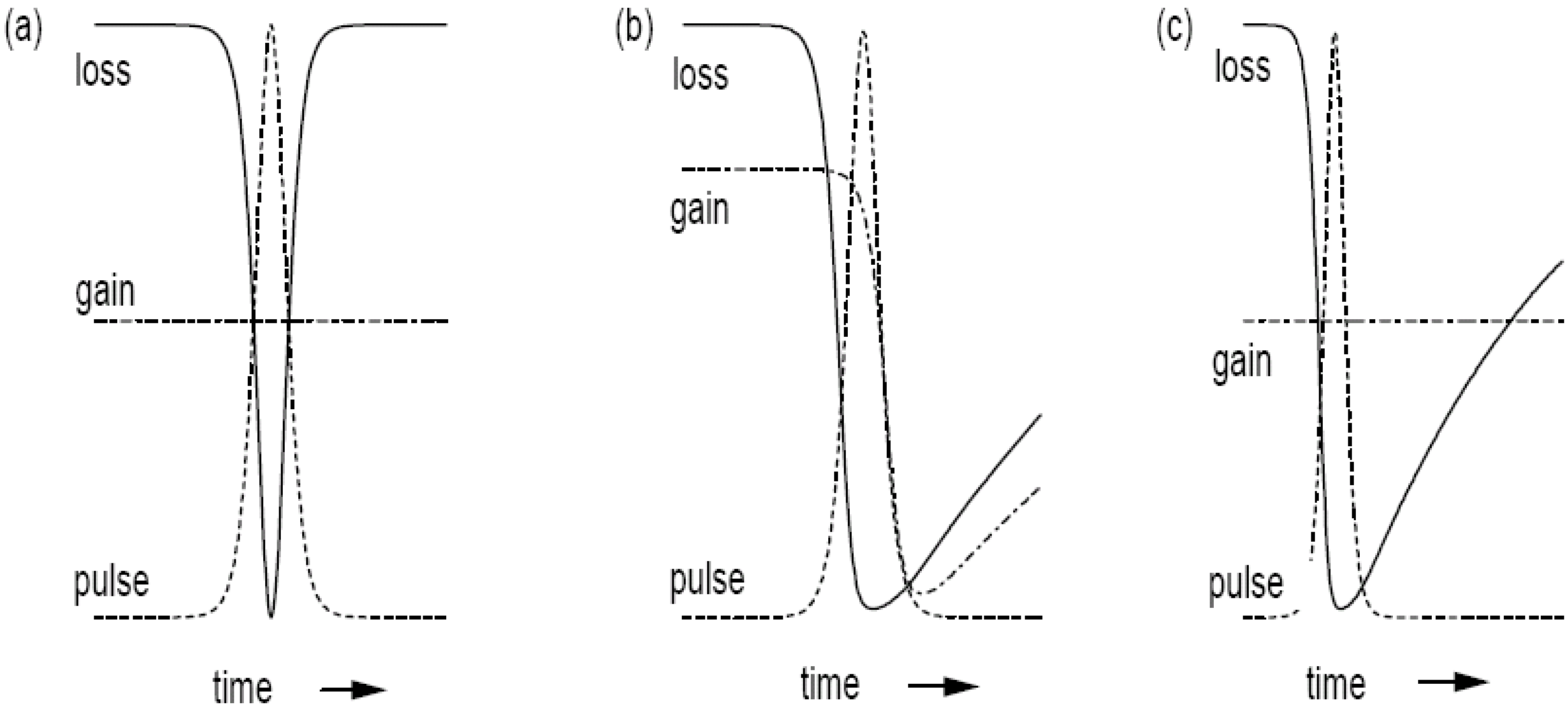
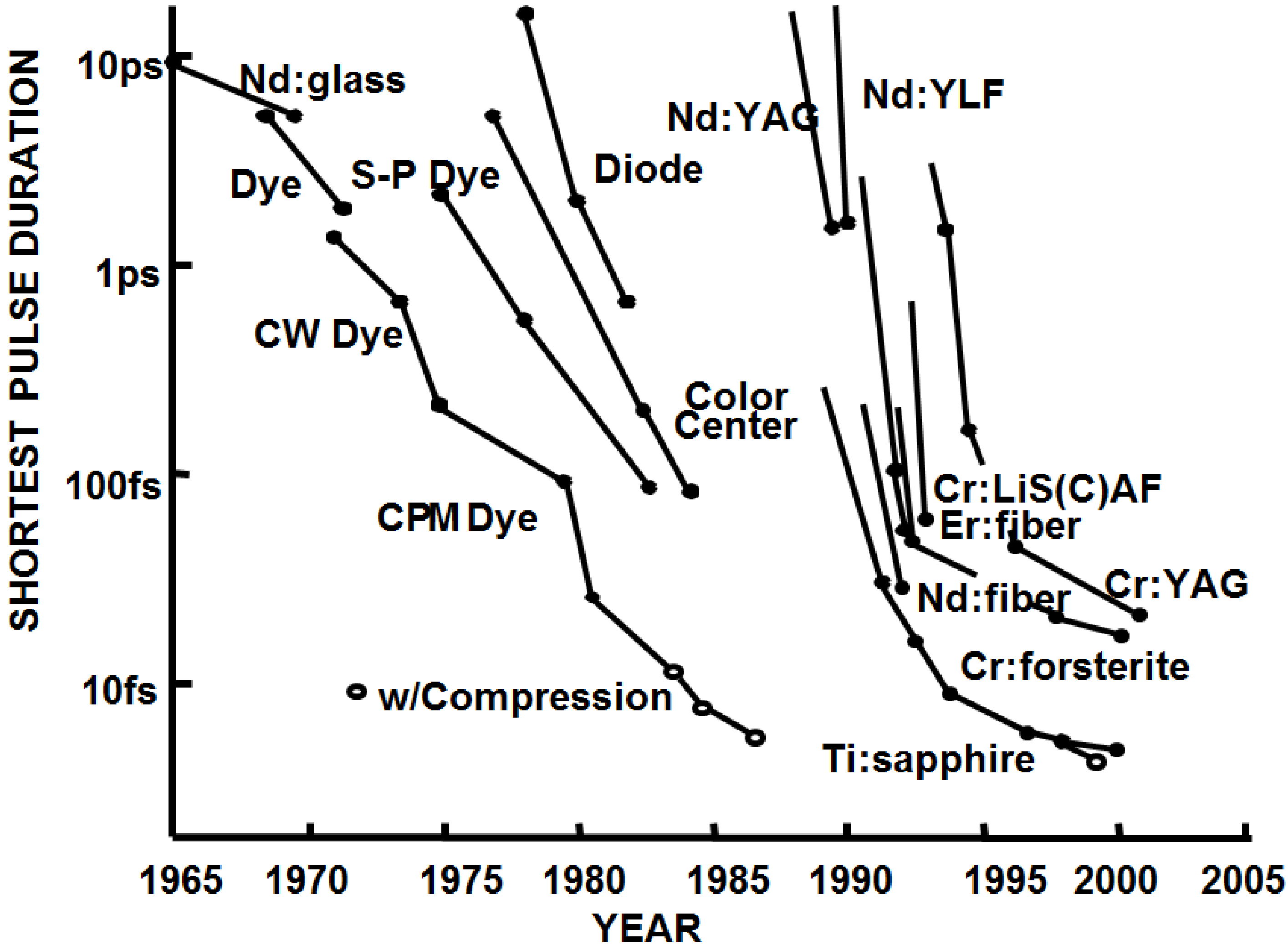


Fig. 6.1: Principles of mode locking

Evolution of shortest pulse duration



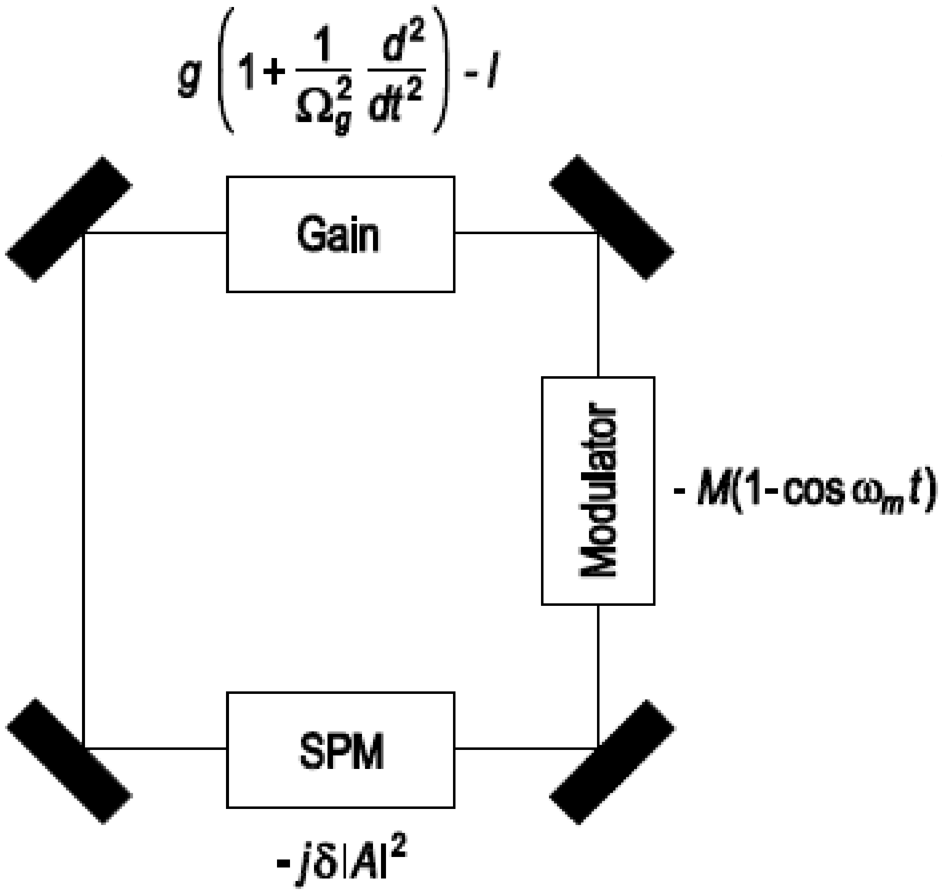
Brief history of mode-locking technology

- 1964— Hargrover, Fork, and Pollack, Locking of He-Ne laser modes induced by synchronous intracavity modulation (Active mode locking)
- 1972— Ippen, Shank, and Dienes, Passive mode locking of the cw dye laser (Passive mode locking using slow saturable absorber (SA))
- 1974— Shank and Ippen, sub-picosecond kilowatt pulses from a mode-locked cw dye laser (Passive mode locking using slow SA)
- 1984— Mollenauer and, The soliton laser (Artificial fast SA mode locking: additive pulse mode locking)
- 1991— Spence, Kean, and Sibbett, 60-fsec pulse generation from a self-mode-locked Ti:Sapphire laser (Artificial fast SA mode locking: Kerr-lens mode locking)
- 1992— Tamura, Haus, and Ippen, Self-starting additive pulse mode-locked erbium fiber ring laser(Artificial fast SA mode locking: nonlinear polarization rotation (NPR) mode locking)
- 1992— Tamura, Ippen, Haus, and Nelson, 77-fs pulse generation from a stretched-pulse mode-locked all-fiber ring laser (Artificial fast SA mode locking: NPR + dispersion managed soliton)

Brief history of mode-locking theory

- 1970— Kuizenga and Siegman, Modulator frequency detuning effects in the FM mode-locked laser (Active mode locking)
- 1975— Haus, Theory of mode locking with a fast saturable absorber (Passive mode locking using fast SA)
- 1975— Haus, Theory of mode locking with a fast saturable absorber (Passive mode locking using slow SA)
- 1984—Martinez, Fork, and Gordon, Theory of passively mode-locked laser including self-phase modulation and group-velocity dispersion (SA mode locking with SPM and GVD)
- 1989— Ippen, Haus, and Liu, Additive pulse mode locking (Artificial fast SA mode locking: additive pulse mode locking)
- 1992— Haus, Fujimoto, and Ippen, Analytic theory of additive pulse and Kerr lens mode locking (Artificial fast SA mode locking)
- 1995— Haus, Tamura, Nelson, and Ippen, Stretched-pulse additive pulse modelocking in fiber ring lasers : Theory and experiment (Artificial fast SA mode locking: NPR)
- 1999— Chen, Kärtner, Morgner, Cho, Haus, Ippen, and Fujimoto, Dispersion-managed mode locking (Artificial fast SA mode locking: dispersion managed)

Master equation of mode-locking



Assume in steady state, the change in the pulse caused by each element in the cavity is small.

$$T_{\mathrm{R}}\frac{\partial A(T,t)}{\partial T}=\sum_i \Delta A_i=0$$

A: the pulse envelope
T_{R}: the cavity round-trip time
T: the time that develops on a time scale of the order of T_R
t: the fast time of the order of the pulse duration
ΔA_i: the changes of the pulse envelope due to different elements in the cavity.

Loss: $$T_R \left. \frac{\partial A(T,t')}{\partial T} \right|_{(loss)} = -l A(T,t') \tag{5.15}$$

Gain: $$T_R \left. \frac{\partial A(T,t')}{\partial T} \right|_{(gain)} = \left(g(T) + D_g \frac{\partial^2}{\partial t'^2} \right) A(T,t'), \qquad D_g = \frac{g(T)}{\Omega_g^2}$$

dispersion

loss

$$\begin{aligned} T_R \frac{\partial A(T,t')}{\partial T} &= -l A(T,t') + j \sum_{n=2}^{\infty} D_n \left(j \frac{\partial}{\partial t'} \right)^n A(T,t') \\ &+ g(T) \left(1 + \frac{1}{\Omega_g^2} \frac{\partial^2}{\partial t'^2} \right) A(T,t') \\ &- q(T,t') A(T,t') - j\delta |A(T,t')|^2 A(T,t'). \end{aligned} \tag{5.21}$$

gain

Mode-locking element

Gain dispersion

Self-phase modulation

Saturable absorber for cavity loss modulation

Saturable absorber: an optical passive device, which introduces large loss for low optical intensities and small loss at high optical intensities.

$$\frac{dq}{dt} = -\frac{q-q_0}{\tau_A} - \frac{q|A(t)|^2}{E_A}$$

$A(t)$ is normalized such that $|A(t)|^2 = power$

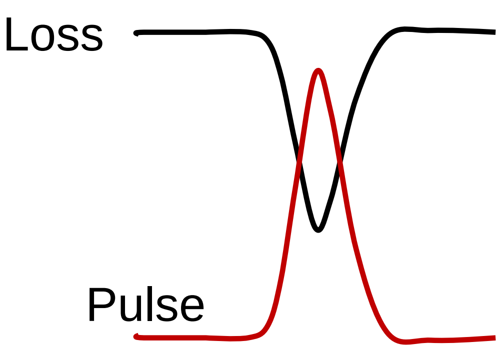
τ_A Relaxation time, 1-100 ps

E_A Saturation energy

Fast saturable absorber $\tau_A << \tau_p$

$$dq/dt = 0$$

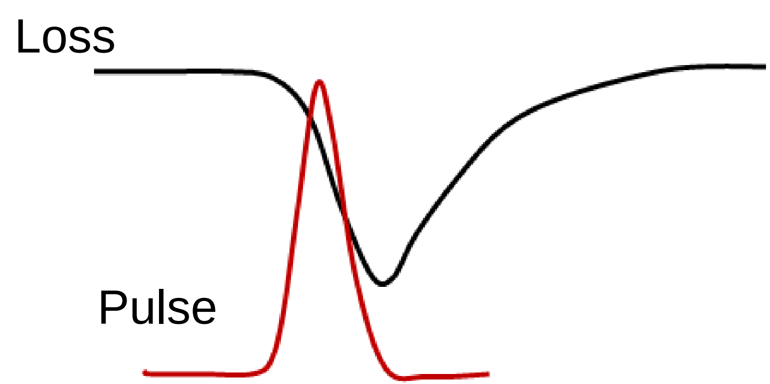
$$q(t) = \frac{q_0}{1+\frac{|A(t)|^2}{E_A/\tau_A}} = \frac{q_0}{1+\frac{|A(t)|^2}{P_A}}$$



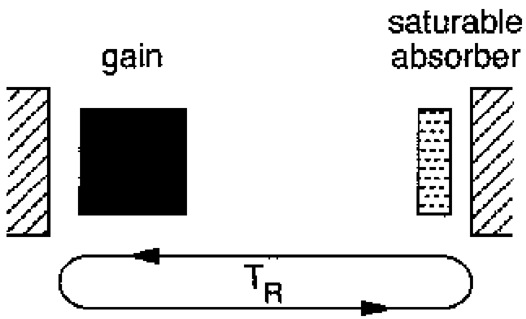
Slow saturable absorber $\tau_A >> \tau_p$

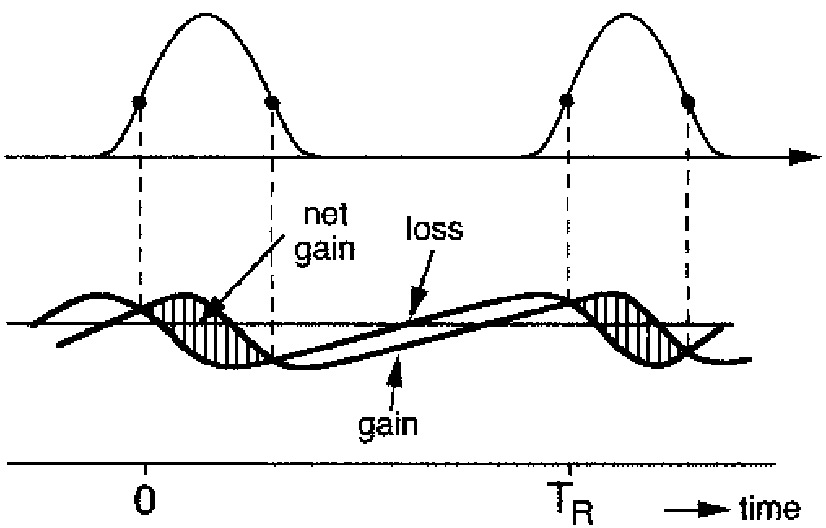
$$\frac{dq}{dt} \approx -\frac{q|A(t)|^2}{E_A} \qquad E(t) = \int_{-T_R/2}^{t} dt\,|A(t)|^2$$

$$q(t) = q_0 \exp[-E(t)/E_A]$$



6.1 Slow SA mode locking





No fast element necessary:
Both absorber and gain
may recover on ns-time scale

Master equation for mode-locking

$$T_R \frac{\partial A(T,t)}{\partial T} = \left[g(t) - q(t) - l + D_f \frac{\partial^2}{\partial t^2} \right] A(T,t)$$

Background loss

Fixed filtering / finite bandwidth

For slow gain medium:

$$\frac{dg}{dt} = -\frac{g - g_0}{\tau_L} - g\frac{|A(t)|^2}{E_L} \qquad \tau_L >> \tau_p$$

$$\frac{dg}{dt} \approx -g\frac{|A(t)|^2}{E_L} \qquad E(t) = \int_{-T_{R/2}}^{t} dt |A(t)|^2$$

$$g(t) = g_i \exp\left[-E(t)/E_L\right]$$

$$\approx g_i \left[1 - (E(t)/E_L) + \frac{1}{2} (E(t)/E_L)^2\right]$$

For slow SA:

$$q(t) = q_0 \exp\left[-E(t)/E_A\right]$$

$$\approx q_0 \left[1 - (E(t)/E_A) + \frac{1}{2} (E(t)/E_A)^2\right]$$

Slow SA mode locking

$$T_R \frac{\partial A(T,t)}{\partial T} = g_i \left[1 - (E(t)/E_L) + \frac{1}{2}(E(t)/E_L)^2\right] A(T,t)$$

$$- q_0 \left[1 - (E(t)/E_A) + \frac{1}{2}(E(t)/E_A)^2\right] A(T,t)$$

$$- \; l \; A(T,t) + D_f \frac{\partial^2}{\partial t^2} \; A(T,t)$$

$$E(t) = \int_{-T_{R/2}}^{t} dt |A(t)|^2$$

Trial solution:

$$A(t,T) = A_o \operatorname{sech}\left(\frac{t}{\tau} + \alpha \frac{T}{T_R}\right)$$

$$T_R \frac{\partial}{\partial T} A(t,T) = -\alpha \tanh\left(\frac{t}{\tau} + \alpha \frac{T}{T_R}\right) A(t,T)$$

$$\frac{1}{\Omega_f^2} \frac{\partial^2}{\partial t^2} A(t,T) = \frac{1}{\Omega_f^2 \tau^2} \left(1 - 2\operatorname{sech}^2\left(\frac{t}{\tau} + \alpha \frac{T}{T_R}\right)\right) A(t,T)$$

α is the fraction of the pulsewidth. The pulse is shifted in each round-trip due to the shaping by loss and gain.

$$E(t) = \int_{-T_{R/2}}^{t} dt |A(t)|^2 = \frac{W}{2} \left(1 + \tanh\left(\frac{t}{\tau} + \alpha \frac{T}{T_R}\right)\right)$$

$$E(t)^2 = \left(\frac{W}{2}\right)^2 \left(2 + 2\tanh\left(\frac{t}{\tau} + \alpha \frac{T}{T_R}\right) - \operatorname{sech}^2\left(\frac{t}{\tau} + \alpha \frac{T}{T_R}\right)\right)$$

