Universität Hamburg
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CFEL SCIENCE

IMPRS UFAST

Course 2: Basic Technologies

Part III: Synchrotron Sources / FEL

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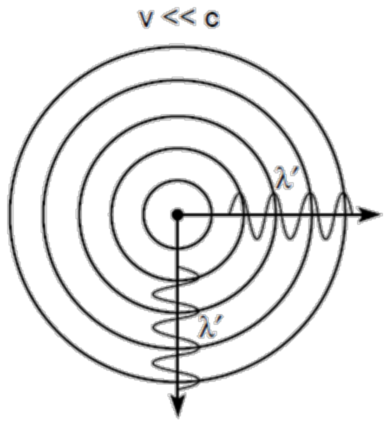
Synchrotron Radiation

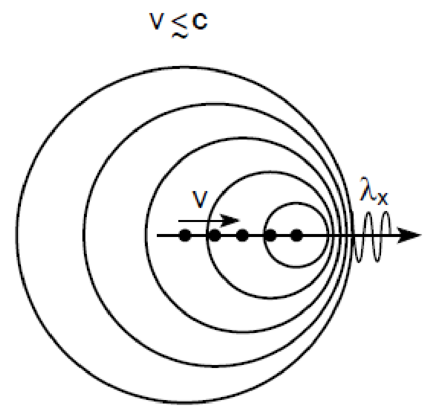
Facts and figures

- Ring accelerator for electrons and positrons
- Length: 2304 metres
- Commissioning: 1978
- 1978-1986: particle physics
- 1987-2007: pre-accelerator for HERA and X-ray radiation source
- Since 2009: most brilliant storage-ring-based X-ray source in the world
- Start of user operation: 2009
- 14 experimental stations with up to 30 instruments

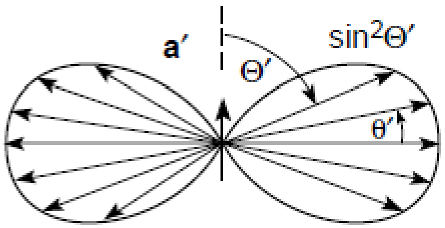
relativistic electrons



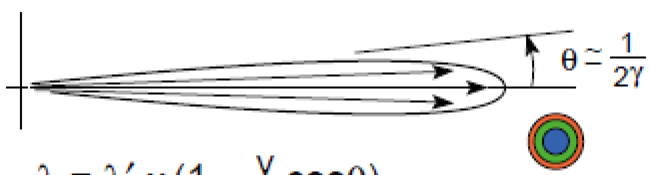
Frame moving with electron



Laboratory frame of reference



$$\lambda = \lambda' \left(1 - \frac{v}{c}\cos\theta\right)$$



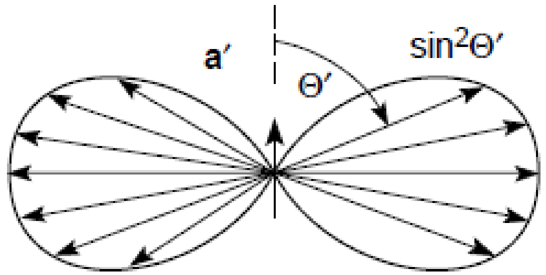
$$\lambda = \lambda' \gamma \left(1 - \frac{v}{c}\cos\theta\right)$$

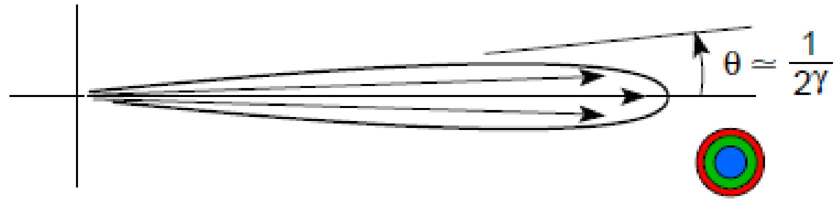
$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\tan\theta = \frac{\sin\theta'}{\gamma(\beta + \cos\theta')}$$

Relativistic Electrons Radiate in a Narrow Forward Cone

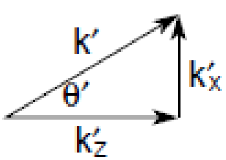
Dipole radiation



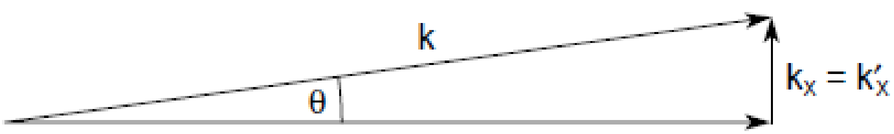


Frame of reference moving with electrons

Laboratory frame of reference



Lorentz transformation

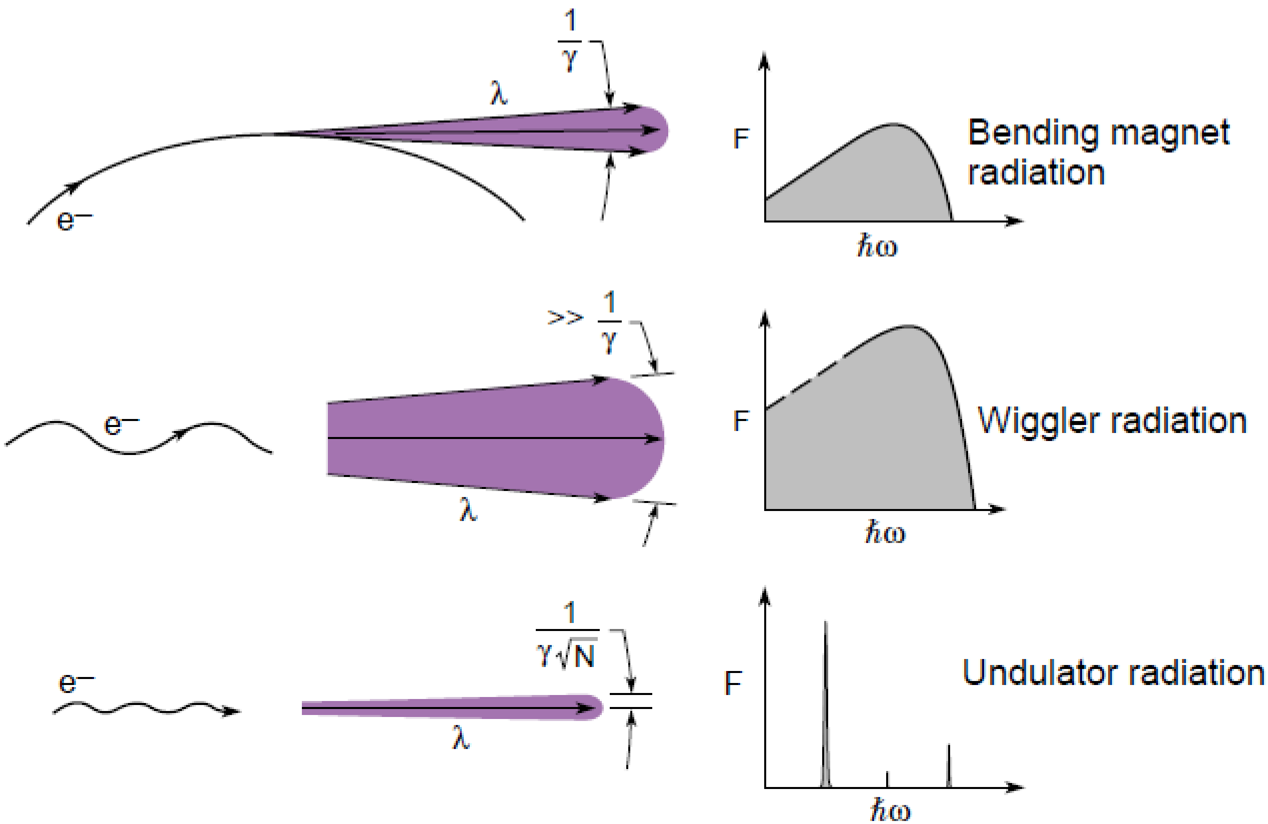


$k' = 2\pi/\lambda'$

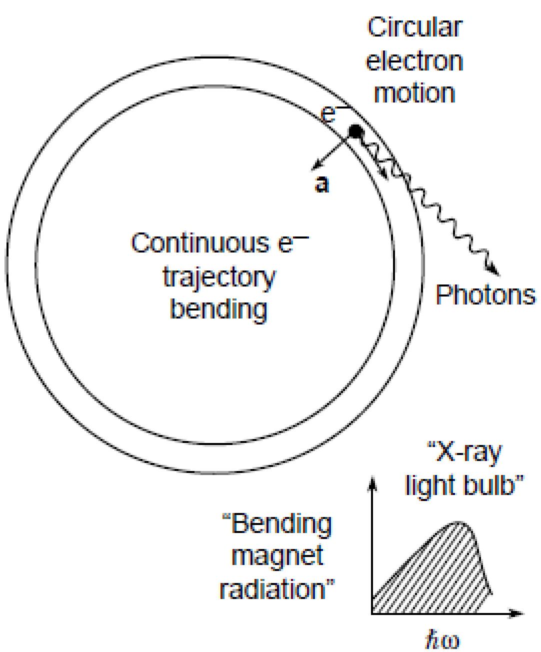
$k_z = 2\gamma k'_z$(Relativistic Doppler shift)

$$\theta \simeq \frac{k_x}{k_z} \simeq \frac{k'_x}{2\gamma k'_z} = \frac{\tan\theta'}{2\gamma} \simeq \frac{1}{2\gamma}$$

