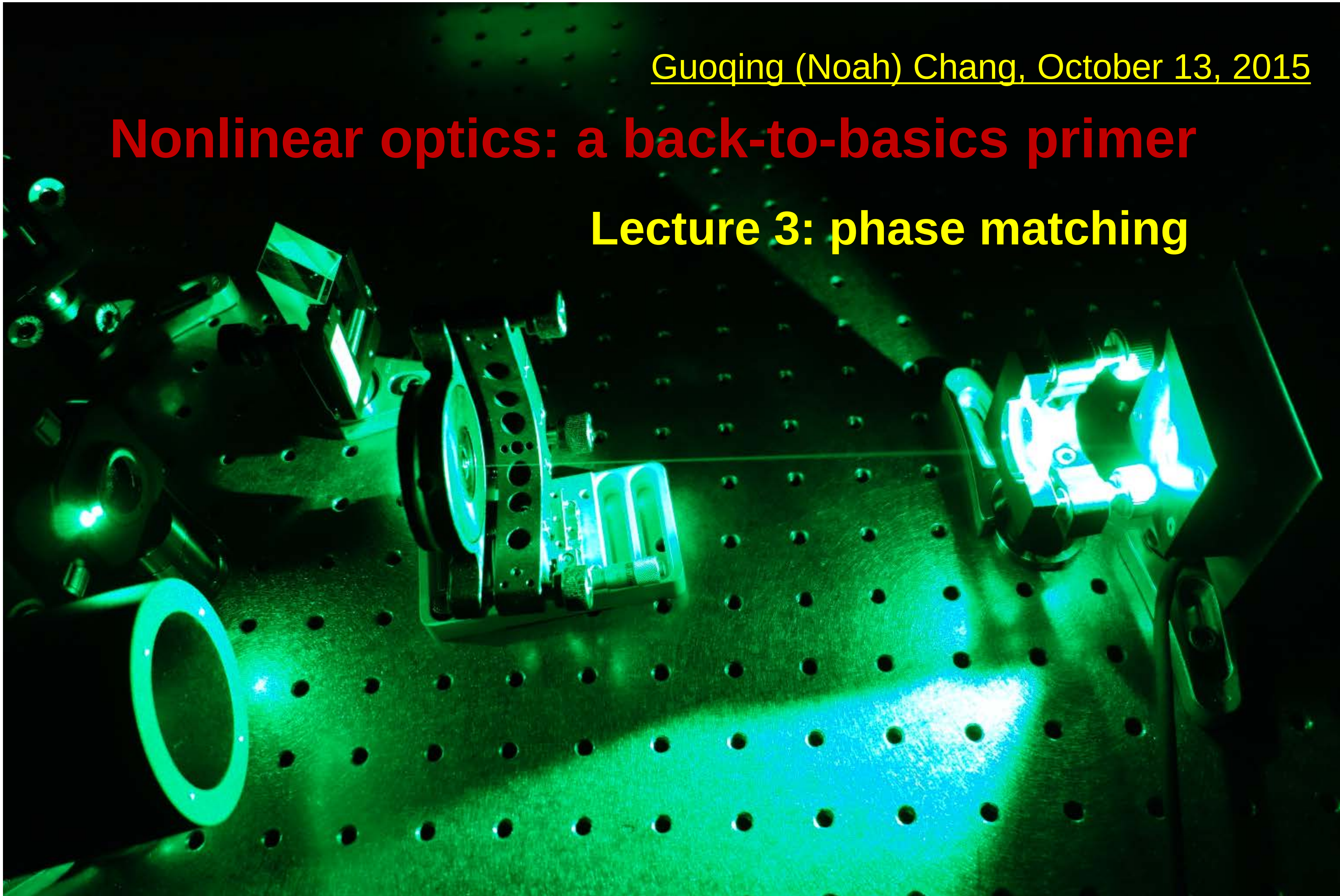
Guoqing (Noah) Chang, October 13, 2015

Nonlinear optics: a back-to-basics primer

Lecture 3: phase matching



Coupled wave equation

$\sqrt{-1} = i$ Physics notation

$\sqrt{-1} = j$ Engineering notation

$$(\nabla^2 - \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2})E = \mu_0 \frac{\partial^2 P}{\partial t^2} = \mu_0 (\frac{\partial^2 P_L}{\partial t^2} + \frac{\partial^2 P_{NL}}{\partial t^2})$$

$$\left\{ \begin{aligned} E(t) &= \sum_n \mathrm{E}(\omega_n) e^{j\omega_n t} + c.c. \\ P_L(t) &= \varepsilon_0 \sum_n \chi^{(1)}(\omega_n)\mathrm{E}(\omega_n) e^{j\omega_n t} + c.c. \\ P_{NL}(t) &= \sum_n P_{NL}(\omega_n) e^{j\omega_n t} + c.c. \end{aligned} \right.$$

$$(\nabla^2 + \frac{\omega_n^2 n^2(\omega_n)}{c_0^2})\mathrm{E}(\omega_n) = -\omega_n^2 \mu_0 P_{NL}(\omega_n) = -\frac{\omega_n^2}{\varepsilon_0 c_0^2} P_{NL}(\omega_n)$$

This is a set of coupled equations.

$$(\frac{d^2}{dz^2} + k_n^2)\mathrm{E}(\omega_n) = -\frac{\omega_n^2}{\varepsilon_0 c_0^2} P_{NL}(\omega_n) \qquad k_n^2 \equiv \frac{\omega_n^2 n^2(\omega_n)}{c_0^2}$$

Plane waves propagating in the +z direction

$$\frac{d^2 A_n}{dz^2} - 2jk_n \frac{dA_n}{dz} = -\frac{\omega_n^2}{\varepsilon_0 c_0^2} P_{NL}(\omega_n) e^{jk_n z} \qquad \mathrm{E}(\omega_n) = A_n(z) e^{-jk_n z}$$

Coupled wave equation

$$\frac{d^2A_n}{dz^2} - 2jk_n\frac{dA_n}{dz} = -\frac{\omega_n^2}{\varepsilon_0 c_0^2}P_{NL}(\omega_n)e^{jk_n z} \qquad \mathrm{E}(\omega_n) = A_n(z)e^{-jk_n z} \qquad \lambda_n = \frac{2\pi}{k_n}$$

Slowly varying amplitude approximation $\left|\frac{d^2A_n}{dz^2}\right| << \left|k_n\frac{dA_n}{dz}\right| \Leftrightarrow \left(\left|\frac{d^2A_n}{dz^2}\right|\lambda_n\right)/\left|\frac{dA_n}{dz}\right| << 2\pi$

$$\frac{dA_n}{dz} = -j\frac{\omega_n^2}{2\varepsilon_0 c_0^2 k_n}P_{NL}(\omega_n)e^{jk_n z} \qquad k_n^2 \equiv \frac{\omega_n^2 n^2(\omega_n)}{c_0^2}$$

$$\frac{dA_n}{dz} = -j\frac{\omega_n}{2\varepsilon_0 n(\omega_n)c_0}P_{NL}(\omega_n)e^{jk_n z}$$

Example: three wave mixing at three different frequencies, $\omega_1, \omega_2, \omega_3$ and $\omega_3 = \omega_1 + \omega_2$

$$P_{NL}(\omega_3) = 4\varepsilon_0 d_{eff}\mathrm{E}_1(\omega_1)\mathrm{E}_2(\omega_2) = 4\varepsilon_0 d_{eff}A_1A_2e^{-j(k_1+k_2)z} \rightarrow \frac{dA_3}{dz} = -j\frac{2\omega_3 d_{eff}}{n(\omega_3)c_0}A_1A_2e^{-j(k_1+k_2-k_3)z}$$

$$P_{NL}(\omega_1) = 4\varepsilon_0 d_{eff}\mathrm{E}_3(\omega_3)\mathrm{E}_2^*(-\omega_2) = 4\varepsilon_0 d_{eff}A_1A_2e^{-j(k_3-k_2)z} \rightarrow \frac{dA_1}{dz} = -j\frac{2\omega_1 d_{eff}}{n(\omega_1)c_0}A_3A_2^*e^{j(k_1+k_2-k_3)z}$$

$$P_{NL}(\omega_2) = 4\varepsilon_0 d_{eff}\mathrm{E}_3(\omega_3)\mathrm{E}_1^*(-\omega_1) = 4\varepsilon_0 d_{eff}A_1A_2e^{-j(k_3-k_1)z} \rightarrow \frac{dA_2}{dz} = -j\frac{2\omega_2 d_{eff}}{n(\omega_2)c_0}A_3A_1^*e^{j(k_1+k_2-k_3)z}$$

Monley-Rowe relations

Example: three wave mixing at three different frequencies, $\omega_1, \omega_2, \omega_3$ and $\omega_3 = \omega_1 + \omega_2$

If we define wave vector mismatch $\Delta k = k_1 + k_2 - k_3$, we can rewrite the coupled wave equations:

$$\frac{dA_3}{dz} = -j\frac{2\omega_3 d_{eff}}{n(\omega_3)c_0}A_1A_2e^{-j\Delta kz} \qquad \frac{dA_1}{dz} = -j\frac{2\omega_1 d_{eff}}{n(\omega_1)c_0}A_3A_2^*e^{j\Delta kz} \qquad \frac{dA_2}{dz} = -j\frac{2\omega_2 d_{eff}}{n(\omega_2)c_0}A_3A_1^*e^{j\Delta kz}$$