W **Total pulse energy**

Slow SA mode locking

The constant term gives the necessary small signal gain:

$$g_i\left[1-\frac{W}{2E_L}+\left(\frac{W}{2E_L}\right)^2\right]=l+q_0\left[1-\frac{W}{2E_A}+\left(\frac{W}{2E_A}\right)^2\right]-\frac{1}{\Omega_f^2\tau^2}$$

The constant in front of the odd tanh −function delivers the timing shift per round-trip:

$$\alpha=\frac{\Delta t}{\tau}=g_i\left[\frac{W}{2E_L}-\left(\frac{W}{2E_L}\right)^2\right]-q_0\left[\frac{W}{2E_A}-\left(\frac{W}{2E_A}\right)^2\right]$$

The constant in front of the sech^2-function determines the pulsewidth:

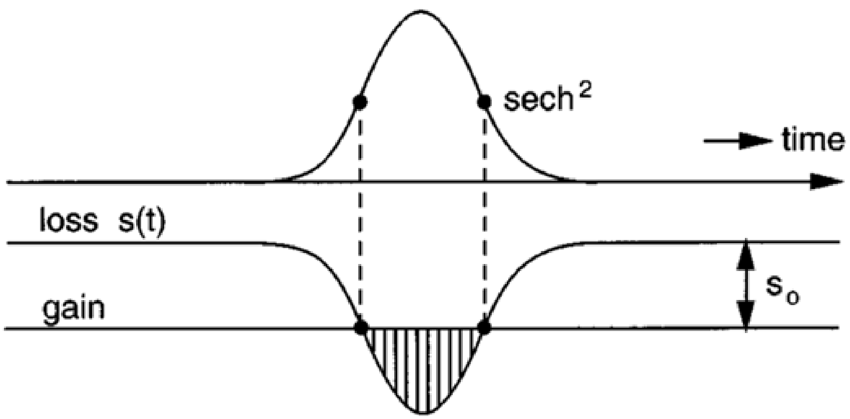
$$\frac{1}{\tau^2}=\frac{\Omega_f^2W^2}{8}\left(\frac{q_0}{E_A^2}-\frac{g_i}{E_L^2}\right)$$

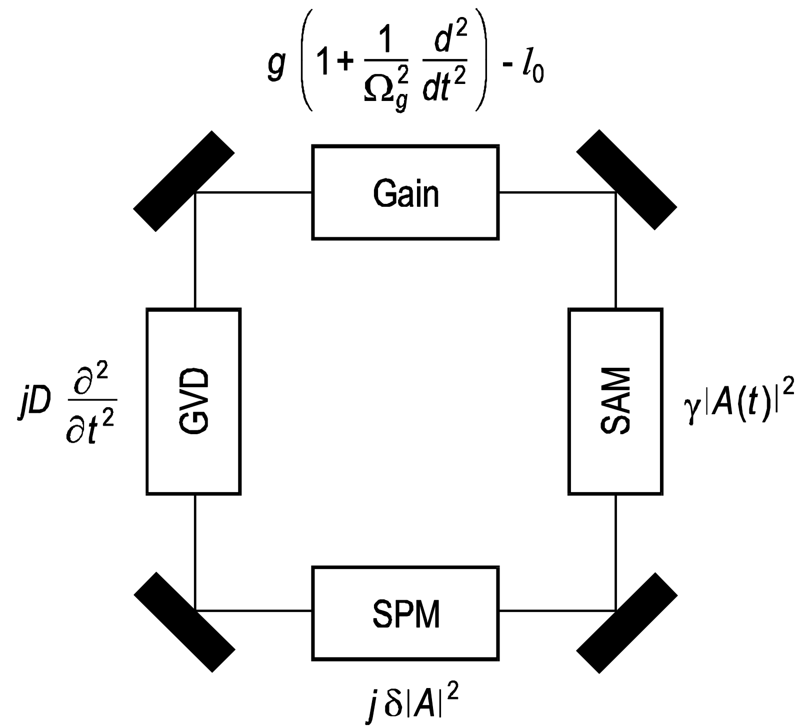
This implies that $q_0/E_A^2 > g_i/E_L^2$

SA must saturate more easily and therefore more strongly than the gain medium to open a net gain window.

Shortest pulse width possible: $\tau=\frac{2\sqrt{2}}{\sqrt{q_0}\Omega_f}\frac{E_A}{W}$

6.2 Fast SA mode locking





SA responds to instantaneous power:

$$q(A) = \frac{q_0}{1 + \frac{|A|^2}{P_A}}$$

Approximately: $q(A) = q_0 - \gamma|A|^2$

$\gamma = q_0/P_A$ is SA modulation coefficient.

$$T_R \frac{\partial A(T,t)}{\partial T} = \left[g - l_0 + D_f \frac{\partial^2}{\partial t^2} + \underbrace{\gamma|A|^2}_{\text{SAM}} + \underbrace{jD_2 \frac{\partial^2}{\partial t^2}}_{\text{Dispersion}} \underbrace{- j\delta|A|^2}_{\text{SPM}} \right] A(T,t)$$

$l_0 = l + q_0$

SAM: self-amplitude modulation

Fast SA mode locking without GDD and SPM

$$T_R \frac{\partial A(T,t)}{\partial T} = \left[g - l_0 + D_f \frac{\partial^2}{\partial t^2} + \gamma |A|^2 \right] A(T,t)$$

$$T_R \frac{\partial A_s(T,t)}{\partial T} = 0 \longrightarrow A_s(T,t) = A_s(t) = A_0 \mathrm{sech}\left(\frac{t}{\tau}\right)$$

$$\left[(g - l_0) + \frac{D_f}{\tau^2} \left[1 - 2\mathrm{sech}^2\left(\frac{t}{\tau}\right) \right] + \gamma |A_0|^2 \mathrm{sech}^2\left(\frac{t}{\tau}\right) \right] \cdot A_0 \mathrm{sech}\left(\frac{t}{\tau}\right) = 0$$

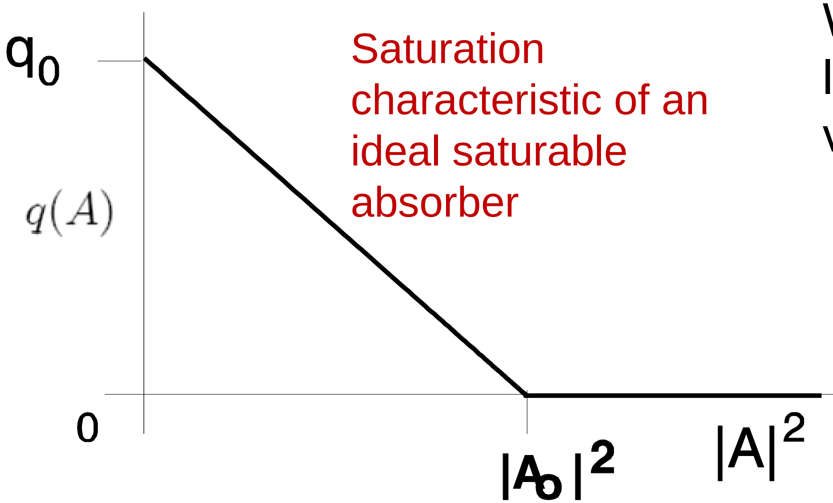
Comparison of the coefficients with the sech- and sech^3-expressions results in the conditions for the pulse peak intensity and pulse width and for the saturated gain:

$$\frac{D_f}{\tau^2} = \frac{1}{2} \gamma |A_0|^2 \qquad g = l_0 - \frac{D_f}{\tau^2}$$

Pulse Energy: $W = 2A_0^2 \tau \longrightarrow \tau = \dfrac{4D_f}{\gamma W}$

This expression is rather similar to the soliton width except that the conservative pulse shaping effects due to GDD and SPM are replaced by gain filtering and saturable absorption.

Fast SA mode locking without GDD and SPM



We assumed that the absorber saturates linearly with intensity up to a maximum value q0:

$$q(A) = q_0 - \gamma |A|^2 = q_0 \left(1 - \frac{|A|^2}{A_0^2}\right)$$

$$q_0 = \gamma A_0^2$$

$$q_0 = \gamma A_0^2$$

$$\frac{D_f}{\tau^2} = \frac{1}{2}\gamma |A_0|^2$$

$$D_f = D_g = \frac{g}{\Omega_g^2}$$

$$\frac{D_f}{\tau^2} = \frac{q_0}{2}$$

$$g = l_0 - \frac{D_f}{\tau^2}$$

$$l_0 = l + q_0$$

$$\tau = \sqrt{\frac{2g}{q_0}\frac{1}{\Omega_g}}$$

$$g = l + \frac{q_0}{2}$$

Minimum Pulse Width:

$$\tau_{\min} = \frac{1}{\Omega_g}$$

Example:
Ti:sapphire laser

$\Omega_g = 270 \text{ THz}$ $\quad \tau_{\min} = 3.7$ $\quad \tau_{FWHM} = 6.5 \text{ fs}$

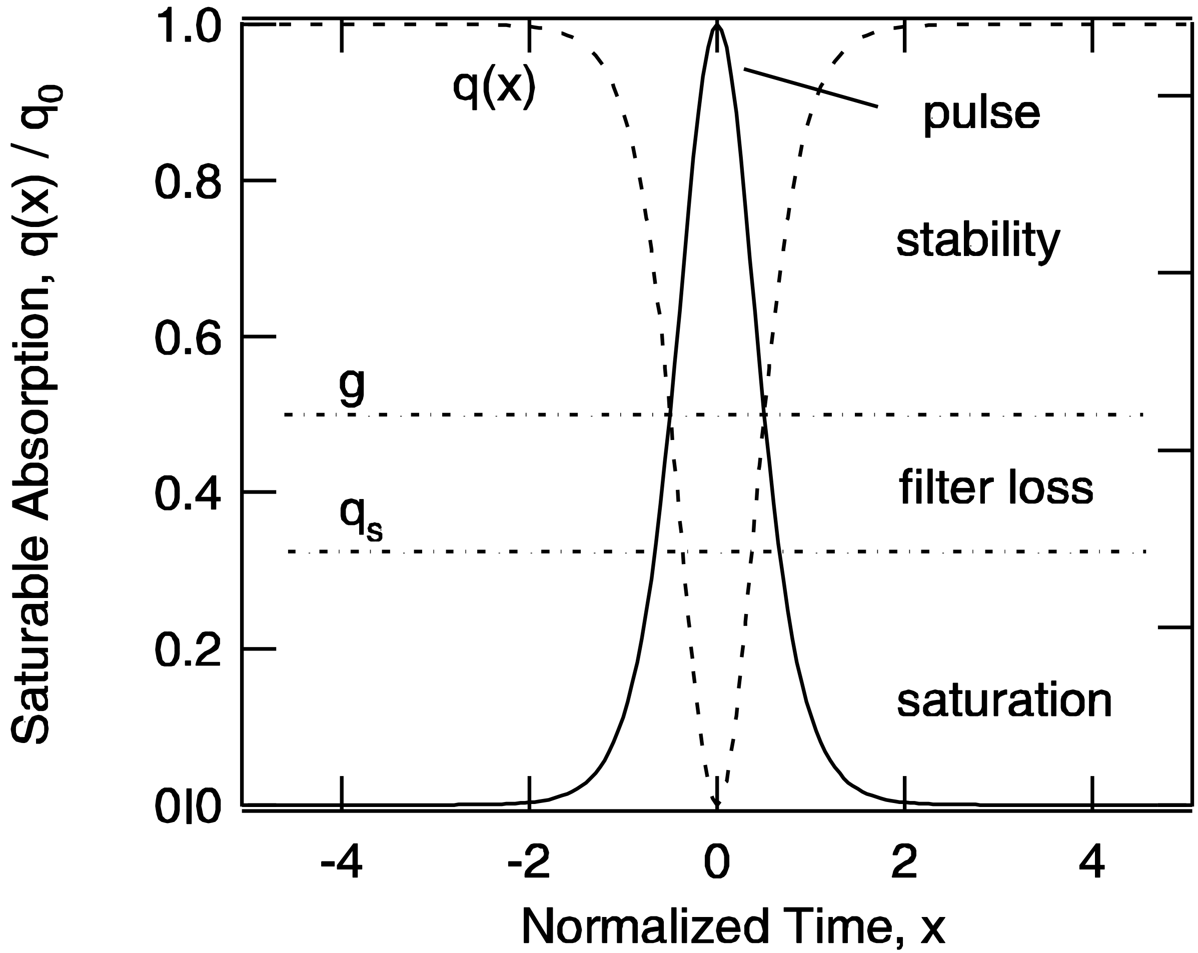


Fig. 6.4: Gain and loss in a fast saturable absorber (FSA) modelocked laser

Fast SA mode locking with GDD and SPM

$$T_R \frac{\partial A(T,t)}{\partial T} = \left[g - l_0 + D_f \frac{\partial^2}{\partial t^2} + \gamma |A|^2 + jD_2 \frac{\partial^2}{\partial t^2} - j\delta |A|^2 \right] A(T,t)$$

Steady-state solution is chirped sech-shaped pulse with 4 free parameters:

$$A_s(T,t) = A_0 \left(\mathrm{sech}\left(\frac{t}{\tau}\right)\right)^{(1+j\beta)} e^{j\psi T/T_R}$$

Pulse amplitude: A_0 or Energy: $W = 2 A_0{}^2 \tau$

Pulse width: τ

Chirp parameter : β

Carrier-Envelope phase shift : ψ

Substitute above trial solution into the master equation and comparing the coefficients to the same functions leads to two complex equations:

$$\frac{1}{\tau^2}(D_f + jD_2)\left(2 + 3j\beta - \beta^2\right) = (\gamma - j\delta)|A_0|^2 \qquad (6.49)$$

$$l_0 - \frac{(1+j\beta)^2}{\tau^2}(D_f + jD_2) = g - j\psi \qquad (6.50)$$

Fast SA mode locking with GDD and SPM

The real part and imaginary part of Eq.(6.49) give

$$\frac{1}{\tau^2}\left[D_f\left(2-\beta^2\right)-3\beta D_2\right] = \gamma|A_0|^2 \qquad (6.52)$$

$$\frac{1}{\tau^2}\left[D_2\left(2-\beta^2\right)+3\beta D_f\right] = -\delta|A_0|^2 \qquad (6.53)$$

Normalized parameters:

Normalized nonlinearity

$$\delta_n = \delta/\gamma$$

Normalized dispersion

$$D_n = D_2/D_f$$

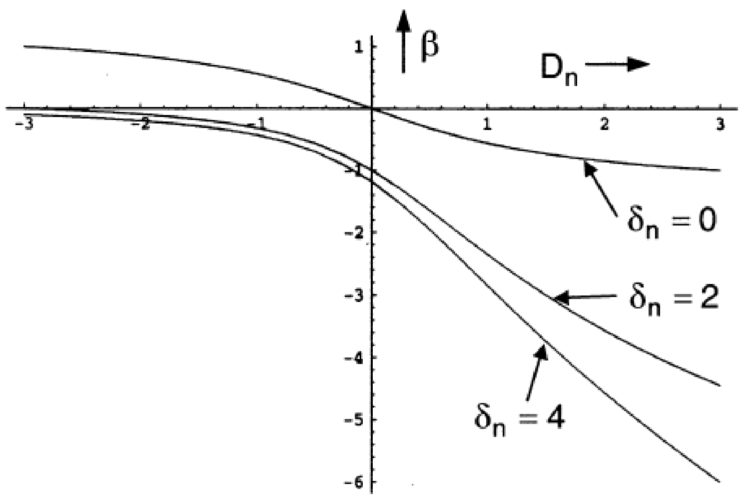
Dividing Eq.(6.53) by (6.52) leads to a quadratic equation for the chirp:

$$\frac{D_n\left(2-\beta^2\right)+3\beta}{\left(2-\beta^2\right)-3\beta D_n} = -\delta_n \longrightarrow \frac{3\beta}{2-\beta^2} = \frac{\delta_n + D_n}{-1+\delta_n D_n} \equiv \frac{1}{\chi} \qquad (6.54)$$

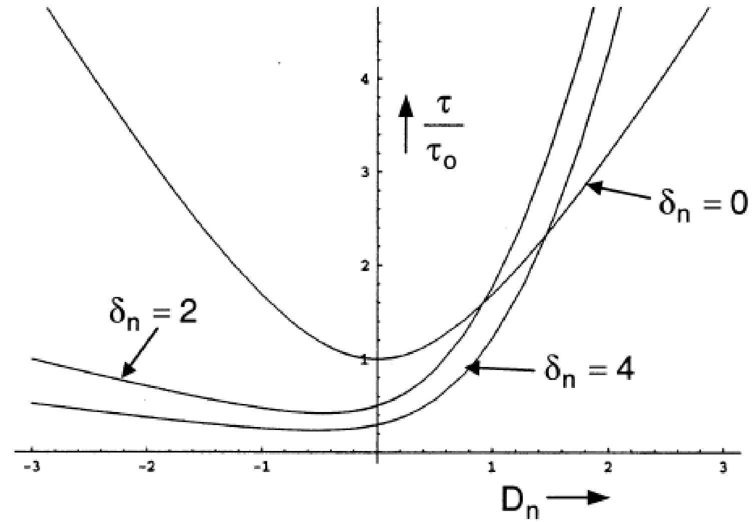
depends only on the system parameters

Fast SA mode locking with GDD and SPM

Chirp $\beta = -\frac{3}{2}\chi \pm \sqrt{\left(\frac{3}{2}\chi\right)^2 + 2}$



Pulse width $\tau = \frac{3\tau_0}{2}\beta\left(\chi - D_n\right)$



- strong soliton-like pulse shaping if $\delta_n \gg 1$ and $-D_n \gg 1$ the chirp is always much smaller than for positive dispersion and the pulses are solitonlike.
- pulses are even chirp free if $\delta_n = -D_n$, with the shortest with directly from the laser, which can be a factor 2-3 shorter than by pure SA modelocking.
- Without SPM and GDD, SA has to shape the pulse. When SPM and GDD included, they can shape the pulse via soliton formation; SA only has to stabilize the pulse.

Fast SA mode locking with GDD and SPM

$$l_0 - \frac{(1+j\beta)^2}{\tau^2}(D_f + jD_2) \;=\; g - j\psi \qquad (6.50)$$

The real part of Eq.(6.50) gives the saturated gain:

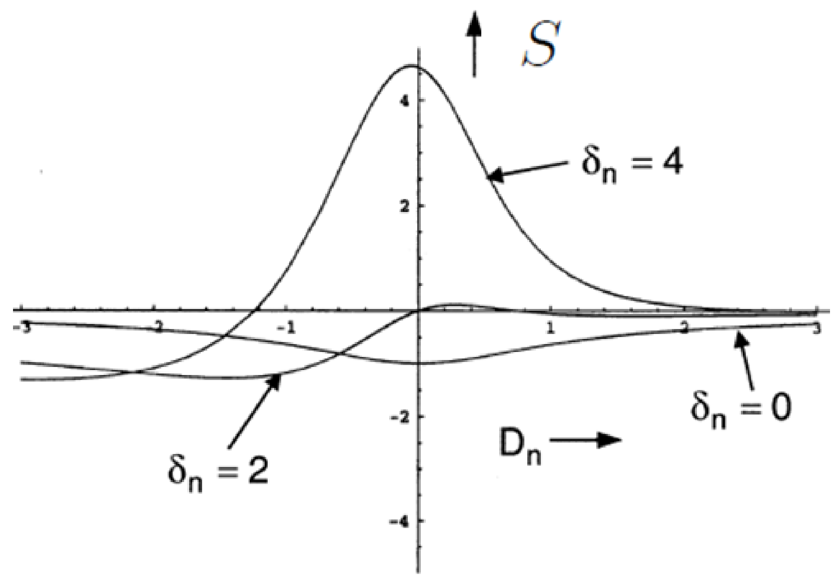
$$g = l_0 - \frac{1-\beta^2}{\tau^2} D_f + \frac{2\beta D_2}{\tau^2}$$

A necessary but not sufficient criterion for the pulse stability is that there must be net loss leading and following the pulse:

$$g - l_0 = -\frac{1-\beta^2}{\tau^2} D_f + \frac{2\beta D_2}{\tau^2} < 0$$

If we define the stability parameter S

$$S = 1 - \beta^2 - 2\beta D_n < 0$$



- Without SPM, the pulses are always stable.
- Excessive SPM can lead to instability near zero dispersion and for positive dispersion.

