Nonlinear Optics (WiSe 2019/20)

Lecture 8: December 6, 2019

8 Optical solitons

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Nonlinear Optics (WiSe 2019/20)
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9 Optical Parametric Amplifiers and Oscillators

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[5] Largely follows the review paper by G. Cerullo *et al.*, "Ultrafast Optical Parametric Amplifiers," Rev. Sci. Instrum. 74, 1-17 (2003)

8 Optical solitons

$$
\begin{aligned}
\mathbf{E}(z,t) &= E(z,t)\mathbf{e}_x \\
E(z,t) &= Re\left\{\hat{E}(\omega)e^{j(\omega t - kz)}\right\} \\
&= |\hat{E}|\cos(\omega t - kz + \varphi),
\end{aligned}
\tag{8.1}
$$

where $k(\omega) = \omega n(\omega)/c_0$, with n the refractive index of the medium. In general the refractive index depends on frequency, and we want to understand the propagation of a pulse with carrier frequency ω_0 (see Fig. 8.1)

$$
E(z,t) = Re\left\{\frac{1}{2\pi}\int_0^\infty \hat{E}(\omega)e^{i(\omega t - k(\omega)z)}d\omega\right\}. \tag{8.2}
$$

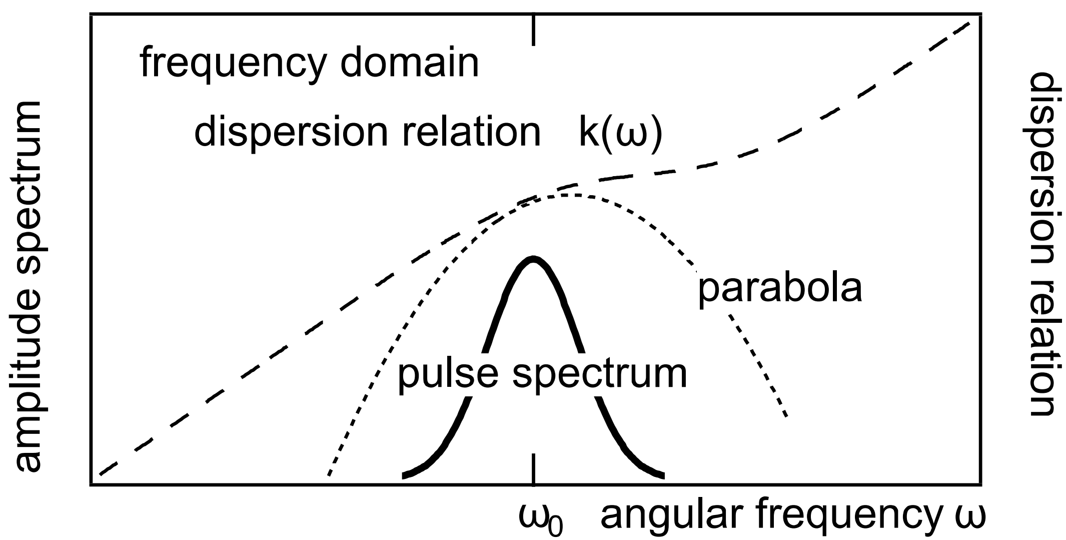


Figure 8.1: Spectral density and dispersion relation for an optical pulse.

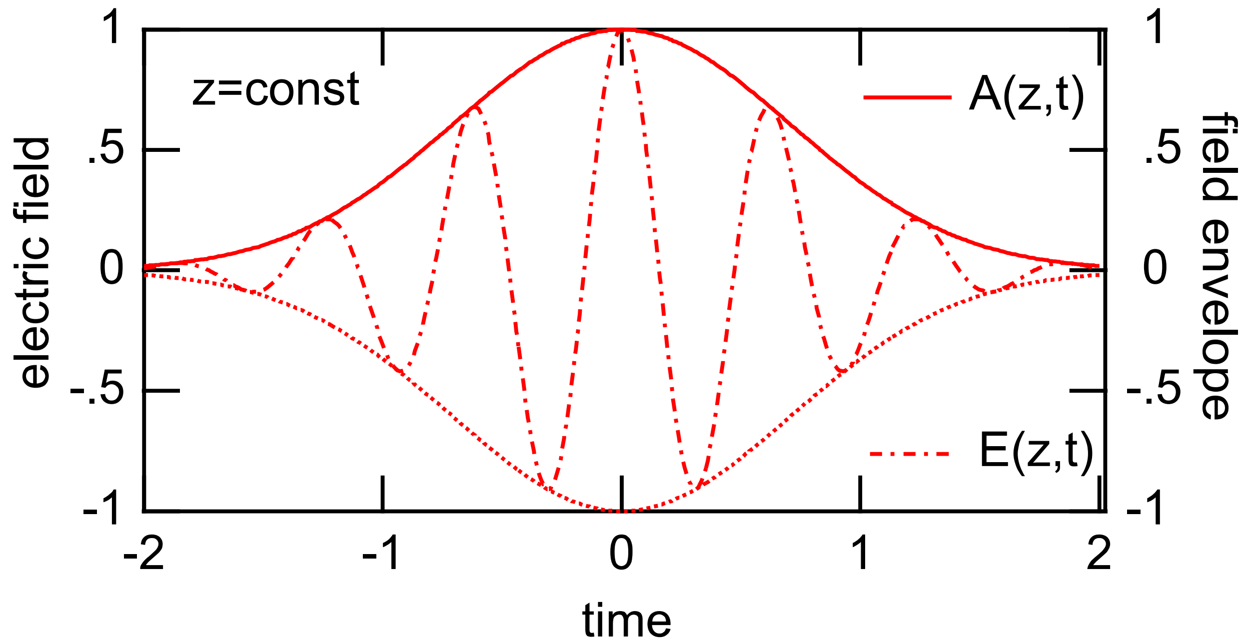


Figure 8.2: Decomposition of a pulse into carrier wave and envelope.

The electric field of the pulse in Eq. (8.2) can be decomposed into a carrier wave and an envelope $A(z,t)$ and we normalize the wave such that its magnitude square is the average intensity

$$E(z,t) = \sqrt{\frac{2Z_0}{n(\omega_0)}} Re \left\{ A(z,t) e^{j(\omega_0 t - k(\omega_0) z)} \right\}. \tag{8.3}$$

The envelope is then defined as

$$A(z,t) = \frac{1}{2\pi}\int_{-\omega_0 \to -\infty}^{\infty} \hat{A}(\Delta\omega) e^{j(\Delta\omega t - \Delta k(\Delta\omega) z)} d\Delta\omega, \tag{8.4}$$

where

$$\Delta\omega = \omega - \omega_0, \tag{8.5}$$

$$\Delta k(\Delta\omega) = k(\omega_0 + \Delta\omega) - k(\omega_0), \tag{8.6}$$

$$\tilde{A}(\Delta\omega) = \tilde{E}(\omega = \omega_0 + \Delta\omega)\sqrt{\frac{2Z_0}{n(\omega_0)}}, \tag{8.7}$$

as shown in Fig. 8.2.

8.1 Dispersion

$$k(\omega) = k(\omega_0) + k'|_{\omega_0}\Delta\omega + \frac{k''|_{\omega_0}}{2}\Delta\omega^2 + \frac{k'''|_{\omega_0}}{6}\Delta\omega^3 + O(\Delta\omega^4). \tag{8.8}$$

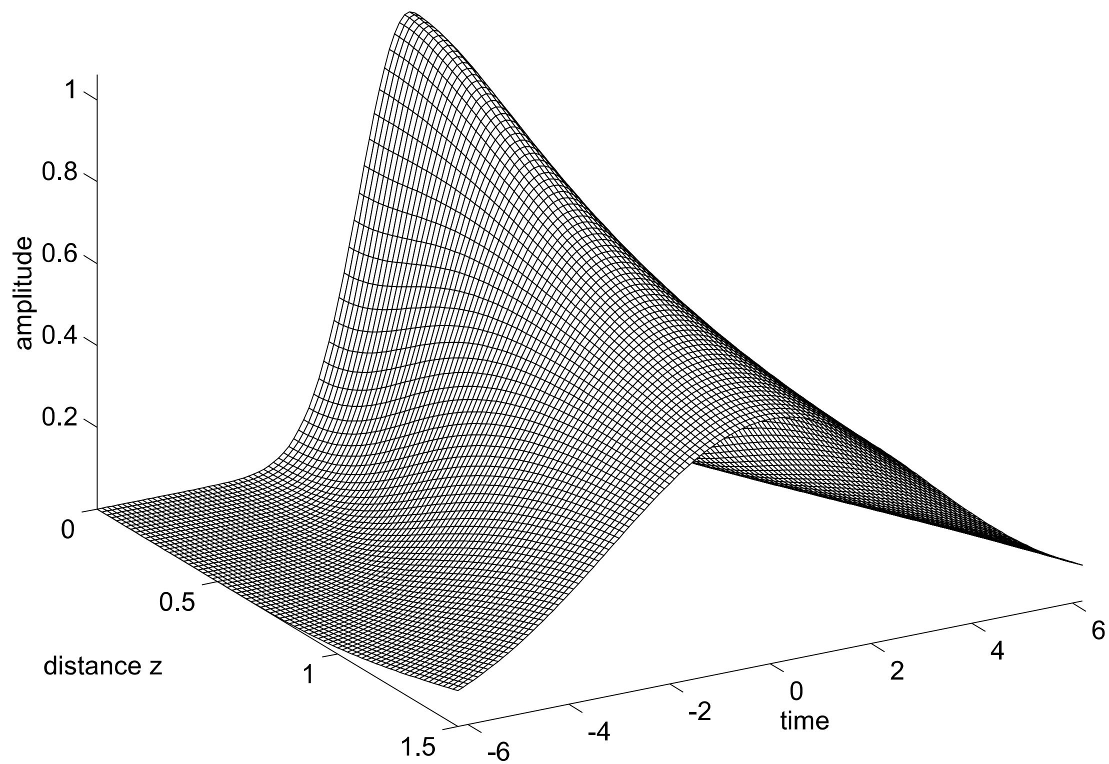


Figure 8.3: Amplitude $|A(z,t)|$ of a Gaussian pulse during propagation due to dispersion.

$$A(z,t) = A(0, t - z/v_g), \tag{8.9}$$

where $v_g = 1/k'$. Again, we introduce the retarded time, $t' = t - z/v_g$, such that

$$A(z,t') = A(0,t'). \tag{8.10}$$

When the spectrum becomes more broadband, then the second term in Eq. (8.8) becomes important, which is the group-velocity dispersion (GVD), i.e., wave packets with different carrier frequency propagate with different speeds (8.4). The envelope obeys the equation

$$\frac{\partial A(z,t')}{\partial z} = j\frac{k''}{2}\frac{\partial^2 A(z,t')}{\partial t'^2}. \tag{8.11}$$

$$\frac{\partial A(z,t')}{\partial z} = -j\sum_{n=2}^{\infty}\frac{k^{(n)}}{n!}\left(-j\frac{\partial}{\partial t'}\right)^n A(z,t'). \tag{8.12}$$

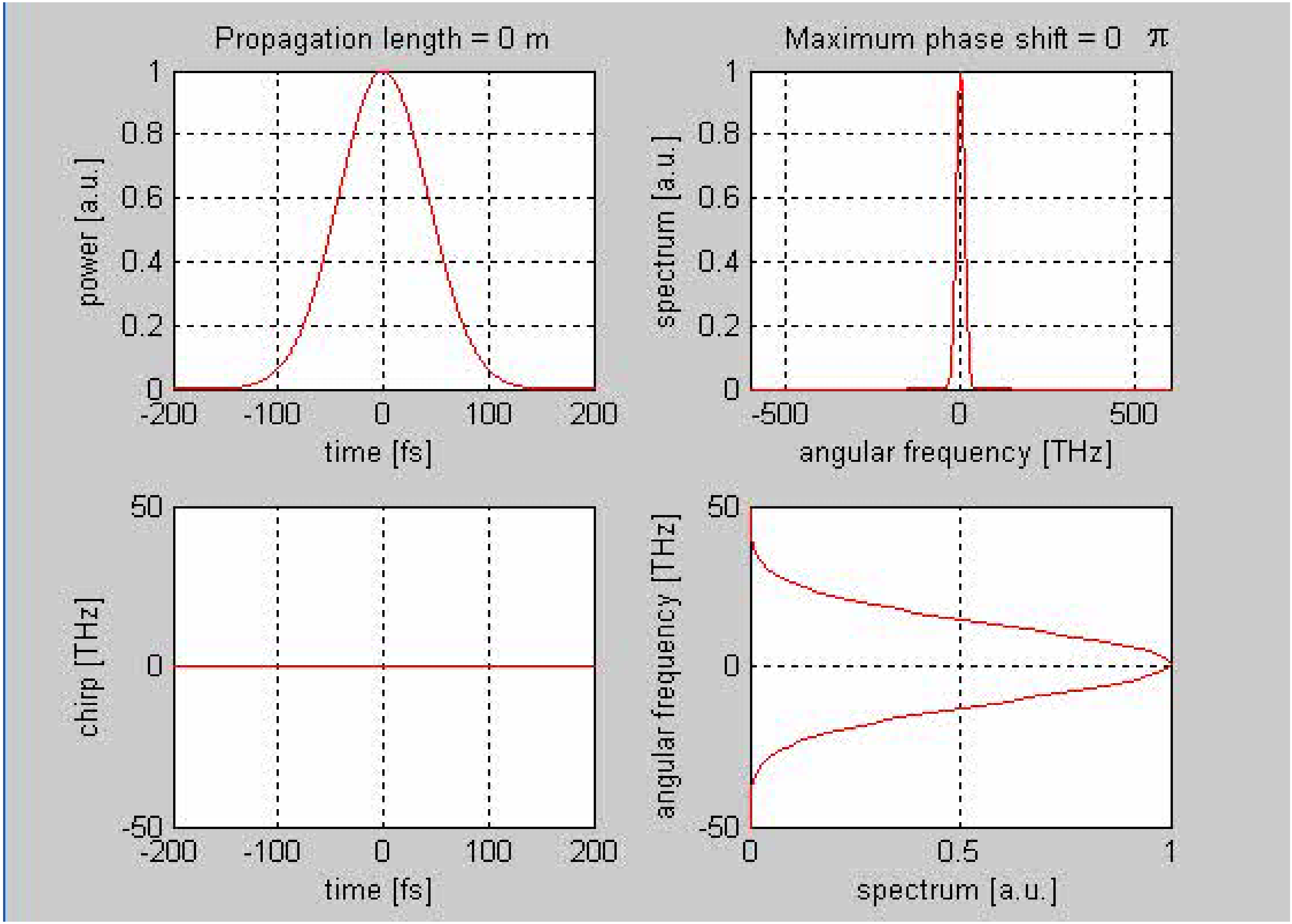
8.2 Self-phase modulation

$$n = n(\omega, |A|^2) \approx n_0(\omega) + n_2^I |A|^2. \tag{8.13}$$

The envelope of the optical pulse then follows

$$\frac{\partial A(z,t)}{\partial z} = -jk_0 n_2 |A(z,t)|^2 A(z,t) = -j\delta |A(z,t)|^2 A(z,t), \tag{8.14}$$

Self-phase modulation



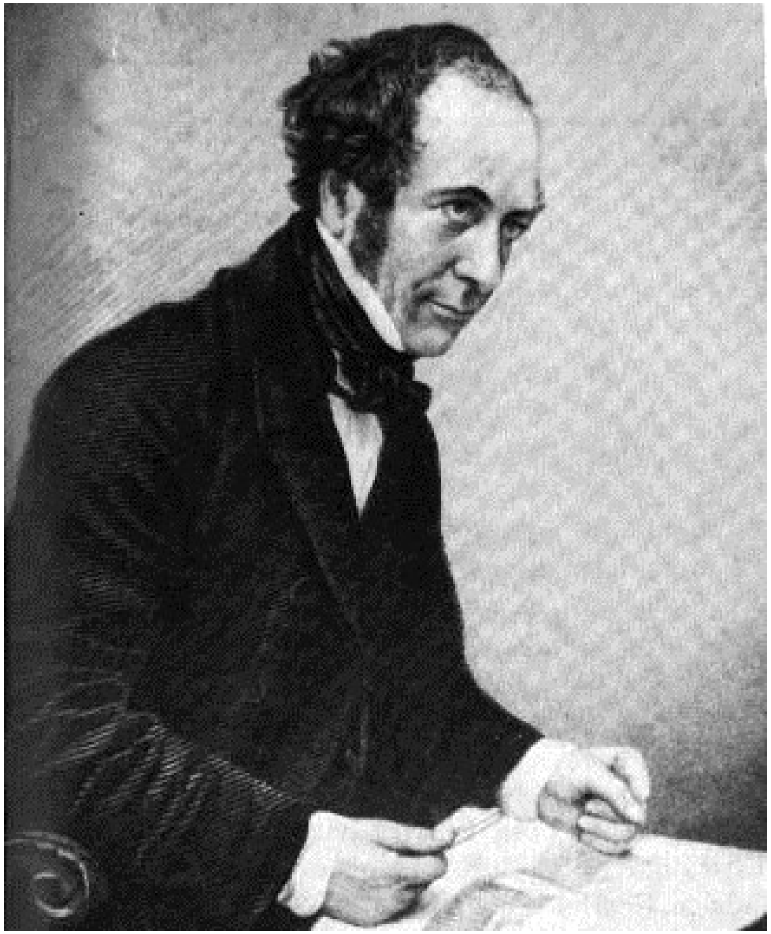
Input: Gaussian pulse, Pulse duration = 100 fs, Peak power = 1 kW

8.3 Nonlinear Schrödinger equation (NLSE)

$$-j\frac{\partial A(z,t)}{\partial z} = \frac{k''}{2}\frac{\partial^2 A}{\partial t^2} - \delta|A|^2 A. \tag{8.15}$$

John Scott Russell

(1808-1882)



8.3 Nonlinear Schrödinger equation (NLSE)

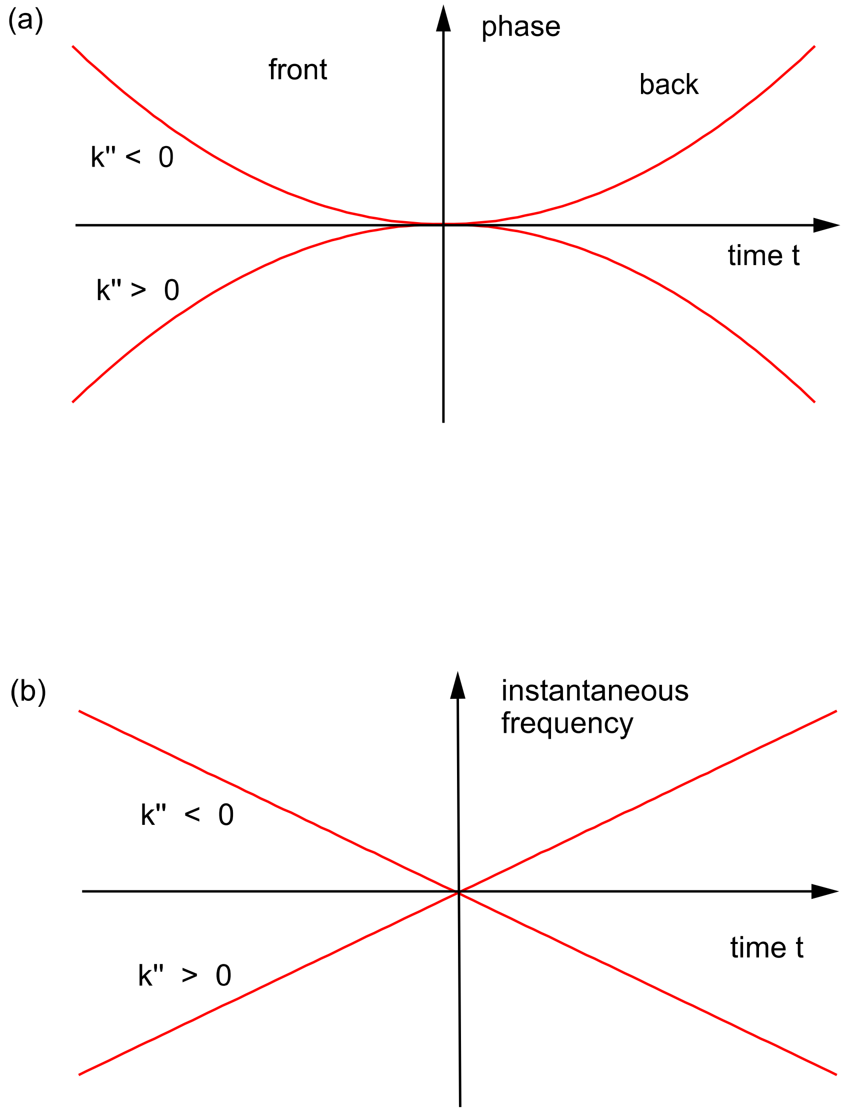


Figure 8.4: (a) Phase, (b) instantaneous frequency in a Gaussian pulse propagating in a positive dispersive medium.

