IMPRS: Ultrafast Source Technologies

Lecture III: April 16, 2013: Ultrafast Optical Sources

Franz X. Kärtner

Is there a time during galopping, when all feet are of the ground? (1872) Leland Stanford

Eadweard Muybridge (* 9. April 1830 in Kingston upon Thames; † 8. Mai 1904, britischer Fotograf & Pionier der Fototechnik.

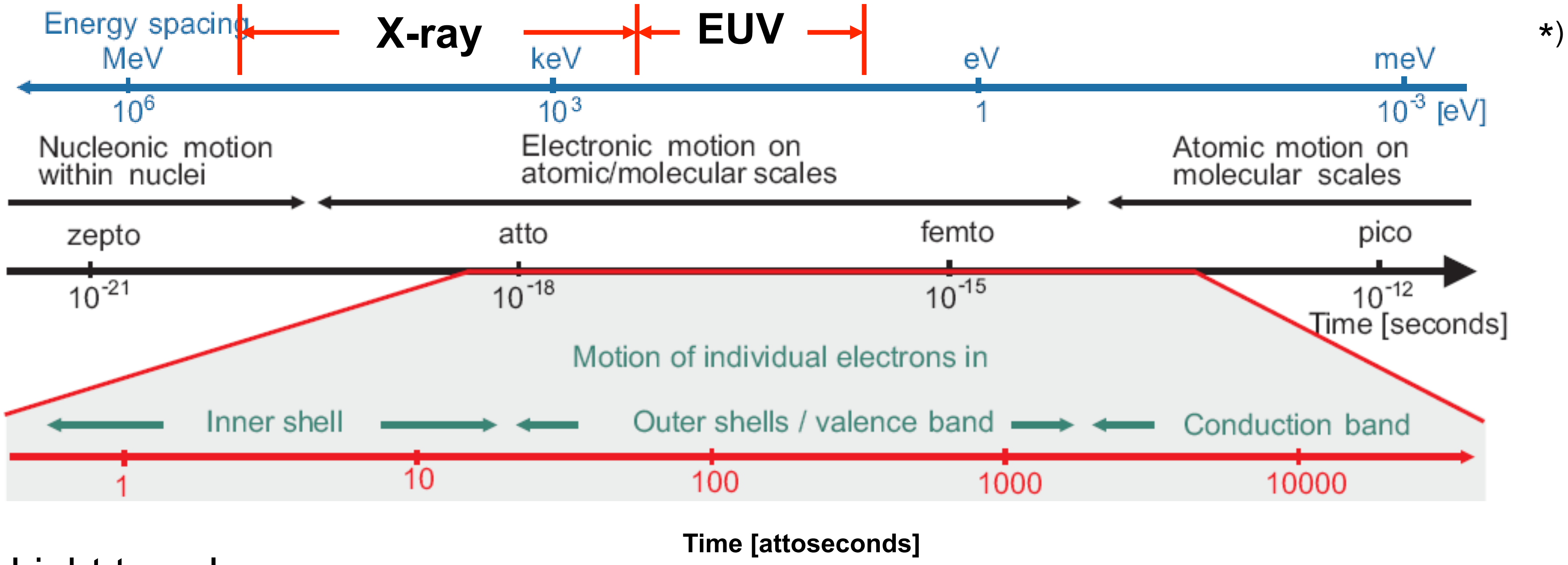
http://www.eadweardmuybridge.co.uk/

What happens when a bullet rips through an apple?

6. April 1903 in Fremont, Nebraska, USA; † 4. Januar 1990 in Cambridge, MA) american electrical engineer, inventor strobe photography.

http://web.mit.edu/edgerton/

Physics on femto- attosecond time scales?



Light travels:

A second: from the moon to the earth

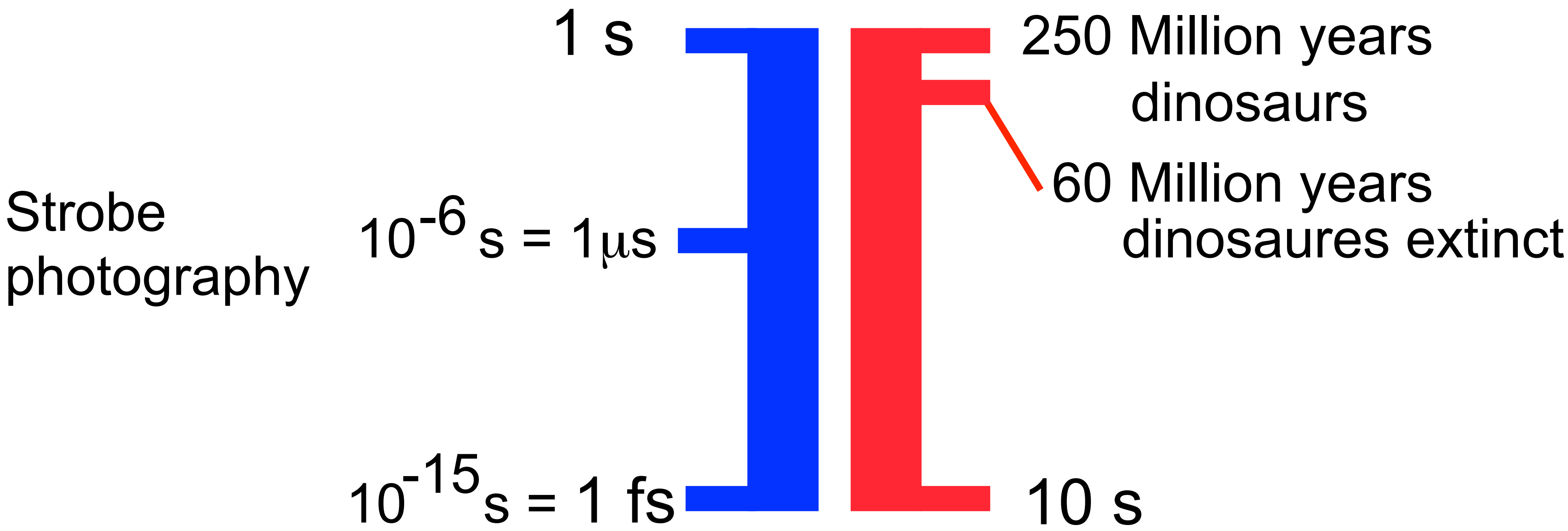
A picosecond: a fraction of a millimeter, through a blade of a knife

A femtosecond: the period of an optical wave, a wavelength

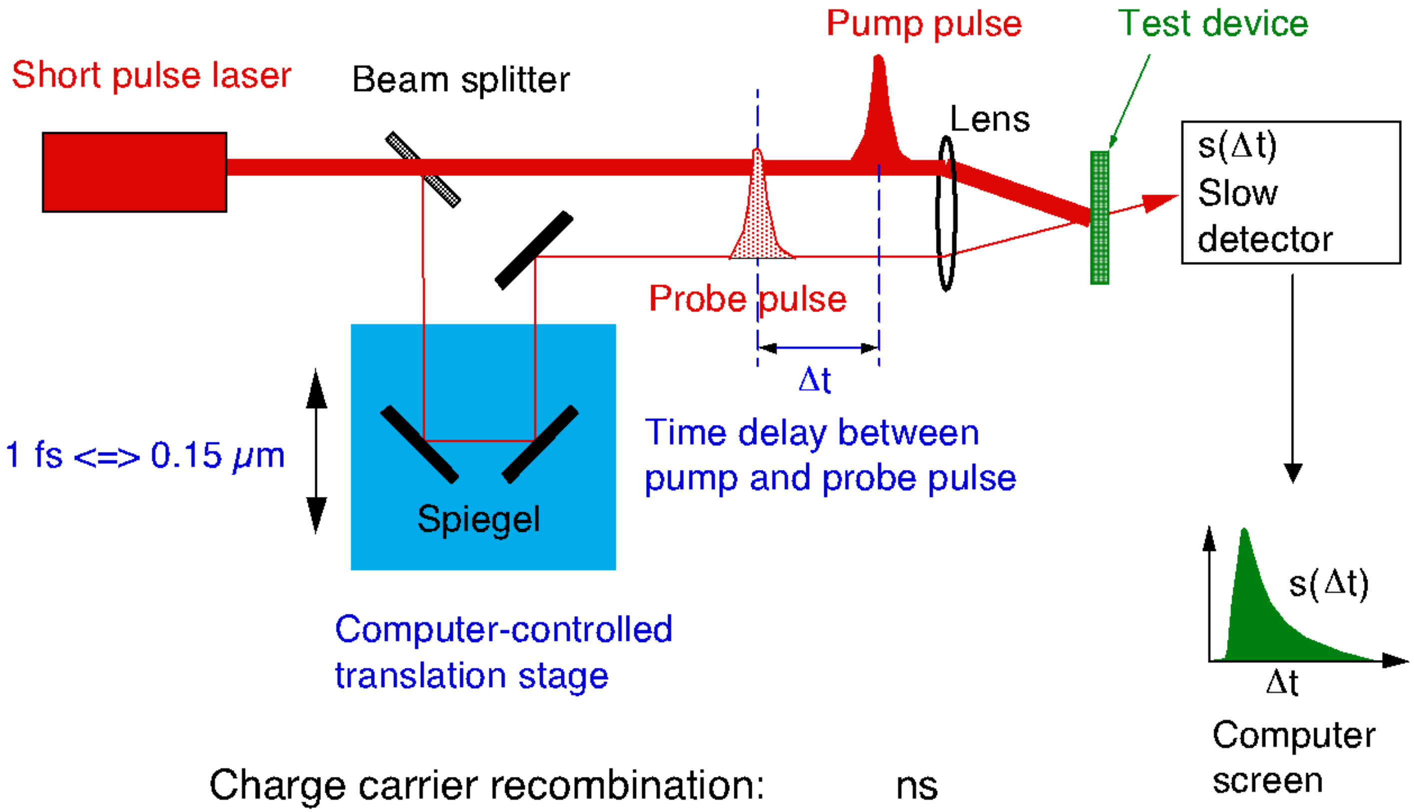
An attosecond: the period of X-rays, a unit cell in a solid

***F. Krausz and M. Ivanov, Rev. Mod. Phys. 81, 163 (2009)**

How short is a Femtosecond



Pump-probe measurement



Charge carrier recombination: ns

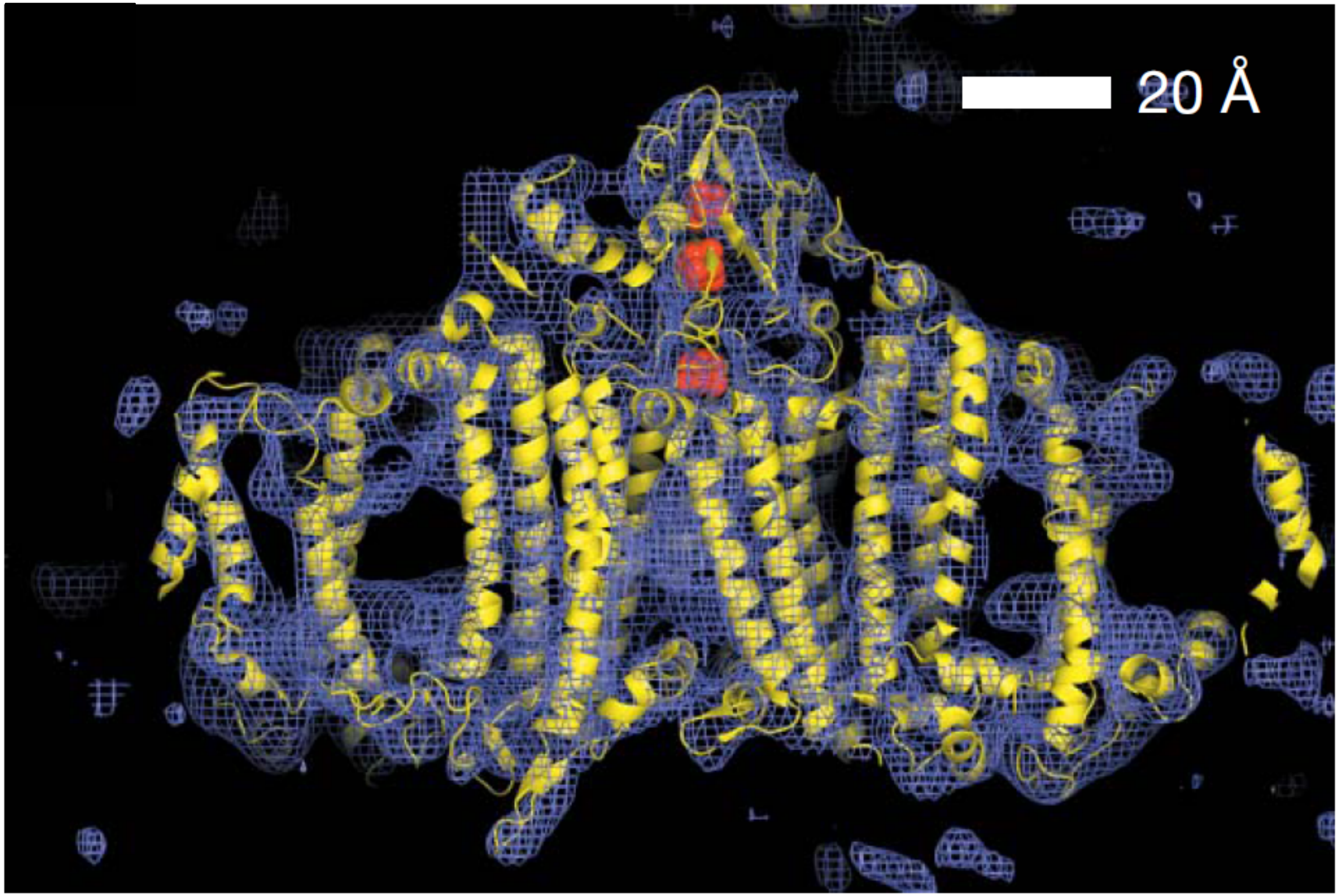
Thermalization electrons with lattice: ps

Thermalization electron gas: 10 -100 fs

Todays Frontiers in Space and Time

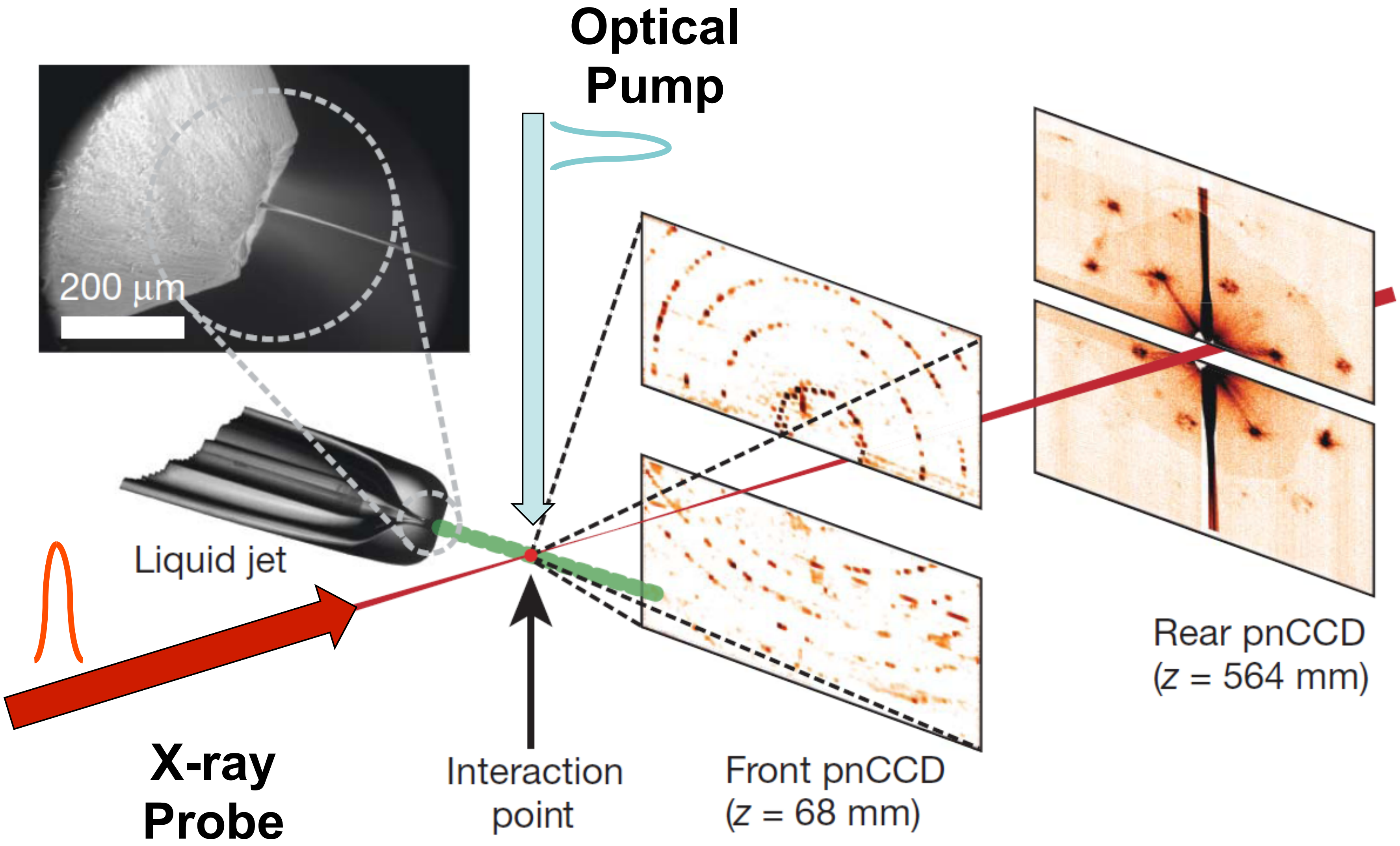
Structure, Dynamics and Function of Atoms and Molecules