Soliton mode locking with slow SA

$$T_R \frac{\partial A(T,t)}{\partial T} = \left[g - l + (D_f + jD)\frac{\partial^2}{\partial t^2} - j\delta |A(T,t)|^2 - q(T,t) \right] A(T,t).$$

In the case of strong soliton-like pulse shaping, the absorber doesn't have to be really fast, because the pulse is shaped by GDD and SPM and the absorber has only to stabilize the soliton against the continuum.

$$\frac{\partial q(T,t)}{\partial t} = -\frac{q - q_0}{\tau_A} - \frac{|A(T,t)|^2}{E_A}.$$

$$A(T,t) = \left(\underbrace{A\,\mathrm{sech}\left(\frac{t}{\tau}\right)}_{\text{soliton}} + \underbrace{a_c(T,t)}_{\text{continuum}} \right) e^{-j\phi_0 \frac{T}{T_R}}$$

g Saturation gain $\quad$ l_c Loss for continuum

q_s Averaged SA saturation loss

Stable modelocking condition: $l_c > g$

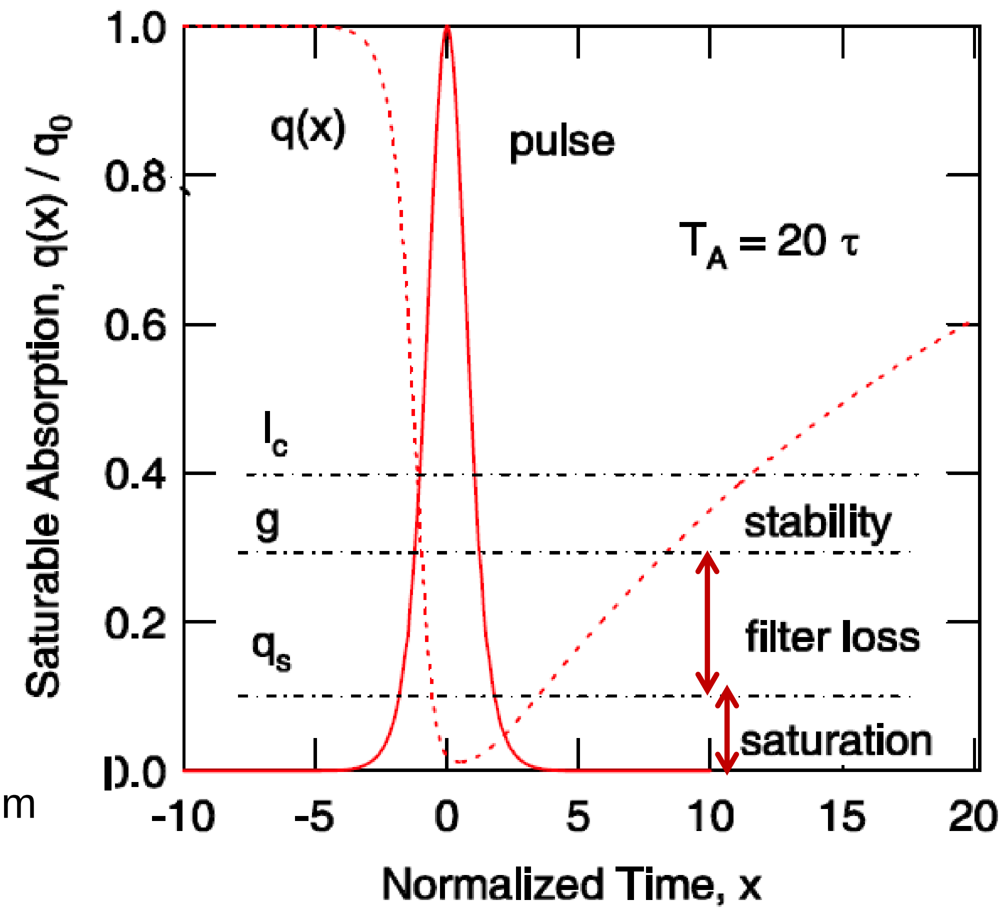
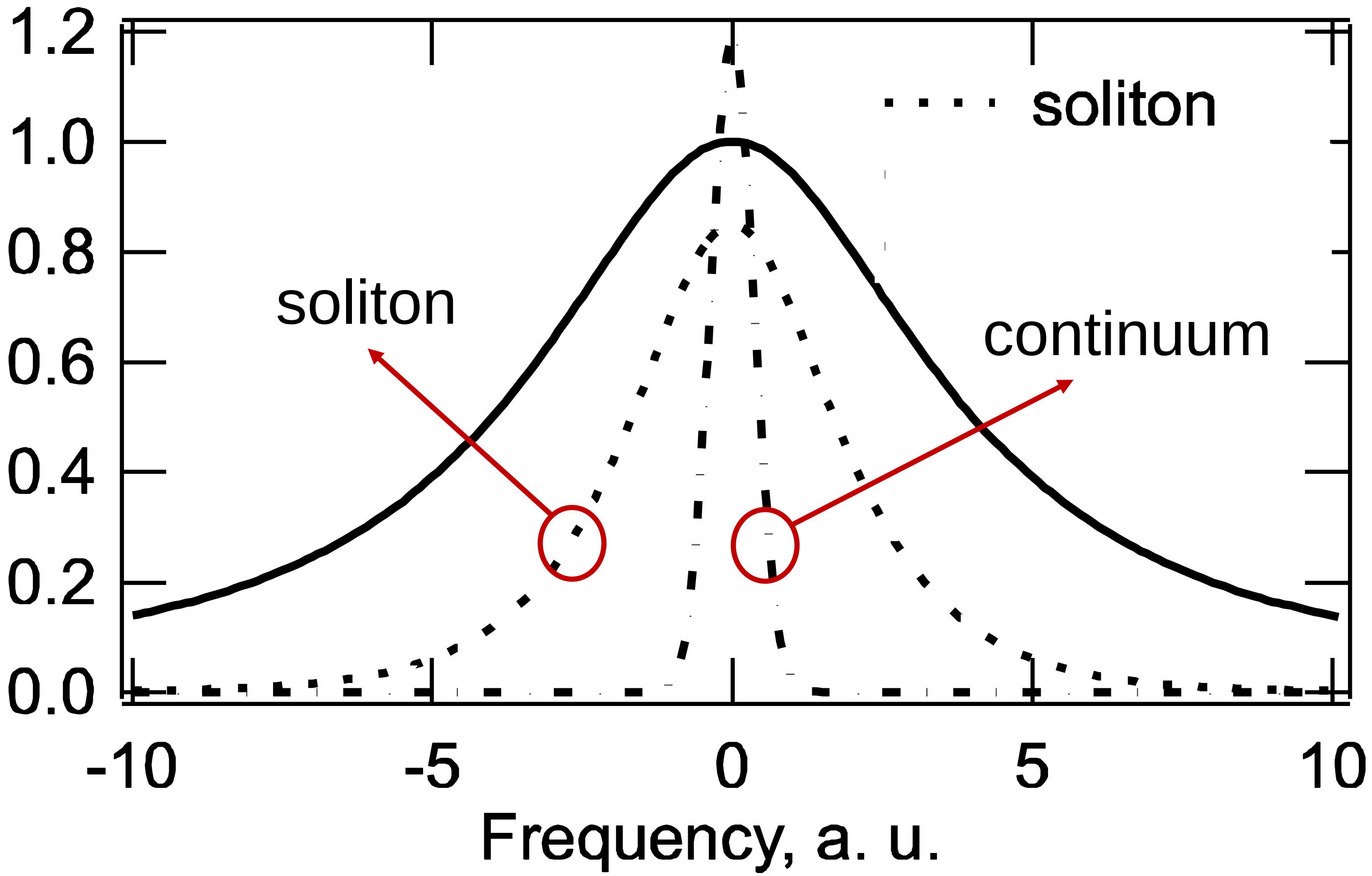


Fig. 6.7: Soliton modelocking

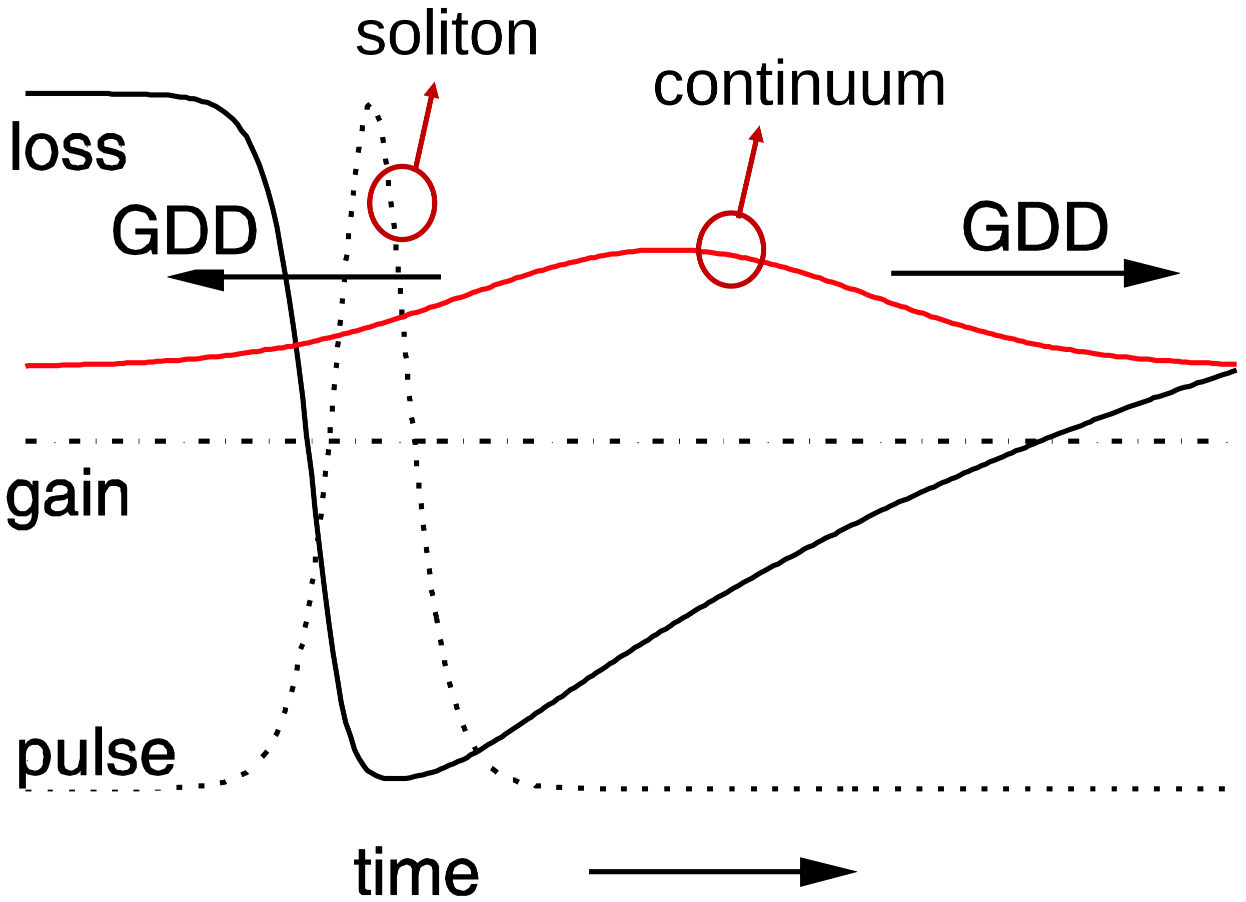
Soliton mode locking with slow SA



The continuum can be viewed as a long pulse competing with the soliton for the available gain. In the frequency domain, the soliton has a broader spectrum compared to the continuum. Therefore, continuum experiences peak of the gain, whereas the soliton on average experiences less gain.

→ **Gain filtering effect leads to faster growing of continuum.**

Soliton mode locking with slow SA



- Advantage in gain of the continuum has to be compensated for in the time domain by the saturable absorber response.
- Whereas for the soliton, there is a balance of the nonlinearity and the dispersion, this is not so for the continuum. Therefore, the continuum is spread by the dispersion into the regions of high absorption.

Soliton mode locking with slow SA: a case study

- Rule of thumb: absorber recovery time can be about 10 times longer than the soliton width.
- Lowering the dispersion increases the bandwidth of the soliton and therefore its loss, while lowering at the same time the loss for the continuum.
- At the dispersion D = −500 fs^2 the laser becomes unstable by break through of the continuum.
- Reducing the dispersion even further might lead again to more stable but complicated spectra related to formation of higher order solitons.

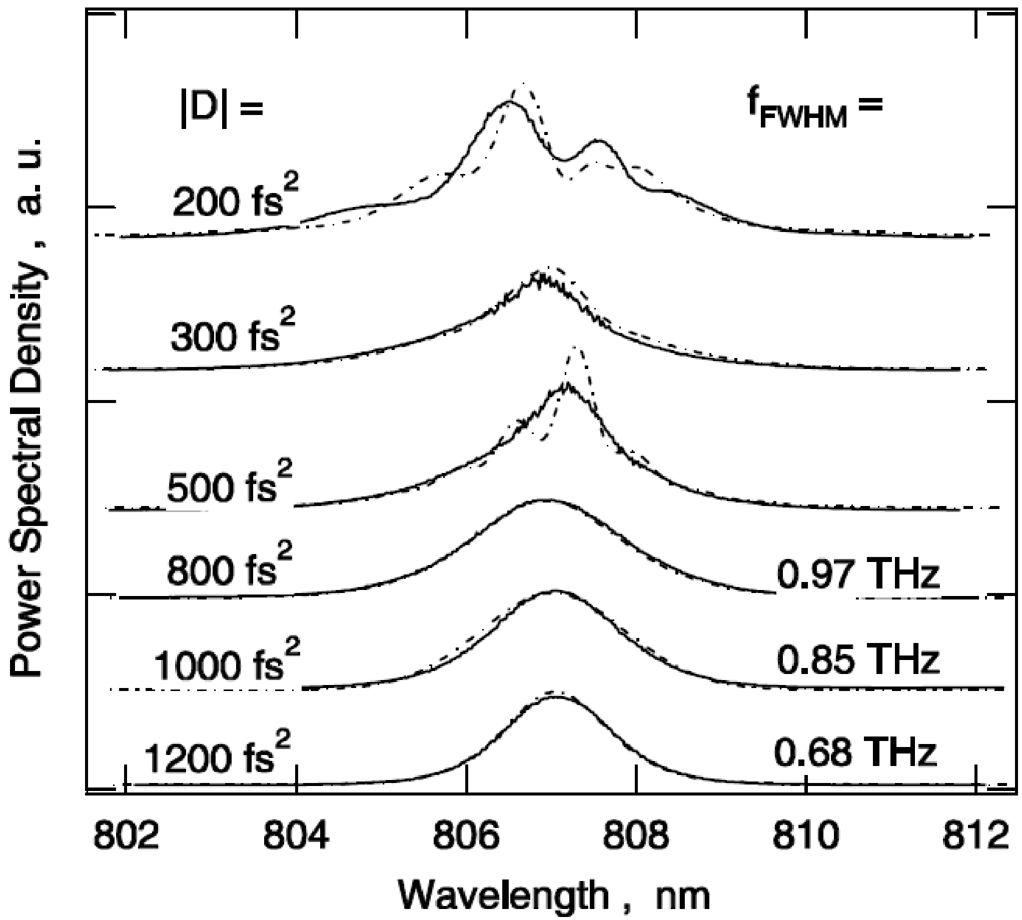


Fig. 6.10: Measured (---) and simulated (- - -) spectra from a semiconductor saturable absorber modelocked Ti:sapphire laser for different net intracavity dispersion.

Soliton mode locking with slow SA: a case study

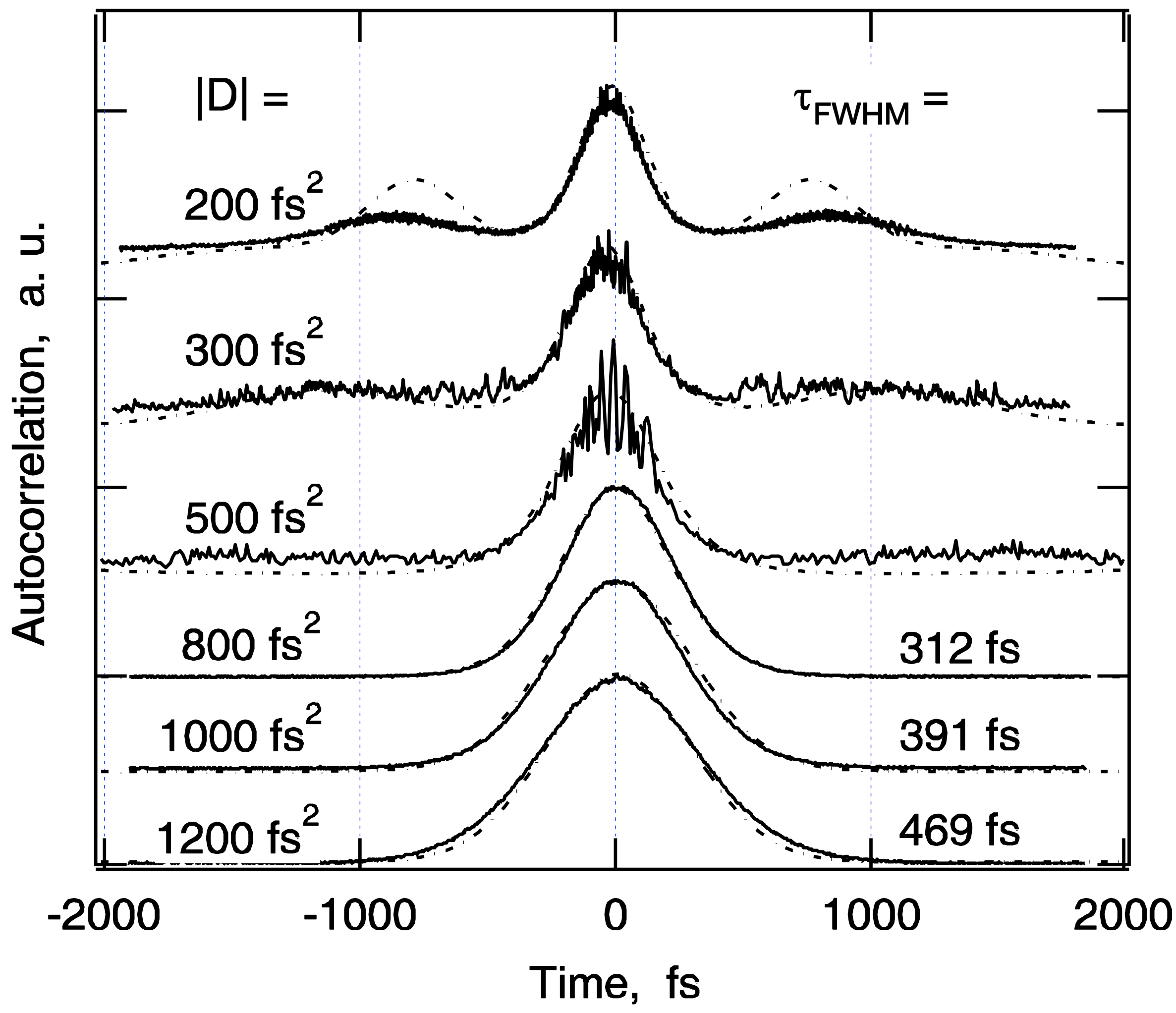
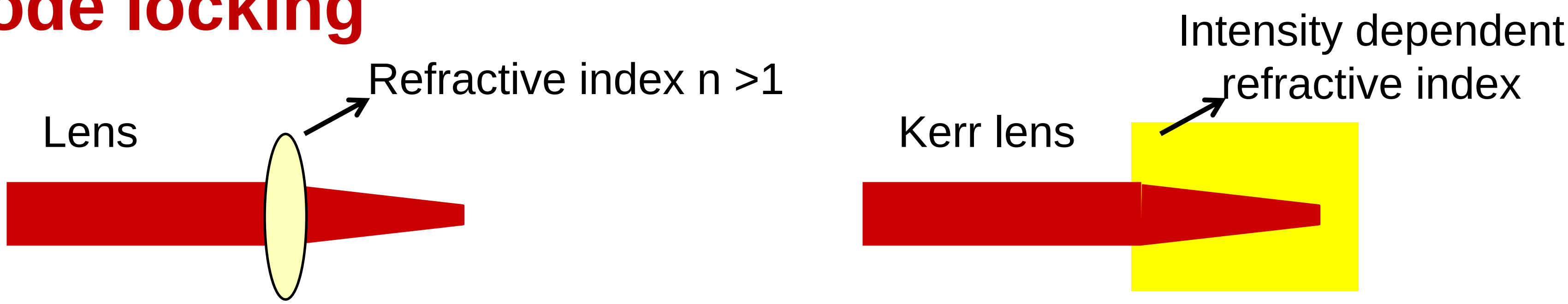
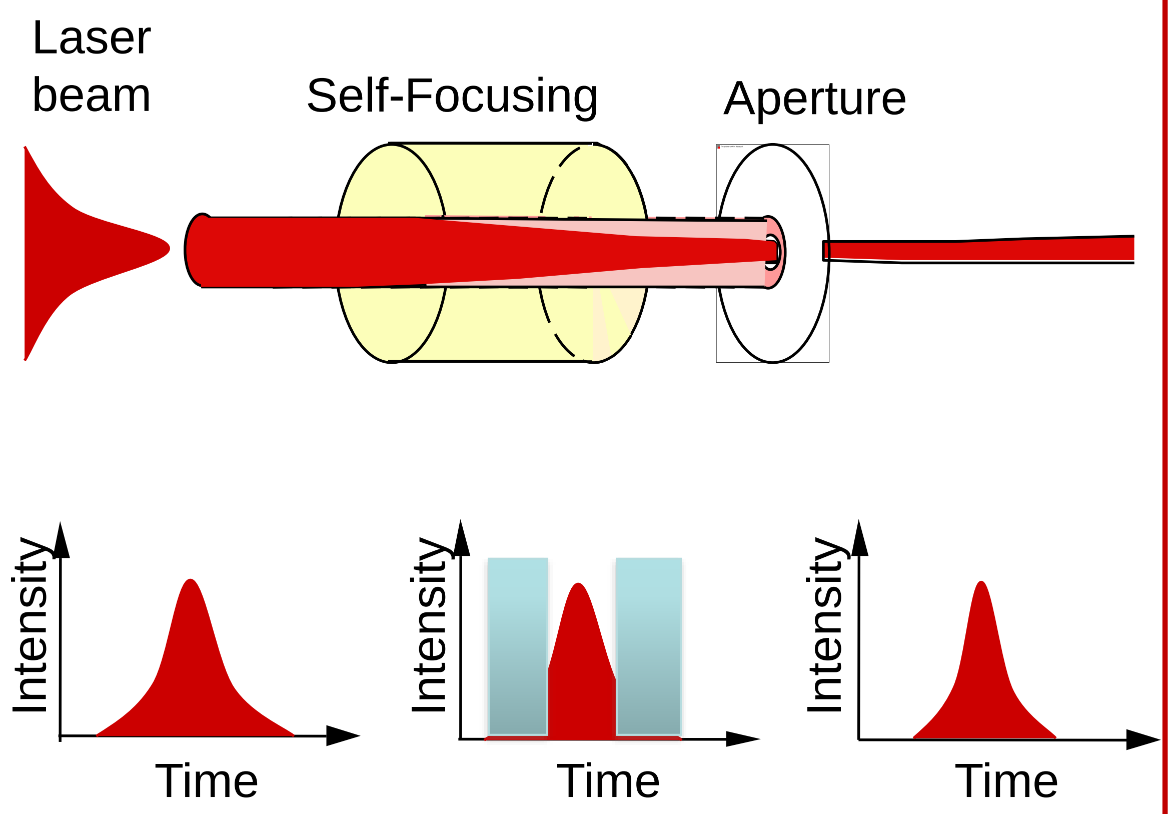