John Scott Russel

In 1834, while conducting experiments to determine the most efficient design for canal boats, John Scott Russell made a remarkable scientific discovery. As he described it in his "Report on Waves":

Report of the fourteenth meeting of the British Association for the Advancement of Science, York, September 1844 (London 1845), pp 311-390, Plates XLVII-LVII).

Russell's report

"I was observing the motion of a boat which was rapidly drawn along a narrow channel by a pair of horses, when the boat suddenly stopped - not so the mass of water in the channel which it had put in motion; it accumulated round the prow of the vessel in a state of violent agitation, then suddenly leaving it behind, rolled forward with great velocity, assuming the form of a large solitary elevation, a rounded, smooth and well-defined heap of water, which continued its course along the channel apparently without change of form or diminution of speed."

Russell's report

"I followed it on horseback, and overtook it still rolling on at a rate of some eight or nine miles an hour, preserving its original figure some thirty feet long and a foot to a foot and a half in height. Its height gradually diminished, and after a chase of one or two miles I lost it in the windings of the channel. Such, in the month of August 1834, was my first chance interview with that singular and beautiful phenomenon which I have called the Wave of Translation."

Scott Russell aqueduct

89.3m long, 4.13m wide,1.52m deep, On the union Canal, Near Heroit-Watt Univ.

*www.spsu.edu/math/txu/research/presentations/**soliton/talk.ppt***

Scott Russell aqueduct

www.spsu.edu/math/txu/research/presentations/***soliton/talk.ppt***

A brief history (mainly for optical soliton)

- 1838 – observation of soliton in water
- 1895 – mathematical description of waves on shallow water surfaces, i.e. KdV equation
- 1972 – optical solitons arising from NLSE
- 1980 – experimental demonstration
- 1990's – soliton control techniques
- 2000's –understanding soliton in the context of supercontinuum generation

8.4 The solitons of the NLSE

Without loss of generality, by normalization of the field amplitude $A = \frac{A'}{\tau}\sqrt{\frac{2D_2}{\delta}}$, the propagation distance $z = z' \cdot \tau^2/D_2$, and the time $t = t' \cdot \tau$, the NLSE (8.15) reads

$$j\frac{\partial A(z,t)}{\partial z} = \frac{\partial^2 A}{\partial t^2} + 2|A|^2 A. \tag{8.16}$$

8.4.1 The fundamental soliton

$$A_s(z,t) = A_0 \mathrm{sech}\left(\frac{t}{\tau}\right) e^{-j\theta}, \tag{8.17}$$

where θ is the nonlinear phase shift of the soliton

$$\theta = \frac{1}{2}\delta A_0^2 z. \tag{8.18}$$

$$\theta = \frac{|k''|}{2\tau^2} z. \tag{8.19}$$

Since the field amplitude $A(z,t)$ is normalized, such that the absolute square is the intensity, the soliton energy fluence is given by

$$w = \int_{-\infty}^{\infty} |A_s(z,t)|^2 dt = 2A_0^2\tau. \tag{8.20}$$

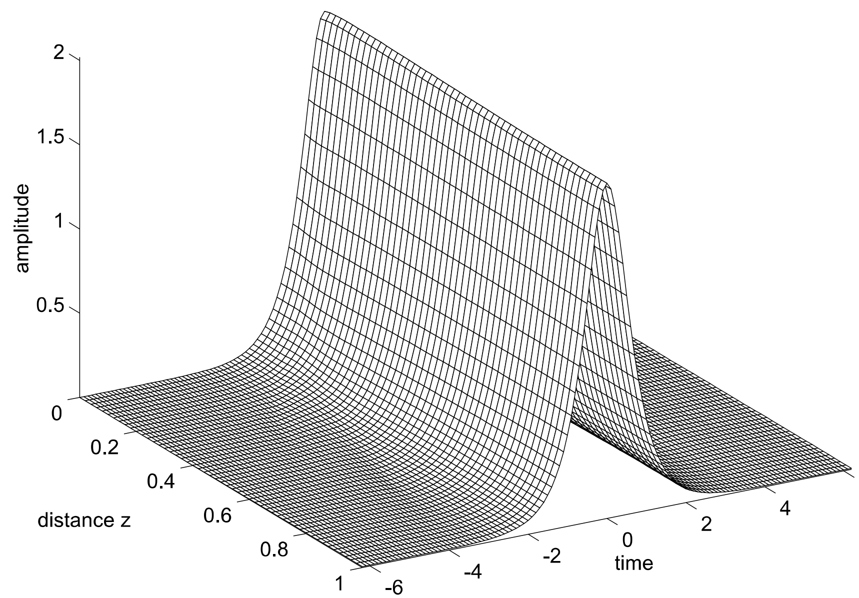
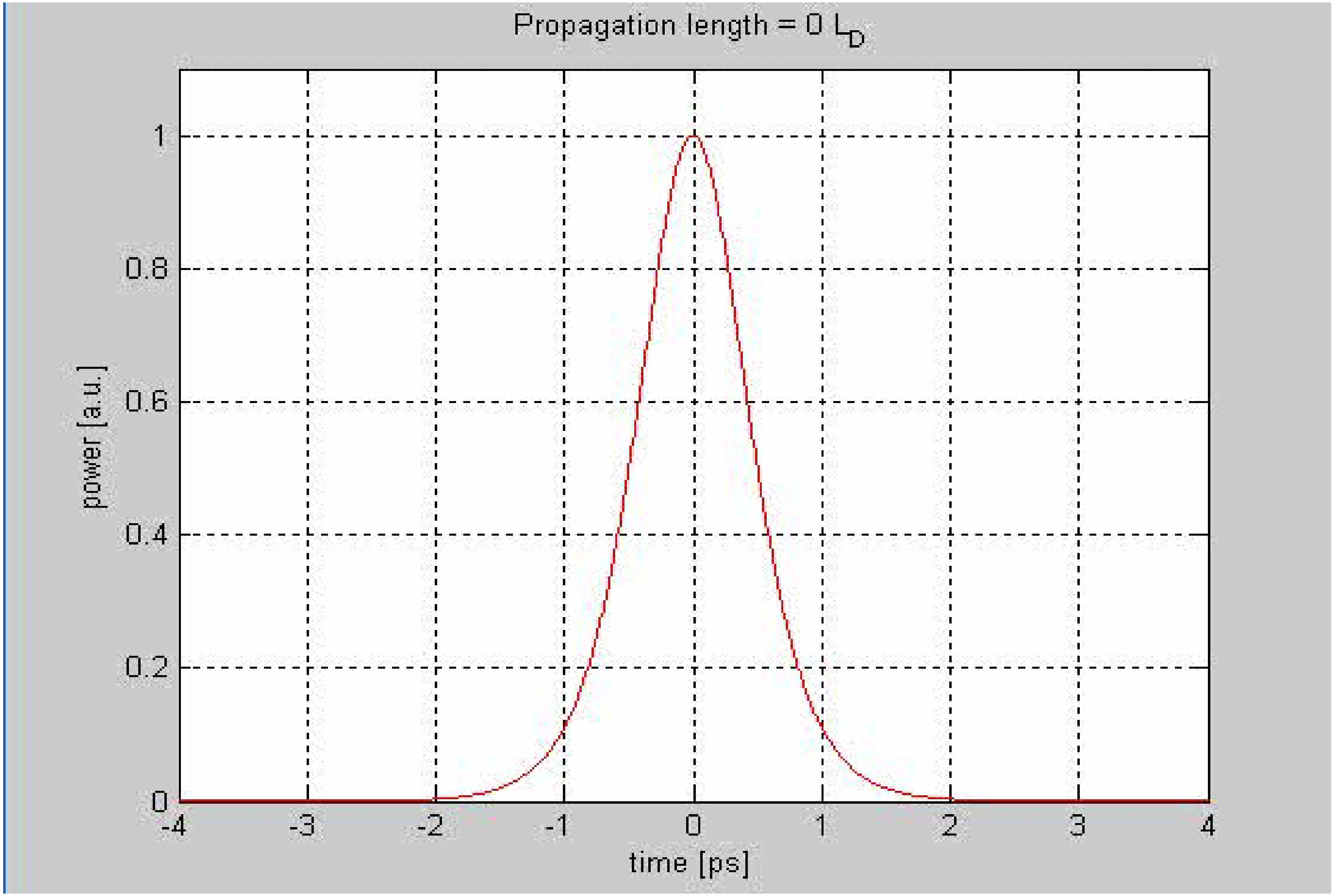


Figure 8.6: Fundamental soliton of the NLSE.

Propagation of fundamental soliton



Input: 1ps soliton centered at 1.55 um; medium: single-mode fiber

Important relations

$$\boxed{\delta A_0^2 = \frac{2|D_2|}{\tau^2} (= \frac{|\beta_2|}{\tau^2})} \Rightarrow \boxed{A_s(z,t) = A_0 \operatorname{sech}\left(\frac{t}{\tau}\right) e^{-j\theta}}$$

(balance between dispersion and nonlinearity)

nonlinear phase shift soliton acquires during propagation:

$$\theta = \frac{1}{2}\delta A_0^2 z = \frac{|D_2|}{\tau^2} z$$

area theorem $\quad \text{Pulse Area} = \int_{-\infty}^{\infty} |A_s(z,t)| dt = \pi A_0 \tau = \pi \sqrt{\frac{2|D_2|}{\delta}}.$

soliton energy: $\quad w = \int_{-\infty}^{\infty} |A_s(z,t)|^2 dt = 2A_0^2 \tau \quad$ **pulse width:** $\quad \tau = \frac{4|D_2|}{\delta w}$

General fundamental soliton

$$A_s(z,t) = A_0 \mathrm{sech}(x(z,t)) e^{-j\theta(z,t)}, \tag{8.23}$$

with

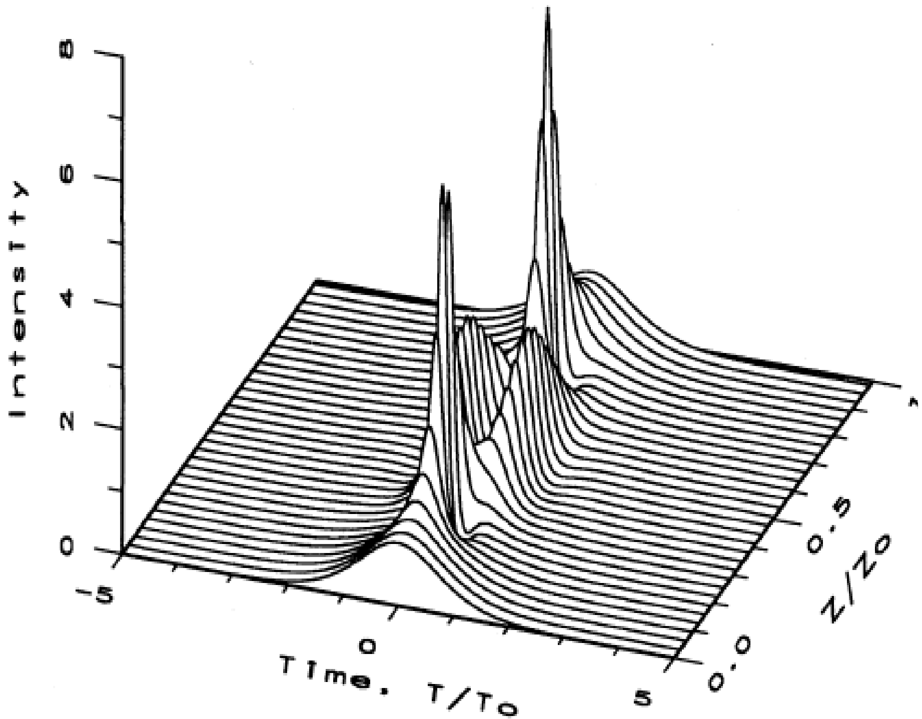
$$x = \frac{1}{\tau}(t - |k''| p_0 z - t_0), \tag{8.24}$$

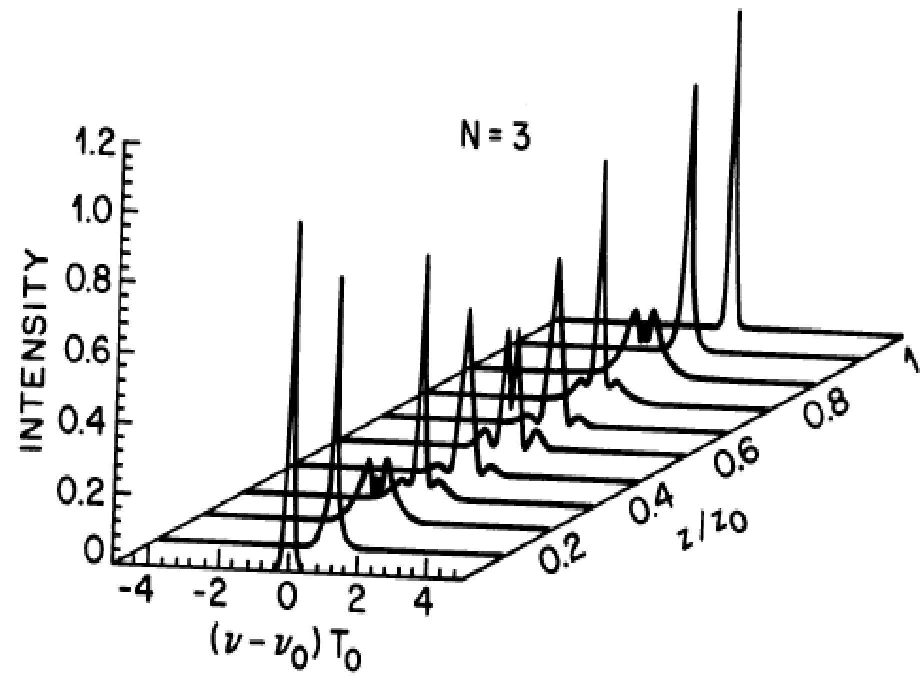
and the generalized phase shift

$$\theta = p_0(t - t_0) + \frac{|k''|}{2}\left(\frac{1}{\tau^2} - p_0^2\right) z + \theta_0. \tag{8.25}$$

Higher-order solitons: periodical evolution in both the time and the frequency domain

$$A_0\tau = N\sqrt{\frac{2|D_2|}{\delta}}, N = 1,2,3\ldots \qquad u(0,\tau) = N\mathrm{sech}(\tau)$$





G. P. Agrawal, *Nonlinear fiber optics* (2001)

Interaction between solitons (soliton collision)

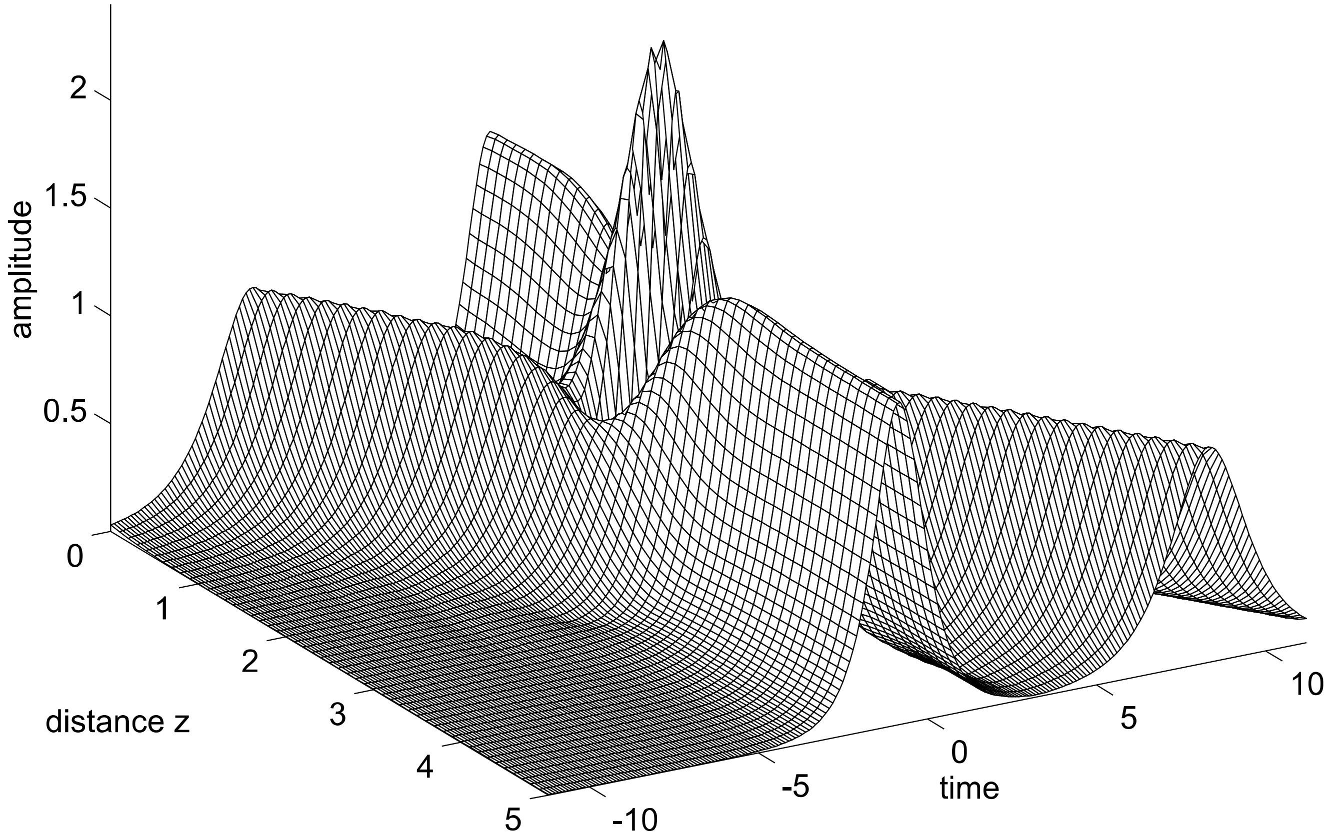
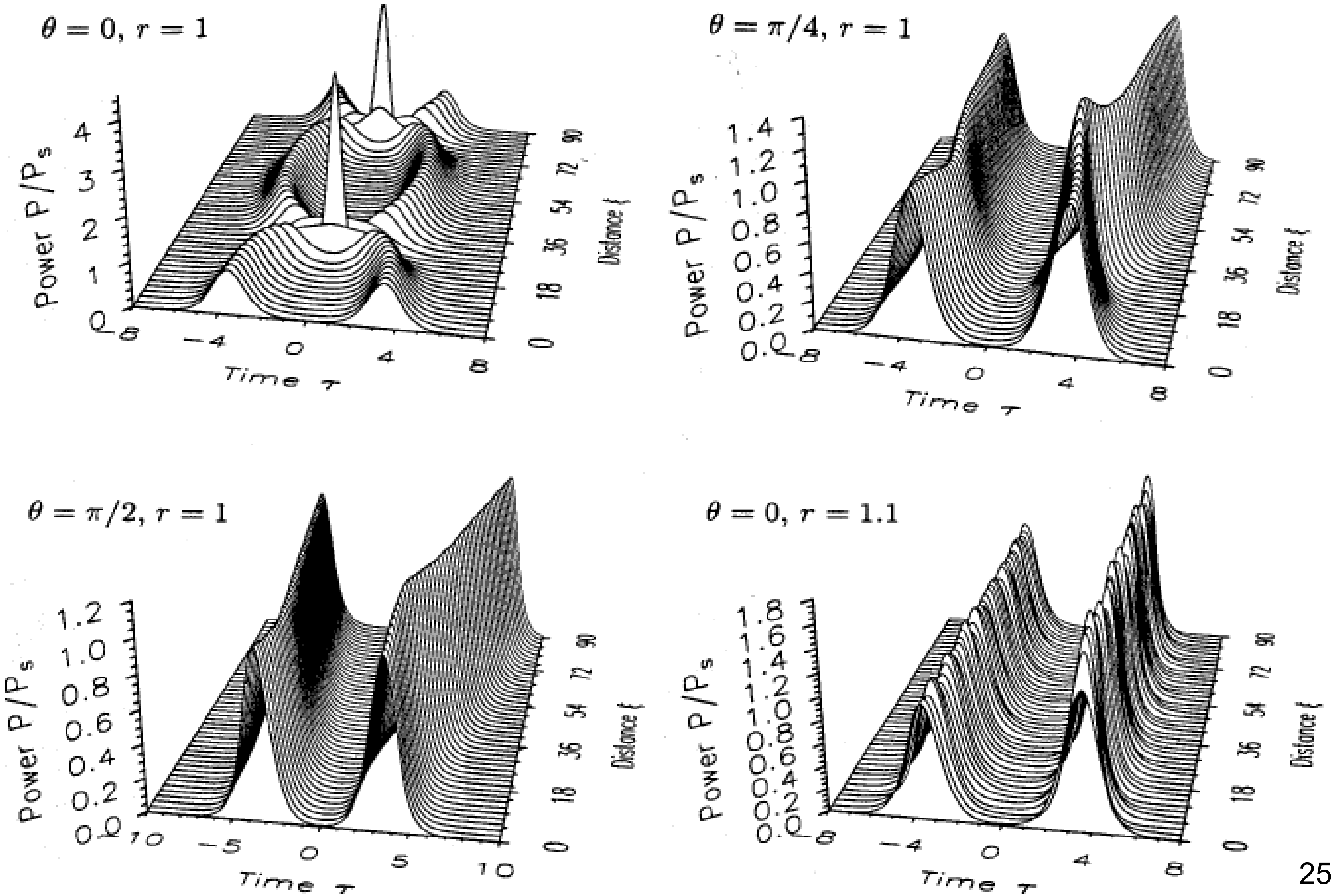


Figure 8.7: Soliton collision, both pulses recover completely.

