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# Course 2: Basic Technologies

Part III: High Harmonic Generation

*The route to sub-fs physics*

Jun.-Prof. Dr. Thorsten Uphues  
Thorsten.Uphues@cfel.de

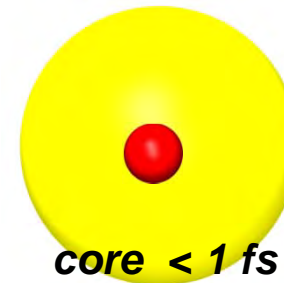
# Outline

- Attosecond Physics
- Strong field ioniation (Keldysh / ADK)
- HHG
  - Wavelength scaling
  - Conversion efficiencies
- Techniques
  - Ultrashort pulses
  - CEP
  - PG
  - DOG
- X-ray optics in pump-probe setups

## timestructure of atomic processes

time  $\leftrightarrow$  energy :  $\Delta t \sim h/\Delta E$

valence  $n = 1 \sim 1 \text{ fs}$



object of study:  
dynamics  
of **inner-shell** processes

but **visible** light:  $T \sim 2 \text{ fs}$



requires: attosecond **XUV**-pulse

Rydberg  $n > 1 \text{ fs} \dots \mu\text{s}$



# Strong field ionization

## Ionization in strong fields

Let us assume the case of the hydrogen atom:

The strength of the Coulomb potential experienced by an electron in the first Bohr orbit of atomic hydrogen is defined by

$$\mathcal{E}_a = \frac{e}{(4\pi\epsilon_0)a_0^2} = 5.14221 \times 10^9 \text{V/cm}$$

and we determine the field strength of a laser electric field from the intensity with

$$I = \frac{1}{2}\epsilon_0 c E^2$$

thus the Coulomb potential of the hydrogen atom corresponds to a laser electric field intensity of  **$\sim 3.51 \cdot 10^{16} \text{ W/cm}^2$**

## Interaction with a time varying electric field

The perturbation of a time varying electric field of the typical strength of a laser interacting with an atom cannot be treated in terms of perturbation theory anymore.

- the perturbation of the Coulomb potential is not weak!
- the strong time varying potential competes with the Coulomb binding potential

The dynamics of the ionization process of a bound electron inside the potential is strongly determined by the instantaneous strength of the applied electric field.

The strength of the electric field can change by orders of magnitude within a cycle of the laser electric field

The Keldysh approach (1965)

## The Keldysh approach (1965)

If we assume a classical particle with binding energy  $I_p$  and a field strength that sufficiently suppresses the Coulomb potential, we can define a „barrier thickness“

$$l = \frac{I_p}{e \mathcal{E}_0}$$

and a corresponding „tunneling time“ the electron needs to penetrate through the rectangular barrier

$$\tau = \frac{\sqrt{2mI_p}}{e \mathcal{E}_0}$$

## Keldysh parameter

Multiplying with the frequency of the laser field leads to the Keldysh parameter

$$\begin{aligned}\gamma = \tau\omega_L &= \omega_L \frac{\sqrt{2mI_p}}{e\mathcal{E}_1} \\ &= \sqrt{\frac{I_P}{2U_P}}\end{aligned}$$

The Keldysh parameter  $\gamma$  distinguishes between the ionization processes in the limiting cases  $\gamma \gg 1$  for multi photon ionization (MPI) and  $\gamma \ll 1$  for tunneling ionization (TI)



## Consequences for ultrashort laser pulses

- that the value of  $\tau$  is determined by the frequency of the laser field. Regarding high frequencies for the applied field there should appear a frequency dependent tunneling probability.
- The ponderomotive potential  $U_p(t)$  defines a time varying relation for either the MPI or TI regime to dominate the ionization process.

**$\gamma$  should be interpreted in terms of the dominance of one process in respect to the other.**

## Towards High Harmonic Generation

The Keldysh interpretation allows calculation of ionization probabilities of atomic bound states in strong laser fields including excitations and resonances and even the dependence on the time evolution of the laser field in the *Quasi-Static-Approximation (QSA)*

$$W_K = \frac{\sqrt{6\pi}}{4} \frac{I_p}{\hbar} \sqrt{1 - \frac{e\mathcal{E}_0\hbar}{m^{1/2}I_p^{3/2}}} \times \exp \left[ -\frac{4\sqrt{2m}I_p^{3/2}}{3e\hbar\mathcal{E}_0} \left( 1 - \frac{m\omega^2 I_p}{5e^2\mathcal{E}_0^2} \right) \right]$$

### Limitations

- low-frequency approximation for the applied electric field
- the final state of the electron is a free electron oscillating in the laser field, which is known as a final nonperturbative Volkov state
- does not include any kind of species dependence in the ionization rate calculation

## ADK Theory (Ammosov, Delone and Krainov)

Description of the ionization of complex atoms and atomic ions in arbitrary states

The ionization rate equation in atomic units is given by

$$W_{m ADK} = |C_{n^*l^*}|^2 f_{lm} I_p \sqrt{\frac{6}{\pi}} \left( \frac{2(2I_p)^{3/2}}{\mathcal{E}} \right)^{2n^* - |m| - 3/2} \\
 \times \exp\left(-\frac{2(2I_p)^{3/2}}{3\mathcal{E}}\right)$$

with

$$f_{lm} = \frac{(2l+1)(l+|m|)!}{2^{|m|}|m|!(l-|m|)!} \\
 |C_{n^*l^*}|^2 = \frac{2^{2n^*}}{n^*\Gamma(n^*+l^*+1)\Gamma(n^*-l^*)}$$

## Ionization Probability

$$W_{mADK} = |C_{n^*l^*}|^2 f_{lm} I_p \sqrt{\frac{6}{\pi}} \left( \frac{2(2I_p)^{3/2}}{\mathcal{E}} \right)^{2n^* - |m| - 3/2} \times \exp\left(-\frac{2(2I_p)^{3/2}}{3\mathcal{E}}\right)$$

$n^*$  effective principal quantum number

$m$  magnetic quantum number

$l$  the angular momentum

$I_p$  is the atomic ionization potential

$E$  the electric field strength of the laser.

For ionization rate calculation the ground state values for  $n^*$  and  $l$  are mainly used, which leads to  $l^* = n^* - 1$ .

## Ionization rate equation

Averaging over all magnetic quantum numbers gives the complete ionization rate

$$W_{ADK} = \frac{1}{2l + 1} \sum_{m=-l}^l W_{mADK}$$

for a gaussian laser pulse

$$\mathcal{E}(t) = \mathcal{E}_0 \exp\left(-\frac{2 \ln 2 t^2}{\tau_{in}}\right) \cos(\omega_0 t)$$

the total number of ions created within a subfraction of the laser pulse starting to act on the atom at time  $t'$

$$\widetilde{W}_{frac} = \int_t^{+\infty} W_{ADK}(\mathcal{E}(t')) dt'$$

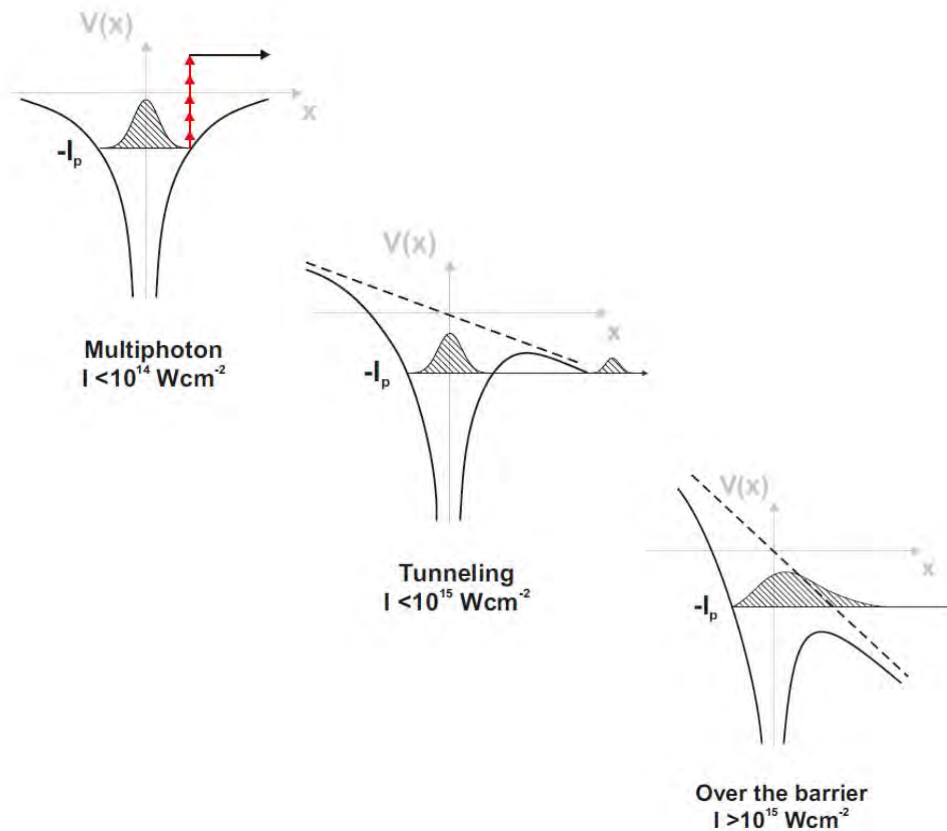
## Ionization rate in the quasistatic limit

$$\Gamma_{QS}(t) = 4 \frac{(2Ip)^{5/2}}{\mathcal{E}(t)} \times \exp \left[ -\frac{2(2Ip)^{3/2}}{3\mathcal{E}(t)} \right]$$

a critical laser intensity  $I_c$  can be defined describing the intensity at which the Coulomb barrier starts to be suppressed

$$\begin{aligned} I_c [W/cm^2] &= \frac{\pi^2 c \epsilon^3 I_p^4}{2 Z^2 e^6} \\ &= 4 \cdot 10^9 I_p^4 [eV] Z^2 \end{aligned}$$

## Laser driven ionization



Coulomb barrier suppression intensity

$$\begin{aligned}
 I_c [\text{W}/\text{cm}^2] &= \frac{\pi^2 c \epsilon^3 I_p^4}{2 Z^2 e^6} \\
 &= 4 \times 10^9 I_p^4 [\text{eV}] Z^2
 \end{aligned}$$

Keldysh parameter

$$\begin{aligned}
 \gamma = \tau \omega_L &= \omega_L \frac{\sqrt{2mI_p}}{e\mathcal{E}_l} \\
 &= \sqrt{\frac{I_p}{2U_P}}
 \end{aligned}$$

$\gamma \gg 1$  for *multi photon ionization (MPI)*  
 $\gamma \ll 1$  for *tunneling ionization (TI)*



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Polarisation Gating



## *High Harmonic Generation*

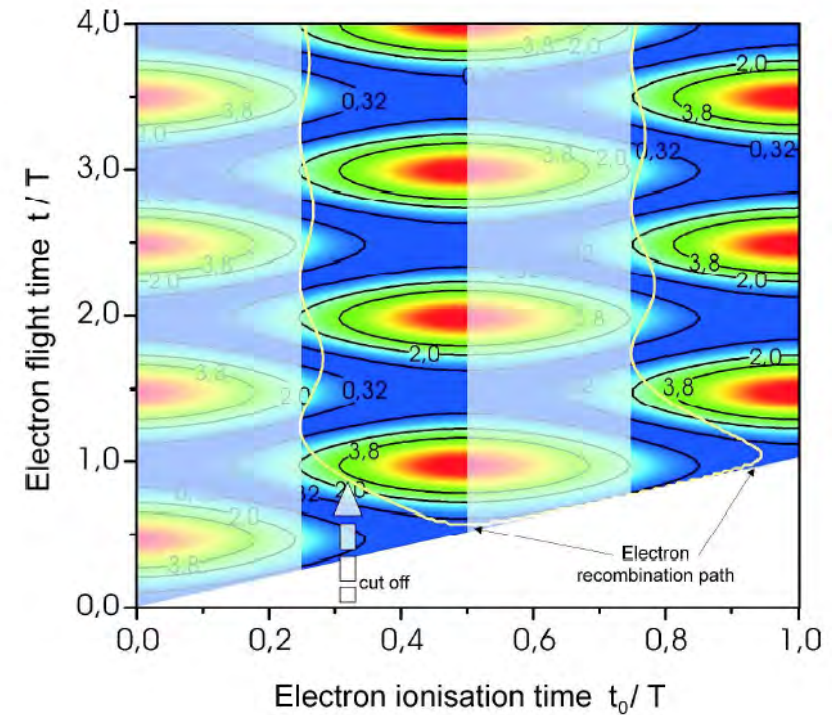
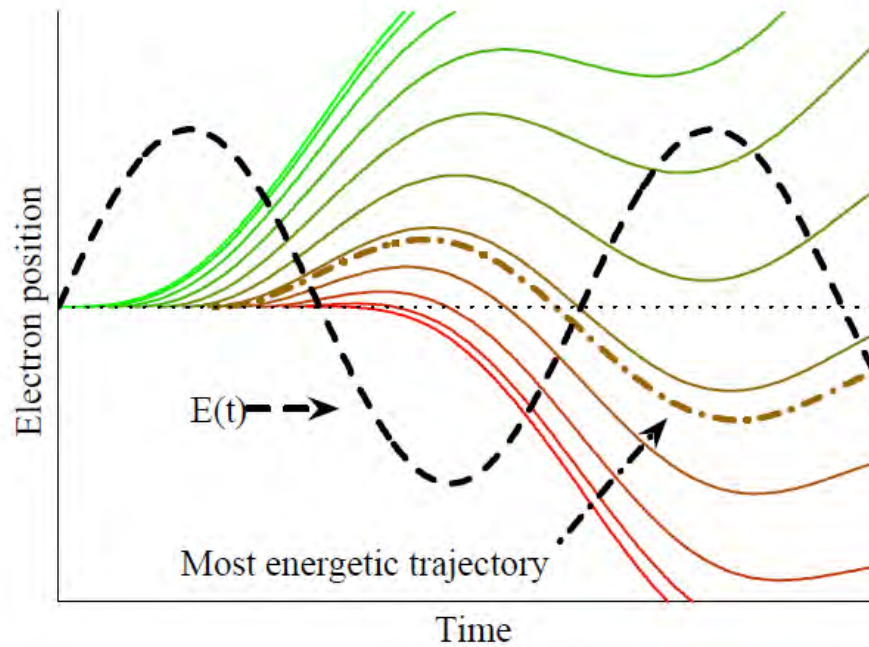
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Thorsten.Uphues@cfel.de



## propagation of a free electron in a laserelectric field

$$\begin{aligned}\ddot{x}(t) &= E_0 \cos \omega t \\ \dot{x}(t) &= \frac{E_0}{\omega} \sin \omega t - \frac{E_0}{\omega} \sin \omega t_0 \\ x(t) &= -\frac{E_0}{\omega^2} \cos \omega t - (t - t_0) \frac{E_0}{\omega} \sin \omega t_0 + \frac{E_0}{\omega^2} \cos \omega t_0,\end{aligned}$$

$t_0$  time at which the electron was released from the atom assumed that the electron is released with zero initial velocity.