Intensity is a more convenient physical quantity, which is related to electric field as

$$I = 2n\varepsilon_0 c_0 |A|^2 = 2n\varepsilon_0 c_0 AA^*$$

Intensity variation is described as:

$$\frac{dI}{dz} = 2n\varepsilon_0 c_0 \left(A\frac{dA^*}{dz} + A^*\frac{dA}{dz}\right)$$

Using coupled wave equations, we can derive the following intensity variation equations:

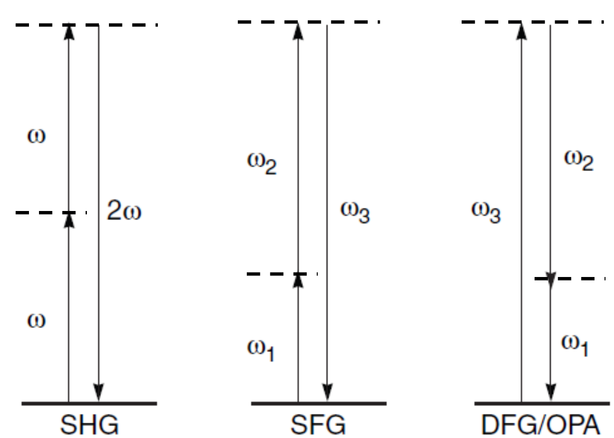
$$\frac{dI_1}{dz} = -8\varepsilon_0 d_{eff}\omega_1 \operatorname{Im}(A_3A_1^*A_2^*e^{j\Delta kz}) \qquad \frac{dI_2}{dz} = -8\varepsilon_0 d_{eff}\omega_2 \operatorname{Im}(A_3A_1^*A_2^*e^{j\Delta kz}) \qquad \frac{dI_3}{dz} = 8\varepsilon_0 d_{eff}\omega_3 \operatorname{Im}(A_3A_1^*A_2^*e^{j\Delta kz})$$

$$\frac{dI_{total}}{dz} = \frac{dI_1}{dz} + \frac{dI_2}{dz} + \frac{dI_3}{dz} = 0$$

Energy conservation in a lossless system

$$\frac{d}{dz}\left(\frac{I_1}{\omega_1}\right) = \frac{d}{dz}\left(\frac{I_2}{\omega_2}\right) = -\frac{d}{dz}\left(\frac{I_3}{\omega_3}\right)$$

The rate at which ω_1 photons are created = The rate at which ω_2 photons are created = The rate at which ω_3 photons are destroyed



Solution with no depletion

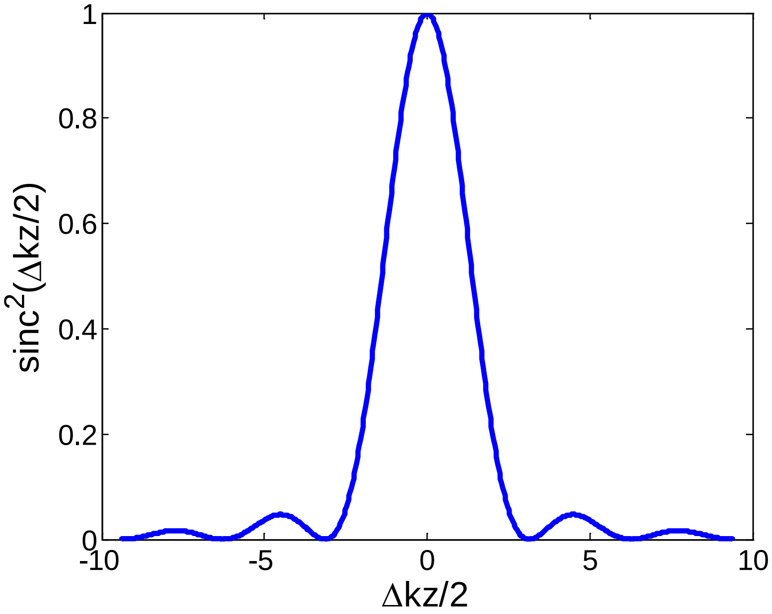
Example: three wave mixing at three different frequencies, $\omega_1, \omega_2, \omega_3$ and $\omega_3 = \omega_1 + \omega_2$

Let's consider the special case that ω_1 field and ω_2 field are not depleted; that is, A_1 and A_2 are constant.

$$\frac{dA_3}{dz} = -j\frac{2\omega_3 d_{eff}}{n(\omega_3)c_0} A_1 A_2 e^{-j\Delta kz}$$

$$A_3(z) = \frac{-2j\omega_3^2 d_{eff} A_1 A_2}{k_3 c_0^2}\left(\frac{1-e^{-j\Delta kz}}{j\Delta k}\right)$$

$$I_3(z) = \frac{8\omega_3^2 d_{eff}^2 I_1 I_2 z^2}{n(\omega_1)n(\omega_2)n(\omega_3)\varepsilon_0 c_0^2}\sin c^2\left(\frac{\Delta kz}{2}\right)$$



Under perfect phase matching condition: $\Delta k = k_1 + k_2 - k_3 = 0$

$$I_3(z) = \frac{8\omega_3^2 d_{eff}^2 I_1 I_2 z^2}{n(\omega_1)n(\omega_2)n(\omega_3)\varepsilon_0 c_0^2}$$

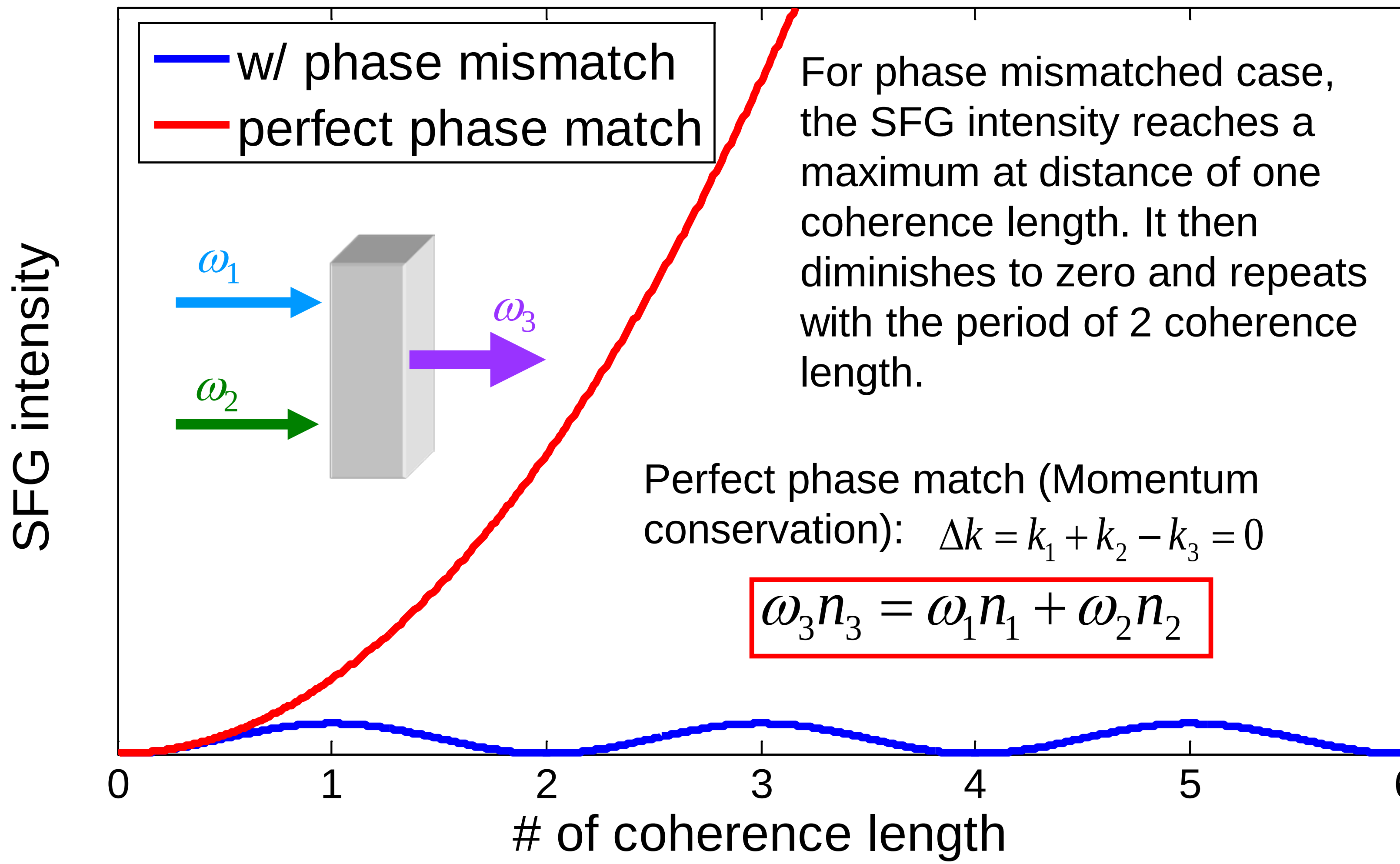
How to select a nonlinear crystal to maximize the nonlinear effect? We can define a figure of merit (FOM) to compare different crystals:

$$FOM = \frac{d_{eff}^2}{n(\omega_1)n(\omega_2)n(\omega_3)}$$

Phase matching

$$I_3(z) = \frac{8\omega_3^2 d_{eff}^2 I_1 I_2 z^2}{n(\omega_1)n(\omega_2)n(\omega_3)\varepsilon_0 c_0^2}\sin c^2\left(\frac{\Delta k z}{2}\right) \qquad L_{coh} \equiv \frac{\pi}{|\Delta k|}$$

Coherence length: the propagation distance at which the three waves accumulate a π phase difference.