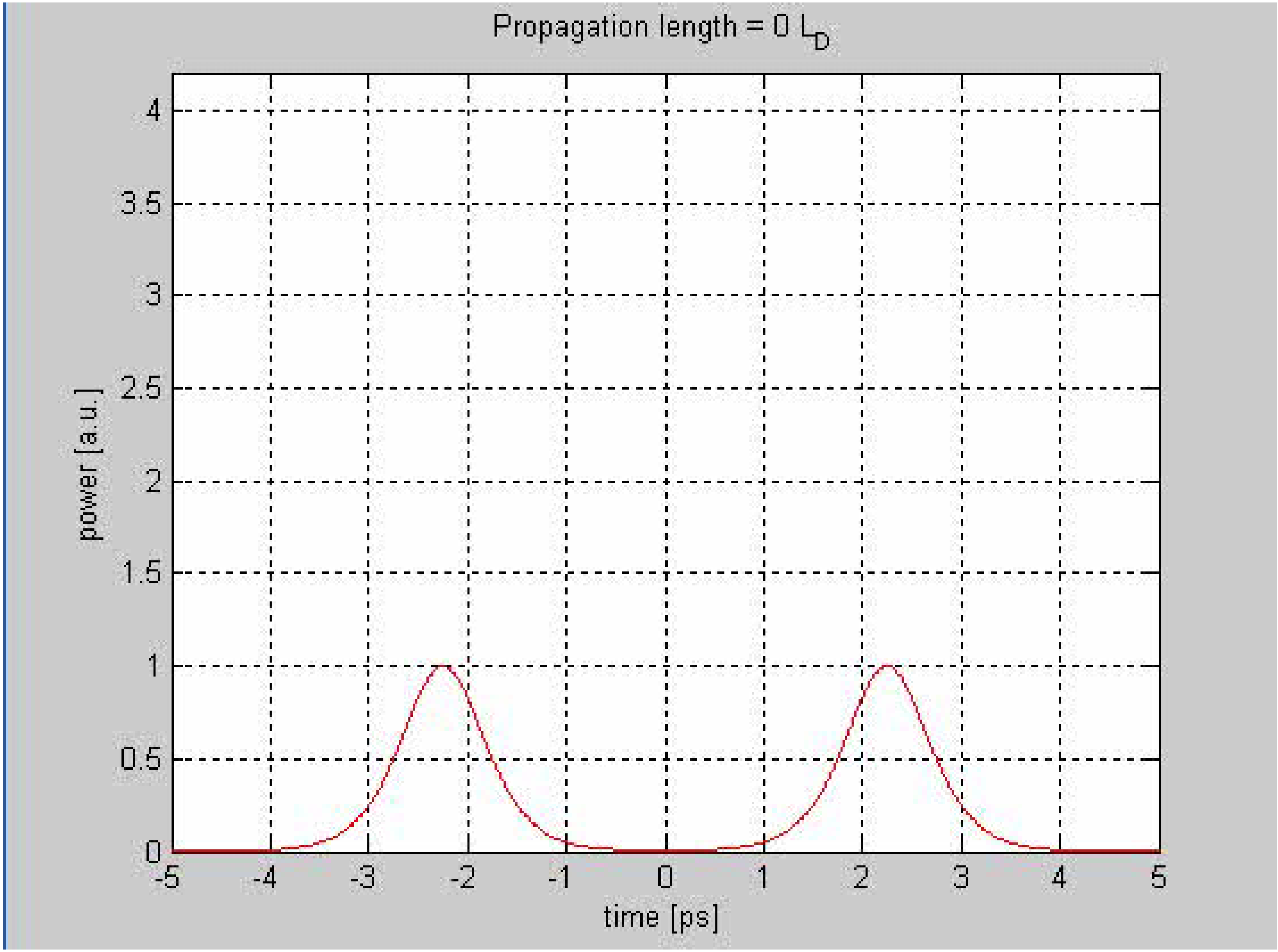
Interaction of two solitons at the same center frequency

Input to NLSE: $u(0,\tau) = \mathrm{sech}(\tau - q_0) + r\,\mathrm{sech}[r(\tau + q_0)]e^{i\theta}$

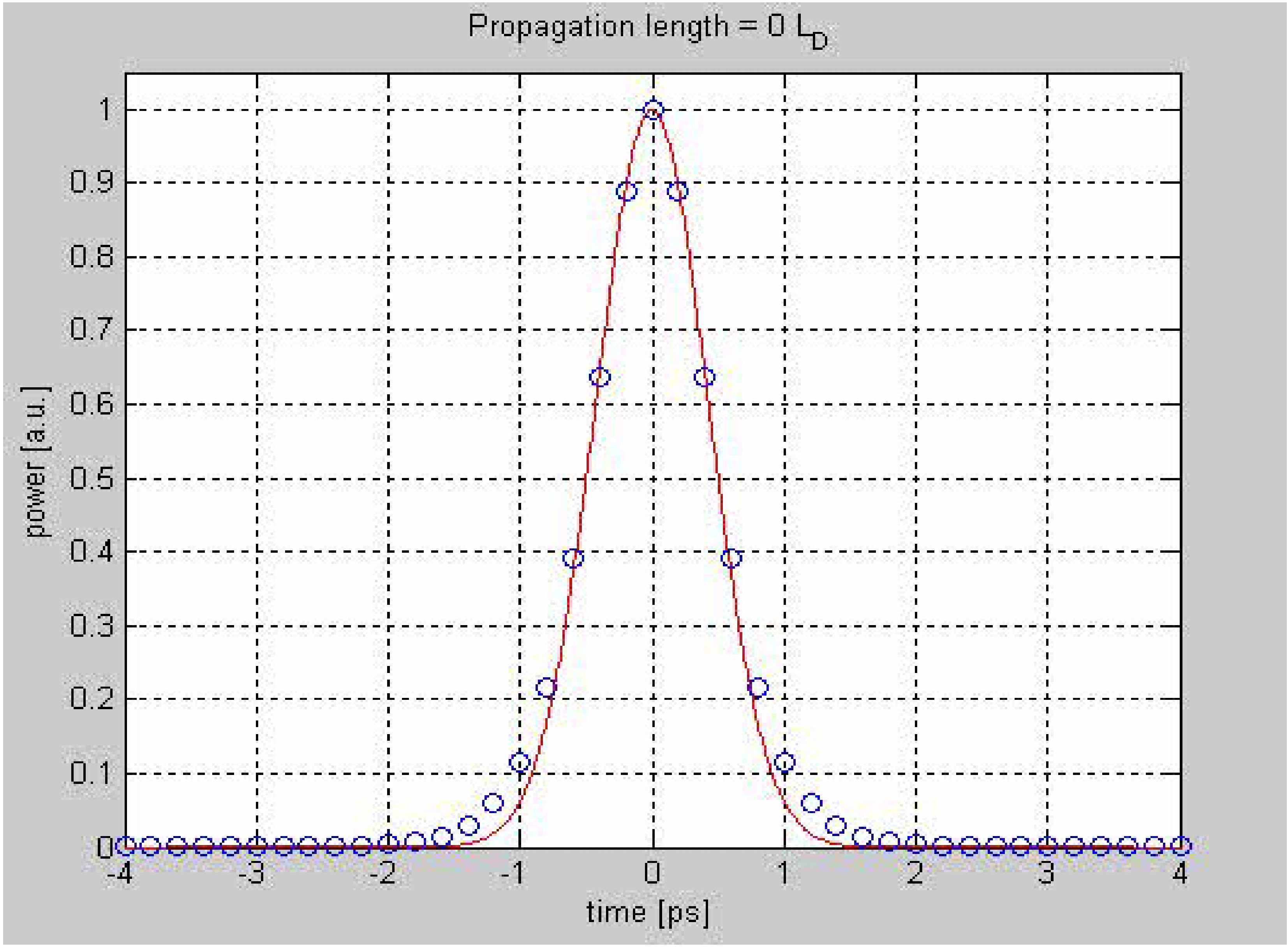


G. P. Agrawal, *Nonlinear fiber optics* (2001)

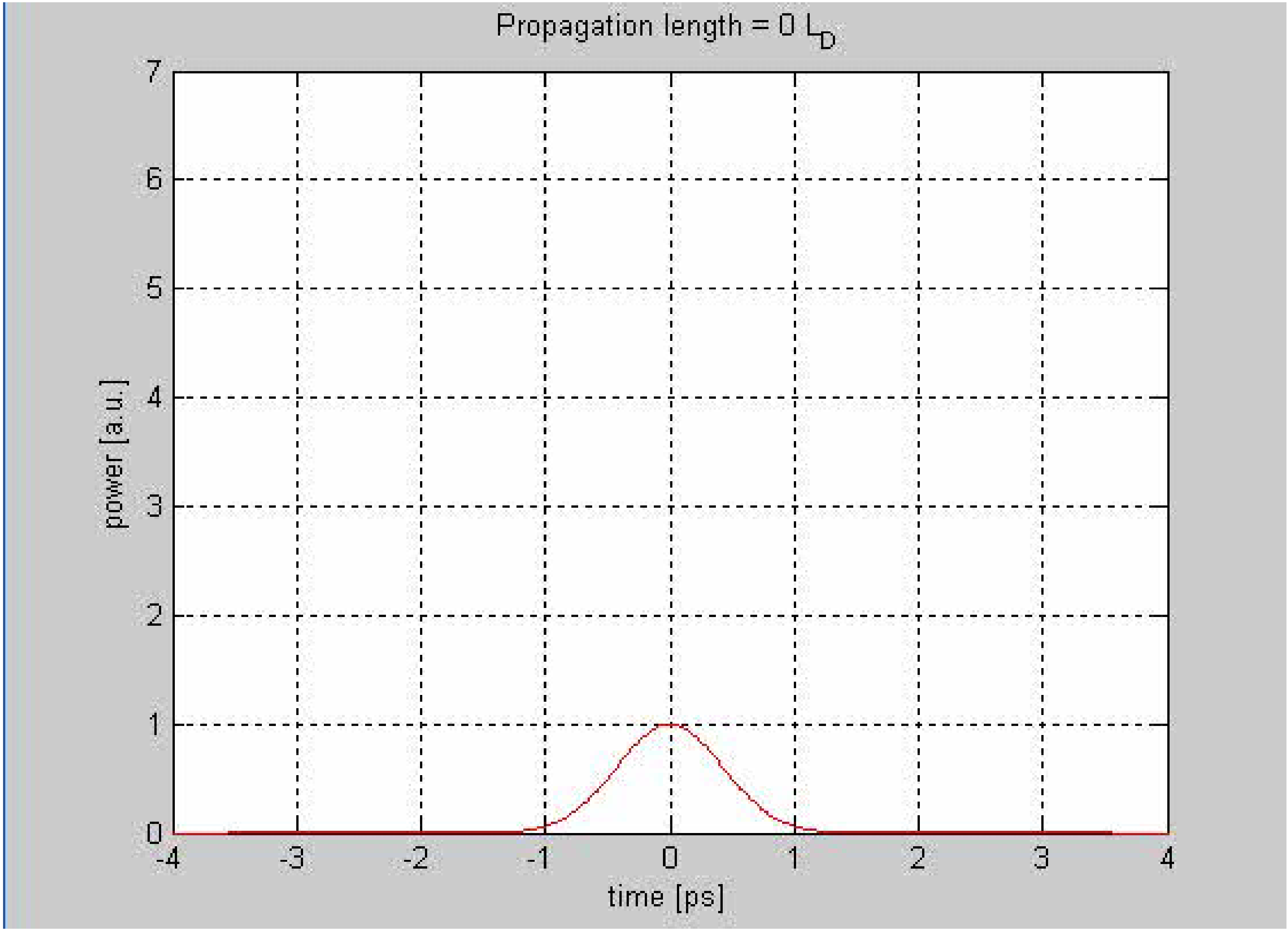
Interactions of two solitons



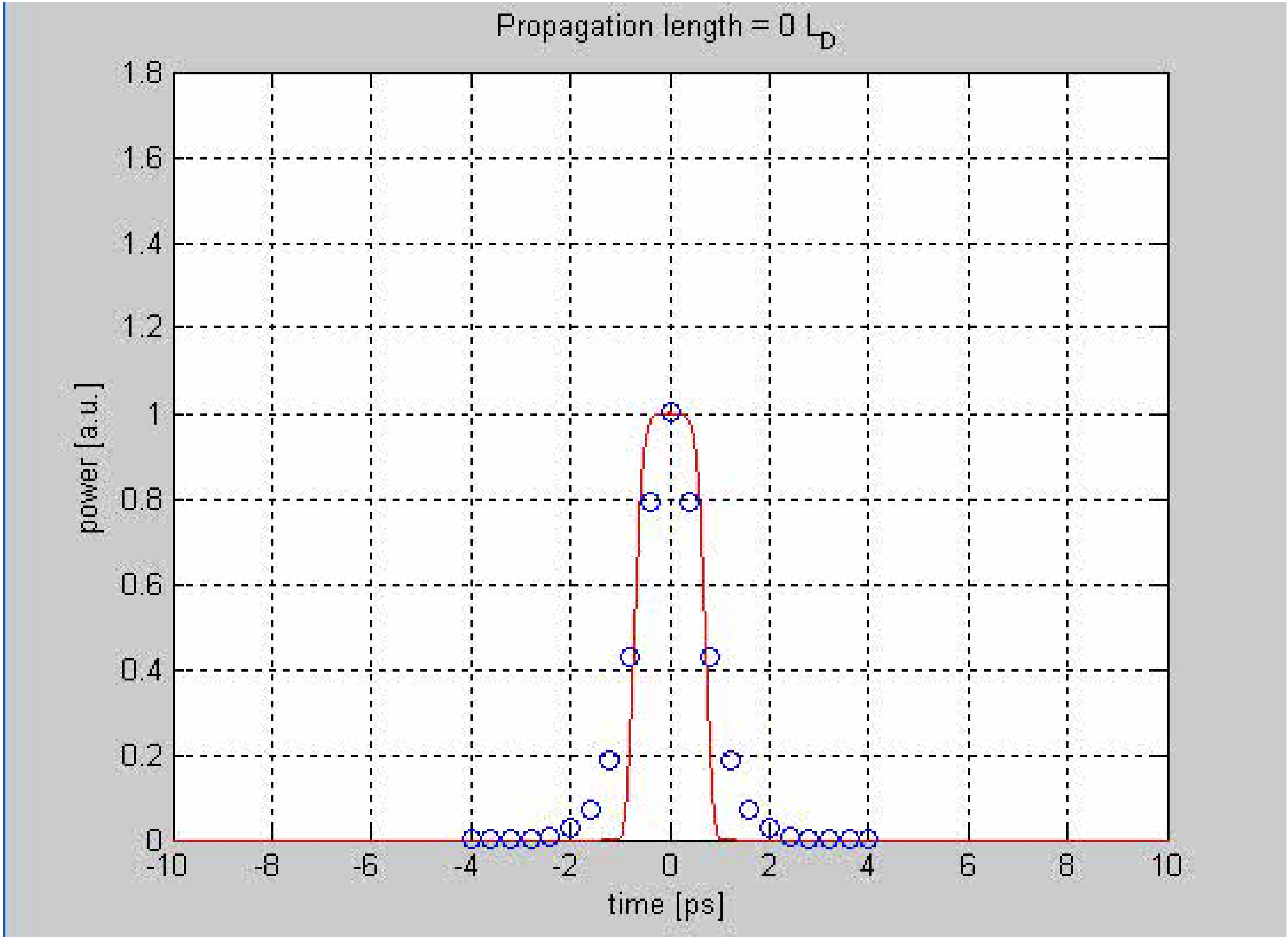
From Gaussian pulse to soliton



Gaussian pulse to 3-order soliton



Evolution of a super-Gaussian pulse to soliton



Rogue wave

find more information from New York Times: http://www.nytimes.com/2006/07/11/science/11wave.html

optical rogue waves: D. R. Solli *et al.*, Nature **450**, 1054 (2007)
D.-I. Yeom *et al.*, Nature **450**, 953 (2007)

One more rogue wave

Standard solution of PDEs

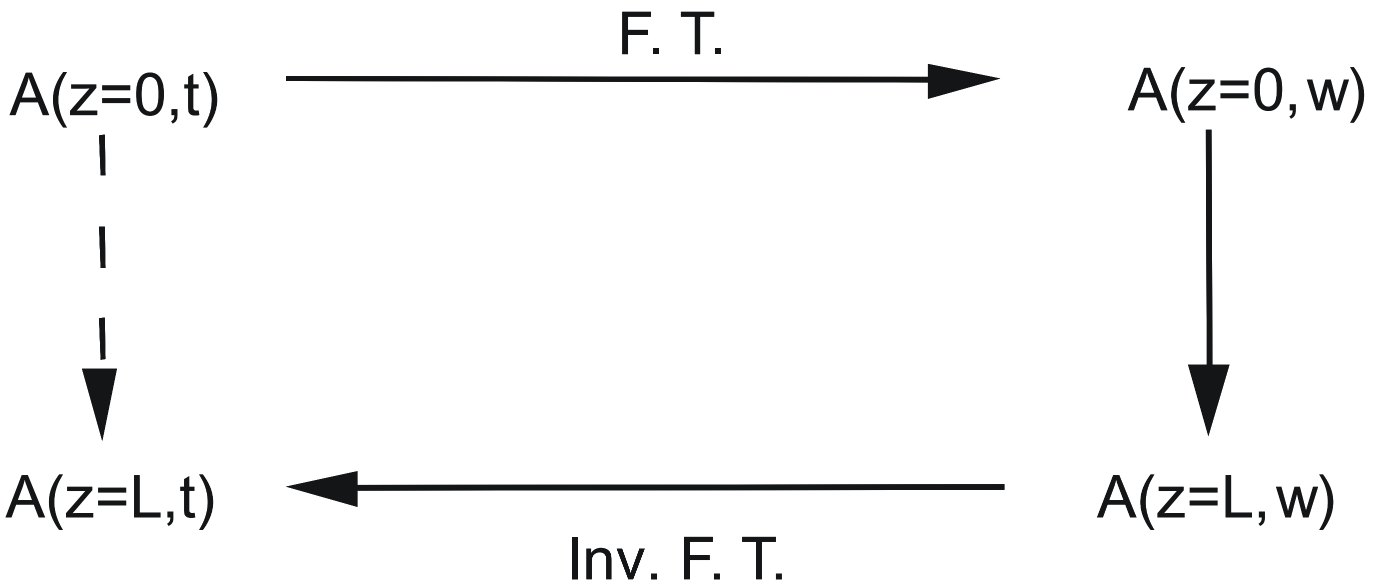


Figure 8.9: Fourier transform method for the solution of linear time invariant PDEs

