

Ultrafast Optical Physics II (SoSe 2020)

Lecture 6, June 12, 2020

Pulse Compression: revisited and finished

5 Active Mode Locking

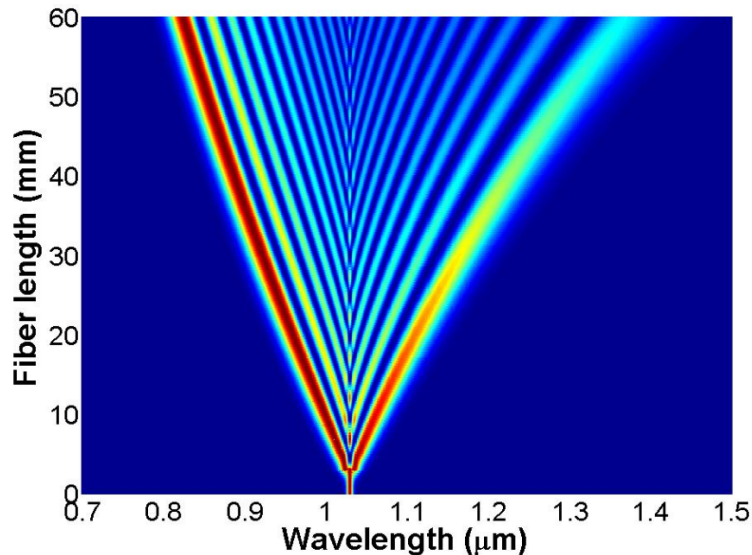
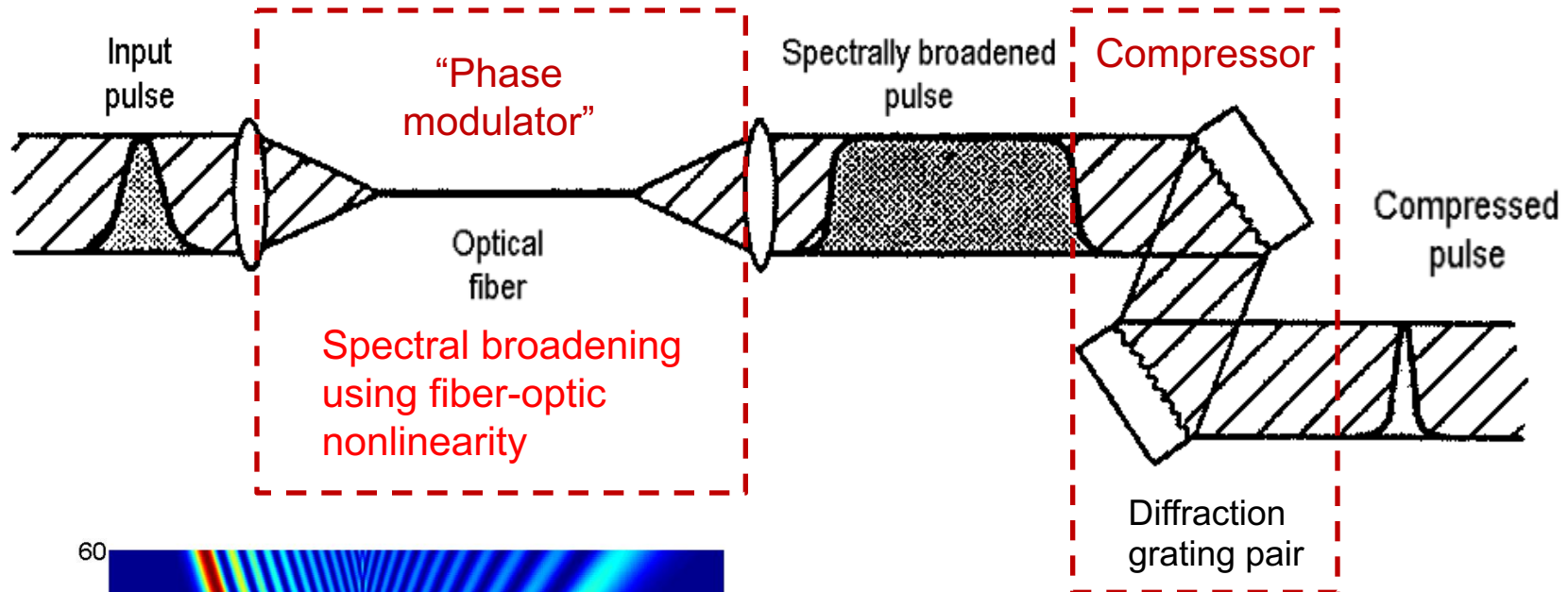
- 5.1 The Master Equation of Mode Locking**
- 5.2 Active Mode Locking by Loss Modulation**
- 5.3 Active Mode Locking by Phase Modulation**
- 5.4 Active Mode Locking with Additional SPM**
- 5.5 Active Mode Locking with Soliton Formation**
- 5.6 Active Mode Locking and Detuning**

Topics for short presentations

1. Dispersion of a prism pair
2. Dispersion of a grating pair
3. Kramers-Kronig Relations
4. White light interferometry (FTS)
5. Lotka-Volterra Model and Q-switching
6. Michelson Spectrometer
7. Dual comb spectroscopy
8. Active mode locking with detuning
9. Semiconductor Saturable Absorber Mode Locking
10. Optical communication and the Er-doped fiber amplifier

Pulse compression: general idea

Spectral broadening followed by dispersion compensation to compress (de-chirp) the pulse



$$T_g(\omega) = \phi'(\omega_0) + \phi''(\omega_0)\Delta\omega + \frac{1}{2}\phi'''(\omega_0)\Delta\omega^2 + \frac{1}{3!}\phi''''(\omega_0)\Delta\omega^3 + \dots$$

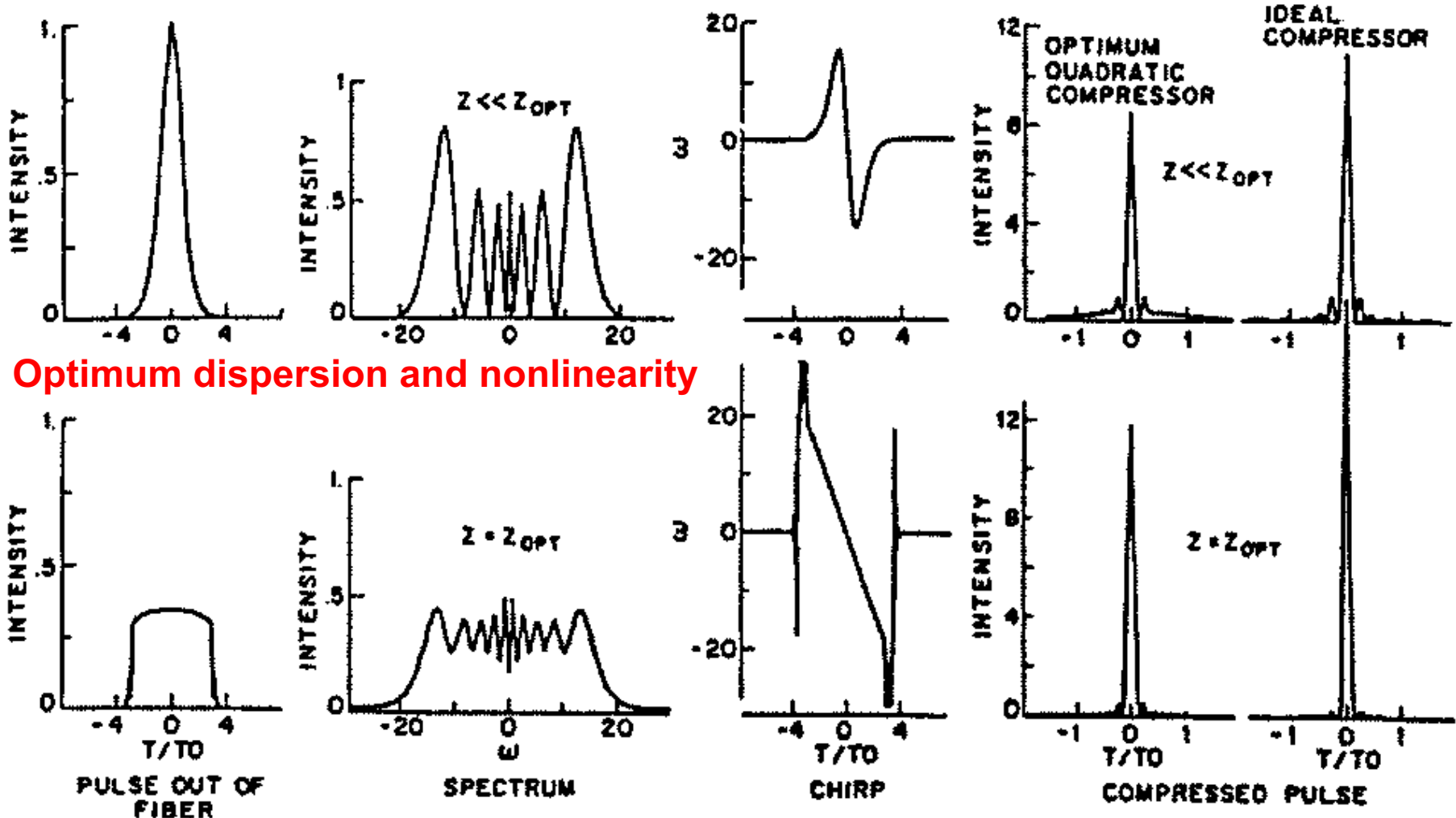
$$\phi''(\omega_0) = \phi''_{\text{modulator}} + \phi''_{\text{compressor}} = 0$$

$$\phi'''(\omega_0) = \phi'''_{\text{modulator}} + \phi'''_{\text{compressor}} = 0$$

Variable dispersion by the grating pair.

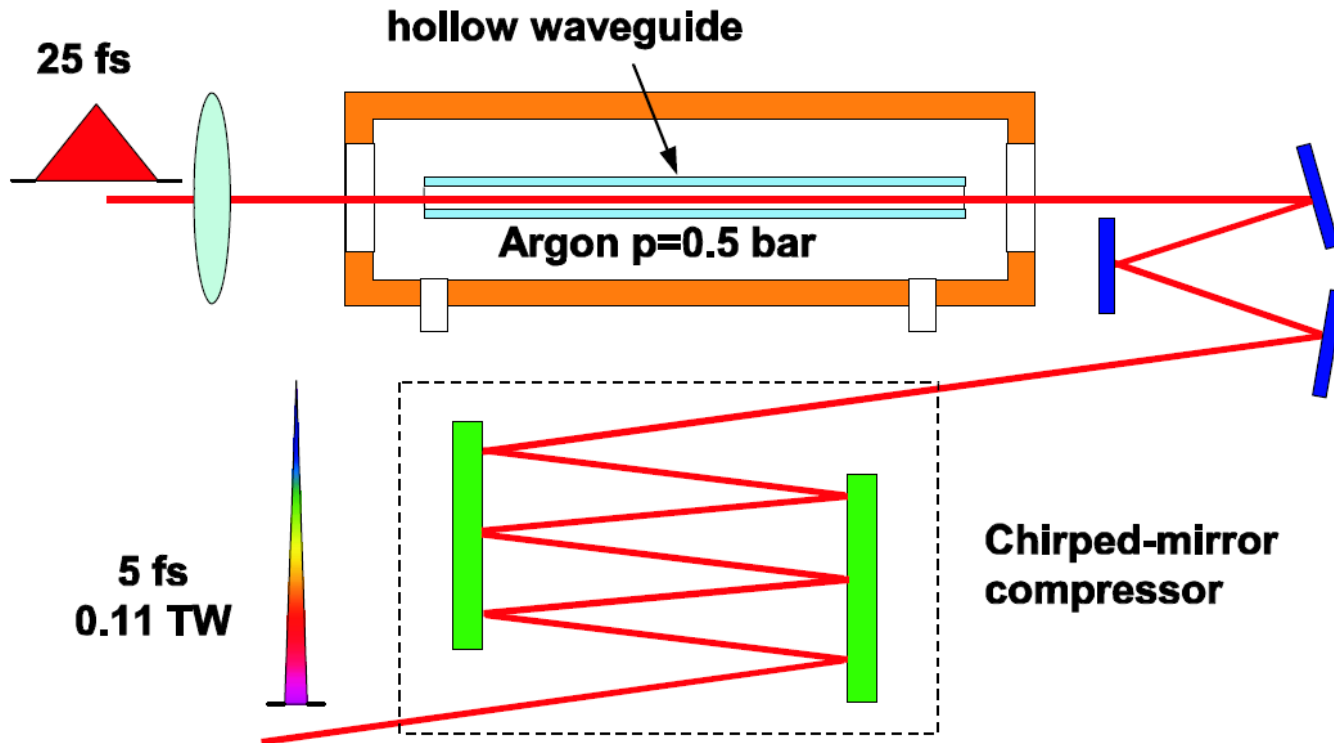
Dispersion matters in spectral broadening

Dispersion negligible using short fiber, SPM dominates



Hollow fiber compression of mili-joule pulses

Self focusing threshold in fused silica is 4 MW. For ~ 100 fs pulse, the pulse energy allowed in a fused silica fiber is ~ 400 nJ before fiber breakdown.



The modes of the hollow fiber are leaky modes, i.e. they experience radiation loss. However, the EH_{11} mode has considerably less loss than the higher order modes and is used for pulse compression. The nonlinear index in the fiber can be controlled with the gas pressure. Typical fiber diameters are 100-500 μm and typical gas pressures are in the range of 0.1-3 bar.

5. Laser Mode Locking

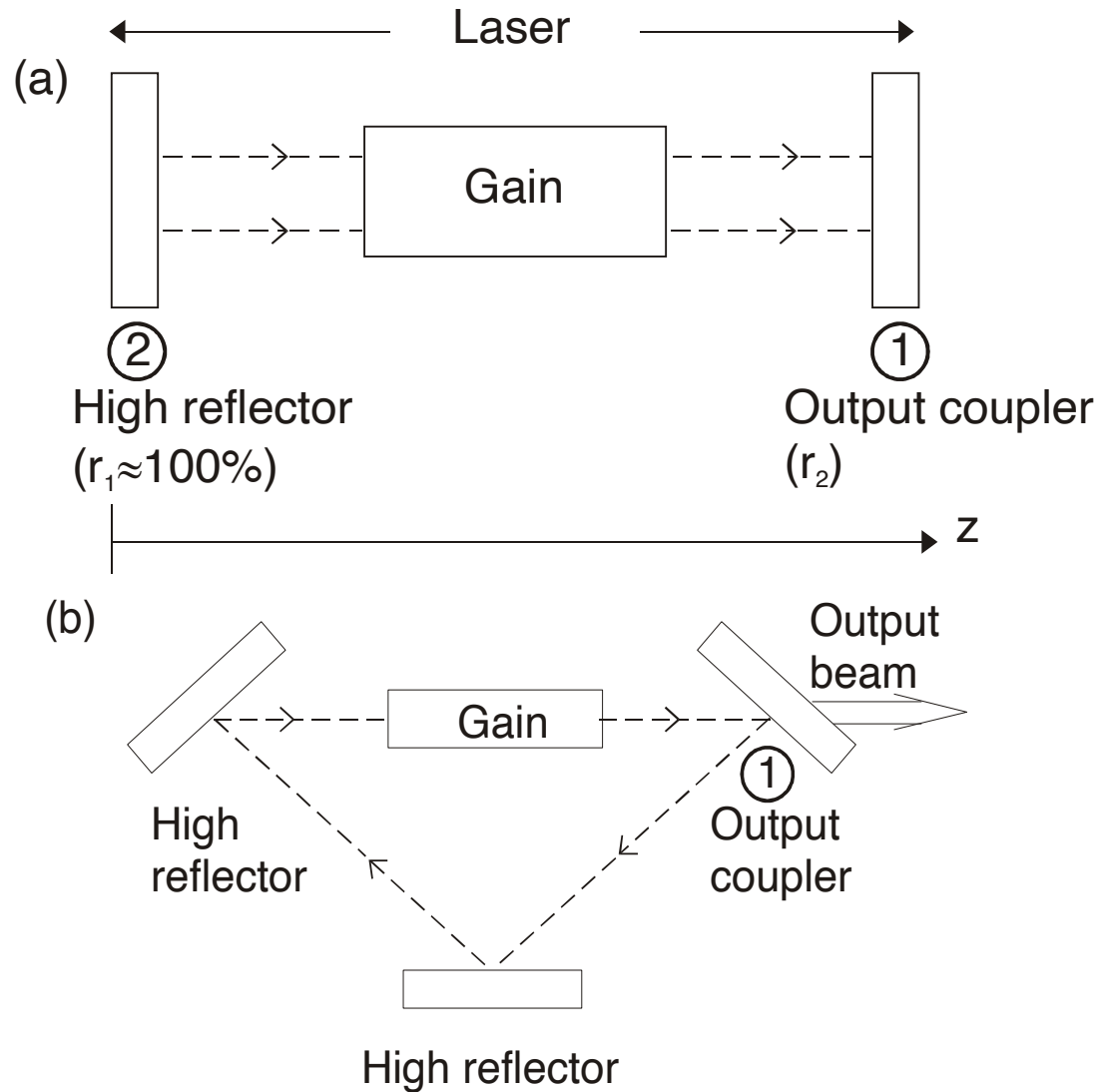


Figure 1.12: Possible cavity configurations

Steady State Lasing

$$E(z, t) = \Re \{ E_0 e^{j(\omega t - kz)} \}$$

$$k = \frac{\omega}{c} n$$

Gain and loss: $n = n' + jn''$

After propagation through gain medium and air path: $\ell = n'_g \ell_g + \ell_a$

$$E = \Re \left\{ E_0 e^{\frac{\omega}{c} n''_g \ell_g} e^{j\omega t} e^{-j\frac{\omega}{c} (n'_g \ell_g + \ell_a)} \right\}$$