Struture of Photosystem I



Chapman, et al. Nature 470, 73, 2011

X-ray Imaging (Time Resolved)

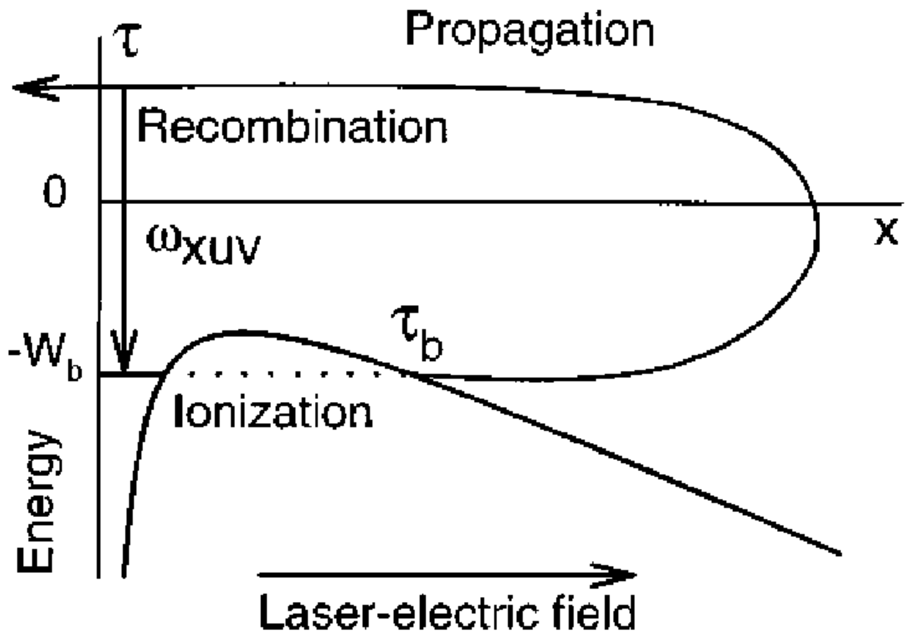


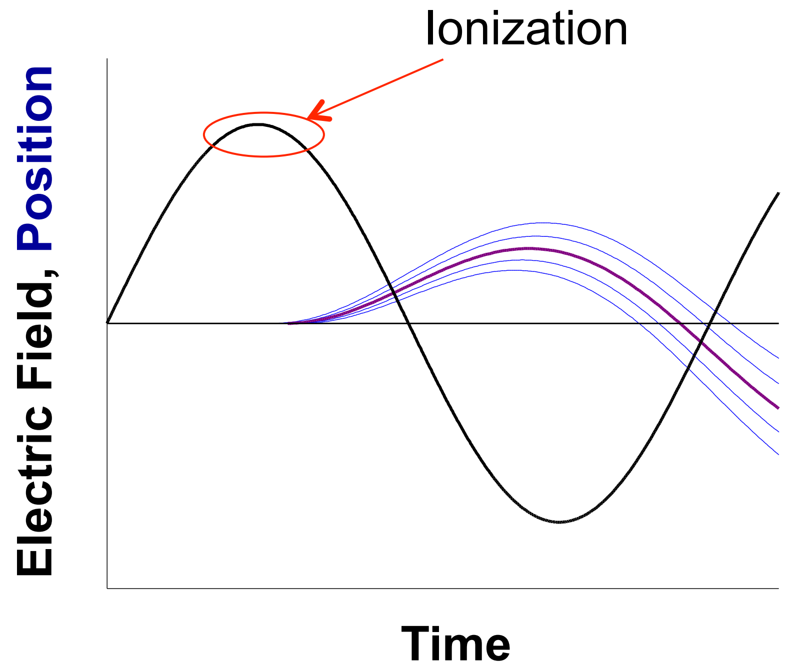
Imaging before destruction: Femtosecond Serial X-ray crystalography

Chapman, et al. Nature 470, 73, 2011

Attosecond Soft X-ray Pulses

Three-Step Model **Corkum, 1993** **Trajectories**





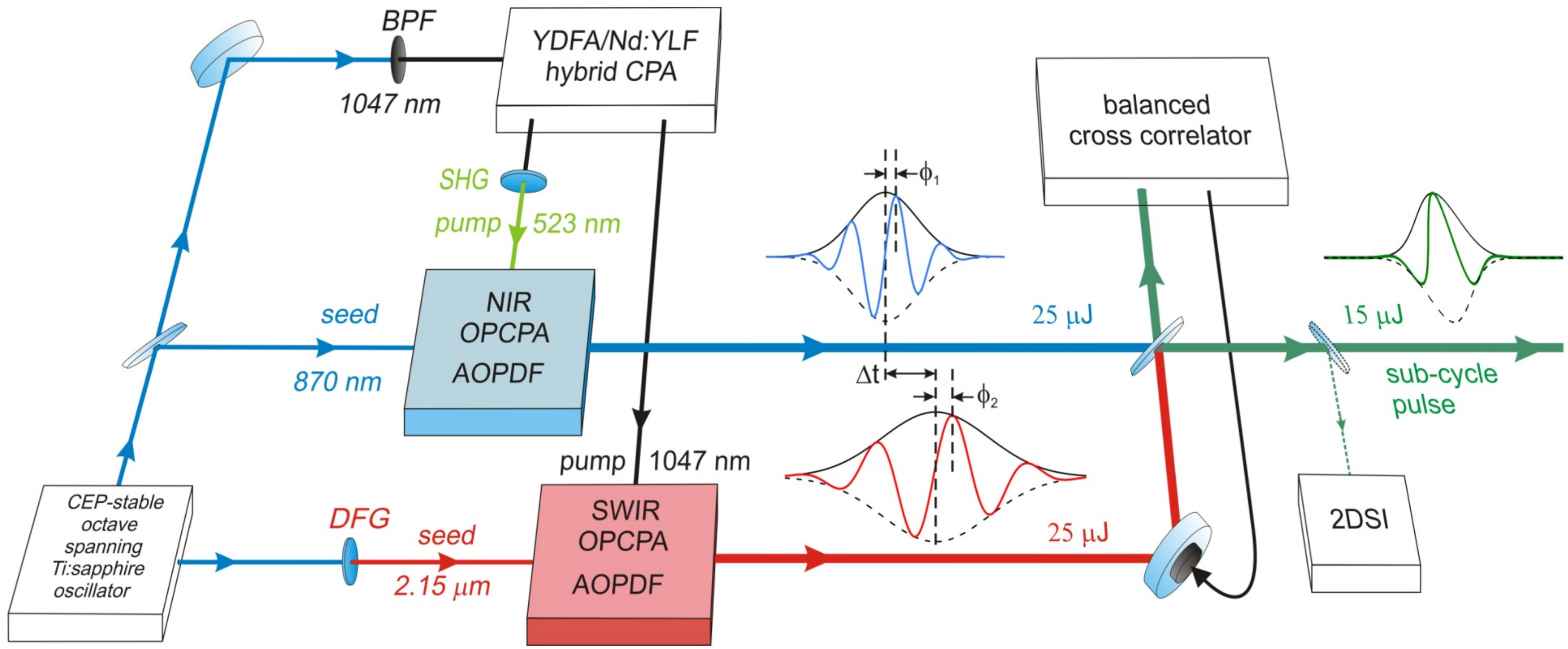
First Isolated Attosecond Pulses: M. Hentschel, et al., Nature 414, 509 (2001)

Hollow-Fiber Compressor: M. Nisoli, et al., Appl. Phys. Lett. 68, 2793 (1996)

High - energy single-cycle laser pulses!

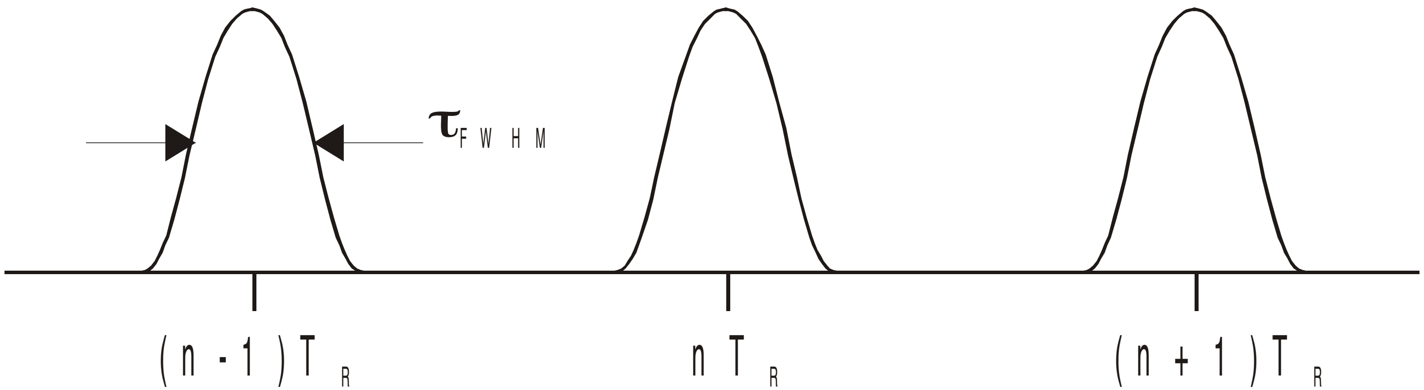
How do we generate them?

High Energy Laser Systems



- **Laser Oscillators (nJ), cw, q-switched, modelocked : Semiconductor, Fiber, Solid-State Lasers**
- **Laser Amplifiers: Solid-State or Fiber Lasers**
  - **Regenerative Amplifiers**
  - **Multipass Amplifiers**
  - **Chirped Pulse Amplification**
  - **Parametric Amplification and Nonlinear Frequency Conversion**

3. Basics of Optical Pulses



T_R : pulse repetition rate

W : pulse energy

$P_{ave} = W/T_R$: average power

τ_{FWHM} : Full Width Half Maximum pulse width

P_p : peak power

Peak Electric Field: $$E_p = \sqrt{2Z_{F0}\frac{P_p}{A_{\text{eff}}}}.$$

$$P_p = \frac{W}{\tau_{\text{FWHM}}} = P_{ave}\frac{T_R}{\tau_{\text{FWHM}}}$$

A_{eff} : effective beam cross section

Z_{Fo} : field impedance, $Z_{Fo} = 377\ \Omega$

average power:

$$P_{ave} \sim 1W - 1kW$$

repetition rates:

$$T_R^{-1} = f_R = \mathrm{mHz} - 100\,\mathrm{GHz}$$

pulse energy:

$$W = 1\mathrm{pJ} - 1\mathrm{kJ}$$

pulse width:

$$\tau_{\mathrm{FWHM}} = \begin{array}{ll} 5\,\mathrm{fs} - 50\,\mathrm{ps}, & \mathrm{modelocked} \\ 30\,\mathrm{ps} - 100\,\mathrm{ns}, & \mathrm{Q-switched} \end{array}$$

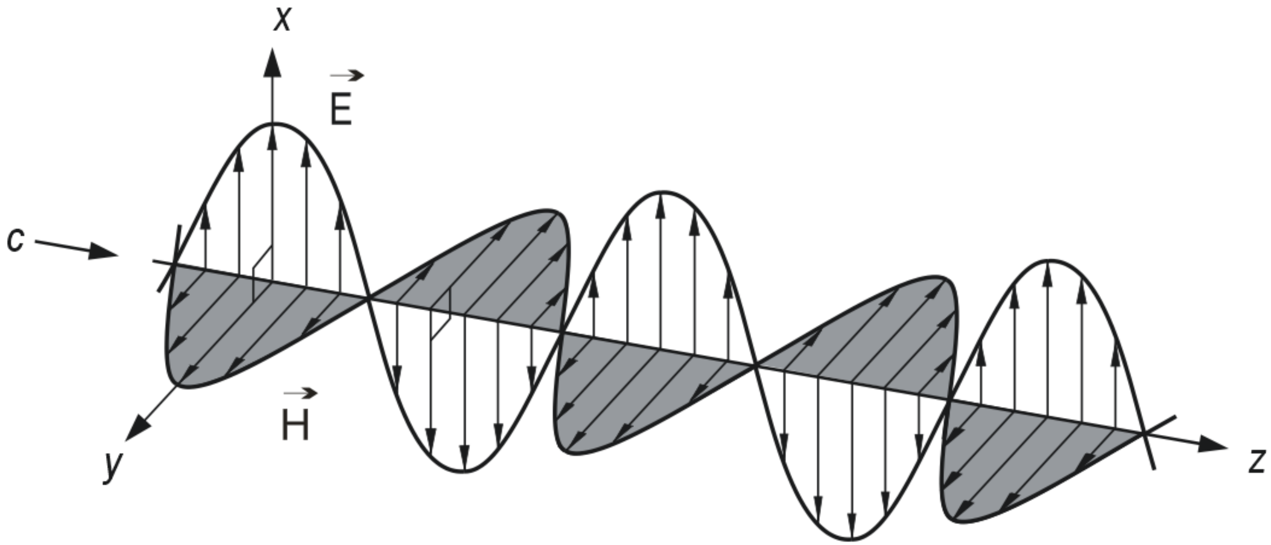
peak power:

$$P_p = \frac{1\,\mathrm{kJ}}{1\,\mathrm{ps}} = \frac{1\,\mathrm{J}}{1\,\mathrm{fs}} \sim 1\,\mathrm{PW},$$

Typical Lab Pulse: $$P_p = \frac{10\,\mathrm{nJ}}{10\,\mathrm{fs}} \sim 1\,\mathrm{MW}$$

$$E_p = \sqrt{2 \times 377 \times \frac{10^6 \times 10^{12}}{\pi \times (1.5)^2}}\,\frac{\mathrm{V}}{\mathrm{m}} \approx 10^{10}\,\frac{\mathrm{V}}{\mathrm{m}} = \frac{10\,\mathrm{V}}{\mathrm{nm}}$$

3.1 Electromagnetic Waves



Transverse electromagnetic wave (TEM) (Teich, 1991)

See Chapter: 2.1.2 Plane-Wave Solutions (TEM-Waves)

3.2 Optical Pulses (propagating along z-axis)

$$\underline{\vec{E}}(\vec{r},t) = \int_0^\infty \frac{d\Omega}{2\pi} \underline{\widetilde{E}}(\Omega) e^{j(\Omega t - K(\Omega) z)} \ \vec{e}_x$$

$$\underline{\vec{H}}(\vec{r},t) = \int_0^\infty \frac{d\Omega}{2\pi Z_F(\Omega)} \underline{\widetilde{E}}(\Omega) e^{j(\Omega t - K(\Omega) z)} \ \vec{e}_y$$

$$\vec{E}(\vec{r},t) = \frac{1}{2}\left(\underline{\vec{E}}(\vec{r},t) + \underline{\vec{E}}(\vec{r},t)^*\right)$$

$$\vec{H}(\vec{r},t) = \frac{1}{2}\left(\underline{\vec{H}}(\vec{r},t) + \underline{\vec{H}}(\vec{r},t)^*\right)$$

$|\underline{\tilde{E}}(\Omega)| e^{j\varphi(\Omega)}$: Wave amplitude and phase

$K(\Omega) = \Omega / c(\Omega) = n(\Omega)\Omega / c_0$: Wave number

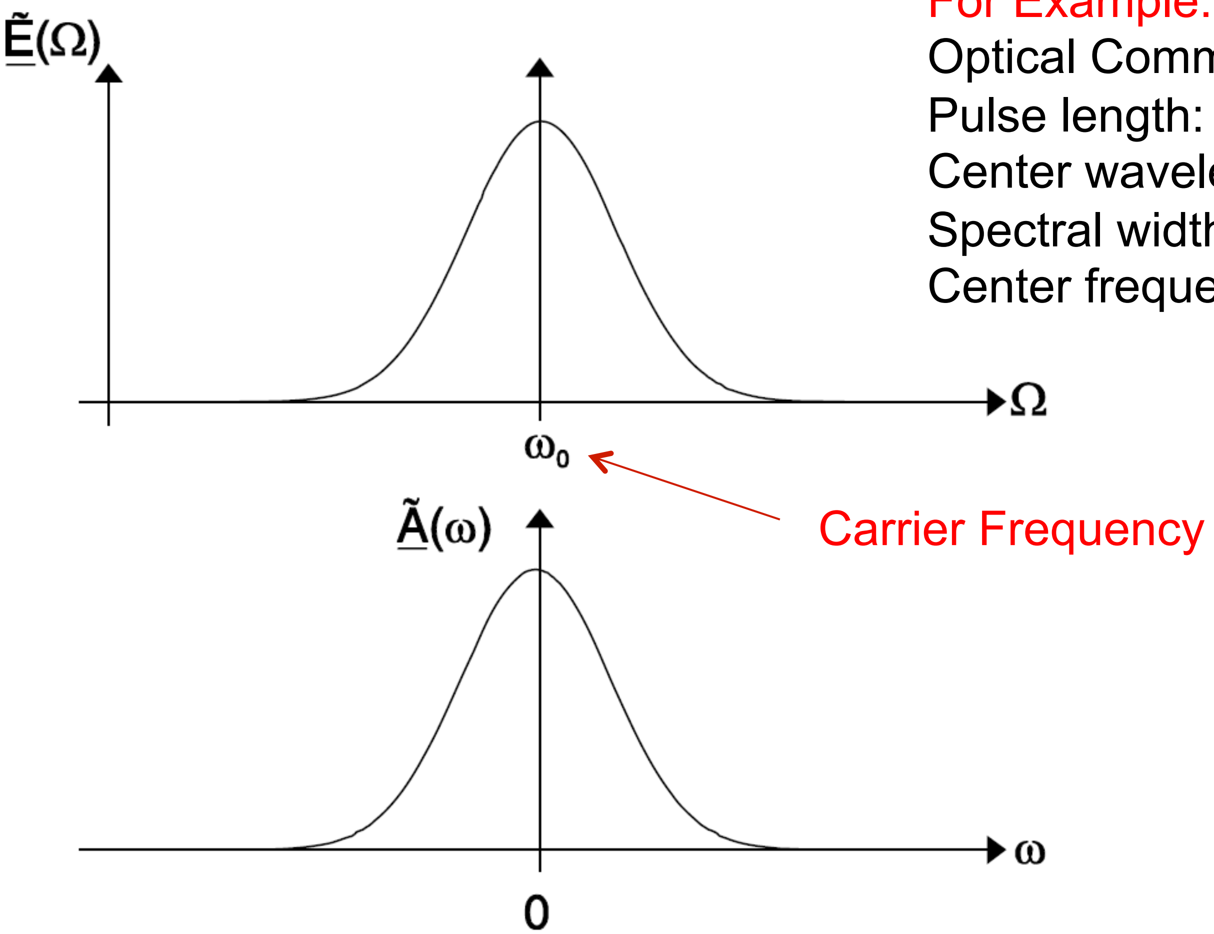
$c(\Omega) = \dfrac{c_0}{n(\Omega)}$: Phase velocity of wave

$$\tilde{n}^2(\Omega) = 1 + \tilde{\chi}(\Omega)$$

At z=0

$$\underline{E}(z=0,t) = \frac{1}{2\pi}\int_0^\infty \underline{\tilde{E}}(\Omega)e^{j\Omega t}d\Omega$$