3.3.4 Inverse scattering theory

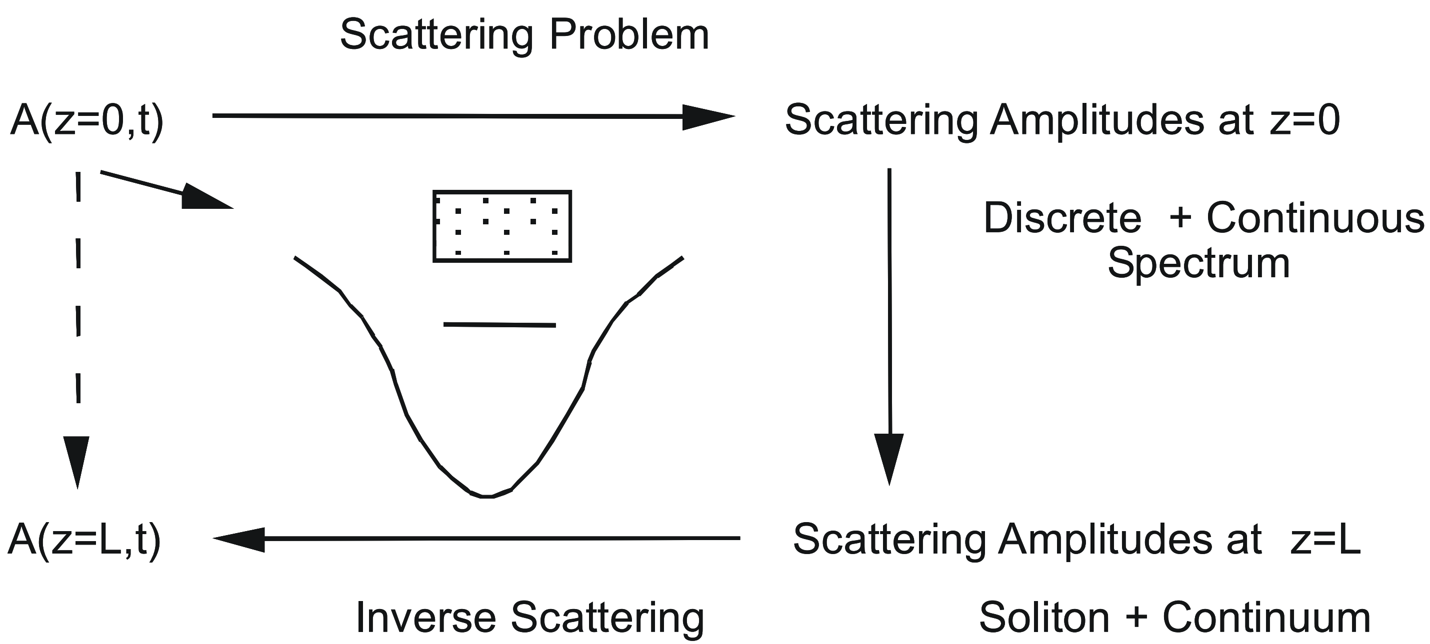


Figure 8.10: Schematic representation for the inverse scattering theory for the solution of integrable nonlinear partial differential equations

Rectangular shaped initial pulse and continuum generation

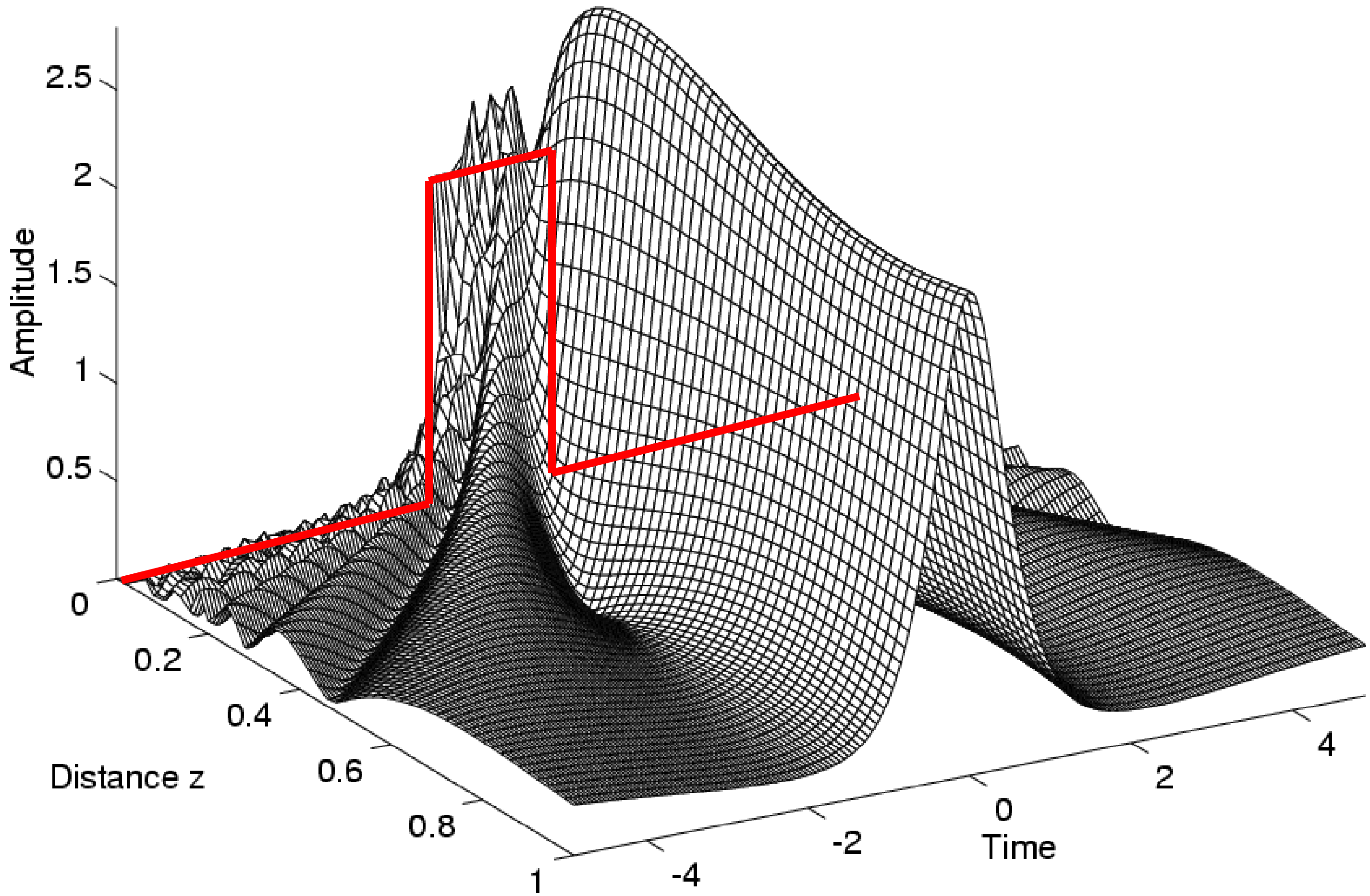
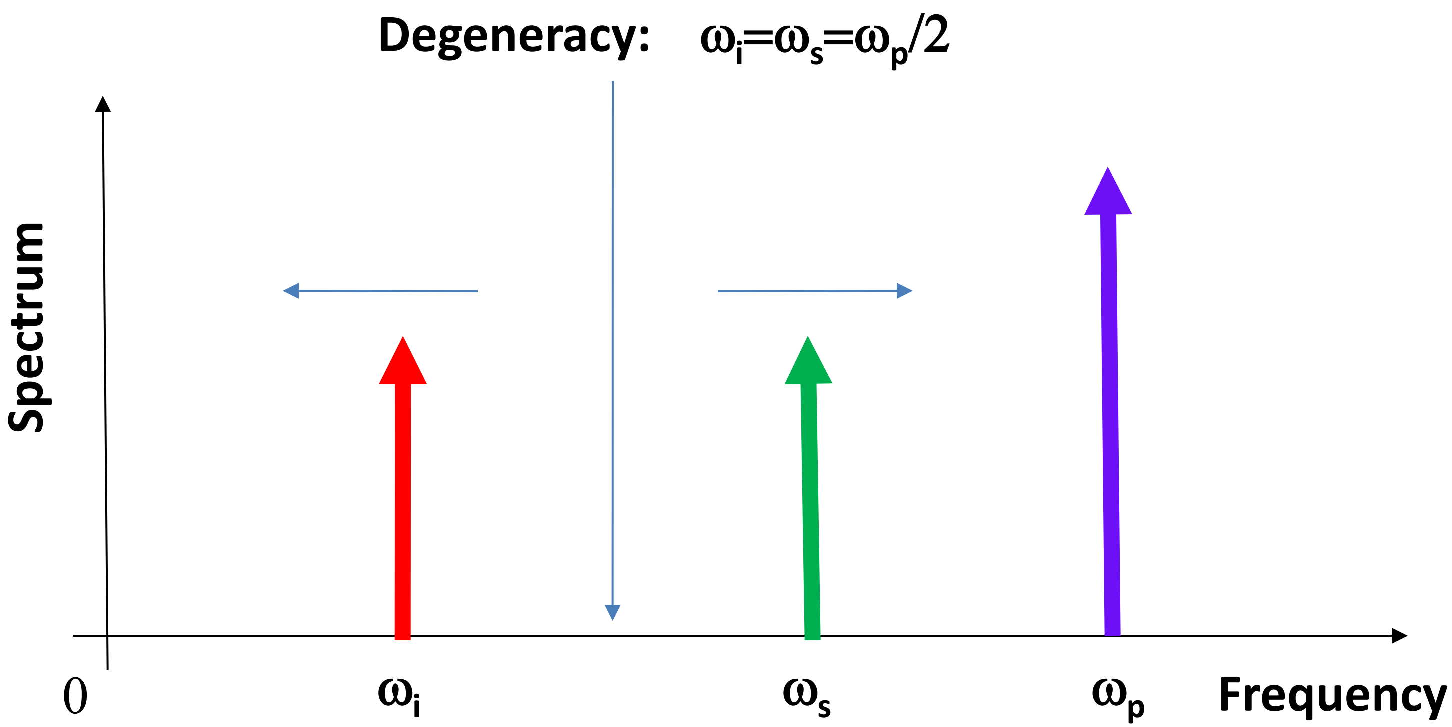


Figure 8.11: Solution of the NSE for a rectangular shaped initial pulse

9 Optical Parametric Amplifiers and Oscillators

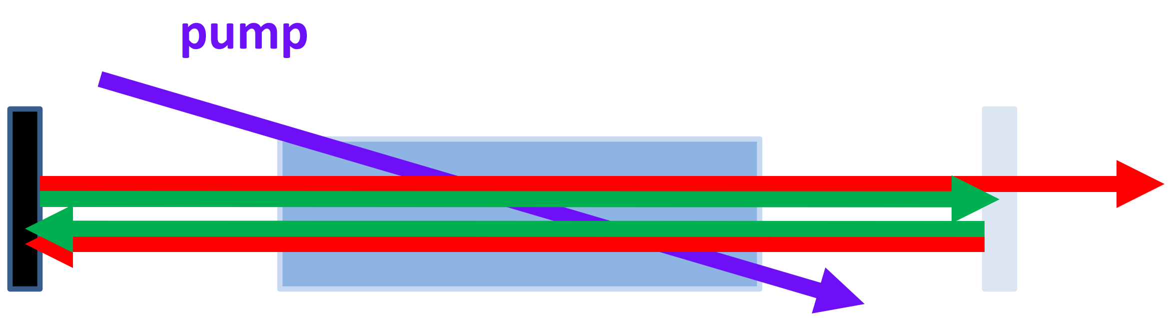
9.1 Optical Parametric Generation (OPG)



energy conservation: $\hbar\omega_p = \hbar\omega_s + \hbar\omega_i.$

momentum conservation: $\hbar\vec{k}_p = \hbar\vec{k}_s + \hbar\vec{k}_i$

Optical Parametric Oscillator (OPO)



double resonant: signal and idler resonant

single resonant: only signal resonant

Advantage: Widely tunable, both signal and idler can be used!

For OPO to operate, less gain is necessary in contrast to an OPA

Nonlinear Optical Susceptibilities

Total field: pump, signal and idler:

$$\vec{E}(\vec{r},t) = \sum_{\omega_a > 0} \sum_{i=1}^{3} \frac{1}{2} \left\{ \hat{E}_i(\omega_a) e^{j(\omega_a t - \vec{k}_a \vec{r})} + c.c. \right\} \vec{e}_i.$$

Drives polarization in medium:

$$\vec{P}(\vec{r},t) = \sum_n \vec{P}^{(n)}(\vec{r},t)$$

Polarization can be expanded in power series of the electric field:

$$\vec{P}^{(n)}(\vec{r},t) = \sum_{\omega_b > 0} \sum_{i=1}^{3} \frac{1}{2} \left\{ P_i^{(n)}(\omega_b) e^{j(\omega_b t - \vec{k}'_b \vec{r})} + c.c. \right\} \vec{e}_i.$$

Defines susceptibility tensor:

$$P_i^{(n)}(\omega_b) = \frac{\varepsilon_0}{2^{m-1}} \sum_P \sum_{j \ldots k} \chi_{ij \ldots k}^{(n)}(\omega_b : \omega_1, \ldots, \omega_n) E_j(\omega_1) \cdots E_k(\omega_n)$$

$$\omega_b = \sum_{i=1}^{n} \omega_i \text{ and } \mathbf{k}'_b = \sum_{i=1}^{n} \mathbf{k}_i$$

Special Cases

$$
\begin{aligned}
\hat{P}_i^{(2)}(\omega_3) &= \varepsilon_0 \sum_{jk} \chi_{ijk}^{(2)}(\omega_3 : \omega_1, -\omega_2) \hat{E}_j(\omega_1) \hat{E}_k^*(\omega_2), \\
\omega_3 &= \omega_1 - \omega_2 \text{ und } \mathbf{k}_3' = \mathbf{k}_1 - \mathbf{k}_2.
\end{aligned}
$$

($\longrightarrow$ Difference Frequency Generation (DFG))

$$
\begin{aligned}
\hat{P}_i^{(2)}(\omega_2) &= \varepsilon_0 \sum_{jk} \chi_{ijk}^{(2)}(\omega_2 : \omega_3, -\omega_1) \hat{E}_j(\omega_3) \hat{E}_k^*(\omega_1), \\
\omega_2 &= \omega_3 - \omega_1 \text{ und } \mathbf{k}_2' = \mathbf{k}_3 - \mathbf{k}_1.
\end{aligned}
$$

($\longrightarrow$ Parametric Generation (OPG))

$$
\begin{aligned}
\hat{P}_i^{(3)}(\omega_4) &= \frac{6\varepsilon_0}{4} \sum_{jkl} \chi_{ijkl}^{(3)}(\omega_4 : \omega_1, \omega_2, -\omega_3) \hat{E}_j(\omega_1) \hat{E}_k(\omega_2) \hat{E}_l^*(\omega_3) \\
\omega_4 &= \omega_1 + \omega_2 - \omega_3 \text{ und } \mathbf{k}_4' = \mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}_3.
\end{aligned}
$$

($\longrightarrow$ Four Wave Mixing (FWM))

9.2 Continuous-wave OPA

Wave equation :

$$\left(\Delta - \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\right)\vec{E} = \mu_0 \frac{\partial^2}{\partial t^2}\vec{P}.$$

Include linear and second-order terms:

$$\left(\Delta - \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\right)\vec{E} = \mu_0 \frac{\partial^2}{\partial t^2}\left(\underbrace{\vec{P}^{(l)}(\vec{r},t)} + \underbrace{\vec{P}^{(2)}(\vec{r},t)}\right)$$

Changes group and phase velocities of waves

Nonlinear interaction of waves

$$.\, k(\omega)$$

z-propagation only:

$$\vec{E}_{p,s,i}(z,t) = \mathrm{Re}\left\{E_{p,s,i}(z)\; e^{j(\omega_{p,s,i}t - k_{p,s,i}\;\, z)}\vec{e}_{p,s,i}\right\}$$

Wave amplitudes

$$\vec{P}^{(2)}_{p,s,i}(z,t) = \mathrm{Re}\left\{ P^{(2)}_{p,s,i}(z)\ e^{j\left(\omega_{p,s,i}t - k'_{p,s,i}z\right)} \vec{e}_{p,s,i} \right\}$$

Separate into three equations for each frequency component:

Slowly varying amplitude approximation:

$$d^2_{p,s,i}E(z)\ /dz^2 << \ 2k\ dE_{p,s,i}(z)\ /dz,$$

$$\frac{\partial E_{p,s,i}(z)}{\partial z} = -\frac{jc_0^2\omega_{p,s,i}}{2n(\omega_{p,s,i})} P^{(2)}_{p,s,i}(z)\ e^{-j\left(k'_{p,s,i} - k_{p,s,i}\right)z}$$

Introduce phase mismatch: $\Delta k = k(\omega_p) - k(\omega_s) - k(\omega_i)$

and effective nonlinearity and coupling coefficients:

$$d_{eff} = \frac{1}{2}\chi^{(2)}_{ijk}(\omega_p : \omega_s, \omega_i), \quad \kappa_{p,s,i} = \omega_{p,s,i}\ d_{eff}/(n_{p,s,i}c_0)$$

Coupled wave equations:

$$\frac{\partial E_p(z)}{\partial z} = -j\kappa_p \; E_s(z)E_i(z) \; e^{j\Delta kz} \; ,$$
$$\frac{\partial E_s(z)}{\partial z} = -j\kappa_s \; E_p(z)E_i^*(z) \; e^{-j\Delta kz}, \qquad \textbf{X} \;\; n_{p,s,i}c_0\varepsilon_0 E^*_{p,s,i}/2$$
$$\frac{\partial E_i(z)}{\partial z} = -j\kappa_i \; E_p(z)E_s^*(z) \; e^{-j\Delta kz}.$$

Intensity of waves: $I_{p,s,i} = \frac{n_{p,s,i}}{2Z_{F_0}} \left|E_{p,s,i}\right|^2$

Manley-Rowe relations: $$-\frac{1}{\omega_p}\frac{dI_p}{dz} = \frac{1}{\omega_s}\frac{dI_s}{dz} = \frac{1}{\omega_i}\frac{dI_i}{dz}$$

9.4 Theory of Optical Parametric Amplification

Undepleted pump approximation: $E_p = const.$

$$\frac{\partial E_s(z)}{\partial z} = -j\kappa_s\, E_p E_i^*(z)\, e^{-j\Delta k z},$$
$$\frac{\partial E_i(z)}{\partial z} = -j\kappa_i\, E_p E_s^*(z)\, e^{-j\Delta k z}.$$

with:

$$E_s(z=0) = E_s(0) \qquad E_i(z=0) = 0$$

$$E_s(z) \tilde{} E_s(0)\; e^{gz - j\Delta k z/2} \text{ and } E_i(z) \tilde{} E_i(0)\; e^{gz - j\Delta k z/2}$$

$$\longrightarrow \quad \begin{vmatrix} g - j\frac{\Delta k}{2} & j\kappa_s\, E_p \\ j\kappa_i\, E_p^* & g + j\frac{\Delta k}{2} \end{vmatrix} = 0$$

$$g = \sqrt{\Gamma^2 - \left(\frac{\Delta k}{2}\right)^2}, \text{ with } \Gamma = \sqrt{\kappa_i\, \kappa_s\, |E_p|^2}.$$

gain

max. gain, when phase matched

Maximum gain

$$\boxed{\Gamma^2 = \frac{\omega_s \omega_i}{n_s n_i c_0^2} d_{eff}^2 \ |E_p|^2 = \frac{2 Z_{F0} \omega_s \omega_i}{n_p n_s n_i c_0^2} d_{eff}^2 \ I_p}$$

$$\boxed{FOM = \frac{d_{eff}}{\sqrt{\lambda_s \lambda_i n_p n_s n_i}}}$$

General solutions:

$$E_s(z) = \{E_s(0)\cosh gz + B\sinh gz\}\, e^{-j\Delta kz/2} \qquad B = -j\frac{\Delta k}{2g}E_s(0) - j\frac{\kappa_1}{g}E_p E_i^*(0)$$

$$E_i(z) = \{E_i(0)\cosh gz + D\sinh gz\}\, e^{-j\Delta kz/2} \qquad D = -j\frac{\Delta k}{2g}E_i(0) - j\frac{\kappa_2}{g}E_p^* E_s^*(0)$$