Three types of radiation



1. Bending magnet radiation (… „old style“)



$$\boxed{R = \frac{\gamma m v}{eB} \simeq \frac{\gamma m c}{eB}}$$

http://henke.lbl.gov/optical_constants/bend2.html

The Lorentz force for a relativistic electron in a constant magnetic field is

$$\mathbf{F} = \frac{d\mathbf{p}}{dt} = -e\mathbf{v} \times \mathbf{B}$$

where $\mathbf{p} = \gamma m\mathbf{v}$. In a fixed magnetic field the rate of change of electron energy is

$$\frac{dE_e}{dt} = \mathbf{v} \cdot \mathbf{F} = \underbrace{-e\mathbf{v} \cdot (\mathbf{v} \times \mathbf{B})}_{=0}$$

thus with $E_e = \gamma mc^2$

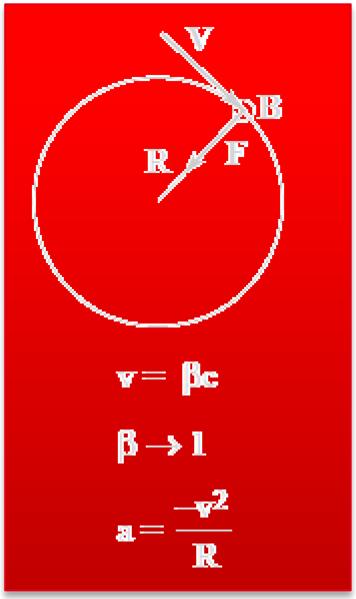
$$\frac{dE_e}{dt} = \frac{d}{dt}(\gamma mc^2) = 0$$

$$\therefore \gamma = \text{constant}$$

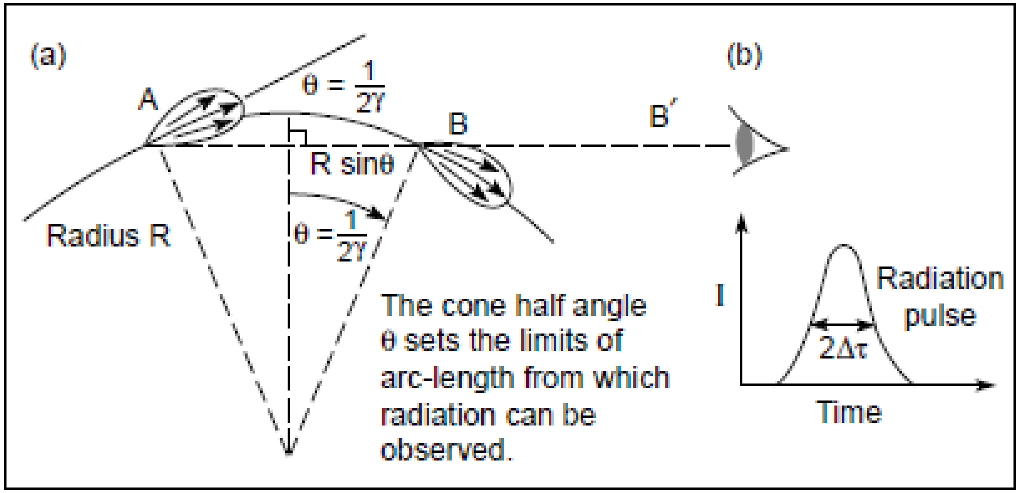
and the force equation becomes

$$\frac{d\mathbf{p}}{dt} = \gamma m \frac{d\mathbf{v}}{dt} = -e\mathbf{v} \times \mathbf{B}$$

$$\gamma m \left(-\frac{v^2}{R}\right) = -evB$$



Bending Magnet Radiation



$$2\,\Delta\tau = \tau_e - \tau_r$$

$$2\,\Delta\tau = \frac{\text{arc length}}{\text{v}} - \frac{\text{radiation path}}{c}$$

$$2\,\Delta\tau \simeq \frac{R \cdot 2\theta}{\text{v}} - \frac{2R\sin\theta}{c}$$

With $\theta \simeq 1/2\gamma$, $\sin\theta \simeq \theta$

$$2\,\Delta\tau \simeq \frac{R}{\gamma \text{v}} - \frac{R}{\gamma c} = \frac{R}{\gamma}\left(\frac{1}{\text{v}} - \frac{1}{c}\right)$$

With $\text{v} = \beta c$

$$2\,\Delta\tau \simeq \frac{R}{\gamma\beta c}(1-\beta) \quad \text{but} \quad (1-\beta) \simeq \frac{1}{2\gamma^2} \quad \text{and} \quad R \simeq \frac{\gamma mc}{eB}$$

$$\boxed{2\Delta\tau = \frac{m}{2eB\gamma^2}}$$

Bending magnet radiation revisited

From Heisenberg's Uncertainty Principle for rms pulse duration and photon energy

$$\Delta E \cdot \Delta\tau \geq \hbar/2$$

thus

$$\Delta \mathrm{E} \geq \frac{\hbar}{2\Delta\tau}$$

$$\Delta \mathrm{E} \geq \frac{\hbar}{\mathrm{m}/2\mathrm{eB}\gamma^2}$$

Thus the single-sided rms photon energy width (uncertainty) is

$$\Delta E \geq \frac{2e\hbar B\gamma^2}{m}$$

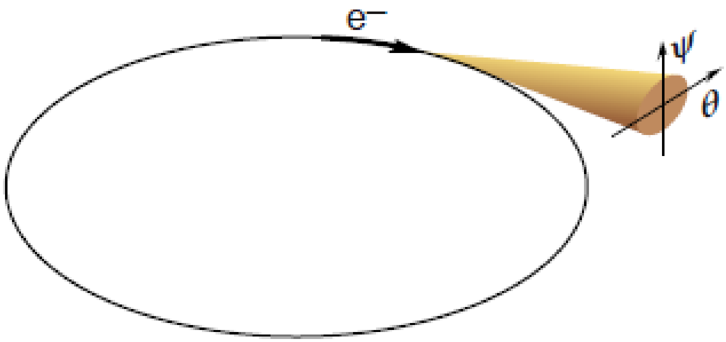
A more detailed description of bending magnet radius finds the critical photon energy

$$\boxed{E_c = \hbar\omega_c = \frac{3e\hbar B\gamma^2}{2m}}$$

In practical units the critical photon energy is

$$E_c(\mathrm{keV}) = 0.6650 E_e^2(\mathrm{GeV}) B(\mathrm{T})$$

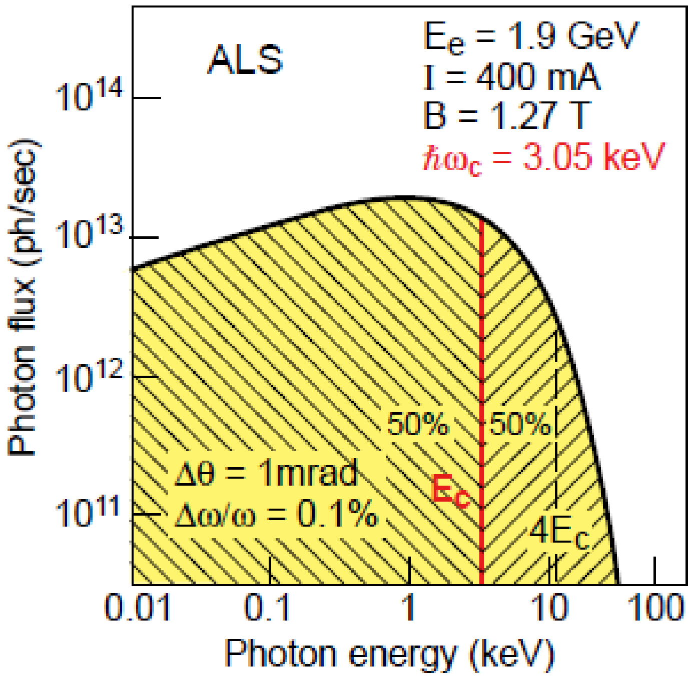
bending magnet - summary



$$E_c = \hbar\omega_c = \frac{3e\hbar B\gamma^2}{2m} \qquad (5.7a)$$

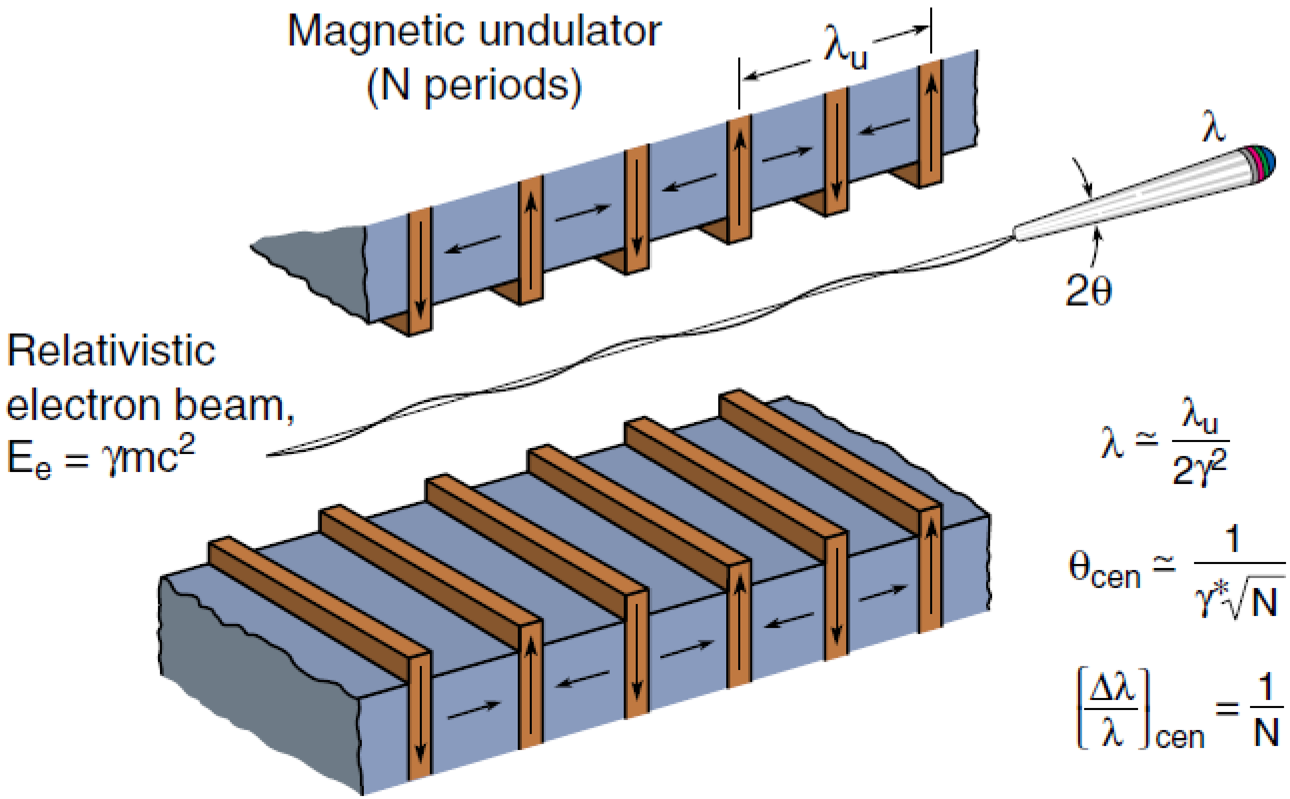
$$E_c(\mathrm{keV}) = 0.6650 E_e^2(\mathrm{GeV}) B(\mathrm{T}) \qquad (5.7b)$$

$$\frac{d^2F_B}{d\theta\, d\omega/\omega} = 2.46 \times 10^{13} E_e(\mathrm{GeV}) I(\mathrm{A}) G_1(E/E_c) \frac{\mathrm{photons/s}}{\mathrm{mrad}\cdot(0.1\%\mathrm{BW})} \qquad (5.8)$$