Steady-State Condition:

$$E = \Re \left\{ r_1 r_2 e^{2\frac{\omega}{c} n''_g \ell_g} E_0 e^{j\omega t - j2\frac{\omega}{c} \ell} \right\} \Rightarrow r_1 r_2 e^{2\frac{\omega}{c} n''_g \ell_g} = 1$$

Mode Condition: $\frac{2\omega \ell}{c} = 2m\pi$

Resonance Frequencies: $\omega_m = \frac{m\pi c}{\ell} \quad f_m = \frac{mc}{2\ell}$

$$\Delta f = f_m - f_{m-1} = \frac{c}{2\ell}$$

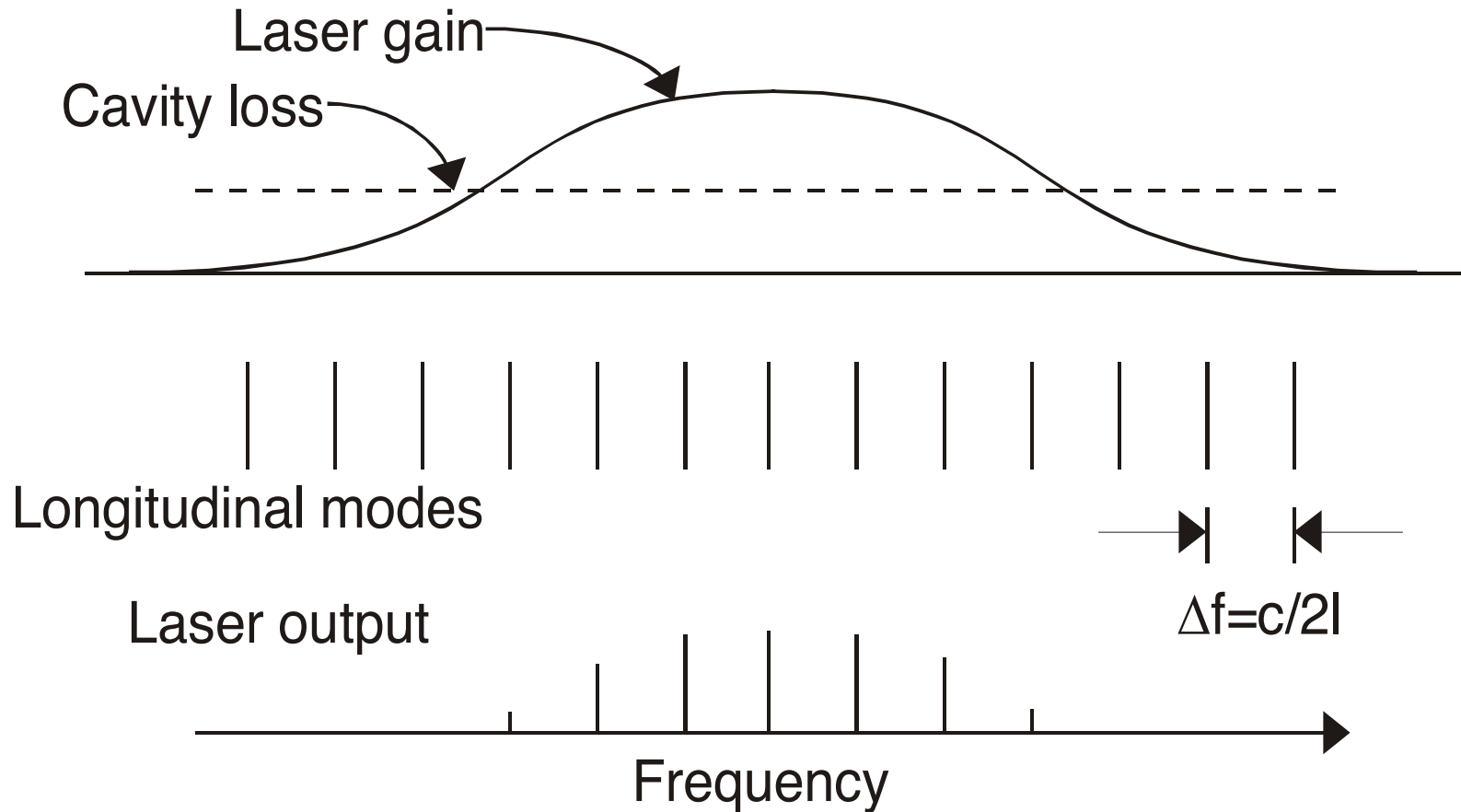


Figure 1.13: Laser gain and cavity loss spectrum

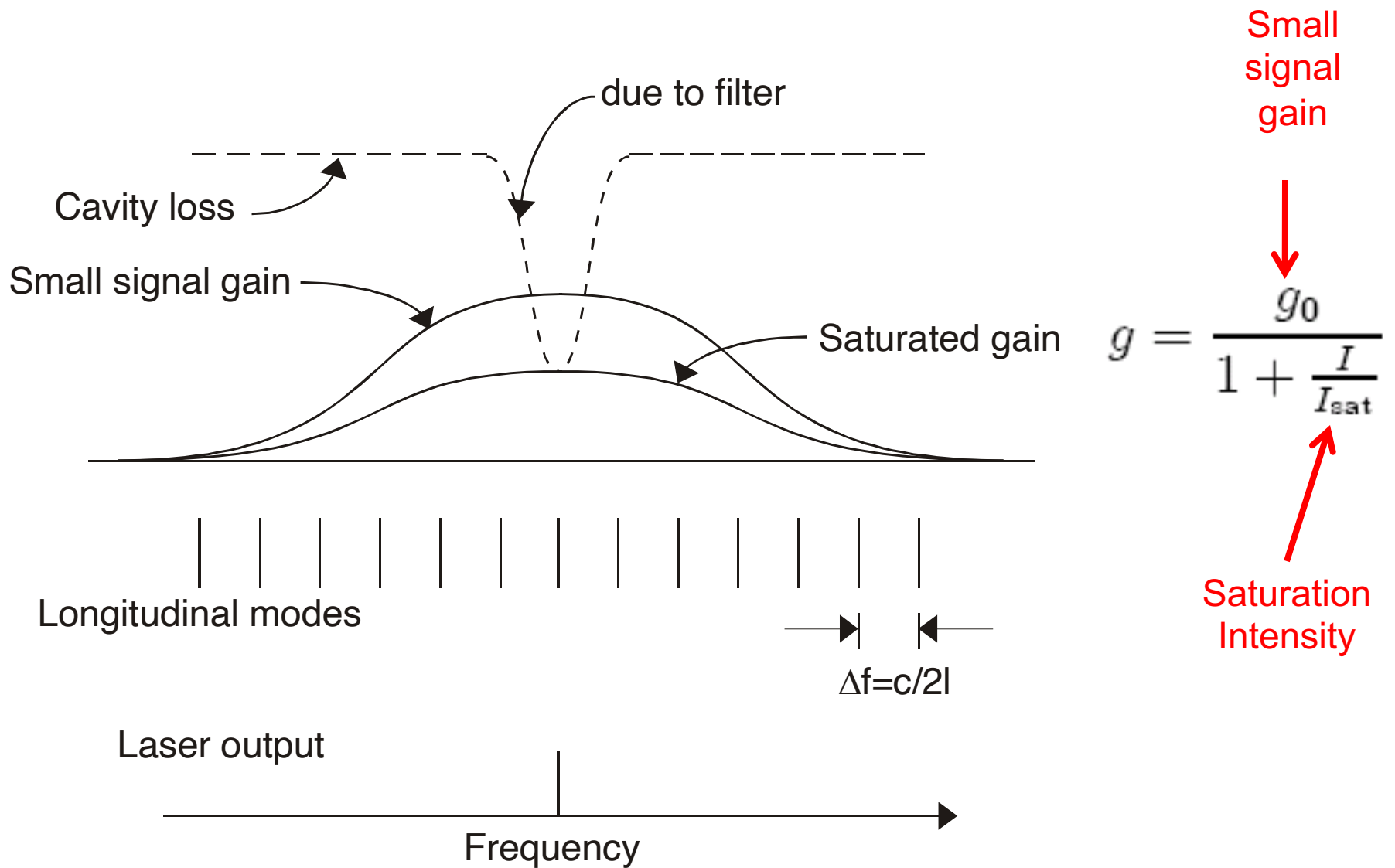
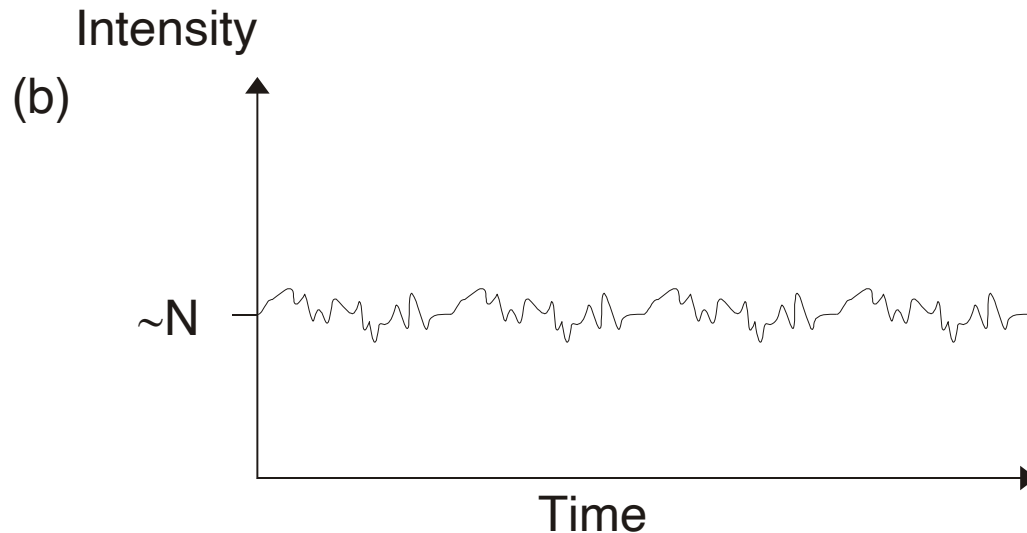
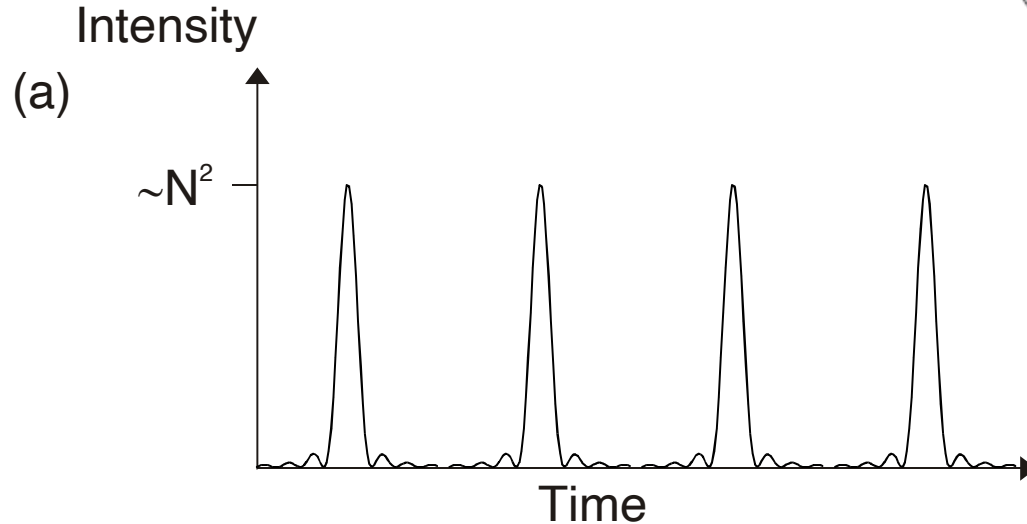


Figure 1.14: Gain and loss spectra.

Mode Locking:



$$E(z, t) = \Re \left[\sum_m \tilde{E}_m e^{j(\omega_m t - k_m z + \phi_m)} \right]$$

$$\omega_m = \omega_0 + m\Delta\omega = \omega_0 + \frac{m\pi c}{\ell},$$

$$k_m = \frac{\omega_m}{c},$$

$$= \Re \left[A(t - z/c) e^{j\omega_0(t - z/c)} \right]$$

Envelope

Carrier

$$\Delta t = \frac{2\pi}{N\Delta\omega} = \frac{1}{N\Delta f}$$

Figure 1.15: (a) mode-locked laser output with constant pulse
(b): with random phase.

Self-consistent Model for Mode-Locked Laser

Medium modeled by pumped two level atoms
(Polarization and Inversion, at each point in space and time)

Medium Polarization is source in the Maxwell's Equations

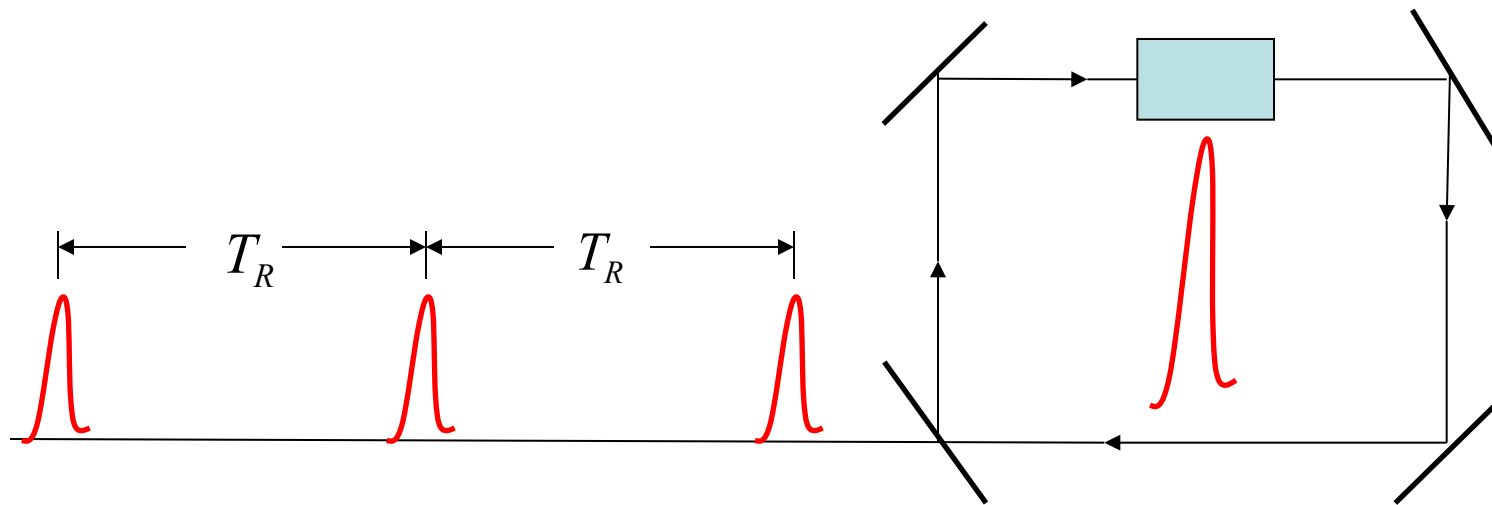


Maxwell Equations describe Field in Resonator

Resonator included by proper boundary conditions for fields at the cavity mirrors.

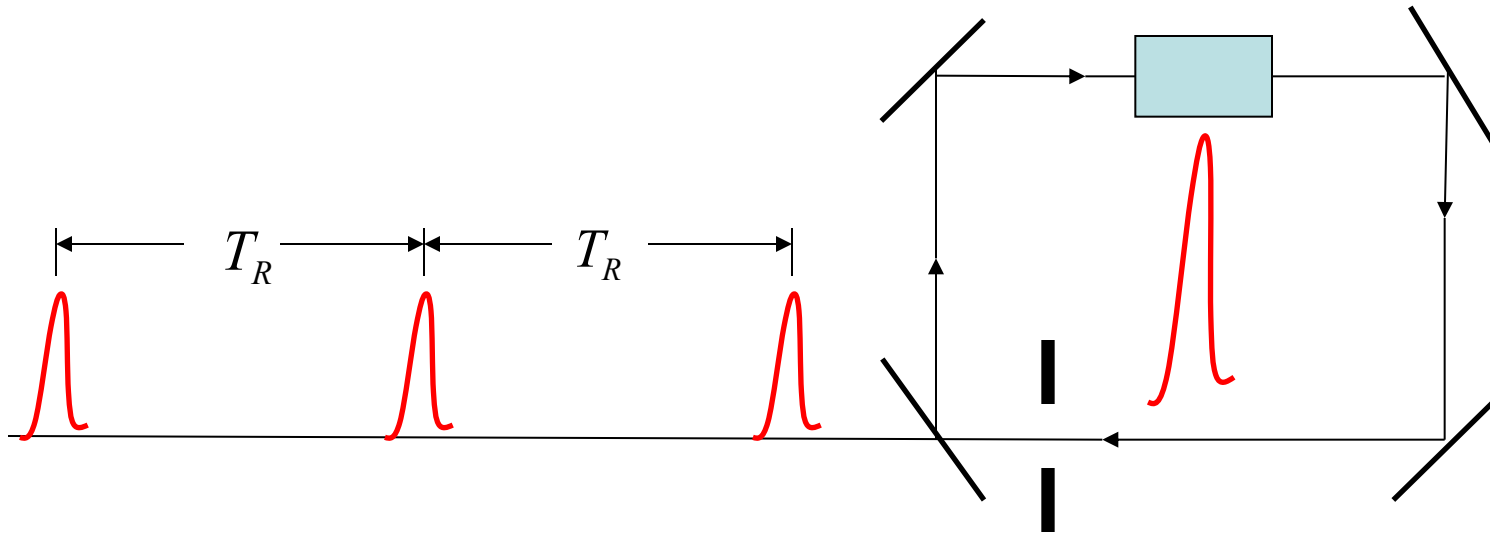
Directly write field as a sum of modes whose amplitudes change slowly with time due to coupling to the gain medium, dispersion,

Time-domain picture of mode-locking



Each time the pulse hits the output coupler, a small fraction of the power is transmitted out of the cavity. The output is a pulse train with repetition rate $1/T_R$.