Here:

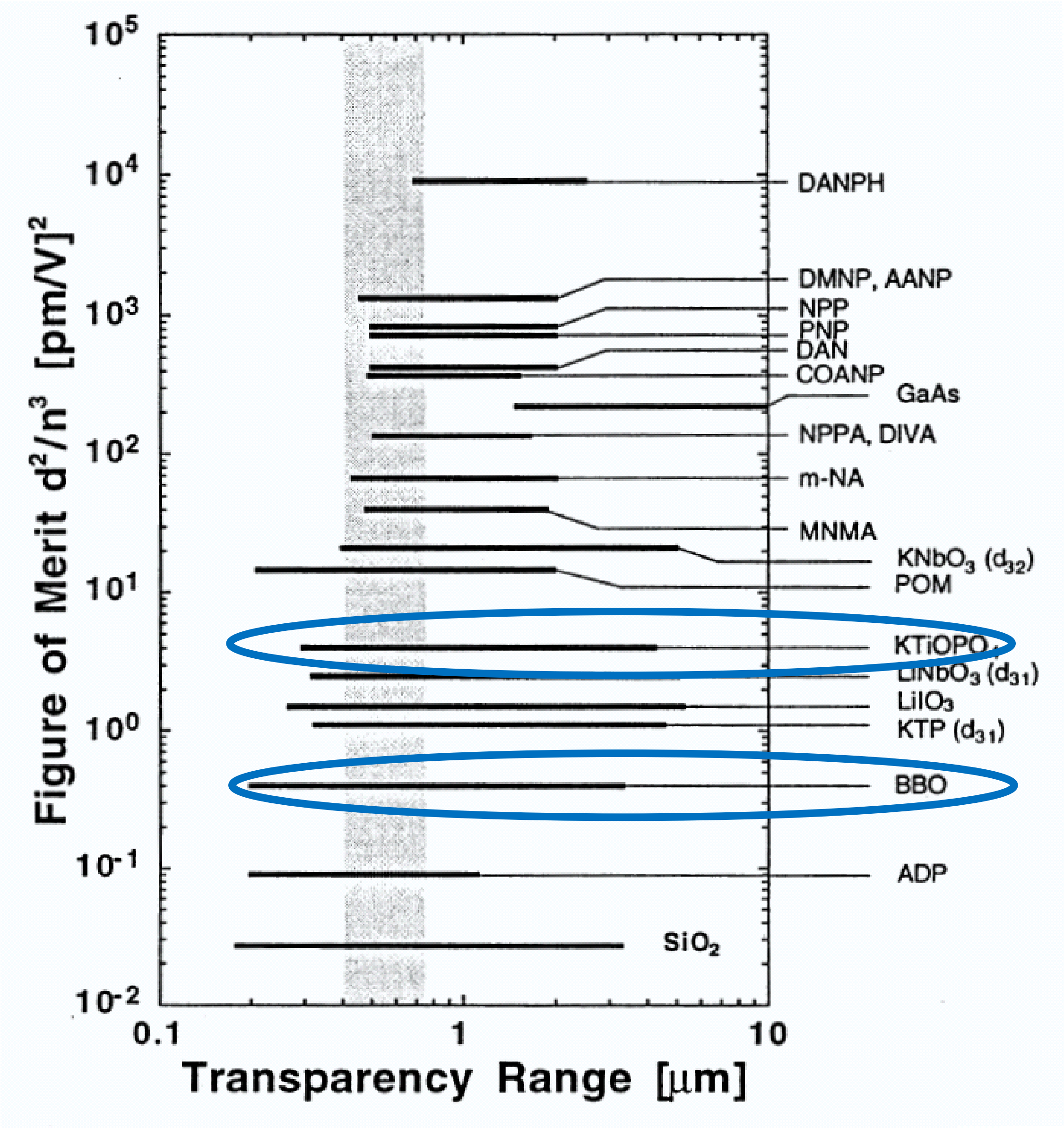
$$\begin{aligned} I_s(L) &= I_s(0)\left[1 + \frac{\Gamma^2}{g^2}\sinh^2 gL\right] \\ I_i(L) &= I_s(0)\,\frac{\omega_i}{\omega_s}\frac{\Gamma^2}{g^2}\sinh^2 gL. \end{aligned}$$

For large gain: $\Gamma L >> 1$

$$\begin{aligned} I_s(L) &= \frac{1}{4} I_s(0)\ e^{2\Gamma L}, \\ I_i(L) &= \frac{1}{4} I_s(0)\ \frac{\omega_i}{\omega_s}\ e^{2\Gamma L} \end{aligned} \longrightarrow \quad G = \frac{I_s(L)}{I_s(0)} = \frac{1}{4}\ e^{2\Gamma L}$$

Figure of merit:

$$FOM = \frac{d_{eff}}{\sqrt{\lambda_s \lambda_i n_p n_s n_i}}$$



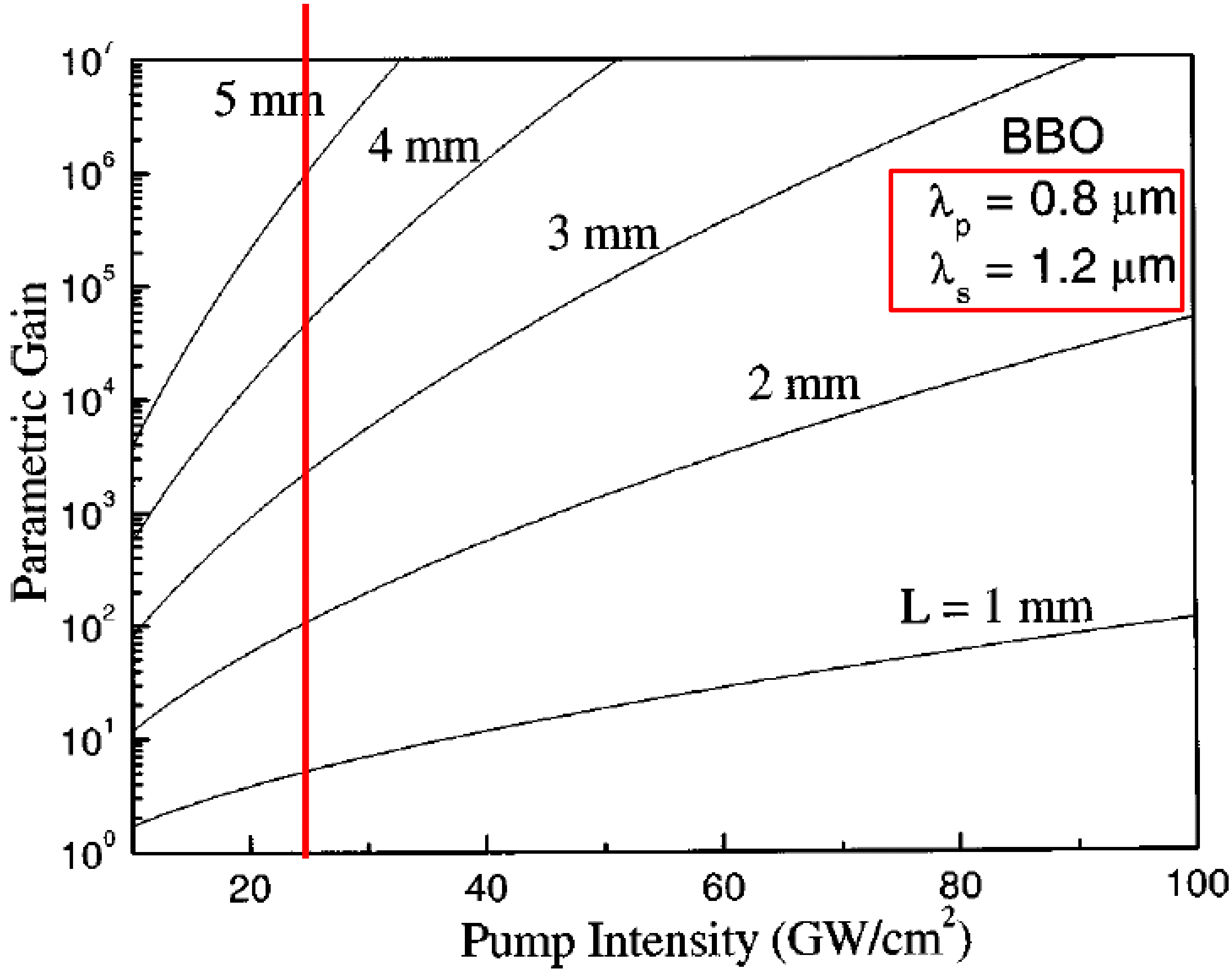


Fig. 9.3 Parametric gain for an OPA at the pump wavelength λ_p = 0.8 μm and the signal wavelength λ_s = 1.2 μm, using type-I phase matching in BBO (d_{eff} = 2 pm/V).

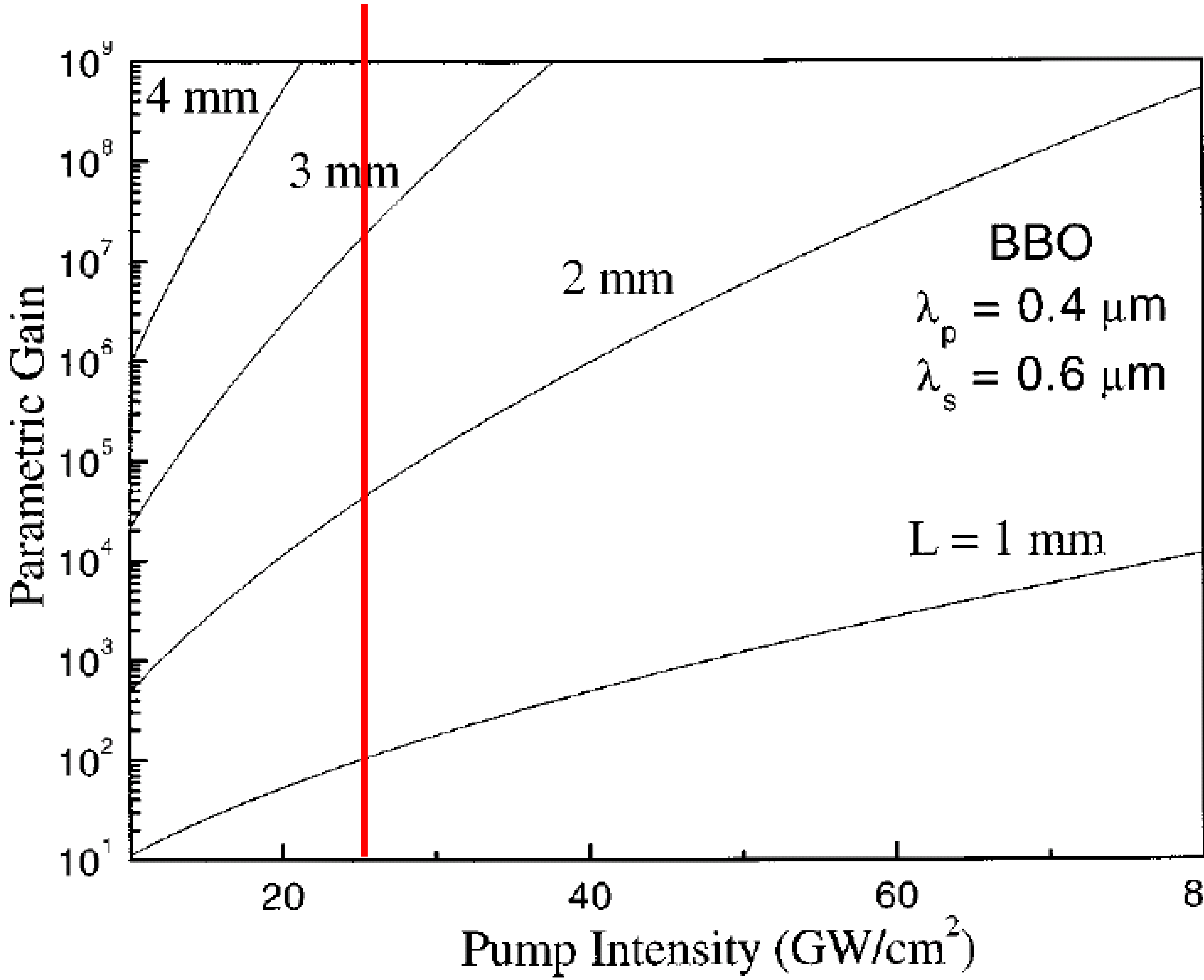


Fig. 9.4 Parametric gain for an OPA at the pump wavelength λ_p = 0.4 μm and the signal wavelength λ_s = 0.6 μm, using type-I phase matching in BBO (d_{eff} = 2 pm/V).

9.4 Phase Matching

$$\Delta k = 0 \longrightarrow n_p = \frac{n_s \omega_s + n_i \omega_i}{\omega_p}$$

Uniaxial crystal: $n_e < n_o$

Type I: noncritical

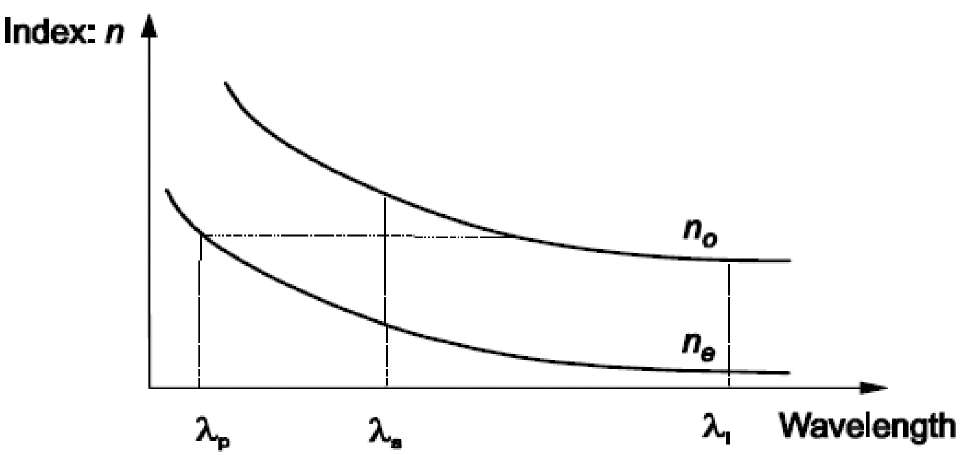


Fig. 9.5 Type-I noncritical phase matching.

Type I: critical

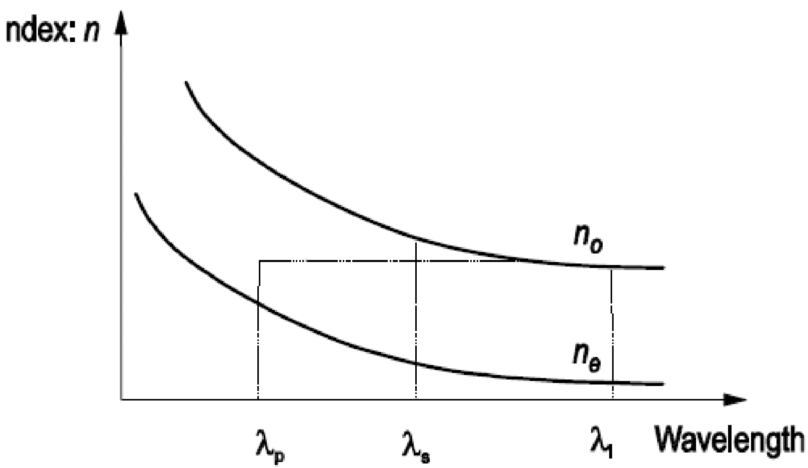
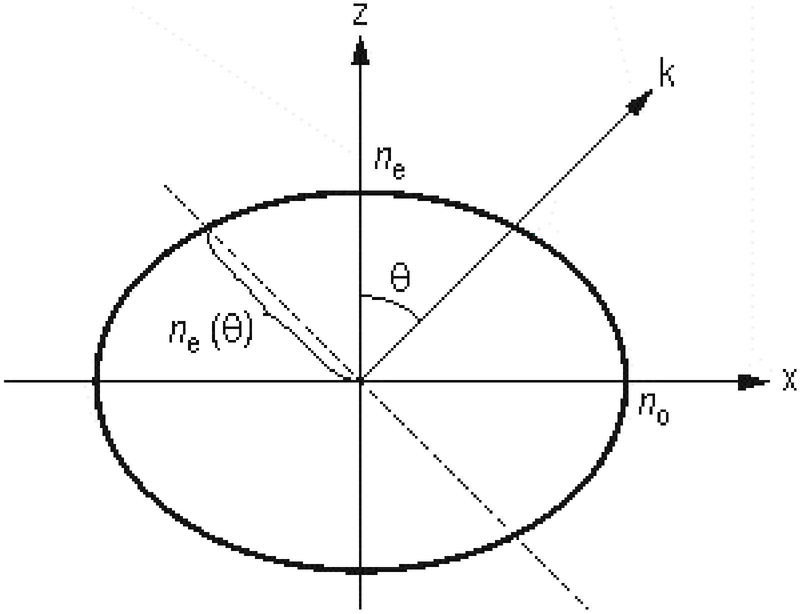


Fig. 9.6 Type-I critical phase matching by adjusting the angle θ between wave vector of the propagating beam and the optical axis.