For Example:
Optical Communication; 10Gb/s
Pulse length: 20 ps
Center wavelength : λ=1550 nm.
Spectral width: ~ 50 GHz,
Center frequency: 200 THz,



Spectrum of an optical wave packet described in absolute and relative frequencies

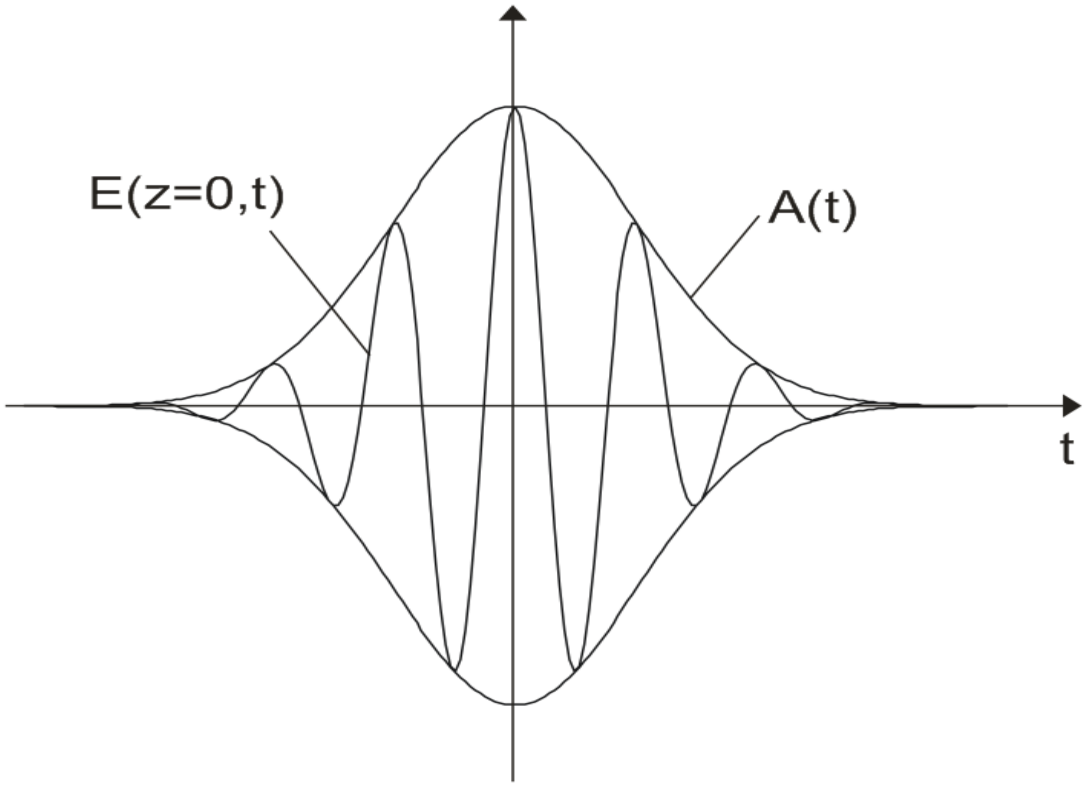
Carrier and Envelope

$$
\begin{aligned}
\underline{E}(z &= 0, t) = \frac{1}{2\pi}\int_{-\omega_0}^{\infty} \underline{\tilde{E}}(\omega_0+\omega)e^{j(\omega_0+\omega)t}d\omega \\
&= \frac{1}{2\pi}e^{j\omega_0 t}\int_{-\omega_0}^{\infty} \underline{\tilde{E}}(\omega_0+\omega)e^{j\omega t}d\omega \\
& \quad A(t)e^{j\omega_0 t}.
\end{aligned}
$$

Carrier Frequency

Envelope:

$$
\begin{aligned}
\underline{A}(t) &= \frac{1}{2\pi}\int_{-\omega_0\to-\infty}^{\infty} \underline{\tilde{A}}(\omega)e^{j\omega t}d\omega \\
&= \frac{1}{2\pi}\int_{-\infty}^{\infty} \underline{\tilde{A}}(\omega)e^{j\omega t}d\omega,
\end{aligned}
$$



Electric field and envelope of an optical pulse

Pulse width: **Full Width at Half Maximum of** $|A(t)|^2$

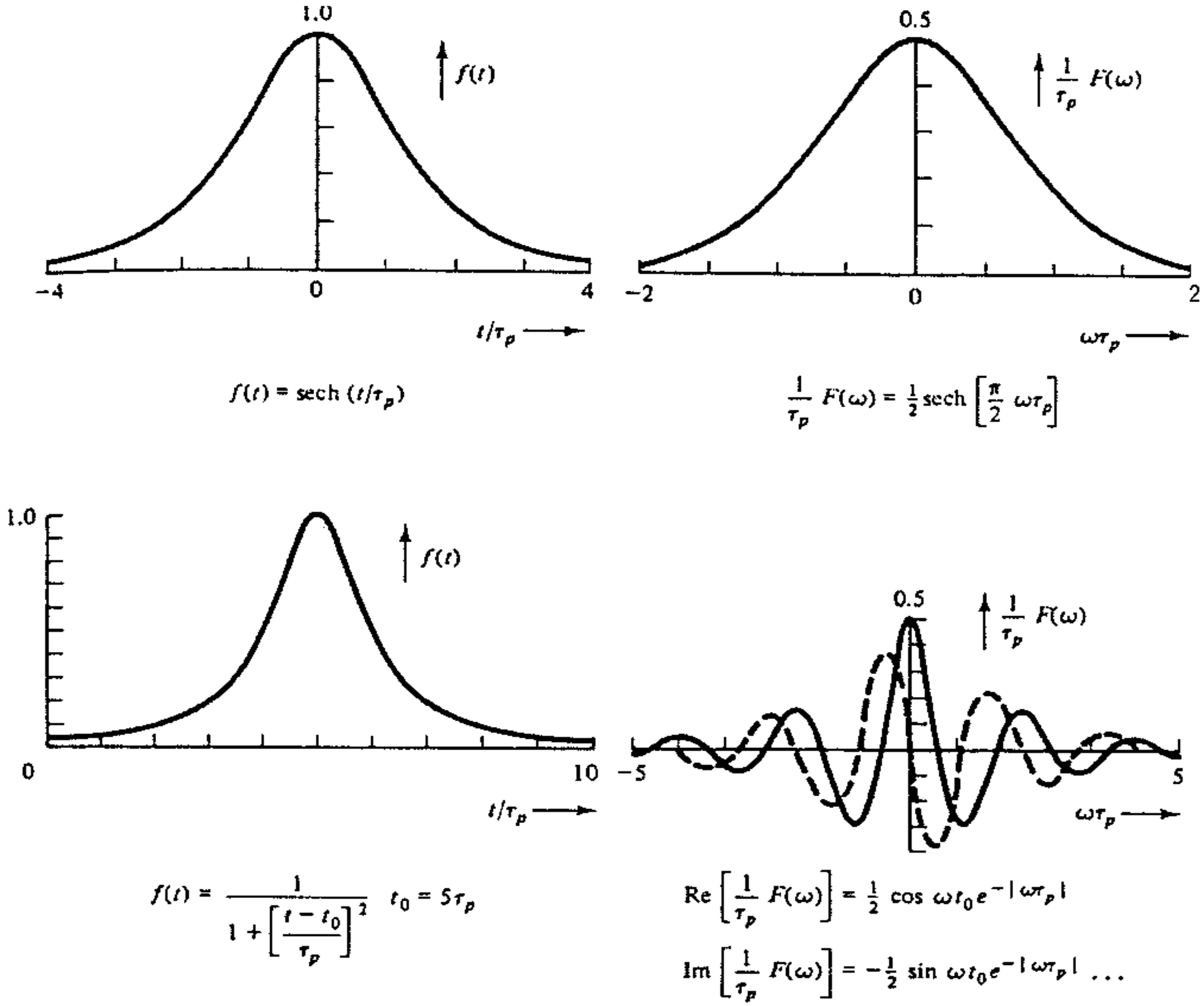
Spectral width : **Full Width at Half Maximum of** $|\underline{\tilde{A}}(\omega)|^2$

Often Used Pulses

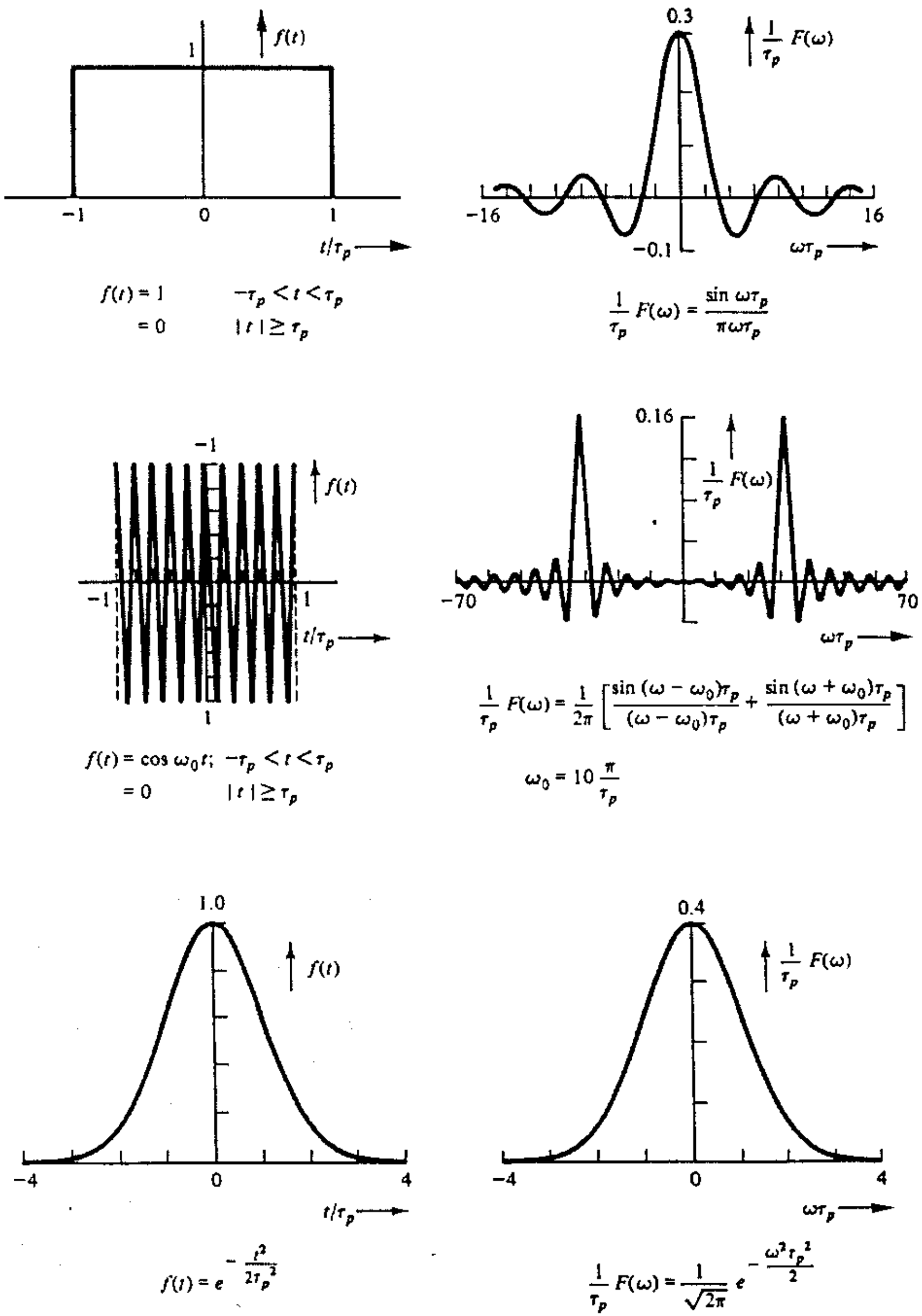
Pulse Shape	Fourier Transform	Pulse Width	Time-Bandwidth Product
$\underline{A}(t)$	$\underline{\tilde{A}}(\omega) = \int_{-\infty}^{\infty} a(t) e^{-j\omega t} dt$	Δt	$\Delta t \cdot \Delta f$
Gaussian: $e^{-\frac{t^2}{2\tau^2}}$	$\sqrt{2\pi}\tau e^{-\frac{1}{2}\tau^2\omega^2}$	$2\sqrt{\ln 2}\tau$	0.441
Hyperbolic Secant: $\mathrm{sech}(\frac{t}{\tau})$	$\frac{\tau}{2}\,\mathrm{sech}\left(\frac{\pi}{2}\tau\omega\right)$	$1.7627\ \tau$	0.315
Rect-function: $\begin{cases} 1, & \lvert t\rvert \le \tau/2 \\ 0, & \lvert t\rvert > \tau/2 \end{cases}$	$\tau\frac{\sin(\tau\omega/2)}{\tau\omega/2}$	τ	0.886
Lorentzian: $\frac{1}{1+(t/\tau)^2}$	$2\pi\tau e^{-\lvert\tau\omega\rvert}$	$1.287\ \tau$	0.142
Double-Exp.: $e^{-\lvert\frac{t}{\tau}\rvert}$	$\frac{\tau}{1+(\omega\tau)^2}$	$\ln 2\ \tau$	0.142

Table 2.2: Pulse shapes, corresponding spectra and time bandwidth products.

Pulse width and spectral width: FWHM



Fourier transforms to pulse shapes listed in table 2.2 [16]



Fourier transforms to pulse shapes listed in table 2.2, continued [16]

3.3 Linear Pulse Propagation

$$\underline{E}(z,t) = \frac{1}{2\pi}\int_0^\infty \underline{\tilde{E}}(\Omega) e^{\mathrm{j}(\Omega t - K(\Omega) z)} d\Omega.$$

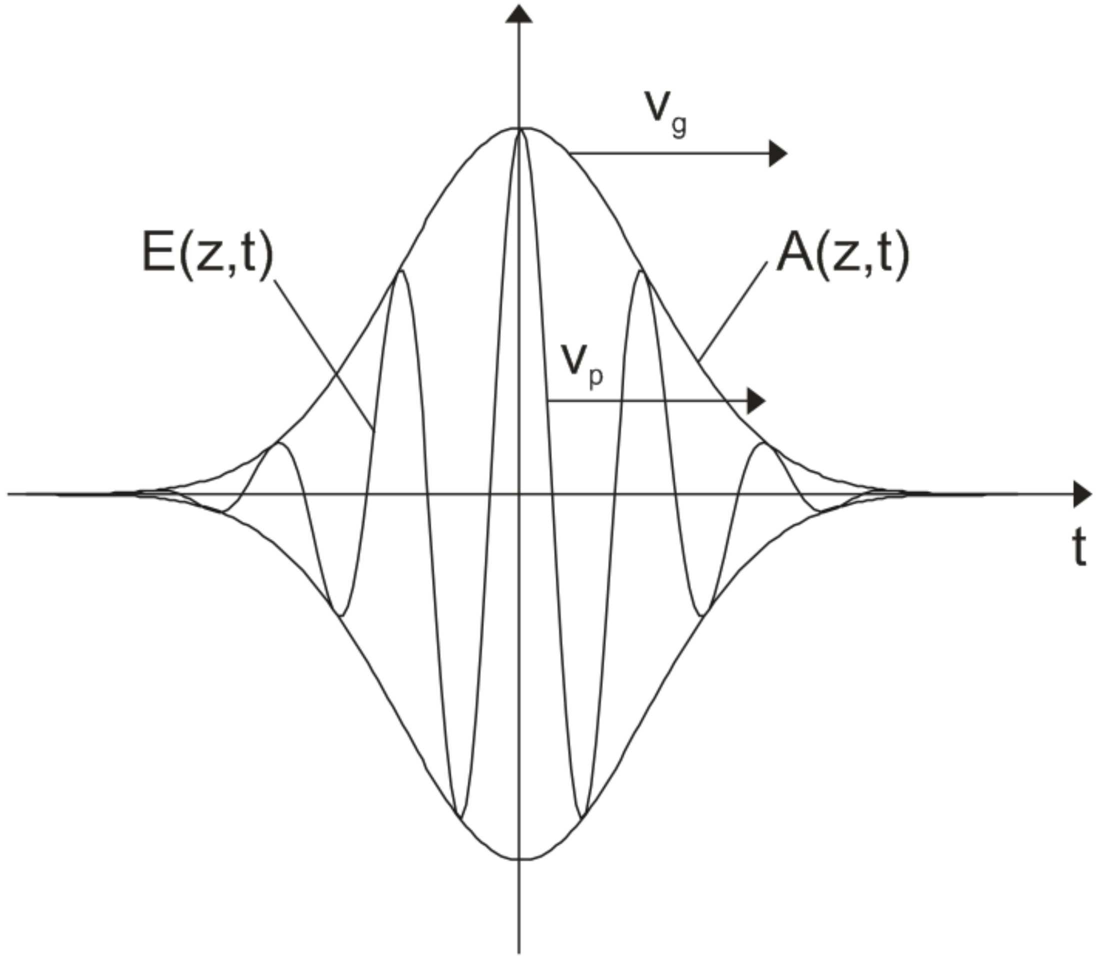
$$\underline{E}(z,t) = \underline{A}(z,t) e^{\mathrm{j}(\omega_0 t - K(\omega_0) z)}$$