The position of the electron as function of time for different "ionization times"  $t_0$ . The "most energetic trajectory" refers to the solution where the electron encounters the nucleus with the maximal kinetic energy

## three step model / Lewenstein 1994

$$i|\dot{\Psi}(\mathbf{x}, t)\rangle = \left[ -\frac{1}{2}\nabla^2 + V(\mathbf{x}) - E \cos(t)x \right] |\Psi(\mathbf{x}, t)\rangle$$

The basic assumptions for this formulation of HHG are:

- the contribution to the evolution of the system of all bound states except the ground state  $|0\rangle$  can be neglected
- the depletion of the ground state can be neglected since  $U_p < U_{sat}$ ,  $U_{sat}$  being the saturation energy that completely ionizes the atom in one optical period
- in the continuum the electron can be treated as a free particle moving in the electric field with no effect of  $V(x)$

## The cut-off law within the Lewenstein model

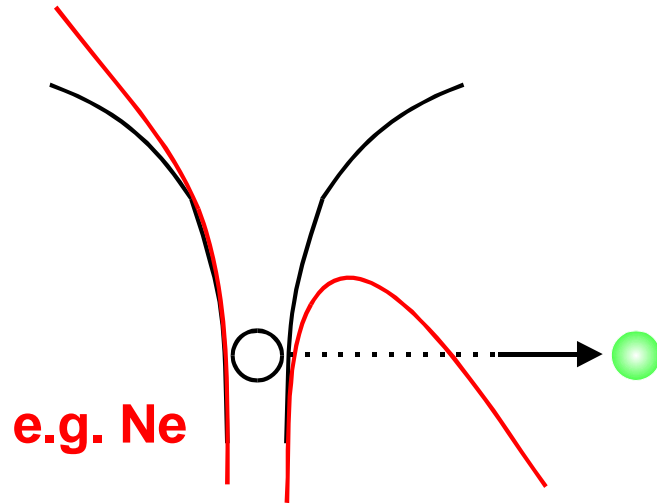
As a result the maximum energy that can be extracted from an electron in the ponderomotive potential of the laser electric field is given by

$$E_{hh}^{cutoff} = I_p + 3.17 \cdot U_p$$

As a result of the more accurate quantum mechanical calculations the *cutoff law* has been corrected giving

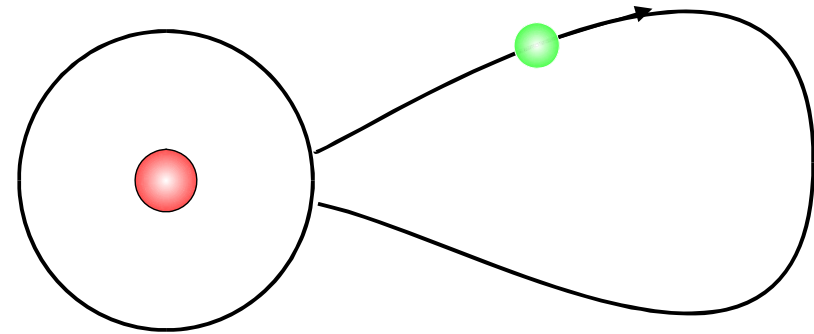
$$E_{hh}^{cutoff} = 3.17 U_p + I_p \cdot F(I_p/U_p)$$

For  $I_p \ll U_p \implies F(I_p/U_p) \simeq 1.32$ .

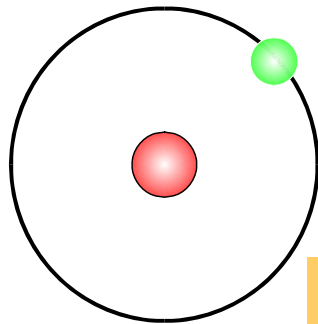


**1. Tunnel-Ionisation**  
 at  $> 10^{14} \text{ W/cm}^2$

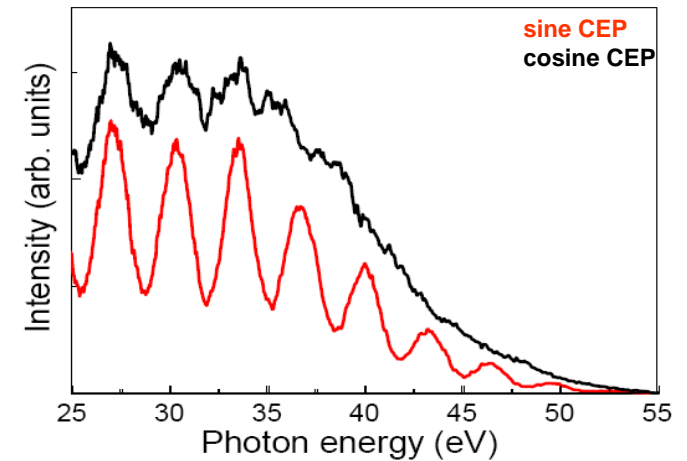
**2. Electron acceleration**  
 in laser electric field



**3. Return to parent ion**  
 and recombination



$$h\nu_X = n * h\nu_{\text{Laser}} \quad (n \text{ uneven})$$

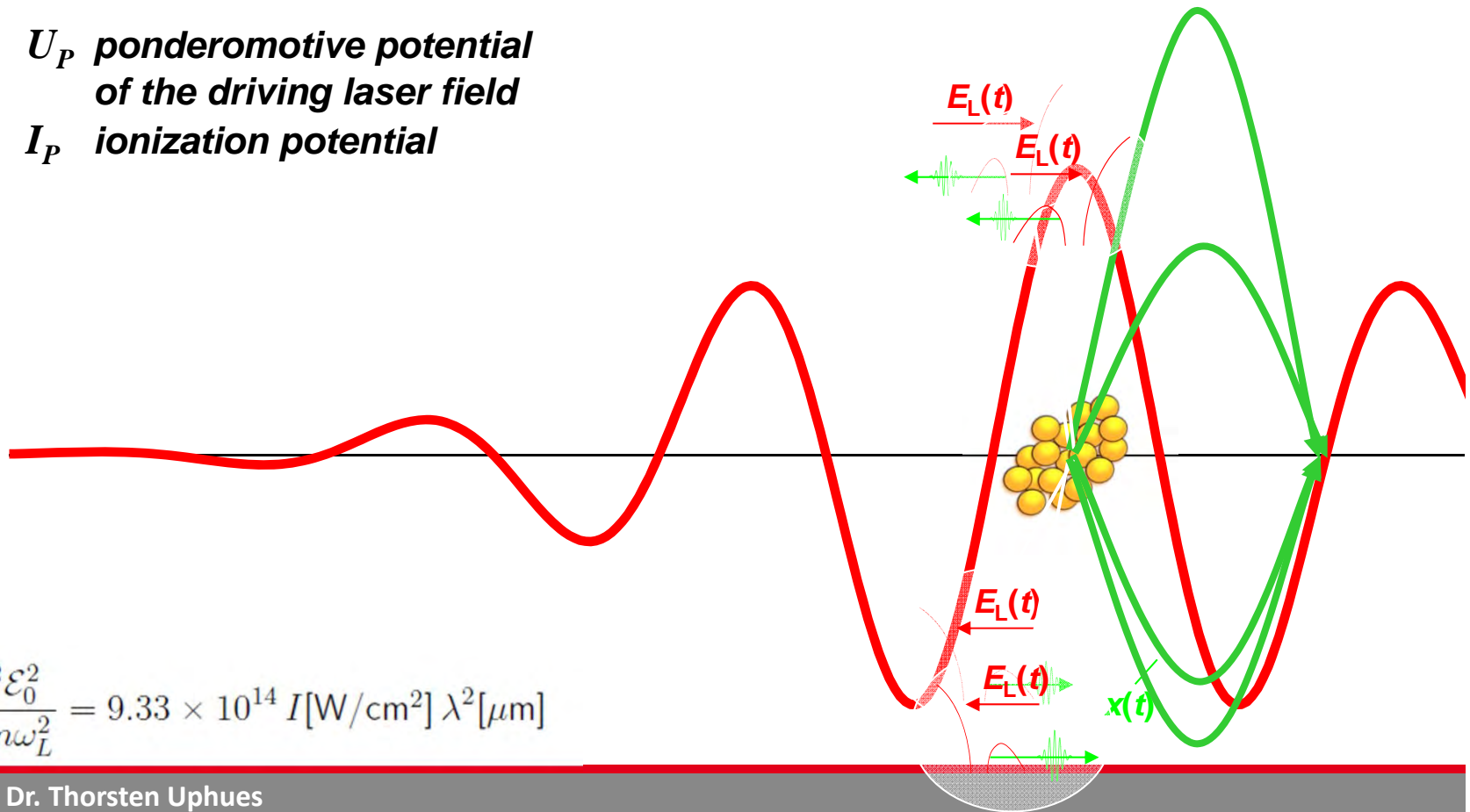


$$E_{hh}^{cutoff} = 3.17 U_p + I_p \cdot F(I_p/U_p)$$

$$I_p \ll U_p \implies F(I_p/U_p) \simeq 1.32$$

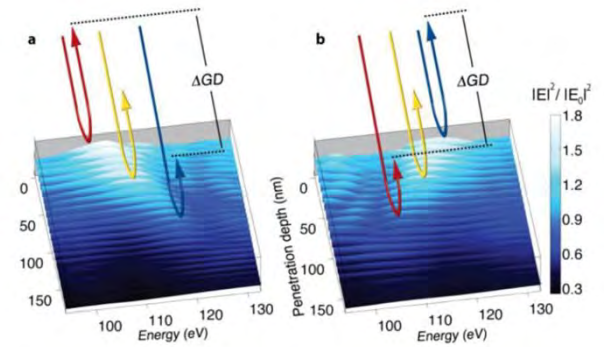
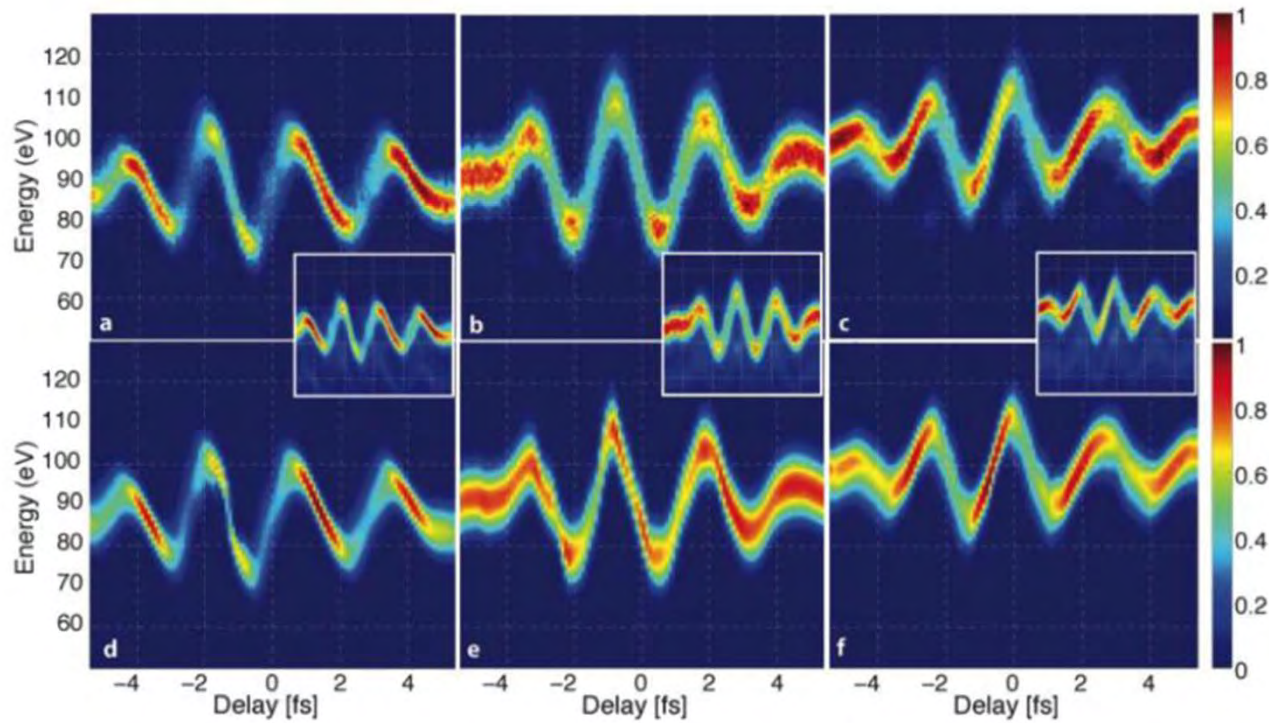
$U_p$  ponderomotive potential  
 of the driving laser field  
 $I_p$  ionization potential