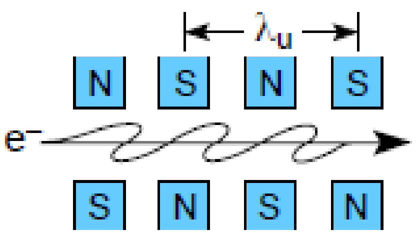
Advantages:	• covers broad spectral range • least expensive • most accessable
Disadvantages:	• limited coverage of hard x-rays • not as bright as undulator

Undulator


Magnetic undulator
(N periods)
λ_u
λ
2θ
Relativistic
electron beam,
$E_e = \gamma mc^2$
$\lambda \simeq \frac{\lambda_u}{2\gamma^2}$
$\theta_{cen} \simeq \frac{1}{\gamma^* \sqrt{N}}$
$\left[\frac{\Delta\lambda}{\lambda}\right]_{cen} = \frac{1}{N}$

undulator radiation

Laboratory Frame of Reference

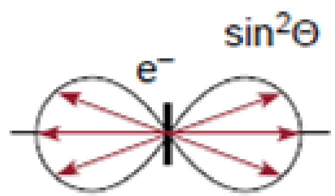


$E = \gamma mc^2$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

N = # periods

Frame of moving e-



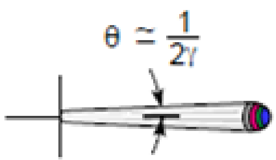
e⁻ radiates at the Lorentz contracted wavelength:

$$\lambda' = \frac{\lambda_u}{\gamma}$$

Bandwidth:

$$\frac{\lambda'}{\Delta\lambda'} \simeq N$$

Frame of observer



Doppler shortened wavelength on axis:

$$\lambda = \lambda'\gamma(1 - \beta\cos\theta)$$

$$\lambda = \frac{\lambda_u}{2\gamma^2}(1 + \gamma^2\theta^2)$$

Accounting for transverse motion due to the periodic magnetic field:

$$\boxed{\lambda = \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K^2}{2} + \gamma^2\theta^2\right)}$$

where $K = eB_0\lambda_u / 2\pi mc$

where does the $\lambda = \lambda_u/2\gamma^2$ come from?

The electron "sees" a Lorentz contracted period

$$\lambda' = \frac{\lambda_u}{\gamma}$$

and emits radiation in its frame of reference at frequency

$$f' = \frac{c}{\lambda'} = \frac{c\gamma}{\lambda_u}$$

Observed in the laboratory frame of reference, this radiation is Doppler shifted to a frequency

$$f = \frac{f'}{\gamma(1 - \beta\cos\theta)} = \frac{c}{\lambda_u(1 - \beta\cos\theta)}$$

On-axis ($\theta = 0$) the observed frequency is

$$\boxed{f = \frac{c}{\lambda_u(1 - \beta)}}$$

$$f = \frac{c}{\lambda_u(1-\beta)}$$

By definition $\gamma = \dfrac{1}{\sqrt{1-\beta^2}}$; $\gamma^2 = \dfrac{1}{(1-\beta)(1+\beta)} \simeq \dfrac{1}{2(1-\beta)}$

thus

$$f = \frac{2\gamma^2 c}{\lambda_u}$$

and the observed wavelength is

$$\boxed{\lambda = \frac{c}{f} = \frac{\lambda_u}{2\gamma^2}}$$

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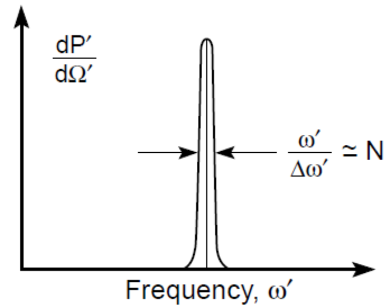
With electrons executing N oscillations as they traverse the periodic magnet structure, and thus radiating a wavetrain of N cycles, it is of interest to know what angular cone contains radiation of relative spectral bandwidth

$$\frac{\Delta\lambda}{\lambda} = \frac{1}{N} \qquad (5.14)$$



Execution of N electron oscillations produces a transform-limited spectral bandwidth, $\Delta\omega'/\omega' = 1/N$.

Write the undulator equation twice, once for on-axis radiation (θ = 0) and o length-shifted radiation off-axis at angle θ:

$$\lambda_0 + \Delta\lambda = \frac{\lambda_u}{2\gamma^2}(1 + \gamma^2\theta^2)$$

$$\lambda_0 = \frac{\lambda_u}{2\gamma^2}$$

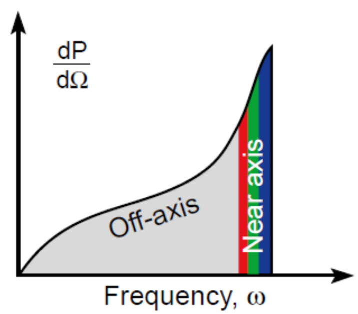
divide and simplify to

$$\frac{\Delta\lambda}{\lambda} \simeq \gamma^2\theta^2 \qquad (5.13)$$

Combining the two equations (5.13 and 5.14)

defines θ_{cen} : $\gamma^2\theta_{cen}^2 = \frac{1}{N}$, which gives $\boxed{\theta_{cen} \simeq \frac{1}{\gamma\sqrt{N}}}$ (5.15)

This is the half-angle of the "central radiation cone", defined as containing radiation of $\Delta\lambda/\lambda = 1/N$.



The Doppler frequency shift has a strong angle dependence, leading to lower photon energies off-axis.

radiation power of an undulator

$$\lambda_x = \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K^2}{2} + \gamma^2\theta^2\right)$$

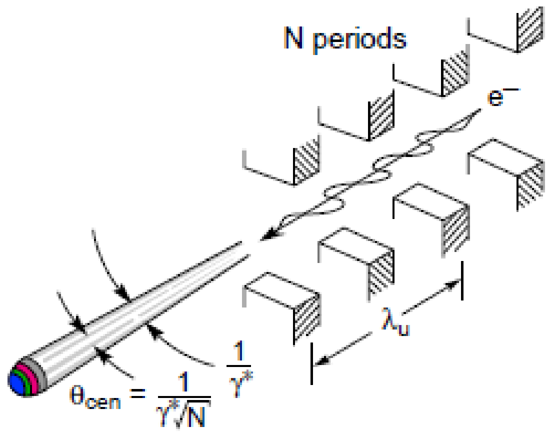
$$\bar{P}_{cen} = \frac{\pi e \gamma^2 I}{\epsilon_0 \lambda_u} \frac{K^2}{\left(1 + \frac{K^2}{2}\right)^2} f(K)$$

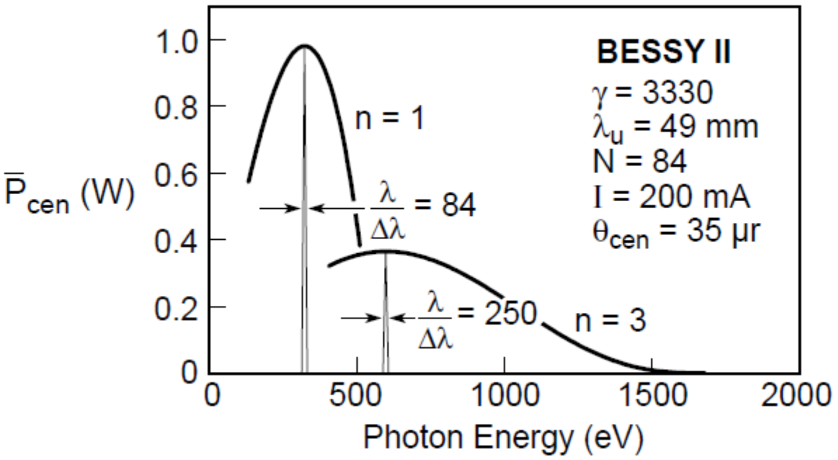
$$\theta_{cen} = \frac{1}{\gamma^* \sqrt{N}}$$

$$\left(\frac{\Delta\lambda}{\lambda}\right)_{cen} = \frac{1}{N}$$

$$K = \frac{e B_0 \lambda_u}{2\pi m_0 c}$$

$$\gamma^* = \gamma / \sqrt{1 + \frac{K^2}{2}}$$





Magnetic fields in the periodic undulator cause the electrons to oscillate and thus radiate. These magnetic fields also slow the electrons axial (z) velocity somewhat, reducing both the Lorentz contraction and the Doppler shift, so that the observed radiation wavelength is not quite so short. The force equation for an electron is

$$\frac{d\mathbf{p}}{dt} = -e(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

where $\mathbf{p} = \gamma m\mathbf{v}$ is the momentum. The radiated fields are relatively weak so that