Envelope + Carrier Wave

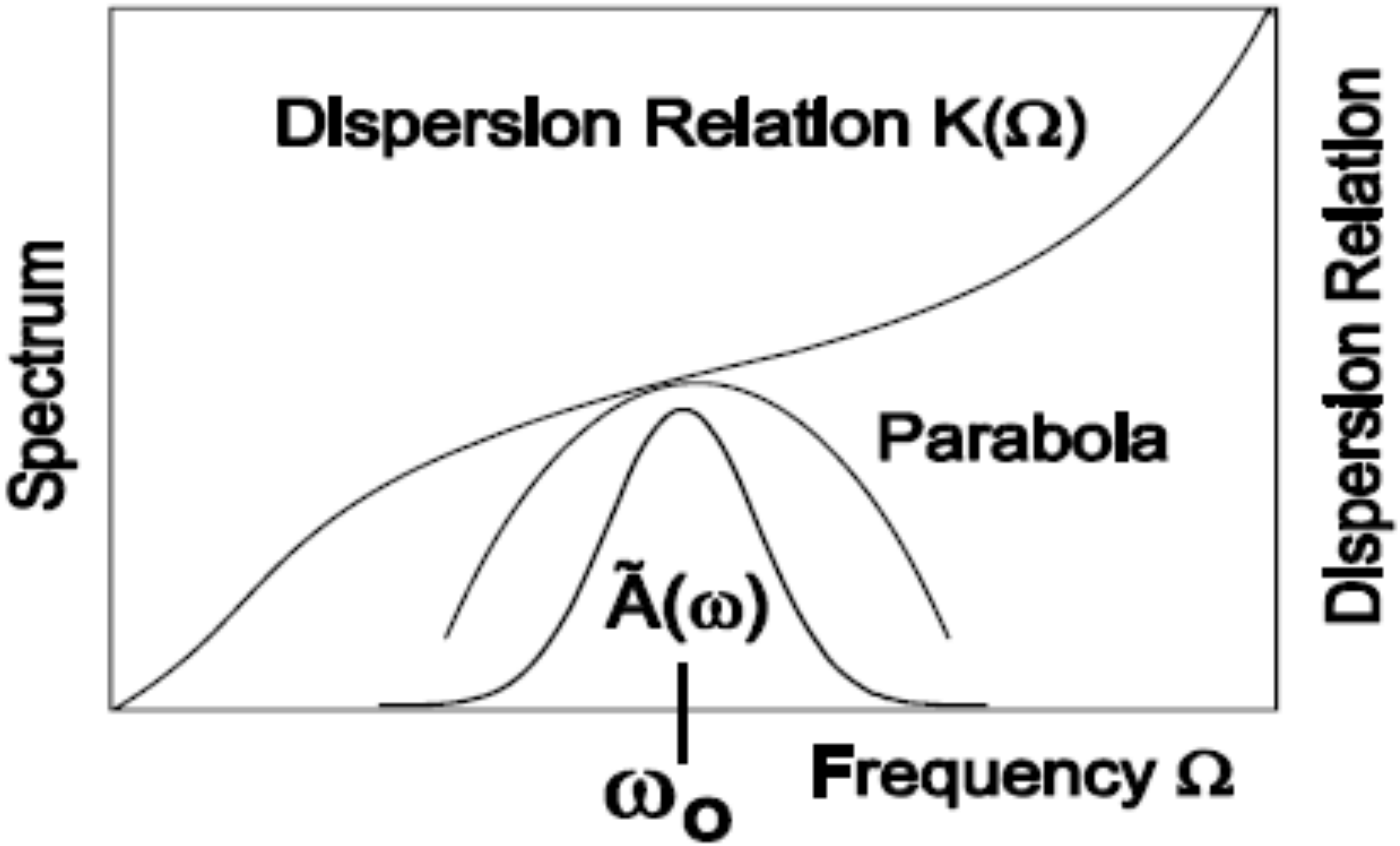
$$\begin{aligned} \omega &= \Omega - \omega_0, \\ k(\omega) &= K(\omega_0 + \omega) - K(\omega_0), \\ \underline{\tilde{A}}(\omega) &= \underline{\tilde{E}}(\Omega = \omega_0 + \omega). \end{aligned}$$

$$\underline{E}(z,t) = \frac{1}{2\pi}\int_{-\infty}^\infty \underline{\tilde{A}}(\omega) e^{\mathrm{j}(\omega t - k(\omega) z)} d\omega \; e^{\mathrm{j}(\omega_0 t - K(\omega_0) z)}$$

$$\underline{A}(z,t) = \frac{1}{2\pi}\int_{-\infty}^\infty \underline{\tilde{A}}(\omega) e^{\mathrm{j}(\omega t - k(\omega) z)} d\omega$$



Electric field and pulse envelope in time domain



Taylor expansion of dispersion relation at the center frequency of the wave packet

3.4 Dispersion

In the frequency domain:

$$\underline{\tilde{A}}(z,\omega) = \underline{\tilde{A}}(z=0,\omega)e^{-\mathrm{j}k(\omega)z}$$

Taylor expansion of dispersion relation:

$$k(\omega) = k'\omega + \frac{k''}{2}\omega^2 + \frac{k^{(3)}}{6}\omega^3 + O(\omega^4)$$

Equation of motion in frequency domain:

$$\frac{\partial \underline{\tilde{A}}(z,\omega)}{\partial z} = -\mathrm{j}k(\omega)\underline{\tilde{A}}(z,\omega)$$

Equation of motion in time domain:

$$\frac{\partial \underline{A}(z,t)}{\partial z} = -\mathrm{j}\sum_{n=1}^{\infty} \frac{k^{(n)}}{n!}\left(-\mathrm{j}\frac{\partial}{\partial t}\right)^n \underline{A}(z,t)$$

i) Keep only linear term:

$$k(\omega) = k'\omega + \cancel{\frac{k''}{2}}\omega^2 + \cancel{\frac{k^{(3)}}{6}}\omega^3 + \cancel{O}(\omega^4)$$

$$\underline{\tilde{A}}(z,\omega) = \underline{\tilde{A}}(z=0,\omega)e^{-jk'\omega z}$$

Time domain:

$$\underline{A}(z,t) = \underline{A}(0, t - z/\upsilon_{g0})$$

Group velocity:

$$\upsilon_{g0} = 1/k' = \left(\left.\frac{dk(\omega)}{d\omega}\right|_{\omega=0} \right)^{-1} = \left(\left.\frac{dK(\Omega)}{d\Omega}\right|_{\Omega=\omega_0} \right)^{-1}$$

Compare with phase velocity:

$$\upsilon_{p0} = \omega_0 / K(\omega_0) = \left(\frac{K(\omega_0)}{\omega_0} \right)^{-1}$$

Retarded time: $t' = t - z/v_{g0}$

$$\underline{A}(z,t) = \underline{A}(0,t')$$

Or start from (2.63)

$$\frac{\partial \underline{A}(z,t)}{\partial z} + \frac{1}{v_{g0}} \frac{\partial \underline{A}(z,t)}{\partial t} = 0$$

Substitute:

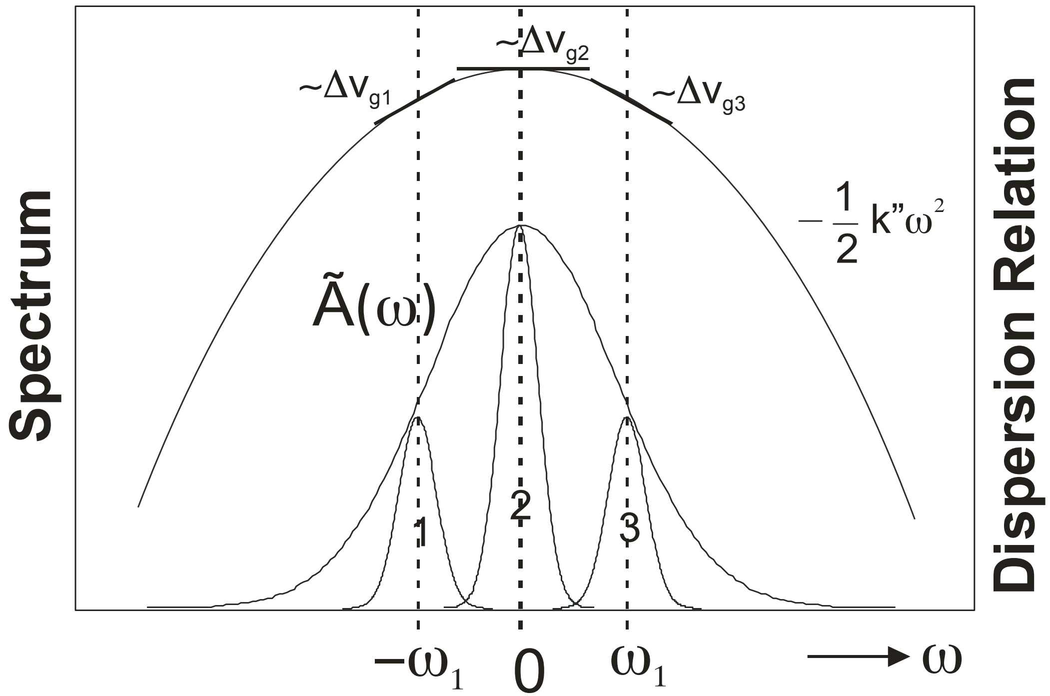
$$\begin{aligned} z' &= z, \\ t' &= t - z/v_{g0}, \end{aligned} \qquad \begin{aligned} \frac{\partial}{\partial z} &= \frac{\partial}{\partial z'} - \frac{1}{v_{g0}} \frac{\partial}{\partial t'} \\ \frac{\partial}{\partial t} &= \frac{\partial}{\partial t'} \end{aligned}$$

$$\boxed{\frac{\partial \underline{A}(z',t')}{\partial z'} = 0}$$

ii) Keep up to second order term:

$$k(\omega) = k'\omega + \frac{k''}{2}\omega^2 + \cancel{\frac{k^{(3)}}{6}\omega^3} + \cancel{O(\omega^4)}$$

$$\frac{\partial \underline{A}(z,t')}{\partial z} = \mathrm{j}\frac{k''}{2}\frac{\partial^2 \underline{A}(z,t')}{\partial t'^2}$$



Decomposition of a pulse into wave packets with different center frequency. In a medium with dispersion the wave packets move at different relative group velocity

Gaussian Pulse:

$$\underline{E}(z = 0, t) = \underline{A}(z = 0, t)e^{j\omega_0 t}$$

$$\underline{A}(z = 0, t = t') = \underline{A}_0 \exp\left[-\frac{1}{2}\frac{t'^2}{\tau^2}\right]$$

Pulse width

$$\frac{\partial \underline{\tilde{A}}(z,\omega)}{\partial z} = -j\frac{k''\omega^2}{2}\underline{\tilde{A}}(z,\omega)$$

Substitute:

$$\underline{\tilde{A}}(z,\omega) = \underline{\tilde{A}}(z = 0,\omega)\exp\left[-j\frac{k''\omega^2}{2}z\right]$$

Gaussian Integral:

$$\boxed{\frac{1}{\sqrt{2\pi\sigma}}\int_{-\infty}^{\infty} e^{-\frac{x^2}{2\sigma}}e^{-jx\varsigma}dx = e^{-\frac{\sigma}{2}\varsigma^2} \text{ for } \operatorname{Re}\{\sigma\} \geq 0}$$

Apply

$$\underline{\tilde{A}}(z = 0,\omega) = A_0\sqrt{2\pi}\tau\exp\left[-\frac{1}{2}\tau^2\omega^2\right]$$

Propagation:

$$\underline{\tilde{A}}(z,\omega) = A_0\sqrt{2\pi}\tau \exp\left[-\frac{1}{2}\left(\tau^2 + \mathrm{j}k''z\right)\omega^2\right]$$

$$\underline{A}(z,t') = A_0\left(\frac{\tau^2}{(\tau^2 + \mathrm{j}k''z)}\right)^{1/2} \exp\left[-\frac{1}{2}\frac{t'^2}{(\tau^2 + \mathrm{j}k''z)}\right]$$

Exponent Real and Imaginary Part:

$$\underline{A}(z,t') = A_0\left(\frac{\tau^2}{(\tau^2 + \mathrm{j}k''z)}\right)^{1/2} \exp\left[-\frac{1}{2}\frac{\tau^2 t'^2}{\left(\tau^4 + (k''z)^2\right)} + \mathrm{j}\frac{1}{2}k''z\frac{t'^2}{\left(\tau^4 + (k''z)^2\right)}\right]$$

z-dependent phase shift

determines pulse width

chirp

FWHM Pulse width:

$$\exp\left[-\frac{\tau^2(\tau'_{FWHM}/2)^2}{\left(\tau^4 + (k''z)^2\right)}\right] = 0.5$$

Initial pulse width:

$$\tau_{FWHM} = 2\sqrt{\ln 2}\;\tau$$

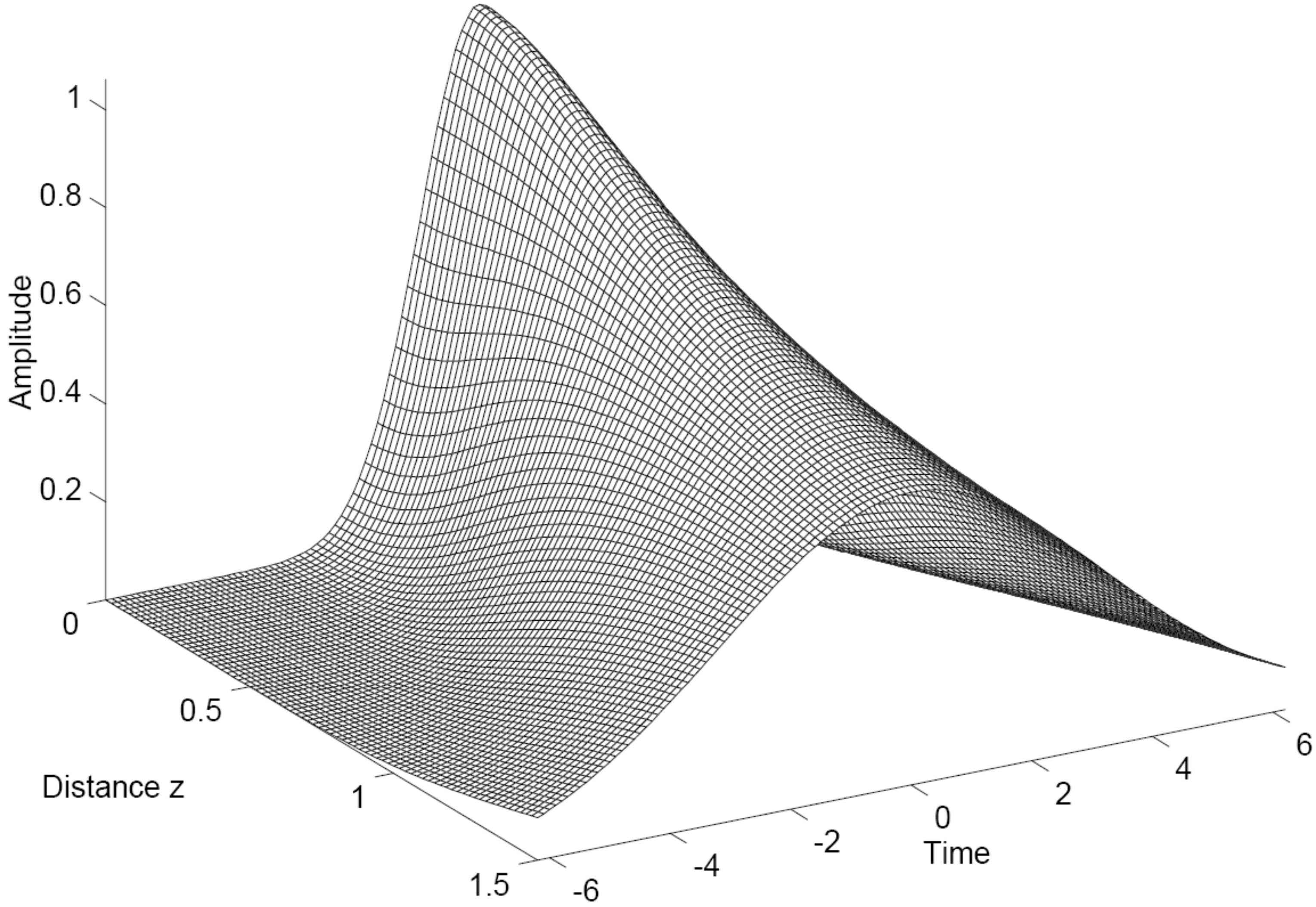
After propagation over a distance z=L:

$$\begin{aligned}\tau'_{FWHM} &= 2\sqrt{\ln 2}\,\tau\sqrt{1+\left(\frac{k''L}{\tau^2}\right)^2} \\ &= \tau_{FWHM}\sqrt{1+\left(\frac{k''L}{\tau^2}\right)^2}\end{aligned}$$