A more intuitive picture: SHG

Z=0 z z+dz Z=L

$$E_1 \exp[j(\omega_1 t - k_1 z)]$$

Input (fundamental) field at z

$$d_{eff} E_1^2 \exp[j(2\omega_1 t - 2k_1 z)]$$

SH polarization of the medium at z

$$d_{eff} E_1^2 \exp[j(2\omega_1 t - 2k_1 z)]dz$$

SH field radiated by SH polarization within z and z+dz

SH field at Z=L radiated by SH polarization at z

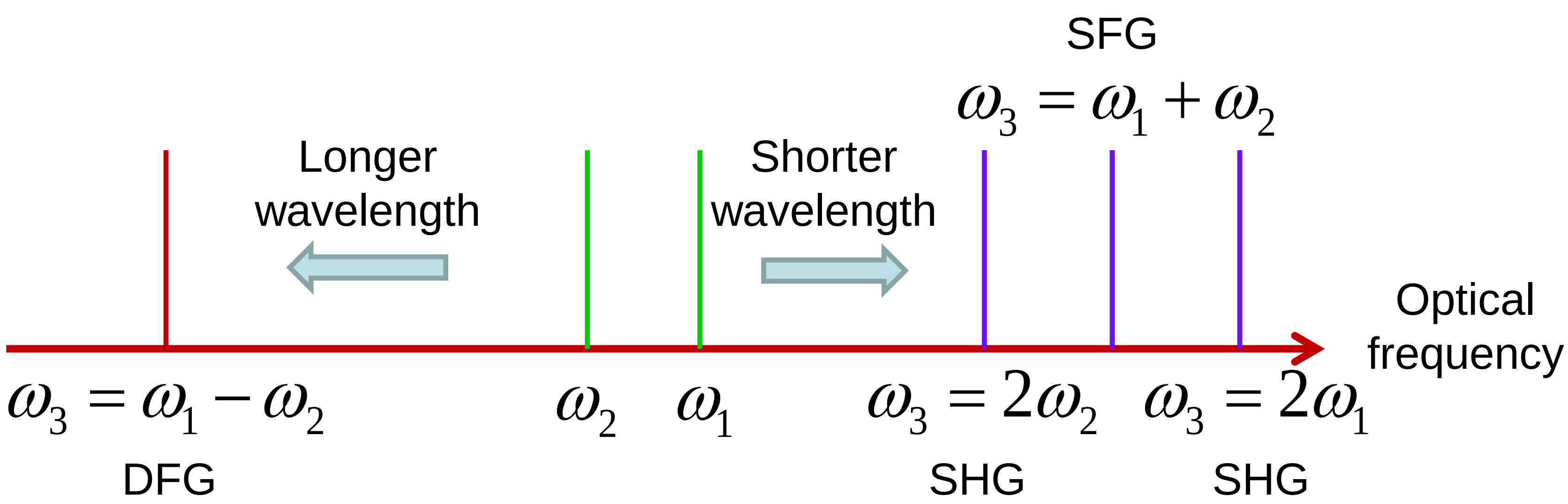
$$d_{eff} E_1^2 \exp[j(2\omega_1 t - 2k_1 z) - jk_2(l - z)]dz$$

$$d_{eff} E_1^2 \exp[j(2\omega_1 t - k_2 l) + \boxed{j(k_2 - 2k_1)z}]dz$$

$$\omega_2 = 2\omega_1 \qquad k_2 = \omega_2 c / n(\omega_2)$$

If the phase matching condition $\Delta k \equiv k_2 - 2k_1 = 0$ is satisfied, the SH field arriving at Z=L is independent on the position z from where the SH field originates. In other words all SH field contribution, from 0 to L, add in phase at z=L, leading to the highest SHG efficiency.

Wavelength conversion using the 2nd order nonlinear optics



	Energy conservation	Momentum conservation	Phase matching condition
SHG	$\omega_3 = 2\omega_1$	$k_3 = 2k_1$	$\frac{\omega_3}{c} n_3 = 2\frac{\omega_1}{c} n_1 \quad n_3 = n_1$
SFG	$\omega_3 = \omega_1 + \omega_2$	$k_3 = k_1 + k_2$	$\omega_3 n_3 = \omega_1 n_1 + \omega_2 n_2$
DFG	$\omega_3 = \omega_1 - \omega_2$	$k_3 = k_1 - k_2$	$\omega_3 n_3 = \omega_1 n_1 - \omega_2 n_2$

Sellmeier equation to model refractive index

If the frequency is far away from the absorption resonance $\left|\omega_0^2 - \omega^2\right| >> 2\omega\gamma$

$$\chi(\omega) = \frac{\omega_p^2}{(\omega_0^2 - \omega^2)} \qquad \omega_p^2 = Ne^2/(m\varepsilon_0)$$

Normally there are multiple resonant frequencies for the electronic oscillators. It means in general the refractive index will have the form

$$\boxed{n^2(\omega) = 1 + \chi(\omega) = 1 + \sum_i A_i \frac{\omega_i^2}{\omega_0^2 - \omega^2} = 1 + \sum_i a_i \frac{\lambda^2}{\lambda^2 - \lambda_i^2}}$$

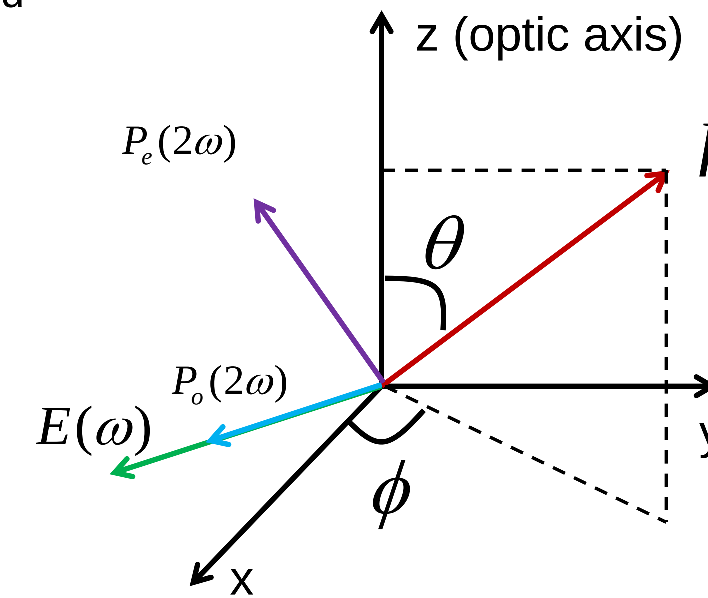
For the frequency (wavelength) far away from absorption resonance, refractive index increases with increasing frequency, which leads to

$$\omega_3 n(\omega_3) = (\omega_1 + \omega_2) n(\omega_3) > \omega_1 n(\omega_1) + \omega_2 n(\omega_2)$$

Therefore, dispersion prevents phase matching in an isotropic medium. How about an anisotropic medium?

Example: SHG of o wave in BBO

A bit more general: the k vector in the arbitrary direction, and the electrical field is in the xy plane; that is, we consider an ordinary wave: $E_x(\omega) = E(\omega)\sin\phi \qquad E_y(\omega) = -E(\omega)\cos\phi$



$$\begin{bmatrix} P_x(2\omega) \\ P_y(2\omega) \\ P_z(2\omega) \end{bmatrix} = 2\varepsilon_0 \begin{bmatrix} 0 & 0 & 0 & 0 & d_{15} & d_{16} \\ d_{16} & -d_{16} & 0 & d_{15} & 0 & 0 \\ d_{31} & d_{31} & d_{33} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} E^2(\omega)\sin^2\phi \\ E^2(\omega)\cos^2\phi \\ 0 \\ 0 \\ 0 \\ -E^2(\omega)\sin 2\phi \end{bmatrix}$$

$$|d_{31}(1.064um)| = 0.04\, pm/V$$

$$|d_{16}(1.064um)| = 2.2\, pm/V$$

$$\boxed{\begin{aligned} P_x(2\omega) &= -2\varepsilon_0 d_{16} E^2(\omega)\sin 2\phi \\ P_y(2\omega) &= -2\varepsilon_0 d_{16} E^2(\omega)\cos 2\phi \\ P_z(2\omega) &= 2\varepsilon_0 d_{31} E^2(\omega) \end{aligned}}$$

E-field direction of o wave:	$(\sin\phi, -\cos\phi, 0)$
E-field direction of e wave:	$(-\cos\theta\cos\phi, -\cos\theta\sin\phi, \sin\theta)$

We can project the polarization onto the direction of o wave and e wave which are normal to k:

$$P_o(2\omega) = -2\varepsilon_0 d_{16} E^2(\omega)\cos 3\phi$$

Case 1: $\phi = 0, \theta = 0$ Fundamental o wave generates SH o wave, demanding phase matching condition of

$$n_o(2\omega) = n_o(\omega)$$

$$P_e(2\omega) = 2\varepsilon_0 (d_{31}\sin\theta + d_{16}\cos\theta\sin 3\phi) E^2(\omega)$$