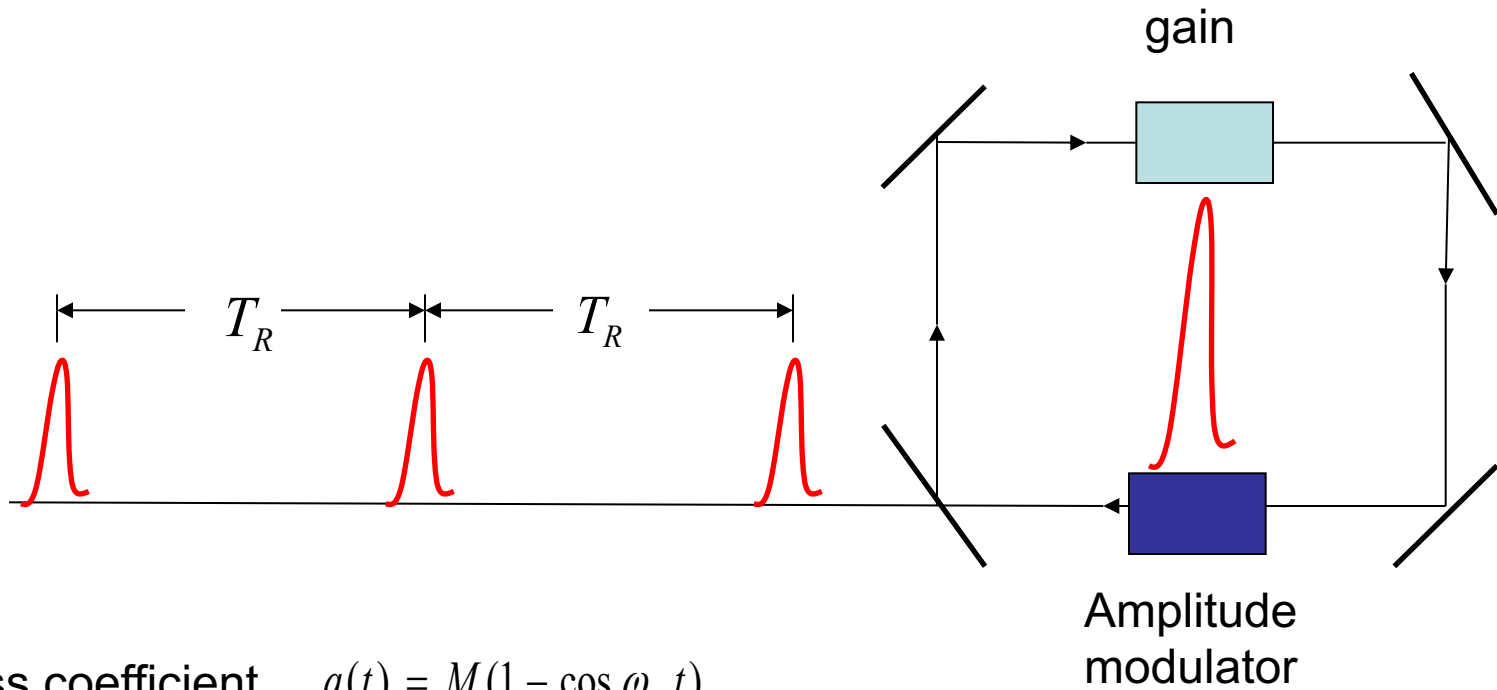
Force laser to generate short-pulse train using a “shutter” to modulate cavity loss



Transient process:

- Shutter is opened, loss is low \rightarrow laser is above threshold \rightarrow peak builds up
- Shutter is closed, loss is high \rightarrow laser is below threshold \rightarrow wings developed

Active mode-locking

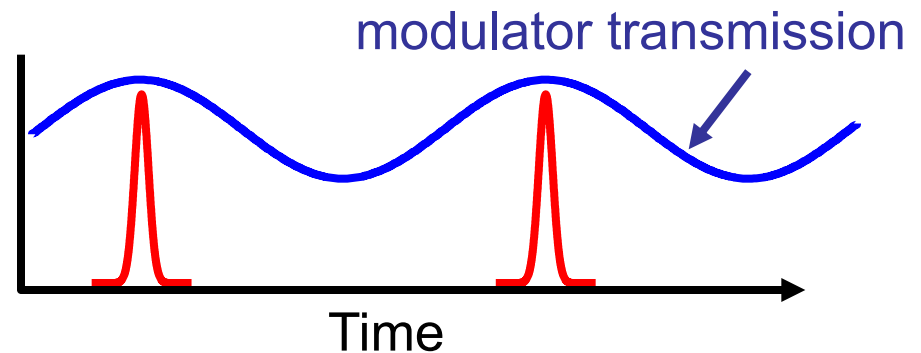


Loss coefficient $q(t) = M(1 - \cos \omega_m t)$

Transmission of the modulator

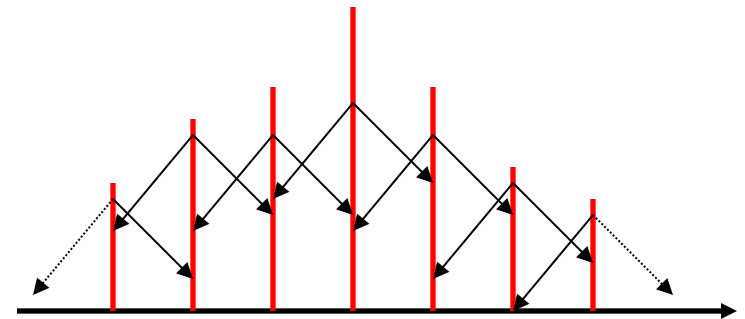
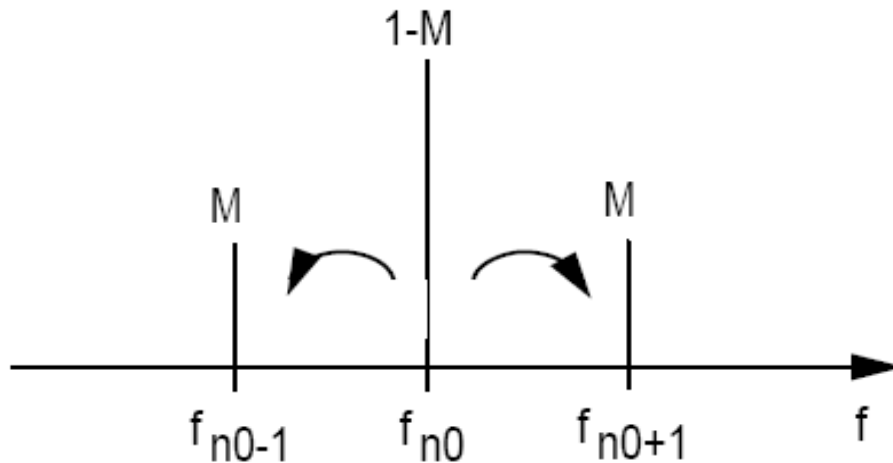
$$T_m = e^{-M(1 - \cos \omega_m t)}$$

$$T_m \approx 1 - M(1 - \cos \omega_m t)$$



Talking among modes

$$\begin{aligned}
 e^{j\omega_{n0}t} T_m &\approx e^{j\omega_{n0}t} (1 - M + M \cos \omega_m t) \\
 &= (1 - M) e^{j\omega_{n0}t} + \frac{M}{2} (e^{j\omega_m t} + e^{-j\omega_m t}) e^{j\omega_{n0}t} \\
 &= (1 - M) e^{j\omega_{n0}t} + \frac{M}{2} e^{j(\omega_{n0} + \omega_m)t} + \frac{M}{2} e^{j(\omega_{n0} - \omega_m)t}
 \end{aligned}$$



The modulator actually takes power out of the central mode and redistributes it to the other modes. This is how mode-locking can occur in a homogeneously broadened laser.

Decompose wave into oscillating modes

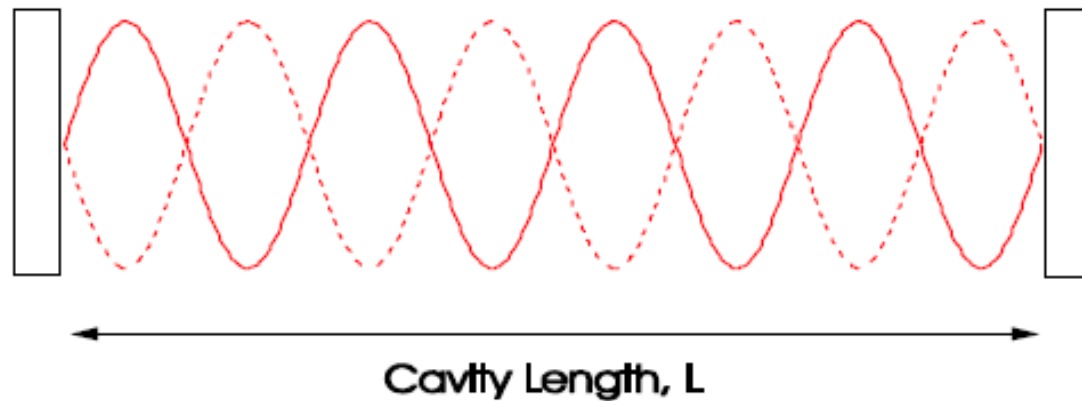


Figure 5.1: Fabry-Perot resonator

$$E^{(left)}(z, t) = \text{Re} \left\{ \sum_{n=0}^{\infty} \hat{E}_n e^{j(\Omega_n t + K_n z)} \right\},$$

$$K_n = n\pi/L$$

$$E^{(right)}(z, t) = \text{Re} \left\{ \sum_{n=0}^{\infty} \hat{E}_n e^{j(\Omega_n t - K_n z)} \right\}.$$

We consider the modes in Eq.(5.2) as a continuum and replace the sum by an integral

$$E^{(right)}(z, t) = \frac{1}{2\pi} \text{Re} \left\{ \int_{K=0}^{\infty} \hat{E}(K) e^{j(\Omega(K)t - Kz)} dK \right\} \quad (5.3)$$

with

$$\hat{E}(K_m) = \hat{E}_m 2L. \quad (5.4)$$

Eq.(5.3) is similar to the pulse propagation discussed in chapter 2 and describes the pulse propagation in the resonator. However, here it is rather an initial value problem, rather than a boundary value problem. Note, the wavenumbers of the modes are fixed, not the frequencies. To emphasize this even more, we introduce a new time variable $T = t$ and a local time frame $t' = t - z/v_{g,0}$, instead of the propagation distance z , where $v_{g,0}$ is the group velocity at the central wave number K_{n_0} of the pulse

$$v_{g,0} = \left. \frac{\partial \omega}{\partial k} \right|_{k=0} = \left. \left(\frac{\partial k}{\partial \omega} \right)^{-1} \right|_{\omega=0}. \quad (5.5)$$

$$k = K - K_{n_0}, \quad (5.6)$$

$$\omega(k) = \Omega(K_{n_0} + k) - \Omega_{n_0}, \quad (5.7)$$

$$\hat{E}(k) = \hat{E}(K_{n_0} + k), \quad (5.8)$$

The temporal evolution of the pulse is then determined by

$$E^{(right)}(z, t) = \frac{1}{2\pi} \text{Re} \left\{ \int_{-K_{n_0} \rightarrow -\infty}^{\infty} \hat{E}(k) e^{j(\omega(k)t - kz)} dk \right\} e^{j(\omega_0 t - k_0 z)}. \quad (5.9)$$

Analogous to chapter 2, we define a slowly varying field envelope, that is already normalized to the total power flow in the beam

$$A(z, t) = \sqrt{\frac{A_{eff}}{2Z_0}} \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{E}(k) e^{j(\omega(k)t - kz)} dk. \quad (5.10)$$

$$A(T, t') = \sqrt{\frac{A_{eff}}{2Z_0}} \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{E}(k) e^{j((\omega(k) - v_{g,0}k)T + kv_{g,0}t')} dk. \quad (5.11)$$

which can be written as

$$T_R \left. \frac{\partial A(T, t')}{\partial T} \right|_{(GDD)} = j \sum_{n=2}^{\infty} D_n \left(-j \frac{\partial}{\partial t'} \right)^n A(T, t'), \quad (5.12)$$

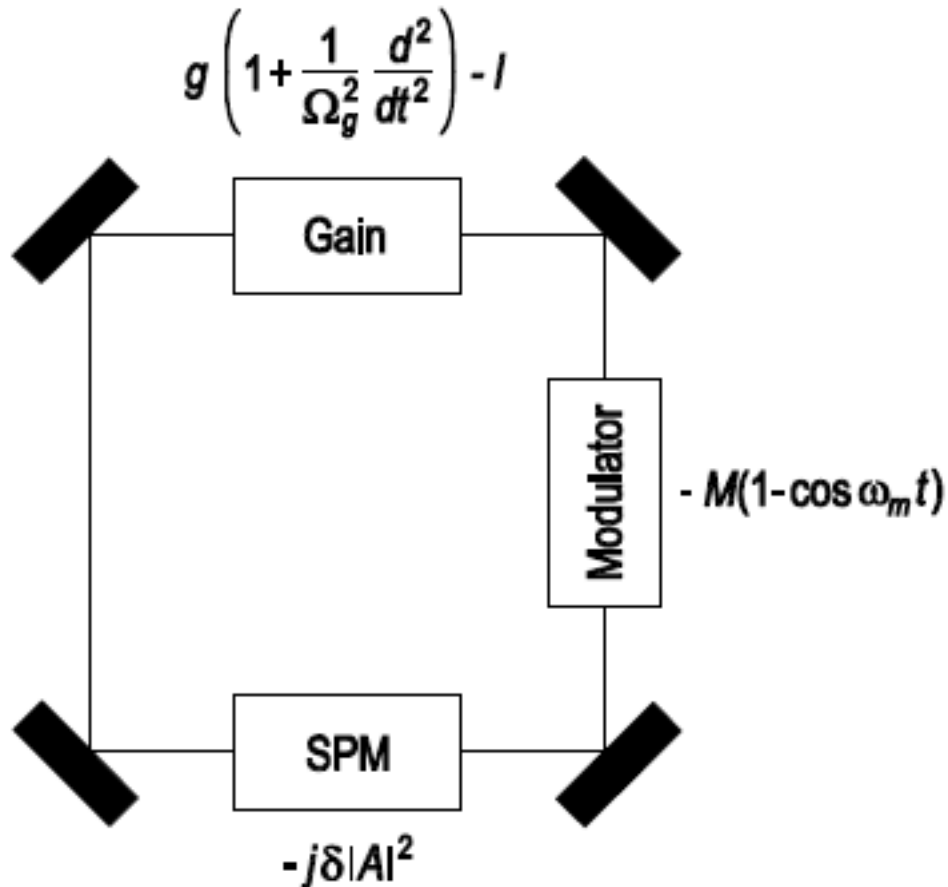
with the dispersion coefficients per resonator round-trip $T_R = \frac{2L}{v_{g,0}}$

$$D_n = \left. \frac{2L}{n! v_{g,0}^{n+1}} \frac{\partial^{n-1} v_g(k)}{\partial k^{n-1}} \right|_{k=0}. \quad (5.13)$$

The dispersion coefficients (5.13) look somewhat suspicious, however, it is not difficult to show, that they are equivalent to derivatives of the roundtrip phase $\phi_R(\Omega) = \frac{\Omega}{c} n(\Omega) 2L$ in the resonator at the center frequency

$$D_n = - \left. \frac{1}{n!} \frac{\partial^n \phi_R^{(n)}(\Omega)}{\partial \Omega^n} \right|_{\Omega=\omega_0}, \quad (5.14)$$

5.1 Master equation of mode-locking



Assume in steady state, the change in the pulse caused by each element in the cavity small.

$$T_R \frac{\partial A(T, t)}{\partial T} = \sum_i \Delta A_i = 0$$

A : the pulse envelope

T_R : the cavity round-trip time

T : the time that develops on a time scale of the order of T_R

t : the fast time of the order of the pulse duration

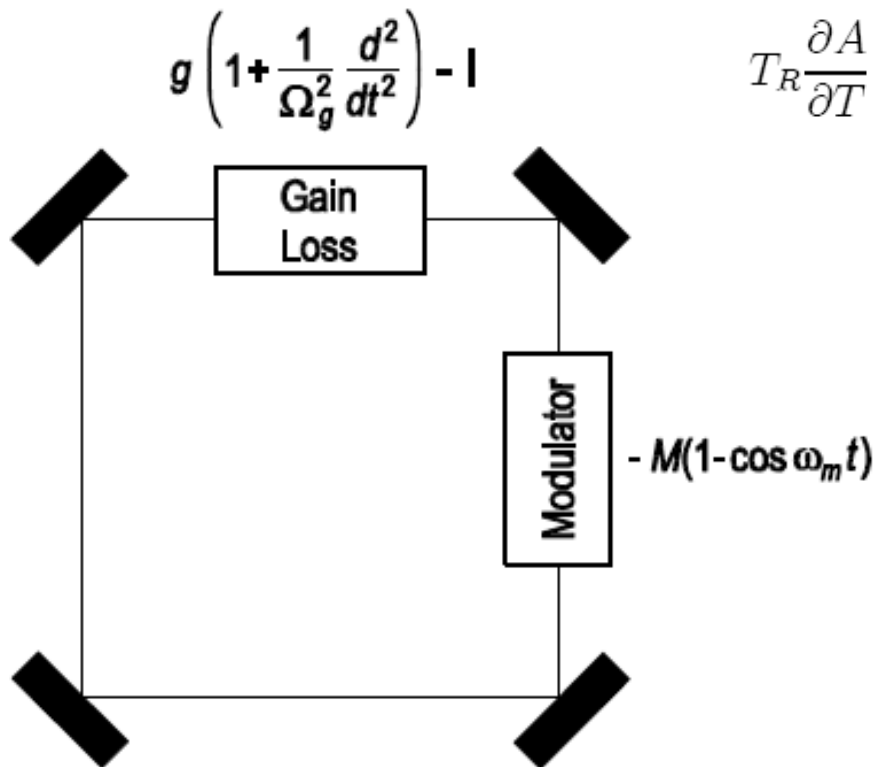
ΔA_i : the changes of the pulse envelope due to different elements in the cavity.