For large distances:

$$\tau'_{FWHM} = 2\sqrt{\ln 2}\,\left|\frac{k''L}{\tau}\right| \quad \text{for} \quad \left|\frac{k''L}{\tau^2}\right| \gg 1$$

$$\left|\frac{k''L}{\tau^2}\right| \gg 1. \qquad \tau'_{FWHM} = 2\sqrt{\ln 2}\,\left|\frac{k''L}{\tau}\right|$$



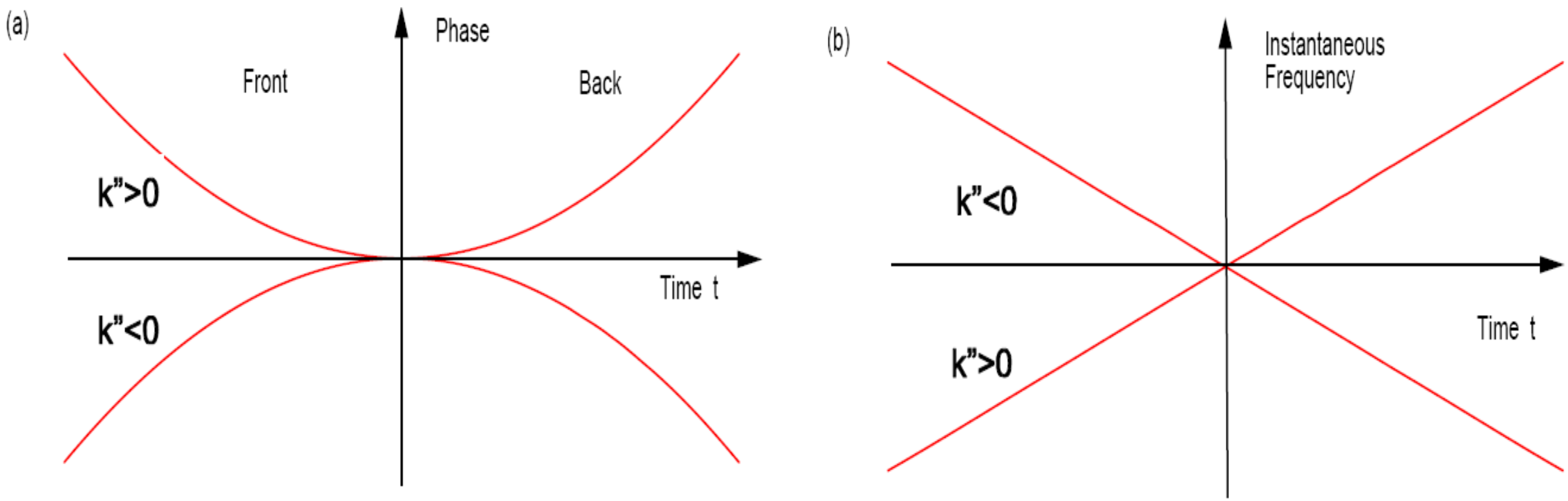
Magnitude of the complex envelope of a Gaussian pulse, $|\underline{A}(z, t')|$, in a dispersive medium

Chirp:

$$\phi(z=L,t') = -\frac{1}{2}\arctan\left[\frac{k''L}{\tau^2}\right] + \frac{1}{2}k''L\frac{t'^2}{\left(\tau^4+(k''L)^2\right)}$$

Instantaneous Frequency:

$$\omega(z=L,t') = \frac{\partial}{\partial t'}\phi(L,t') = \frac{k''L}{\left(\tau^4+(k''L)^2\right)}t'$$



k”>0: Postive Group Velocity Dispersion (GVD), low frequencies travel faster and are in front of the pulse

(a) Phase and (b) instantaneous frequency of a Gaussian pulse during propagation through a medium with positive or negative dispersion

3.5 Sellmeier Equations and Kramers-Kroenig Relations

Causality of medium impulse response: $\chi(t) = 0, \text{ for } t < 0$

Leads to relationship between real and imaginary part of susceptibility

$$\chi_r(\Omega) = \frac{2}{\pi}\int_0^\infty \frac{\omega\chi_i(\omega)}{\omega^2-\Omega^2}d\omega = n_r^2(\Omega) - 1$$

$$\chi_i(\Omega) = -\frac{2}{\pi}\int_0^\infty \frac{\Omega\chi_r(\omega)}{\omega^2-\Omega^2}d\omega.$$

Approximation for absorption spectrum in a medium:

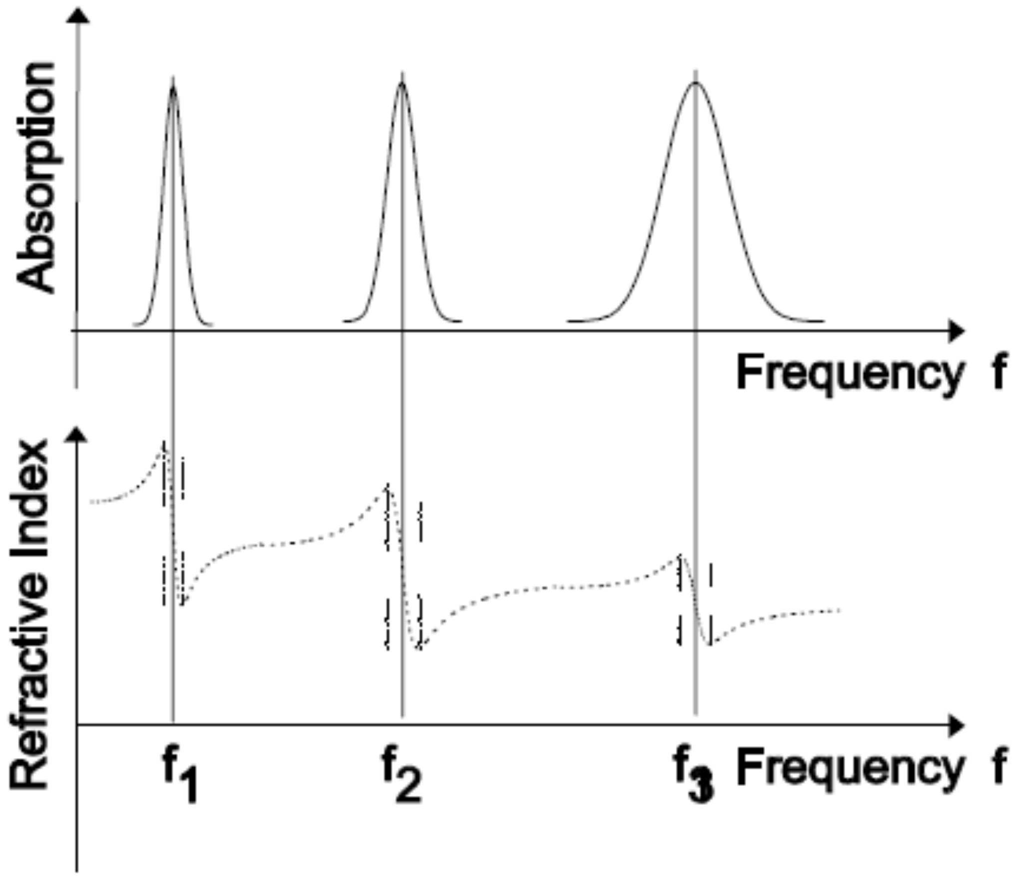
$$\chi_i(\Omega) = \sum A_i\delta\left(\omega-\omega_i\right)$$

$$n^2(\Omega) = 1 + \underbrace{\sum_i A_i\frac{\omega_i}{\omega_i^2-\Omega^2} = 1 + \sum_i a_i\frac{\lambda}{\lambda^2-\lambda_i^2}}_{\chi_r(\Omega)}$$

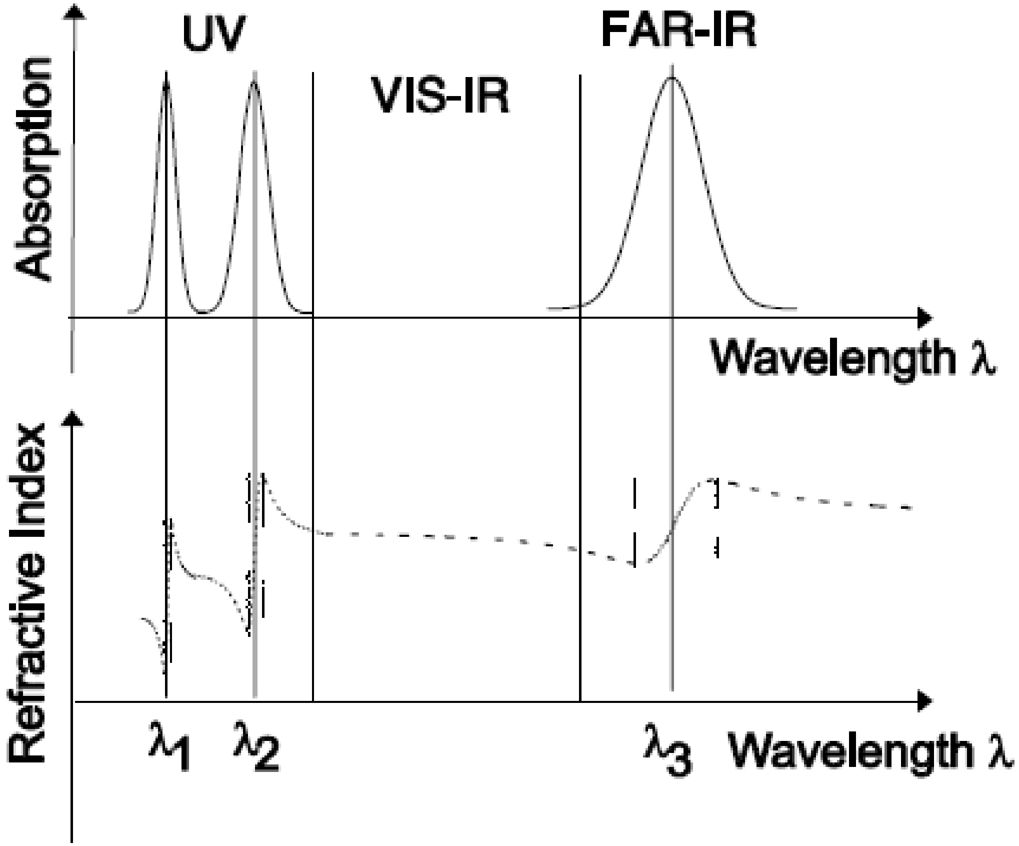
Example: Sellmeier Coefficients for Fused Quartz and Sapphire

	Fused Quartz	Sapphire
a_1	0.6961663	1.023798
a_2	0.4079426	1.058364
a_3	0.8974794	5.280792
λ_1^2	$4.679148 \cdot 10^{-3}$	$3.77588 \cdot 10^{-3}$
λ_2^2	$1.3512063 \cdot 10^{-2}$	$1.22544 \cdot 10^{-2}$
λ_3^2	$0.9793400 \cdot 10^{2}$	$3.213616 \cdot 10^{2}$

Table 2.3: Table with Sellmeier coefficients for fused quartz and sapphire.

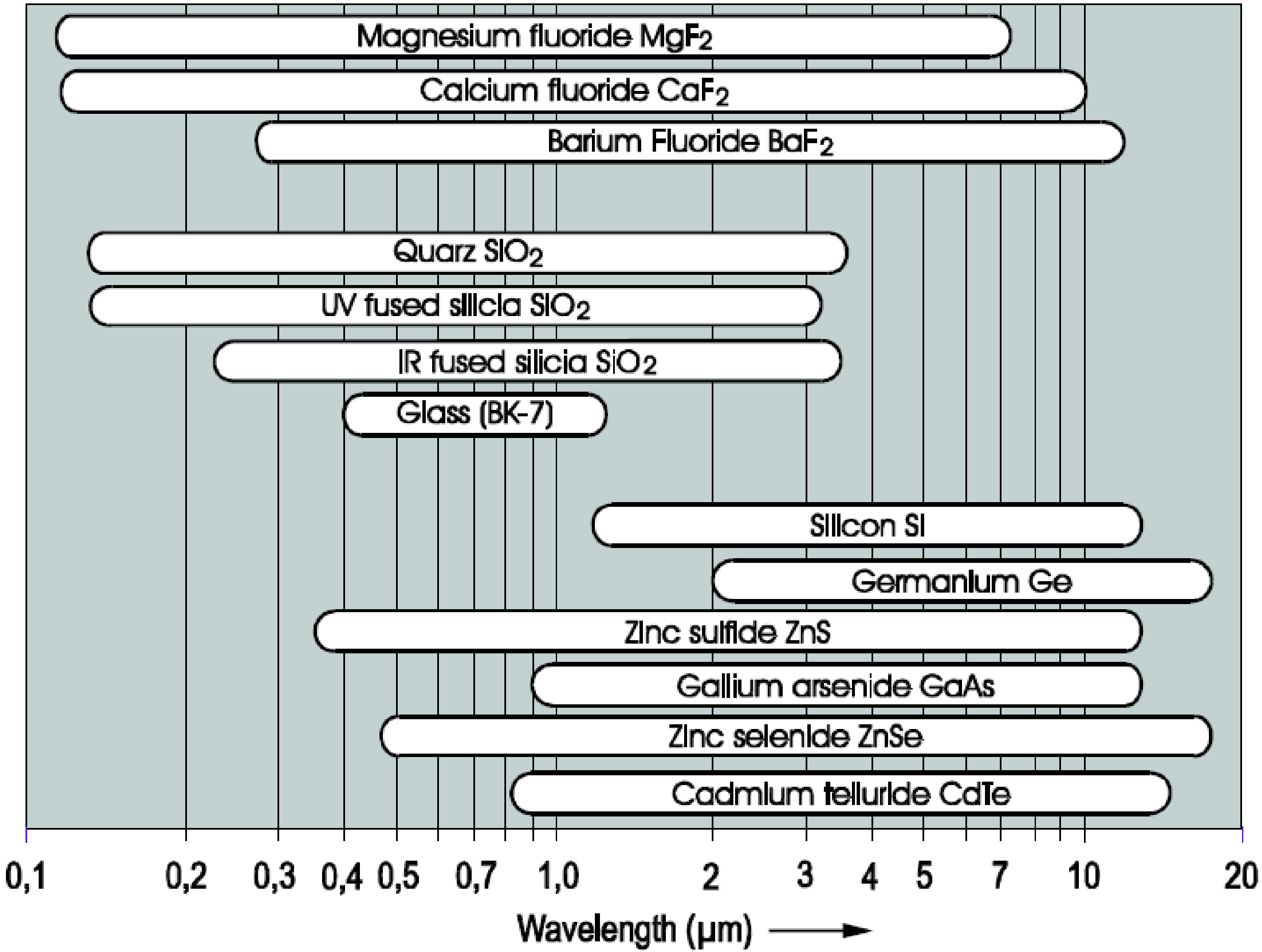


Contribution of absorption lines to index changes



Typical distribution of absorption lines in medium transparent in the visible.

$$\frac{dn}{d\lambda} < 0 : \text{normal dispersion (blue refracts more than red)}$$
$$\frac{dn}{d\lambda} > 0 : \text{abnormal dispersion}$$



Transparency range of some materials according to Saleh and Teich, Photonics p. 175.

Group Velocity and Group Delay Dispersion

$$GVD = \left.\frac{d^2k(\omega)}{d\omega^2}\right|_{\omega=0} = \left.\frac{d}{d\omega}\frac{1}{\upsilon_g(\omega)}\right|_{\omega=0}$$

$$GDD = \left.\frac{d^2k(\omega)}{d\omega^2}\right|_{\omega=0} L = \left.\frac{d}{d\omega}\frac{L}{\upsilon_g(\omega)}\right|_{\omega=0} = \frac{d}{d\omega}T_g(\omega)\Big|_{\omega=0}$$

Group Delay: $T_g(\omega) = L/\upsilon_g(\omega)$

Dispersion Characteristic	Definition	Comp. from $n(\lambda)$
medium wavelength: λ_n	$\frac{\lambda}{n}$	$\frac{\lambda}{n(\lambda)}$
wavenumber: k	$\frac{2\pi}{\lambda_n}$	$\frac{2\pi}{\lambda}n(\lambda)$
phase velocity: υ_p	$\frac{\omega}{k}$	$\frac{c_0}{n(\lambda)}$
group velocity: υ_g	$\frac{d\omega}{dk}$; $d\lambda = \frac{-\lambda^2}{2\pi c_0}d\omega$	$\frac{c_0}{n}\left(1-\frac{\lambda}{n}\frac{dn}{d\lambda}\right)^{-1}$
group velocity dispersion: GVD	$\frac{d^2k}{d\omega^2}$	$\frac{\lambda^3}{2\pi c_0^2}\frac{d^2n}{d\lambda^2}$
group delay: $T_g = \frac{L}{\upsilon_g} = \frac{d\phi}{d\omega}$	$\frac{d\phi}{d\omega} = \frac{d(kL)}{d\omega}$	$\frac{n}{c_0}\left(1-\frac{\lambda}{n}\frac{dn}{d\lambda}\right)L$
group delay dispersion: GDD	$\frac{dT_g}{d\omega} = \frac{d^2(kL)}{d\omega^2}$	$\frac{\lambda^3}{2\pi c_0^2}\frac{d^2n}{d\lambda^2}L$

Table 2.4: Table with important dispersion characteristics and how to compute them from the wavelength dependent refractive index $n(\lambda)$.