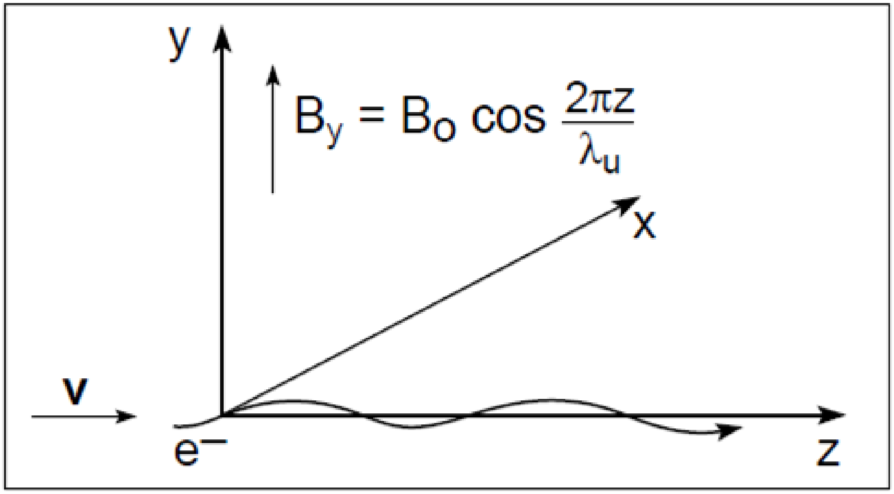
$$\frac{d\mathbf{p}}{dt} \simeq -e(\mathbf{v} \times \mathbf{B})$$

Taking to first order $\mathrm{v} \simeq \mathrm{v}_z$, motion in the x-direction is

$$m\gamma \frac{d\mathrm{v}_x}{dt} = +e\mathrm{v}_z B_y$$

$$m\gamma \frac{d\mathrm{v}_x}{dt} = e\frac{dz}{dt} \cdot B_0 \cos\left(\frac{2\pi z}{\lambda_u}\right) \qquad (0 \le z \le N\lambda_u)$$

$$m\gamma\, d\mathrm{v}_x = e\, dz\, B_0 \cos\left(\frac{2\pi z}{\lambda_u}\right)$$



$$m\gamma\, d\mathrm{v}_x = e\, dz\, B_0 \cos\left(\frac{2\pi z}{\lambda_u}\right)$$

integrating both sides

$$m\gamma \mathrm{v}_x = e B_0 \frac{\lambda_u}{2\pi} \int \cos\left(\frac{2\pi z}{\lambda_u}\right) \cdot d\left(\frac{2\pi z}{\lambda_u}\right)$$

$$m\gamma \mathrm{v}_x = \frac{e B_0 \lambda_u}{2\pi} \sin\left(\frac{2\pi z}{\lambda_u}\right)$$

$$\boxed{\mathrm{v}_x = \frac{K c}{\gamma} \sin\left(\frac{2\pi z}{\lambda_u}\right)}$$

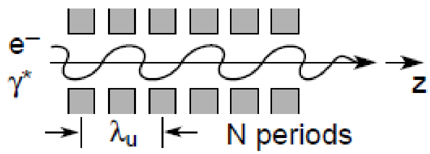
$$\boxed{K \equiv \frac{e B_0 \lambda_u}{2\pi m c} = 0.9337\, B_0(\mathrm{T}) \lambda_u(\mathrm{cm})}$$

is the non-dimensional "magnetic deflection parameter."
The "deflection angle", θ, is

$$\theta = \frac{\mathrm{v}_x}{\mathrm{v}_z} \simeq \frac{\mathrm{v}_x}{c} = \frac{K}{\gamma}\, \sin k_u z$$

calculating the radiation power of the central cone

x, z, t laboratory frame of reference



Lorentz transformation

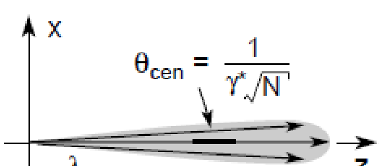
Determine x, z, t motion:

$$\frac{d\mathbf{p}}{dt} = -e\,(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

$$m\gamma \frac{dv_x}{dt} = e\,\frac{dz}{dt} B_0 \cos\frac{2\pi z}{\lambda_u}$$

$$v_x(t);\ a_x(t) = \cdots$$

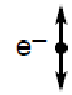
$$v_z(t);\ a_z(t) = \cdots$$



$\theta_{cen} = \frac{1}{\gamma^* \sqrt{N}}$

$\frac{\lambda}{\Delta\lambda} = N$

x', z', t' frame of reference moving with the average velocity of the electron



$\lambda_u' = \frac{\lambda_u}{\gamma^*}$

x', z', t' motion
$a'(t')$ acceleration

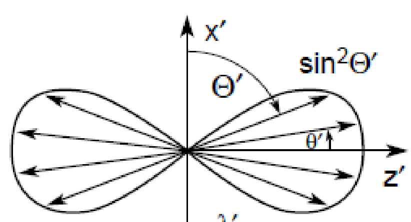
$$\boxed{\bar{P}_{cen} = \frac{\pi e \gamma^2 I}{\epsilon_0 \lambda_u} \frac{K^2}{(1 + \frac{K^2}{2})^2} f(K)}$$

Dipole radiation:

$$\frac{dP'}{d\Omega'} = \frac{e^2 a'^2 \sin^2\Theta'}{16\pi^2 \epsilon_0 c^3}$$

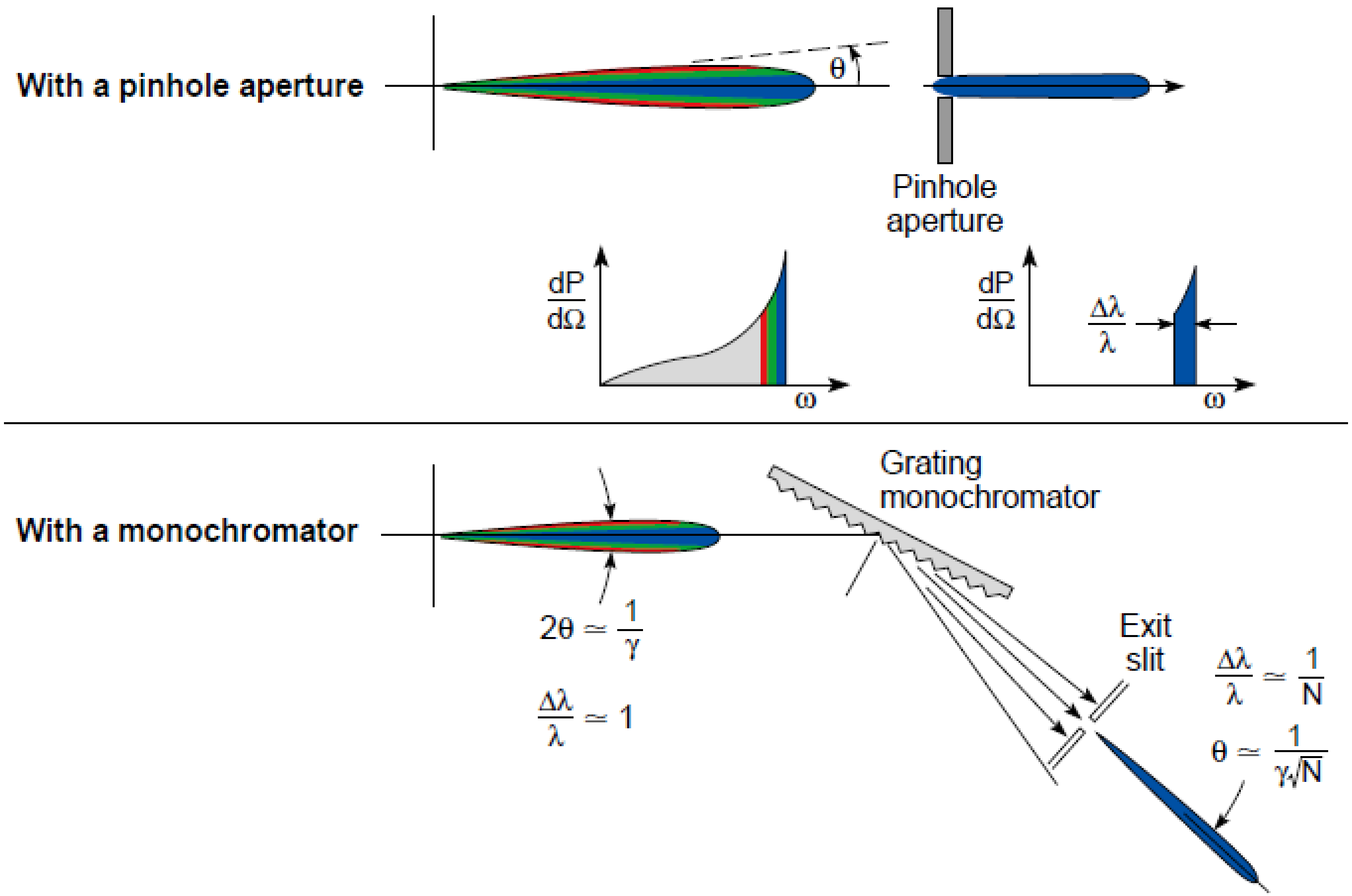
$$\frac{dP'}{d\Omega'} = \frac{e^2 c \gamma^2}{4\epsilon_0 \lambda_u^2} \frac{K^2}{(1 + K^2/2)^2} (1 - \sin^2\theta' \cos^2\phi') \cos^2\omega_u' t'$$