Loss: $T_R \frac{\partial A(T, t')}{\partial T} \Big|_{(loss)} = -lA(T, t')$ (5.15)

Gain: $T_R \frac{\partial A(T, t')}{\partial T} \Big|_{(gain)} = \left(g(T) + D_g \frac{\partial^2}{\partial t'^2} \right) A(T, t'), \quad D_g = \frac{g(T)}{\Omega_g^2}$

$$\begin{aligned}
 T_R \frac{\partial A(T, t')}{\partial T} = & \text{loss} \quad -lA(T, t') + \text{dispersion} \quad j \sum_{n=2}^{\infty} D_n \left(j \frac{\partial}{\partial t'} \right)^n A(T, t') \\
 & + \text{gain} \quad g(T) \left(1 + \frac{1}{\Omega_g^2} \frac{\partial^2}{\partial t'^2} \right) A(T, t') \\
 & - \text{Mode-locking element} \quad q(T, t') A(T, t') - \text{Gain dispersion} \quad j \delta |A(T, t')|^2 A(T, t') - \text{Self-phase modulation}
 \end{aligned} \tag{5.21}$$

5.2 Active mode-locking by loss modulation



Gain tends to narrow the spectrum,
spectrum broadened by modulator.

$$T_R \frac{\partial A}{\partial T} = \left[g(T) + D_g \frac{\partial^2}{\partial t^2} - l - M(1 - \cos(\omega_M t)) \right] A.$$



$$T_R \frac{\partial A}{\partial T} = \left[g - l + D_g \frac{\partial^2}{\partial t^2} - M_s t^2 \right] A.$$

$$D_g = \frac{g}{\Omega_g^2},$$

$$M_s = \frac{M\omega_M^2}{2}.$$

Hermite-Gaussian Solutions

$$A_n(T, t) = A_n(t)e^{\lambda_n T/T_R} \quad A_n(t) = \sqrt{\frac{W_n}{2^n \sqrt{\pi} n! \tau_a}} H_n(t/\tau_a) e^{-\frac{t^2}{2\tau_a^2}}$$

$$\tau_a = \sqrt[4]{D_g/M_s} \longrightarrow \tau_a = \sqrt[4]{2} \left(\frac{g}{M}\right)^{\frac{1}{4}} \frac{1}{\sqrt{\Omega_g \omega_M}}$$

- 1) Larger modulation depth, M, and higher modulation frequency will give shorter pulses because the “low loss” window becomes narrower, thus shortening the pulses.
- 2) A broader gain bandwidth yields shorter pulses because the filtering effect of gain narrowing is lower and more modes are lasing.

$$\lambda_n = g_n - l - 2M_s \tau_a^2 \left(n + \frac{1}{2}\right).$$

For given g, the eigen solution with n=0 has the largest gain per round-trip and saturate the gain to

$$g_s = l + M_s \tau_a^2$$

All other modes will decay.

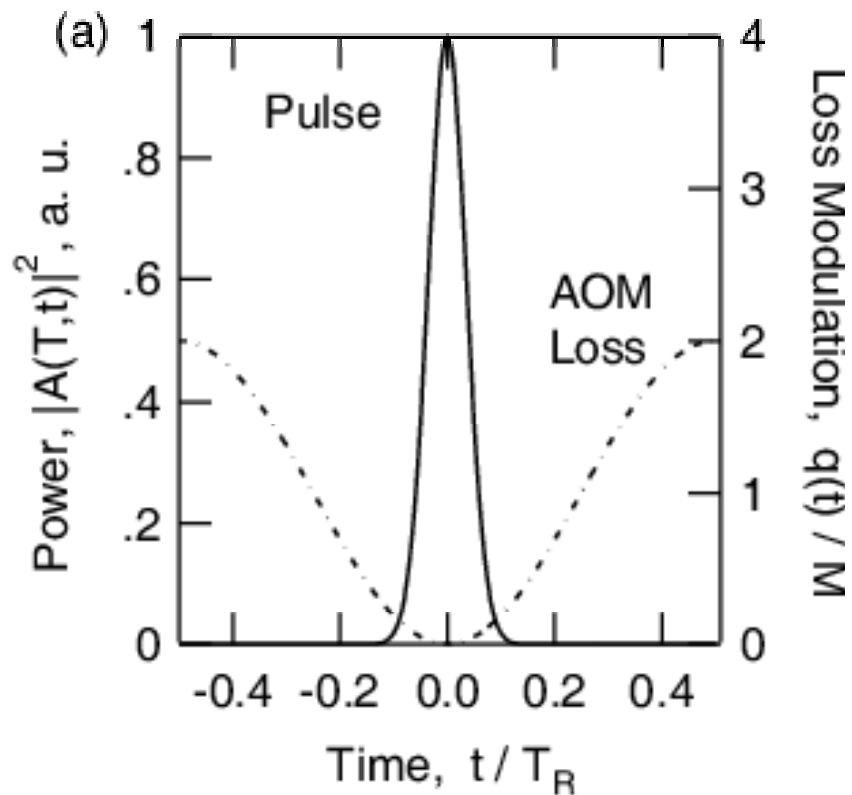


Fig. 5.4: Loss modulation results in pulse shortening in each roundtrip

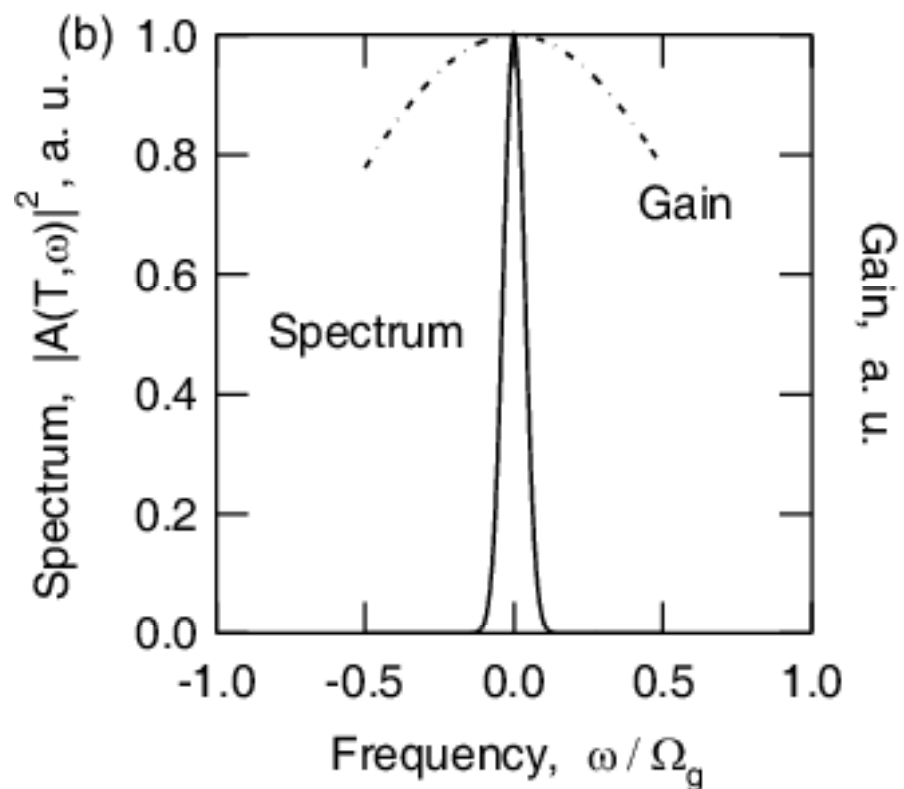


Fig. 5.5: Gain filtering broadens the pulse in each roundtrip

Example:

Nd:YAG; $2l=2g=10\%$,

$$\Omega_g = \pi \Delta f_{\text{FWHM}} = 0.65 \text{ THz},$$

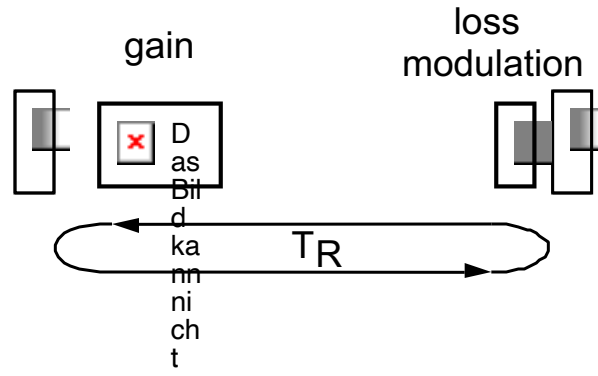
$$M = 0.2, f_R = 100 \text{ MHz},$$

$$D_g = 0.24 \text{ ps}^2,$$

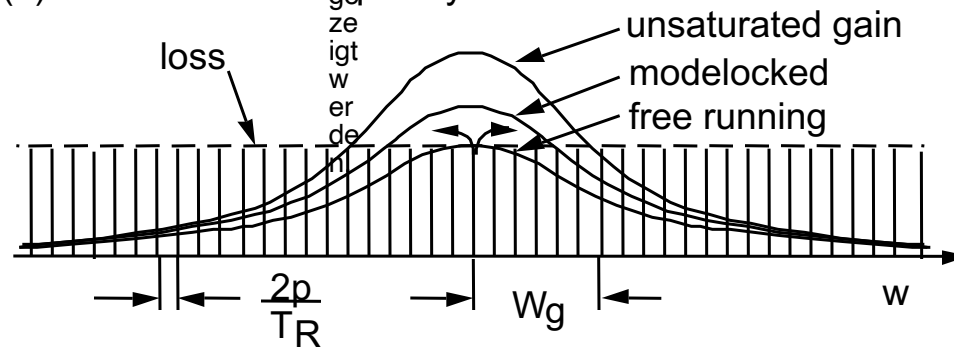
$$M_s = 4 \times 10^{16} / \text{s}^2.$$

$$\tau = 99 \text{ ps}.$$

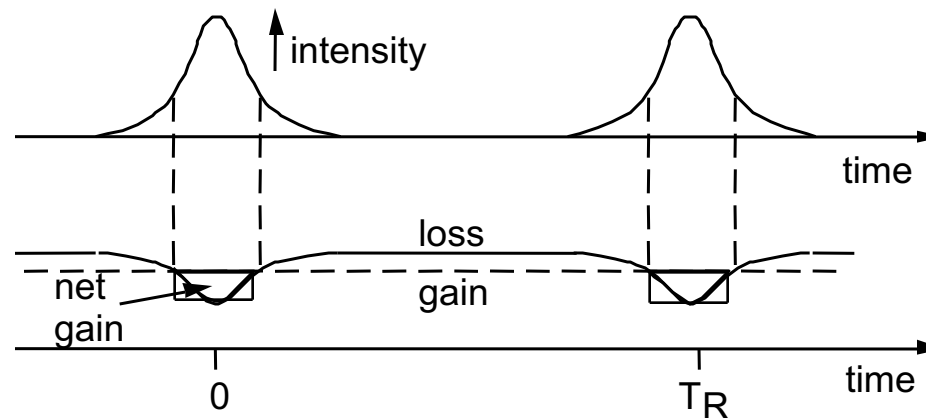
(a) Laser Cavity



(b) Frequency Domain



(c) Time Domain



5.2 Active mode-locking by phase modulation

It can be modeled using master equation by replacing M by jM

$$T_R \frac{\partial A}{\partial T} = \left[g(T) + D_g \frac{\partial^2}{\partial t^2} - l - jM (1 - \cos(\omega_M t)) \right] A$$

$$\tau'_a = \sqrt[4]{-j} \sqrt[4]{D_g/M_s}$$

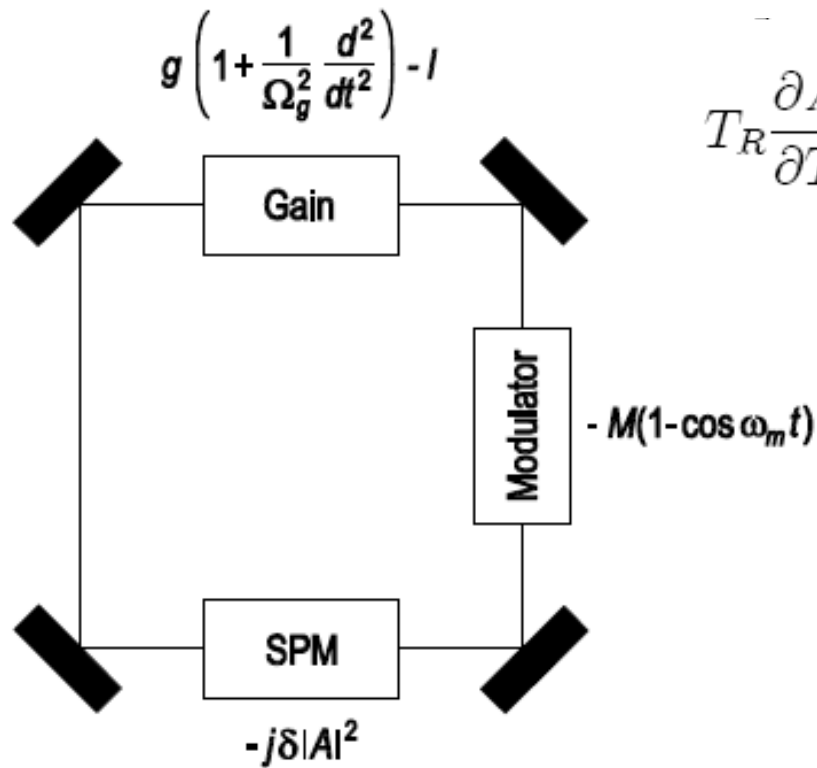
$$A_0(t) = \sqrt{\frac{W_s}{2^n \sqrt{\pi} n! \tau'_a}} e^{-\frac{t^2}{2\tau_a^2} \frac{1}{\sqrt{2}} (1+j)}$$

Chirp pulse

Pulse duration

$$\tau_a = \sqrt[4]{D_g/M_s}$$

5.4 Active mode-locking with additional SPM

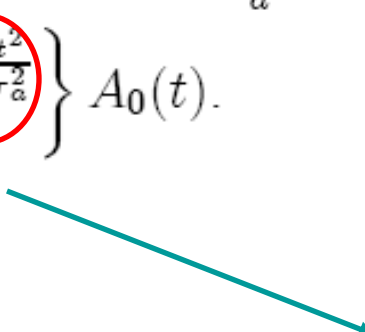


$$T_R \frac{\partial A}{\partial T} = \left[g(T) + D_g \frac{\partial^2}{\partial t^2} - l - M_s t^2 - j\delta |A|^2 \right] A.$$

$$A_0(t) = A e^{-\frac{t^2}{2\tau_a^2}(1+j\beta) + j\Psi T/T_R}$$



$$j\Psi A_0(t) = \left\{ g - l + D_g \left[\frac{t^2}{\tau_a^4} (1 + j\beta)^2 - \frac{1}{\tau_a^2} (1 + j\beta) \right] - M_s t^2 - j\delta |A|^2 e^{-\frac{t^2}{\tau_a^2}} \right\} A_0(t).$$

$$j\Psi A_0(t) = \left\{ g - l + D_g \left[\frac{t^2}{\tau_a^4} (1 + j\beta)^2 - \frac{1}{\tau_a^2} (1 + j\beta) \right] - M_s t^2 - j\delta |A|^2 e^{-\frac{t^2}{\tau_a^2}} \right\} A_0(t).$$


$$j\Psi = g - l + D_g \left[\frac{t^2}{\tau_a^4} (1 + j\beta)^2 - \frac{1}{\tau_a^2} (1 + j\beta) \right] - M_s t^2 - j\delta |A|^2 \left(1 - \frac{t^2}{\tau_a^2} \right)$$

$$0 = \frac{D_g}{\tau_a^4} (1 - \beta^2) - M_s$$

$$g - l = \frac{D_g}{\tau_a^2},$$

$$0 = 2\beta \frac{D_g}{\tau_a^4} + \frac{\phi_0}{\tau_a^2},$$

$$\Psi = D_g \left[-\frac{1}{\tau_a^2} \beta \right] - \phi_0$$

$$\begin{aligned}
 0 &= \frac{D_g}{\tau_a^4} (1 - \beta^2) - M_s \\
 0 &= 2\beta \frac{D_g}{\tau_a^4} + \frac{\phi_0}{\tau_a^2},
 \end{aligned}
 \quad \longrightarrow \quad
 \begin{aligned}
 \beta &= -\frac{\phi_0 \tau_a^2}{2D_g} \\
 \tau_a^4 &= \frac{D_g}{M_s + \frac{\phi_0^2}{4D_g}}.
 \end{aligned}$$

The resulting pulse is shorter due to additional SPM. However, as the pulse shortens by a factor of 2, SPM drives the mode-locking unstable. It is why broadband laser media can not simply generate femtosecond pulses via active mode-locking, since SPM drives them unstable.