3.6 Nonlinear Pulse Propagation

3.6.1 The Optical Kerr Effect

Without derivation, there is a nonlinear contribution to the refractive index:

$$n = n(\omega, |A|^2) \approx n_0(\omega) + n_{2,L}|A|^2$$

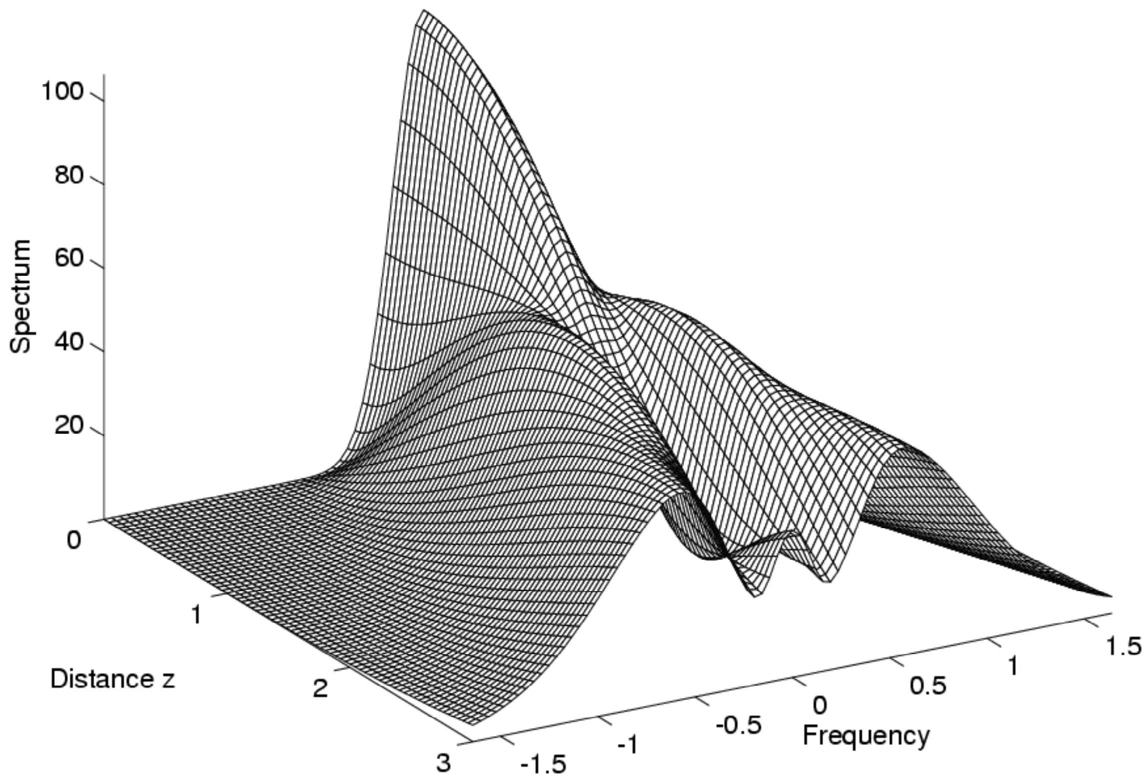
Polarization dependent

Material	Refractive index n	$n_{2,L}[cm^2/W]$
Sapphire (Al_2O_3)	1.76 @ 850 nm	$3{\cdot}10^{-16}$
Fused Quarz	1.45 @ 1064 nm	$2.46{\cdot}10^{-16}$
Glass (LG-760)	1.5 @ 1064 nm	$2.9{\cdot}10^{-16}$
YAG ($Y_3Al_5O_{12}$)	1.82 @ 1064 nm	$6.2{\cdot}10^{-16}$
YLF ($LiYF_4$), n_e	1.47 @ 1047 nm	$1.72{\cdot}10^{-16}$
Si	3.3 @ 1550 nm	$4{\cdot}10^{-14}$

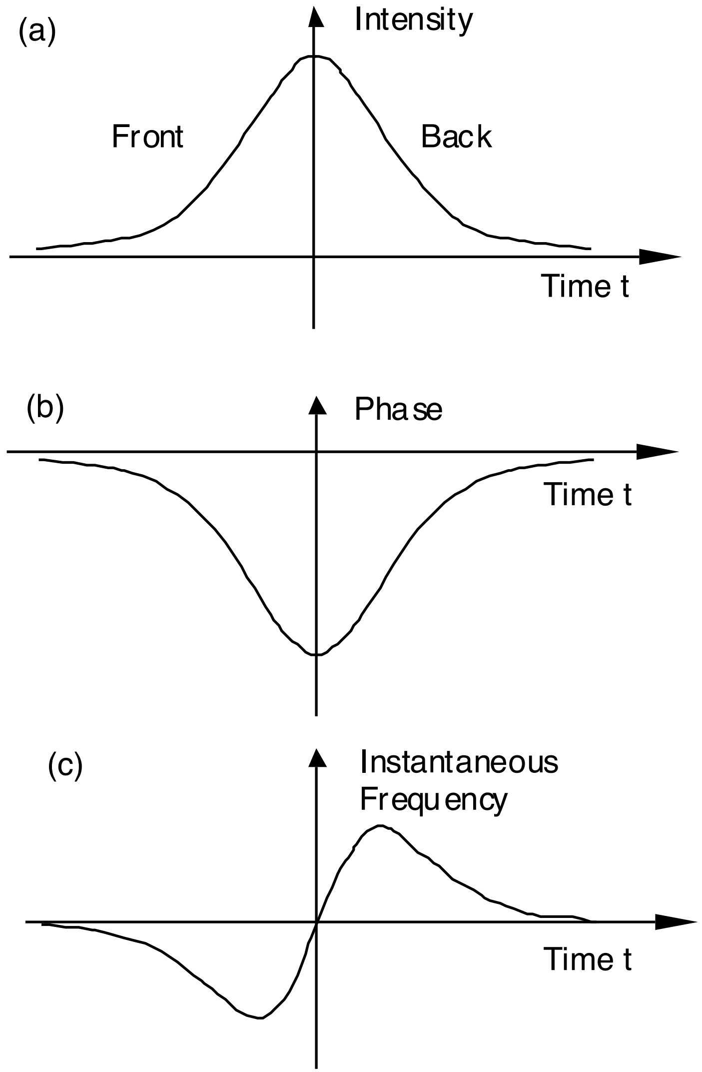
Table 3.1: Nonlinear refractive index of some materials

3.6.2 Self-Phase Modulation (SPM)

$$\frac{\partial A(z,t)}{\partial z} = -jk_0 n_{2,L}|A(z,t)|^2 A(z,t) = -j\delta|A(z,t)|^2 A(z,t).$$



Spectrum of a Gaussian pulse subject to self-phase modulation



(a) Intensity, (b) phase and c) instantaneous frequency of a Gaussian pulse during propagation

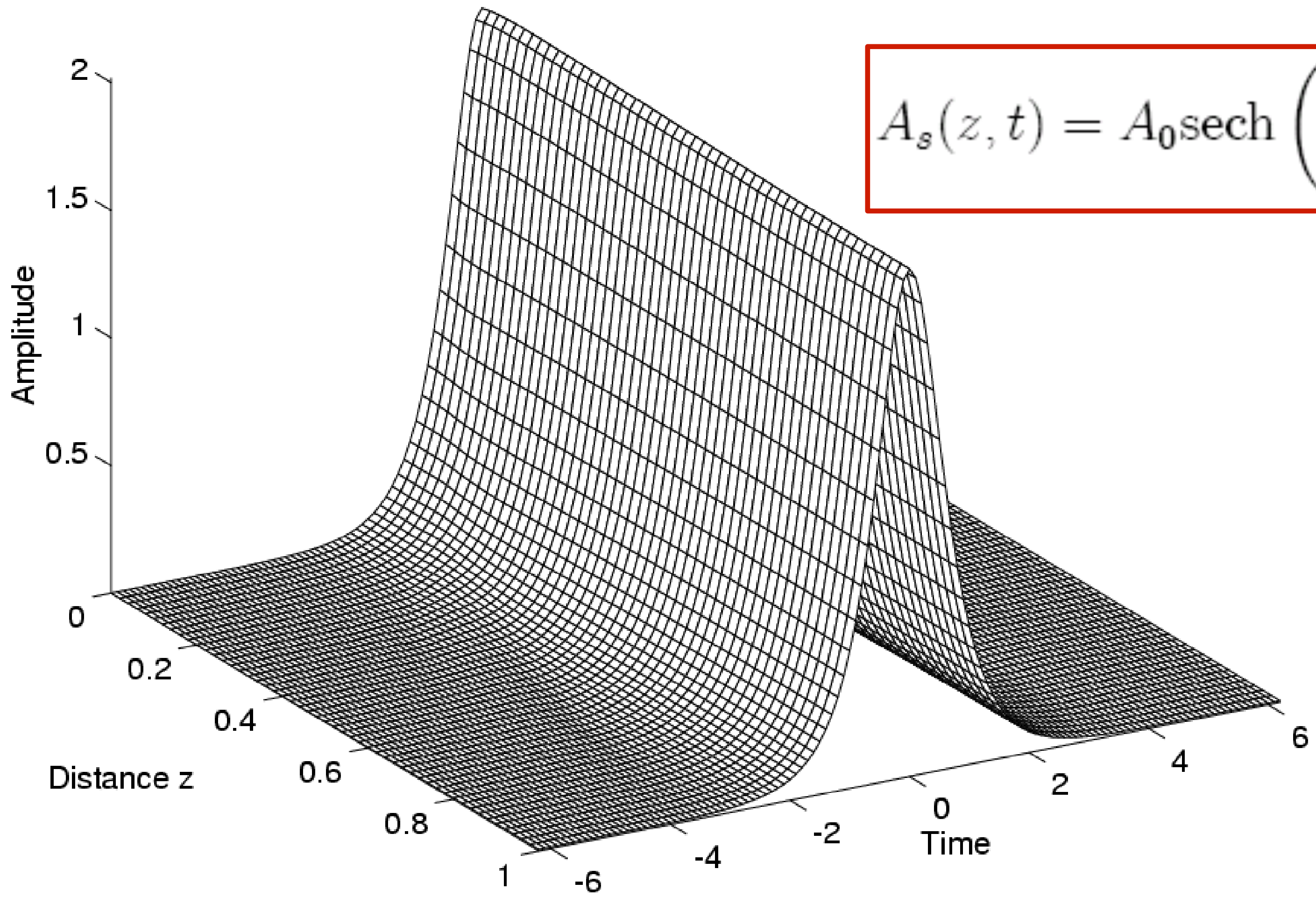
3.6.3 Nonlinear Schroedinger Equation (NSE)

$$j\frac{\partial A(z,t)}{\partial z} = -D_2\frac{\partial^2 A}{\partial t^2} + \delta |A|^2 A$$

$$j\frac{\partial A'(z',t)}{\partial z'} = \frac{\partial^2 A'}{\partial t'^2} + 2|A'|^2 A'$$

3.6.3.1 Solitons of the Nonlinear Schroedinger Equation

3.6.3.2 The Fundamental Soliton



$$A_s(z,t) = A_0 \mathrm{sech}\left(\frac{t}{\tau}\right) e^{-j\theta}$$

Propagation of a fundamental soliton

Important Relations

Nonlinear phase shift soliton aquires during propagation: $$\theta = \frac{1}{2}\delta A_0^2 z$$

Balance between dispersion and nonlinearity: $$\theta = \frac{|D_2|}{\tau^2} z.$$

Soliton Energy: $$w = \int_{-\infty}^{\infty} |A_s(z,t)|^2 dt = 2A_0^2\tau$$

Pulse width: $$\tau = \frac{4|D_2|}{\delta w}$$

Area Theorem

$$\text{Pulse Area} = \int_{-\infty}^{\infty} |A_s(z,t)| dt = \pi A_0 \tau = \pi \sqrt{\frac{2|D_2|}{\delta}}.$$

General Fundamental Soliton Solution

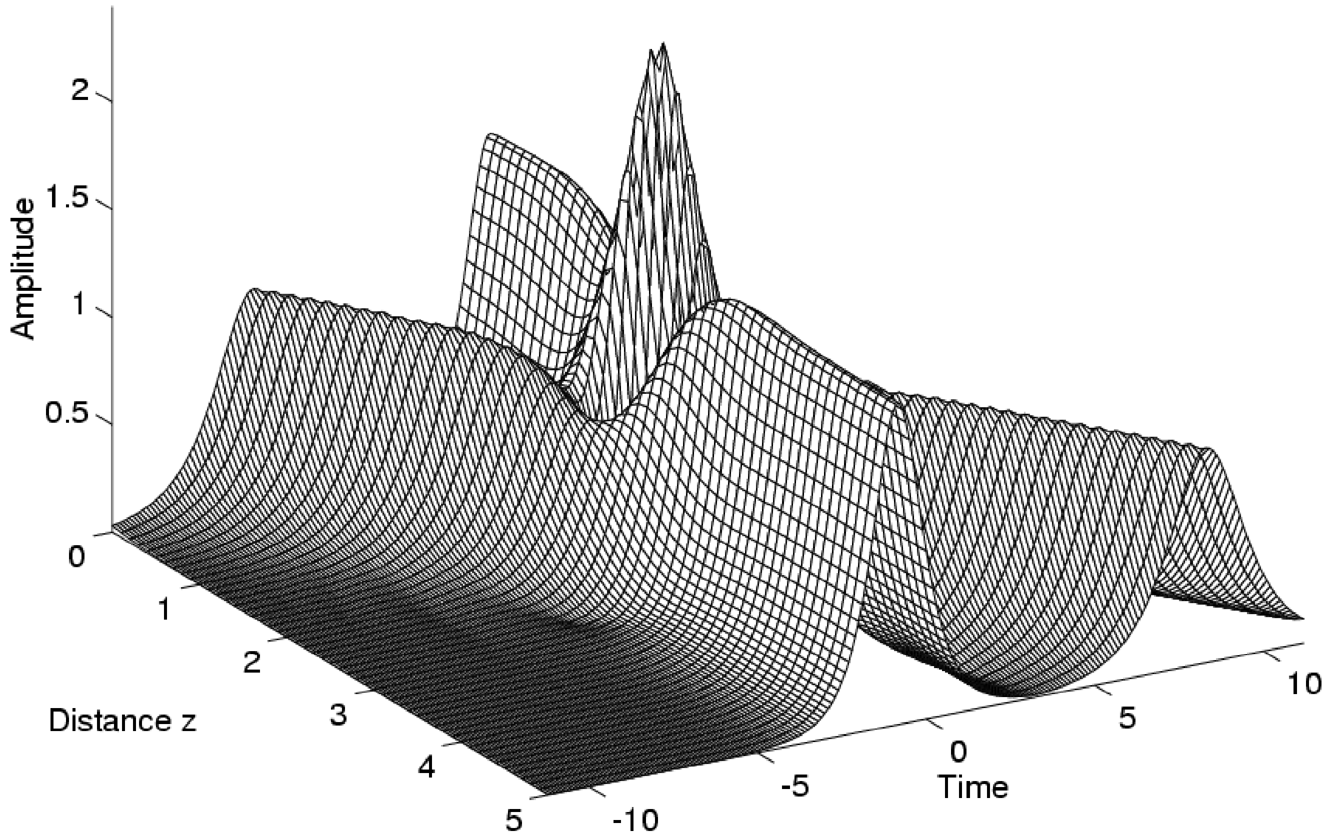
$$A_s(z,t) = A_0 \text{sech}(x(z,t)) e^{-j\theta(z,t)}$$

$$x = \frac{1}{\tau}(t - 2|D_2|p_0 z - t_0)$$

Change of center frequency!