Lorentz transformation



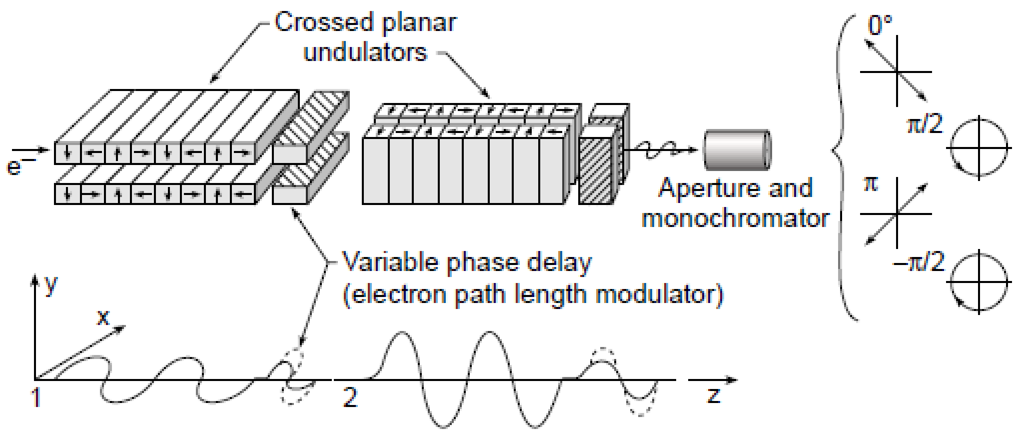
$\frac{\lambda'}{\Delta\lambda'} = N$

bandwith of the undulator radiation

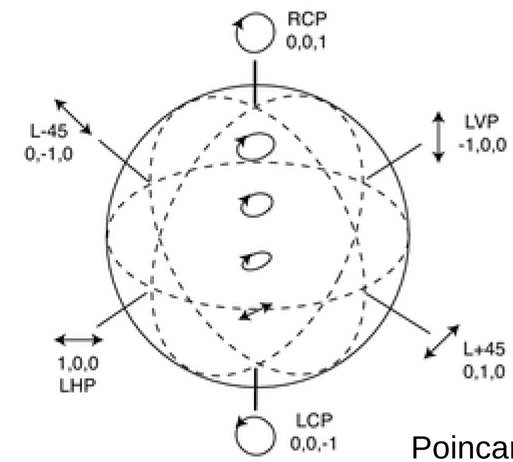


The Narrow (1/N) Spectral Bandwidth of Undulator Radiation Can be Recovered in Two Ways

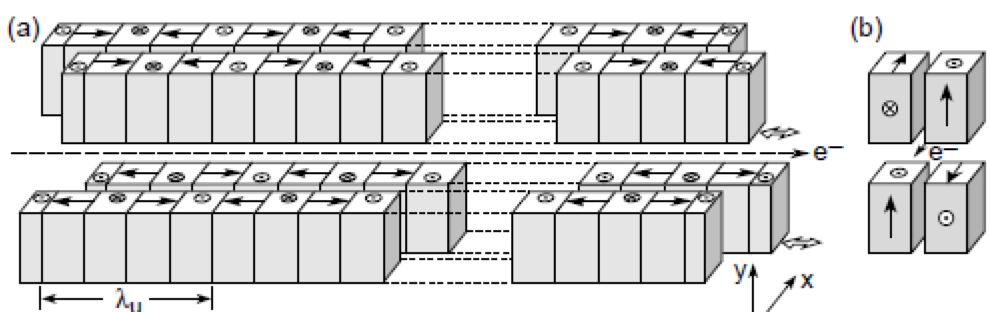
Polarisation properties



(Courtesy of Kwang-Je Kim)

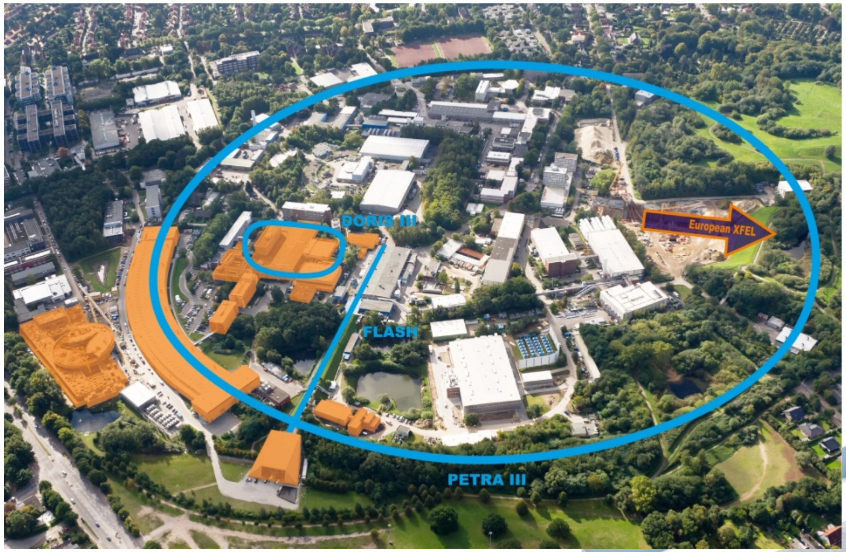


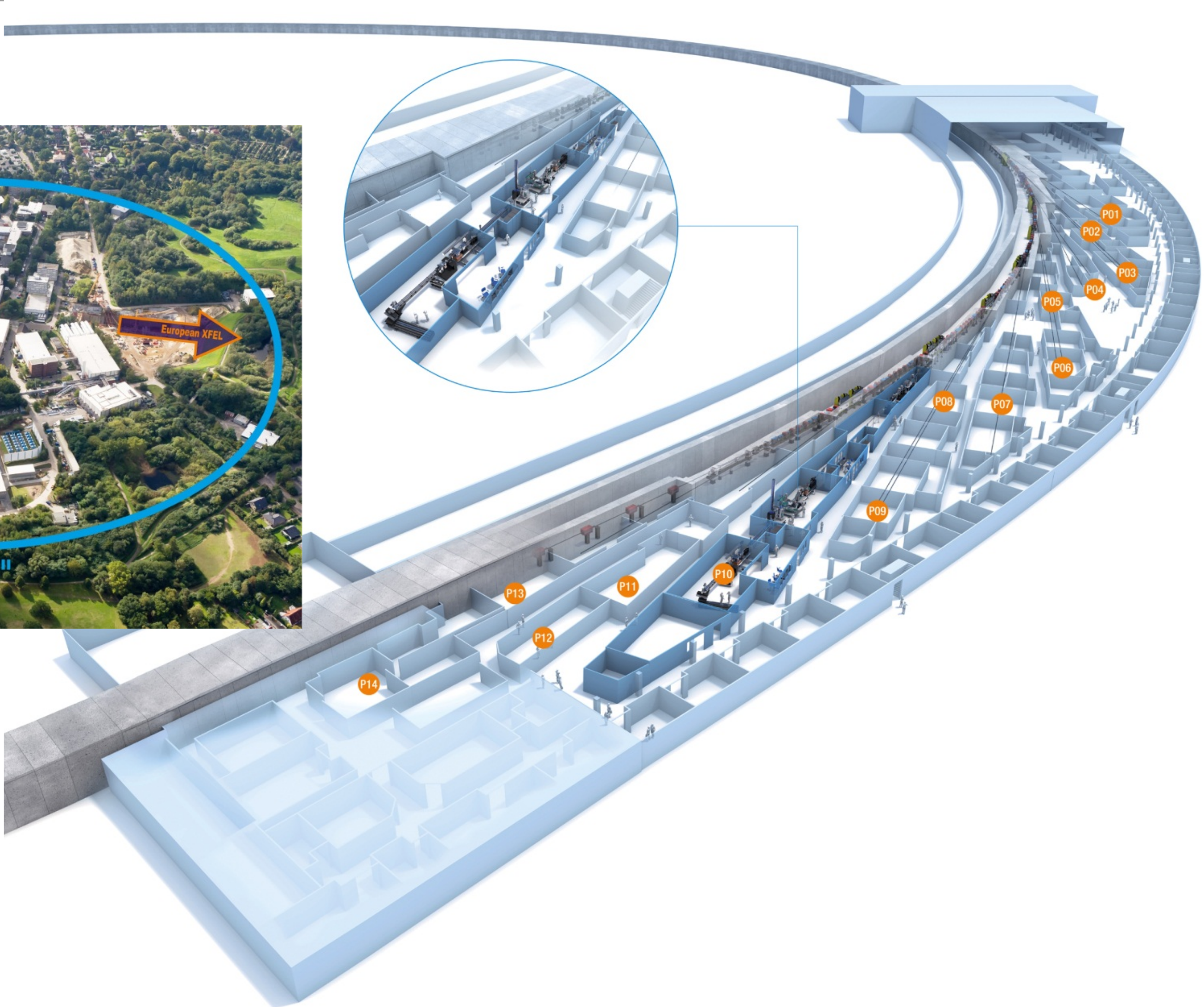
Poincare



(Following S. Sasaki)

PETRA III


DORIS III
European XFEL
FLASH
PETRA III


P01
P02
P03
P04
P05
P06
P07
P08
P09
P10
P11
P12
P13
P14

Benefits of synchrotron radiation

- Spectrum (tunability, bandwith…)
- Prediction
- Polarisation (very clean)
- time structure (electron bunches)
- UHV compatible
- Intensity

<u>High optical power with very good focusability</u>

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Comparison

Bending magnet radiation	Wiggler radiation	Undulator radiation
Broad spectrum	Higher photon energies	Brighter radiation
Good photon flux	More photon flux	Smaller spot size
No heat load	Expensive magnet structure	Partial coherence
Less expensive	Expensive cooled optics	Expensive
Easy access	Less access	Less access