Case 2: $\phi = 90^\circ, \theta = 0$ Fundamental o wave generates SH e wave, demanding phase matching condition of

$$n_e(2\omega, \theta = 0) = n_o(\omega)$$

Example: SHG of o wave in BBO

We can project the polarization onto the direction of o wave and e wave which are normal to k:

$$P_o(2\omega) = -2\varepsilon_0 d_{16} E^2(\omega)\cos 3\phi$$

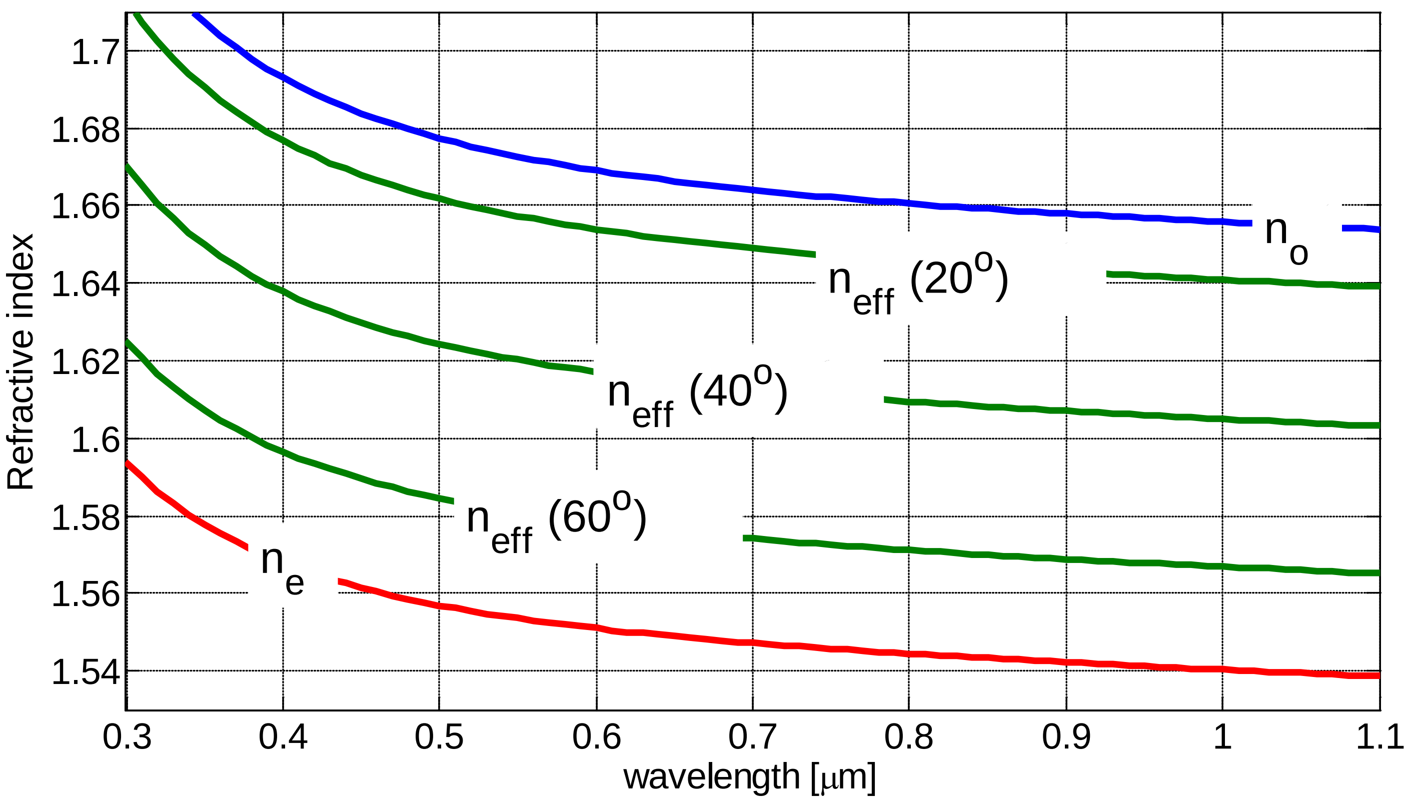
Case 1: $\phi = 0, \theta = 0$ Fundamental o wave generates SH o wave (o→o), demanding phase matching condition of

$$n_o(2\omega) = n_o(\omega)$$

$$P_e(2\omega) = 2\varepsilon_0 (d_{31}\sin\theta + d_{16}\cos\theta\sin 3\phi) E^2(\omega)$$

Case 2: $\phi = 90^\circ, \theta = 0$ Fundamental o wave generates SH e wave(o→e), demanding phase matching condition of

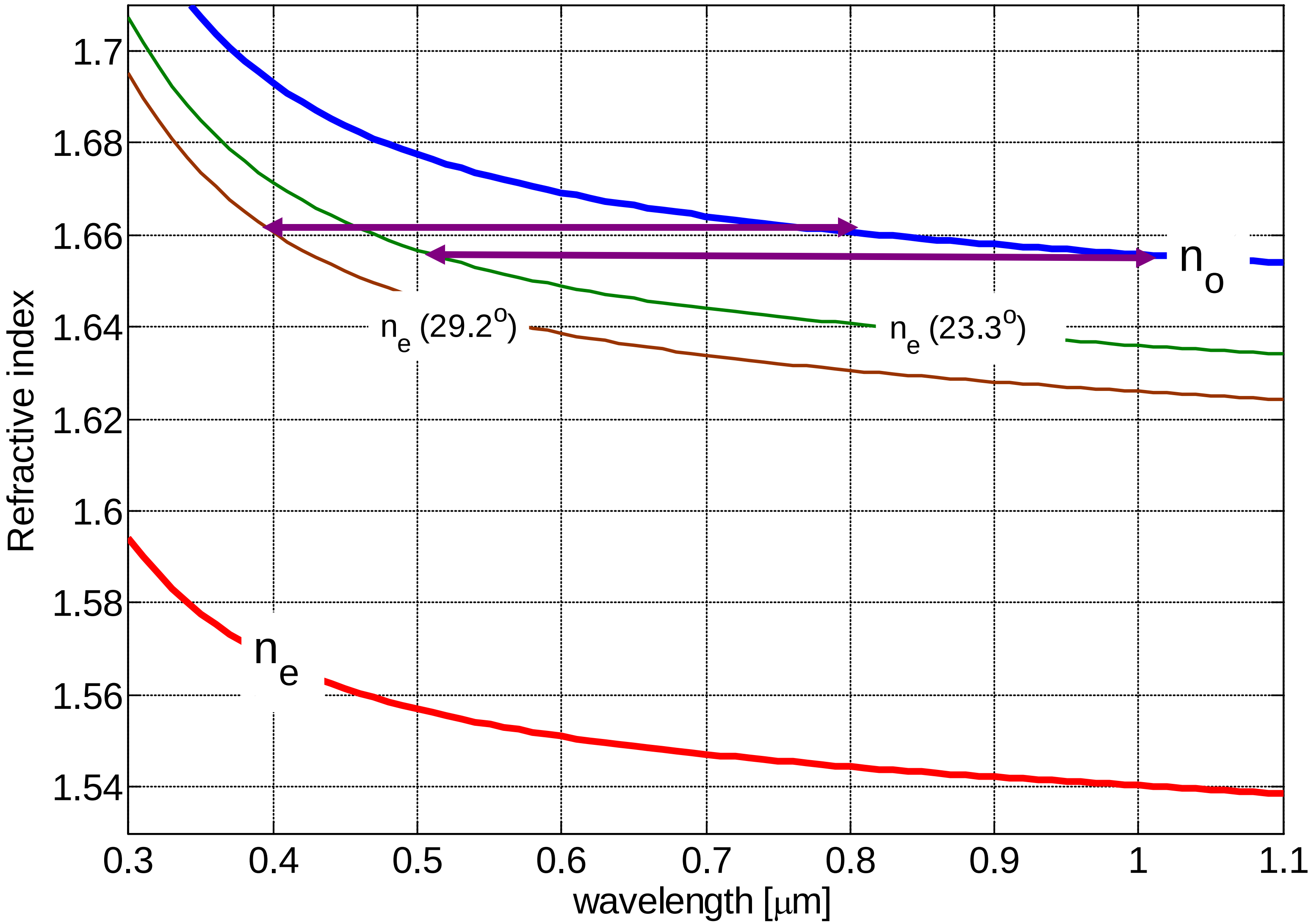
$$n_e(2\omega, \theta = 0) = n_o(\omega)$$