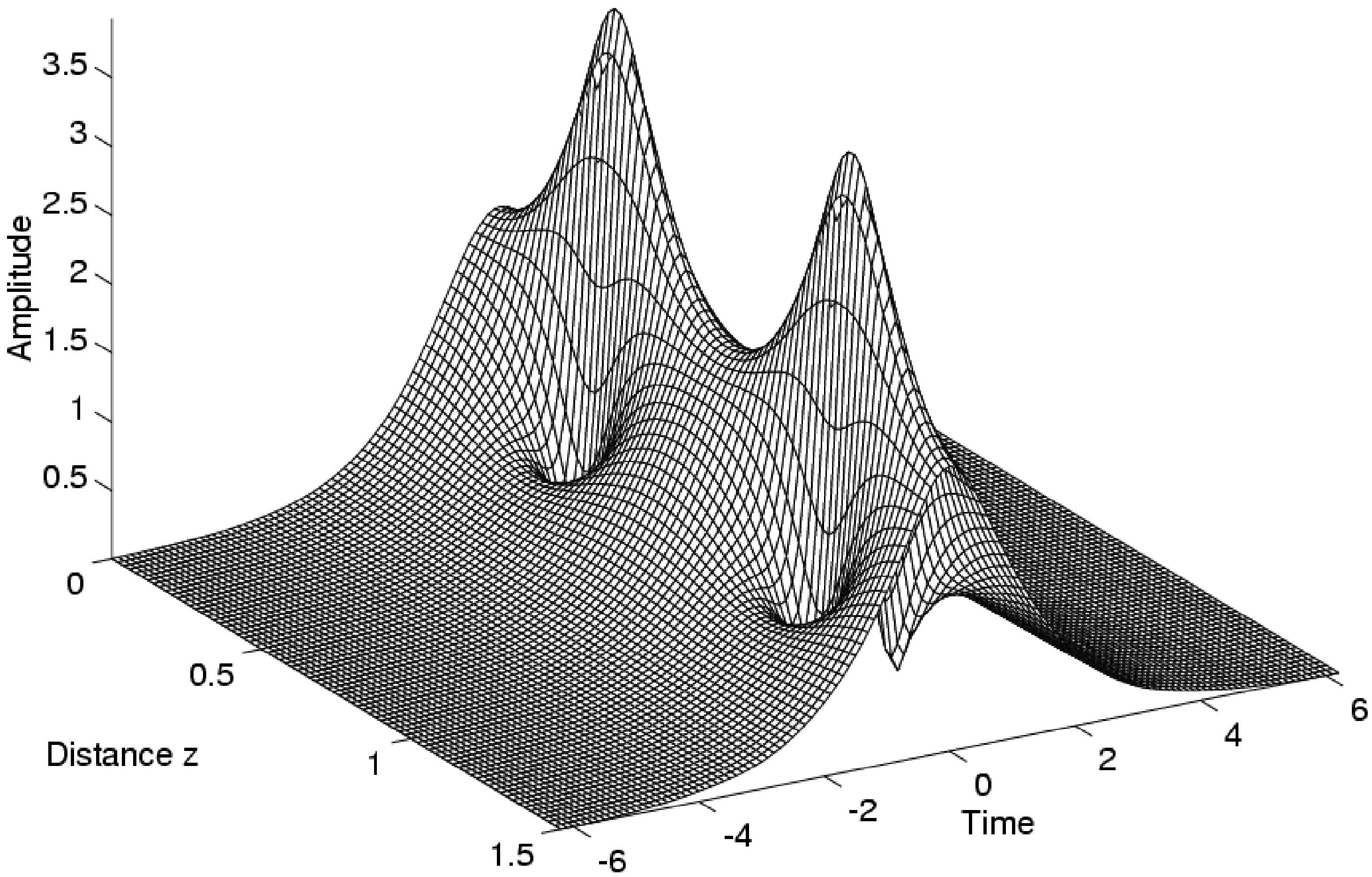
$$\theta = p_0(t - t_0) + |D_2|\left(\frac{1}{\tau^2} - p_0^2\right) z + \theta_0$$

3.6.3.3 Higher Order Soliton (Soliton Collision)



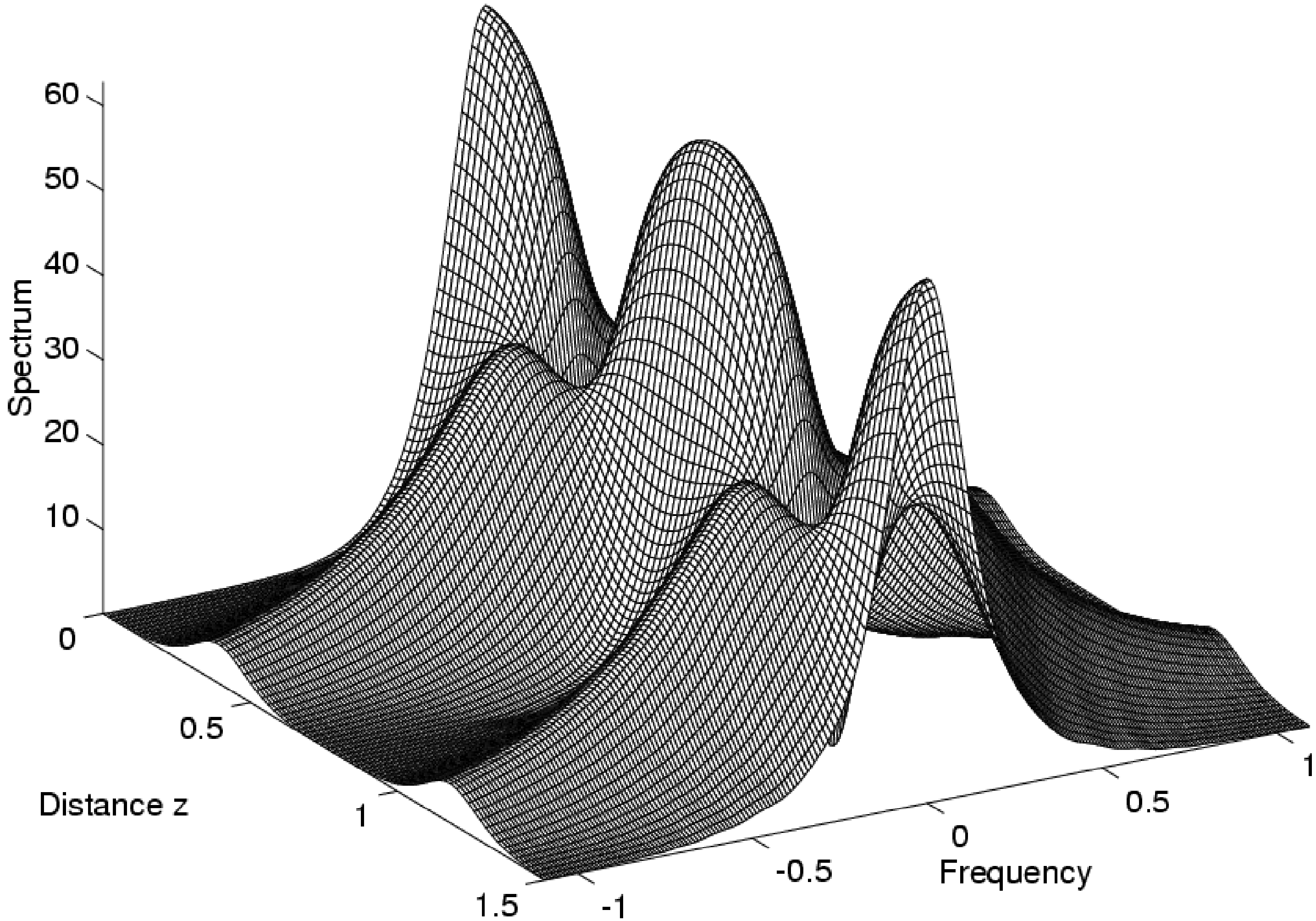
A soliton with high carrier frequency collides with a soliton of lower carrier frequency.

3.6.3.3 Higher Order Soliton (Breather Soliton)



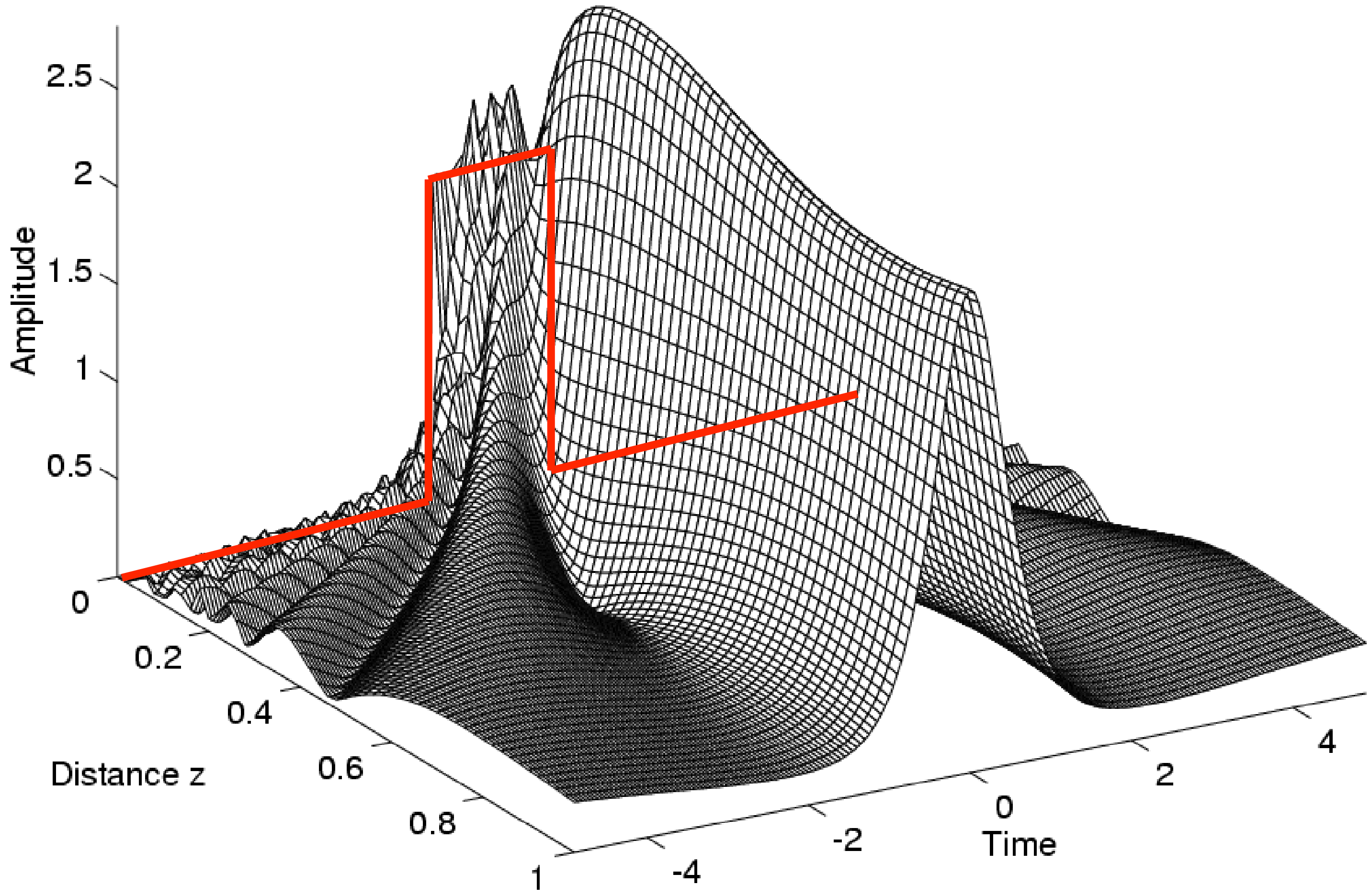
Amplitude of higher order soliton composed of
two fundamental solitons with the same carrier freuqency

3.6.3.3 Higher Order Soliton (Breather Soliton)



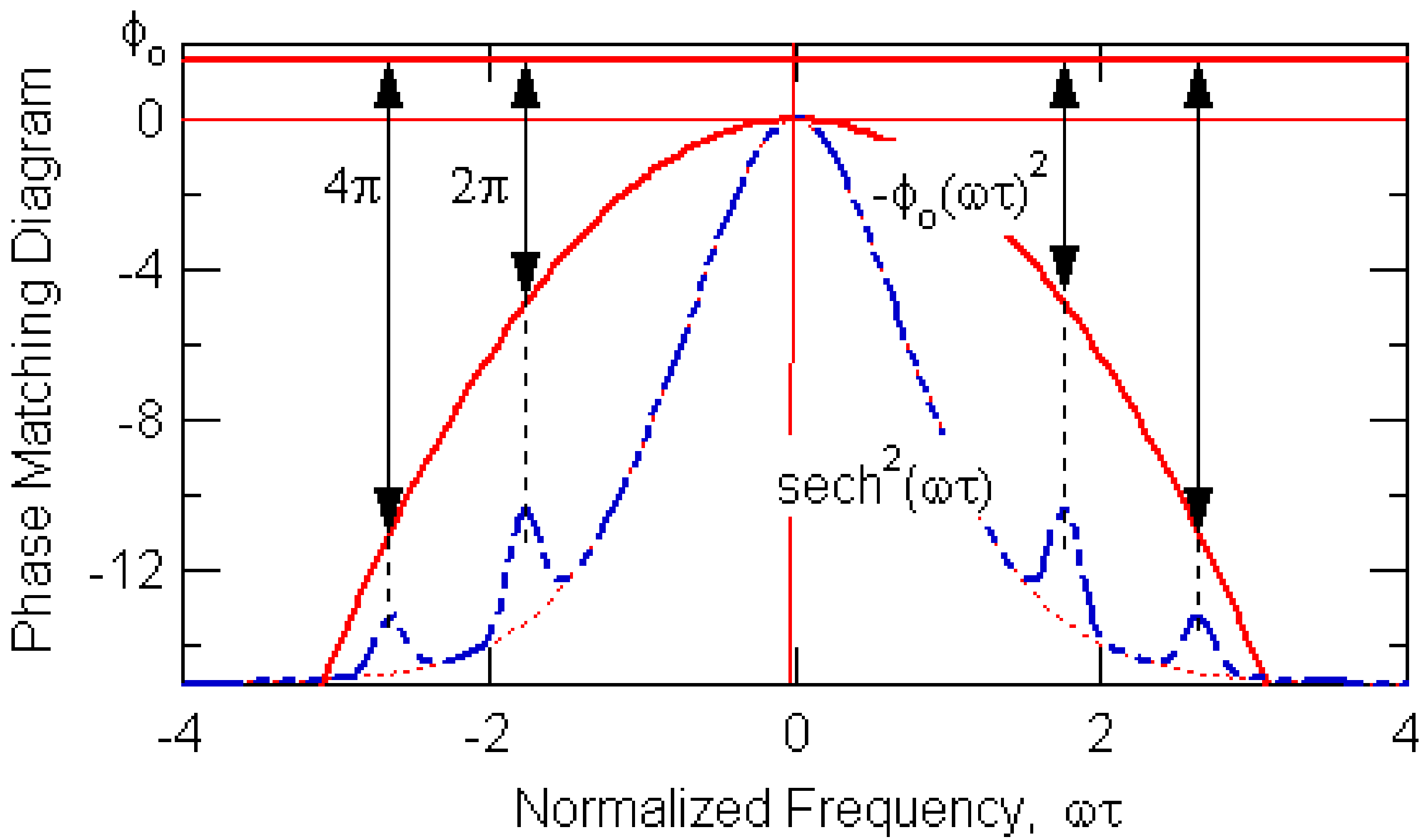
Spectrum of higher order soliton composed of
two fundamental solitons with the same carrier freuqency

Rectangular Shaped Initial Pulse and Continuum Generation



Solution of the NSE for a rectangular shaped initial pulse

Kelly Sidebands



Phase matching of soliton and continuum

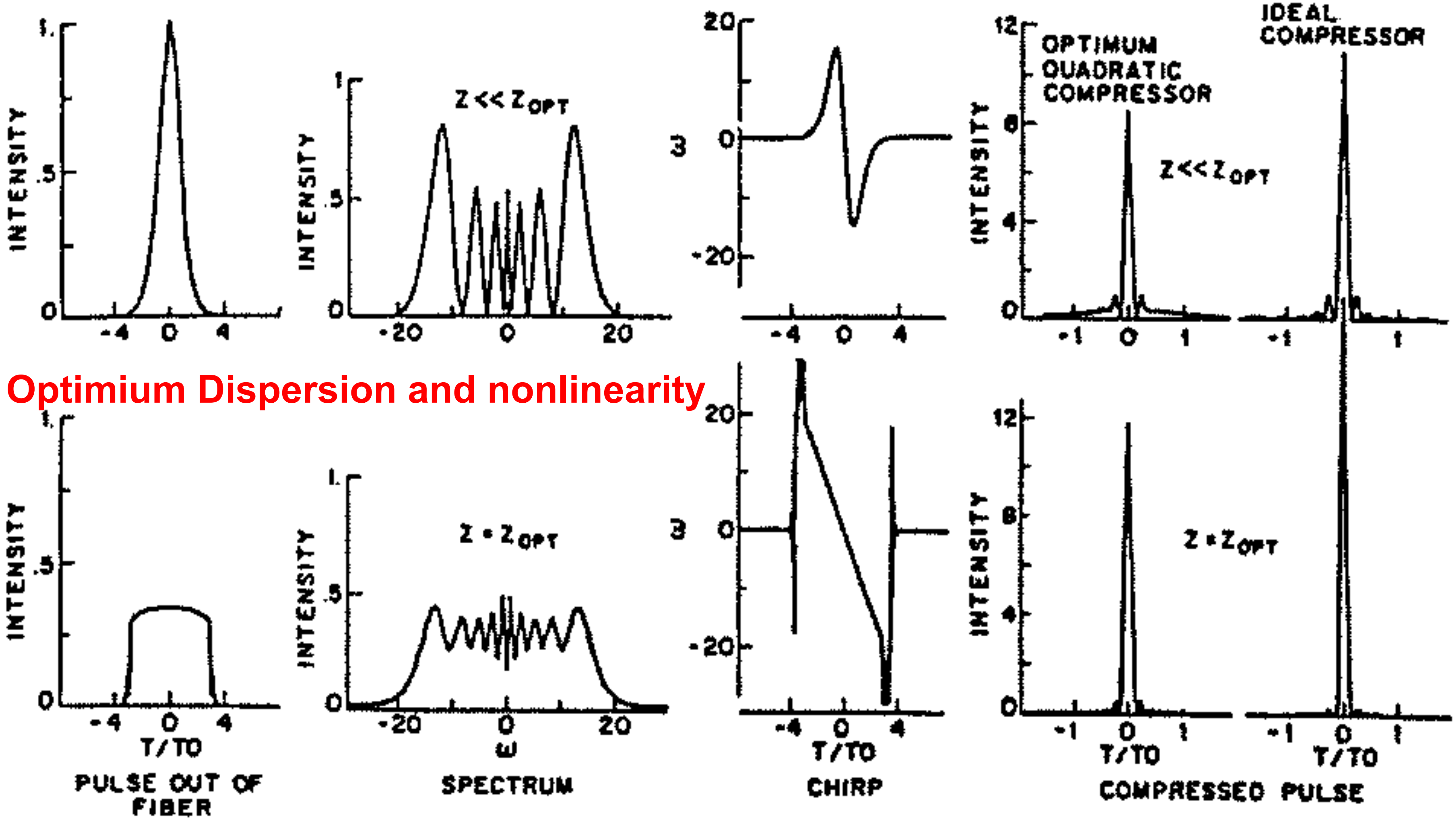
Avoid resonance catastrophy for: $\omega_m \gg \frac{1}{\tau} \longrightarrow \phi_0 \ll \pi/4$