## Maximum Energy Gain

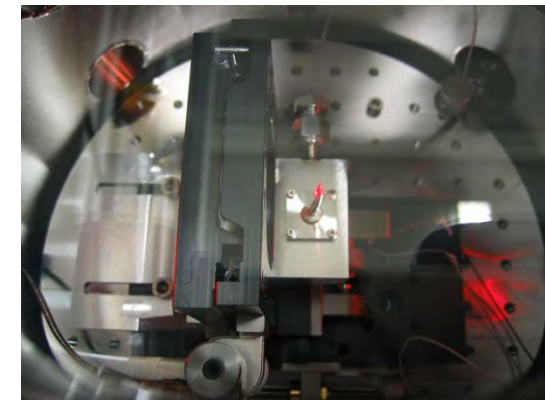
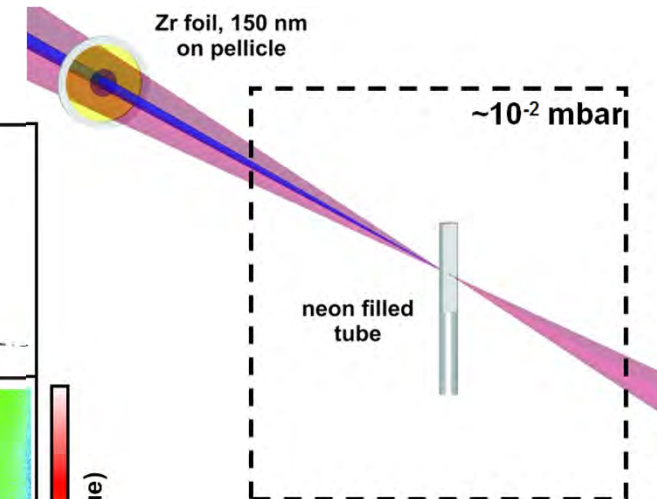
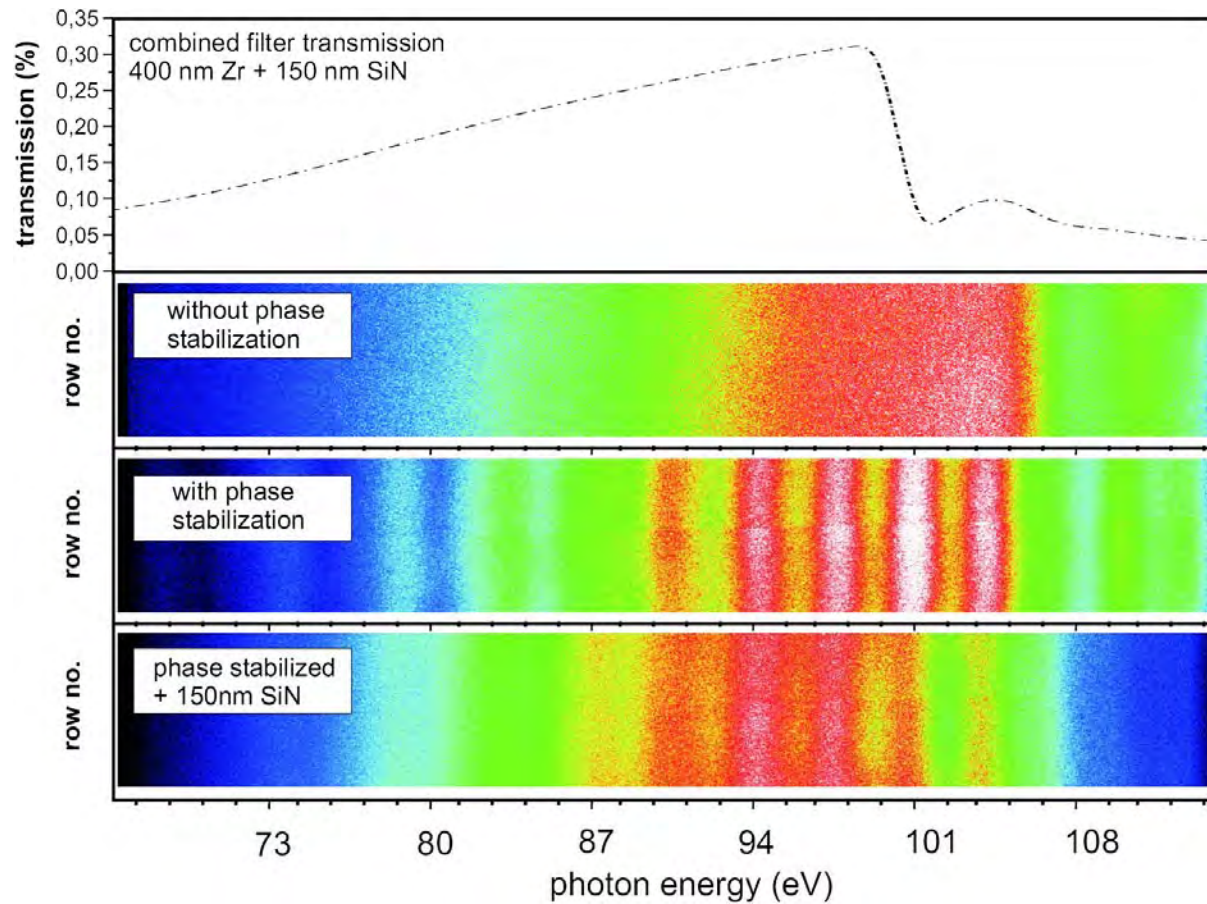


$$U_p[\text{eV}] = \frac{e^2 \mathcal{E}_0^2}{4 m \omega_L^2} = 9.33 \times 10^{14} I[\text{W}/\text{cm}^2] \lambda^2[\mu\text{m}]$$

# harmonics are chirped!

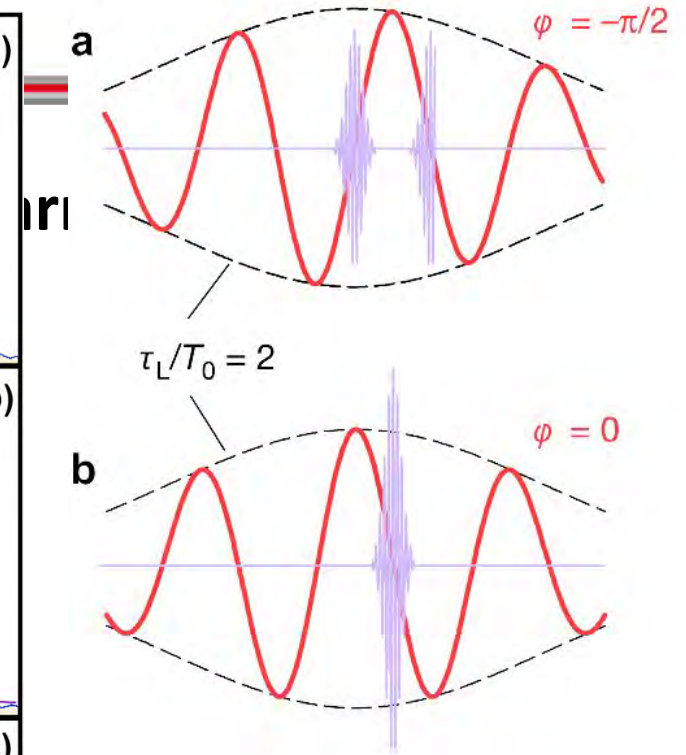
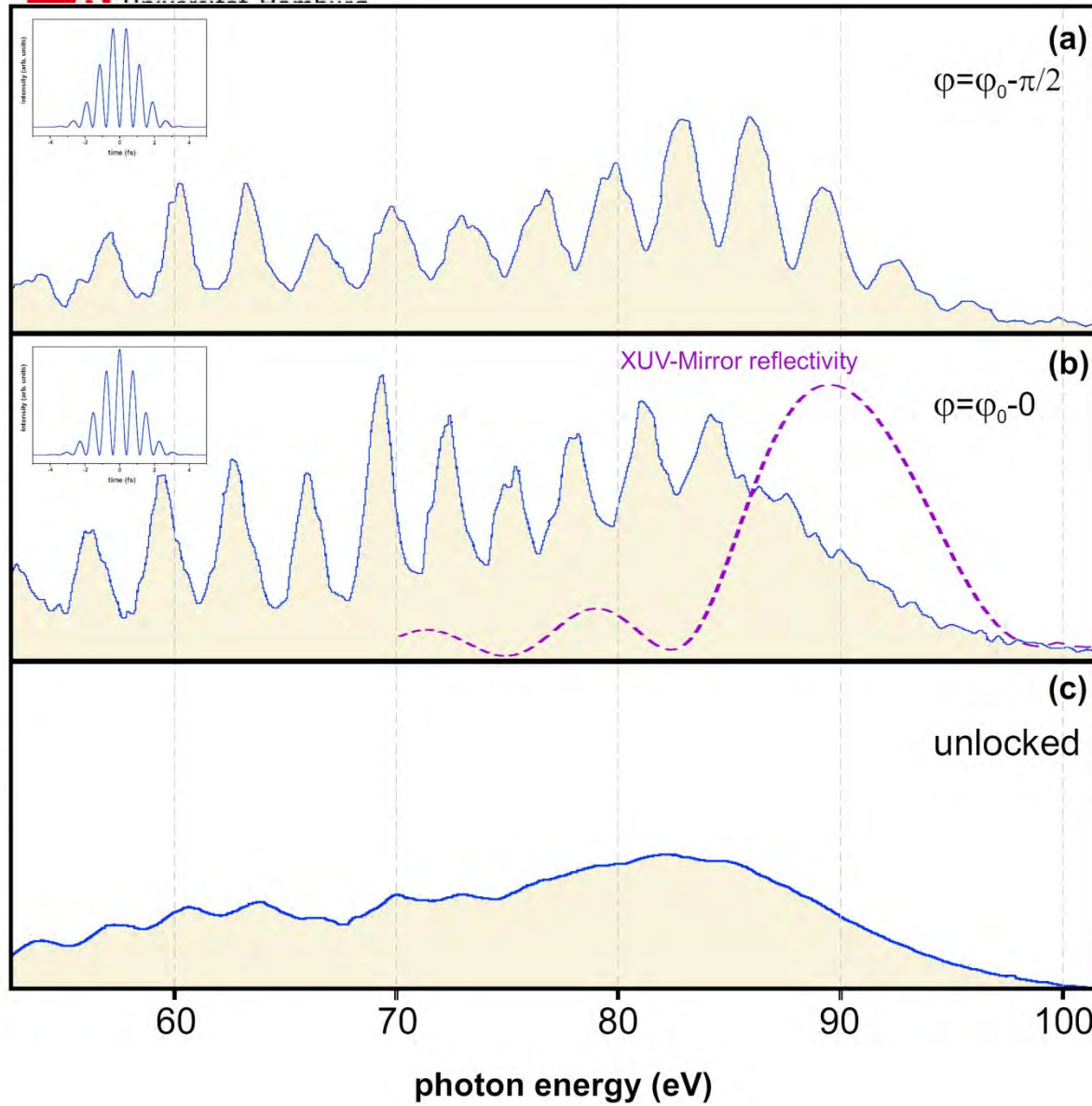


# HHG Generation





soft x-ray spectral intensity (arb. units)



**Careful adjustment of the Carrier Envelope Phase (CEP) necessary to generate isolated attosecond pulses**

## conversion efficiency

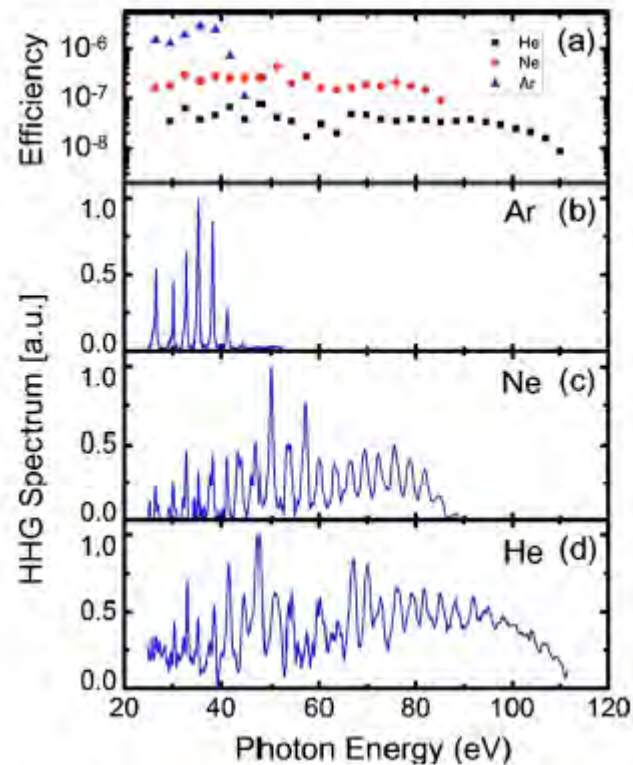


FIG. 2. (Color online) HHG driven by 800 nm, 35 fs driver pulses: (a) conversion efficiencies for Ar (50 mbar, 0.6 mJ), Ne (300 mbar, 2.0 mJ), and He (2.0 bar, 2.0 mJ) using a nozzle positioned 1 cm before the focus for all the gases; (b)-(d) HHG spectra, respectively, for Ar, Ne, and He.

$$\eta = 0.0236 \frac{\sqrt{2I_p\omega_0^5}|a_{\text{rec}}|^2|g(\Delta k, L)|^2}{E_0^{16/3}\Omega_{\text{cutoff}}^2\sigma^2(\Omega_{\text{cutoff}})} \frac{1 - \beta^{4(N-1)}}{(1 - \beta^4)N} \times |1 + \beta|^2 \kappa_0 w[E(tb_{\text{cutoff}})],$$

where

$$g(\Delta k, L) = [e^{i(\Delta k \cdot L)} - e^{-L/(2 \cdot L_{\text{abs}})}] / [1 + 2i(\Delta k \cdot L_{\text{abs}})]$$

phase-mismatch form factor with a phase mismatch of  $\Delta k$ .

Falcão-Filho, (2010). *Appl. Phys. Lett.* 97(6), 061107



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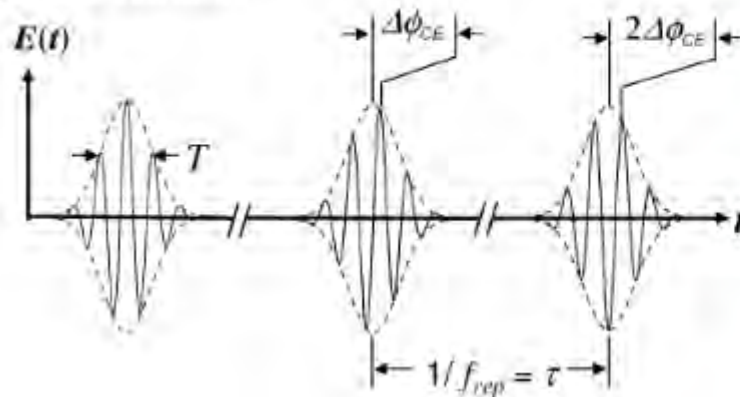


# CEP Stabilization

Jun.-Prof. Dr. Thorsten Uphues  
Thorsten.Uphues@cfel.de

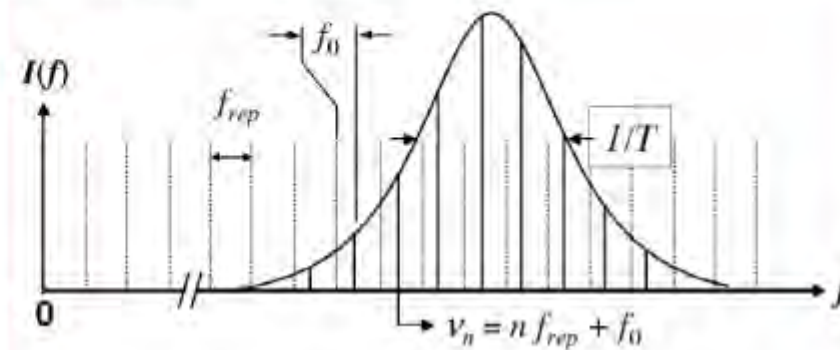
## CE Phase

a) Time domain



IEEE J. Sel. Top. Quant. Electron. 9,1002 (2003)

(b) Frequency domain



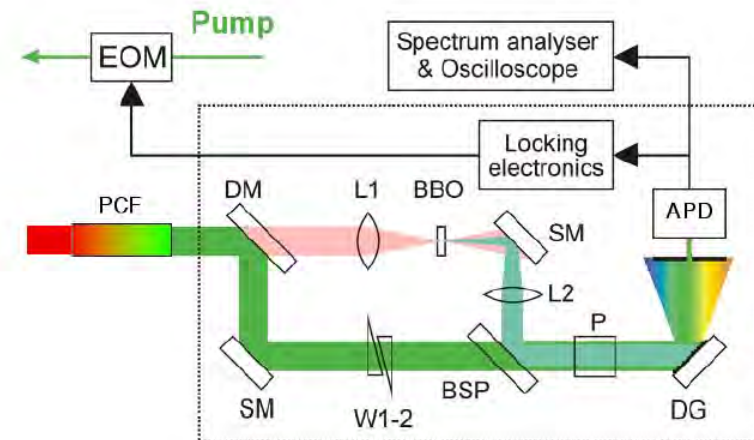
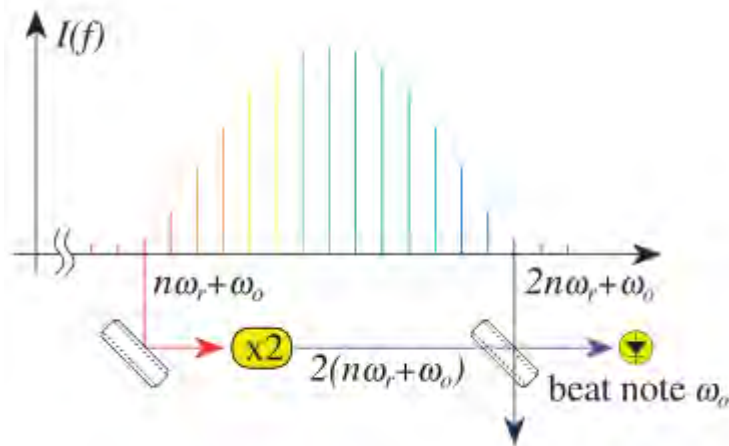
$$f_0 = f_{rep} \cdot \Delta\varphi_{CE} / 2\pi$$

### ■ Oscillator output

- Comb = modes
- $f_0$  = comb frequency **offset**
- $\Delta\varphi_{CE}$  = CE **Offset (CEO)**

Frequency domain  
representation

## stabilization of the CEP phase



$$E_{\text{fund}}(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \sqrt{I_{\text{fund}}(\omega)} \exp[i(\varphi_{\text{fund}}(\omega) - \omega t + \phi)] d\omega + cc$$

$$E_{\text{SH}}(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \sqrt{I_{\text{SH}}(\omega)} \exp[i(\varphi_{\text{SH}}(\omega) - \omega(t + \tau) + 2\phi)] d\omega + cc$$

$$S(\omega) = I_{\text{fund}}(\omega) + I_{\text{SH}}(\omega) + 2\sqrt{I_{\text{fund}}(\omega)I_{\text{SH}}(\omega)} \cos(\varphi_{\text{SH}}(\omega) - \varphi_{\text{fund}}(\omega) + \omega\tau + \phi)$$

## ■ f-2f interferometer

### □ Spectral broadening

- $\nu_n$  and  $\nu_{2n}$
- CEP preserving?