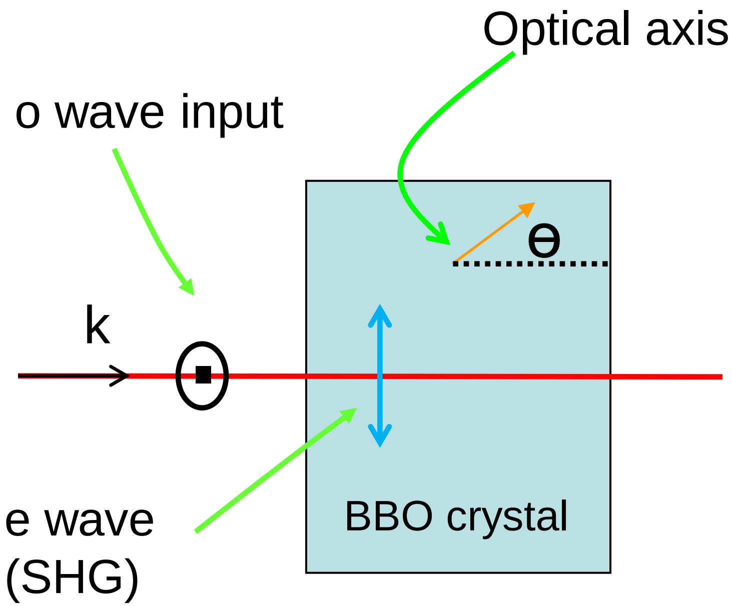
Phase matching for o→e SHG in BBO

$$n_e(0.515um, \theta = 23.3^{\circ}) = n_o(1.03um)$$

$$n_e(0.4um, \theta = 29.2^{\circ}) = n_o(0.8um)$$



Angle tuning for phase-matching



$$\begin{cases} n_o(\theta,\omega) = n_o(\omega) \\ \dfrac{1}{n_e^{\,2}(\theta,\omega)} = \dfrac{\cos^2(\theta)}{n_o^{\,2}(\omega)} + \dfrac{\sin^2(\theta)}{n_e^{\,2}(\omega)} \end{cases}$$

Two special cases:

$$n_e(0^0,\omega) = n_o(\omega)$$

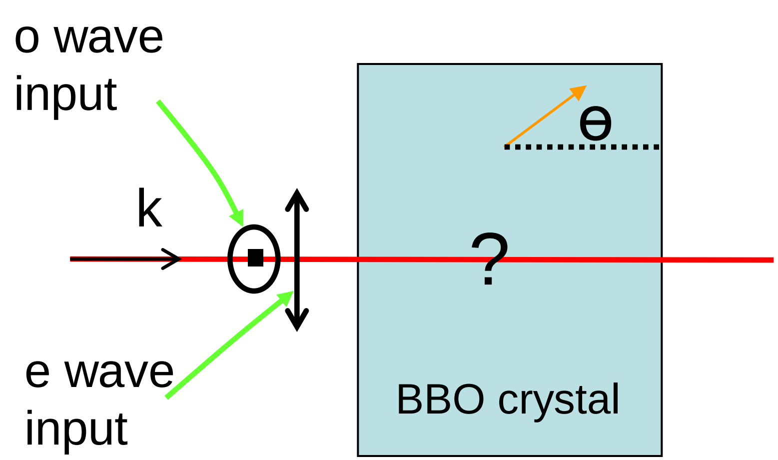
$$n_e(90^0,\omega) = n_e(\omega)$$

For second harmonic generation, the wave with lower frequency is called fundamental wave. From photon picture, two o wave photons generate one e wave SHG photon. So we note this type of SHG process as $o + o \rightarrow e$

How about other possible combinations?

$$e + e \rightarrow o \quad o + e \rightarrow o \quad o + e \rightarrow e$$

Can they satisfy phase matching condition in BBO?



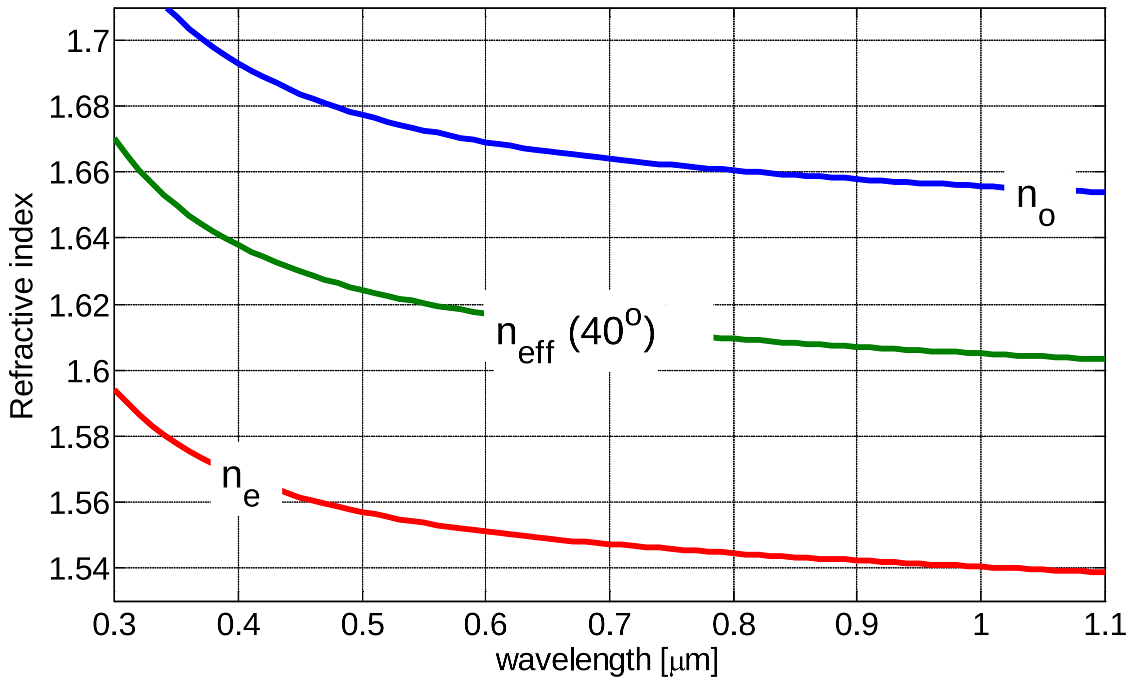
Example: SHG phase matching in BBO

Phase matching condition:

$$e+e\rightarrow o \quad k_e(\omega)+k_e(\omega)-k_o(2\omega)=0 \Leftrightarrow \frac{\omega c_0}{n_e(\omega)}+\frac{\omega c_0}{n_e(\omega,\theta)}-\frac{2\omega c_0}{n_o(2\omega)}=0 \Leftrightarrow \boxed{n_e(\omega,\theta)=n_o(2\omega)}$$

$$o+e\rightarrow o \quad k_o(\omega)+k_e(\omega)-k_o(2\omega)=0 \Leftrightarrow \frac{\omega c_0}{n_o(\omega)}+\frac{\omega c_0}{n_e(\omega,\theta)}-\frac{2\omega c_0}{n_o(2\omega)}=0 \Leftrightarrow \boxed{\frac{1}{n_o(\omega)}+\frac{1}{n_e(\omega,\theta)}=\frac{2}{n_o(2\omega)}}$$

$$o+e\rightarrow e \quad k_o(\omega)+k_e(\omega)-k_e(2\omega)=0 \Leftrightarrow \frac{\omega c_0}{n_o(\omega)}+\frac{\omega c_0}{n_e(\omega,\theta)}-\frac{2\omega c_0}{n_e(2\omega,\theta)}=0 \Leftrightarrow \boxed{\frac{1}{n_o(\omega)}+\frac{1}{n_e(\omega,\theta)}=\frac{2}{n_e(2\omega,\theta)}}$$



BBO is negative uniaxial crystal.

$$n_e(\omega,\theta)<n_o(2\omega) \qquad \frac{1}{n_o(\omega)}+\frac{1}{n_e(\omega,\theta)}<\frac{2}{n_o(2\omega)}$$

Therefore $e+e\rightarrow o$ $o+e\rightarrow o$ are not allowed. Only $o+o\rightarrow e$ and $o+e\rightarrow e$ can take place.

For positive uniaxial crystal,

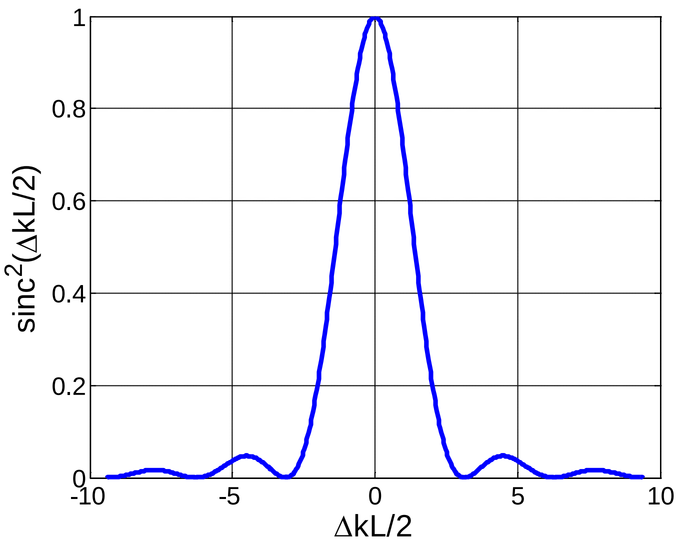
$e+e\rightarrow o$ $o+e\rightarrow o$ are allowed

$o+o\rightarrow e$ $o+e\rightarrow e$ are forbidden.

Acceptance angle (angular phase-matching bandwidth)

Phase matching using birefringence requires to align the input optical beam at some angle with respect to the crystal's optical axis. How accurately the angle should be? Let θ_{pm} be the phase matching angle.

$\Delta k(\theta_{pm}) = 0$ $\quad \mathrm{sinc}^2(\frac{\Delta k L}{2}) = 1$ $\quad$ *L* is the crystal length.