Fig. 6.11: Measured (----) and simulated (- - -) autocorrelations corresponding to the spectra shown in Figure 6.10

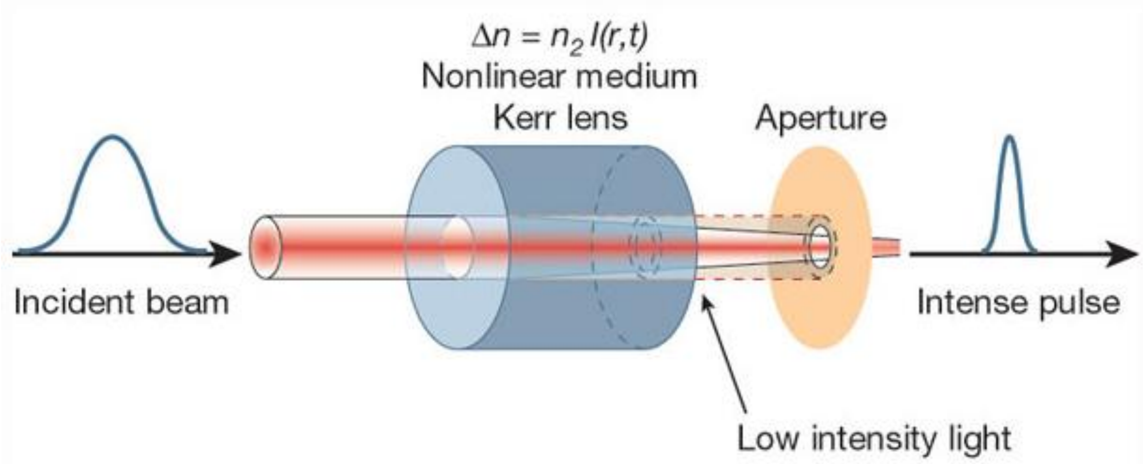
Mode locking using artificial fast SA: Kerr-lens mode locking



- A spatial-temporal laser pulse propagating through the Kerr medium has a time dependent mode size: pulse peak corresponds to smaller beam size than the wings.
- A hard aperture placed at the right position in the cavity strips of the wings of the pulse, shortening the pulse.
- The combined mechanism is equivalent to a fast saturable absorber.

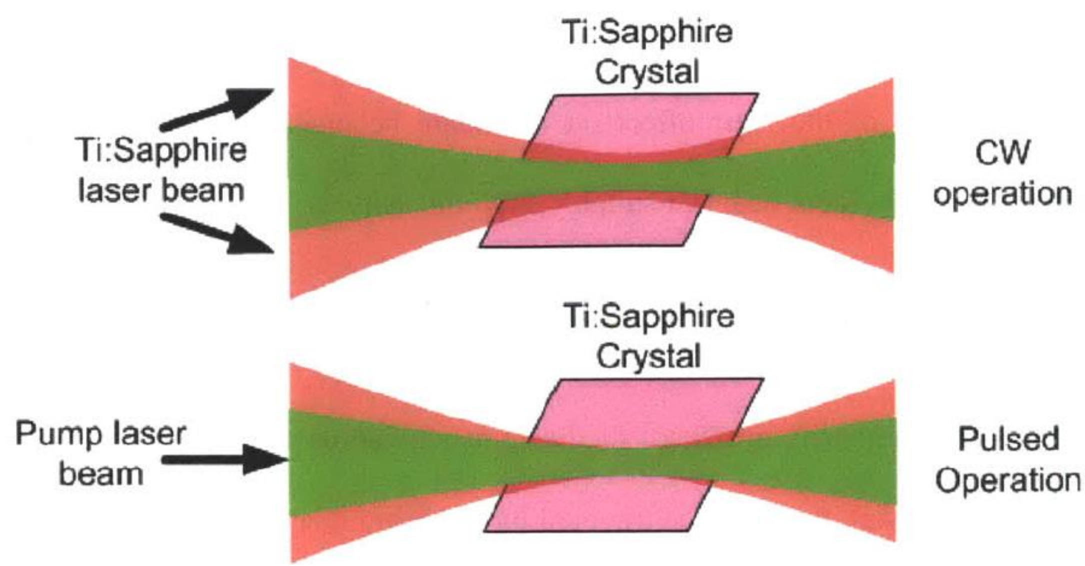


Kerr-lens mode locking: hard aperture versus soft aperture



Hard-aperture Kerr-lens mode-locking: a hard aperture placed at the right position in the cavity attenuates the wings of the pulse, shortening the pulse.

Soft-aperture Kerr-lens mode-locking: gain medium can act both as a Kerr medium and as a soft aperture (i.e. increased gain instead of saturable absorption). In the CW case the overlap between the pump beam and laser beam is poor, and the mode intensity is not high enough to utilize all of the available gain. The additional focusing provided by the high intensity pulse improves the overlap, utilizing more of the available gain.



Mode locking using artificial fast SA: additive pulse mode locking

- A small fraction of the light emitted from the main laser cavity is injected externally into a nonlinear fiber. In the fiber strong SPM occurs and introduces a significant phase shift between the peak and the wings of the pulse. In the case shown the phase shift is π
- A part of the modified and heavily distorted pulse is reinjected into the main cavity in an interferometrically stable way, such that the injected pulse interferes constructively with the next cavity pulse in the center and destructively in the wings.
- This superposition leads to a shorter intracavity pulse and the pulse shaping generated by this process is identical to the one obtained from a fast saturable absorber.

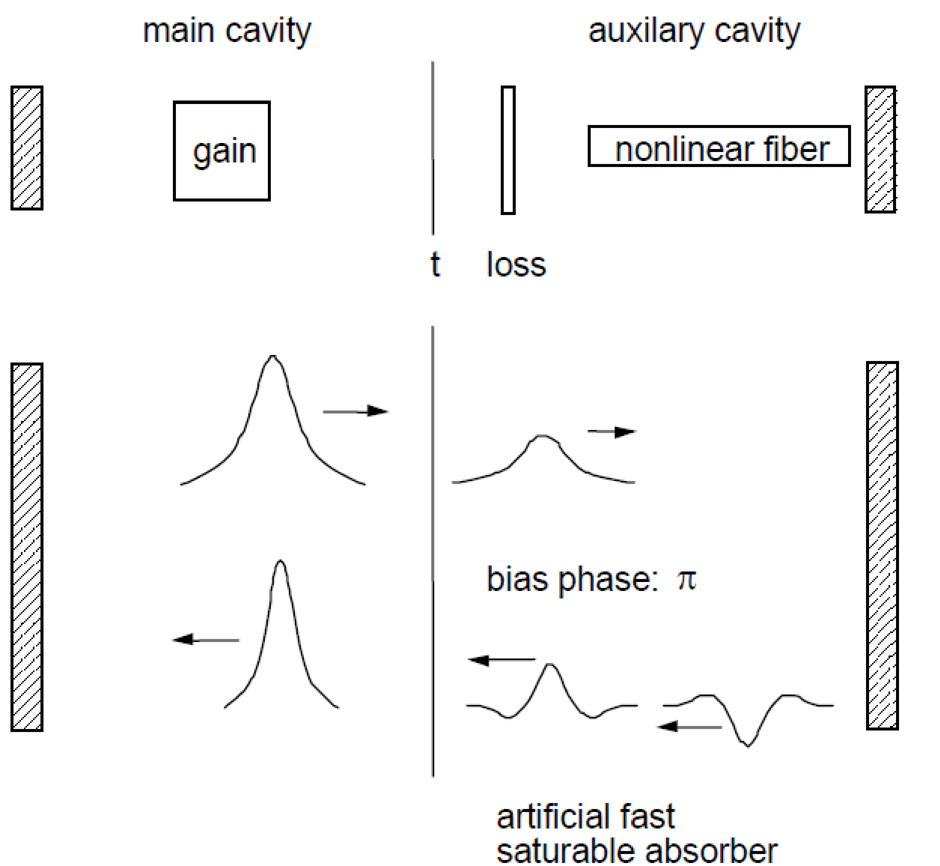
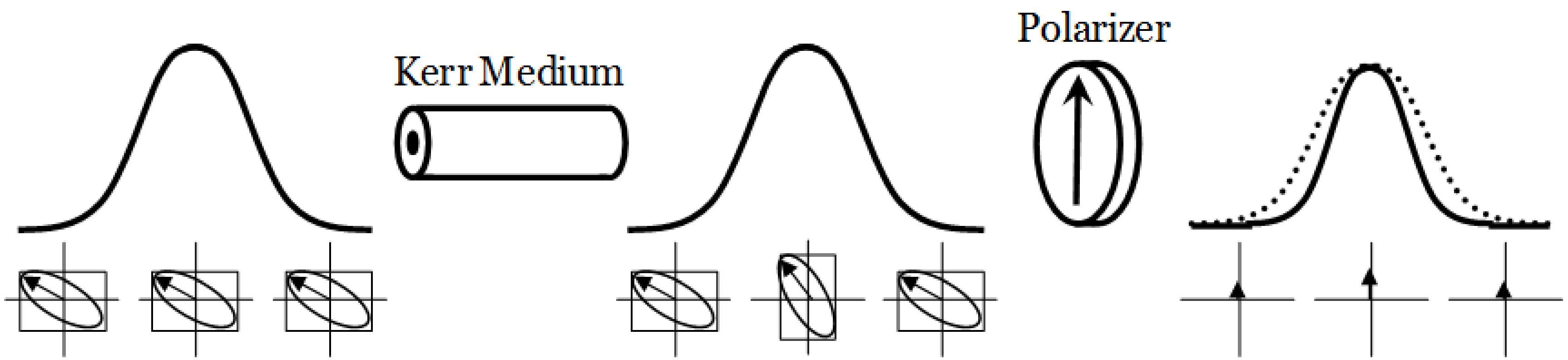


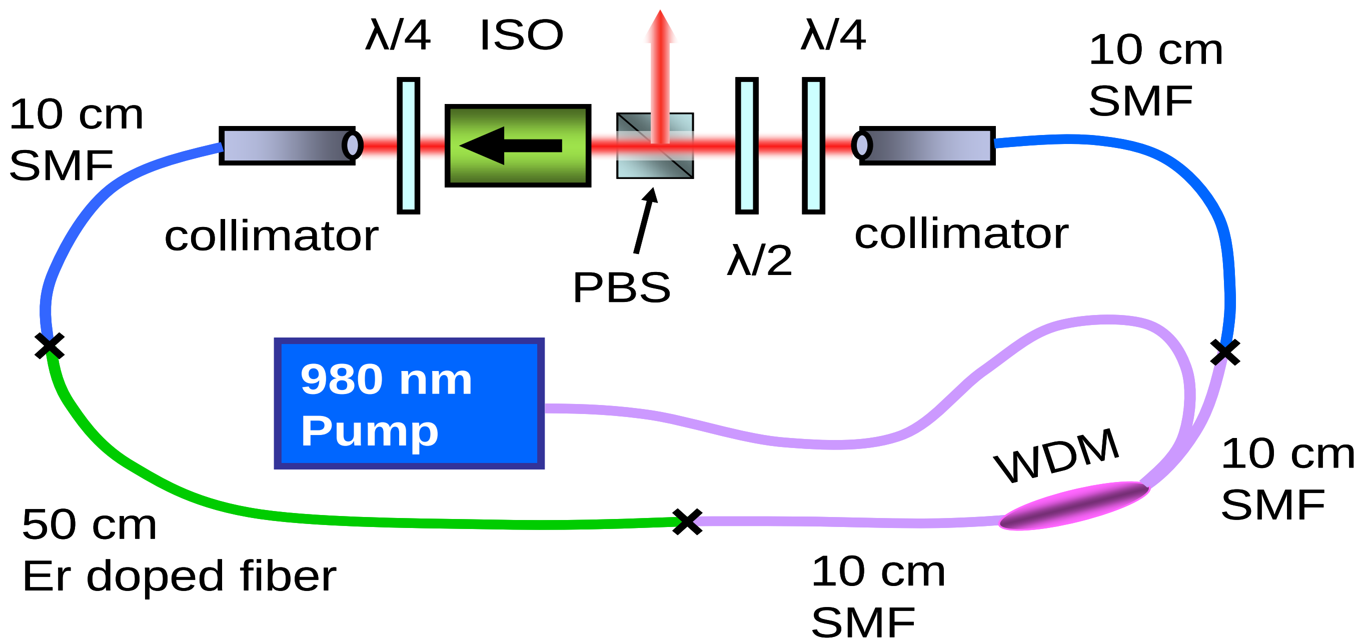
Fig. 7.17: Principle mechanism of additive pulse mode locking

Additive pulse mode locking using nonlinear polarization rotation in a fiber



- When an intense optical pulse travels in an isotropic optical fiber, intensity-dependent change of the polarization state can happen.
- The polarization state of the pulse peak differs from that of the pulse wings after the fiber section due to Kerr effect.
- If a polarizer is placed after the fiber section and is aligned with the polarization state of the pulse peak, the pulse wings are attenuated more by the polarizer and the pulse becomes shorter.

200 MHz soliton Er-fiber laser by additive pulse mode locking

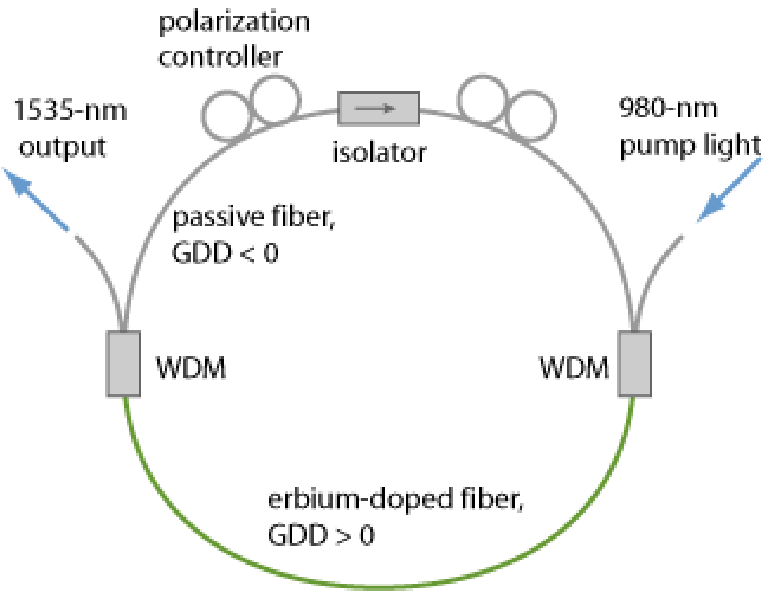


- 167 fs pulses
- 400 pJ intracavity pulse energy
- 200 pJ output pulse energy

K. Tamura et al. Opt. Lett. 18, 1080 (1993).

J. Chen et al, Opt. Lett. 32, 1566 (2007).

Dispersion managed soliton formation in fiber lasers



Setup of a stretched-pulse fiber ring laser (from RP Photonics)

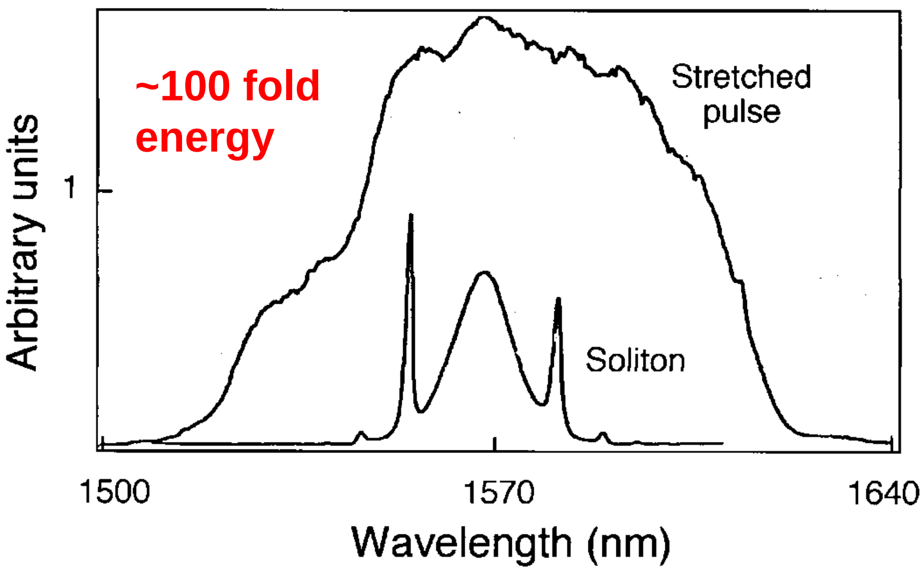
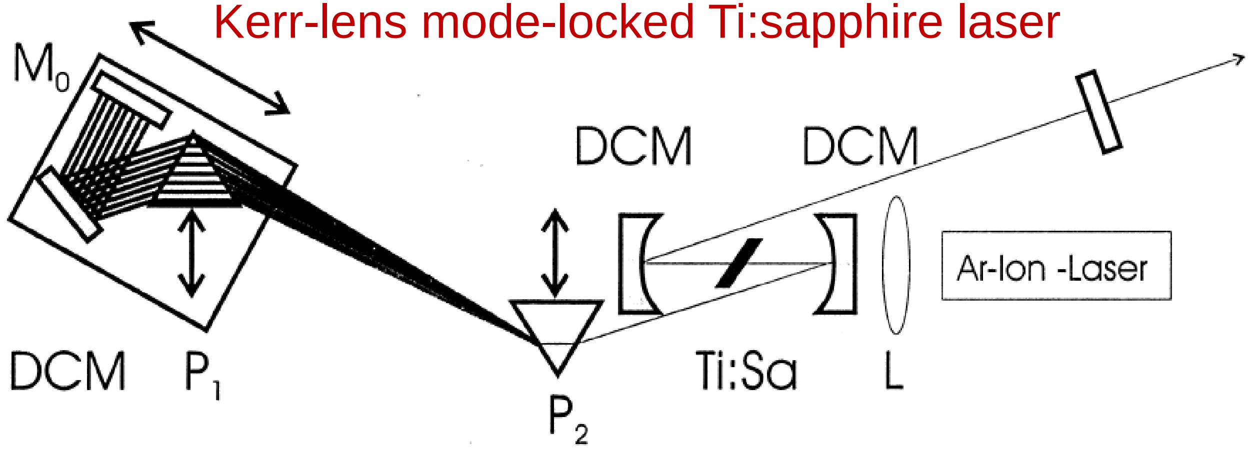


Fig. 6.12: Stretched pulse or dispersion managed soliton mode locking

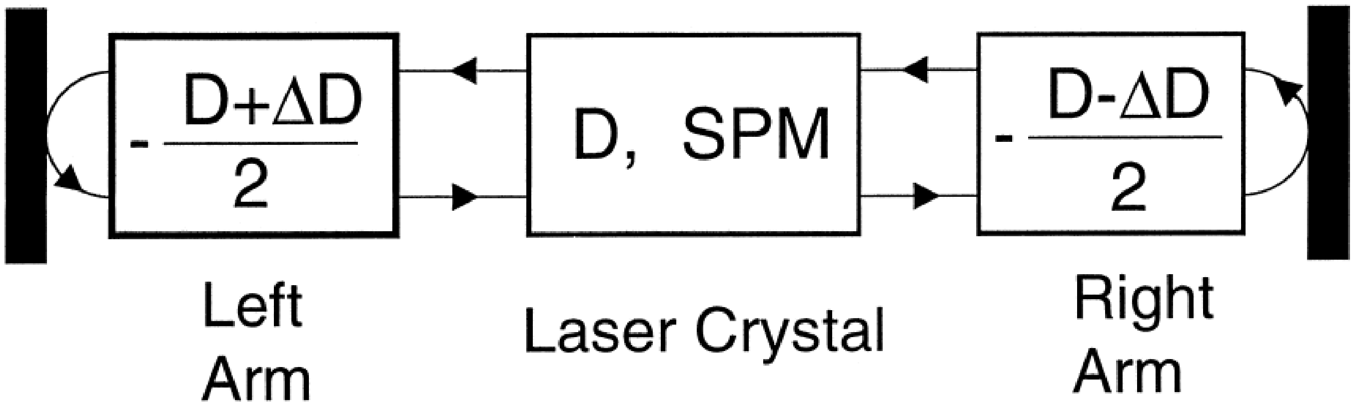
- The positive dispersion in the Er-doped fiber section of a fiber ring laser was balanced by a negative dispersive passive fiber.
- The pulse circulating in the ring was stretched and compressed by as much as a factor of 20 in one roundtrip.
- One consequence of this behavior was a dramatic decrease of the nonlinearity and thus increased stability against the SPM induced instabilities.