5.5 Active Modelocking with Soliton Formation

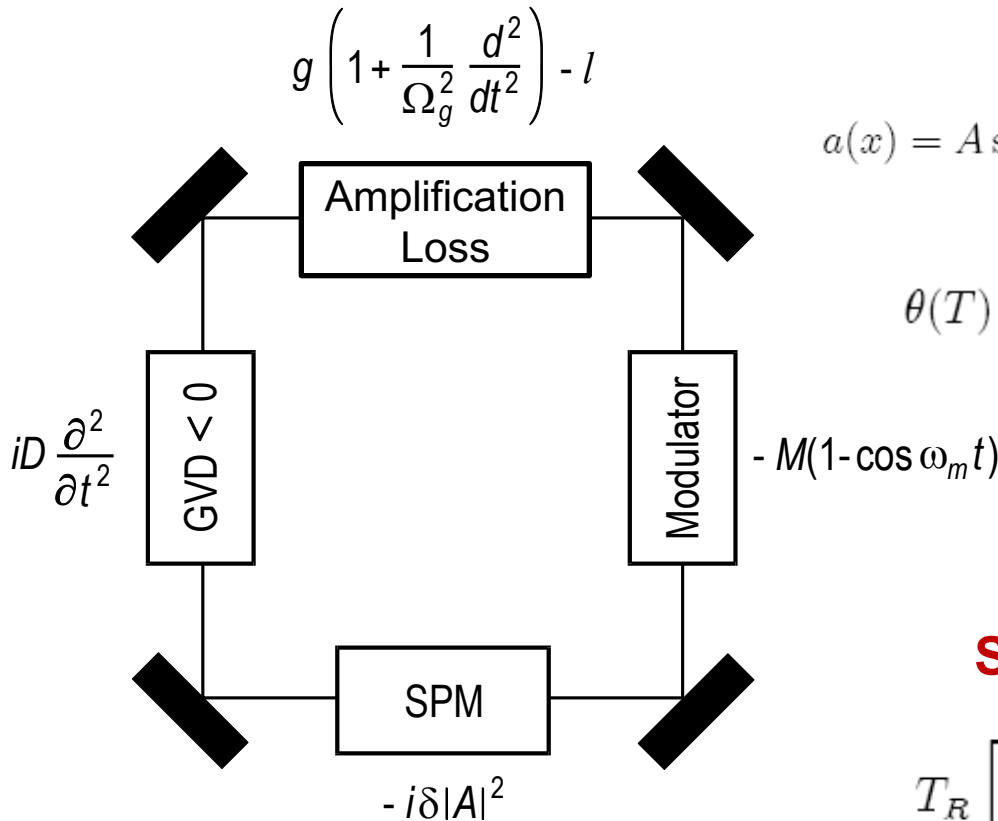


Fig. 5.8: Active Modelocking with Soliton Formation

$$A(T, t) = (a(x)e^{jpt} + a_c(T, t)) e^{-j\theta}$$

$$a(x) = A \operatorname{sech}(x), \quad \text{and} \quad x = \frac{1}{\tau} \left(t + 2D \int_0^T p(T') dT' - t_0 \right)$$

$$\theta(T) = \theta_0(T) - \frac{D}{T_R} \int_0^T \left(\frac{1}{\tau(T')^2} - p(T')^2 \right) dT'$$

$$\frac{|D|}{\tau(T)^2} = \frac{\delta A(T)^2}{2}.$$

Soliton Perturbation Theory

$$\begin{aligned} & T_R \left[\frac{\partial a_c}{\partial T} + \frac{\partial W}{\partial T} \mathbf{f}_w + \frac{\partial \Delta \theta}{\partial T} \mathbf{f}_\theta + \frac{\partial \Delta p}{\partial T} \mathbf{f}_p + \frac{\partial \Delta t}{\partial T} \mathbf{f}_t \right] \\ &= \phi_0 \mathbf{L} (\mathbf{a}_c + \Delta p \mathbf{f}_p) + \mathbf{R} (\mathbf{a} + \Delta p \mathbf{f}_p + \mathbf{a}_c) \\ & \quad - M \omega_M \sin(\omega_M \tau x) \Delta t \mathbf{a}(x) \end{aligned}$$

$$\mathbf{R} = g \left(1 + \frac{1}{\Omega_g^2 \tau^2} \frac{\partial^2}{\partial x^2} \right) - l - M (1 - \cos(\omega_M \tau x)),$$

5.5.1 Steady State and Stability Conditions

Pulse width of stable solitons will satisfy: $\omega_M \tau \ll 1 \ll \Omega_g \tau$

Soliton sheds energy into continuum, but we can neglect coupling back of the continuum to the soliton for large normalized dispersion

Normalized Dispersion: $D_n = |D| \Omega_g^2 / g$.

Generated continuum:

$$T_R \frac{\partial G}{\partial T} = \left[g - l + j\Phi_0 + \frac{g}{\Omega_g^2} (1 - jD_n) \frac{\partial^2}{\partial t^2} - M (1 - \cos(\omega_M t)) \right] G + \mathcal{F}^{-1} \left\{ \begin{aligned} &< \mathbf{f}_k^{(+)} | \mathbf{R} \mathbf{a}_0(x) > \\ &- < \mathbf{f}_k^{(+)} | M \omega_M \sin(\omega_M \tau x) \mathbf{a}_0(x) > \Delta t \end{aligned} \right\}$$

Drive terms

Stability of Continuum

Condition for stability: net loss per roundtrip for all continuum modes:

$$\text{Re}\{\lambda_m\} < 0 \text{ for } m \geq 0$$

with: $\xi = (\tau_a/\tau)^2$

$$\xi^2 - 3\text{Re}\{\sqrt{(1 - jD_n)}\}\xi + \frac{\pi^2}{4} < 0.$$

Minimum amount of negative dispersion:

$$D_{n,crit} = 0.652.$$

For even larger amounts of dispersion, we define the pulse shortening factor in terms of FWHM pulse width

$$R = \frac{1.66}{1.76} \sqrt{\xi}.$$

$$R_{crit} \approx 1.2$$

Pulse width reduction

$$\xi < \sqrt{\frac{9D_n}{2}} \quad \text{or} \quad R < \frac{1.66}{1.76} \sqrt[4]{\frac{9D_n}{2}}.$$

$$\tau^2 = \frac{|D|}{\Phi_0}.$$

Nonlinear phase shift per roundtrip is limited.

$$R_{max} = \frac{1.66}{1.76} \sqrt[12]{\frac{(9\Phi_0/2)^2}{D_g M_s}}$$

$$\tau_{min} = \sqrt[6]{\frac{2D_g^2}{9\Phi_0 M_s}}.$$

$$D_n = \frac{2}{9} \sqrt[3]{\frac{(9\Phi_0/2)^2}{D_g M_s}}.$$

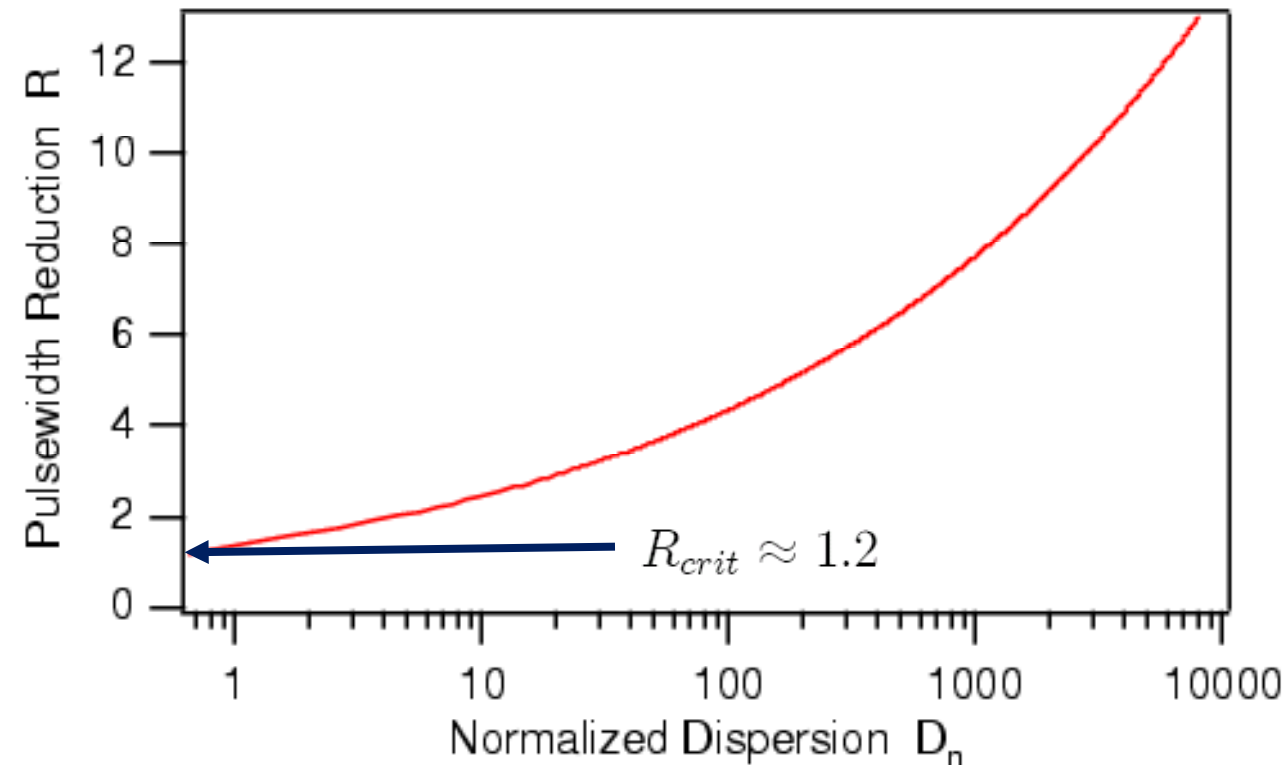


Fig. 5.9: Pulse width reduction

gain material	$\frac{\Omega_g}{2\pi (THz)}$	M	$\frac{\omega_M}{2\pi (MHz)}$	$\frac{D_g}{(ps^2)}$	$M_s \cdot (ps^2)$	$\frac{\tau_{a,FWHM}}{(ps)}$
Nd:YAG	0.06	0.2	250	0.7	$2.5 \cdot 10^{-7}$	68
Nd:glass	4	0.2	250	$158 \cdot 10^{-6}$	$2.5 \cdot 10^{-7}$	8.35
Cr:LiSAF	32	0.2	250	$2.4 \cdot 10^{-6}$	$2.5 \cdot 10^{-7}$	3
Ti:sapphire	43	0.01	100	$1.4 \cdot 10^{-6}$	$2 \cdot 10^{-9}$	8.5

gain material	R_{max}	$\frac{\tau_{min,FWHM}}{(ps)}$	D_n	$\frac{\tau_{trans}}{T_R}$	
Nd:YAG	3	22.7	23.4	702	
Nd:glass	6	1.4	385	11,538	
Cr:LiSAF	8.6	0.35	1563	46,600	
Ti:sapphire	13.5	0.63	9367	281,000	

Table 5.1: Maximum pulsewidth reduction and necessary normalized GVD for different laser systems. In all cases we used for the saturated gain $g = 0.1$ and the soliton phase shift per roundtrip $\Phi_0 = 0.1$. For the broadband gain materials the last column indicates rather long transient times which calls for regenerative mode locking.

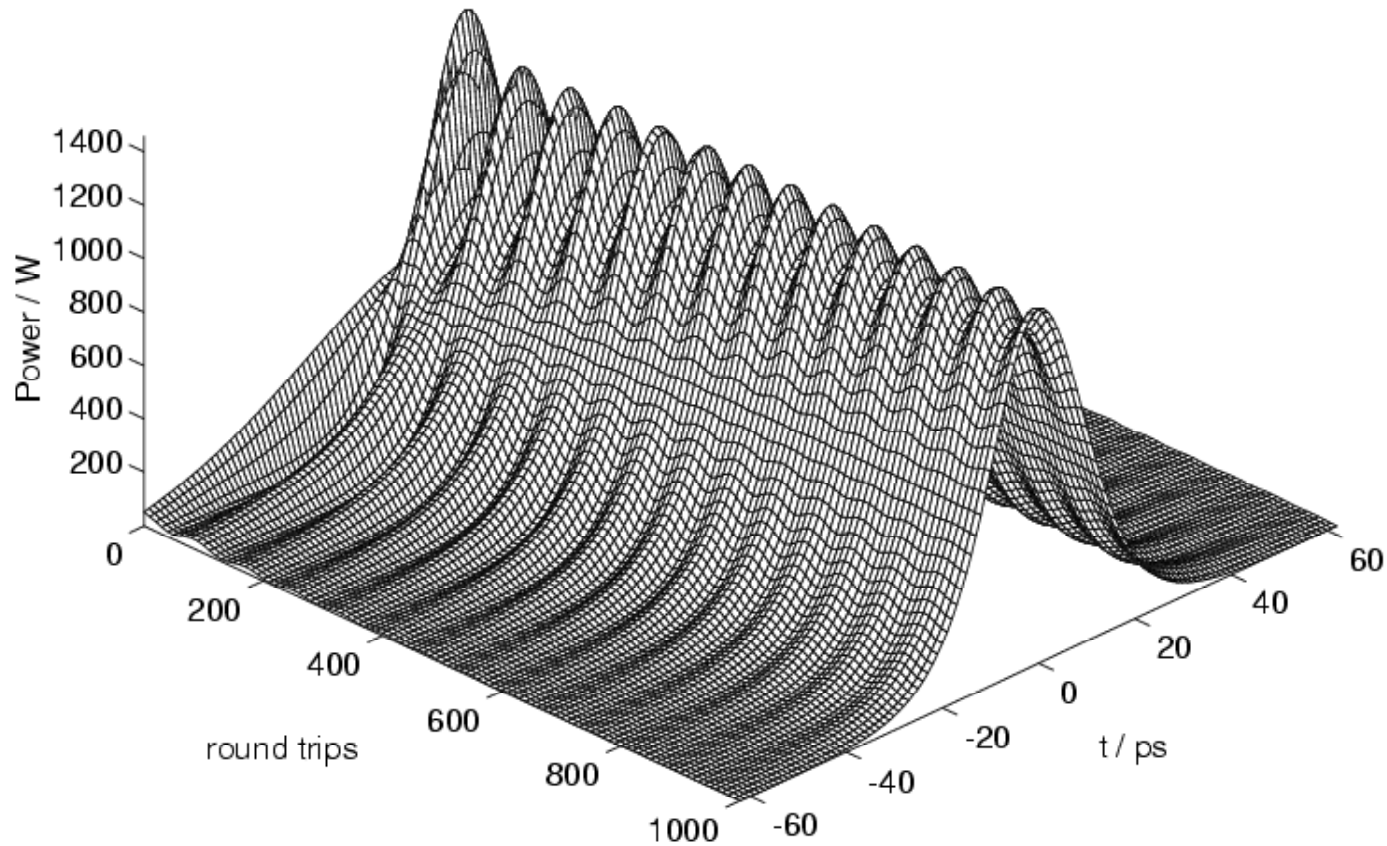


Fig. 5.10: Time evolution in a Nd:YAG laser $D = -17\text{ps}^2$ with initial pulsewidth 68 ps.