### □ Frequency doubling of “red” part

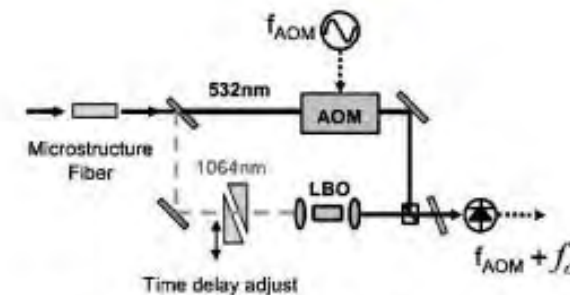
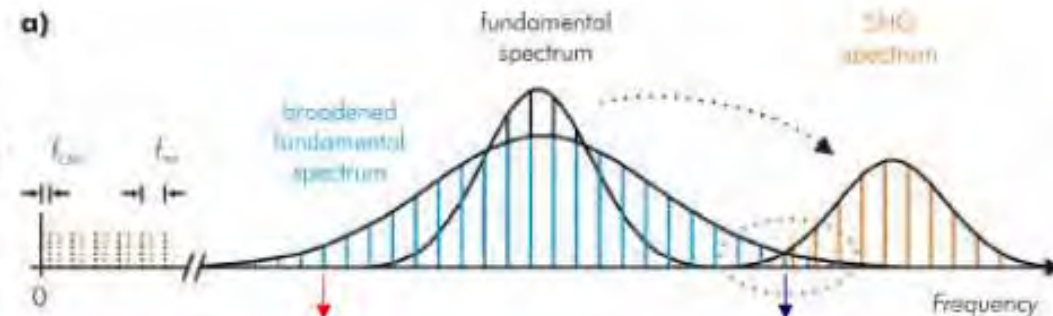
$$\nu_n = n \cdot f_{rep} + f_0 \rightarrow 2\nu_n = 2(n \cdot f_{rep} + f_0)$$

### □ Beat note with “blue” part

$$2\nu_n - \nu_{2n} = 2(n \cdot f_{rep} + f_0) - (2n \cdot f_{rep} + f_0) = f_0$$

- Mach-Zehnder interferometer
- Single point detection

### □ Self referencing: loss of absolute phase value



J. Opt. Soc. Am. B 21, 1098 (2004)

## ■ Broadening in PCF

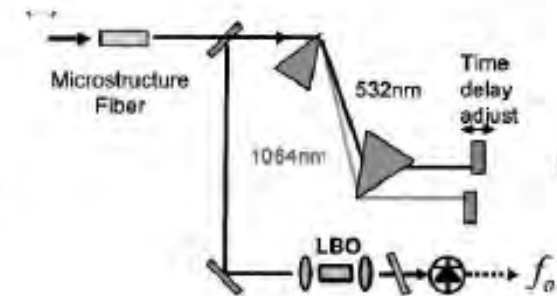
- AM/PM in fiber *Opt. Lett.* 27, 445 (2002)
- Octave spanning oscillators

## ■ MZ stability

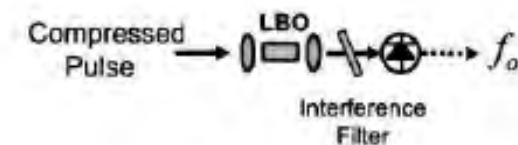
- Michelson *Opt. Lett.* 31, 1011 (2006)
  - Still bulky
- Common path interferometer *J. Opt. Soc. Am. B* 21, 1098 (2004)
  - Small group delay compensation range
- Wollaston prisms *Opt. Lett.* 35, 1209 (2010)
- Stabilization *Opt. Express* 14, 9758 (2006)

## ■ 2 steps in 1

- Broadening + SHG in ZnO layers *Opt. Lett.* 27, 2127 (2002)
- Broadening + DFG in PPLN *Opt. Lett.* 30, 332 (2005)
  - No fiber, no interferometer
  - Useful spectrum
  - <7fs pulses



*J. Opt. Soc. Am. B* 21, 1098 (2004)



## alternative measurement method: stereo ATI

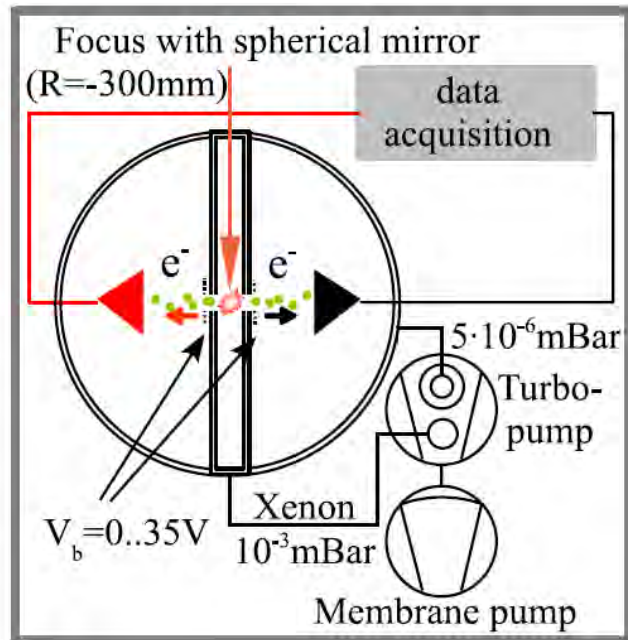
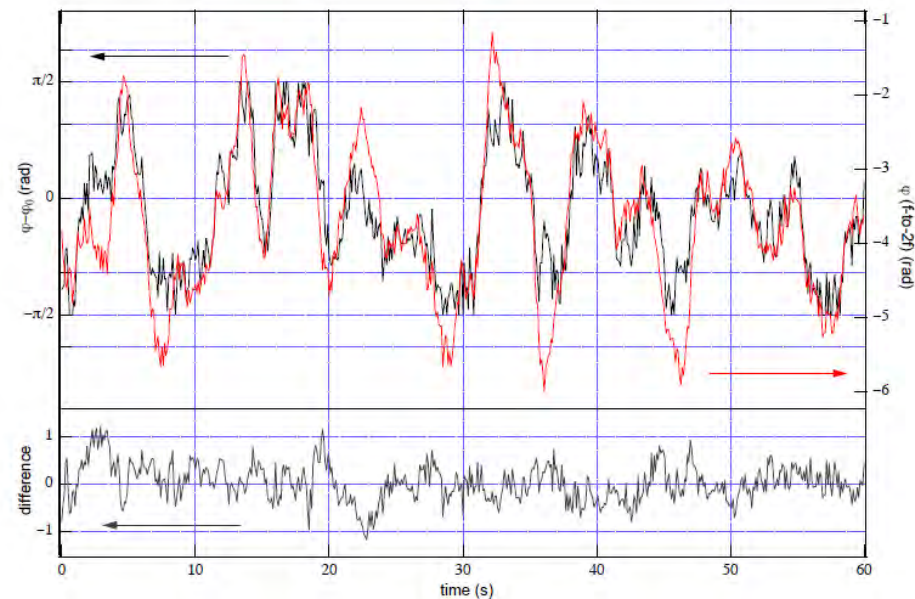


Figure 2.8: Schematic of the mini stereo ATI apparatus.

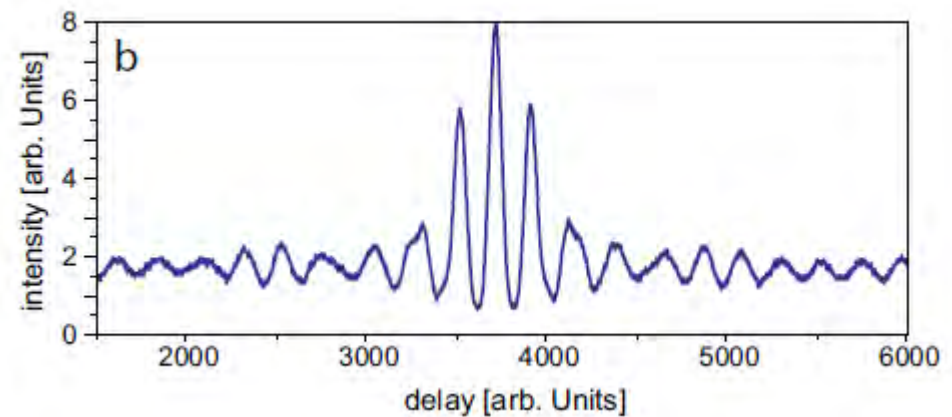
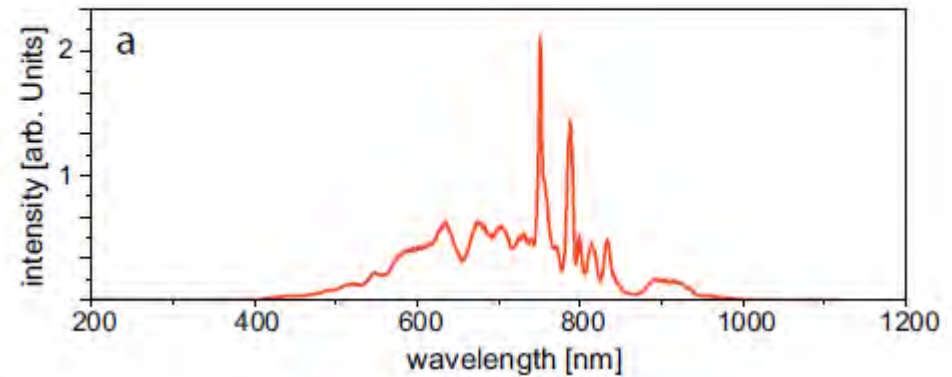
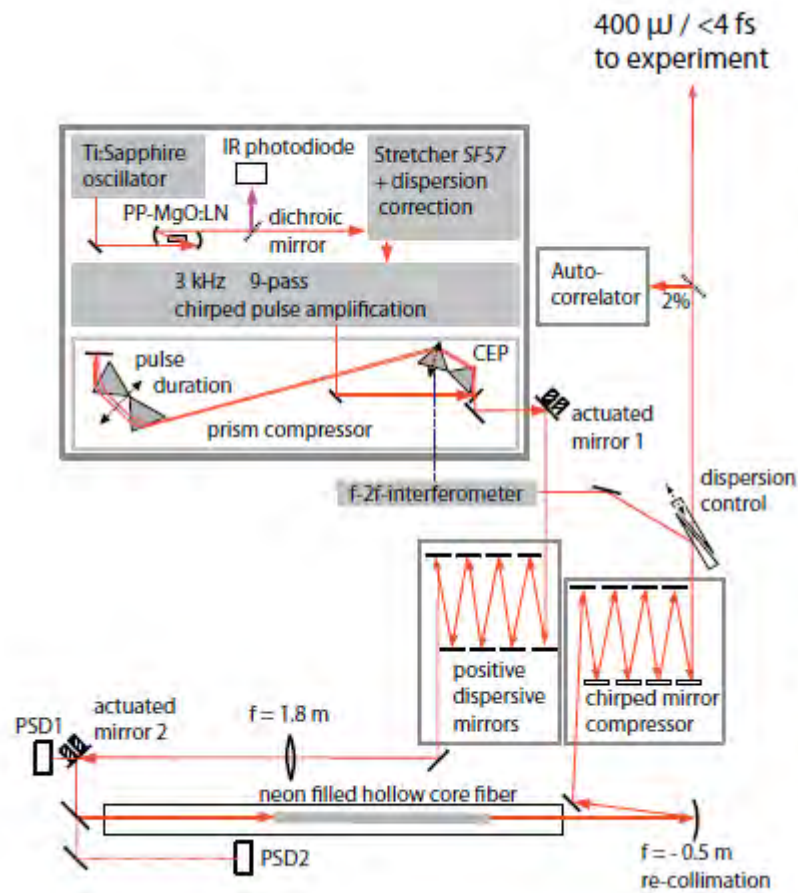
$$\phi = \arcsin \left( A \frac{N_{\text{left}} - N_{\text{right}}}{N_{\text{left}} + N_{\text{right}}} \right) + \phi_0$$