Dispersion managed soliton formation in solid-state lasers

Kerr-lens mode-locked Ti:sapphire laser

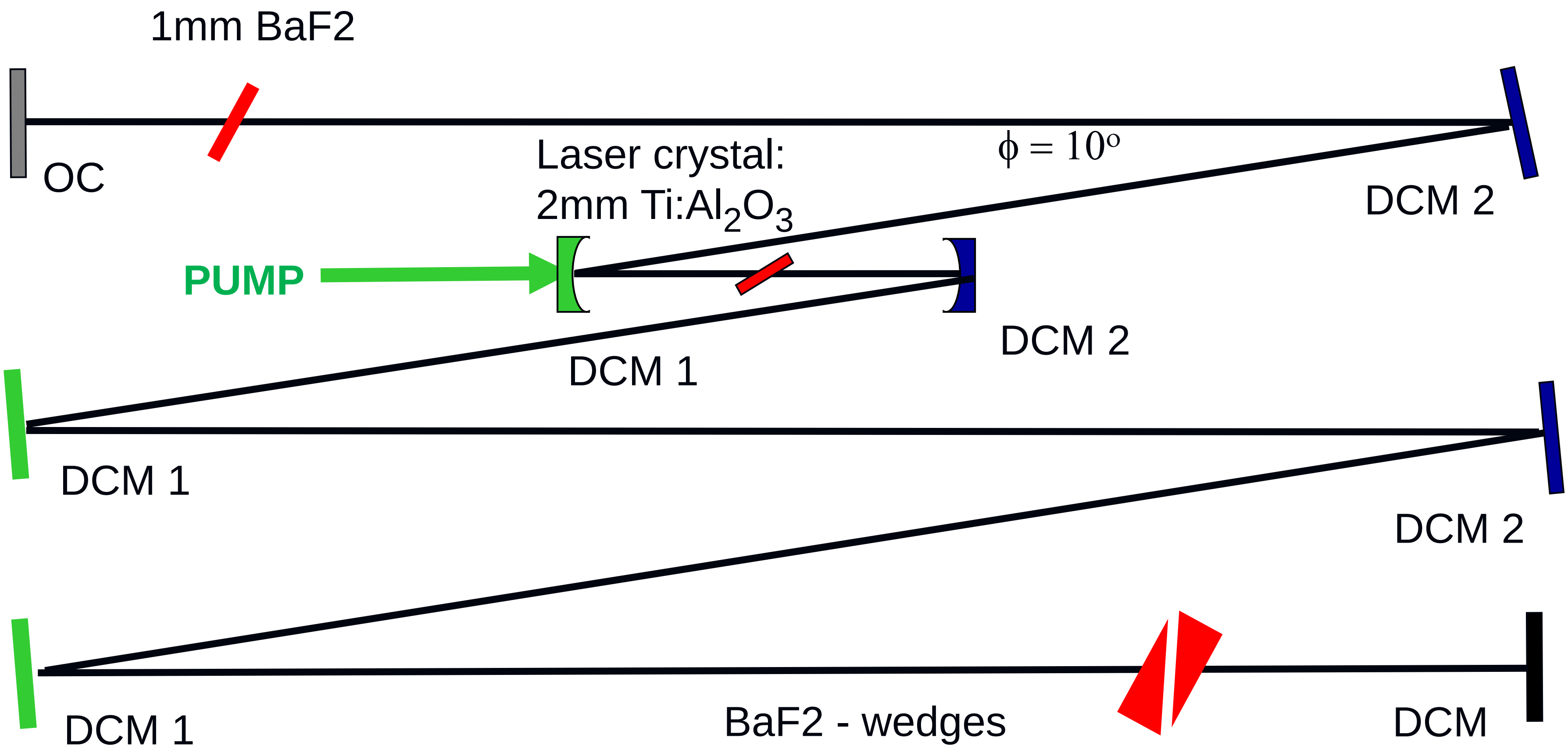


Correspondence with dispersion-managed fiber transmission



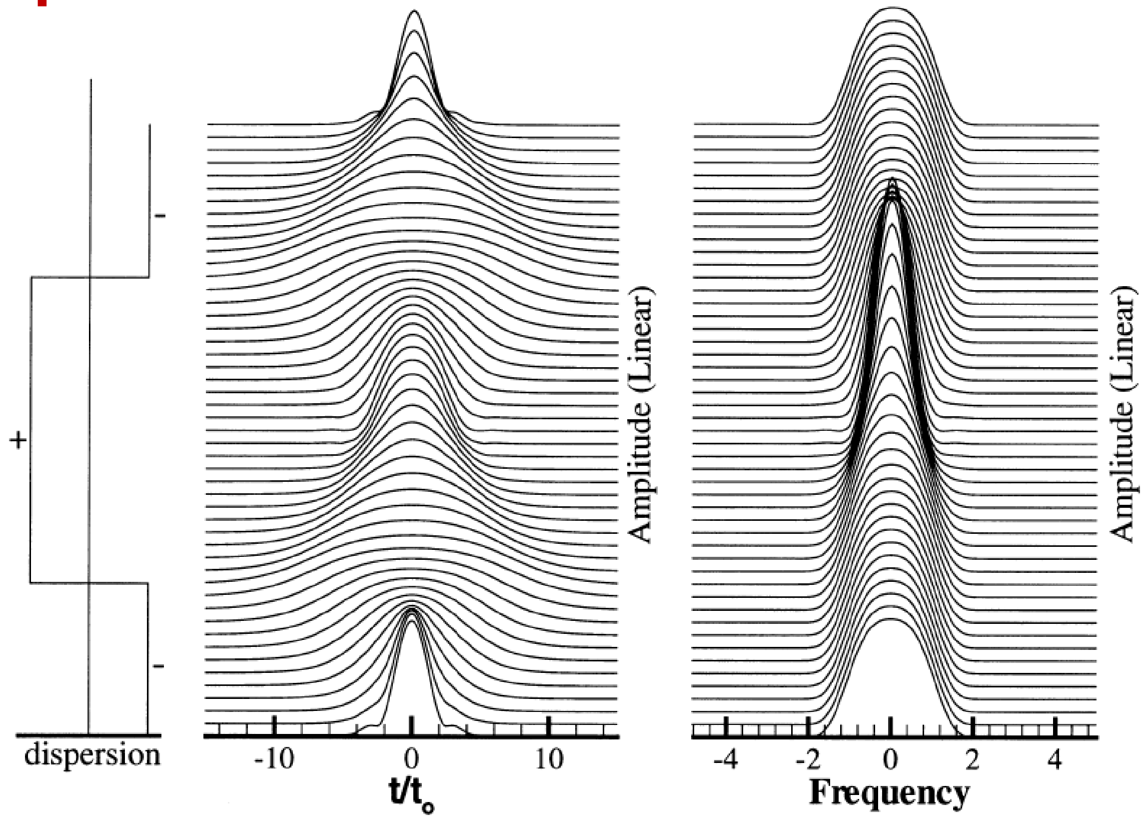
- Ti:sapphire lasers can generate pulses as short as 5 fs directly from the laser.
- At such short pulse lengths the pulse is stretched up to a factor of ten when propagating through the laser crystal creating a dispersion managed soliton.

Today's broadband, prismless Ti:sapphire lasers



It can directly emit 5-fs pulses.

Dispersion managed soliton formation: Pulse shaping in one round trip



- By symmetry the pulses are chirp free in the middle of the dispersion cells.
- A chirp free pulse starting in the center of the gain crystal (i.e. nonlinear segment) is spectrally broadened by the SPM and disperses in time due to the GVD, which generates a rather linear chirp over the pulse.
- After the pulse is leaving the crystal it experiences negative GVD during propagation through the left or right resonator arm, and is compressed back to its transform limit at the end of the arm, where an output coupler can be placed.

Dispersion managed soliton formation: steady state at the center of negative dispersion segment

Dispersion managed soliton resembles Gaussian pulse down to about −10 dB from the peak, but then shows rather complicated structures.

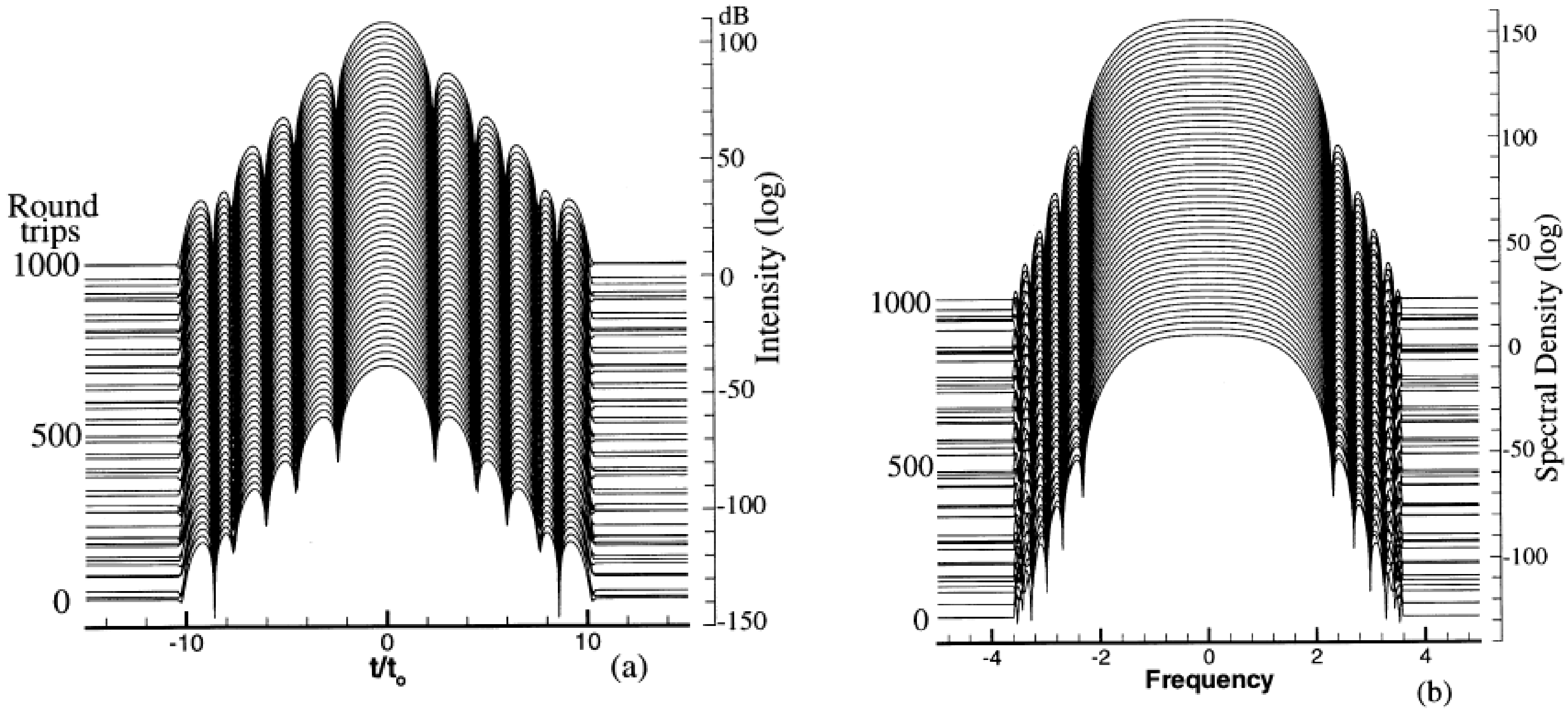


Fig. 6.15: the steady state intensity profiles are shown at the center of the negative dispersion segment over 1000 roundtrips

Dispersion managed soliton formation: effect of self-phase modulation

- Increasing SPM generates shorter pulses.
- The shortest pulse can be approximately three times shorter than the pulse without SPM.
- The behavior is similar to the fast SA case with conventional soliton formation.

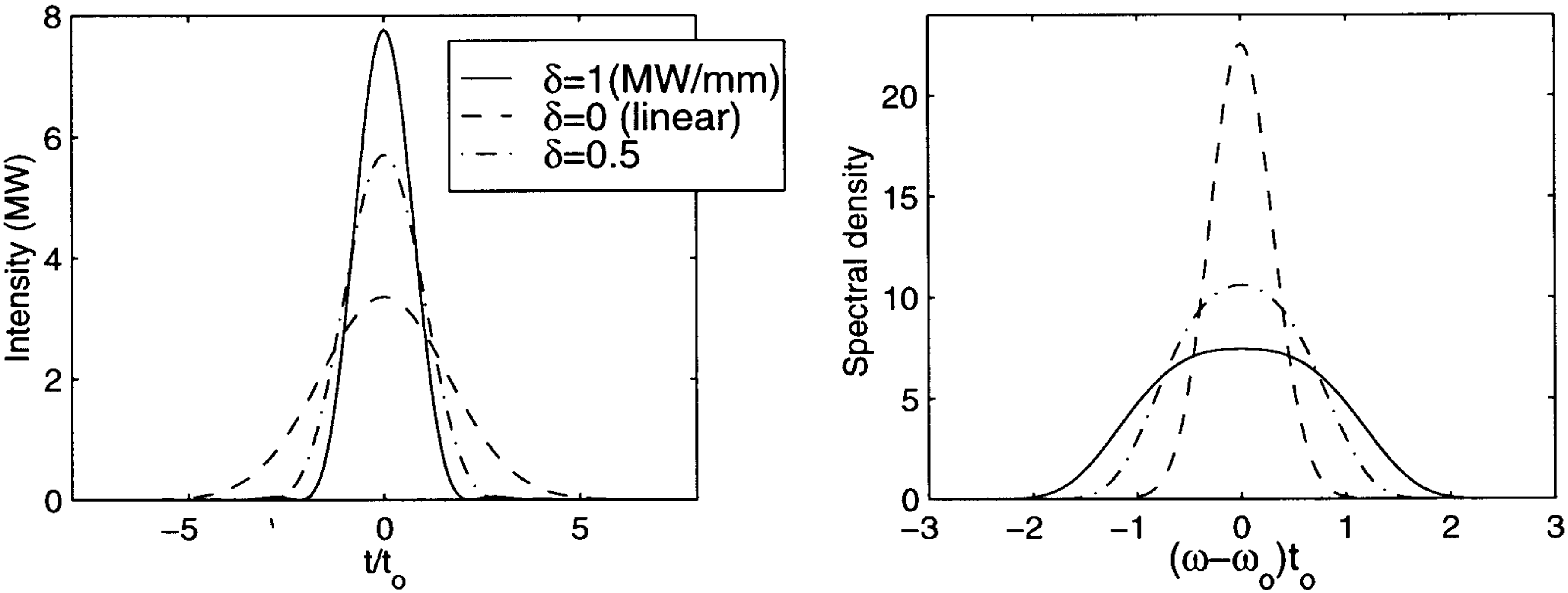


Fig. 6.16: Pulse shortening due to dispersion managed soliton formation. Simulation takes into account gain, loss, saturable absorption, and gain filtering.

8. Semiconductor Saturable Absorbers

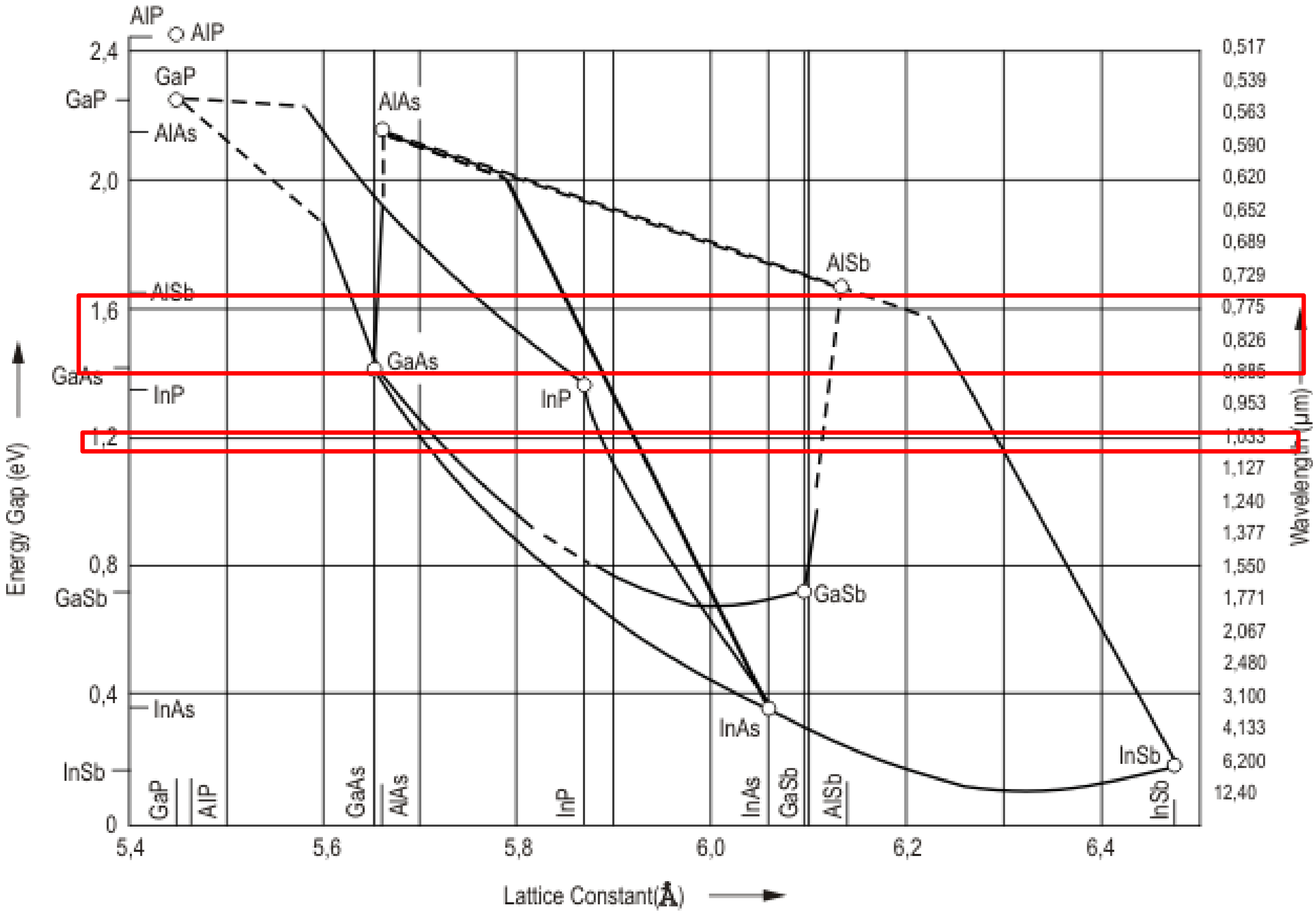


Fig. 8.1: Band Gap and lattice constant for various compound semiconductors. Dashed lines indicate ind. transitions.