9.4 Phase Matching



Critical Phase Matching

$$n_{ep}(\theta)\omega_p = n_{os}\omega_s + n_{oi}\omega_i$$

$$\frac{1}{n_{ep}(\theta)^2} = \frac{\sin^2\theta}{n_{ep}^2} + \frac{\cos^2\theta}{n_{op}^2}$$

$$\theta = \arcsin\left[\frac{n_{ep}}{n_{ep}(\theta)}\sqrt{\frac{n_{op}^2 - n_{ep}^2(\theta)}{n_{op}^2 - n_{ep}^2}}\right]$$

9.4 Phase Matching

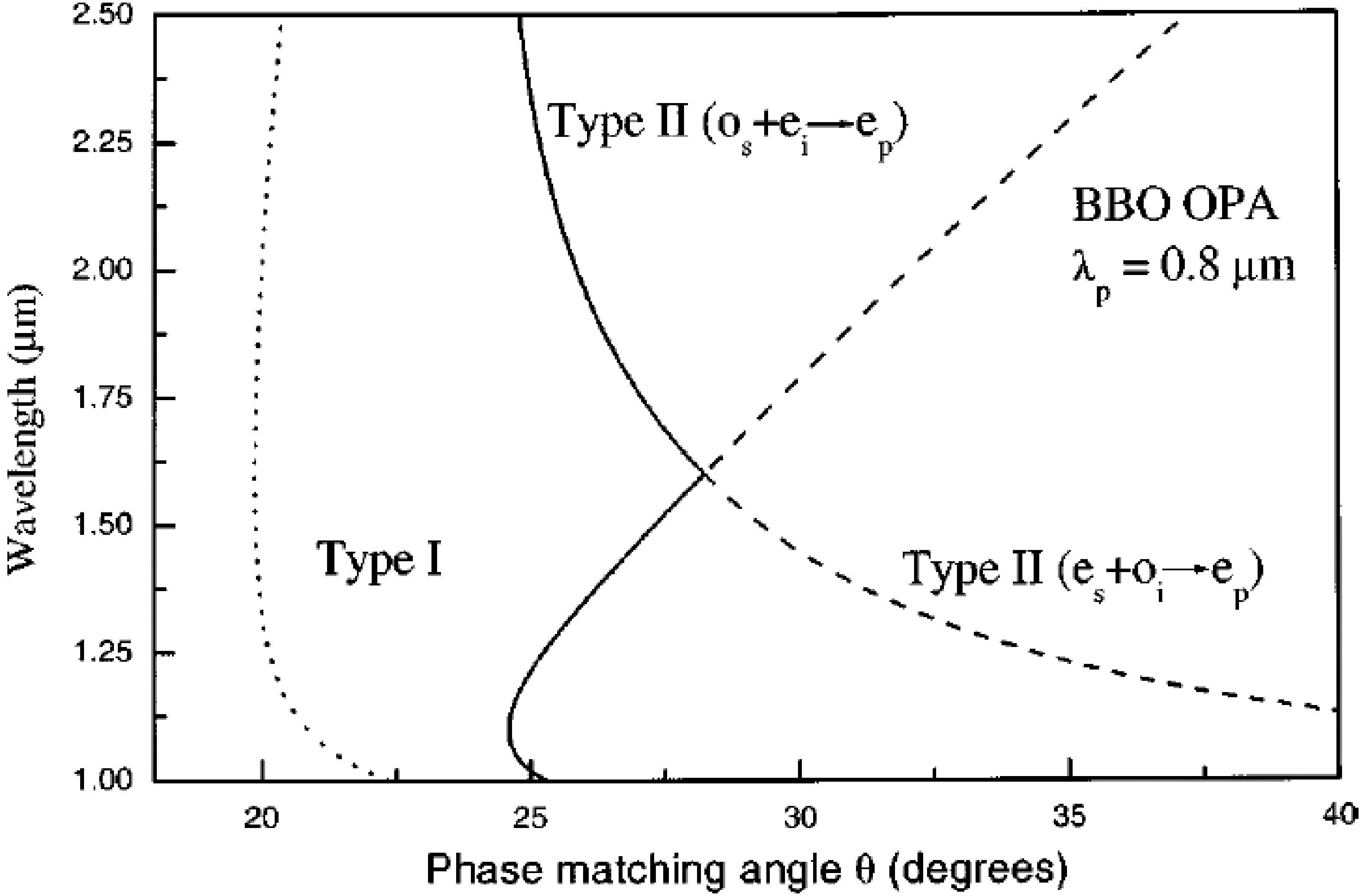


Fig. 9.7 Angle tuning curves for a BBO OPA at the pump wavelength λ_p=0.8 μm for type-I phase matching (dotted line), type-II ($o_s + e_i \rightarrow e_p$) phase matching (solid line), and type-II ($e_s + o_i \rightarrow e_p$) phase matching (dashed line).

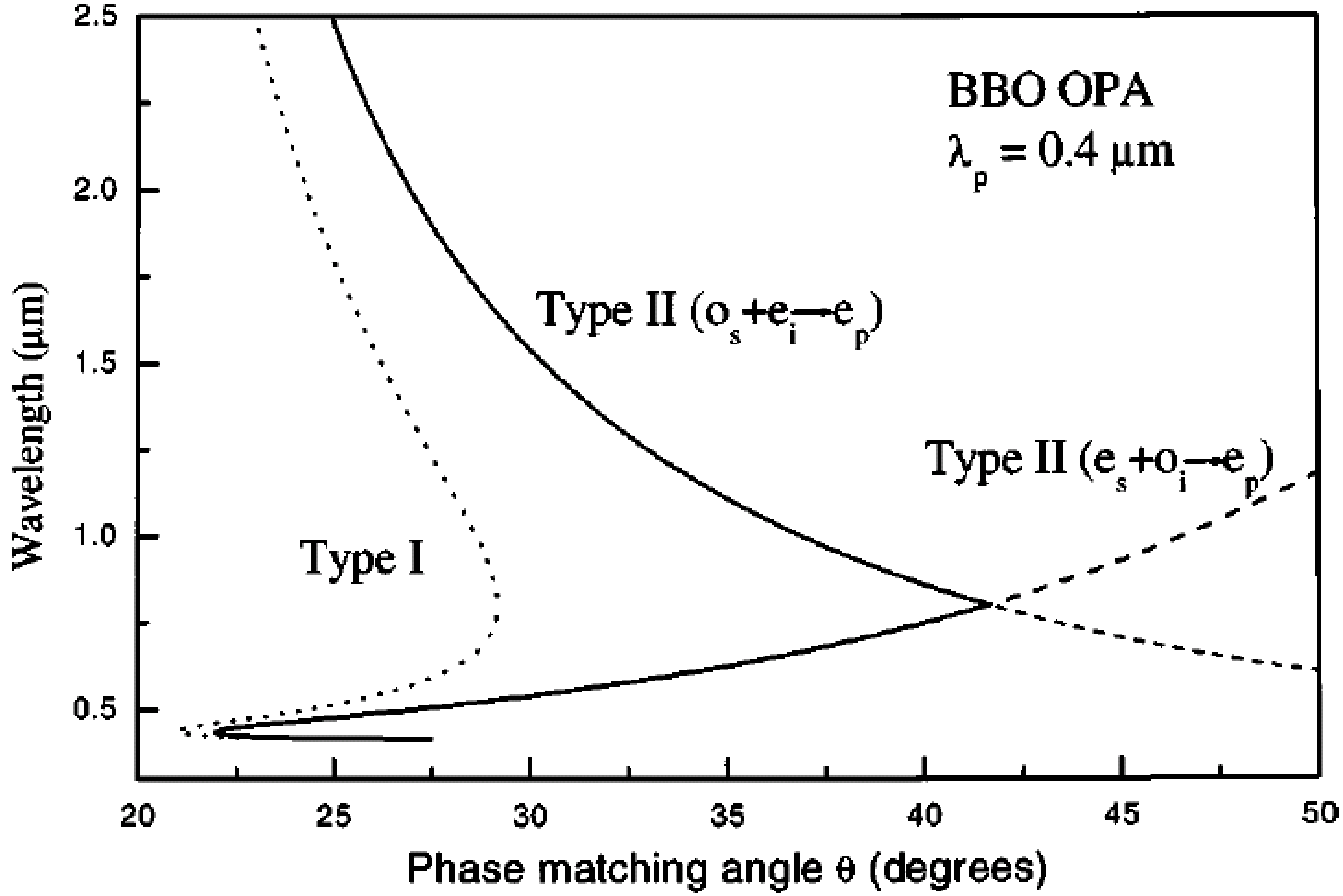


Fig. 9.8 Angle tuning curves for a BBO OPA at the pump wavelength λ_p=0.4 µm for type-I phase matching (dotted line), type-II ($o_s + e_i \rightarrow e_p$) phase matching (solid line), and type-II ($e_s + o_i \rightarrow e_p$) phase matching (dashed line).

9.5 Quasi Phase Matching

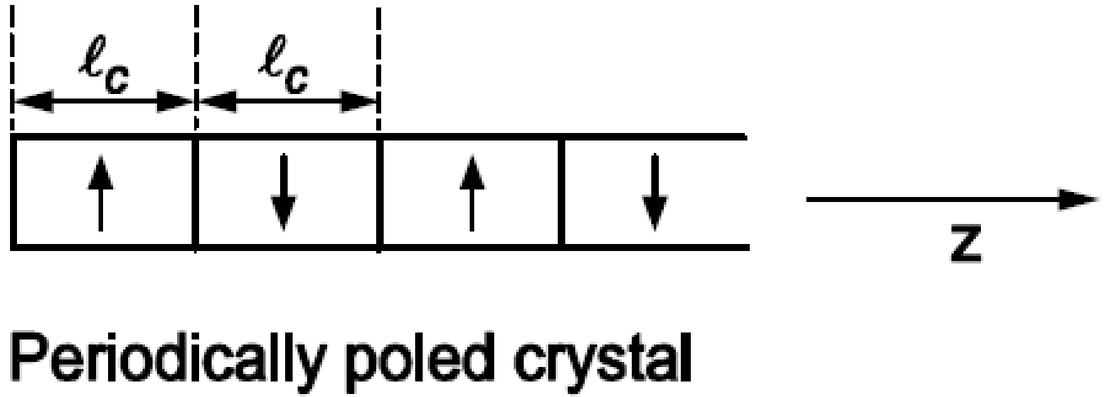


Fig.12.30: Variation of d_{eff} in a quasi phase matched material as a function of propagation distance.

$$d_{eff}(z) = \sum_{m=-\infty}^{+\infty} d_m e^{jm\kappa z}$$

$$\frac{\partial E_p(z)}{\partial z} = -j\kappa_p \, E_s(z) E_i(z) \, e^{j\Delta k z}$$

9.6 Ultrashort-Pulse Optical Parametric Amplification

$$\vec{E}_{p,s,i}(z,t) = \mathrm{Re}\left\{E_{p,s,i}(z,t)\ e^{j(\omega_{p,s,i}t - k_{p,s,i}\ z)}\vec{e}_{p,s,i}\right\}$$

Pulse envelopes

$$\begin{aligned}
\frac{\partial E_p}{\partial z} + \frac{1}{v_p}\frac{\partial E_p}{\partial t} &= -j\kappa_p\ E_s E_i\ e^{j\Delta k z}\ , \\
\frac{\partial E_s}{\partial z} + \frac{1}{v_s}\frac{\partial E_s}{\partial t} &= -j\kappa_s\ E_p E_i^*\ e^{-j\Delta k z}, \\
\frac{\partial E_i}{\partial z} + \frac{1}{v_i}\frac{\partial E_s}{\partial t} &= -j\kappa_i\ E_p E_s^*\ e^{-j\Delta k z},
\end{aligned}$$

$v_{p,s,i} = dk/d\omega|_{\omega_{p,s,i}}$ are the corresponding group velocities

$t' = t - z/v_p$

$$\begin{aligned}
\frac{\partial E_p}{\partial z} &= -j\kappa_p\ E_s E_i\ e^{j\Delta k z}\ , \\
\frac{\partial E_s}{\partial z} + \left(\frac{1}{v_s} - \frac{1}{v_p}\right)\frac{\partial E_s}{\partial t} &= -j\kappa_s\ E_p E_i^*\ e^{-j\Delta k z}, \\
\frac{\partial E_i}{\partial z} + \left(\frac{1}{v_i} - \frac{1}{v_p}\right)\frac{\partial E_s}{\partial t} &= -j\kappa_i\ E_p E_s^*\ e^{-j\Delta k z}.
\end{aligned}$$

Temporal walkoff
Group Velocity Mismatch (GVM)

Pump pulse width

$$\ell_{jp} = \frac{\tau}{\delta_{jp}}, \quad \text{with } \delta_{jp} = \left(\frac{1}{v_j} - \frac{1}{v_p}\right)$$

j=s,i

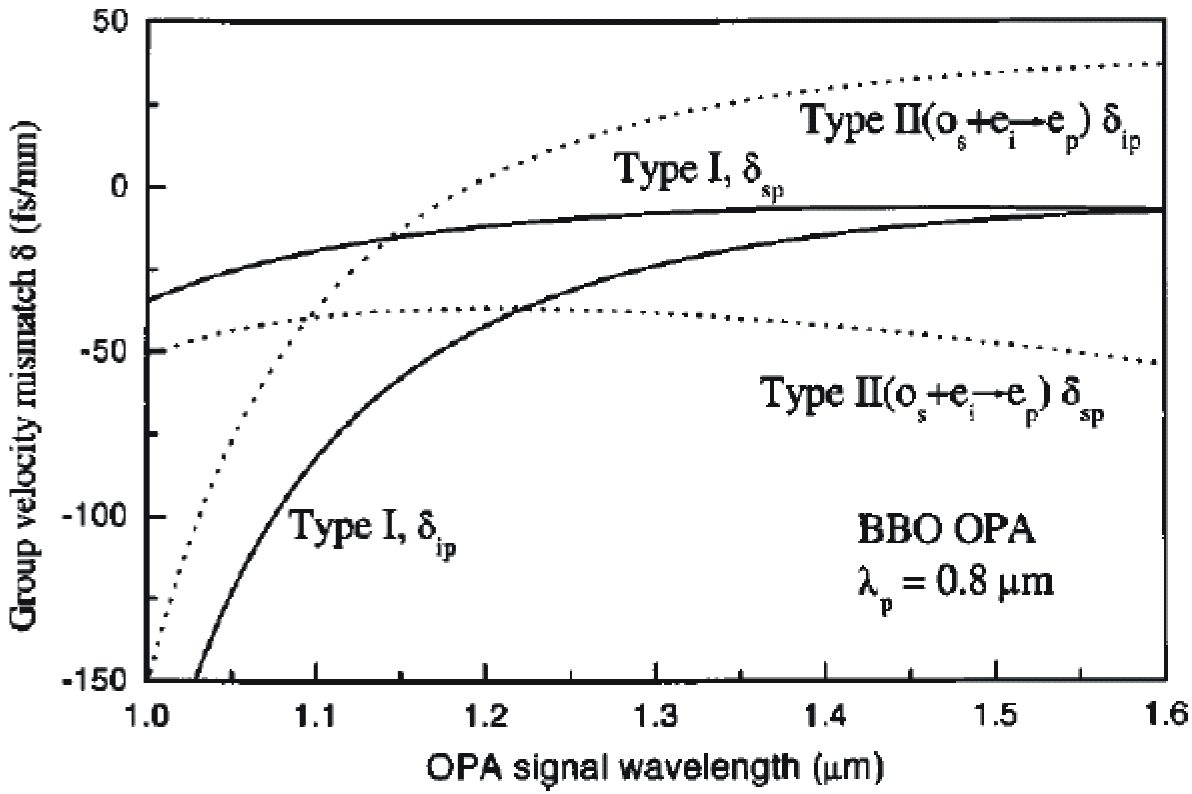


Fig. 9.9: Pump-signal (δ_{sp}) and pump-idler (δ_{ip}) group velocity mismatch curves for a BBO OPA at the pump wavelength λ_p=0.8 μm for type-I phase matching (solid line) and type-II ($o_s + e_i \rightarrow e_p$) phase matching (dashed line).

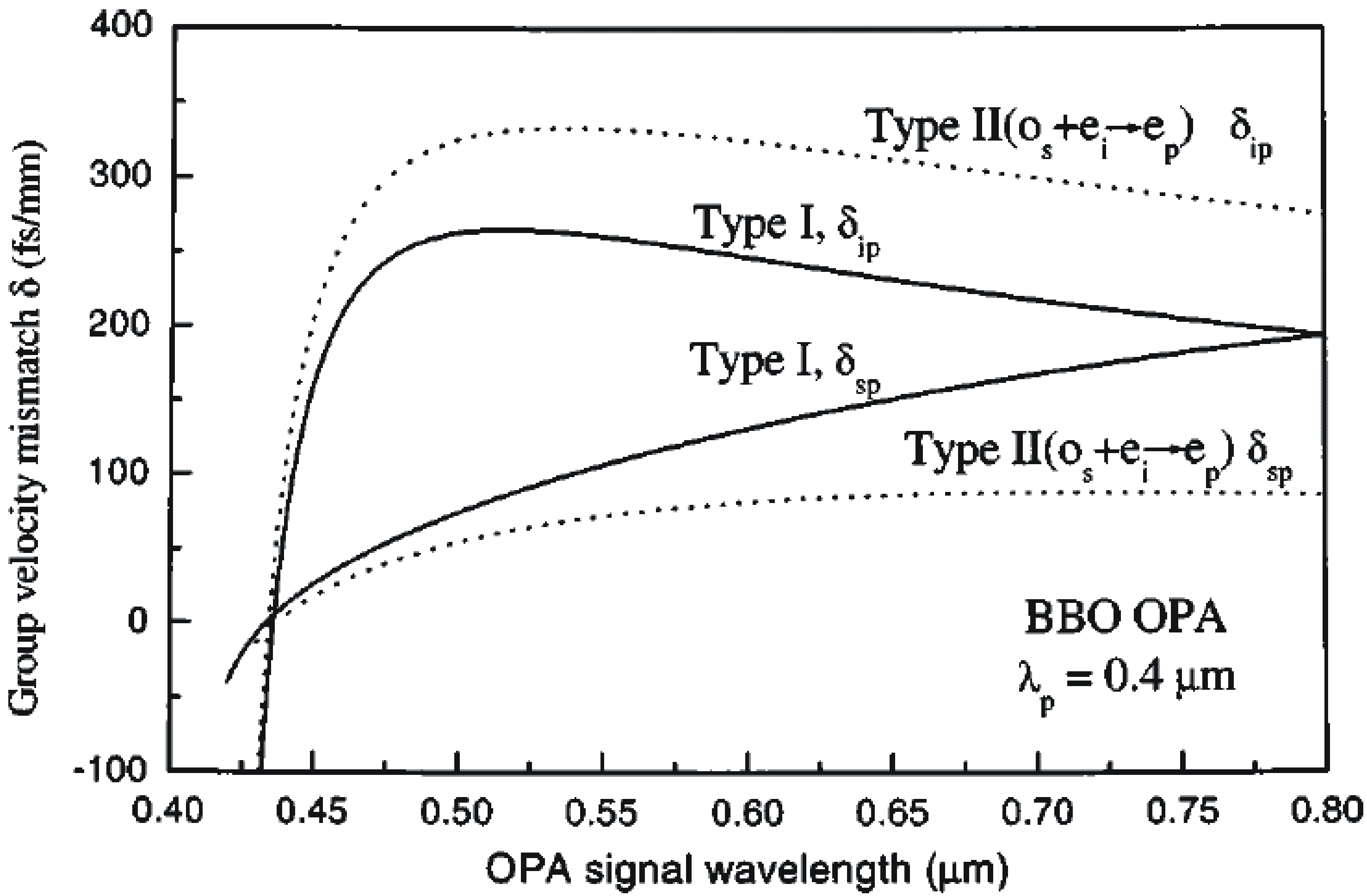


Fig. 9.10: Pump-signal (δ_{sp}) and pump-idler (δ_{ip}) group velocity mismatch curves for a BBO OPA at the pump wavelength λ_p=0.4 µm for type-I phase matching (solid line) and type-II ($o_s + e_i \rightarrow e_p$) phase matching (dashed line).