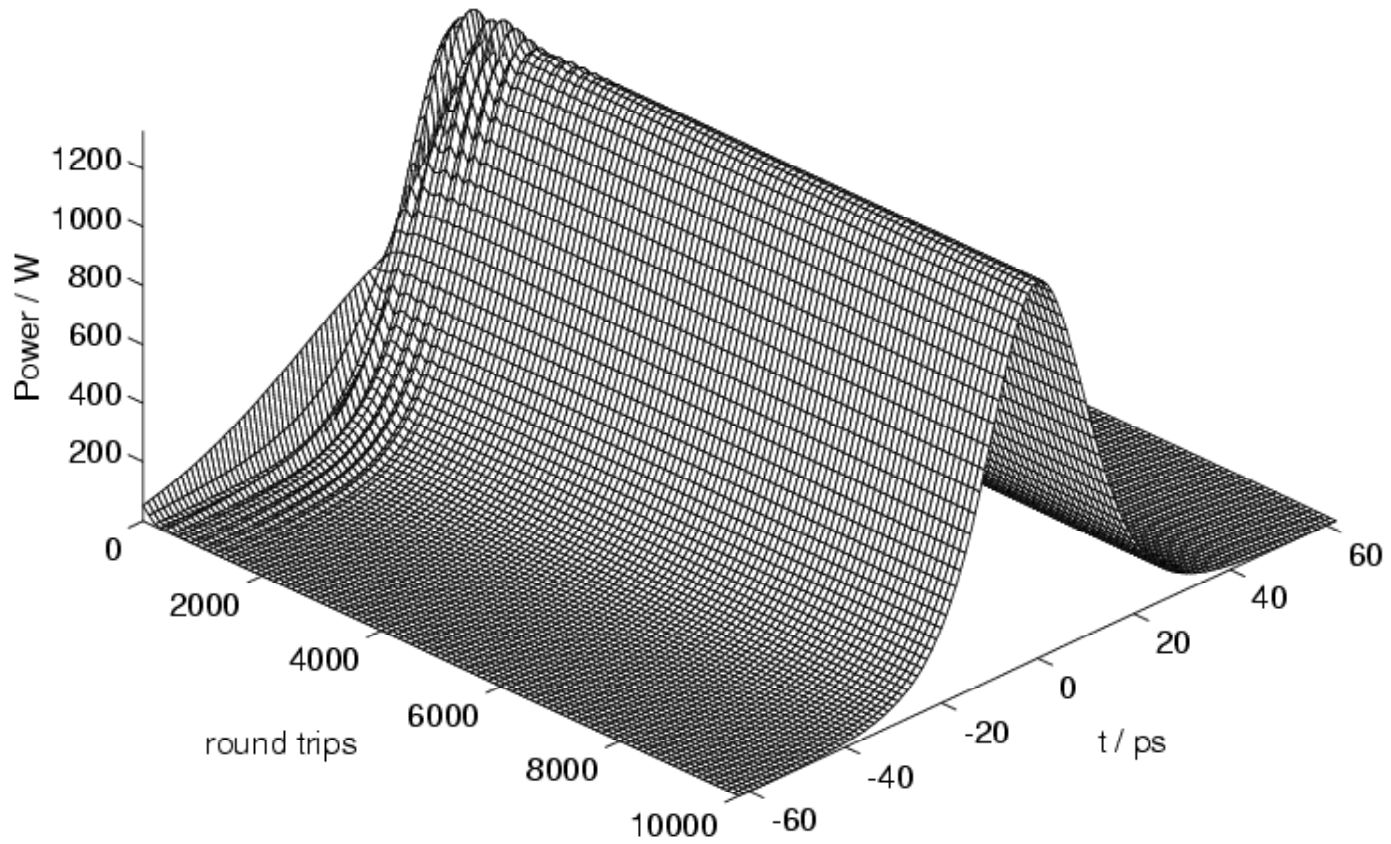


Fig. 5.11: 10000 roundtrips

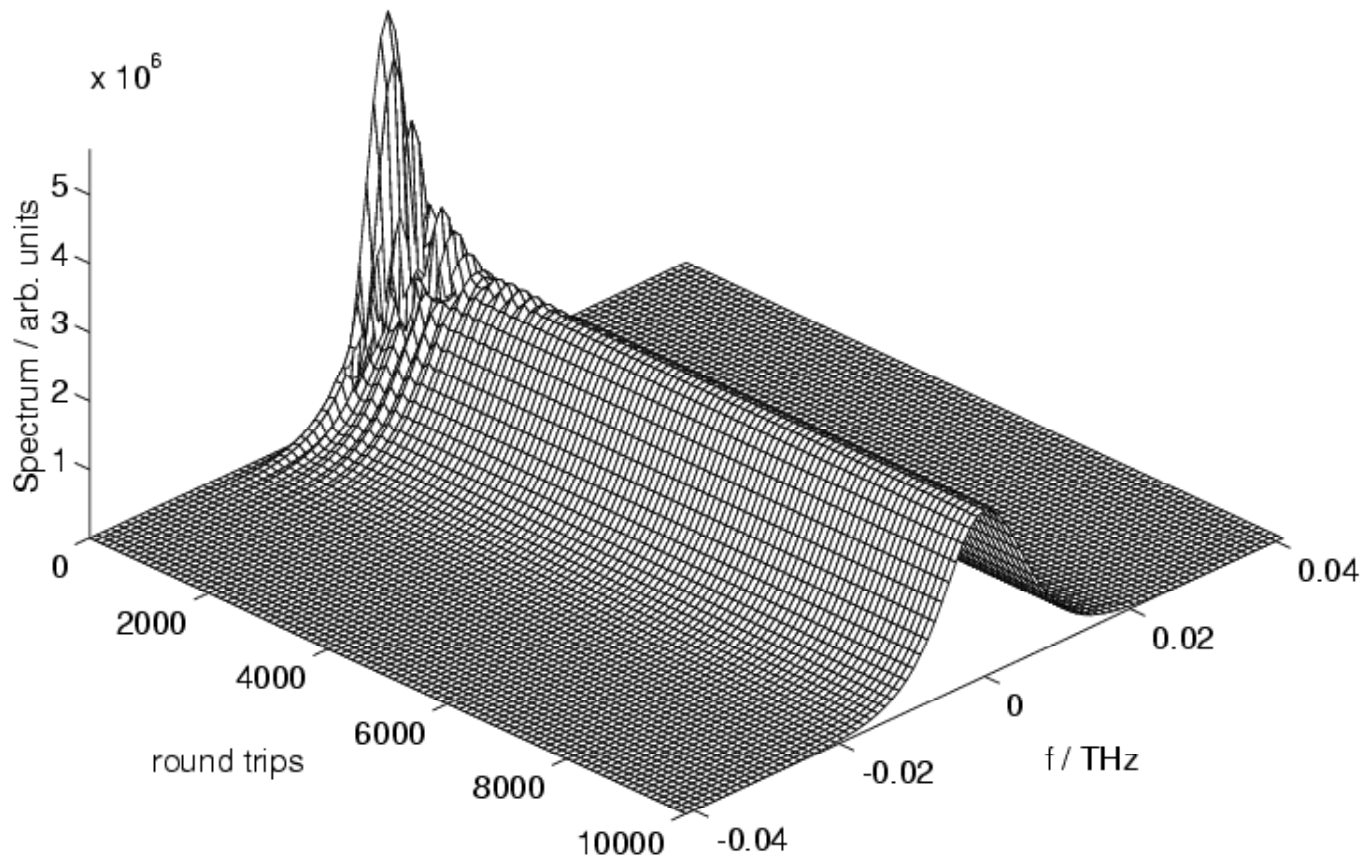


Fig. 5.11: 10000 roundtrips

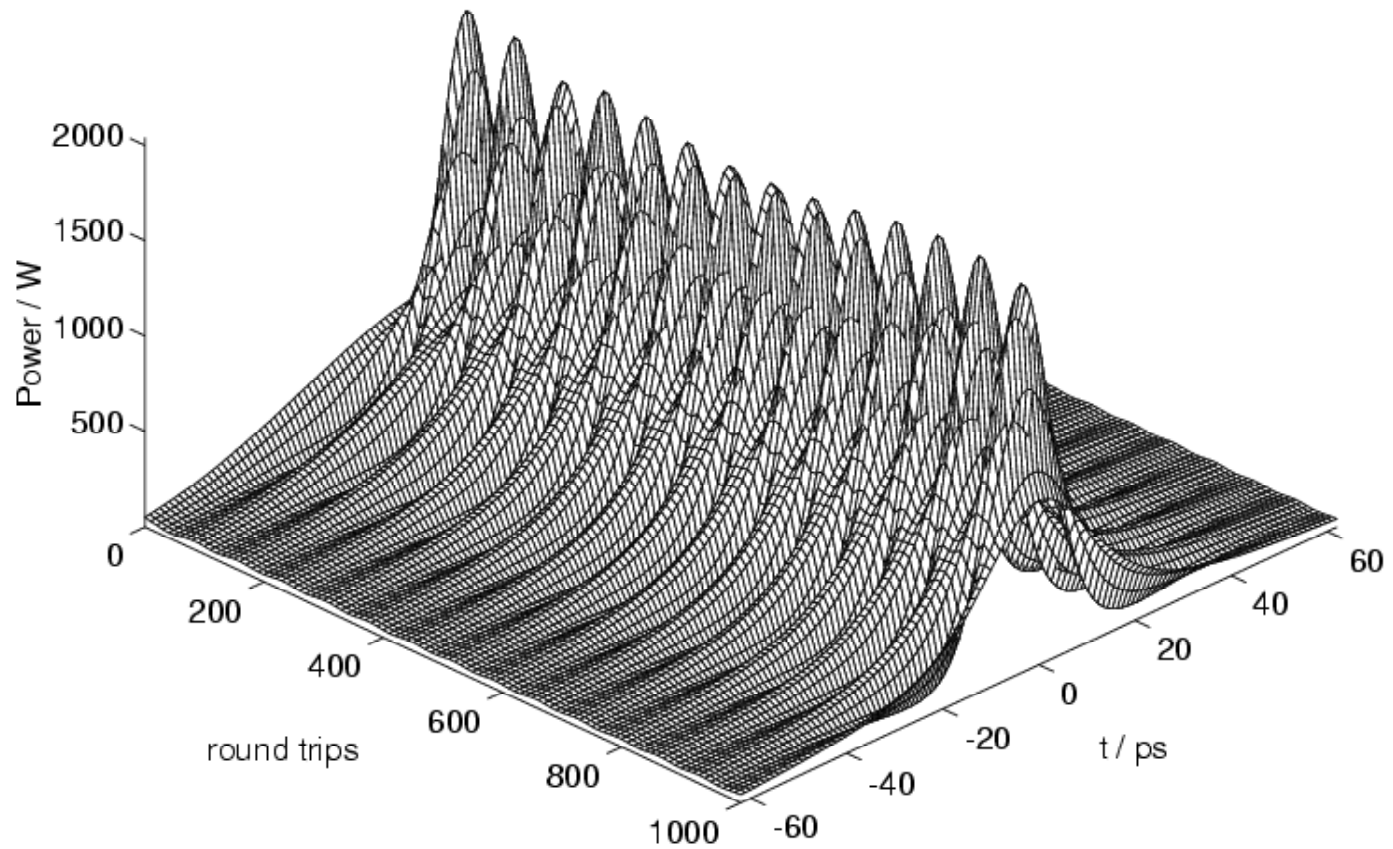


Fig. 5.12a: $D = -10\text{ps}^2$

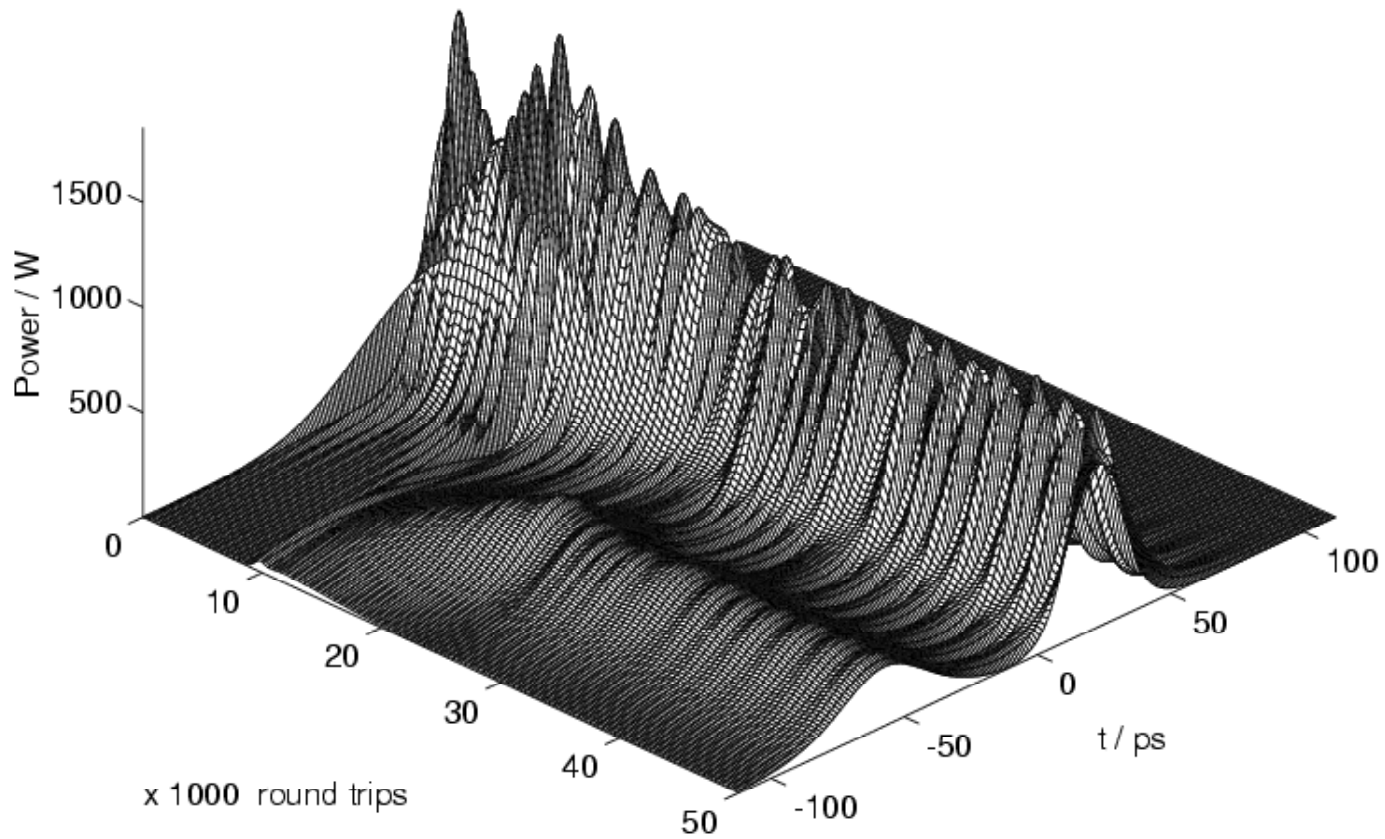


Fig. 5.12 b: 50000 roundtrips

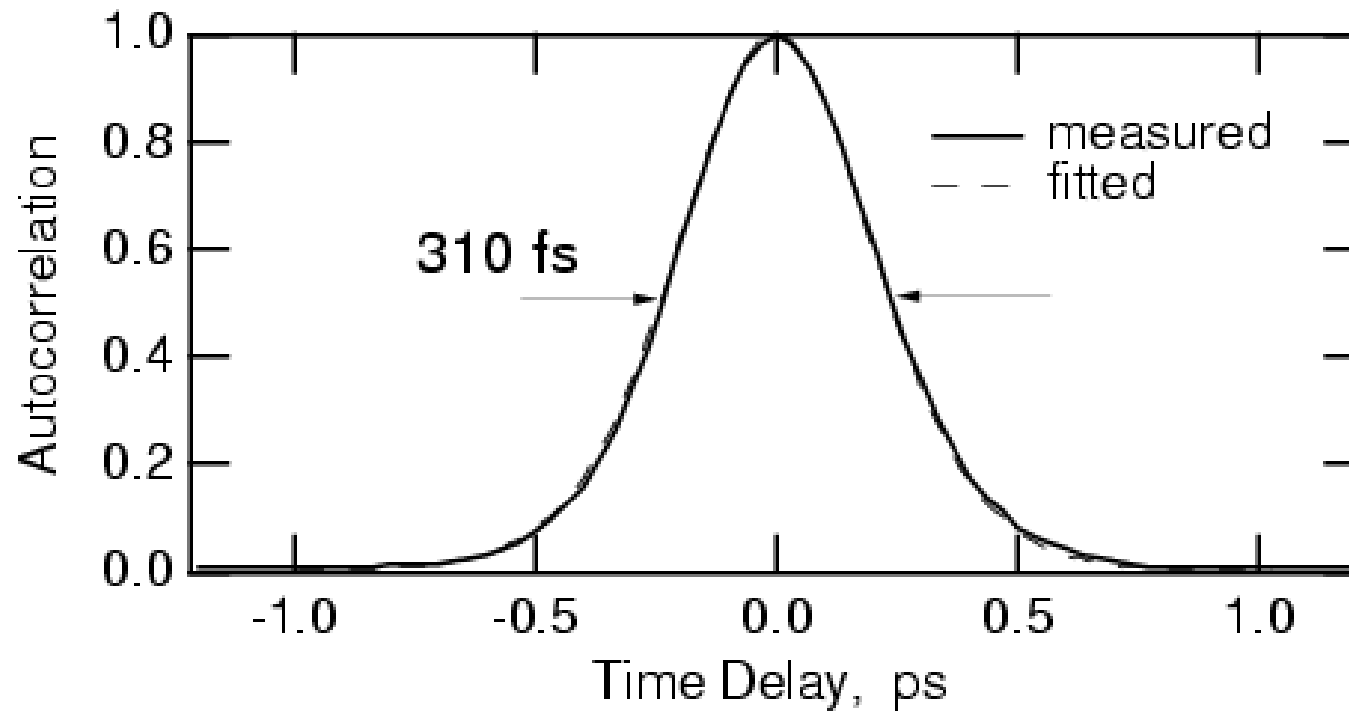


Fig. 5.13: Autocorrelation of the actively mode-locked laser

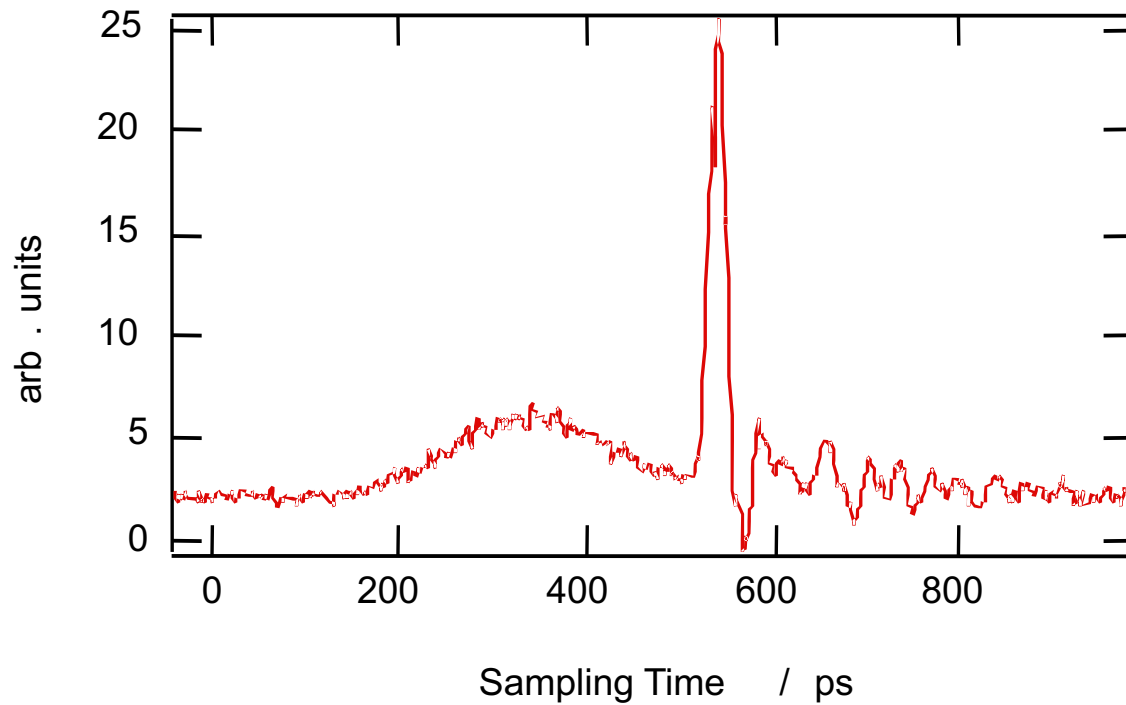


Fig. 5.14: Sampling oscilloscope trace

5.6 Active Modelocking with Detuning

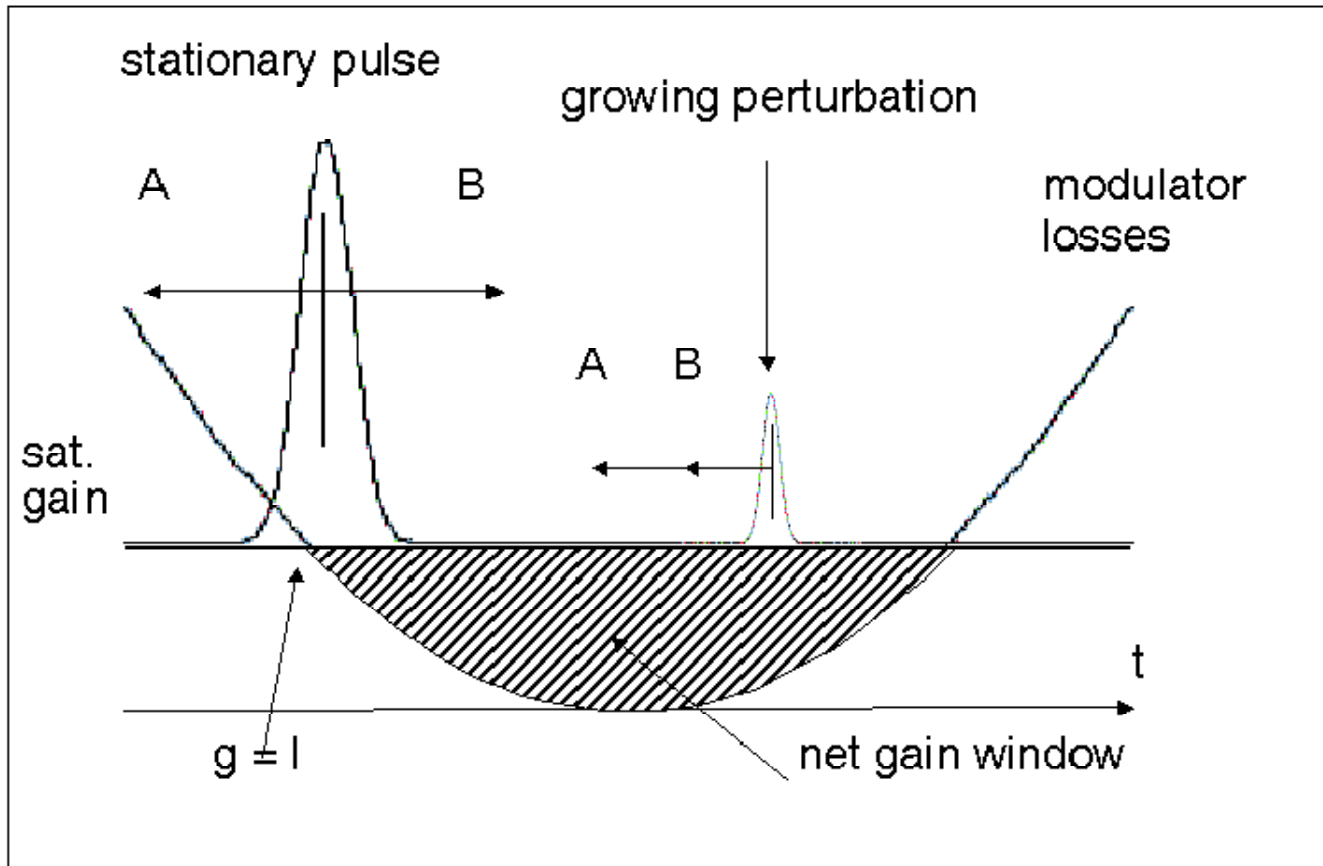


Fig. 5.15: Drifting pulse dynamics in a detuned laser