We define acceptance angle $\Delta\theta_{pm}$ be the phase matching angle such that:

$\Delta k(\theta_{pm} + \Delta\theta_{pm})L = \pi$ $\quad \mathrm{sinc}^2(\frac{\pi}{2}) \approx 0.4$

The SHG power drops by 60% from its peak value achieved at phase matching angle.

For type I phase matching:

$$\Delta k(\theta_{pm} + \Delta\theta_{pm}) = \frac{2\omega}{c_0}[n_o(\omega) - n_e(2\omega, \theta_{pm} + \Delta\theta_{pm})] = \frac{2\omega}{c_0}[n_o(\omega) - n_e(2\omega, \theta_{pm} + \Delta\theta_{pm}) - \left.\frac{\partial n_e(2\omega, \theta)}{\partial\theta}\right|_{\theta_{pm}} \Delta\theta_{pm}]$$

After some mathematics, we have

$$\Delta\theta_{pm} \approx \frac{\omega}{4Lc_0}\frac{1}{[n_e(2\omega) - n_o(2\omega)]\sin 2\theta_{pm}} = \frac{\lambda_0}{4L}\frac{1}{[n_e(2\omega) - n_o(2\omega)]\sin 2\theta_{pm}}$$

Phase matching: critical Vs noncritical

$$\Delta\theta_{pm} \approx \frac{\omega}{4Lc_0}\frac{1}{[n_e(2\omega)-n_o(2\omega)]\sin 2\theta_{pm}} = \frac{\lambda_0}{4L}\frac{1}{[n_e(2\omega)-n_o(2\omega)]\sin 2\theta_{pm}} \qquad (1)$$

SHG from 800 nm light using 1 mm BBO crystal: $\lambda_0 = 800nm \quad \theta_{pm} = 29.2^\circ \quad n_o(400nm) = 1.6903 \quad n_e(400nm) = 1.5679$

$$\Rightarrow \Delta\theta_{pm} = 3.28mrad \approx 0.2^\circ$$

There is a trade-off between the acceptance angle and crystal length. For a fixed crystal length, acceptance angle limits how tight we can focus the incident beam onto the crystal—smaller focal spot leads to larger divergence angle.

One specific case: $\theta_{pm} = 90^\circ$, Eq. (1) diverges, and we need to use higher order term in the Taylor expansion.

After some mathematics, we have $\Delta\theta_{pm} \approx \{\frac{\lambda_0}{4L}\frac{1}{|n_e(2\omega)-n_o(2\omega)|}\}^{1/2}$ In this case, the acceptance angle is normally one order of magnitude larger. Phase matching might be achieved by temperature tuning.

For phase matching with $\theta_{pm} \neq 90^\circ$, acceptance angle (or angular phase-matching bandwidth) is smaller. We call this type of birefringence-enabled phase matching as critical phase matching.

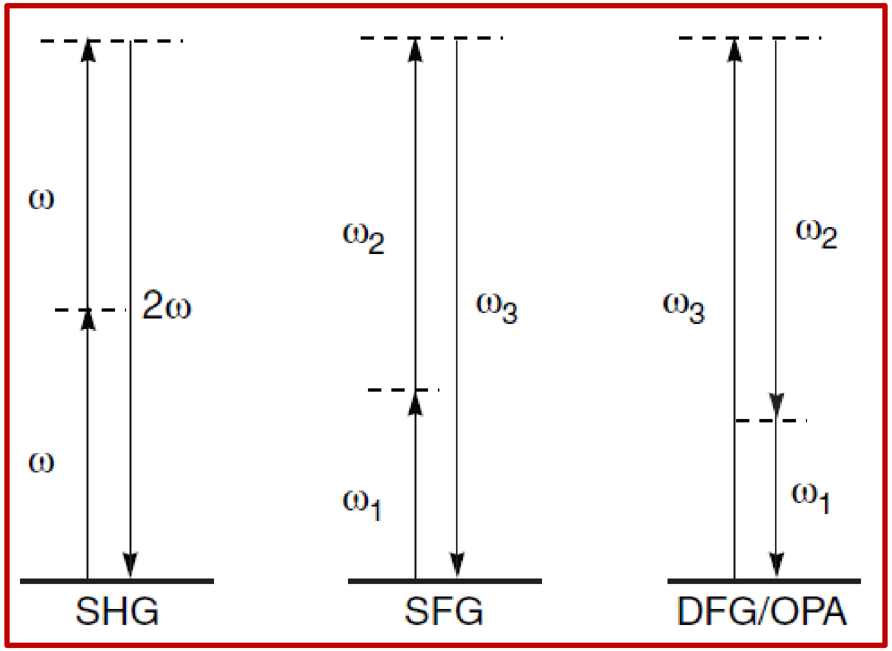
In contrast, phase matching with $\theta_{pm} = 90^\circ$, acceptance angle (or angular phase-matching bandwidth) is larger. We call this type of birefringence-enabled phase matching as noncritical phase matching.

Phase matching: type I Vs. type II

In general, second-order nonlinear effects involve three waves with frequencies linked by the equation

$$\omega_1 + \omega_2 = \omega_3$$

Here ω_3 is the highest frequency of the three.



<u>Type I phase matching:</u>

ω_1 wave and ω_2 wave have the same polarization; that is, they are both ordinary waves or extraordinary waves:

$$o + o \rightarrow e \quad \text{or} \quad e + e \rightarrow o$$

<u>Type II phase matching:</u>

ω_1 wave and ω_2 wave have different polarization:

$$o + e \rightarrow e \qquad o + e \rightarrow o$$

$$e + o \rightarrow o \qquad e + o \rightarrow e$$

TABLE 2.3.2 Phase-matching methods for uniaxial crystals

	Positive uniaxial ($n_e > n_0$)	Negative uniaxial ($n_e < n_0$)
Type I	$n_3^o\omega_3 = n_1^e\omega_1 + n_2^e\omega_2$	$n_3^e\omega_3 = n_1^o\omega_1 + n_2^o\omega_2$
Type II	$n_3^o\omega_3 = n_1^o\omega_1 + n_2^e\omega_2$	$n_3^e\omega_3 = n_1^e\omega_1 + n_2^o\omega_2$

Robert Boyd, <u>*Nonlinear optics*</u>, chapter 2

Type 0 phase matching to maximize nonlinearity

$$d_{np} = \begin{bmatrix} 0 & 0 & 0 & 0 & d_{15} & d_{16} \\ d_{16} & -d_{16} & 0 & d_{15} & 0 & 0 \\ d_{31} & d_{31} & d_{33} & 0 & 0 & 0 \end{bmatrix}$$

For BBO crystal:

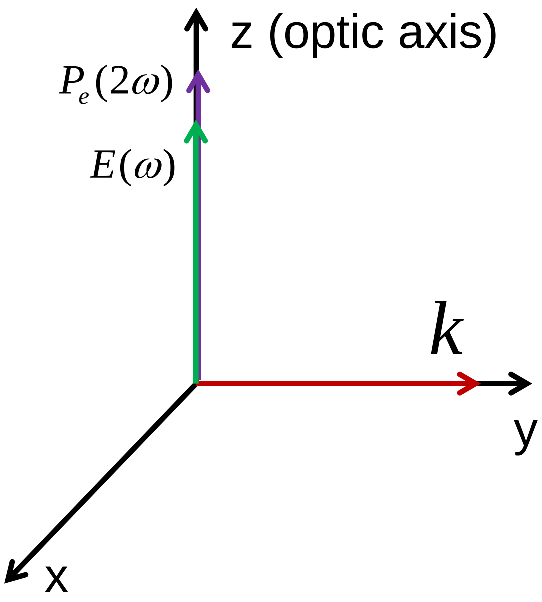
$$\boxed{d_{16}(1.064um) = 2.2\,pm/V} \quad d_{15}(1.064um) = 0.03\,pm/V$$

$$d_{31}(1.064um) = 0.04\,pm/V \quad d_{33}(1.064um) = 0.04\,pm/V$$

$LiNbO_3$ belongs to the same group:

$$d_{16}(1.064um) = -2.1\,pm/V \quad d_{15}(1.064um) = -4.3\,pm/V$$

$$d_{31}(1.064um) = -4.3\,pm/V \quad \boxed{d_{33}(1.064um) = -27.2\,pm/V}$$



To use d_{33} to maximize the nonlinearity, we need to align the E-field along z. Take SHG as an example:

$$P_x(2\omega) = 0 \quad P_y(2\omega) = 0 \quad P_z(2\omega) = 2\varepsilon_0 d_{33} E^2(\omega)$$