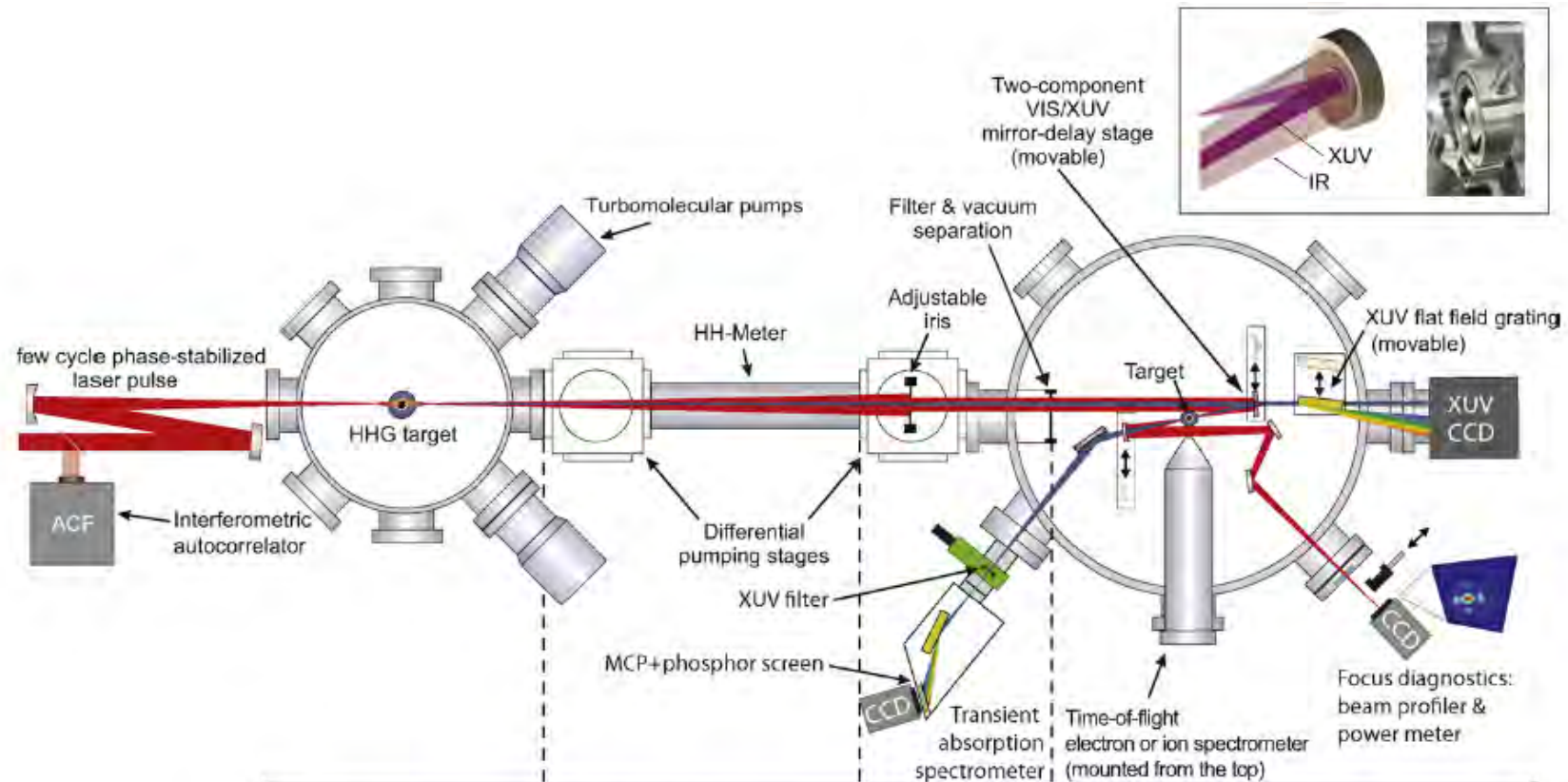


red: f-to-2f    black: Stereo ATI



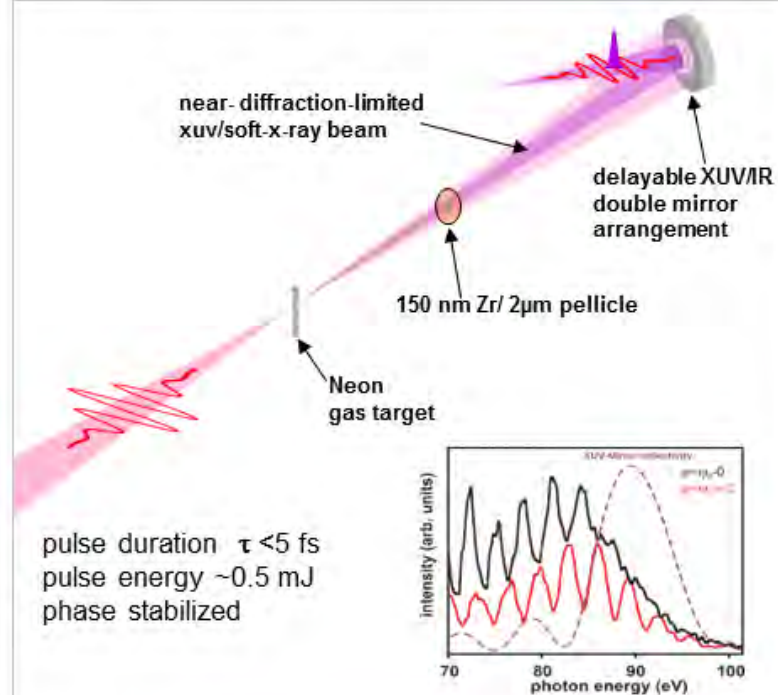
# typical laser systems for sub-fs spectroscopy





Background pressure: $5 \times 10^{-7}$ mbar	Background pressure: $< 1 \times 10^{-6}$ mbar	Background press.: $< 5 \times 10^{-4}$ mbar	Background pressure: $< 1 \times 10^{-4}$ mbar
Operating pressure: $< 3 \times 10^{-2}$ mbar HHG target pressure: $\sim 350$ mbar	Operating pressure: $< 4 \times 10^{-4}$ mbar	Operating pressure: $< 1 \times 10^{-6}$ mbar	Operating pressure: $< 1 \times 10^{-5}$ mbar (electron and ion spectroscopy) $1 \times 10^{-3}$ mbar (XUV absorption spectroscopy)

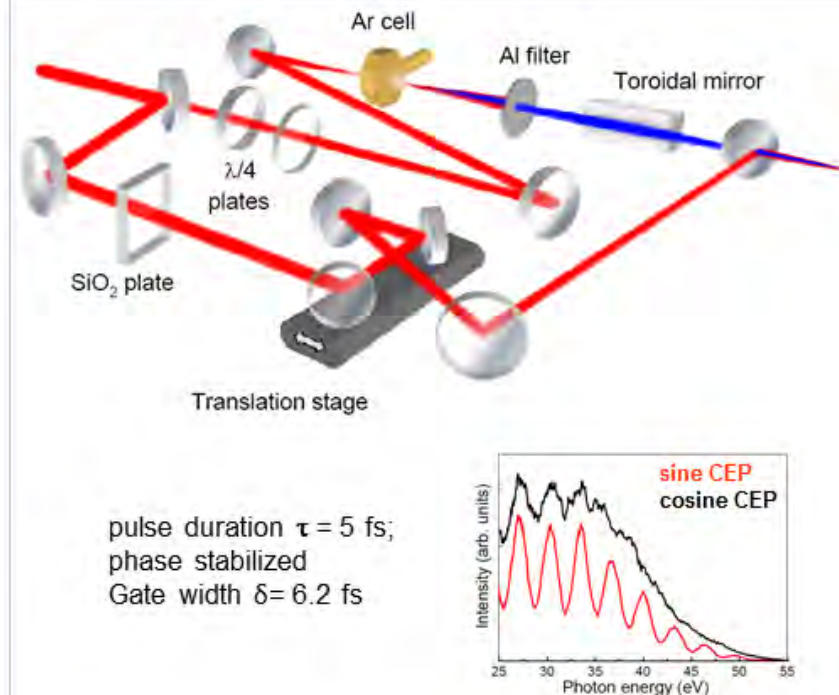
### bandwidth filter for cut-off hamonics



- Co-Linear propagation
- high photon energy
- virtually jitter-free setup
- XUV bandwidth determined by pulse duration
- cut-off very sensitive on pulse fluctuations
- minimum pulse duration determined by mirror reflectivity

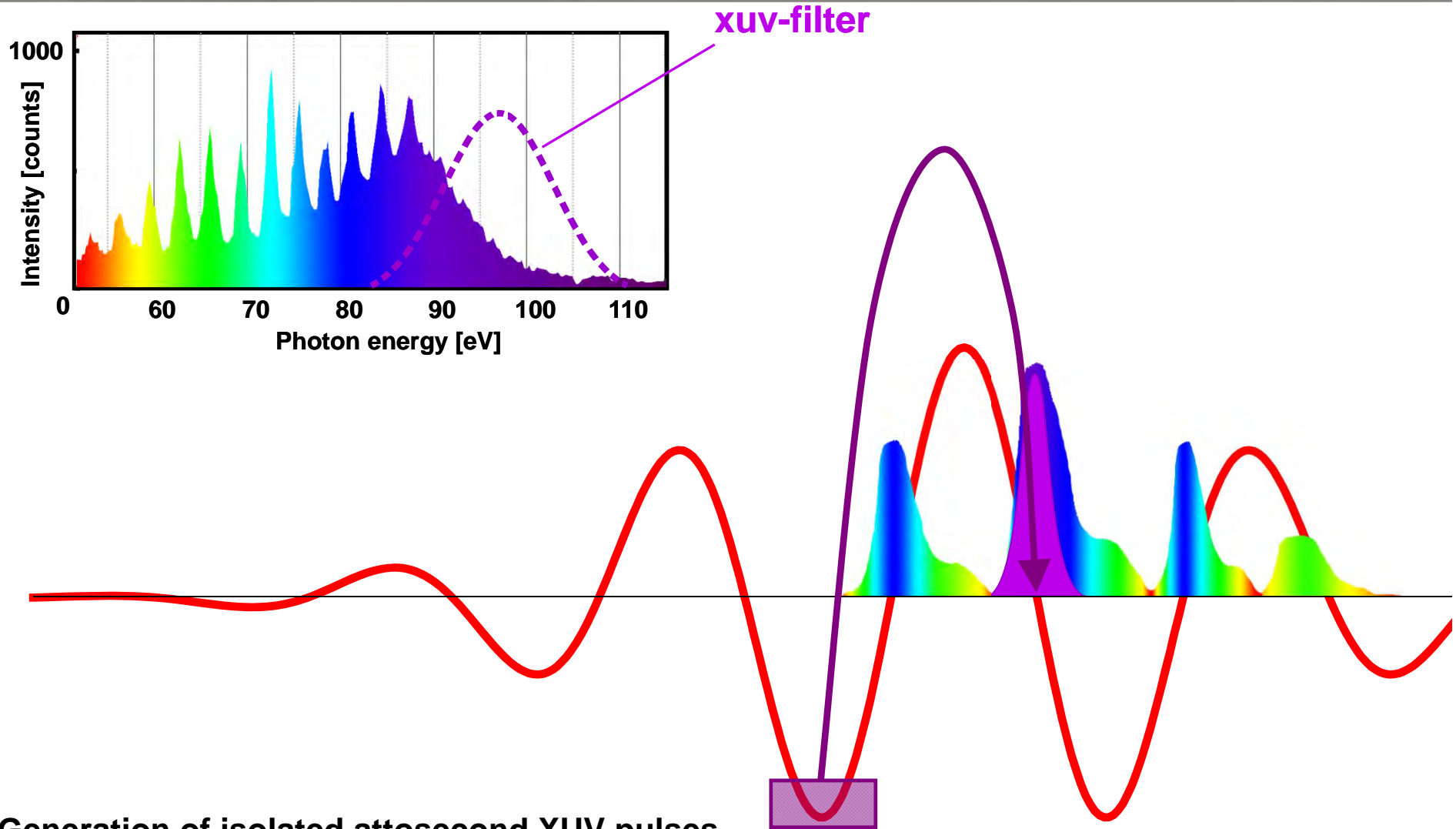


### Polarization Gating



- interferometric setup
- high intensity losses due to the technique
- low photon energy
- XUV bandwidth determined by gate width
- pulse duration only limited by low pass filter
- sensitive on mechanical vibrations





## Generation of isolated attosecond XUV pulses

## Multilayer mirror – principle of constructive interference

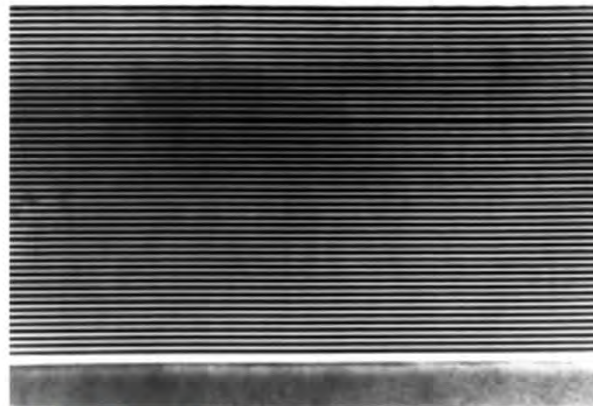
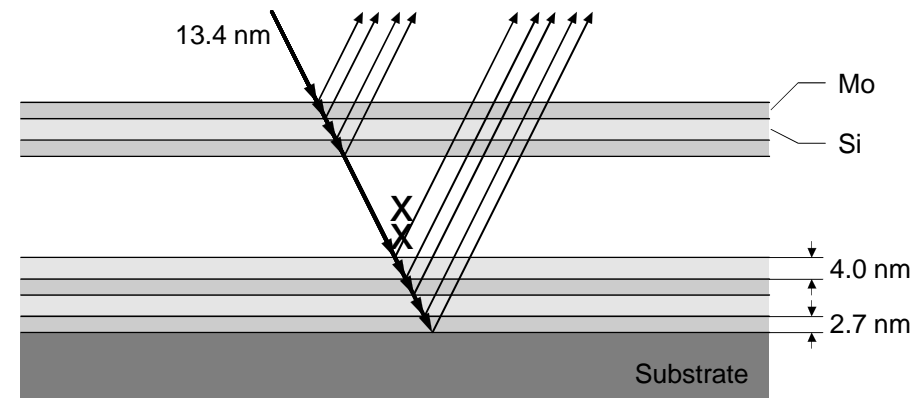
**Bragg:**