$$T_M \frac{\partial A(T, t)}{\partial T} = \left[g(T) - l + D_f \frac{\partial^2}{\partial t^2} - M (1 - \cos(\omega_M t)) + T_d \frac{\partial}{\partial t} \right] A(T, t)$$

Normalized detuning: $\Delta = \frac{1}{2\sqrt{2D_f M_s}} \frac{T_d}{\tau_a}$.

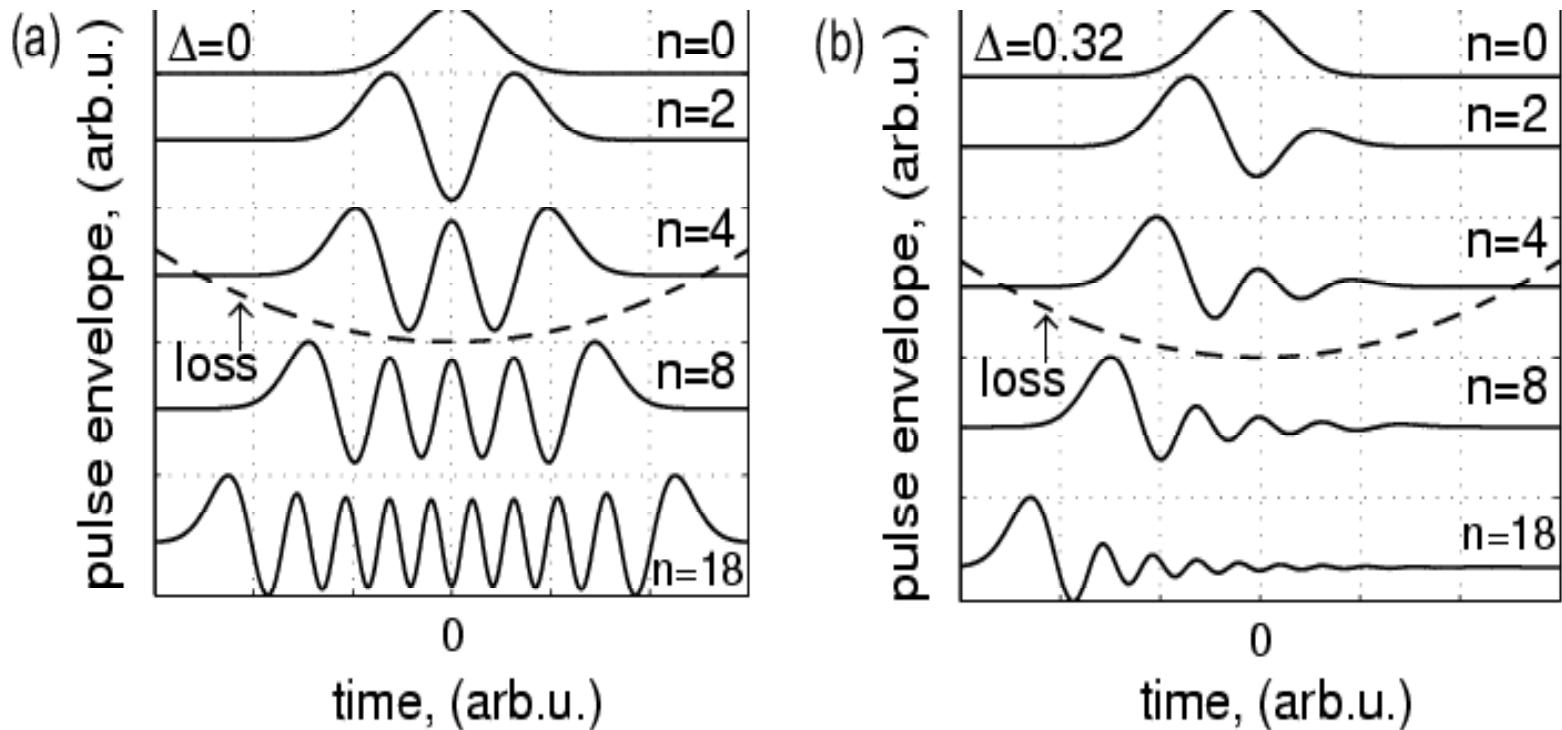


Fig. 5.16: Lower order eigenmodes

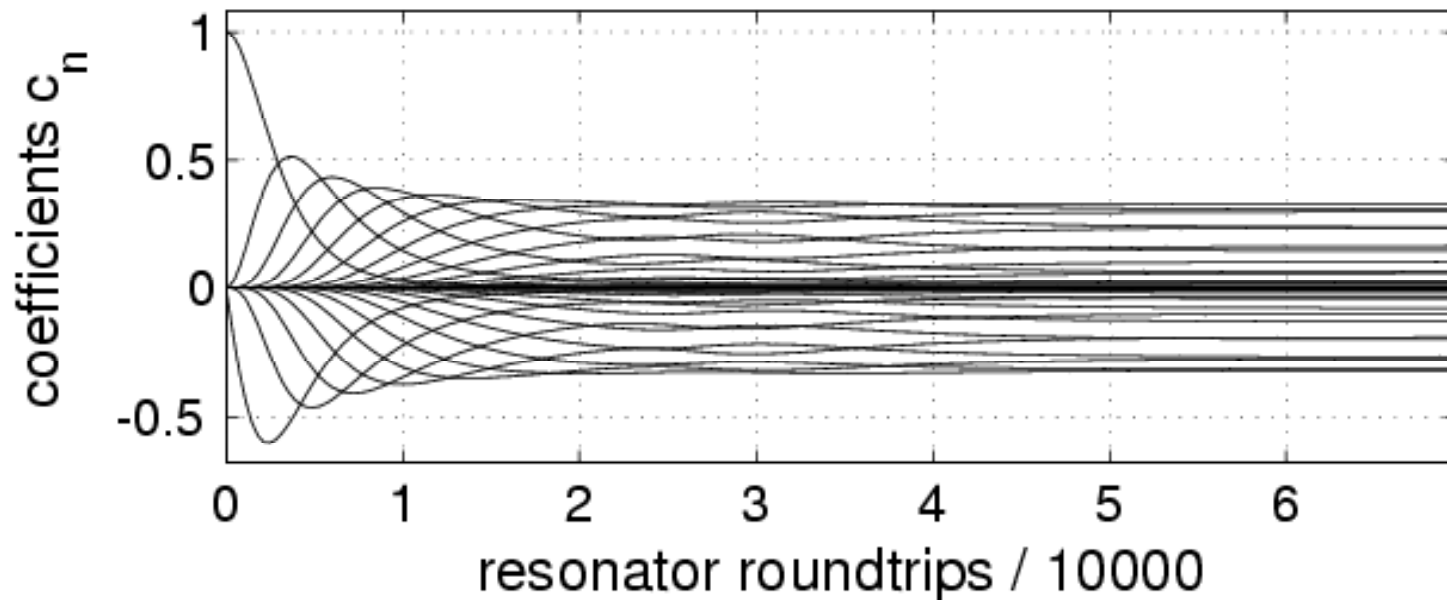


Fig. 5.17: Coefficients of envelope in a Hermite-Gaussian Basis centered at $t=0$ for $\Delta = 3.5$

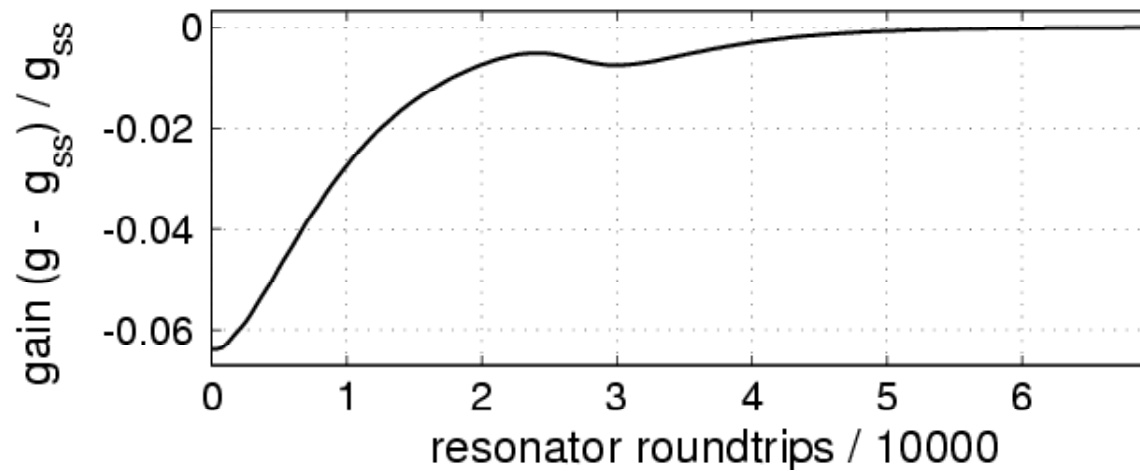


Fig. 5.18: Normalized deviation from steady state gain
for $\Delta = 3.5$

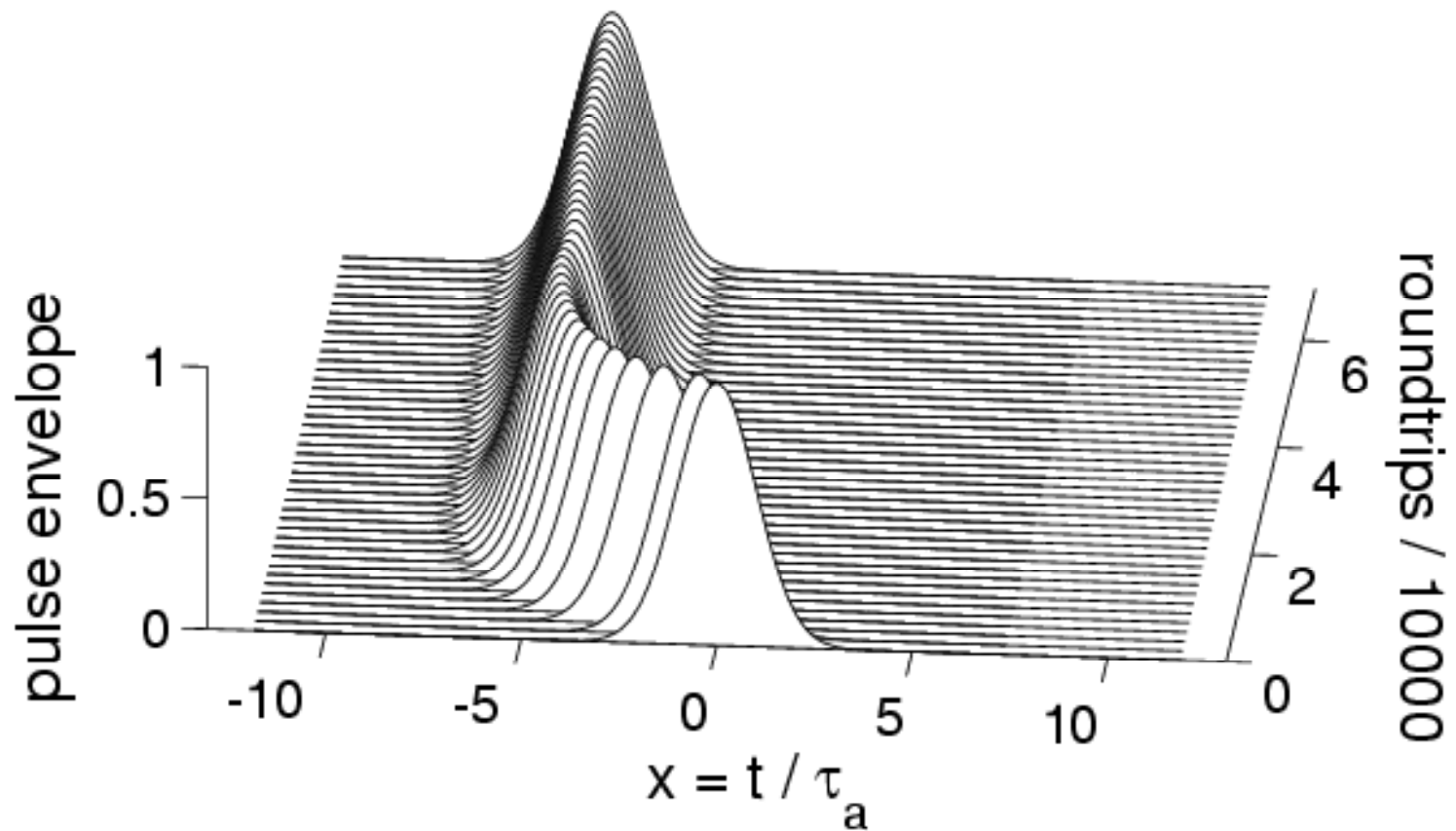


Fig. 5.19: Temporal evolution to steady state position at
to $2^{1/2} \Delta = 4.9$