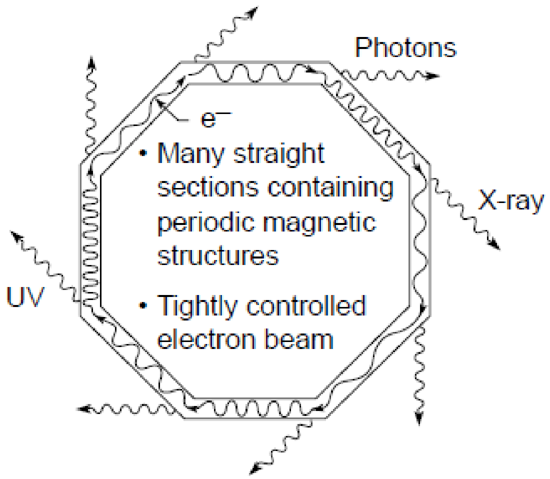
Further information

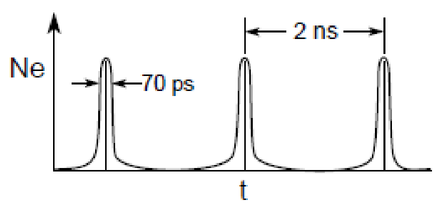
Prof. David Attwood

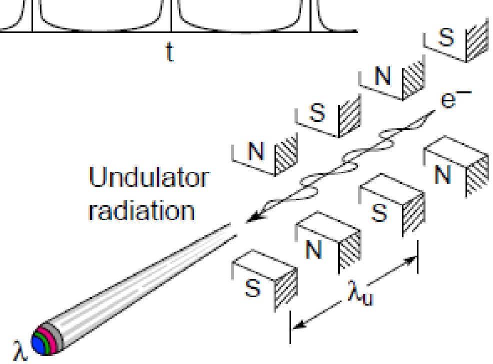
Soft X-Ray and Extreme Ultraviolett Radiation, Cambridge University Press, 1999

http://ast.coe.berkeley.edu/srms/

Synchrotron radiation sources







Bending Magnet:

$$\hbar\omega_c = \frac{3e\hbar B\gamma^2}{2m}$$

Wiggler:

$$\hbar\omega_c = \frac{3e\hbar B\gamma^2}{2m}$$

$$n_c = \frac{3K}{4}\left(1 + \frac{K^2}{2}\right)$$

$$P_T = \frac{\pi e K^2\gamma^2 I N}{3\epsilon_0\lambda_u}$$

Undulator:

$$\lambda = \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K^2}{2} + \gamma^2\theta^2\right)$$

$$K = \frac{eB_0\lambda_u}{2\pi mc}$$

$$\theta_{\text{cen}} = \frac{1}{\gamma^*\sqrt{N}}$$

$$\left.\frac{\Delta\lambda}{\lambda}\right|_{\text{cen}} = \frac{1}{N}$$

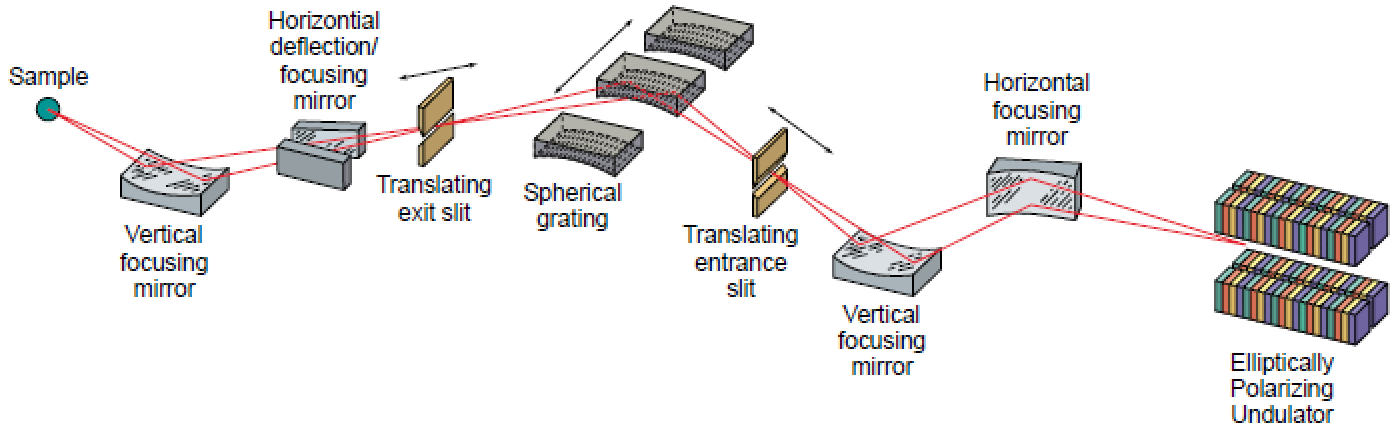
$$\bar{P}_{\text{cen}} = \frac{\pi e\gamma^2 I}{\epsilon_0\lambda_u}\frac{K^2}{\left(1 + \frac{K^2}{2}\right)^2} f(K)$$

For a particle moving at a velocity v with a total energy E

$$E = \gamma m_o c^2$$

with $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$

short reminder…



FEL – Free Electron Lasers

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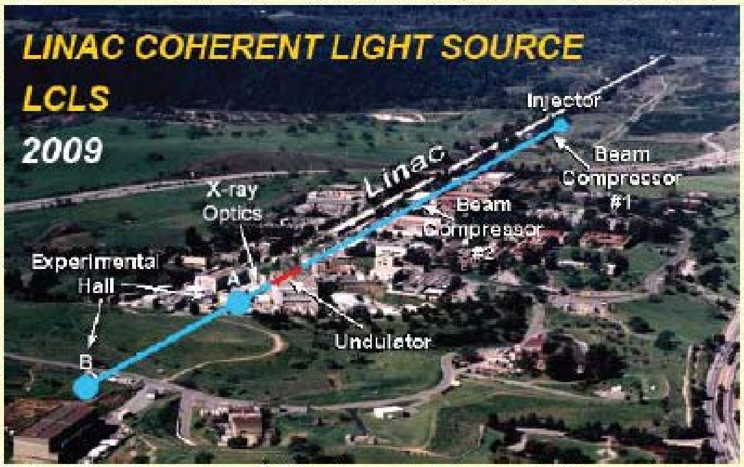
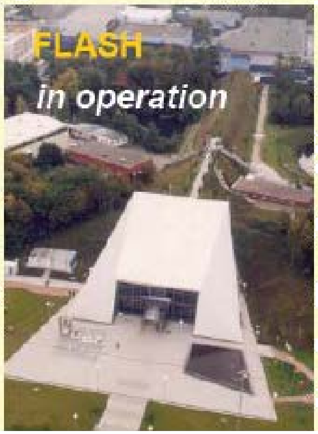
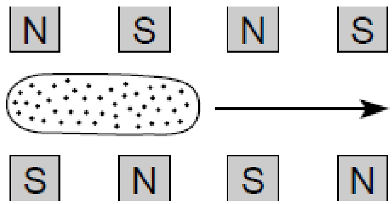
Some Free Electron Lasers

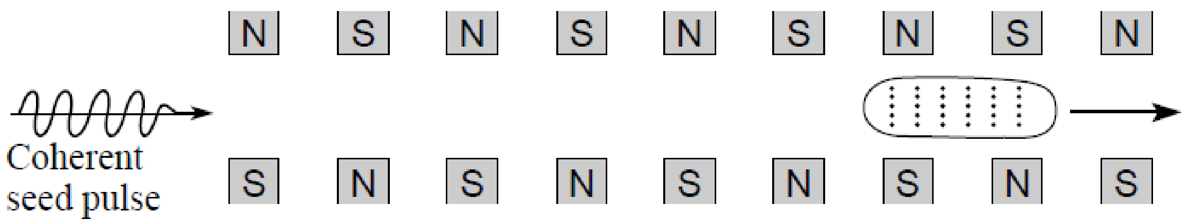




Undulator vs. FEL



Undulator – uncorrelated electron positions, radiated fields uncorrelated, intensities add, limited coherence, power ~ N.



Free Electron Laser (FEL) – very long undulator, electrons are "microbunched" by their own radiated fields into strongly correlated waves of electrons, all radiated electric fields now add, spatially coherent, power ~ N^2

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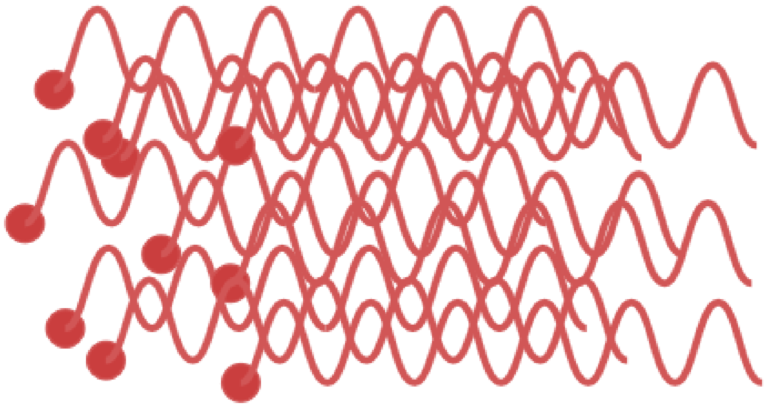
SR or ERL