The generated SHG wave is e wave, so we end up with

$$e + e \rightarrow e$$

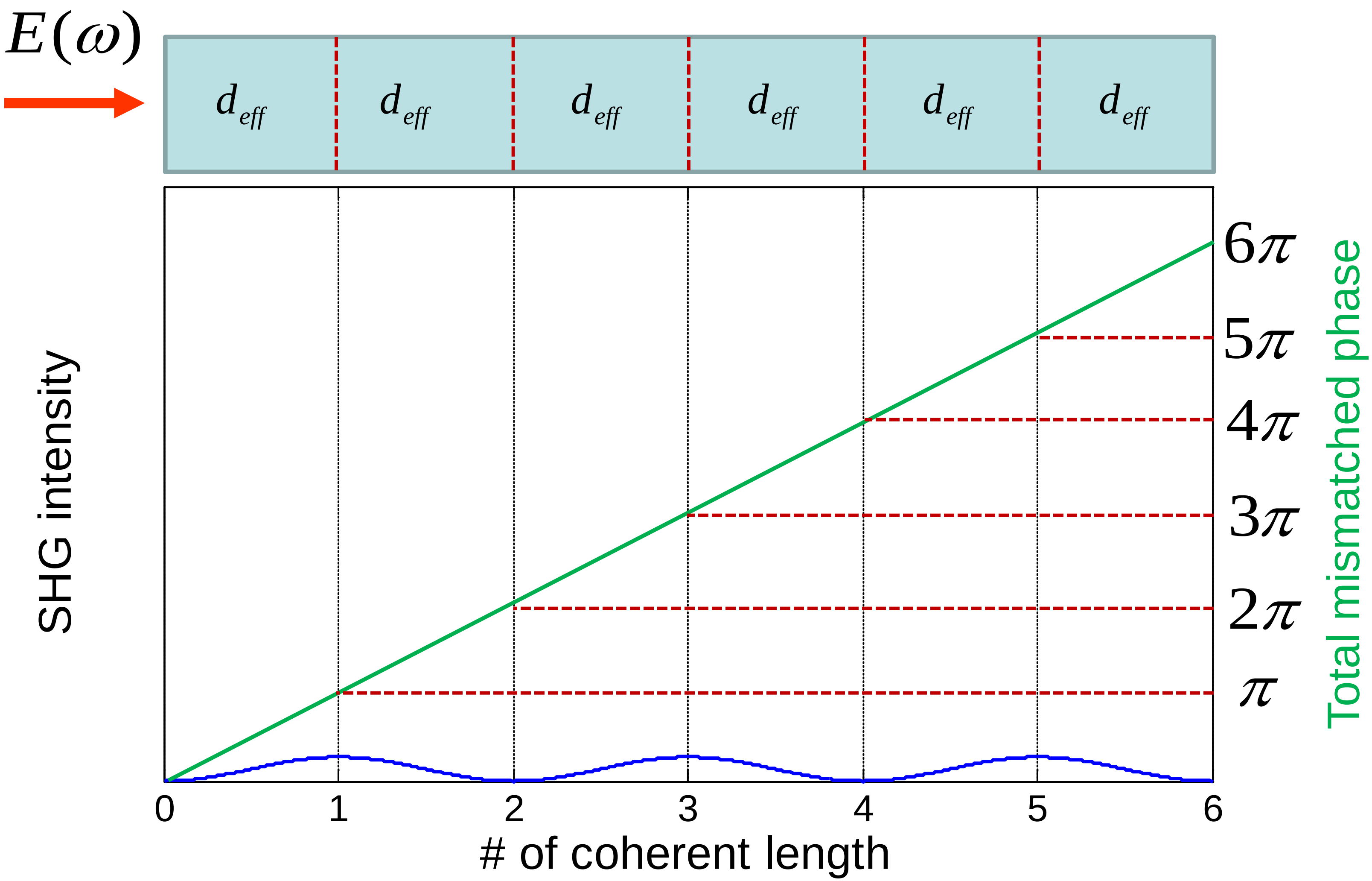
When all waves are in the same polarization, we can it Type 0 phase matching.

Perfect phase matching $\Delta k = 0$ is impossible.

Quasi phase matching (QPM) by periodic arrangement of nonlinearity

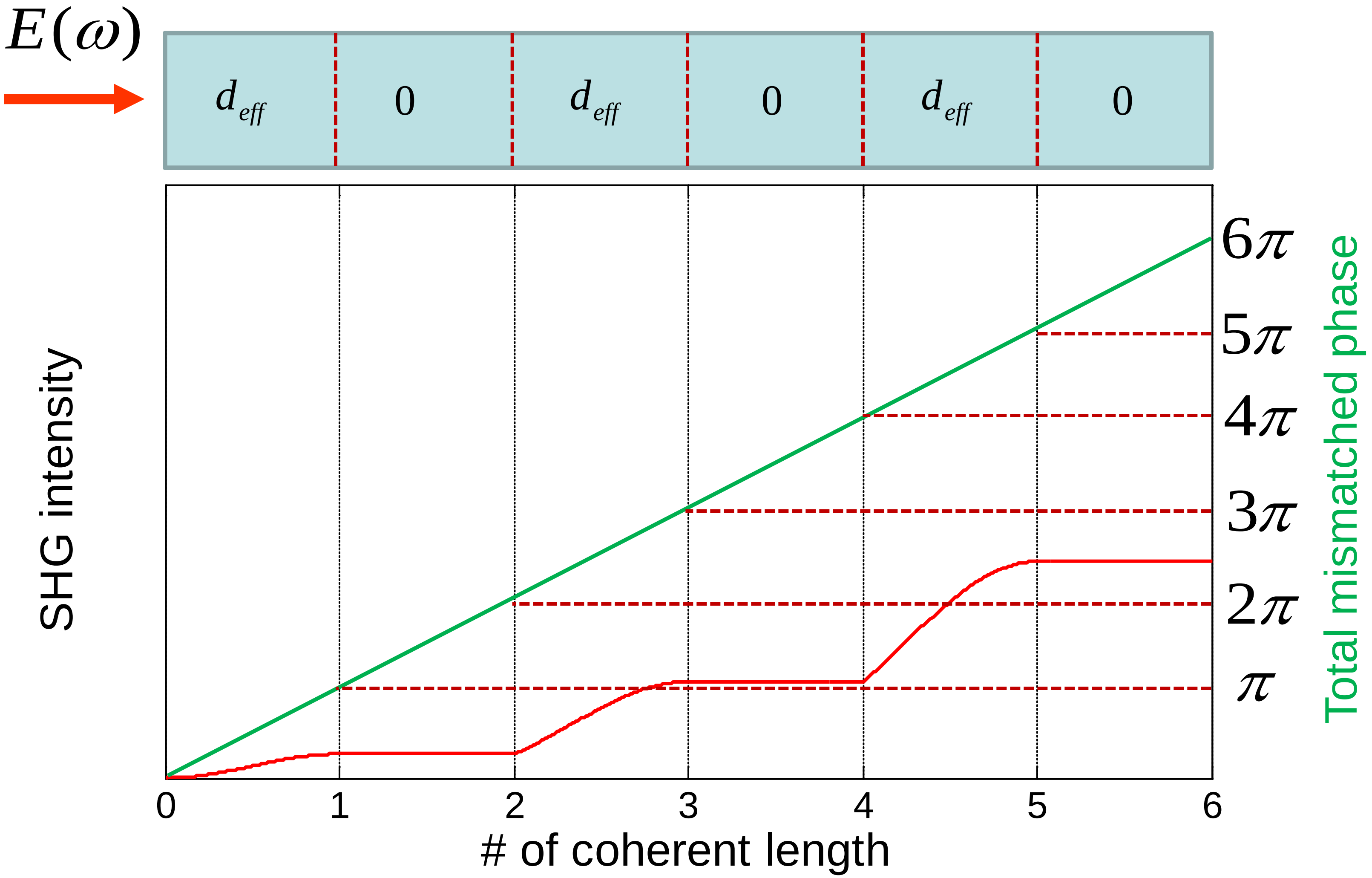
$$\frac{dA_3}{dz} = -j\frac{2\omega d_{eff}}{n(2\omega)c_0}A_1^2 e^{-j\Delta kz} \quad \text{(SHG)}$$

When the mismatched phase accumulates to π, 3π, 5π …, energy starts to flow back from SH to the fundamental.