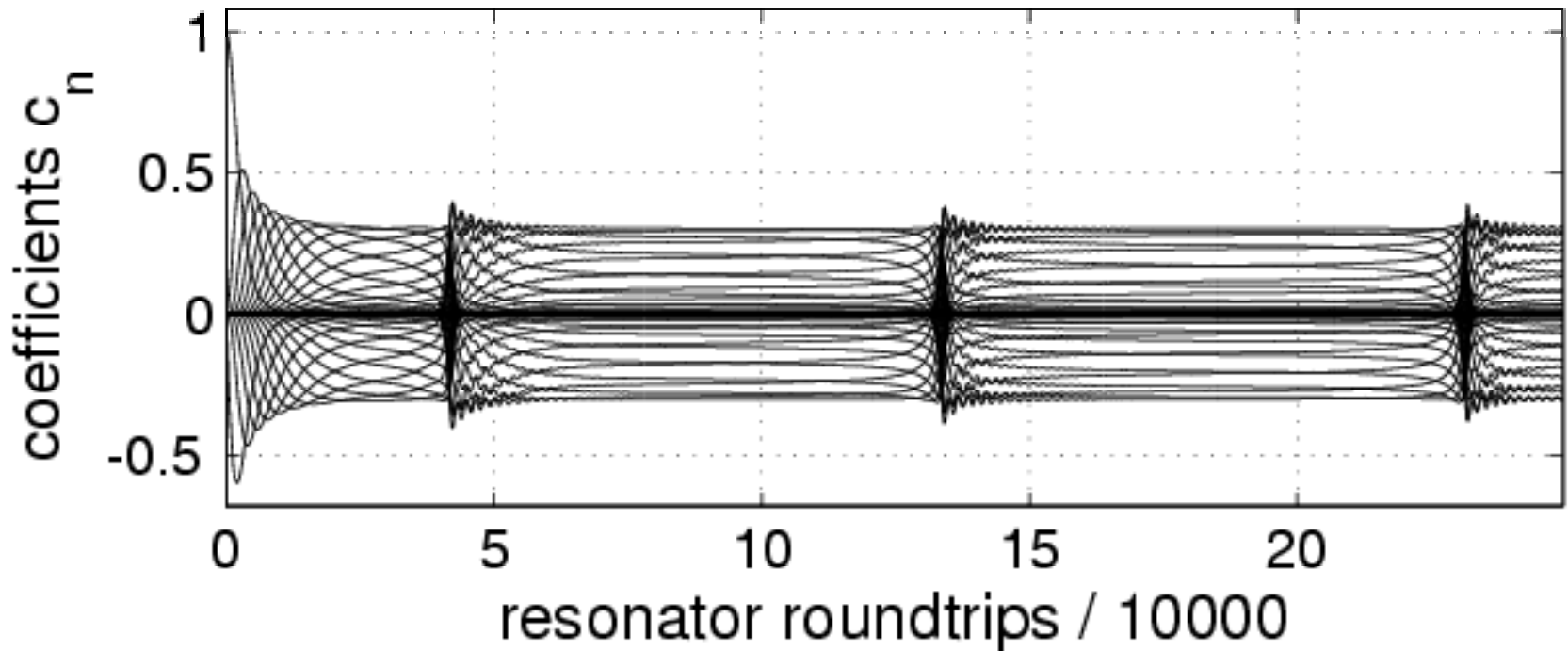


Fig. 5.20: Coefficients of envelope in a Hermite-Gaussian Basis centered at $t=0$ for $\Delta = 4$

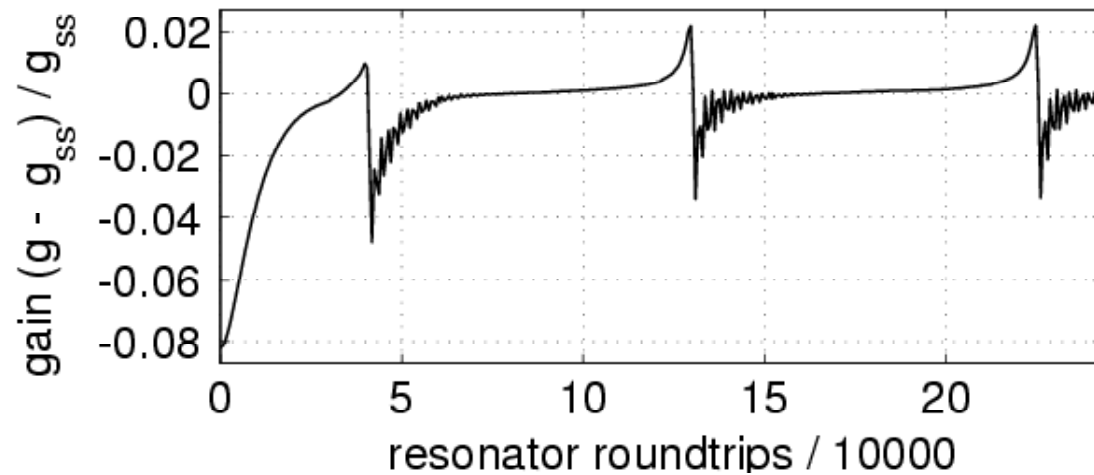


Fig. 5.21: Temporal evolution of gain deviation from quasi steady state

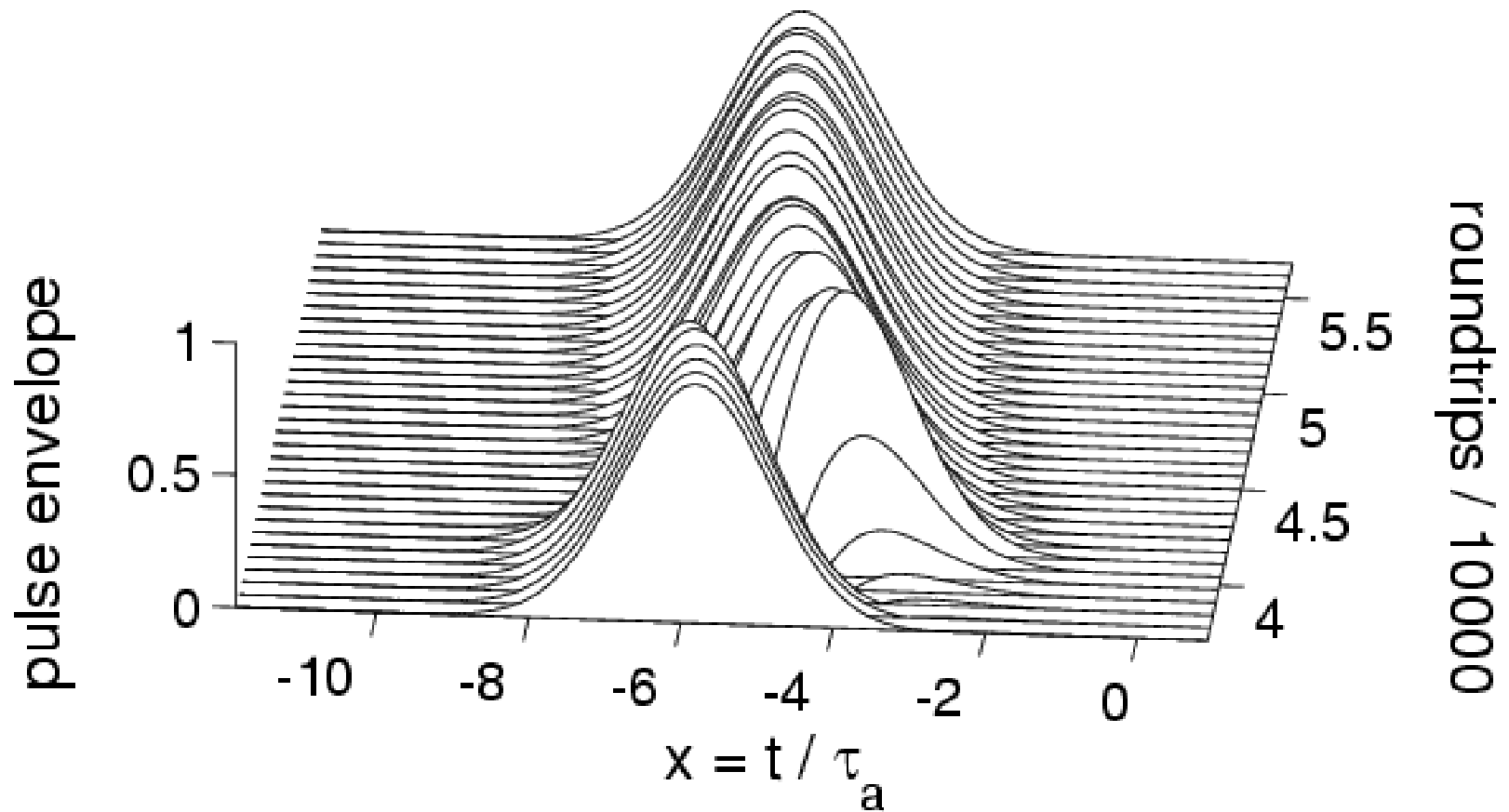


Fig. 5.22: Time evolution of pulse envelope for $\Delta = 4$

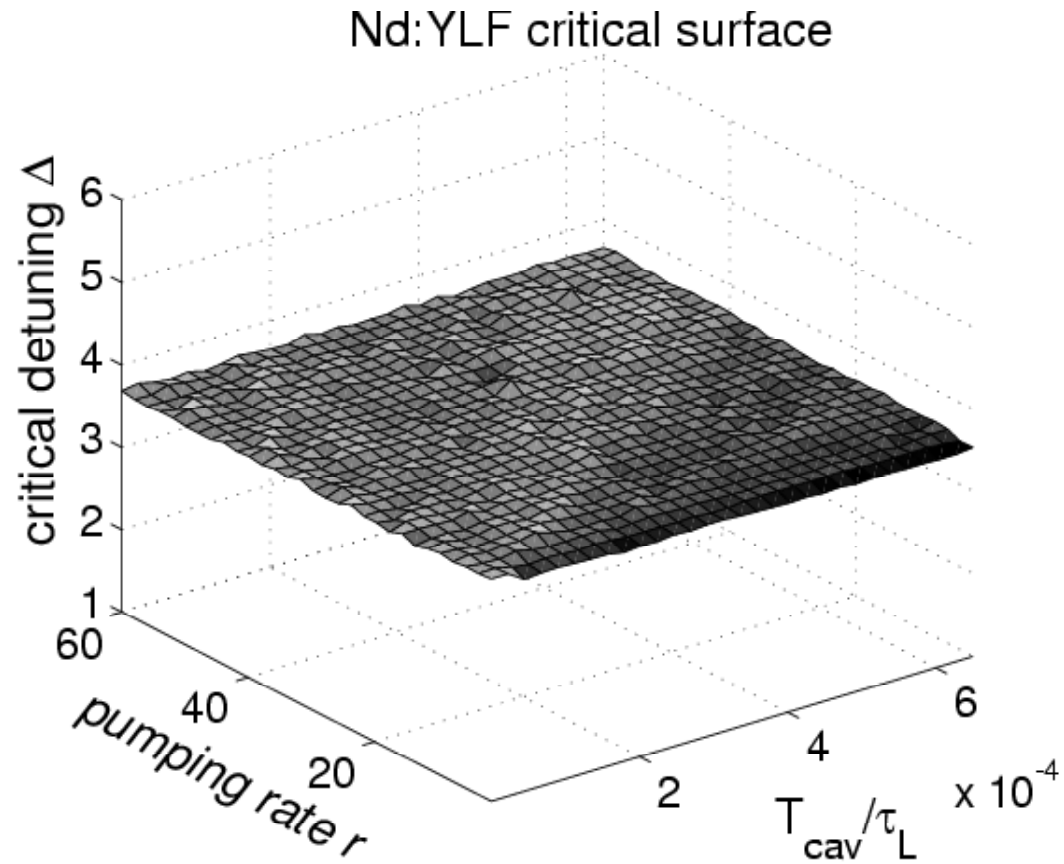


Fig. 5.25: Critical detuning ~ 3.65

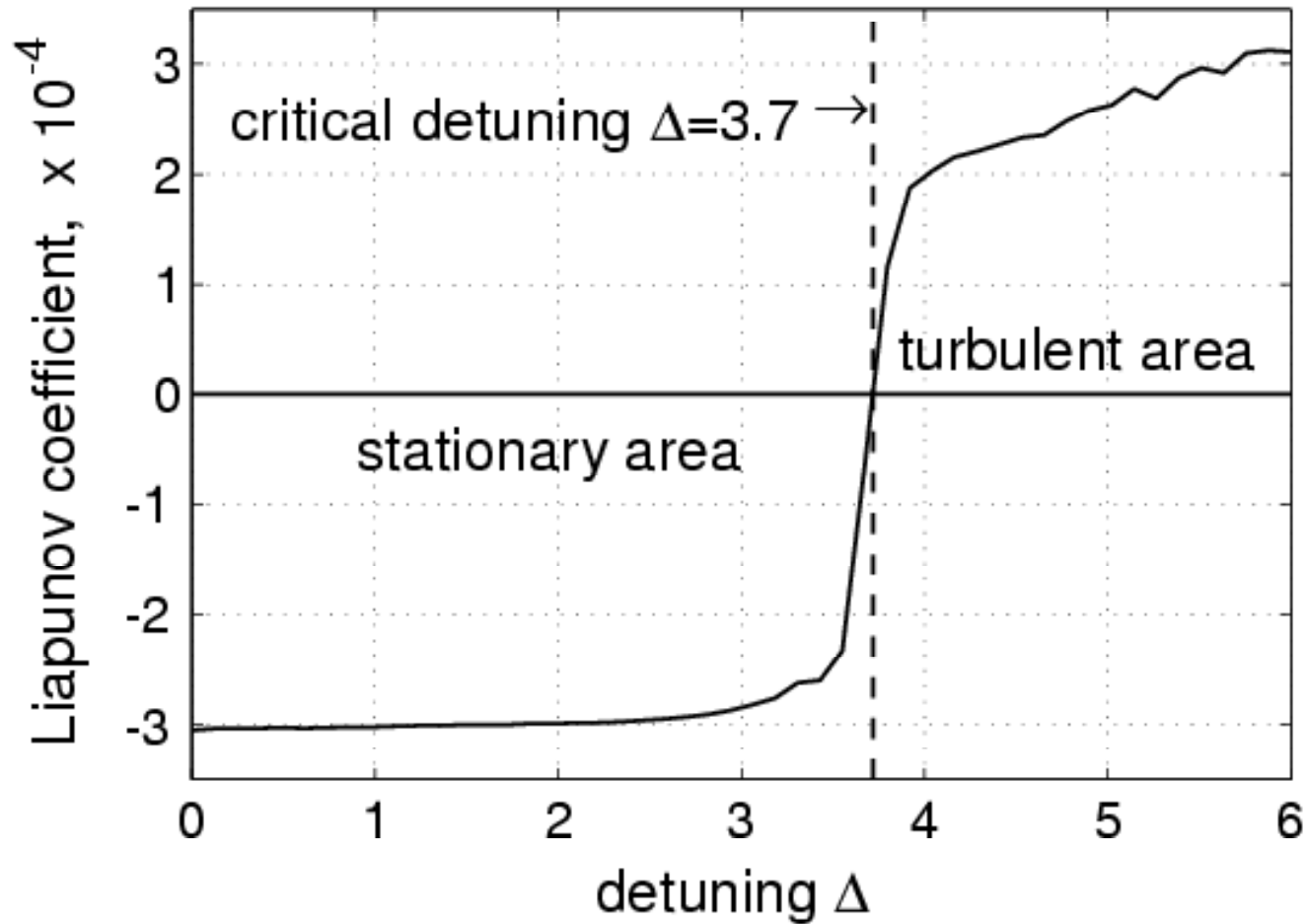


Fig. 5.26: Liapunov coefficient over normalized detuning