Spontaneous Radiation



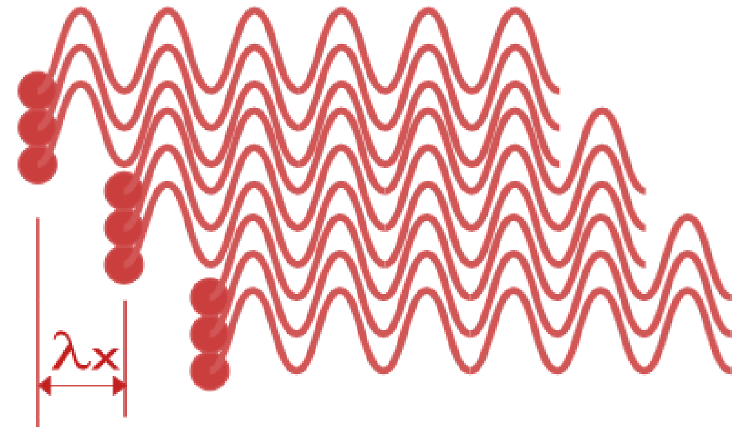
***N*-electrons**
random distribution

$E_{spt} \sim \sqrt{N} E_1$

$P_{spt} \sim N P_1$

FEL: Free Electron Laser

Coherent Radiation

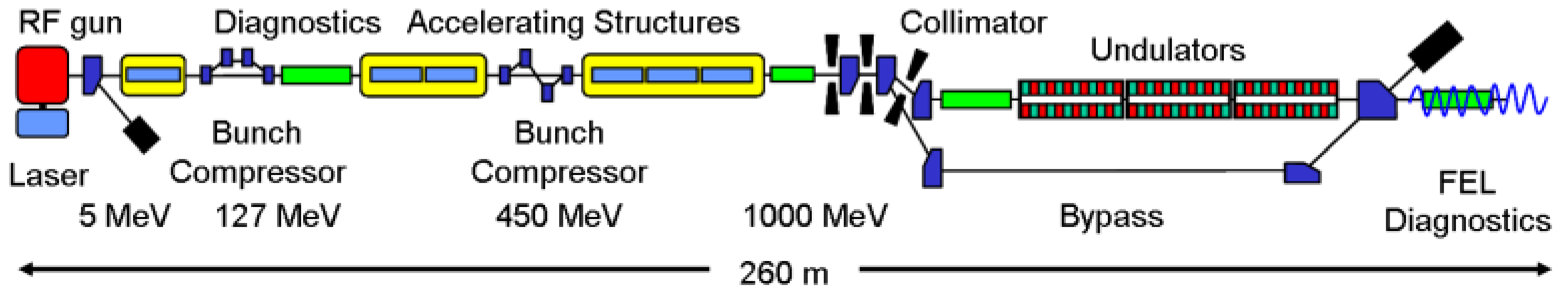


***N*-electrons**
micro-bunched

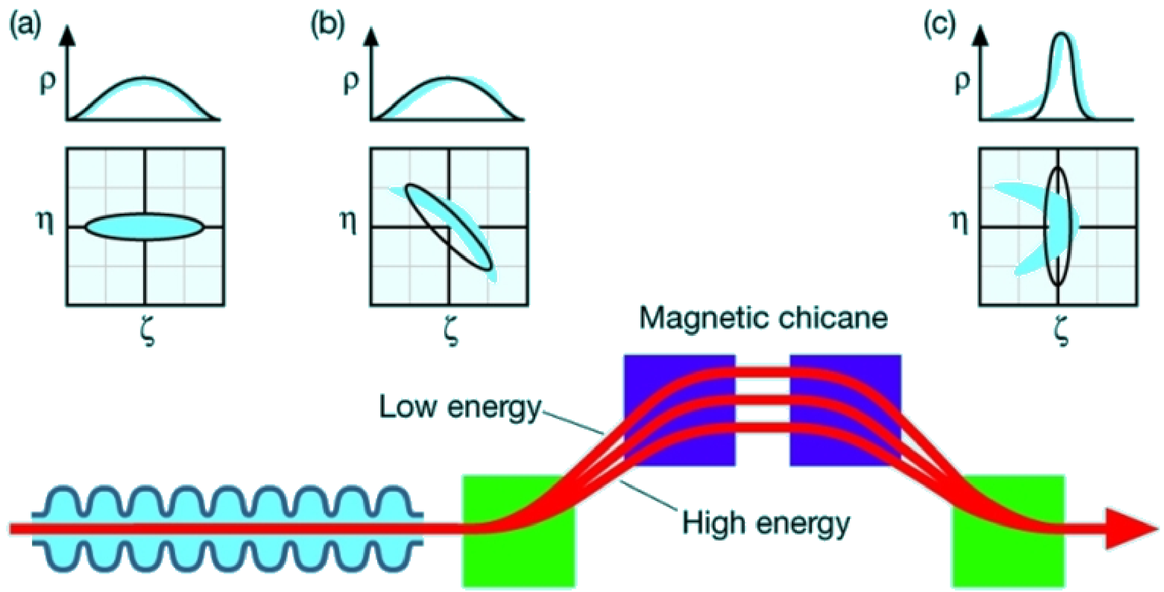
$E_{coherent} \sim N E_1$

$P_{coherent} \sim N^2 P_1$

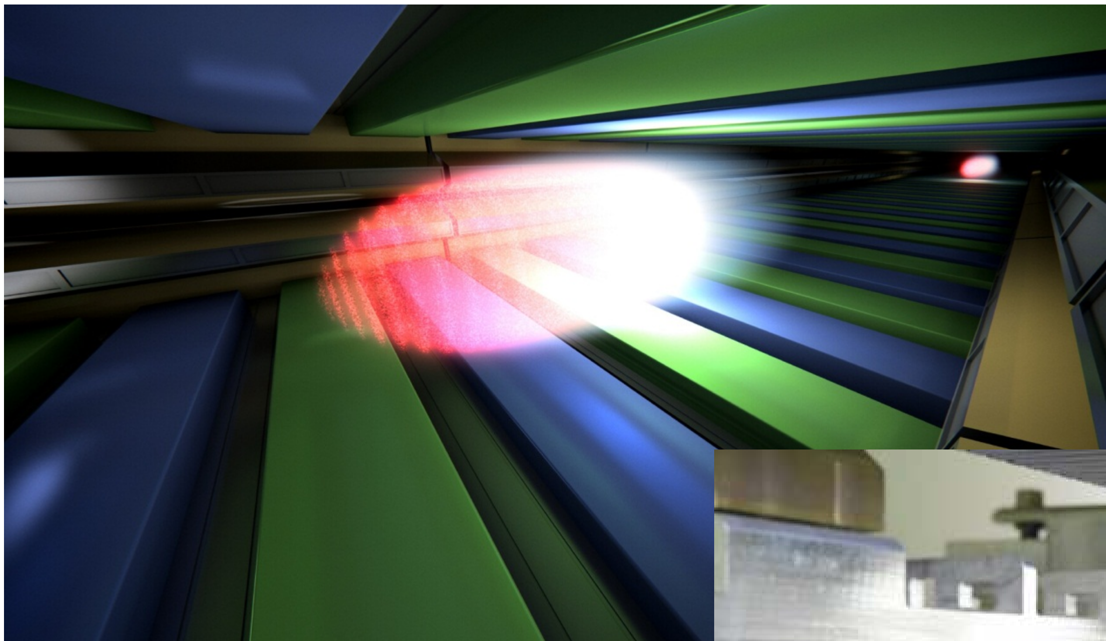
Flash Design layout

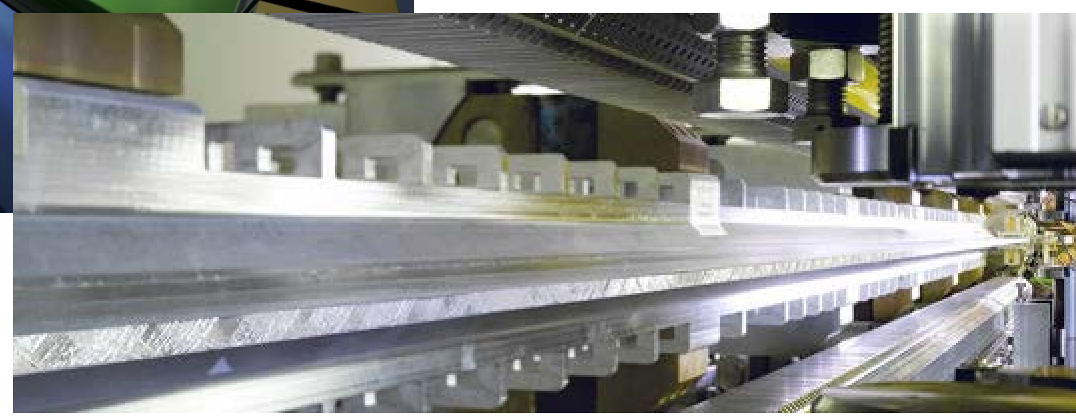


bunch compressor



electron bunch in the undulator





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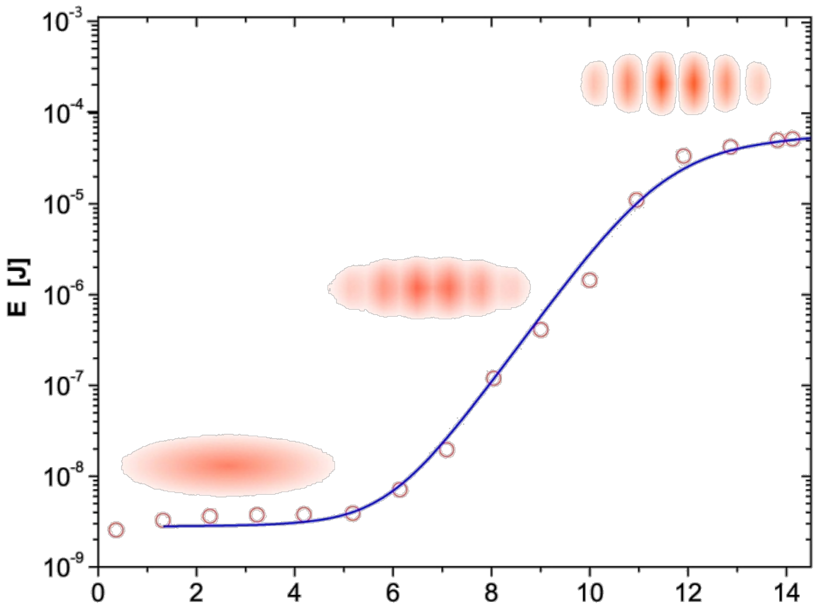
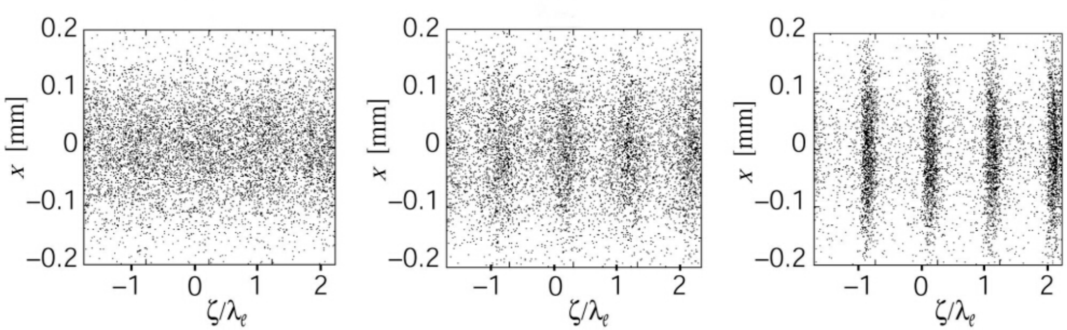
CFEL SCIENCE

radiation power vs. undulator length





Power grows exponentially with undulatordistance z. For a 1-D, mono-energetic beam

$$P \propto e^{\frac{z}{L_{G0}}} \text{ where } L_{G0} = \frac{\lambda_u}{4\sqrt{3\pi\rho}}$$