Quasi phase matching (QPM) by periodic arrangement of nonlinearity

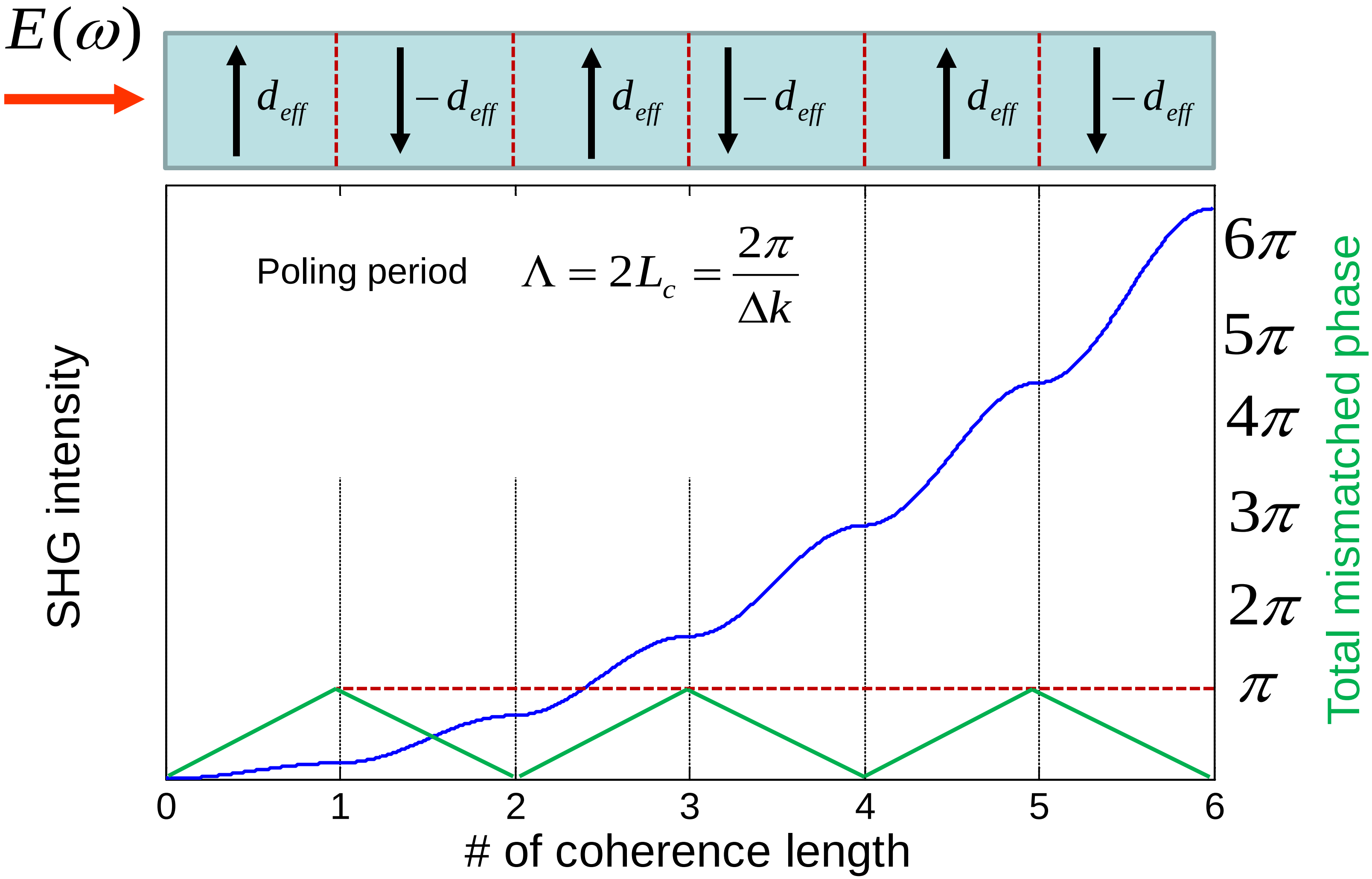
When the mismatched phase accumulates to π, 3π, 5π..., energy starts to flow back from SH to the fundamental. <u>How about we shut down nonlinearity to avoid this back conversion?</u>



QPM by periodically poled $LiNbO_3$ (PPLN)

We can even do a better job by periodically flip the sign of $\chi^{(2)}$

$$\frac{dA_3}{dz} = -j\frac{2\omega d_{eff}}{n(2\omega)c_0}A_1^2 e^{-j\Delta kz}$$



Homemade QPM

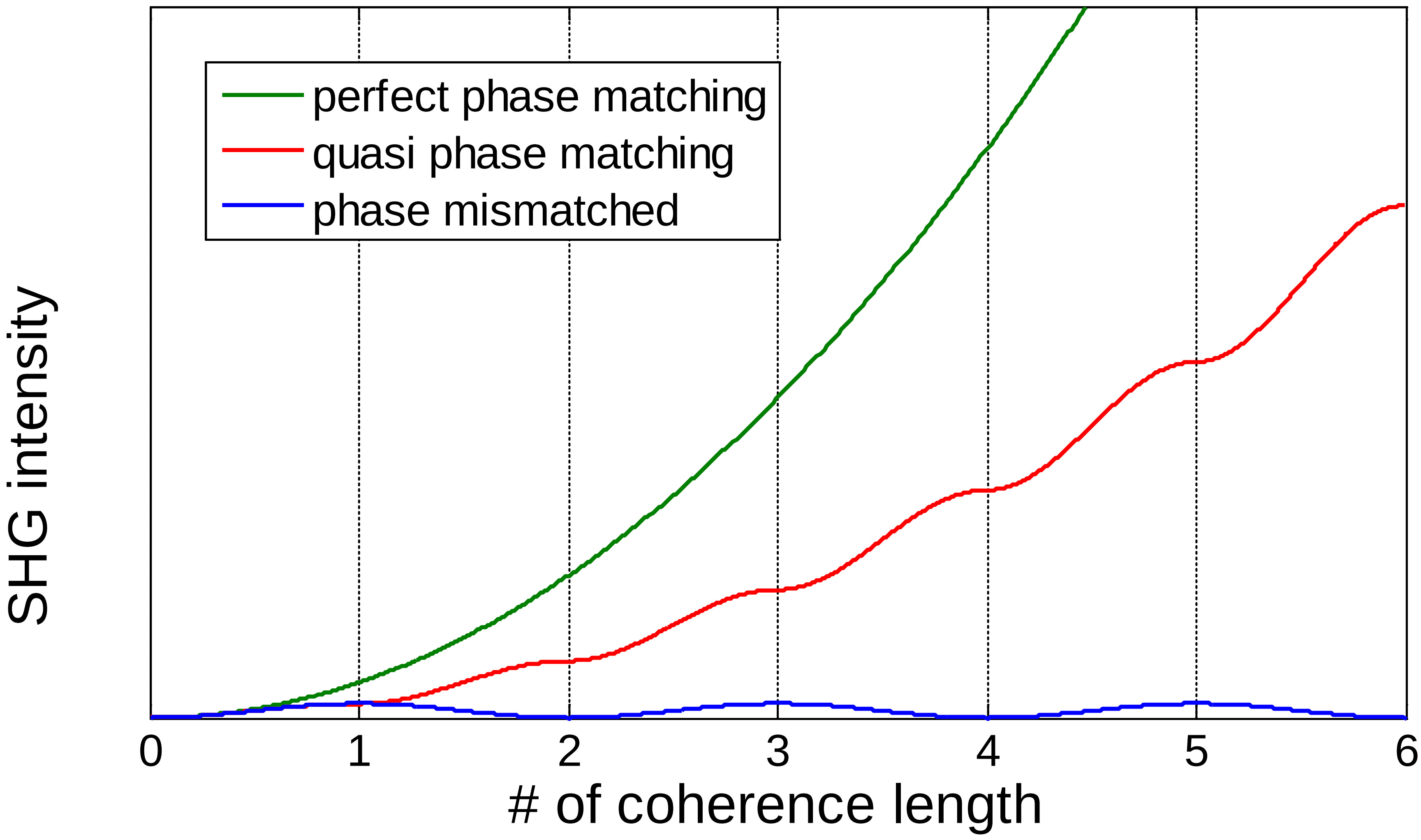
GaAs has a second-order nonlinear coefficient about 7 times larger than $LiNbO_3$. However it is an isotropic medium, and thus we cannot use birefringence enabled phase matching. $LiNbO_3$ works up to 5 um and GaAs is transparent up to 17 um.

$$d_{np} = \begin{bmatrix} 0 & 0 & 0 & d_{36} & 0 & 0 \\ 0 & 0 & 0 & 0 & d_{36} & 0 \\ 0 & 0 & 0 & 0 & 0 & d_{36} \end{bmatrix}$$

$$\boxed{d_{36}(1.064um) = 170\,pm/V}$$

Orientation patterned GaAs (OP-GaAs) or GaP (OP-GaP): material growing with opposite orientation.

Summary of phase matching conditions



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Interactions between Light Waves in a Nonlinear Dielectric*

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The induced nonlinear electric dipole and higher moments in an atomic system, irradiated simultaneously by two or three light waves, are calculated by quantum-mechanical perturbation theory. Terms quadratic and cubic in the field amplitudes are included. An important permutation symmetry relation for the nonlinear polarizability is derived and its frequency dependence is discussed. The nonlinear microscopic properties are related to an effective macroscopic nonlinear polarization, which may be incorporated into Maxwell's equations for an infinite, homogeneous, anisotropic, nonlinear, dielectric medium. Energy and power relationships are derived for the nonlinear dielectric which correspond to the Manley-Rowe relations in the theory of parametric amplifiers. Explicit solutions are obtained for the coupled amplitude equations, which describe the interaction between a plane light wave and its second harmonic or the interaction between three plane electromagnetic waves, which satisfy the energy relationship $\omega_3=\omega_1+\omega_2$, and the approximate momentum relationship $\mathbf{k}_3=\mathbf{k}_1+\mathbf{k}_2+\Delta\mathbf{k}$. Third-harmonic generation and interaction between more waves is mentioned. Applications of the theory to the dc and microwave Kerr effect, light modulation, harmonic generation, and parametric conversion are discussed.

The 22-page paper describes most of the basic principles of nonlinear optics as we know it today.

Then why the field is still growing?

New laser technology, novel optical materials, and emerging applications.

Take-home message

- Coupled wave equations describe the wave-mixing process.
- Phase matching is critical in maximizing the power conversion efficiency in the wave-mixing process.
- Phase matching can be achieved using birefringence in an anisotropic medium.
- Quasi-phase matching allows type 0 phase matching to access to the largest tensor element.

Suggested reading

Coupled wave equation

-- Robert Boyd, *Nonlinear optics*, chapter 2

-- George Stegemann and Robert Stegemann, *Nonlinear optics*, chapter 2 (chapter 4 presents detailed analytical solution)

Phase matching

-- Geoffrey New, *Introduction to nonlinear optics*, chapter 2

-- George Stegemann and Robert Stegemann, *Nonlinear optics*, chapter 3

-- Robert Boyd, *Nonlinear optics*, chapter 2