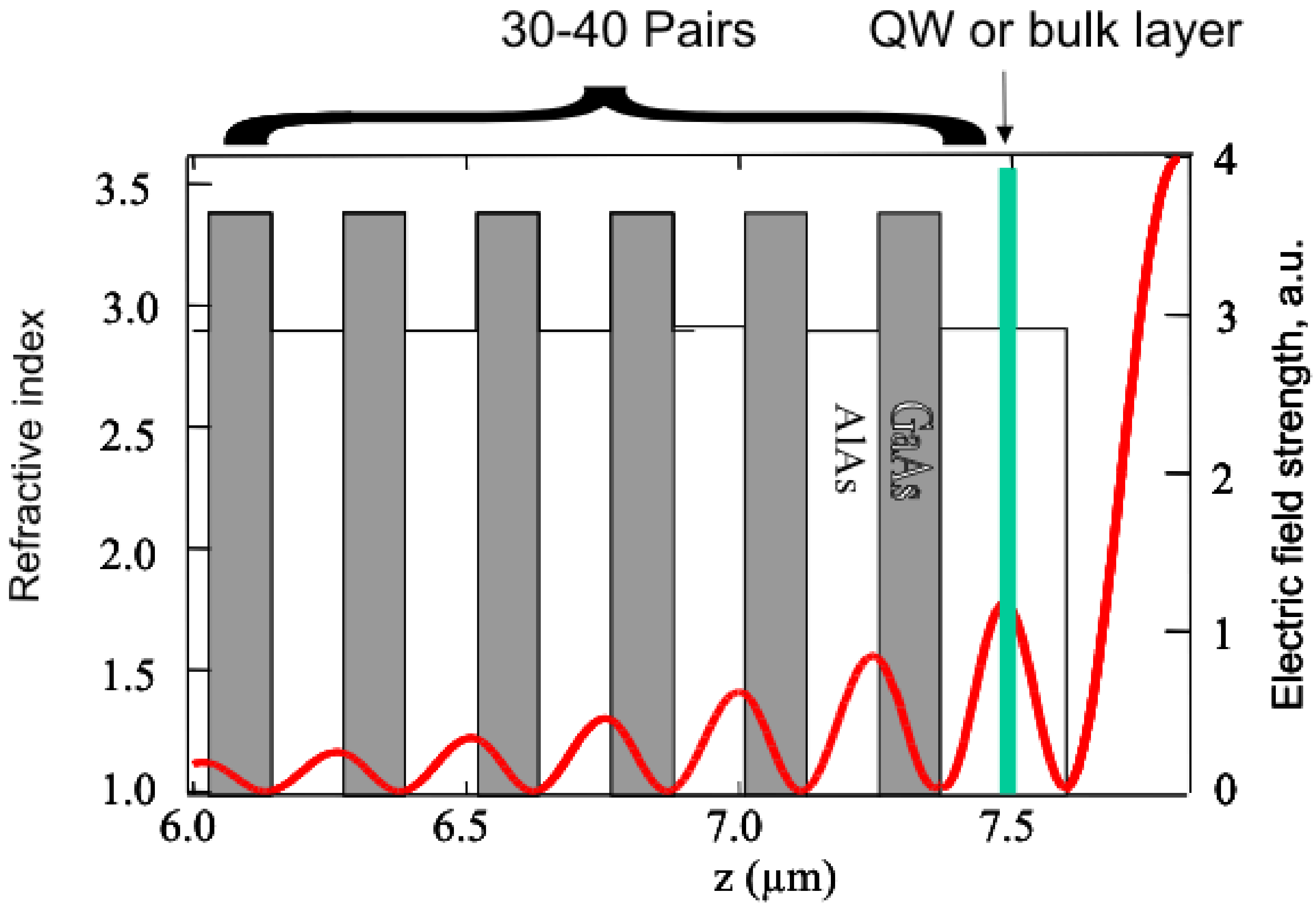


Fig. 8.2: Semiconductor saturable absorber mirror (SESAM) or Semiconductor Bragg mirror (SBR)

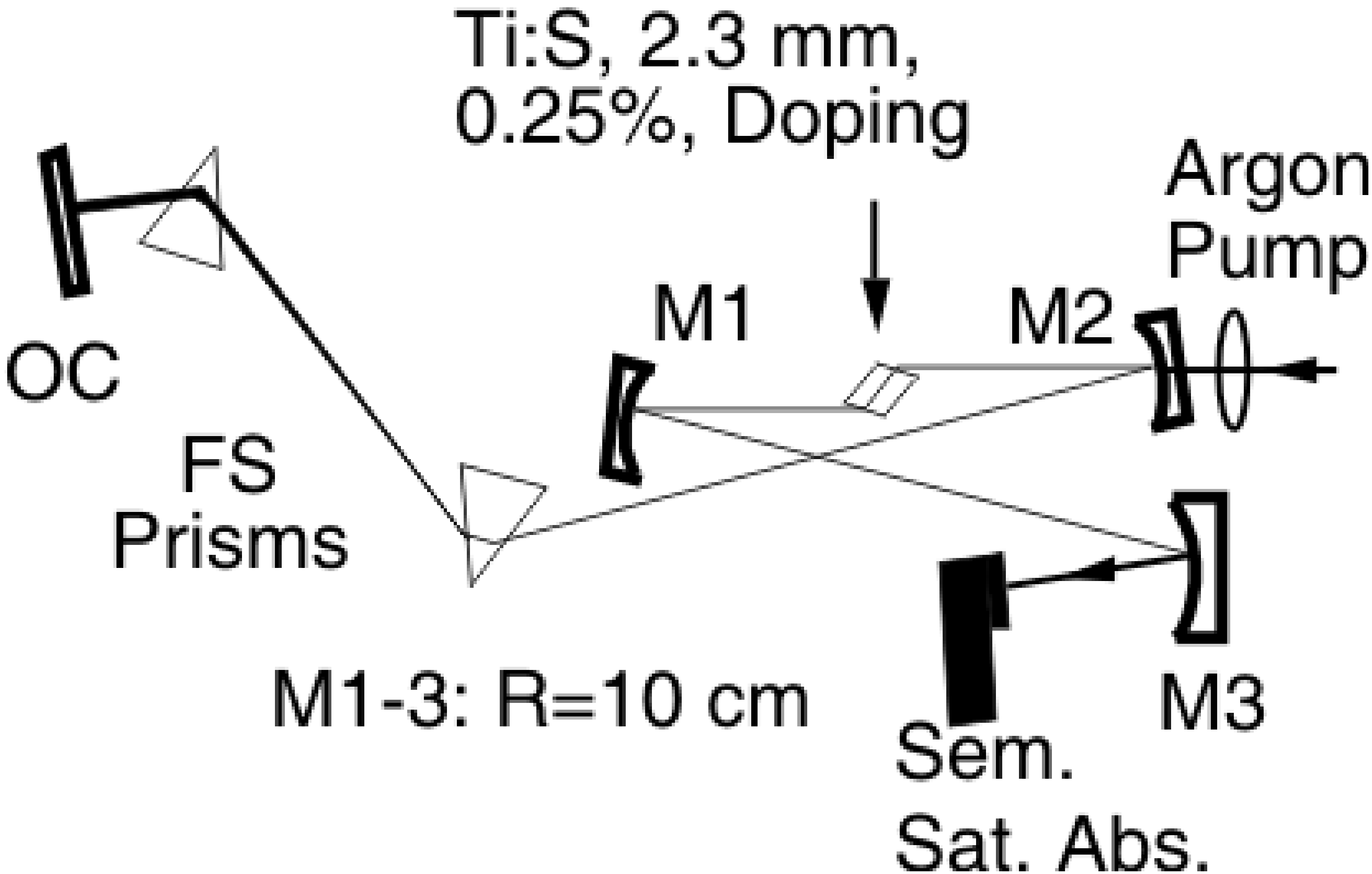


Fig. 8.3: Ti:sapphire laser modelocked by SBR

8.1 Carrier dynamics in semiconductors

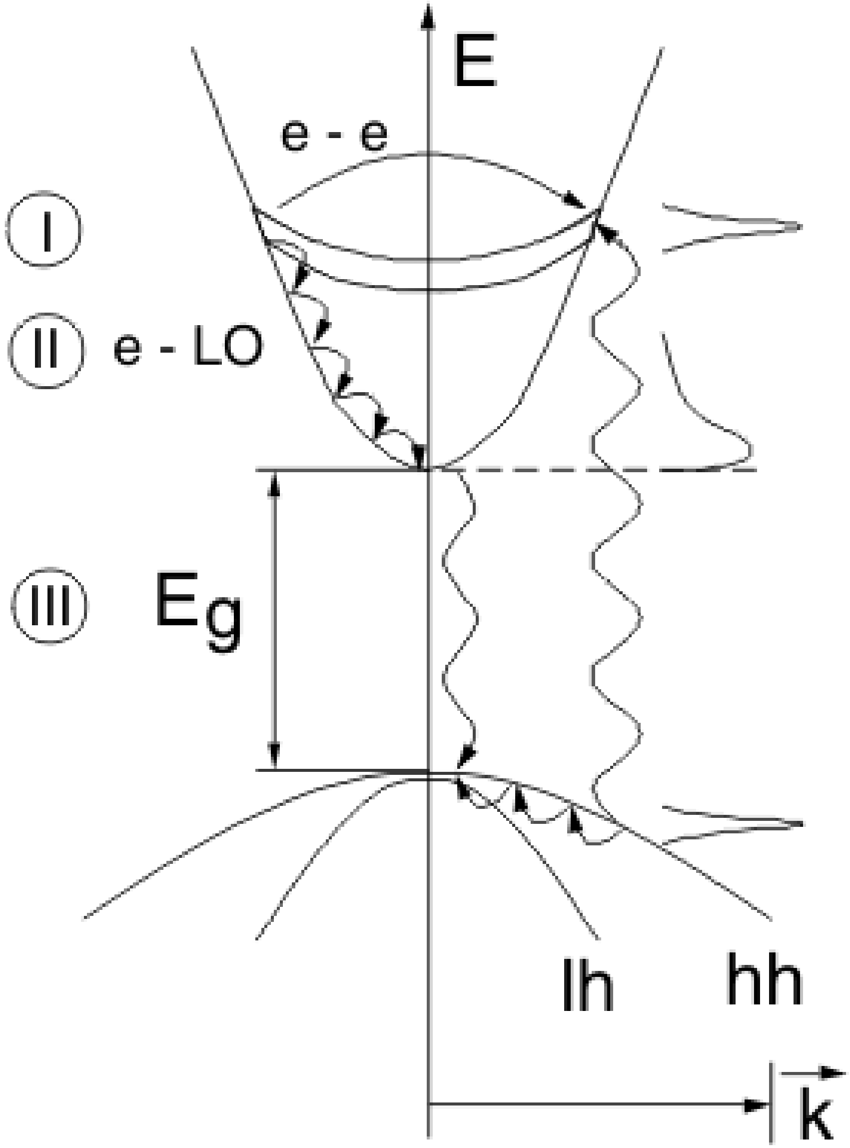


Table 8.4: Carrier dynamics in semiconductors

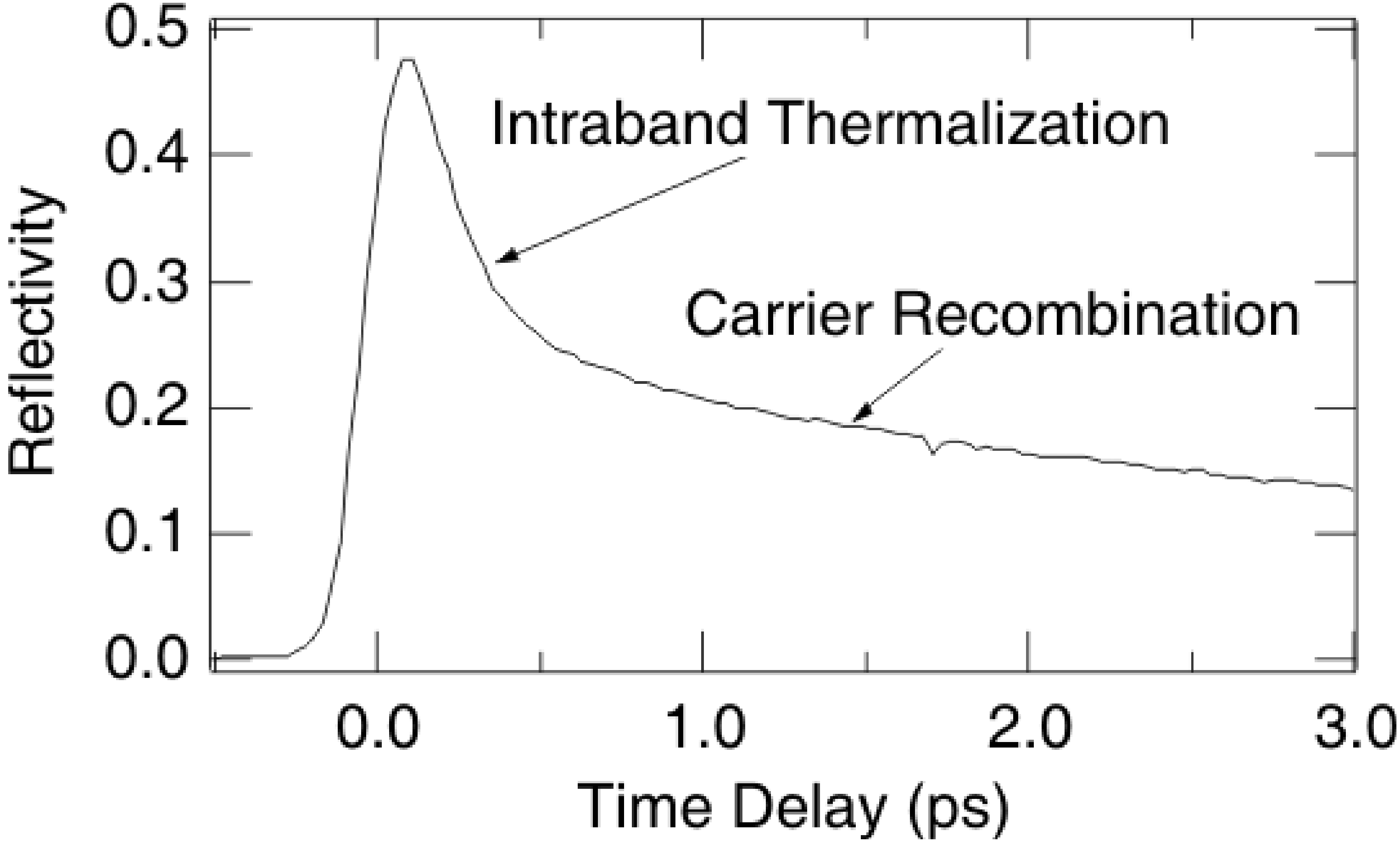


Fig. 8.5: Pump probe of a InGaAs multiple QW absorber

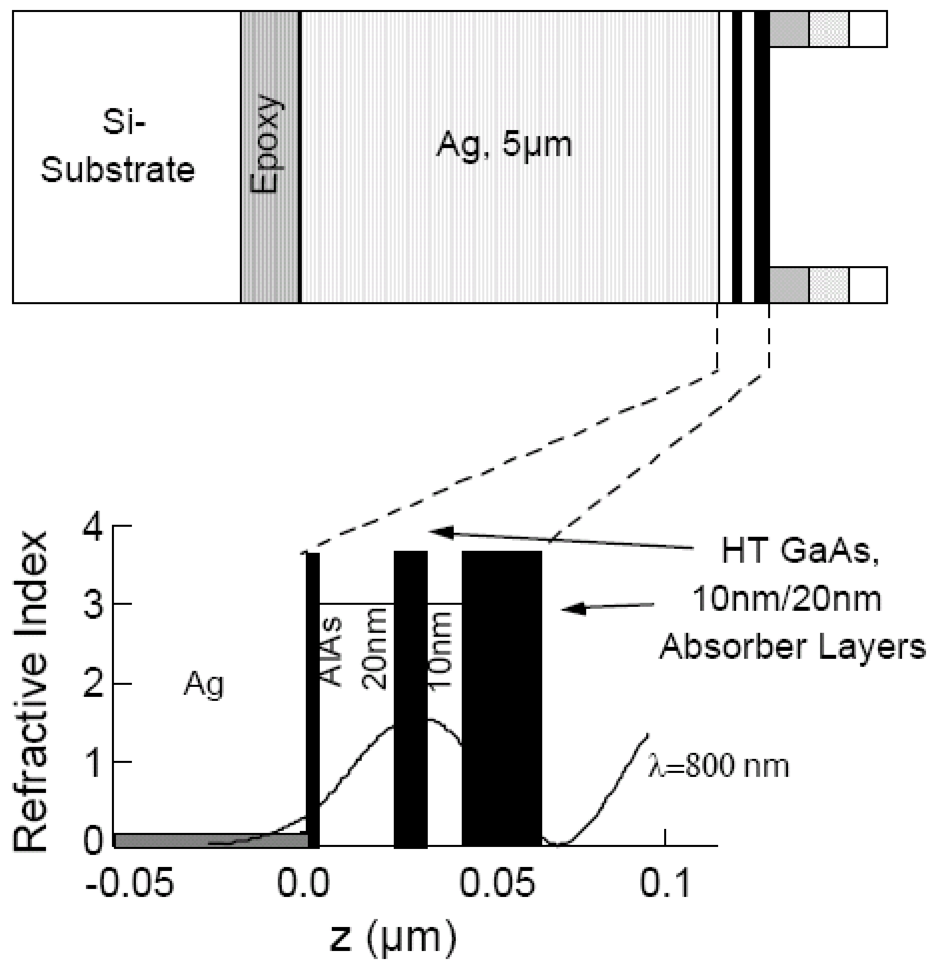


Fig. 8.7: GaAs saturable absorber on metal mirror

Saturation fluence :

$$d = 0.5 \text{ nm}$$

$$T_2 = 20 \text{ fs}$$

$$n_0 = 3$$

$$F_A = \frac{hf}{\sigma_A} = I_A \tau_A = \frac{\hbar^2}{2 T_2 Z_F \left|\vec{M}\right|^2}$$

$$= \frac{\hbar^2 n_0}{2 T_2 Z_{F0} \left|\vec{M}\right|^2} = 35 \frac{\mu J}{cm^2}$$