$\delta_{sp}\delta_{ip} > 0$

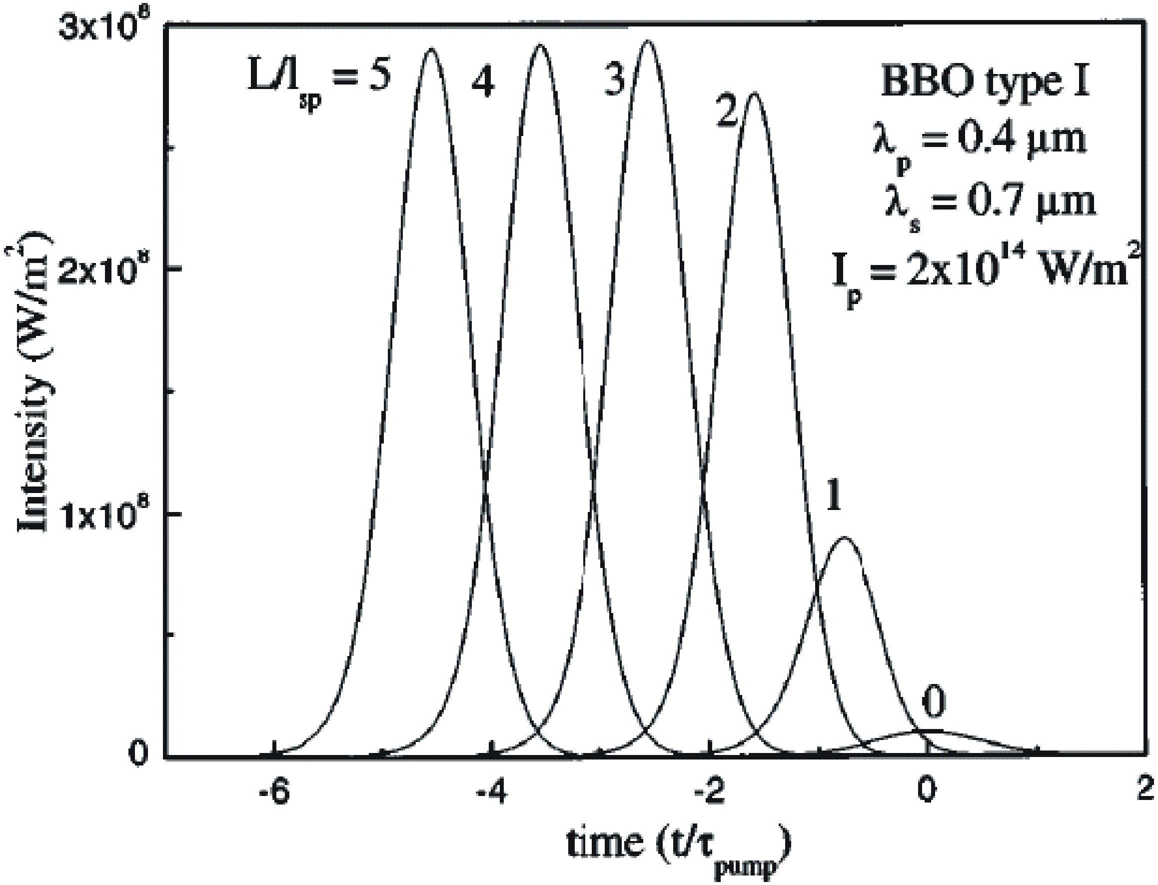


Fig. 9.11: Signal pulse evolution for a BBO type-I OPA with λ_p = 0.4 µm, λ_s = 0.7 µm, for different lengths L of the nonlinear crystal. Pump intensity is 20 GW/cm^2. Time is normalized to the pump pulse duration and the crystal length to the pump-signal pulse splitting length. [5]

$$\delta_{sp}\delta_{ip} < 0$$

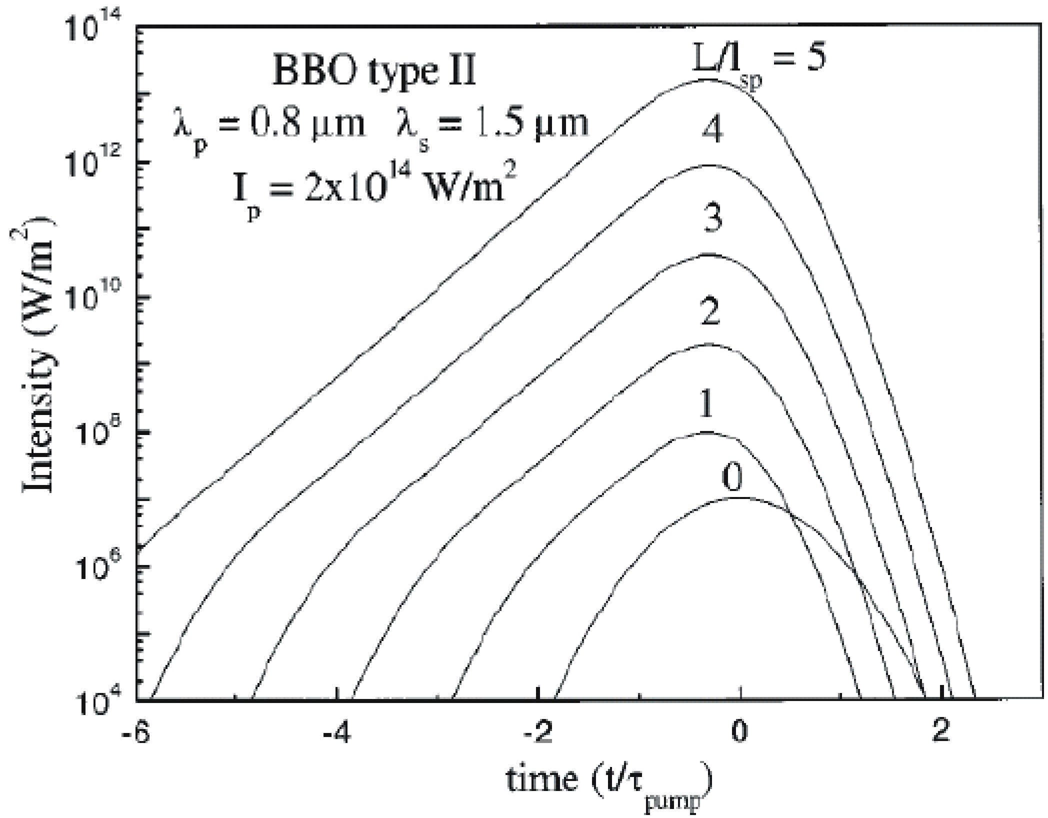


Figure 9.12: Signal pulse evolution for a BBO type-I OPA with λ_p = 0.8 µm, λ_s = 1.5 µm, for different lengths L of the nonlinear crystal. Pump intensity is 20 GW/cm^2. Time is normalized to the pump pulse duration and the crystal length to the pump-signal pulse splitting length. [5]

OPA Bandwidth

$$\omega_s \longrightarrow \omega_s + \Delta\omega \qquad \omega_i \longrightarrow \omega_i - \Delta\omega$$

$$\Delta k = -\frac{dk_s}{d\omega}\Delta\omega + \frac{dk_i}{d\omega}\Delta\omega = \left(\frac{1}{v_i} - \frac{1}{v_s}\right)\Delta\omega$$

Bandwidth limitation due to GVM

$$\Delta f = -\frac{2\sqrt{\ln 2}}{\pi}\sqrt{\frac{\Gamma}{L}}\frac{1}{\left|\frac{1}{v_i} - \frac{1}{v_s}\right|}$$

For signal-idler group velocity matching:

$$\Delta f = -\frac{2\sqrt[4]{\ln 2}}{\pi}\sqrt[4]{\frac{\Gamma}{L}}\frac{1}{\left|\frac{d^2k_s}{d\omega^2} + \frac{d^2k_s}{d\omega^2}\right|}.$$

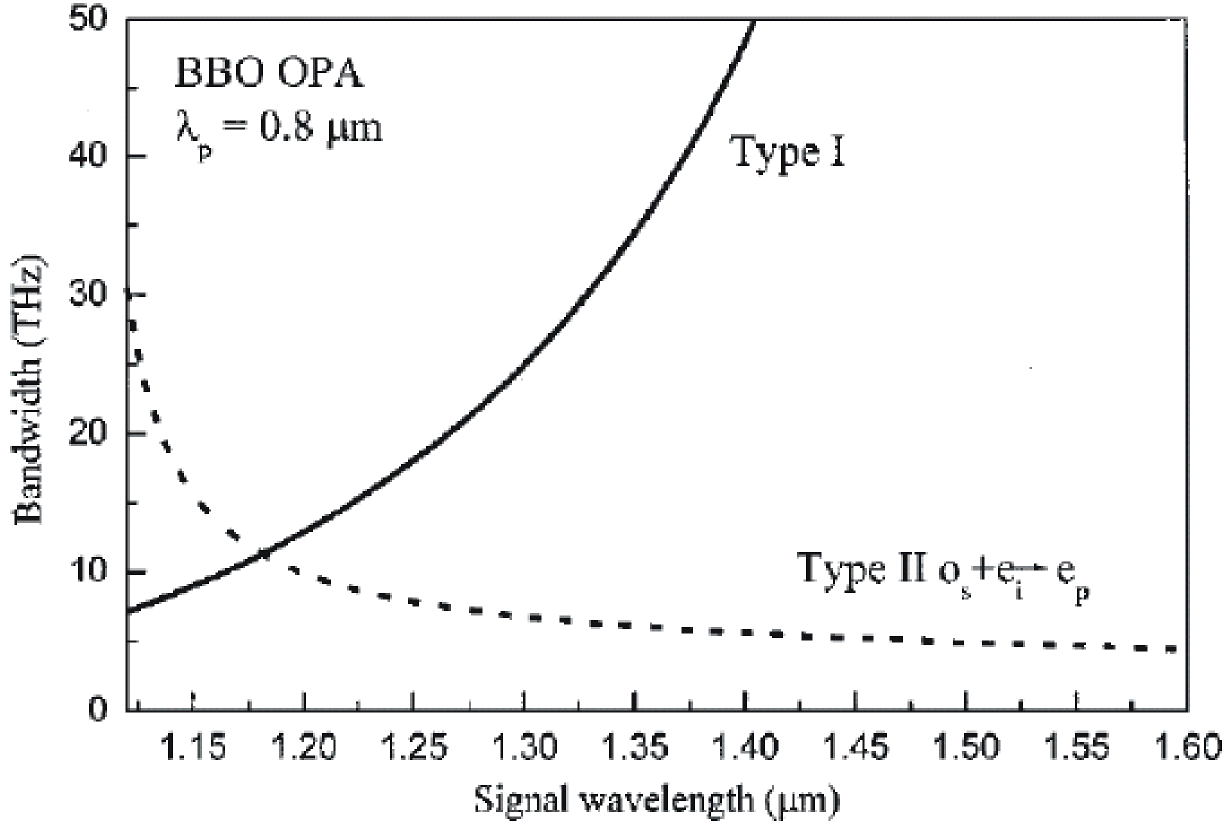


Fig. 9.13: Phase matching bandwidth for a BBO OPA at the pump wavelength λ_p=0.8 µm for type-I phase matching (solid line) and type-II ($o_s + e_i \rightarrow e_p$) phase matching (dashed line). Crystal length is 4 mm and pump intensity 50 GW/cm^2.

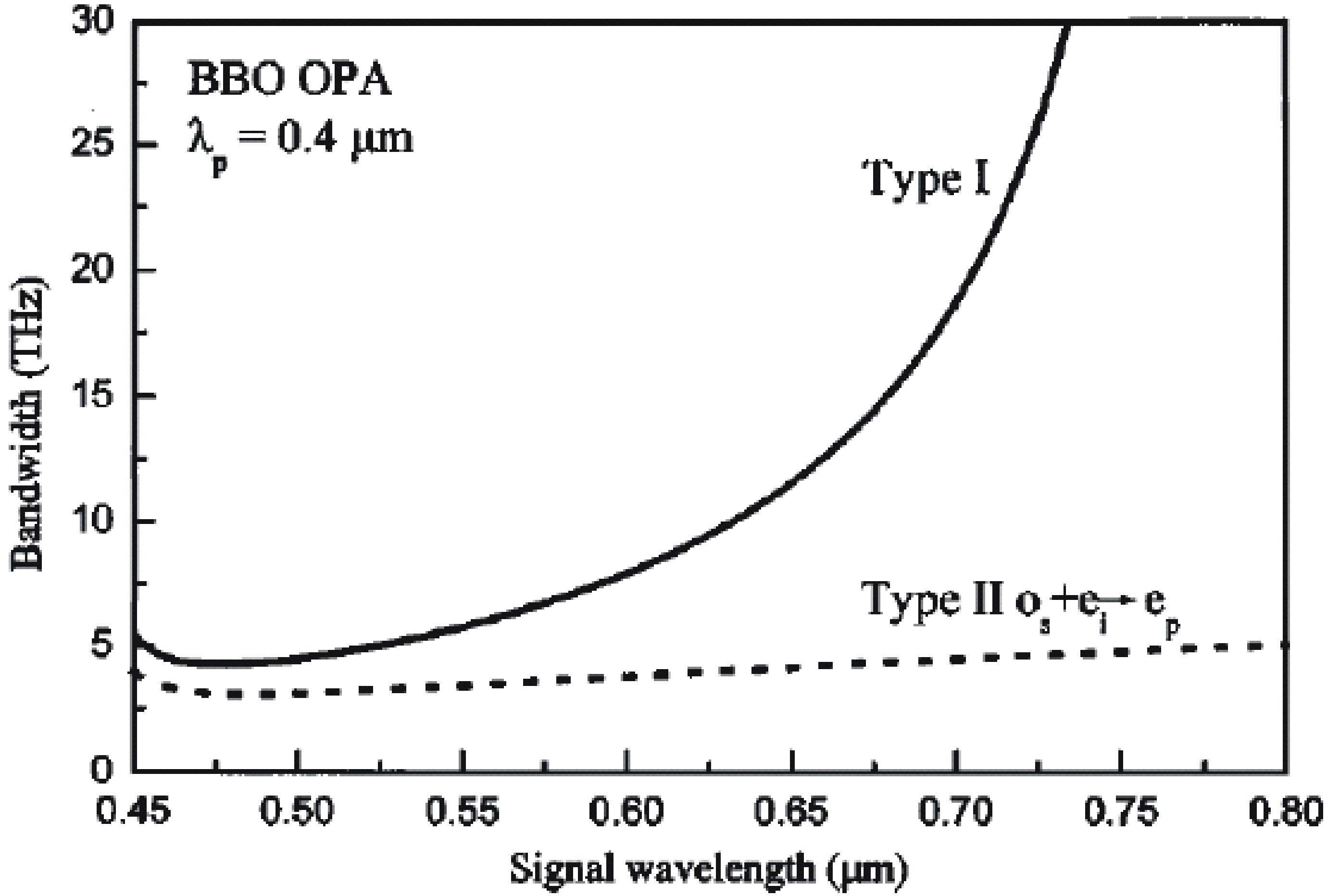


Fig. 9.14: Phase matching bandwidth for a BBO OPA at the pump wavelength λ_p=0.4 μm for type-I phase matching (solid line) and type-II ($o_s + e_i \rightarrow e_p$) phase matching (dashed line). Crystal length is 2 mm and pump intensity 100 GW/cm^2.

9.7 Optical Parametric Amplifier Designs

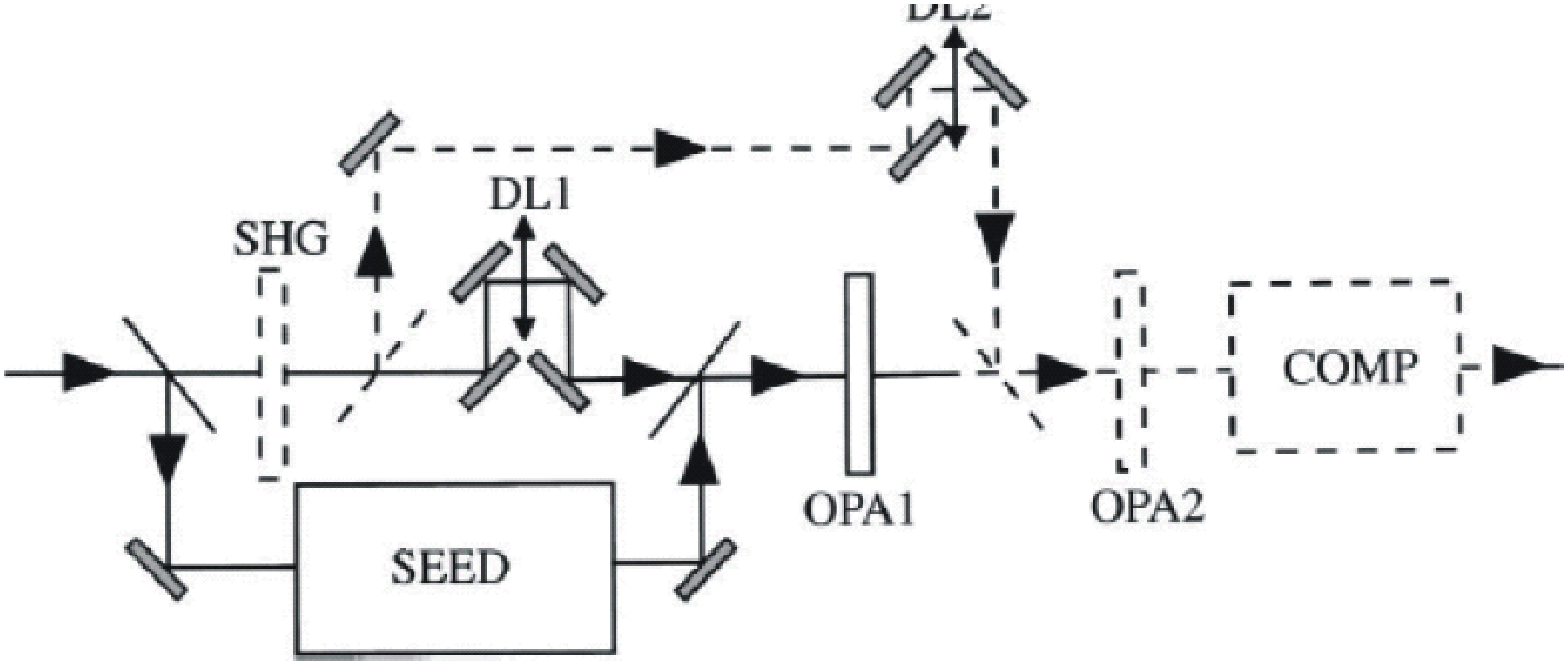


Fig. 9.15: Scheme of an ultrafast optical parametric amplifier. SEED: seed generation stage; DL1, DL2: delay lines; OPA1, OPA2 parametric amplification stages; COMP: compressor.

Near-IR OPA

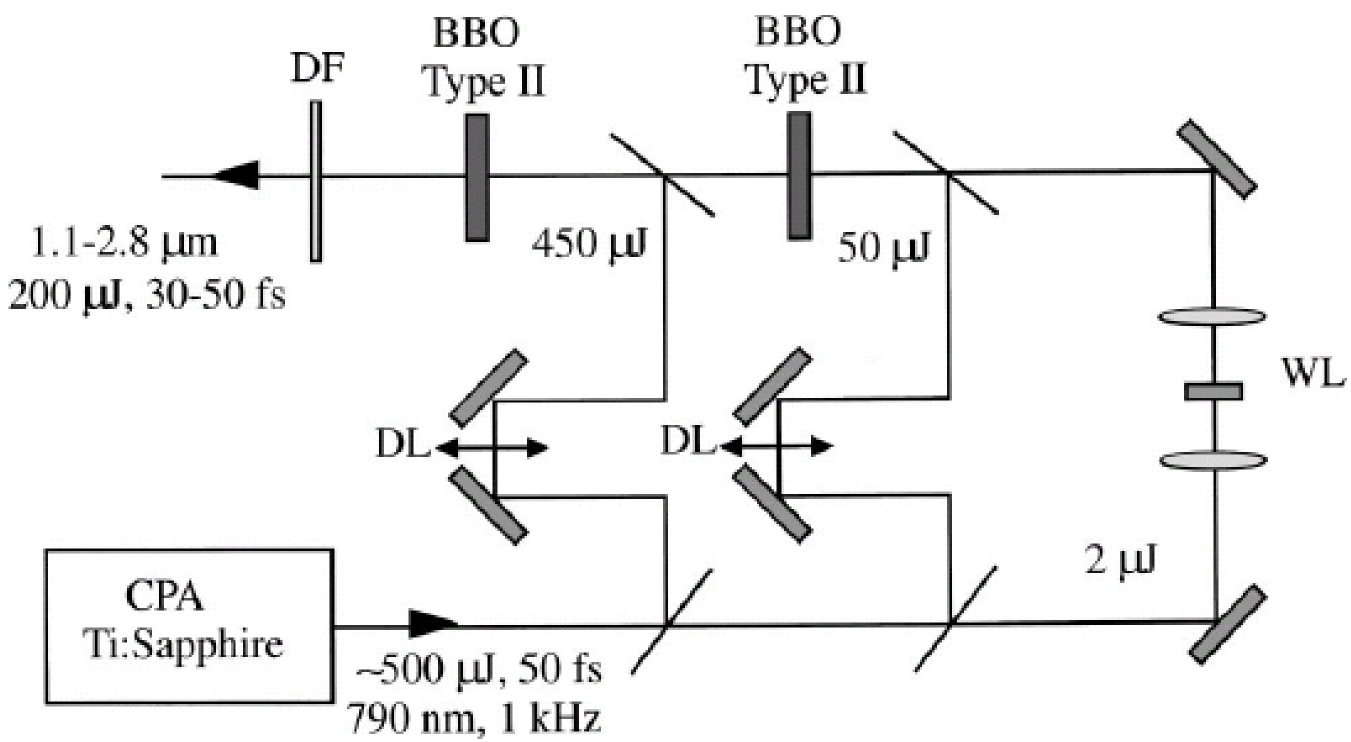


Fig. 9.16: Scheme of a near-IR OPA. DL: delay lines; WL: white light generation stage; DF: dichroic filter. [5]

9.8 Noncollinear Optical Parametric Amplifier (NOPA)

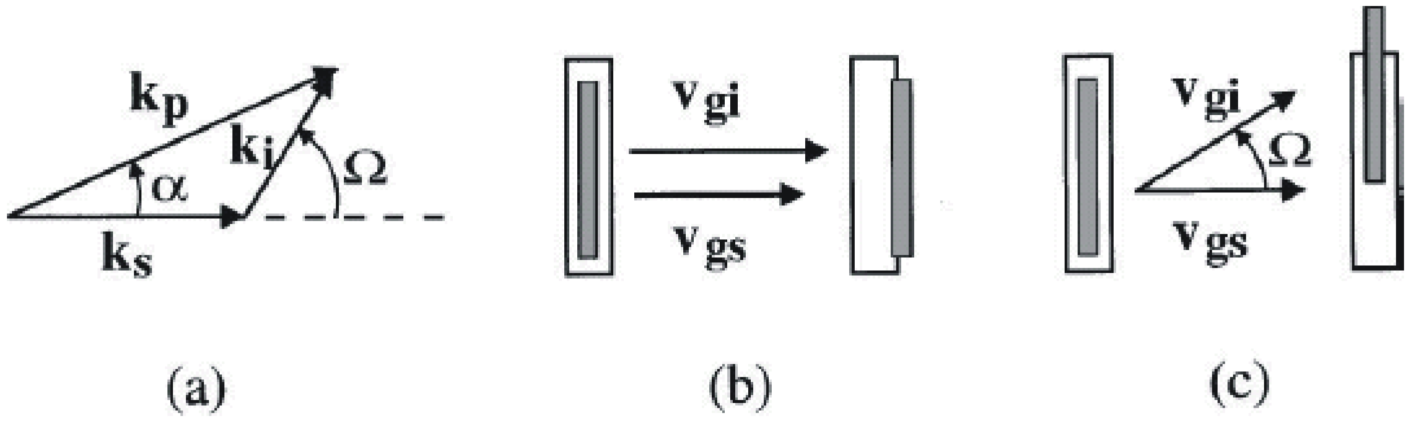


Fig. 9.17: a) Schematic of a noncollinear interaction geometry; b) representation of signal and idler pulses in the case of collinear interaction; and c) same as b) for noncollinear interaction.