$$\lambda = 2d \cos \alpha$$

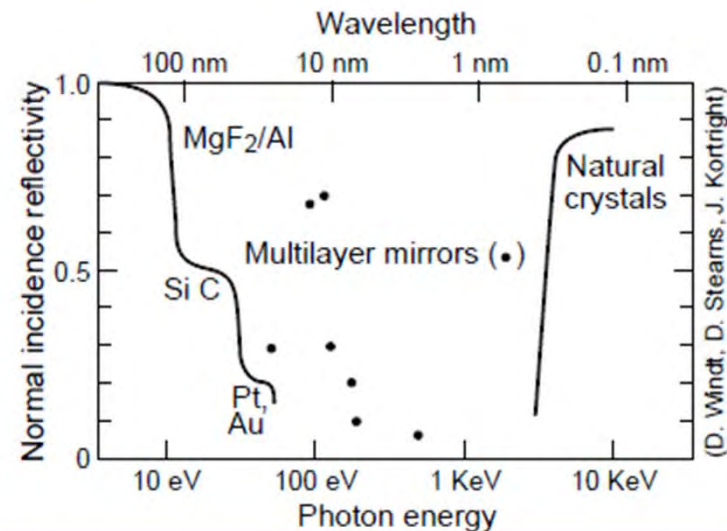
$$d \sim 1/\cos \alpha$$

$d$  ... optical thickness of period

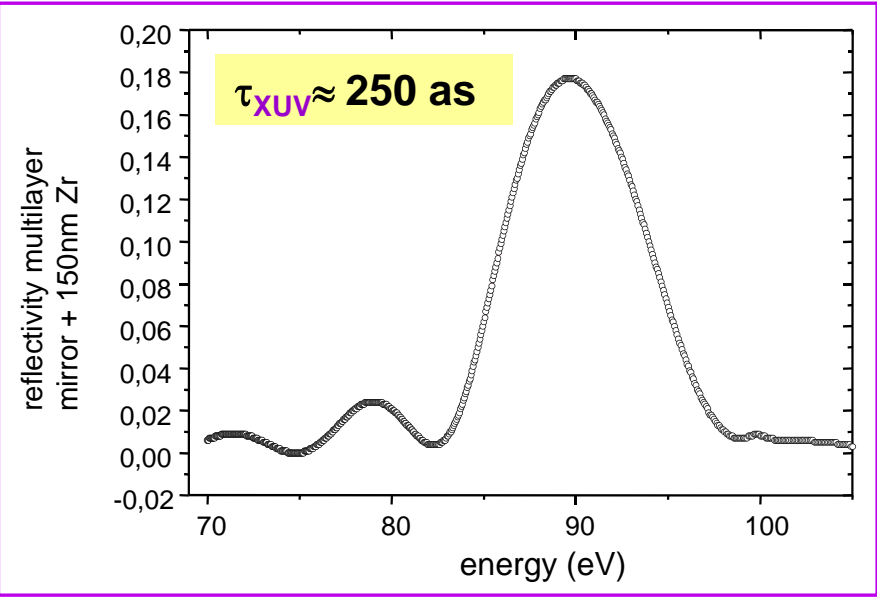
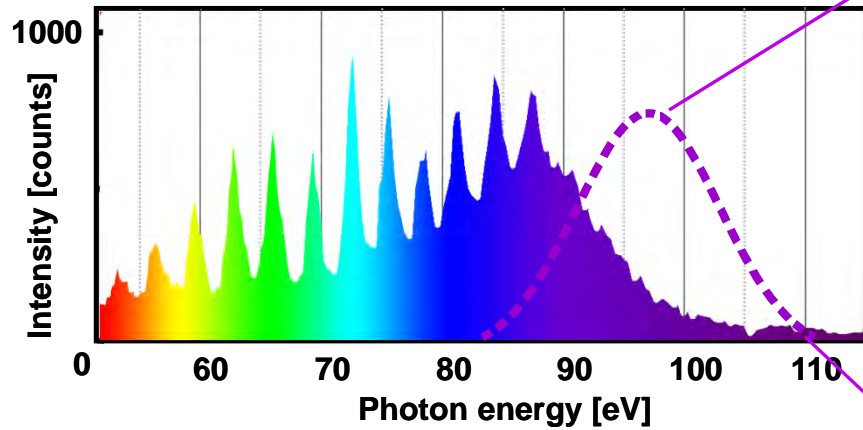
$\alpha$  ...angle of incidence

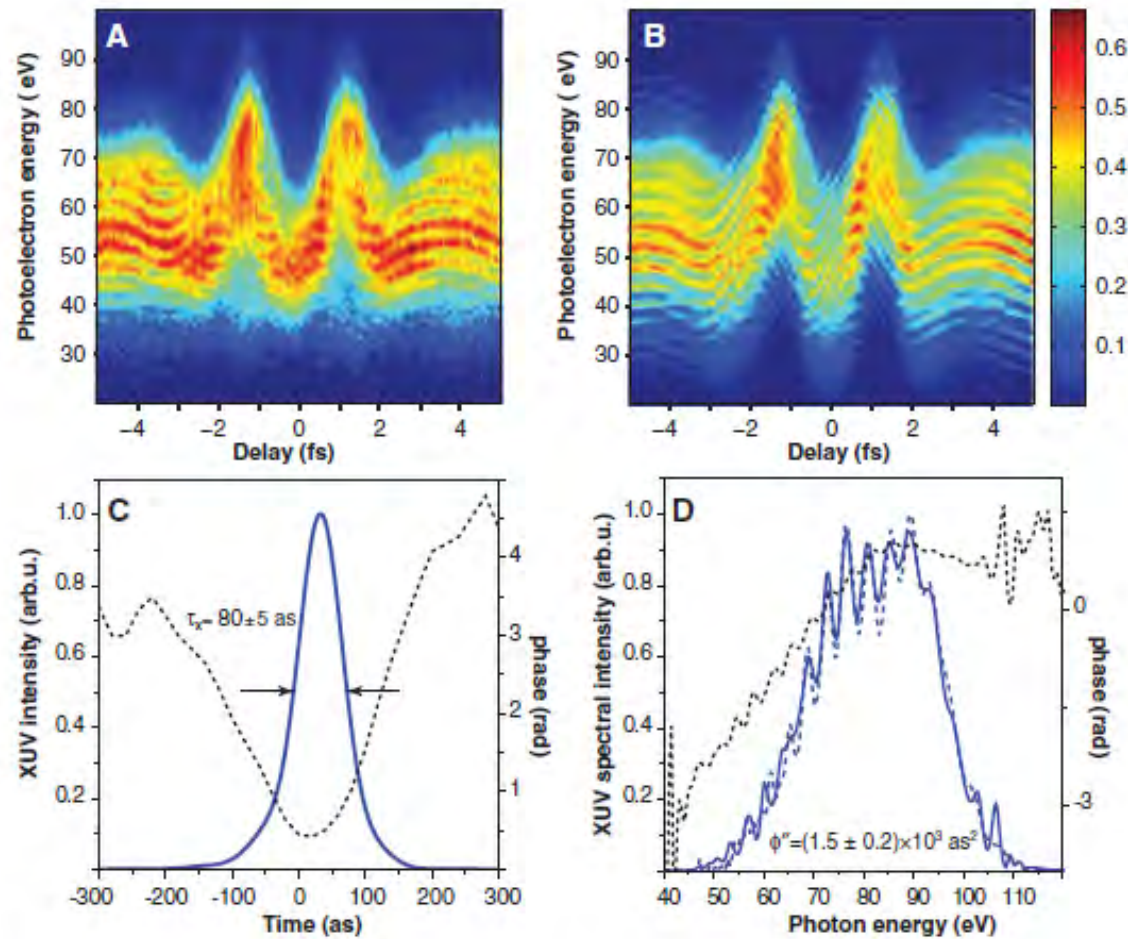


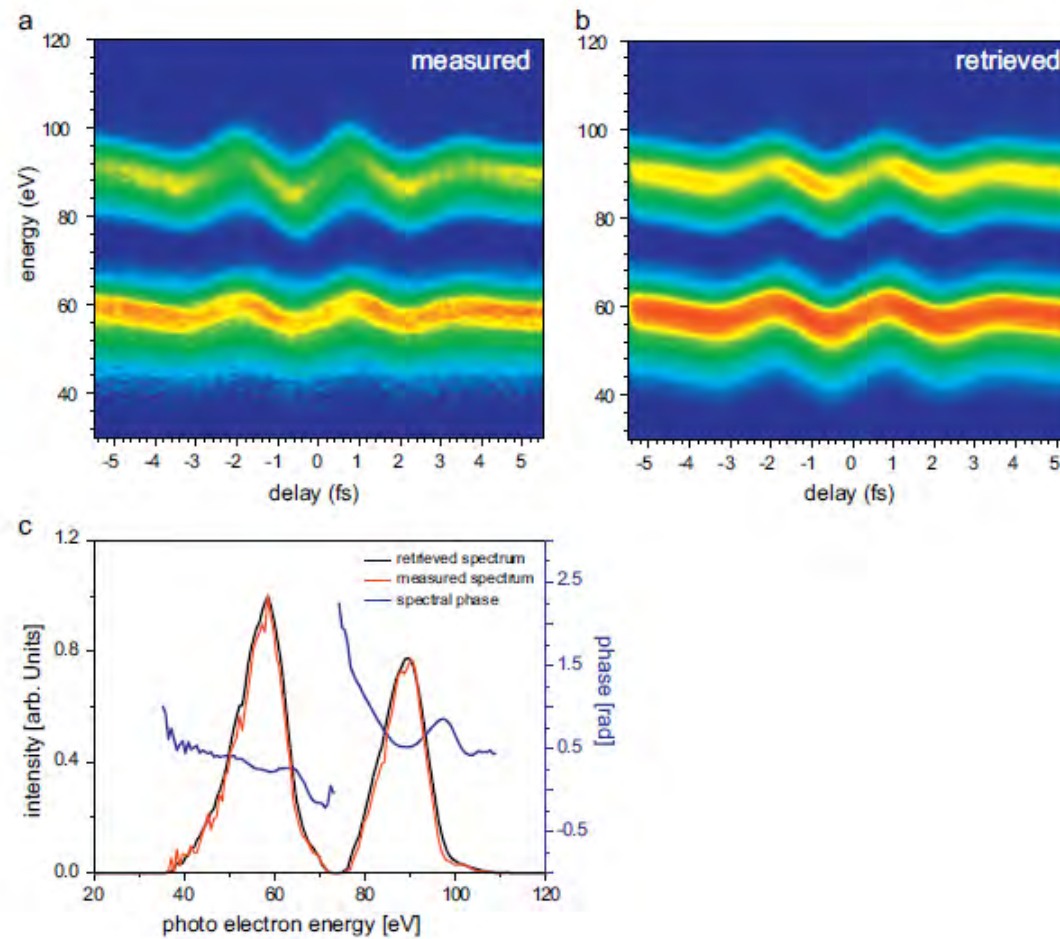
(W/C, T. Nguyen)



(D. Windt, D. Stearns, J. Kortright)

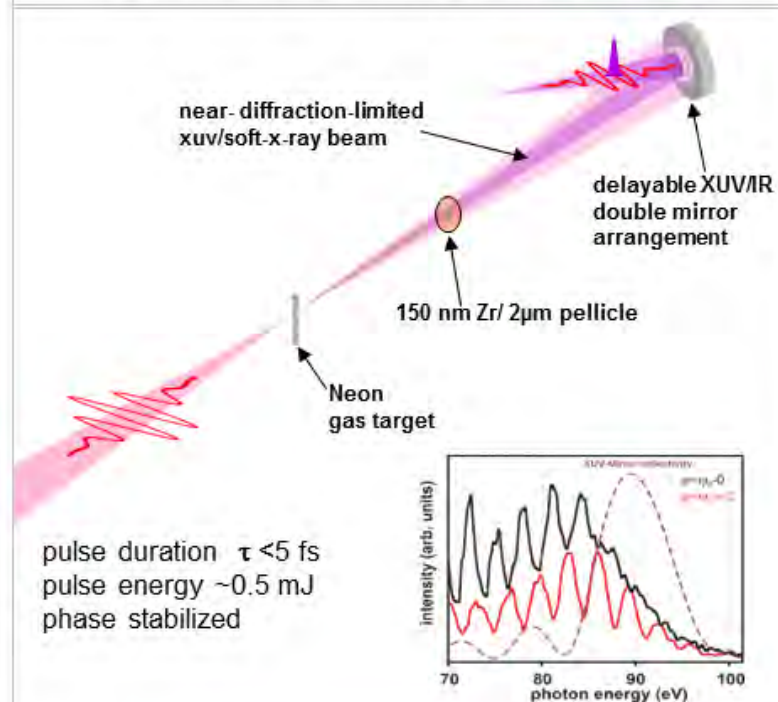








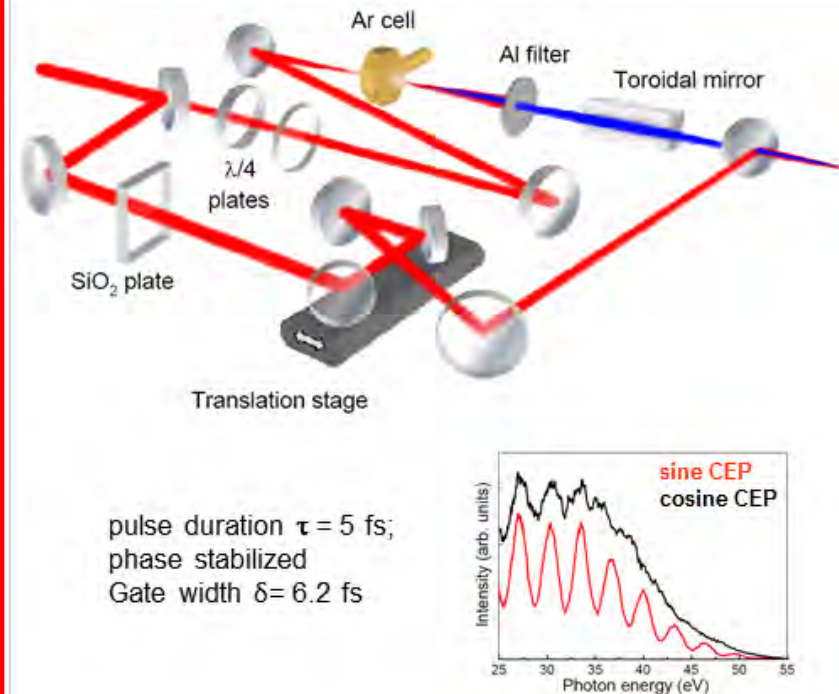
### bandwidth filter for cut-off hamonics



- Co-Linear propagation
- high photon energy
- virtually jitter-free setup
- XUV bandwidth determined by pulse duration
- cut-off very sensitive on pulse fluctuations
- minimum pulse duration determined by mirror reflectivity



### Polarization Gating



- interferometric setup
- high intensity losses due to the technique
- low photon energy
- XUV bandwidth determined by gate width
- pulse duration only limited by low pass filter
- sensitive on mechanical vibrations