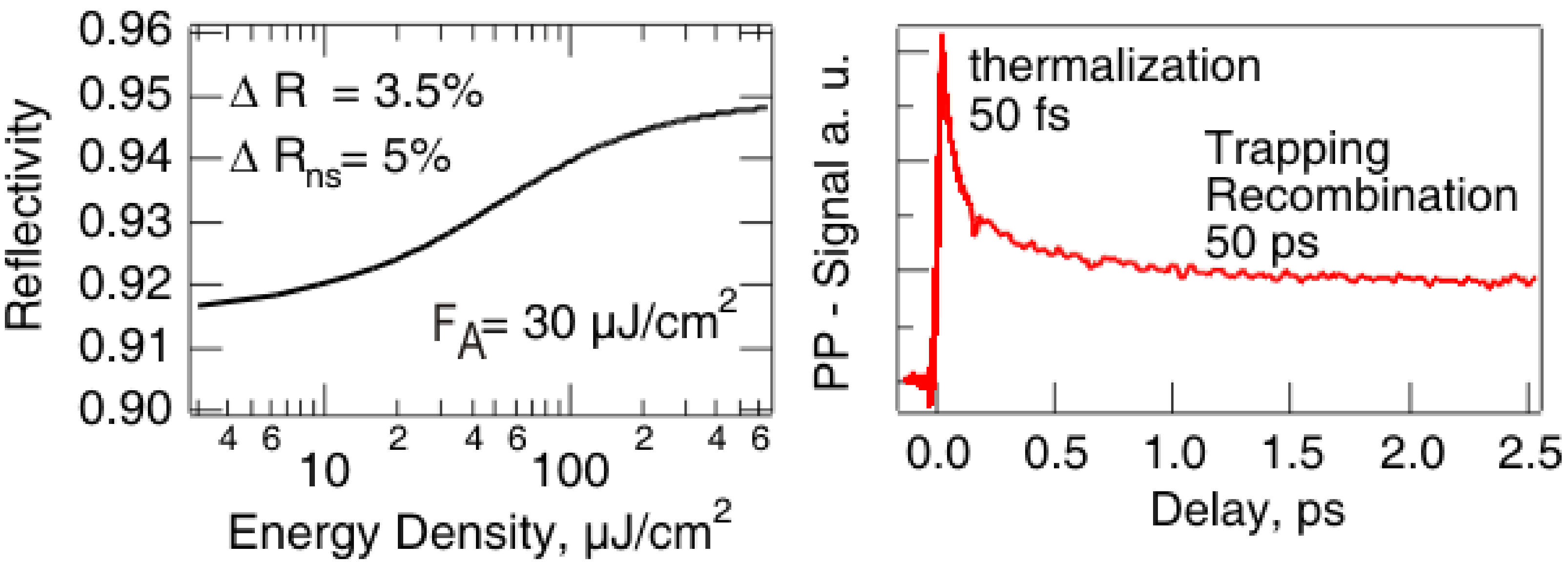


Fig. 8.6: Saturation fluence and pump probe measured with 10 fs pulses

8.2 High Fluence Effects

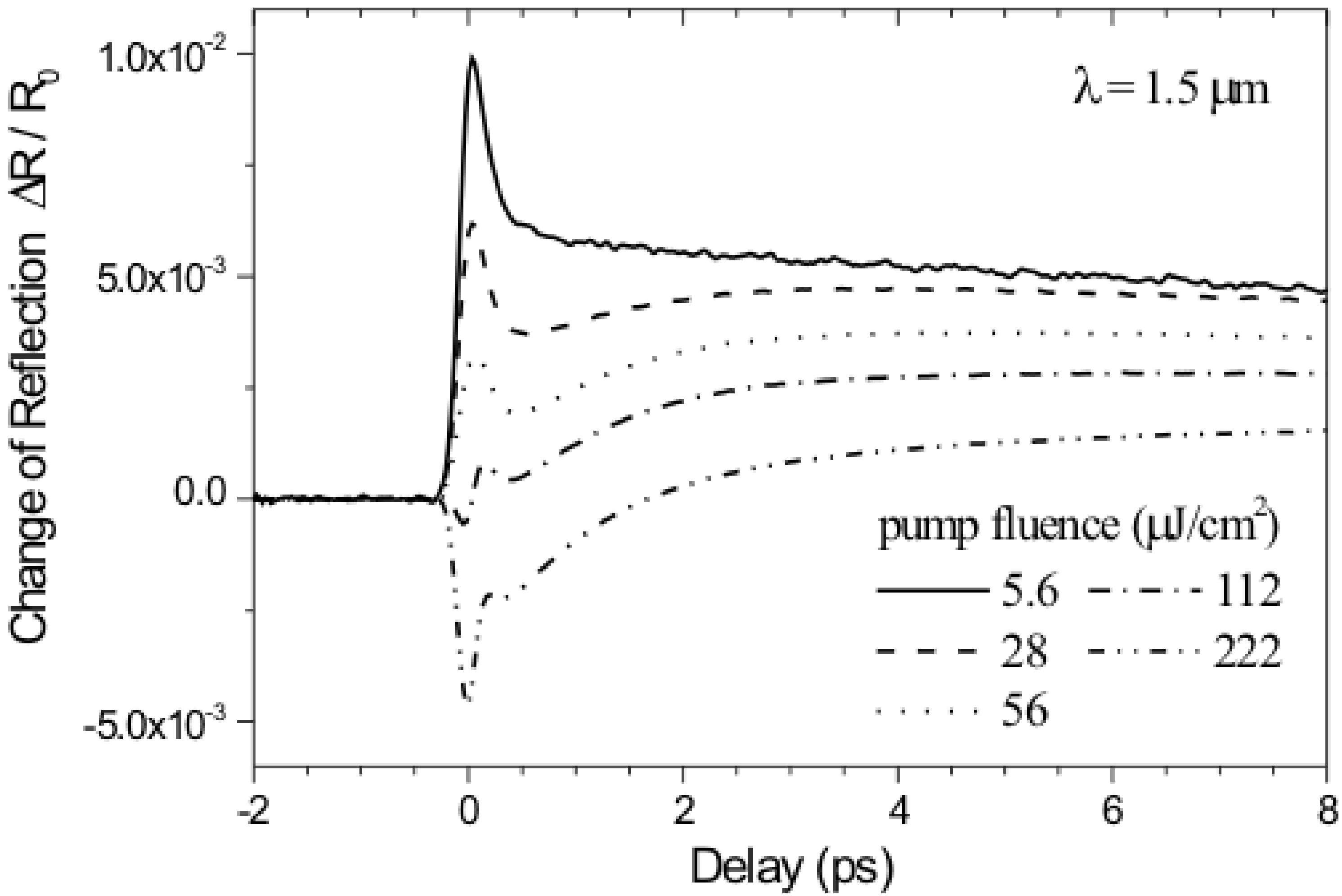


Fig. 8.8: Pump probe with low and high fluence

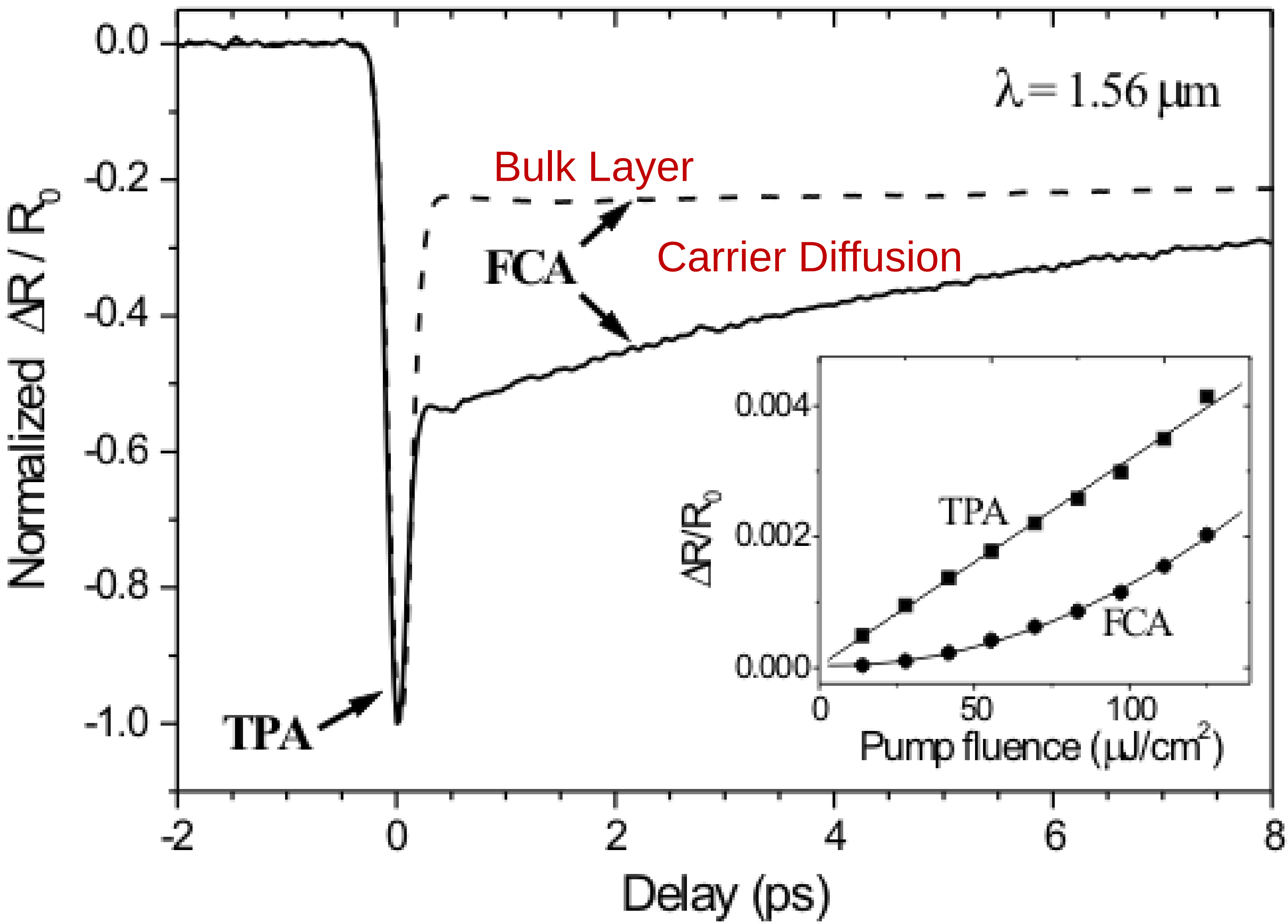


Fig. 8.9: TPA and FCA

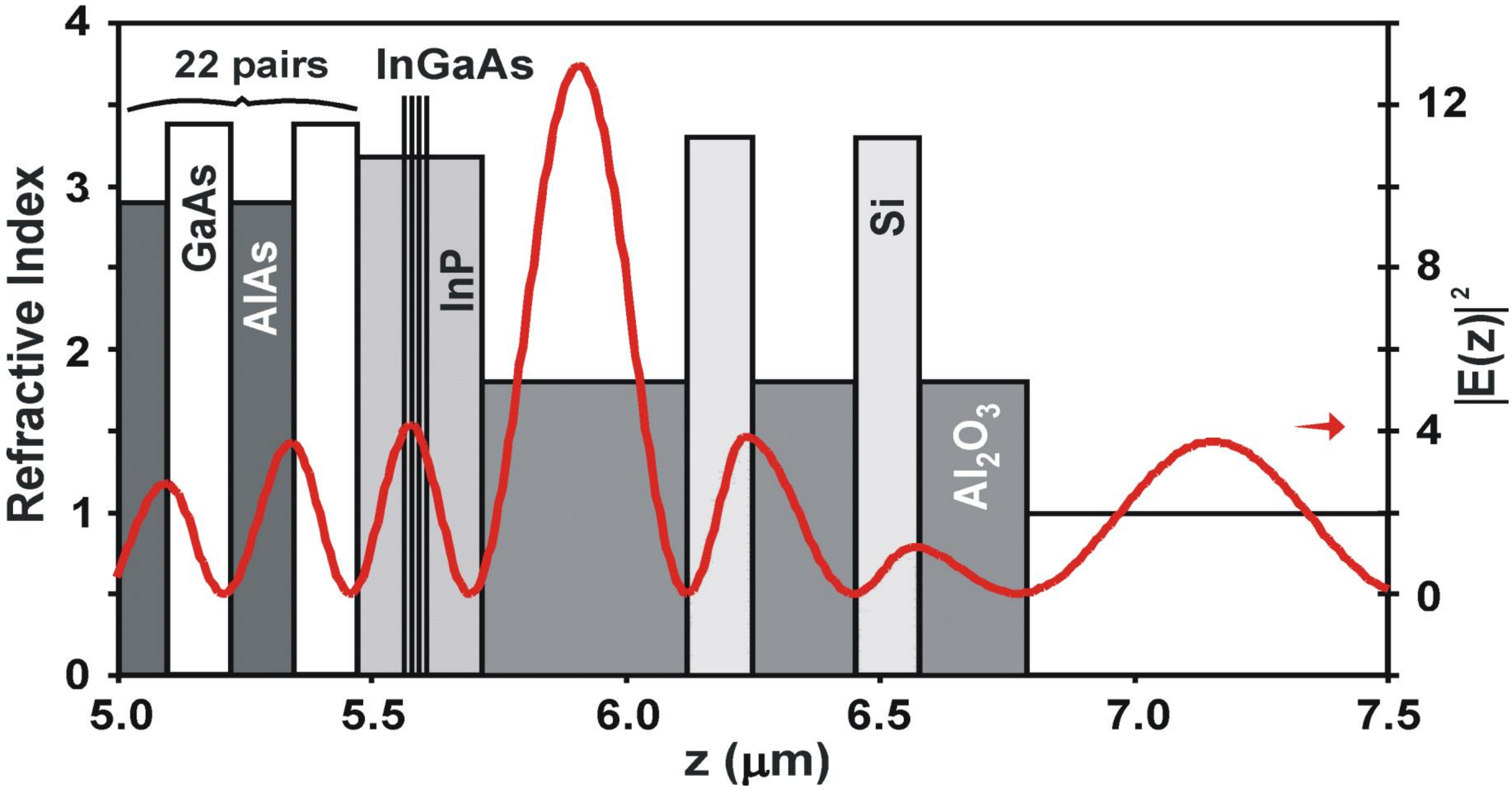


Fig. 8.10: Resonantly enhanced SBR

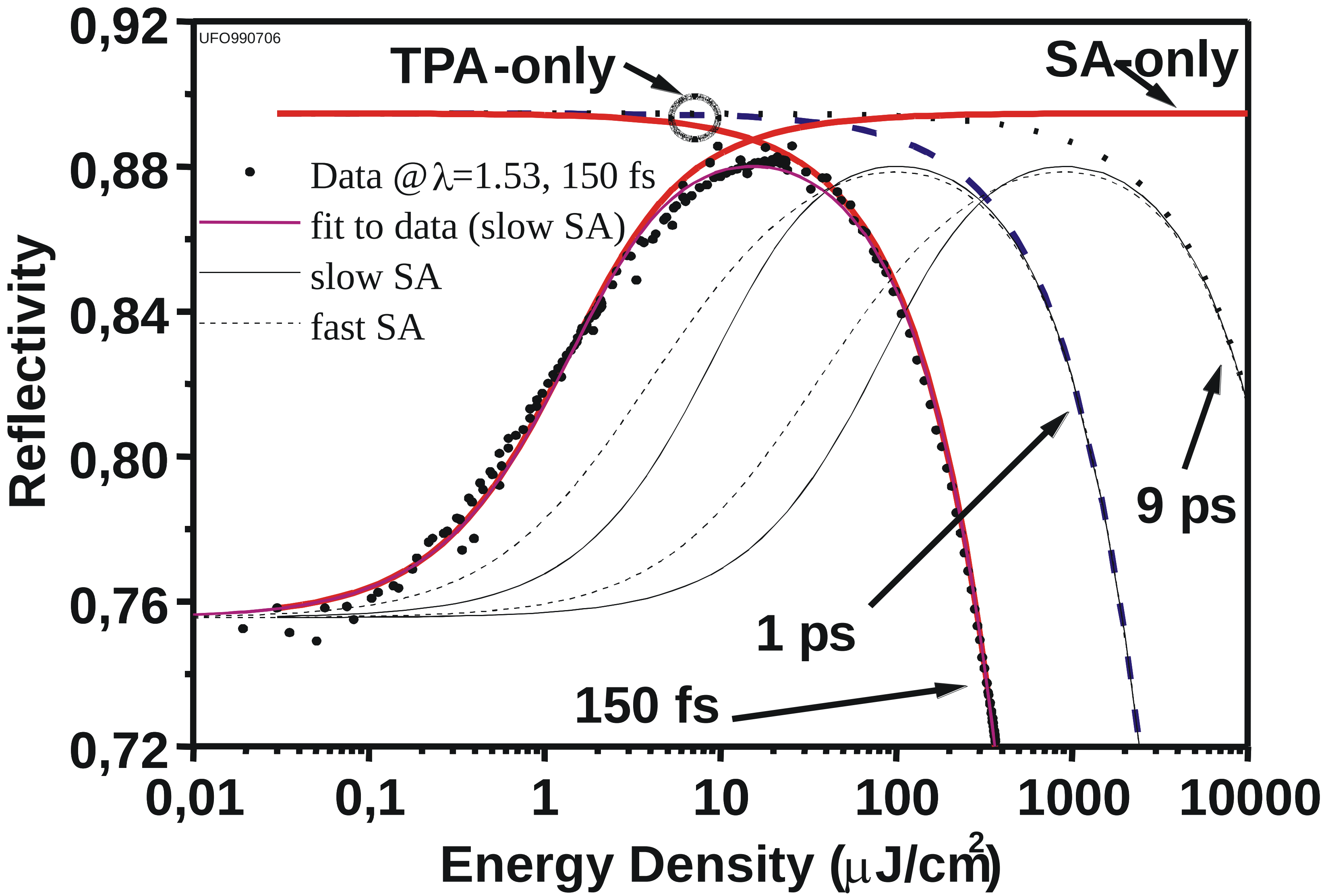


Fig. 8.11: Saturation fluence measurement of resonant absorber

8.3 Break-up into multiple pulses

$$l_m = \frac{D_f}{3\tau_m^2} + q_s(W_m)$$

$$l_1 = \frac{D_f}{3\tau_1^2} + q_s(W_1),$$

$$l_2 = \frac{D_f}{12\tau_1^2} + q_s(\frac{W_1}{2}).$$

$$\frac{D_f}{4\tau_1^2} < \Delta q_s(W) = q_s(\frac{W}{2}) - q_s(W)$$

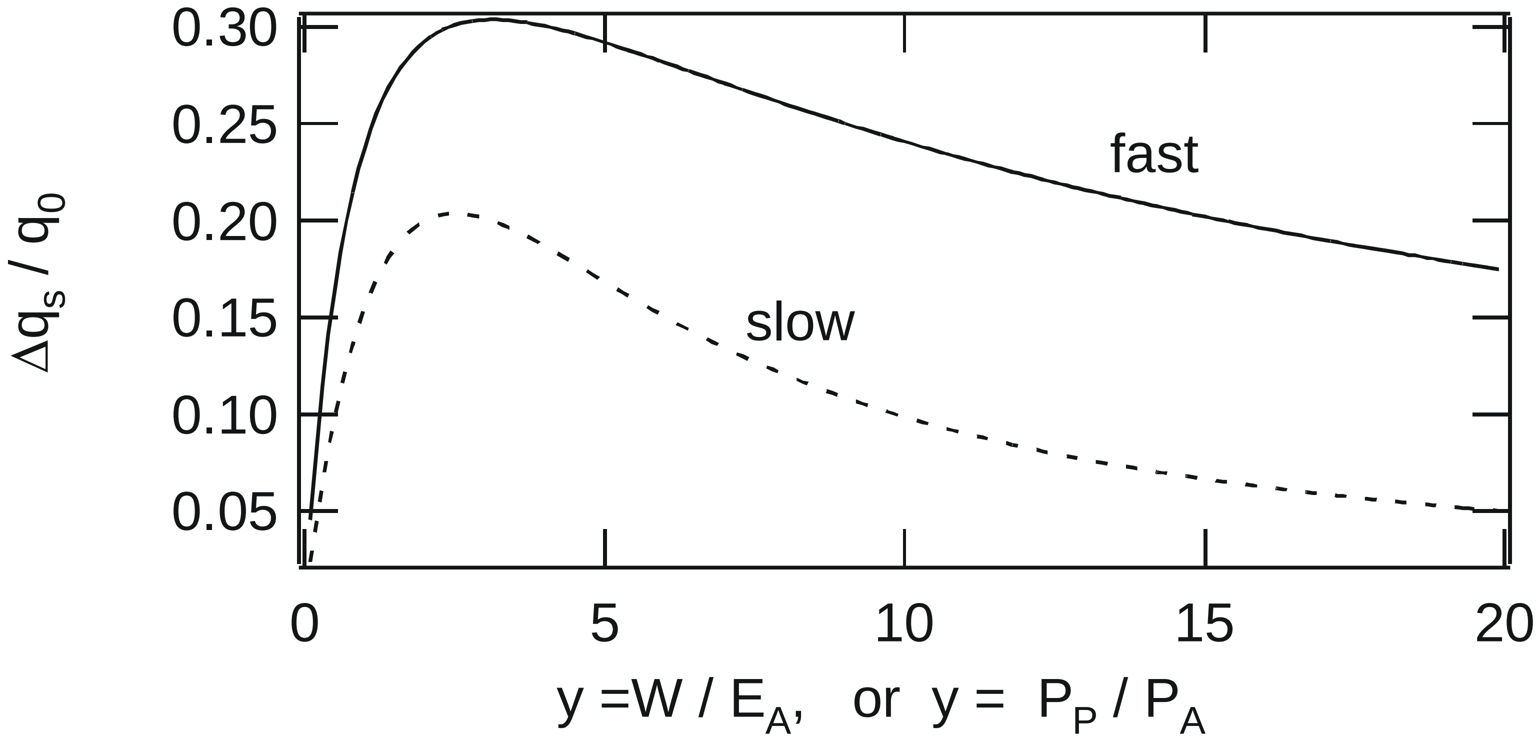


Fig. 8.12: Difference in loss experienced by a sech-shaped pulse in a slow (- - -) and a fast (_____) saturable absorber for a given pulse energy or peak power, respectively.

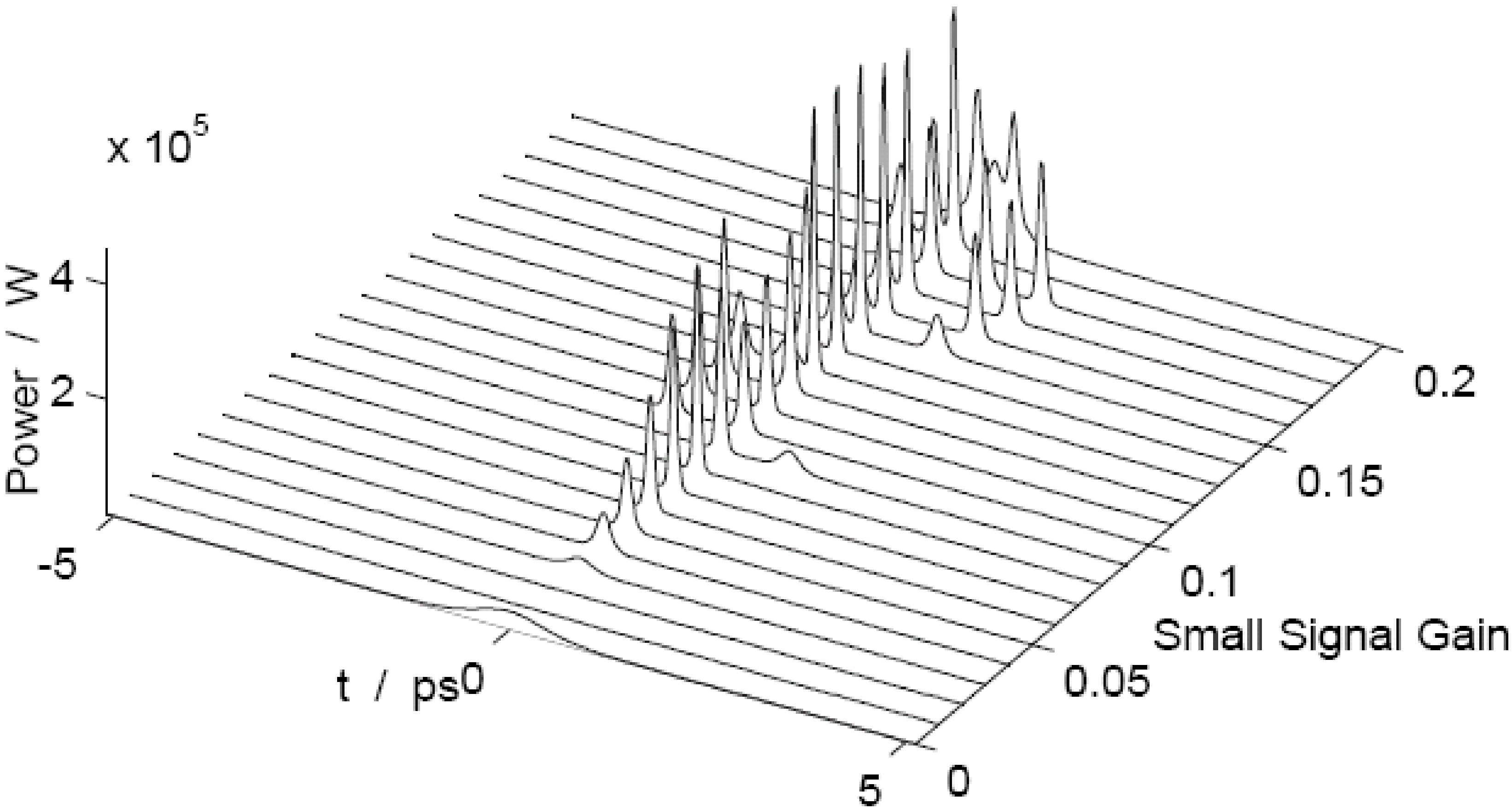


Fig. 8.13: Pulse intensity profiles after 20,000 round-trips each. Laser modelocked with sat. abs with recovery time $\tau_A = 200$ fs.