Phase-matching condition: vector condition:

$$\begin{aligned} \Delta k_{par} &= k_p \cos\alpha - k_s - k_i \cos\Omega = 0 \\ \Delta k_{perp} &= k_p \sin\alpha - k_i \sin\Omega = 0 \end{aligned}$$

Variation on phase matching condition by $\Delta\omega$

$$\Delta k_{par} = -\frac{dk_s}{d\omega_s}\Delta\omega + \frac{dk_i}{d\omega_i}\cos\Omega\ \Delta\omega - k_i\sin\Omega\frac{d\Omega}{d\omega_i}\Delta\omega = 0 \qquad \mathbf{X}\quad \cos(\Omega)$$

$$\Delta k_{perp} = \frac{dk_i}{d\omega_i}\sin\Omega\ \Delta\omega + k_i\cos\Omega\frac{d\Omega}{d\omega_i}\Delta\omega = 0 \qquad \mathbf{X}\quad \sin(\Omega)$$

and addition

$$\frac{dk_i}{d\omega_i} - \cos\Omega\frac{dk_s}{d\omega_s} = 0$$

Correct index

$$\boxed{v_{gs} - v_{gi}\ \cos\Omega = 0}$$

Only possible if:

$$v_{gi} > v_{gs}$$

$$\alpha = \arcsin\left[\frac{1-\frac{v_s^2}{v_i^2}}{1+2v_s n_s\lambda_i/v_i n_i\lambda_s + (n_s\lambda_i/n_i\lambda_s)^2}\right]$$

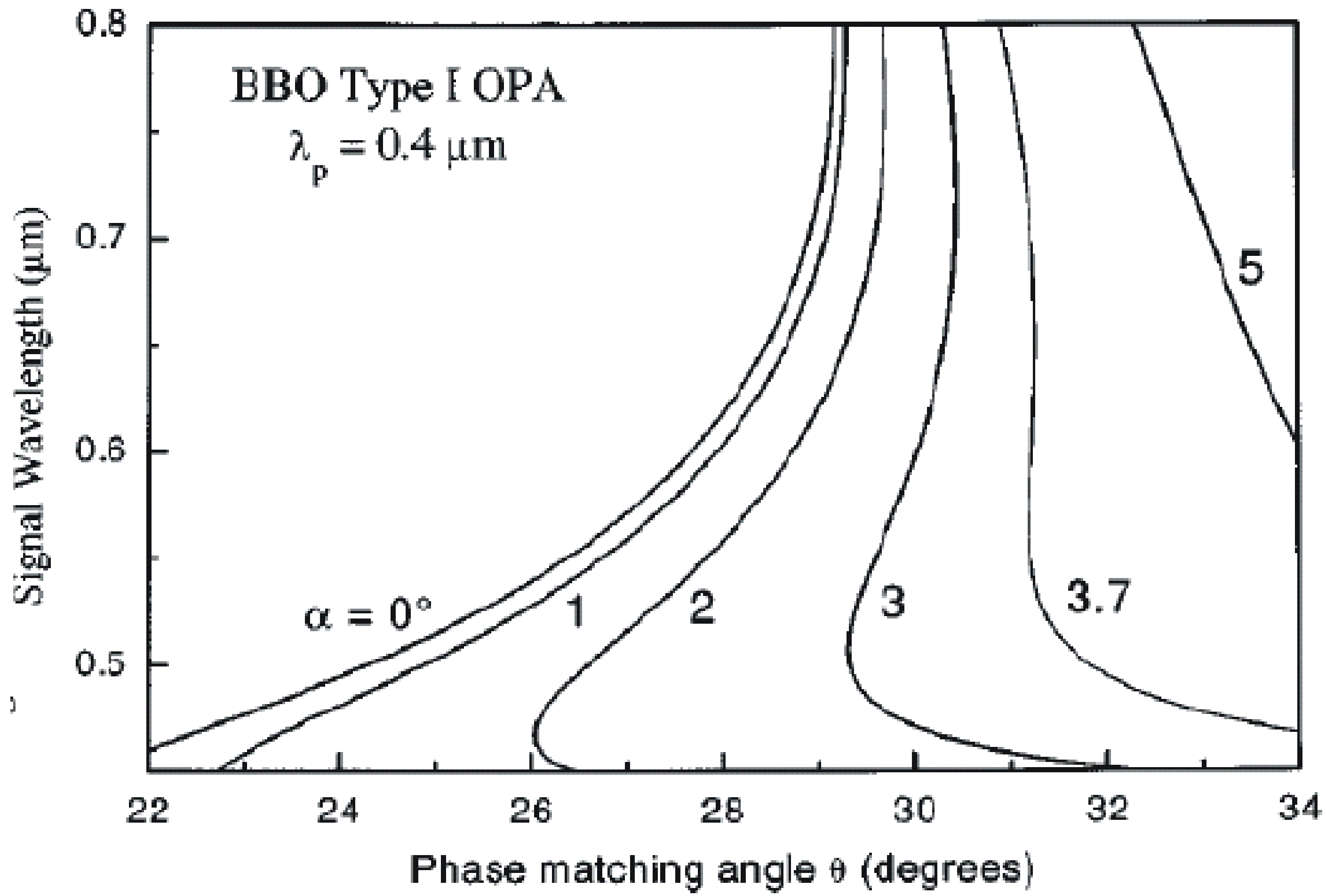
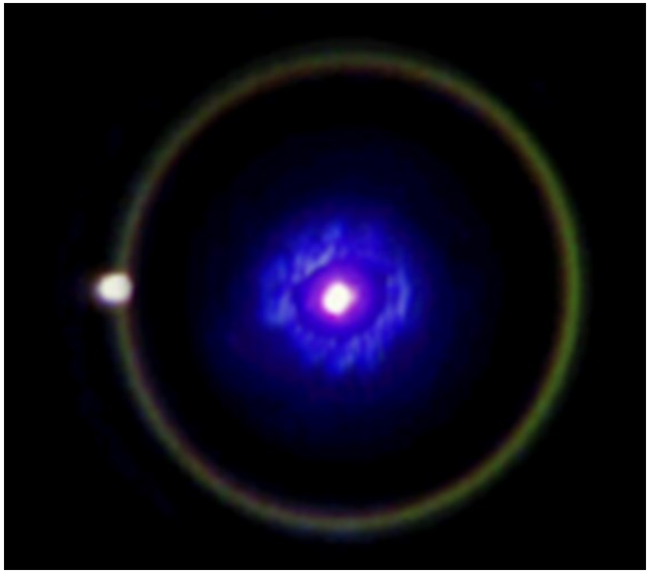


Fig. 9.18: Phase-matching curves for a noncollinear type-I BBO OPA pumped at λ_p=0.4 μm, as function of the pump-signal angle α. [5]

NOPA Layout



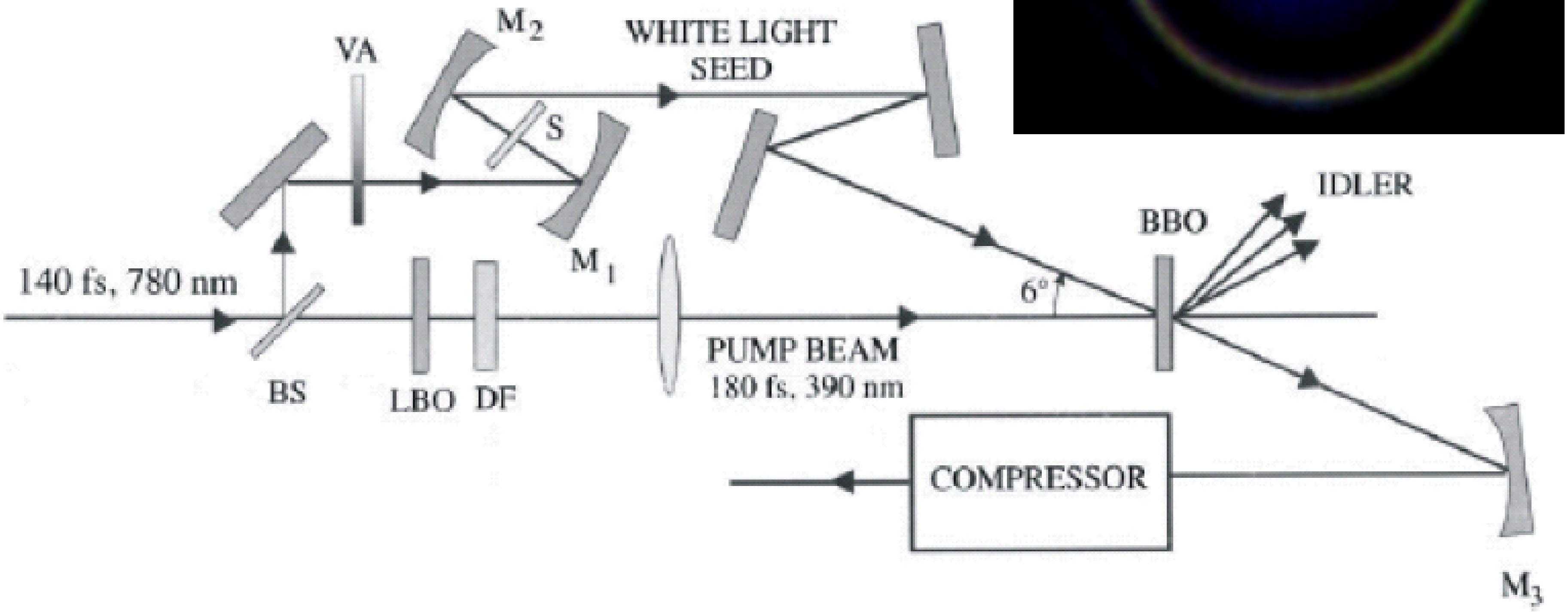


Fig. 9.19: Scheme of a noncollinear visible OPA. BS: beam splitter; VA: variable attenuator; S: 1-mm-thick sapphire plate; DF: dichroic filter; M1 ,M2 , M3 , spherical mirrors. [5]

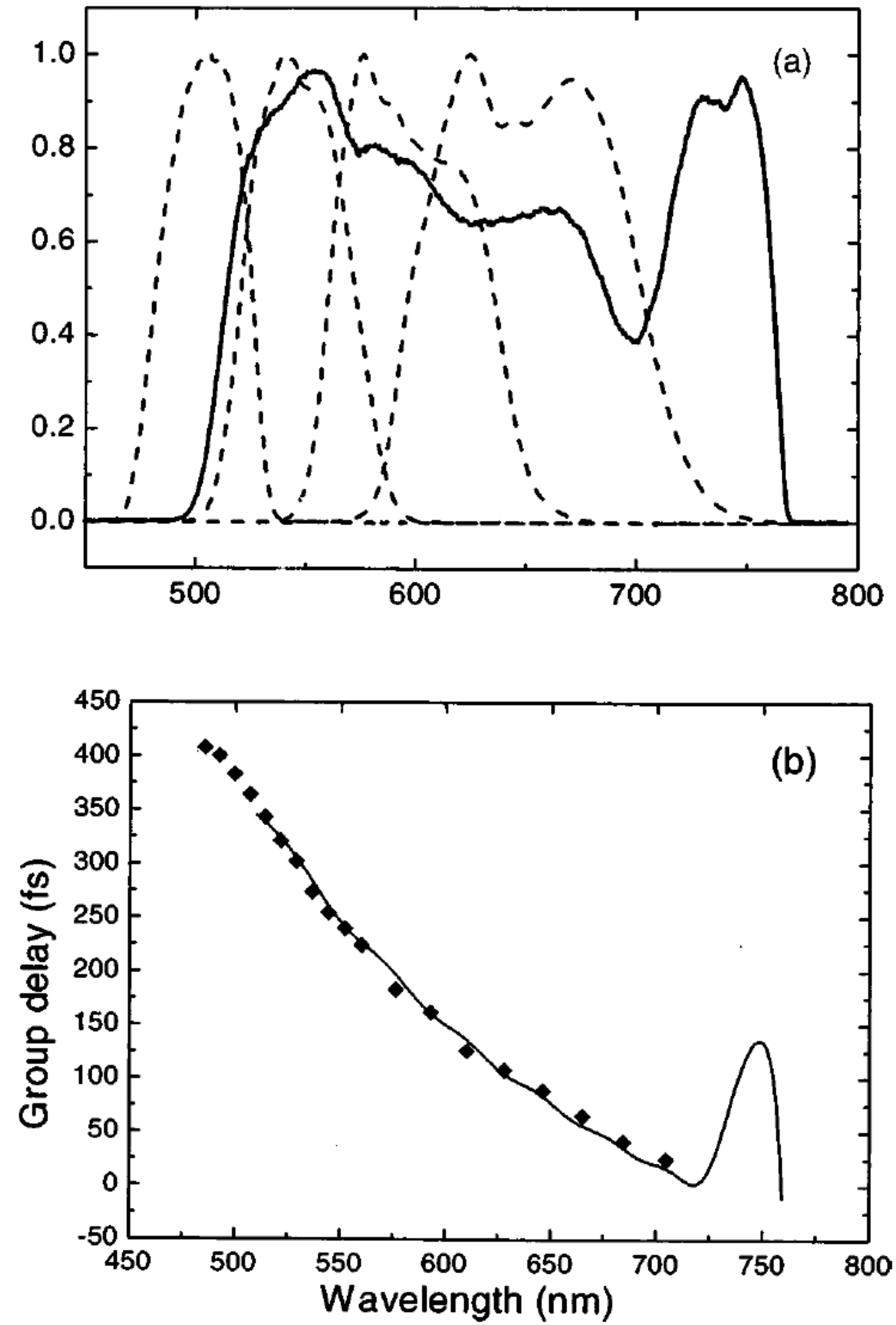


Fig. 9.20: a) Solid line: NOPA spectrum under optimum alignment conditions; dashed line: sequence of spectra obtained by increasing the white light chirp;
b) points: measured group delay (GD) of the NOPA pulses; dashed line: GD after ten bounces on the ultrabroadband chirped mirrors.

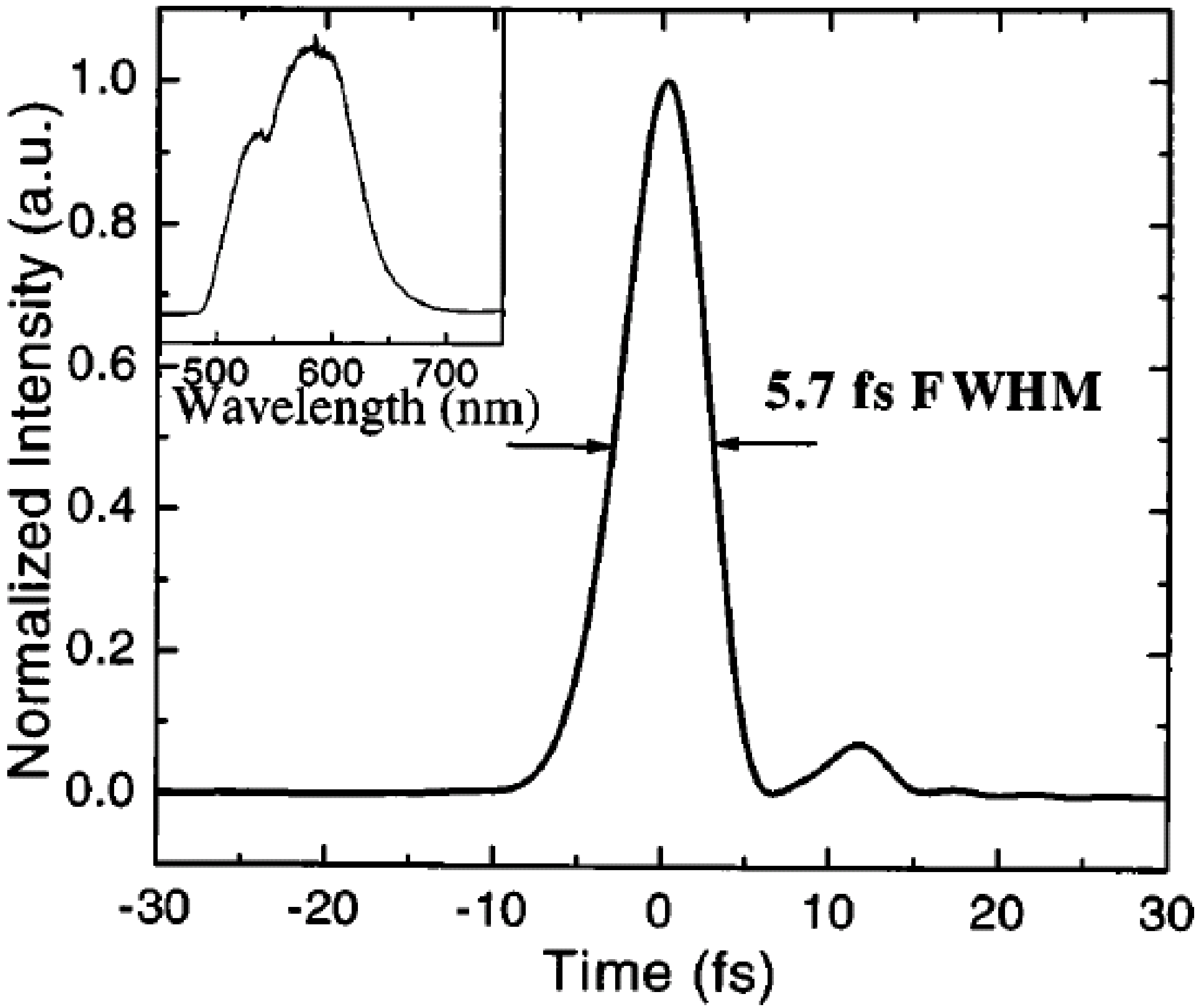


Fig. 9.21: Reconstructed temporal intensity of the compressed NOPA pulse measured by the SPIDER technique. The inset shows the corresponding pulse spectrum. [5]

9.9 Optical Parametric Chirped-Pulse Amplifier (OPCPA)

2-μm OPCPA