Universität Hamburg

DER FORSCHUNG | DER LEHRE | DER BILDUNG

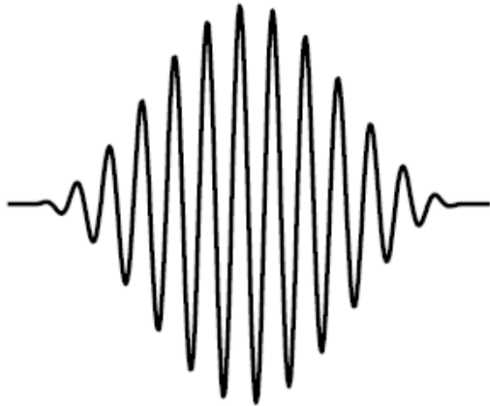
Polarisation Gating



*Generation of isolated  
Attosecond Pulses by  
„Polarization Gating“*

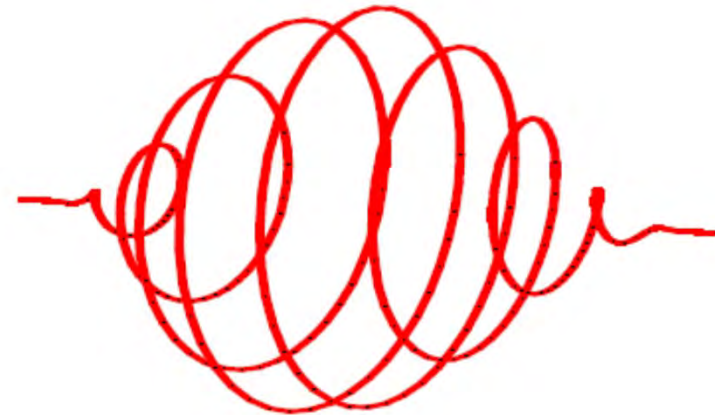
Jun.-Prof. Dr. Thorsten Uphues  
Thorsten.Uphues@cfel.de

linear polarisation



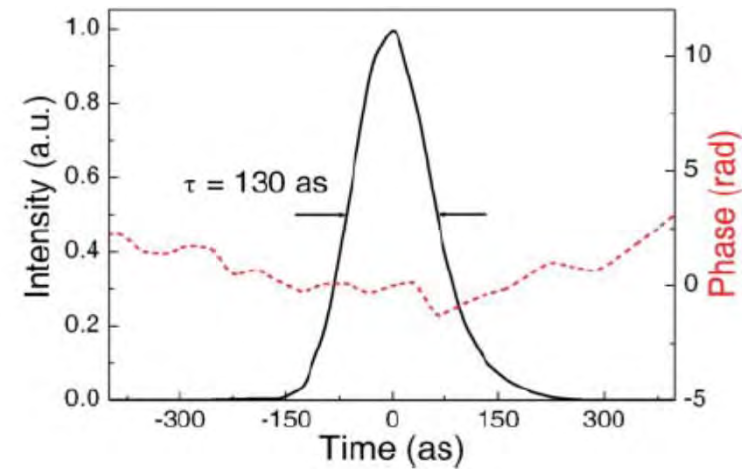
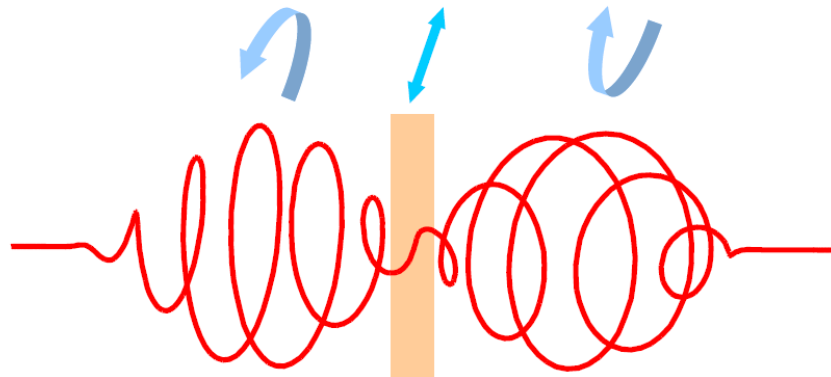
electron returns to the parent ion:  
emission of high harmonic radiation  
**possible**

circular polarisation



electron does not  
return to the parent ion:  
emission of high harmonic radiation  
**hardly suppressed**

## time-dependent polarisation gate



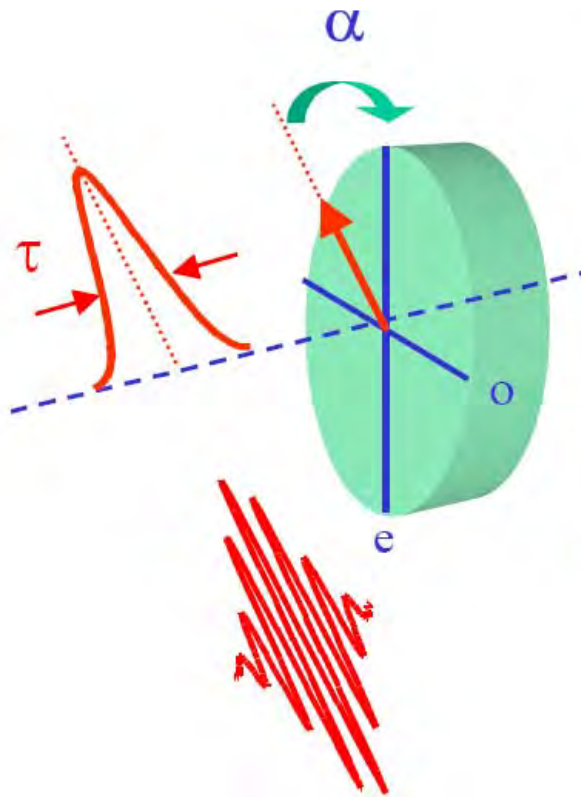
Generation of XUV continuum with PG applying:

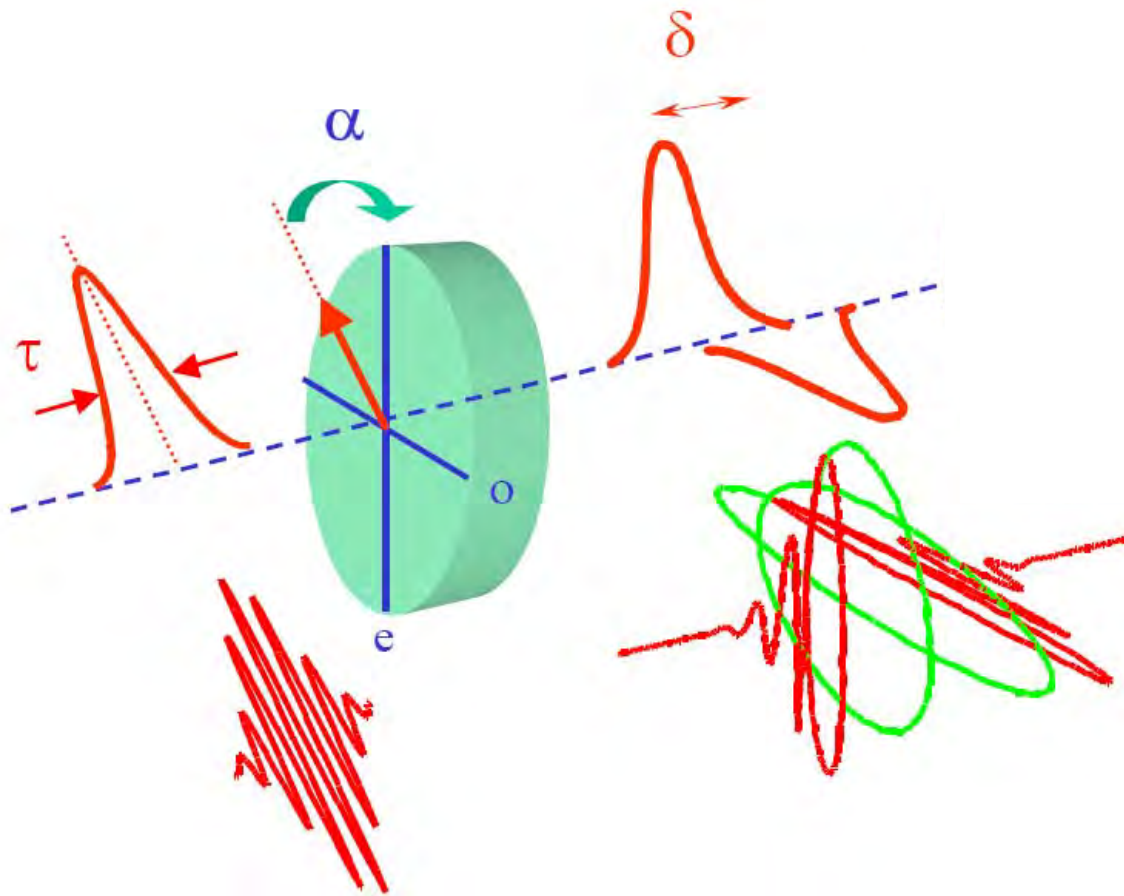
- few-cycle pulses
- Carrier Envelope Phase stabilization
- dispersion control

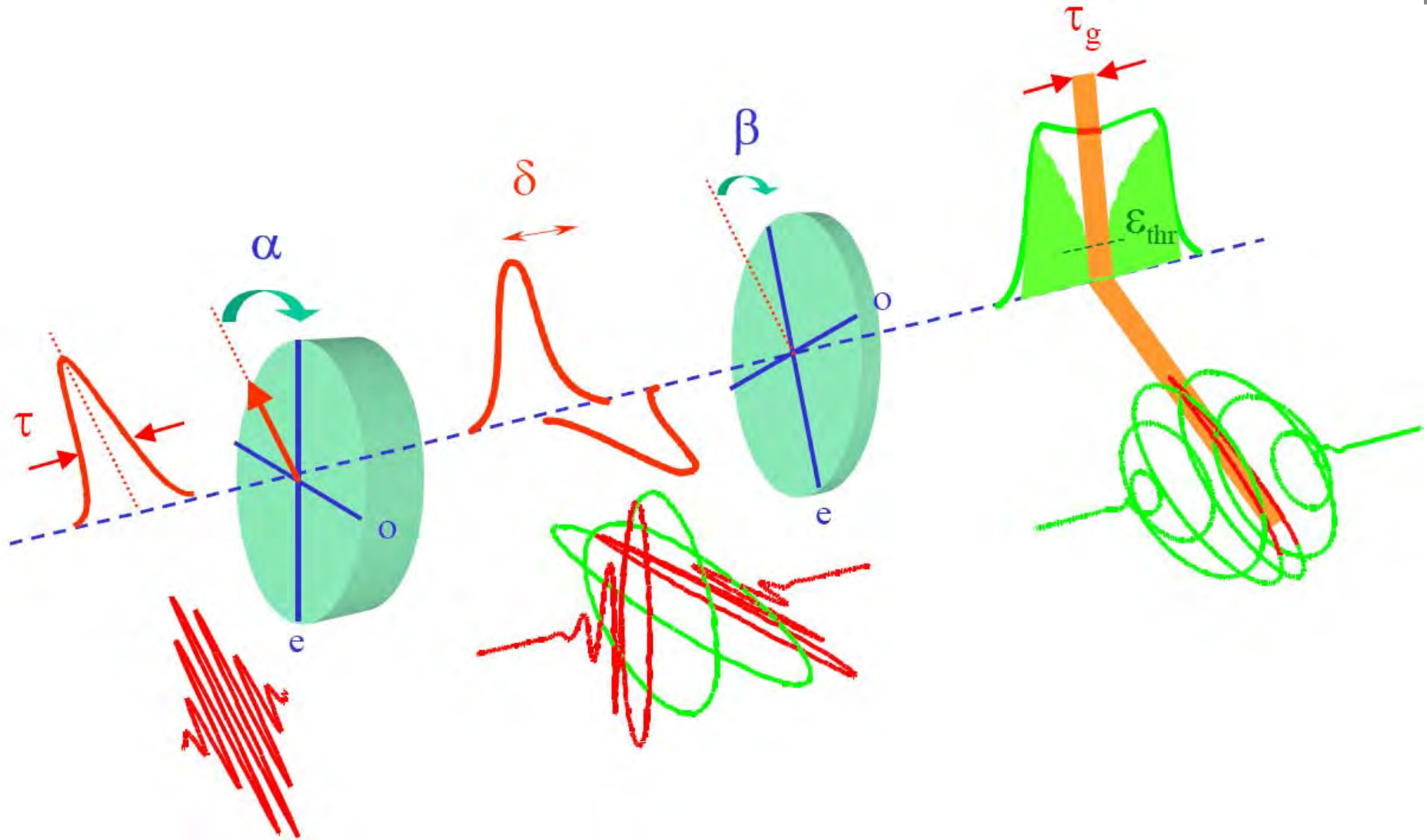
G. Sansone et al., Science 314,443 (2006)

P. Corkum *et al.*, Opt. Lett. **19**,1870 (1994)

O. Tcherbakoff *et al.*, Phys. Rev. A **68**,043804 (2003)







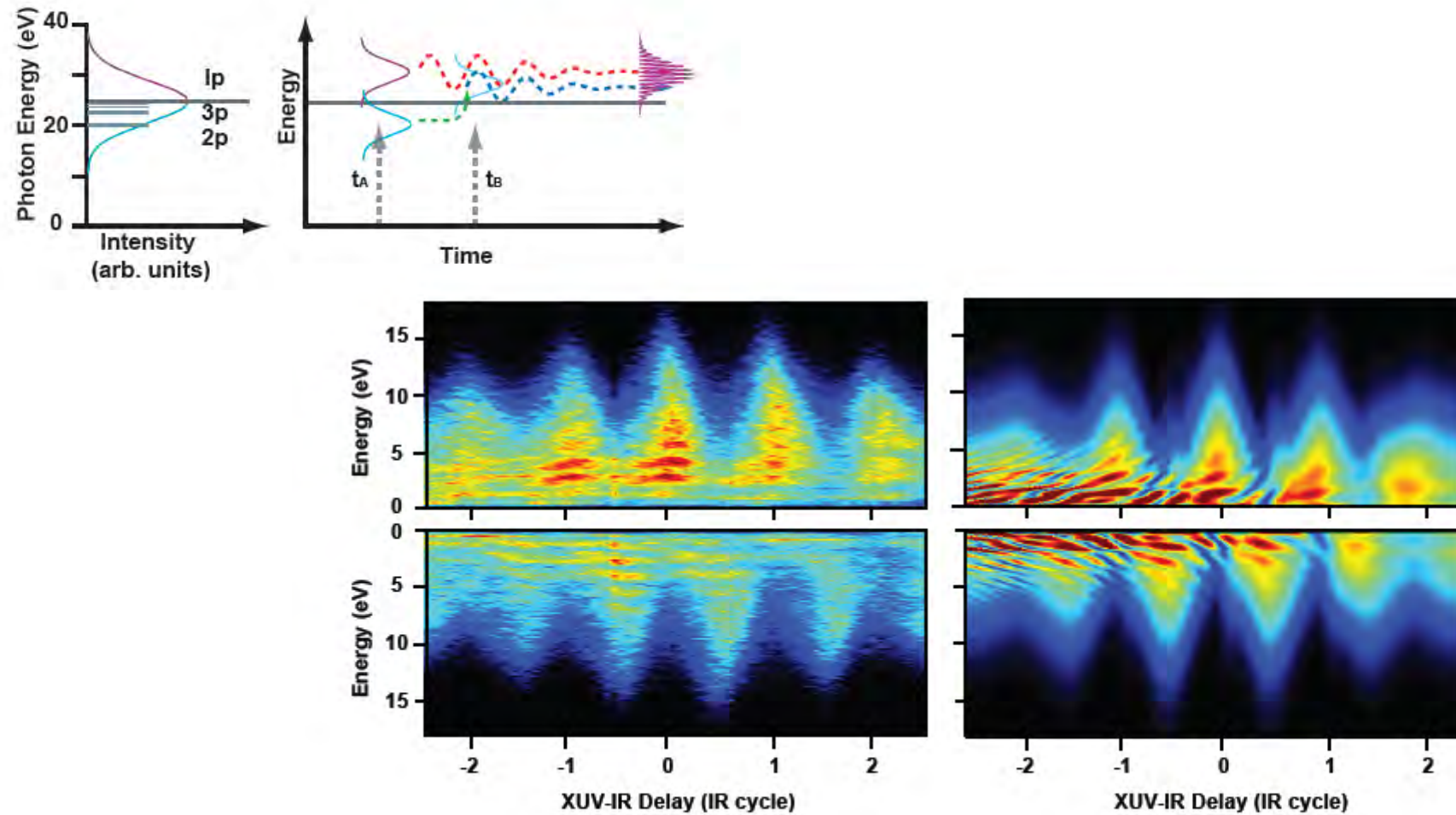
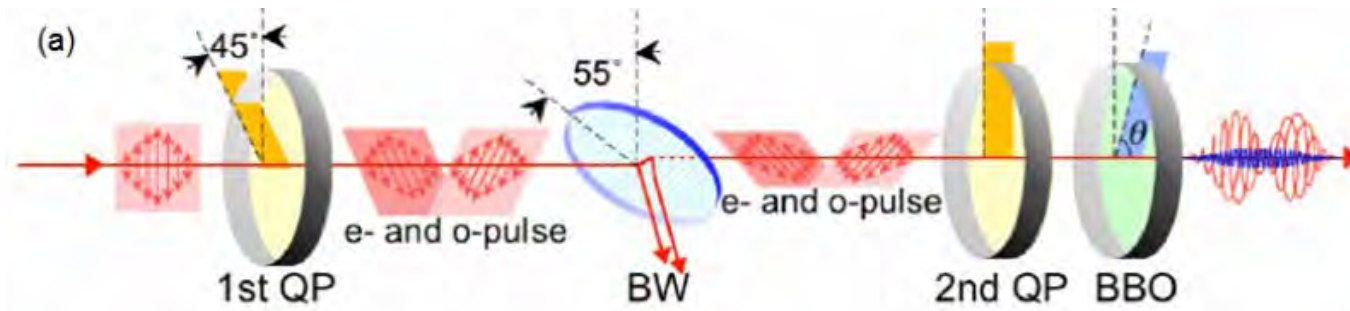