Pierce Parameter $\rho \sim \frac{radiation\ power}{beam\ power}$

peak current

$$\rho = \frac{1}{4\gamma_0}\left[\frac{I\,\lambda_u^2 K^2 [JJ]^2}{I_A \pi^2 \epsilon_x \beta_x}\right]$$

17 kA

beta function

emittance

Pierce Parameter

Pierce parameter ρ says how many undulator periods must electrons travel in order to increase FEL power 2e-times

Important FEL parameters are expressed by ρ e.g.

$$spectrum \quad bandwith \approx \rho \qquad P_{sat} \approx P_{beam} \cdot \rho$$

$$Coherence\ time \quad \tau_c = \frac{1}{\omega\rho}$$

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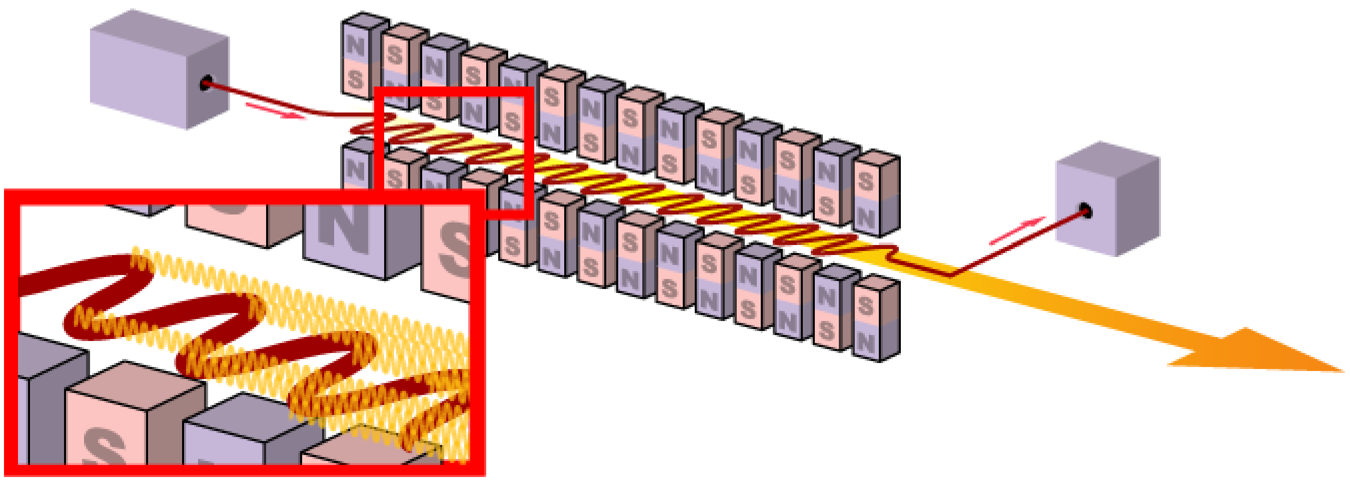
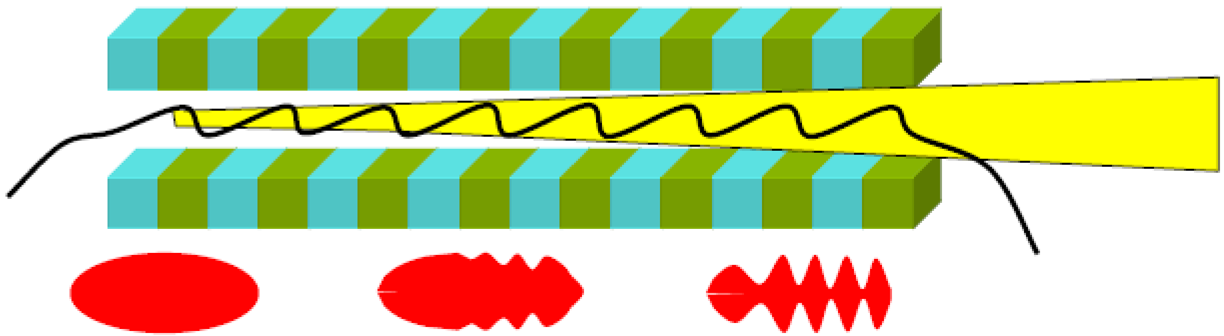
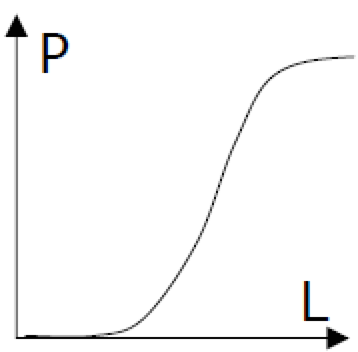
undulator radiation in a long undulator



http://en.wikipedia.org/wiki/Free-electron_laser

SASE – Self Amplified Stimulated Emission





- From noice
- No mirrors (X-rays)
- Tunable
- ″Spiky″ (t and λ)

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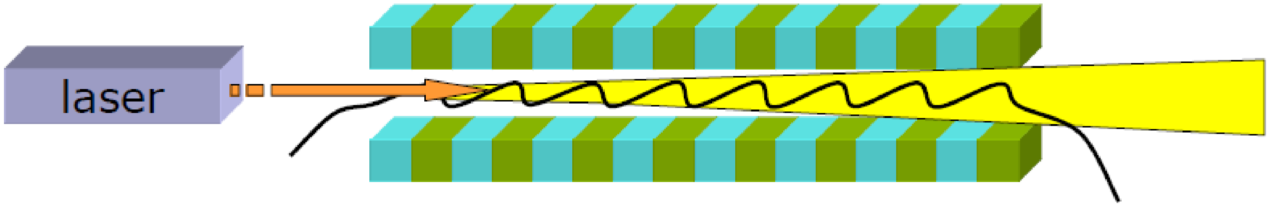
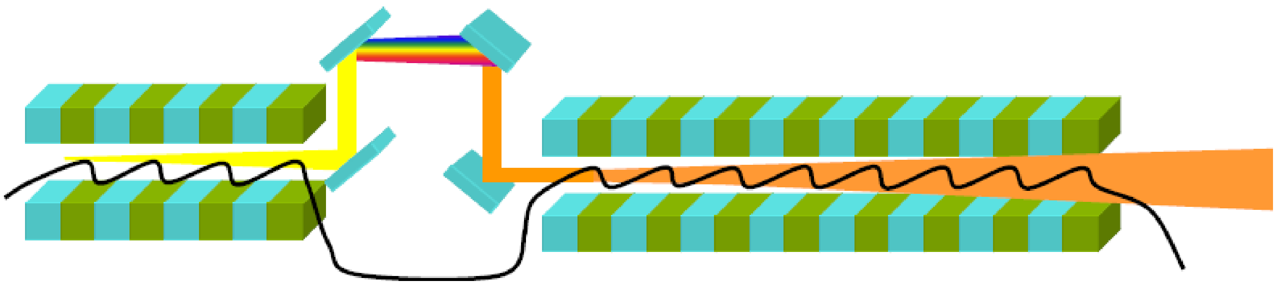
Seeding



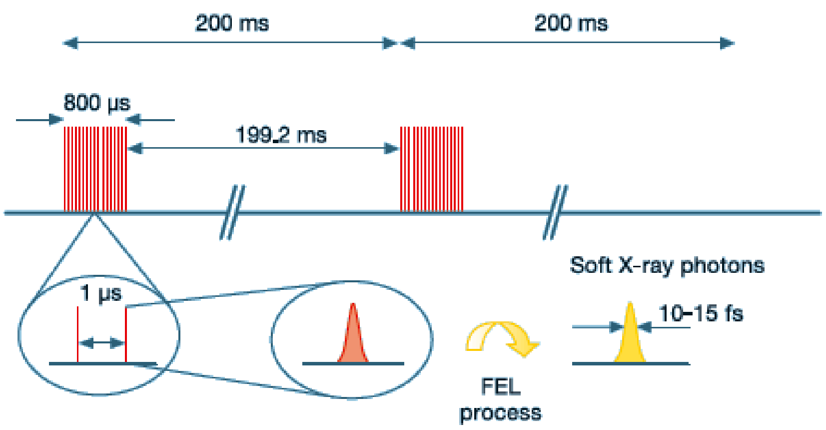
- Remove instabilities
- CHG with gain HGHG



power gain $$P(z) = \frac{P_0}{9} \exp(z / L_g) \;\; for \;\; z \geq 2L_g$$

Key figures of FLASH

http://flash.desy.de/



Performance of the FEL radiation	2005 – 2007
Wavelength range of the fundamental	13 - 47 nm (from fall 2007: 6.5 nm)
Higher harmonics	3rd 4.6 nm
	5th 2.7 nm
	(7th 1.9 nm)
Average pulse energy	up to 100 μJ
Peak pulse energy	170 μJ
Peak power	5 GW
Average power	100 mW
Pulse duration	10 - 50 fs
Spectral width	0.5 - 1%
Peak brilliance	10^{29} - 10^{30} [photons/(s mrad^2 mm^2 0.1% BW)]

http://pr.desy.de/sites2009/site_pr/content/e113/e48/column-objekt221/lbox/infoboxContent222/FLASH_en_2007_ger.pdf

Review of techniques for attosecond X-ray Pulse generation (FEL based)

Main schemes

- emittance spoiler (P. Emma et al. PRL, 2004)
- slicing: wavelength selection (Saldin et al., Opt. Comm. 2004)
- slicing: current enhancement (Zholents et al., PRSTAB 2005)
- slicing: from a Harmonic Cascade FEL (Zholents et al., PRL 2004)
- slicing: energy chirp revisited: tapered undulator (Saldin et al., PRSTAB 2006)
- ultra short electron bunches: single spike (C. Pellegrini et al., 2007)
- energy chirp (C. Schroeder et al., NIMA 2002)
- attosecond trains from mode locking (N. Thompson et al., 2007)

Thank you!