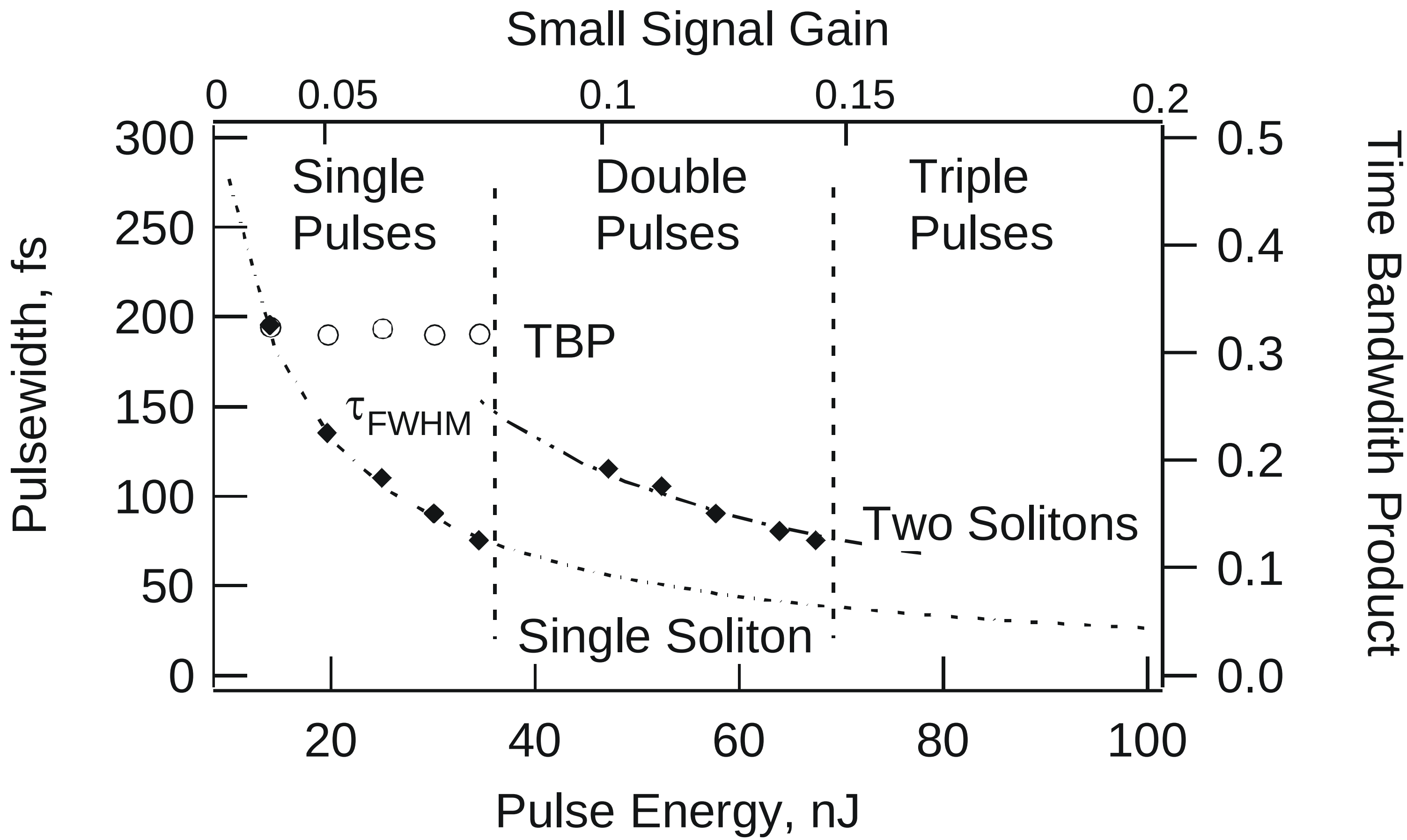


Fig. 8.14: Steady state pulse width and time-bandwidth product

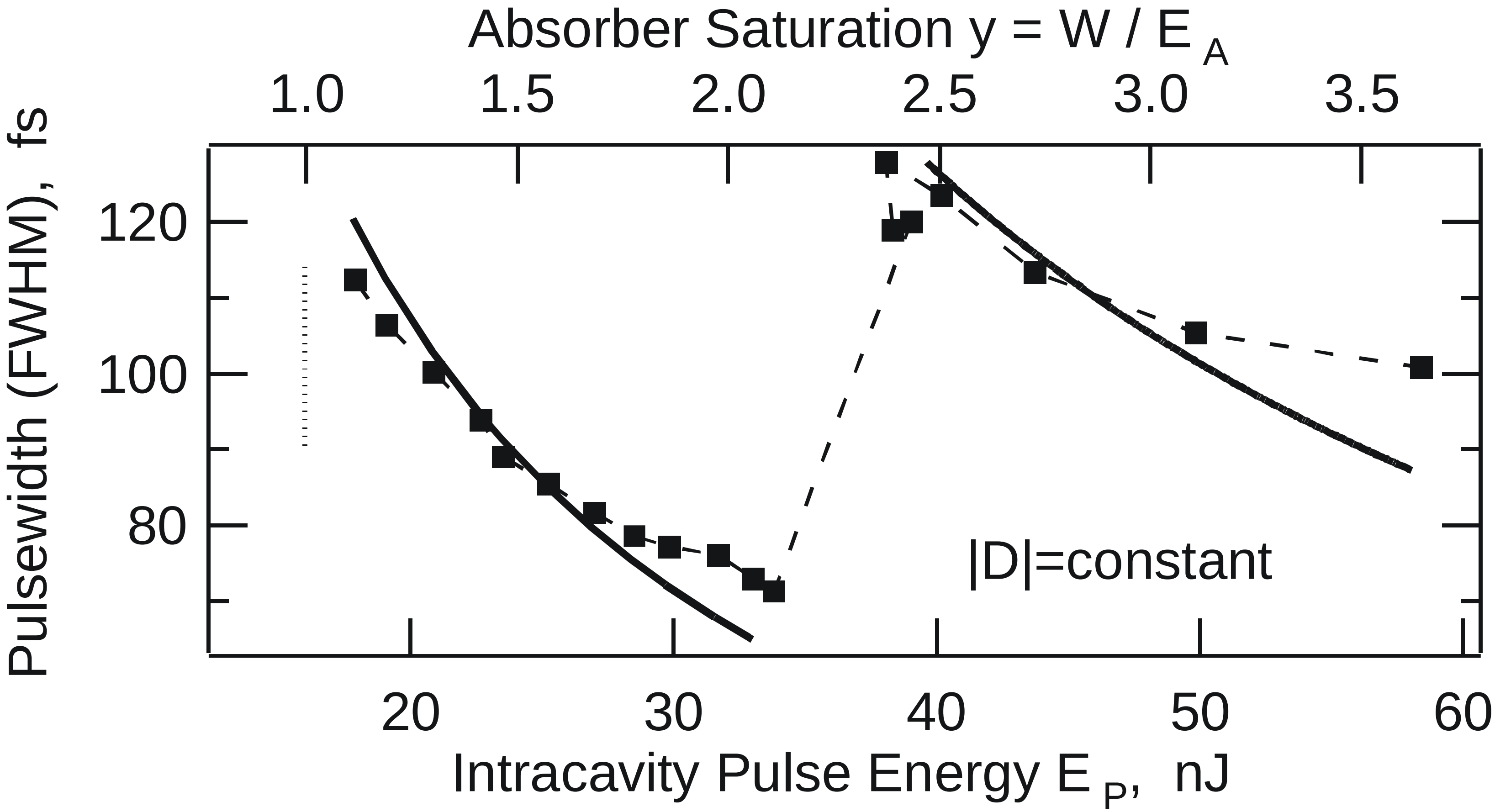


Fig. 8.15: Pulse width of Nd:glas laser.

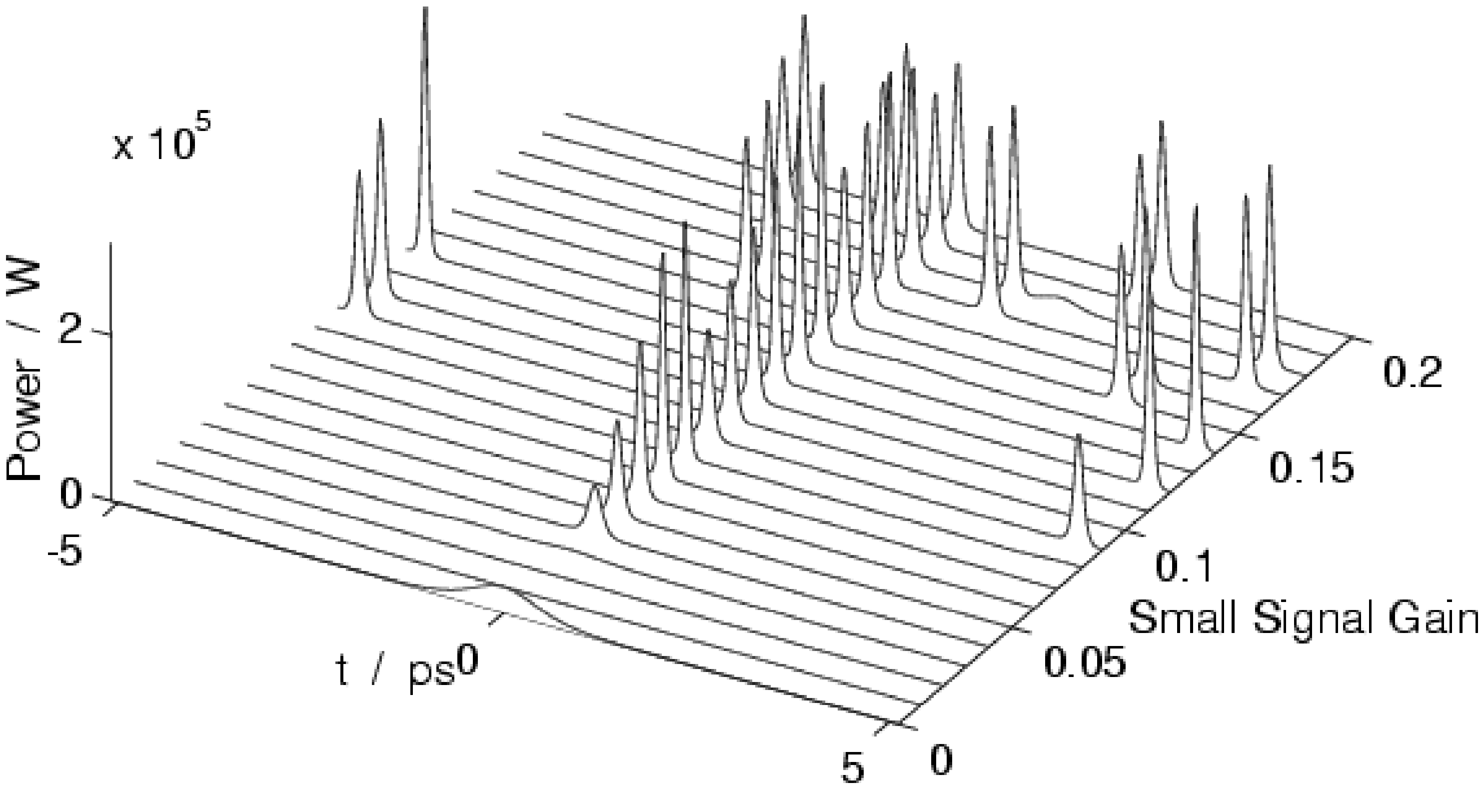


Fig. 8.16: Pulse intensity profiles after 20,000 round-trips each. Laser modelocked with sat. abs with shorter recovery time τ_A = 100 fs.

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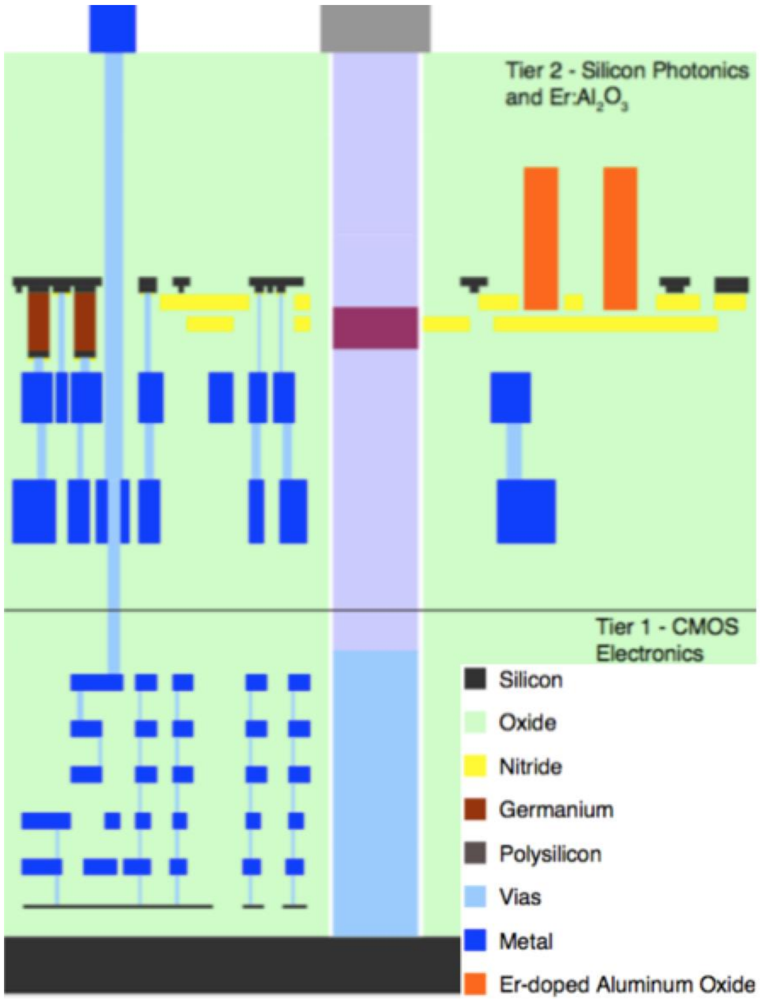

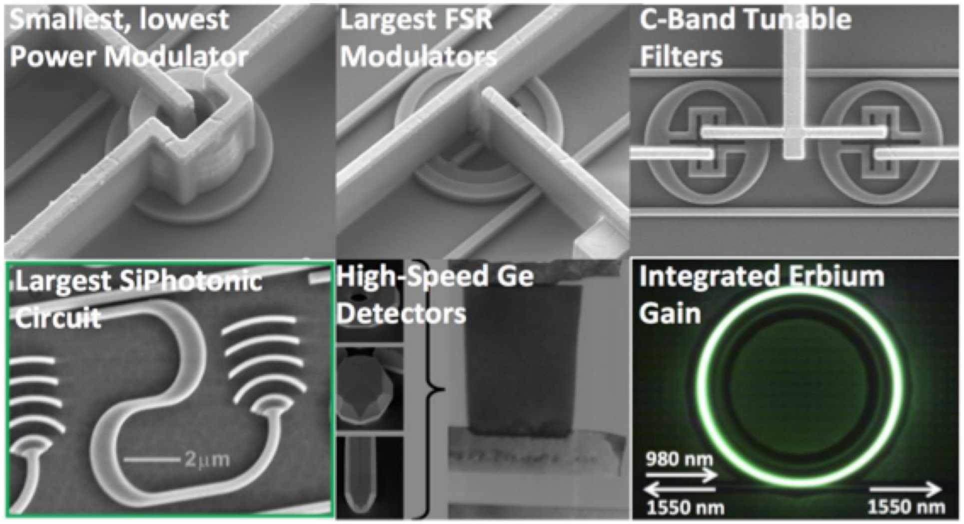

Comprehensive Device Library:

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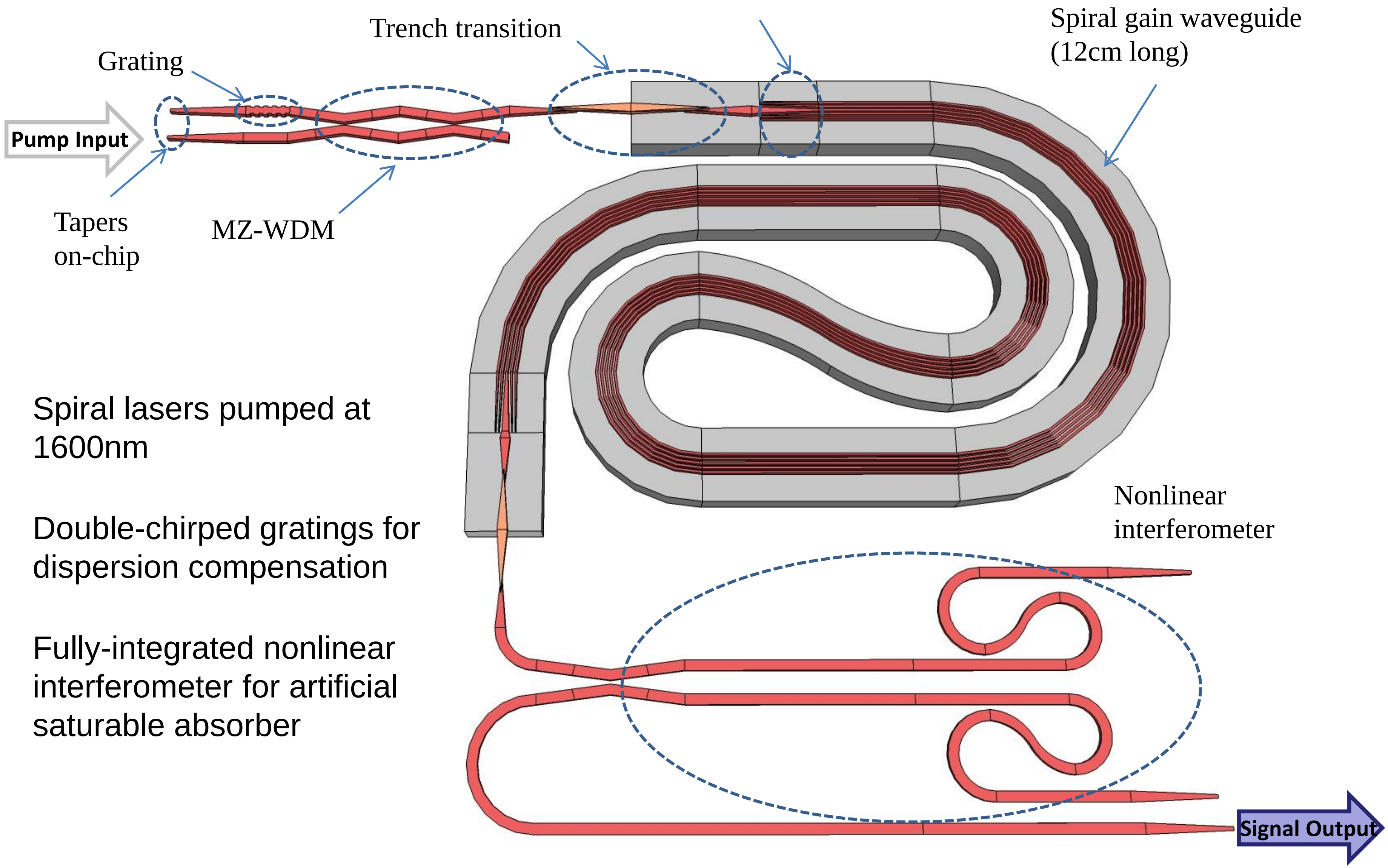
- Ultralow power modulators, phase shifters, switches, tuners
- High speed Ge-detectors
- Low loss Si – SiN – transitions
- Low loss fiber-to-chip couplers
- Erbium/Thulium on-chip gain, lasers
- Largest Si-photonic circ. (phased array)

Courtesy Mike Watts, MIT Microphotonics Group

Integrated Thulium Mode-Locked Laser



- Spiral lasers pumped at 1600nm
- Double-chirped gratings for dispersion compensation
- Fully-integrated nonlinear interferometer for artificial saturable absorber