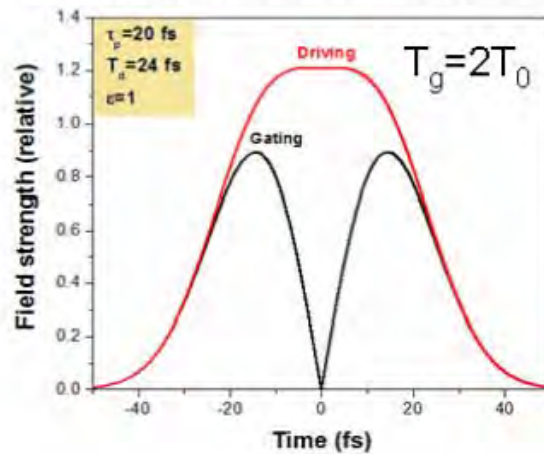
Fig. 2. experimental (left) and TDSE (right) results



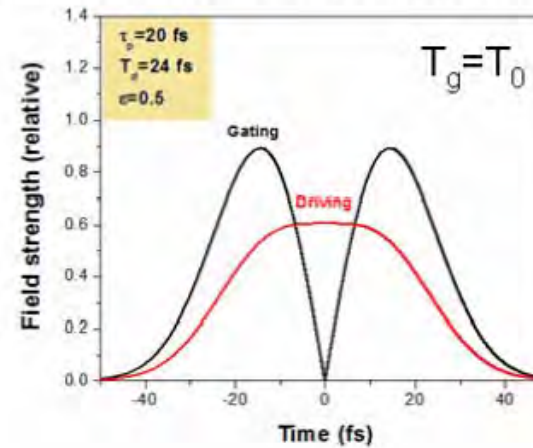
# generalized double optical gating



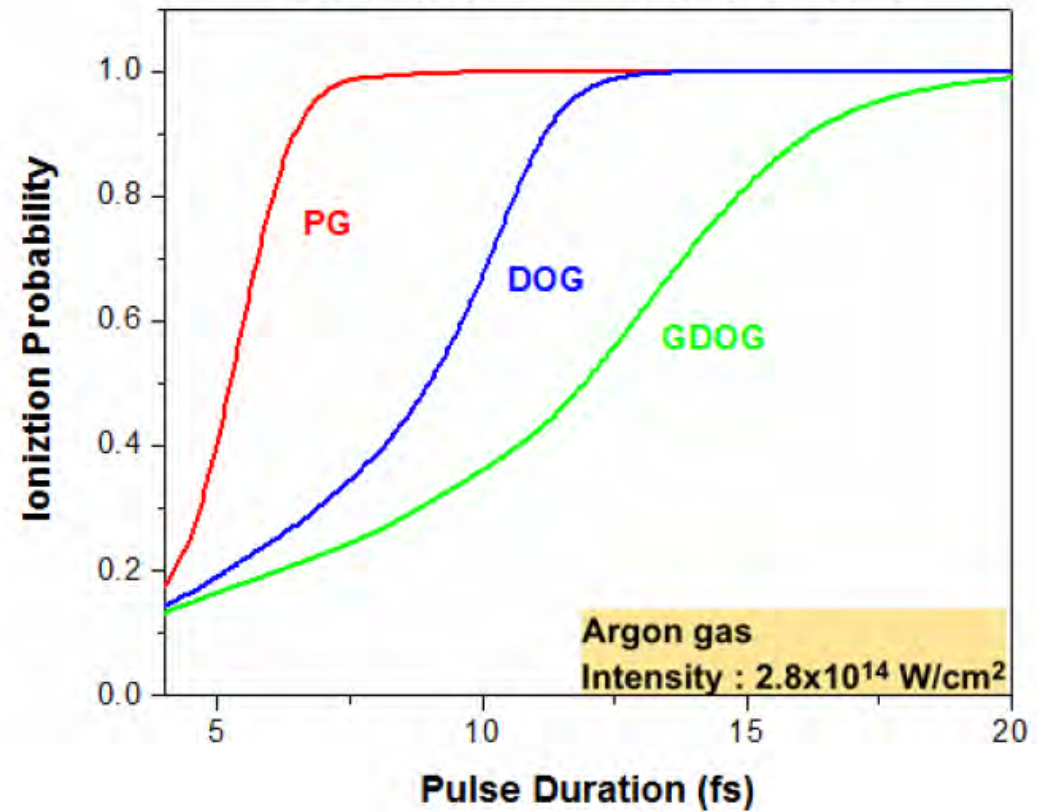
(b) Without Brewster window

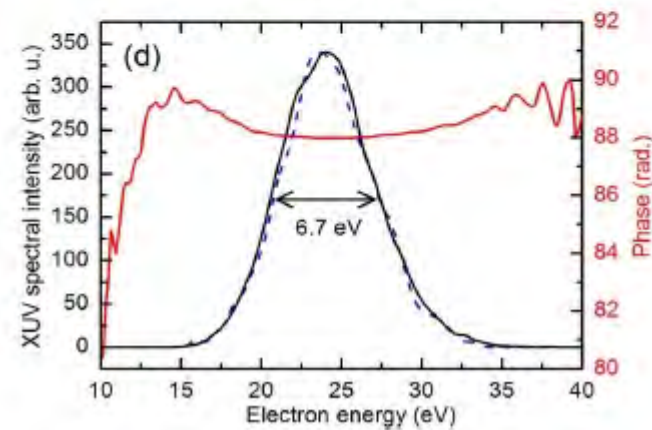
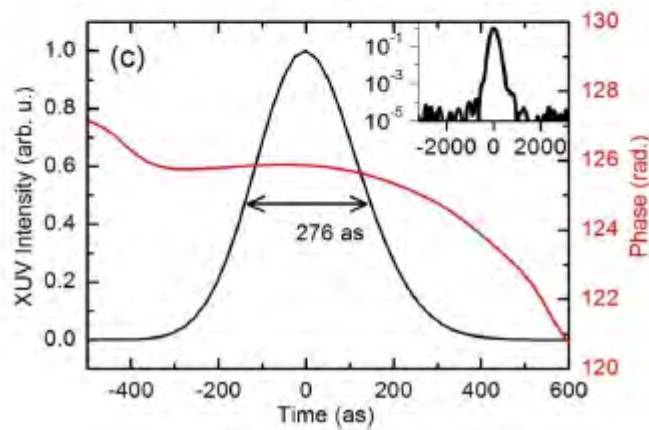
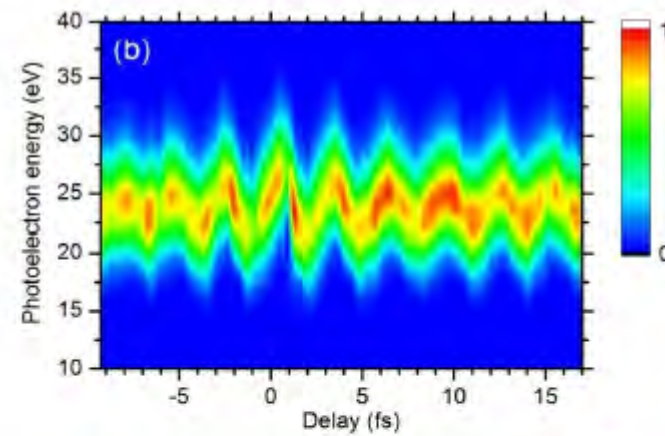
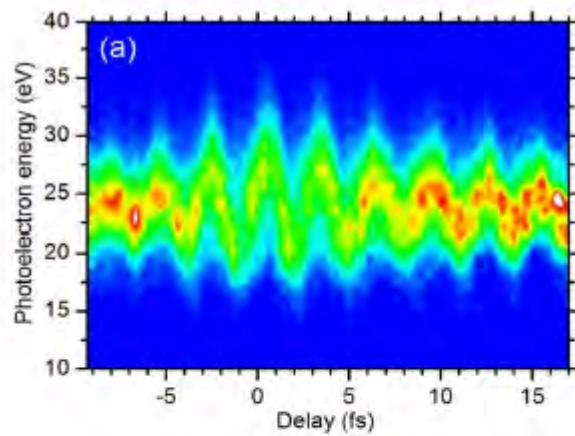


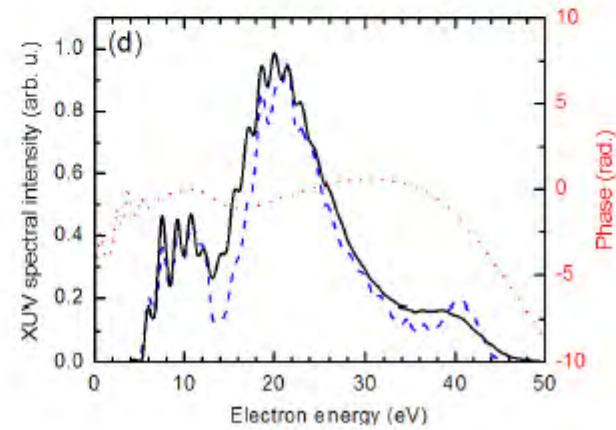
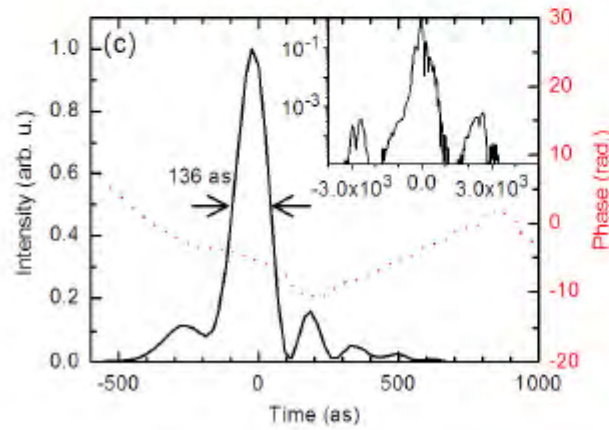
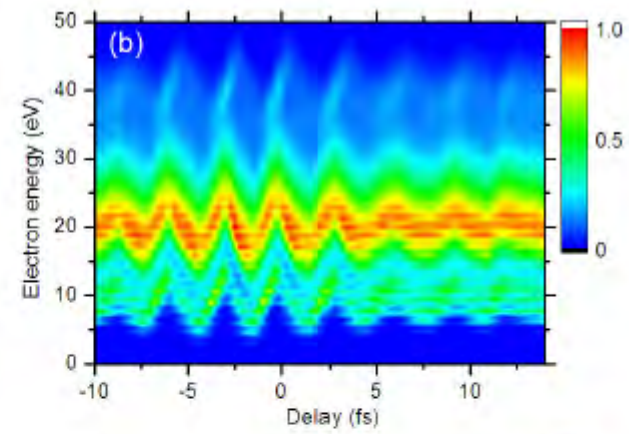
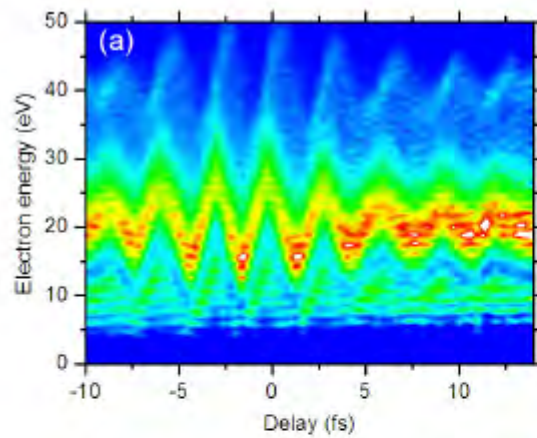
(c) With Brewster window



### Ionization Probability from ADK







## Comparison of different methods

**Table 1 | Relevant parameters of current attosecond sources based on HHG in gases.**

Method	Gas	$\hbar\omega$ (eV)	$\tau$ (as)	$E$	Type	Efficiency	Reference
AG	Neon	80	80	0.5 nJ	IAP	$1.7 \times 10^{-6}$	22
PG	Argon	35	130	70 pJ	IAP	$3.5 \times 10^{-7}$	26,27
PG	Neon	60	—	1 pJ	IAP	$5 \times 10^{-9}$	26
DOG	Argon	38	—	6.5 nJ	IAP	$6 \times 10^{-6}$	31
DOG	Neon	45	107	170 pJ	IAP	$1.5 \times 10^{-7}$	31,85
IG	Xenon	27	155	9 nJ	IAP	$2.6 \times 10^{-5}$	50
IPG	Xenon	50	<1,000	20 nJ	IAP	$2 \times 10^{-5}$	36
LF	Xenon	20	320	10 $\mu$ J	APT	$7 \times 10^{-4}$	16

$\hbar\omega$  is the central energy of the XUV spectrum,  $\tau$  is the measured pulse duration and  $E$  is the pulse energy at the source (in the case of isolated pulses,  $E$  corresponds to the XUV energy in the spectral region characterized by a continuous profile). The efficiency is defined as the ratio between the XUV pulse energy and the driving laser pulse energy. AG, amplitude gating; PG, polarization gating; DOG, double-optical gating; IG, ionization gating; IPG, interferometric polarization gating; LF, loose-focusing; IAP, isolated attosecond pulses; and APT, attosecond pulse trains.

# Thank you!