3.7 Pulse Compression

3.7.1 General Pulse Compression Scheme

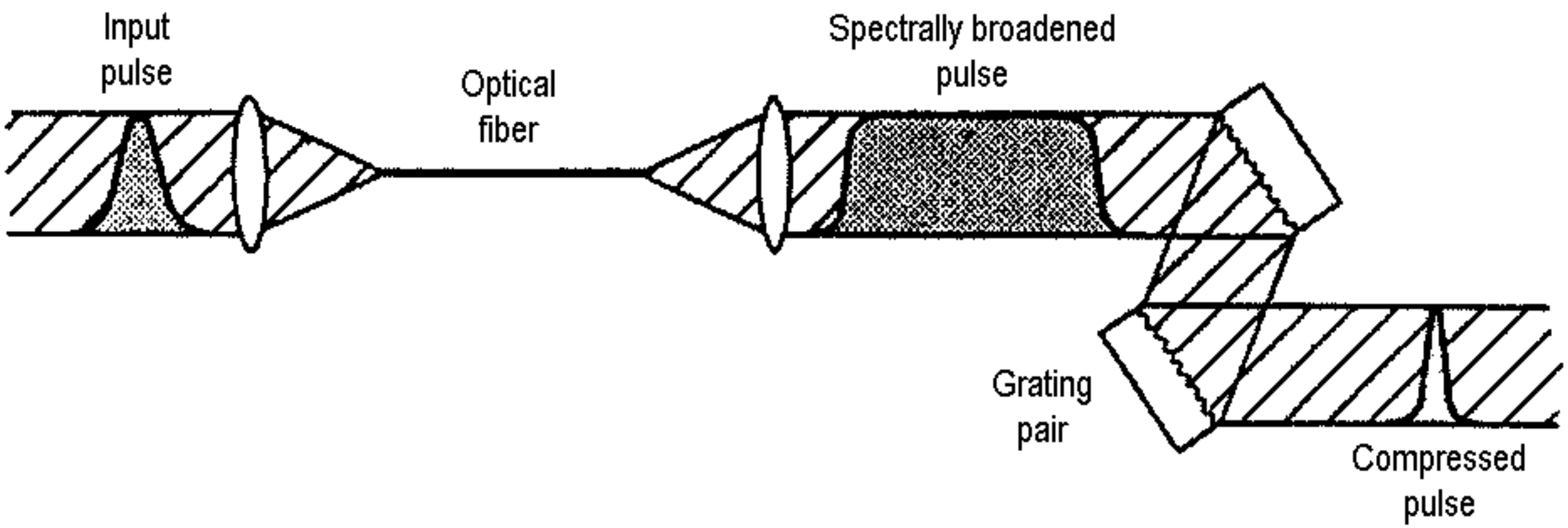
Dispersion negligible, only SPM

Optimium Dispersion and nonlinearity



Pulse compression

3.7.2 Spectral Broadening with Guided Modes



Fiber-grating pulse compressor to generate femtosecond pulses

3.7.3 Dispersion Compensation Techniques

$$T_g(\omega) = \phi'(\omega_0) + \phi''(\omega_0)\Delta\omega + \frac{1}{2}\phi'''(\omega_0)\Delta\omega^2 + \frac{1}{3!}\phi''''(\omega_0)\Delta\omega^3 + \cdots$$

Pulse Compression:

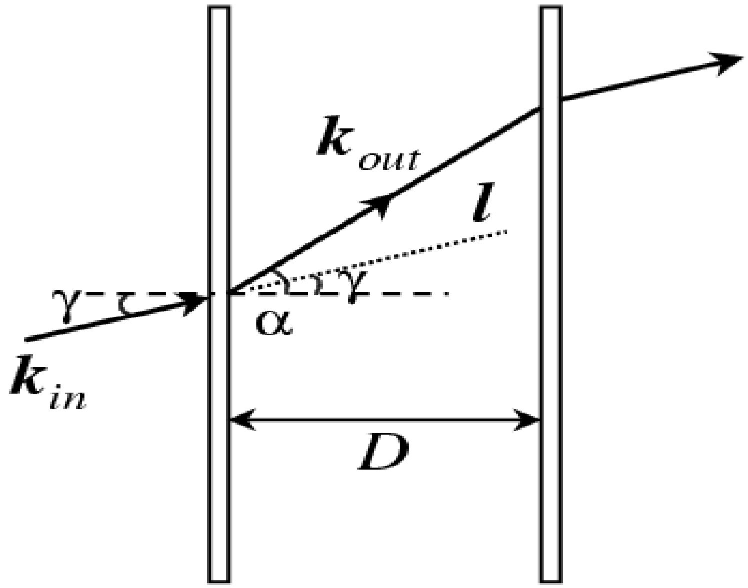
$$\begin{aligned}
\phi''(\omega_0) &= \phi''_{modulator} + \phi''_{compresssor} = 0 \\
\phi'''(\omega_0) &= \phi'''_{modulator} + \phi'''_{compresssor} = 0
\end{aligned}$$

Variable dispersion by grating and prism pairs

Grating Pair

Phase difference between scattered beam and reference beam"

$$\phi(\omega) = \mathbf{k}_{out}(\omega) \cdot \mathbf{l}.$$



$$\phi(\omega) = \frac{\omega}{c}|\mathbf{l}|\cos[\gamma - \alpha(\omega)] = \frac{\omega}{c}\frac{D}{\cos(\gamma)}\cos[\gamma - \alpha(\omega)]$$

$$m\frac{2\pi c}{\omega d} = [\sin\alpha(\omega) - \sin\gamma]$$

$$\phi''(\omega) = -\frac{4\pi^2 cD}{\omega^3 d^2 \cos^3\alpha(\omega)}m^2$$

$$\cos\alpha(\omega)\frac{d\alpha}{d\omega} = -\frac{2\pi c}{\omega^2 d}m$$

$$\phi'''(\omega) = \frac{12\pi^2 cD}{\omega^4 d^2 \cos^3\alpha(\omega)}\left(1 + \frac{2\pi c\sin\alpha(\omega)}{\omega d\cos^2\alpha(\omega)}\right)m^3$$

Disadvantage of grating pair: Losses ~ 25%

Prism Pair

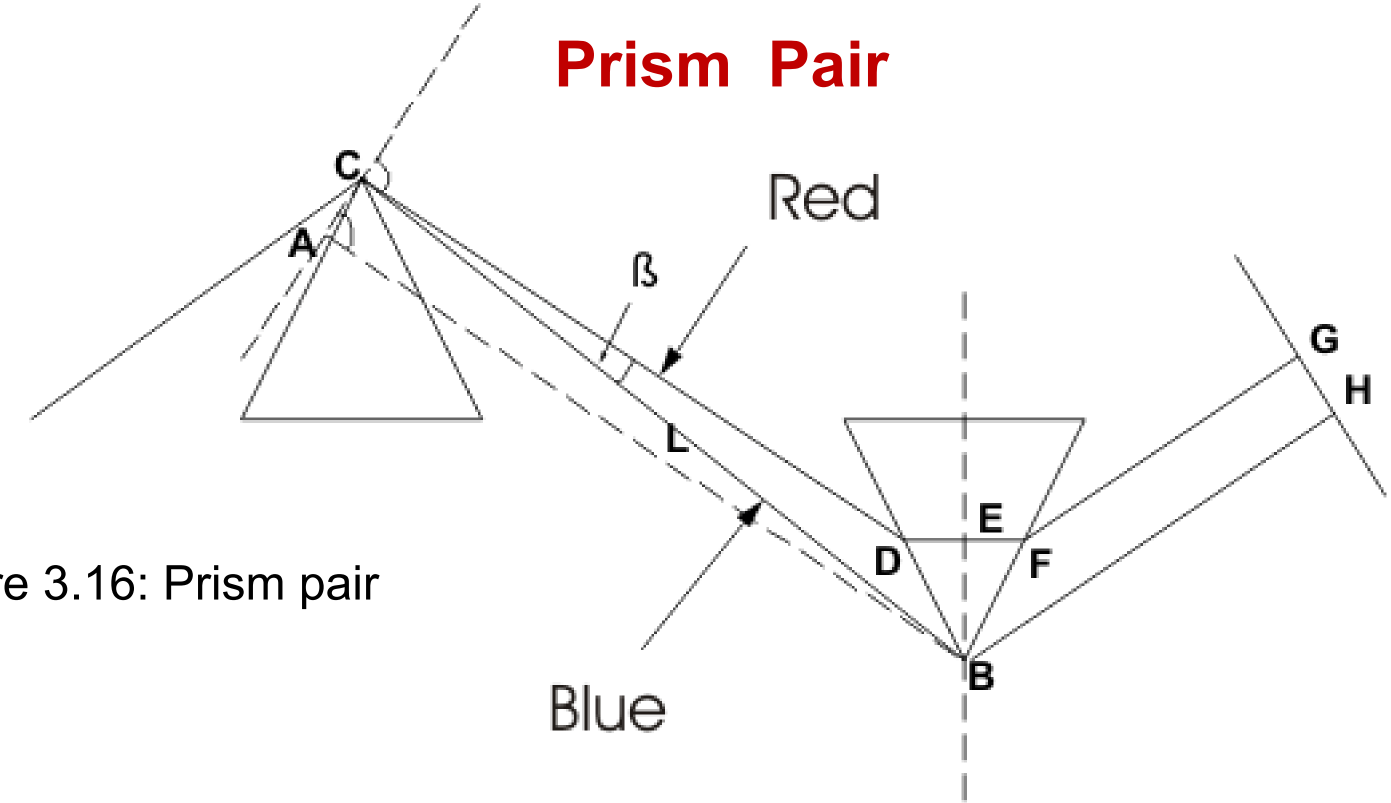


Figure 3.16: Prism pair

$$\phi(\omega) = \frac{\omega}{c}\ell_p \cos\beta(\omega) \qquad P(\lambda) = \ell_p \cos\beta(\lambda)$$

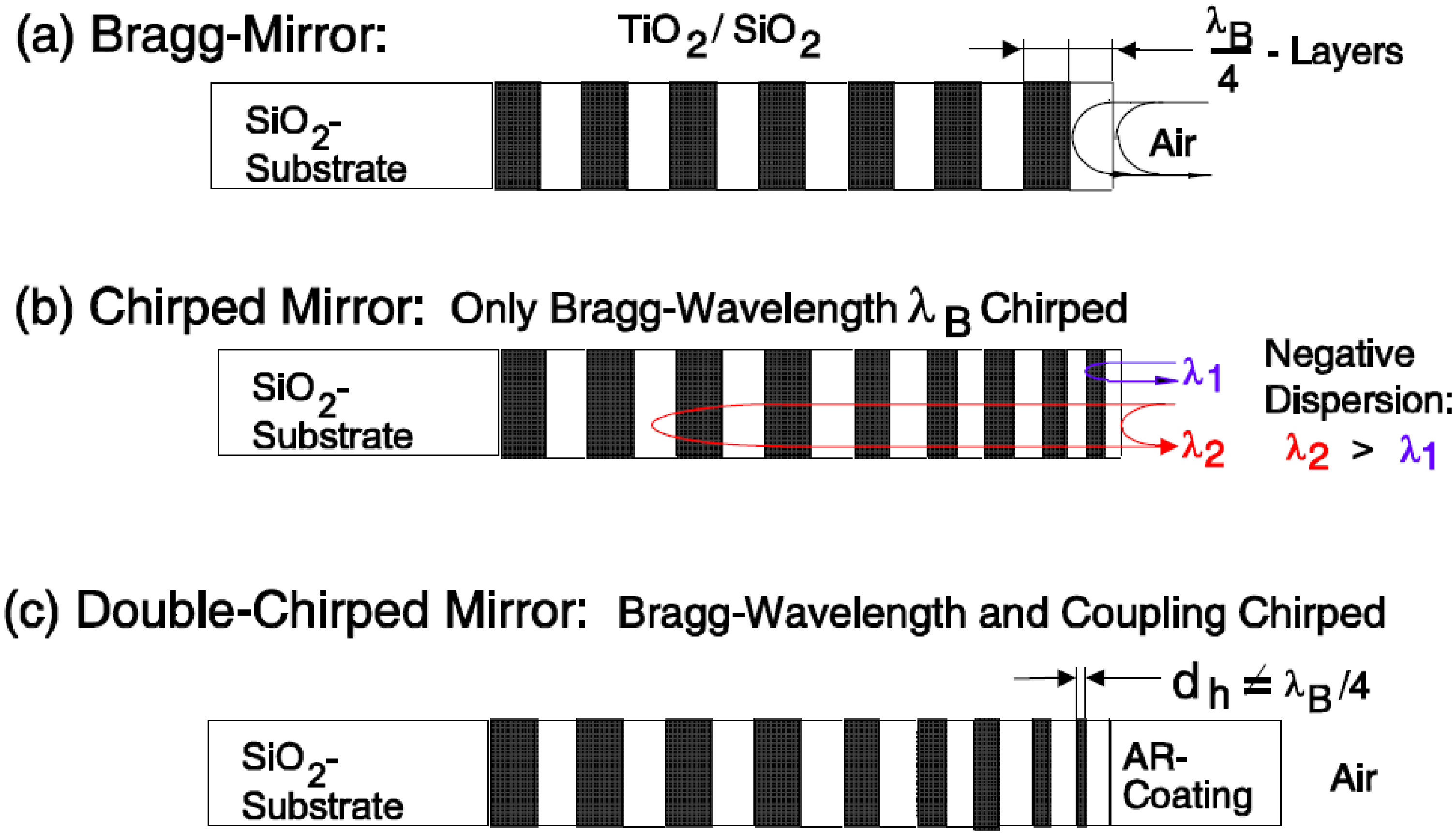
$$\phi''(\omega) = \frac{\lambda^3}{2\pi c^2}\frac{d^2P}{d\lambda^2} \qquad \frac{d^2P}{d\lambda^2} = 2[n'' + (2n - n^{-3})(n')^2]\ell_p \sin\beta - 4(n')^2\ell_p\cos\beta$$

$$\phi'''(\omega) = -\frac{\lambda^4}{4\pi^2c^3}\left(3\frac{d^2P}{d\lambda^2} + \lambda\frac{d^3P}{d\lambda^3}\right) \quad \frac{d^3P}{d\lambda^3} = [6(n')^3(n^{-6} + n^{-4} - 2n^{-2} + 4n^2) + 12n'n''(2n - n^{-3}) + 2n''']\ell_p \sin\beta + 12[(n^{-3} - 2n)(n')^3 - n'n'']\ell_p\cos\beta$$

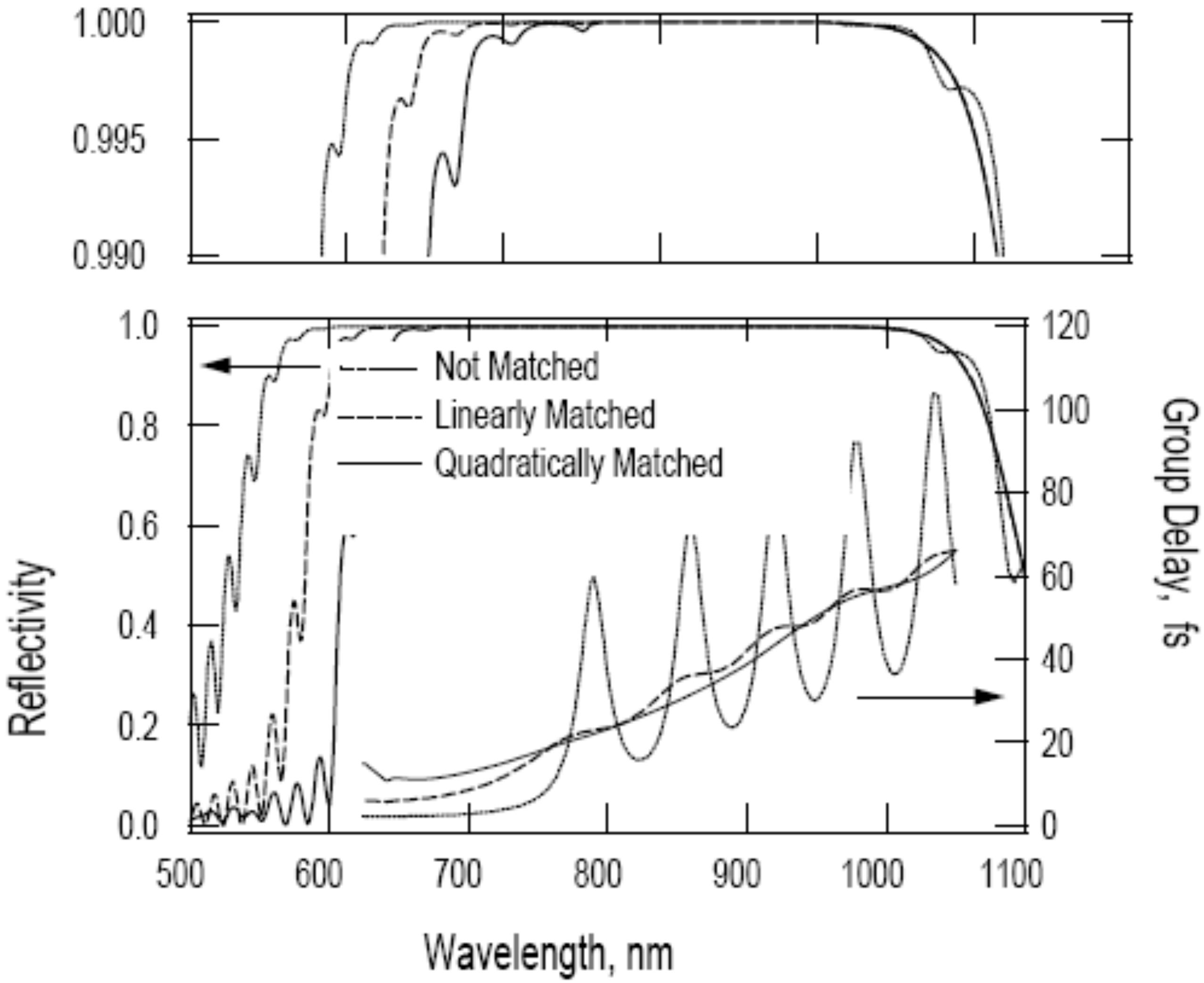
Combination of grating and prims pairs can eliminate 3rd order dispersion

3.7.4 Dispersion Compensating Mirrors

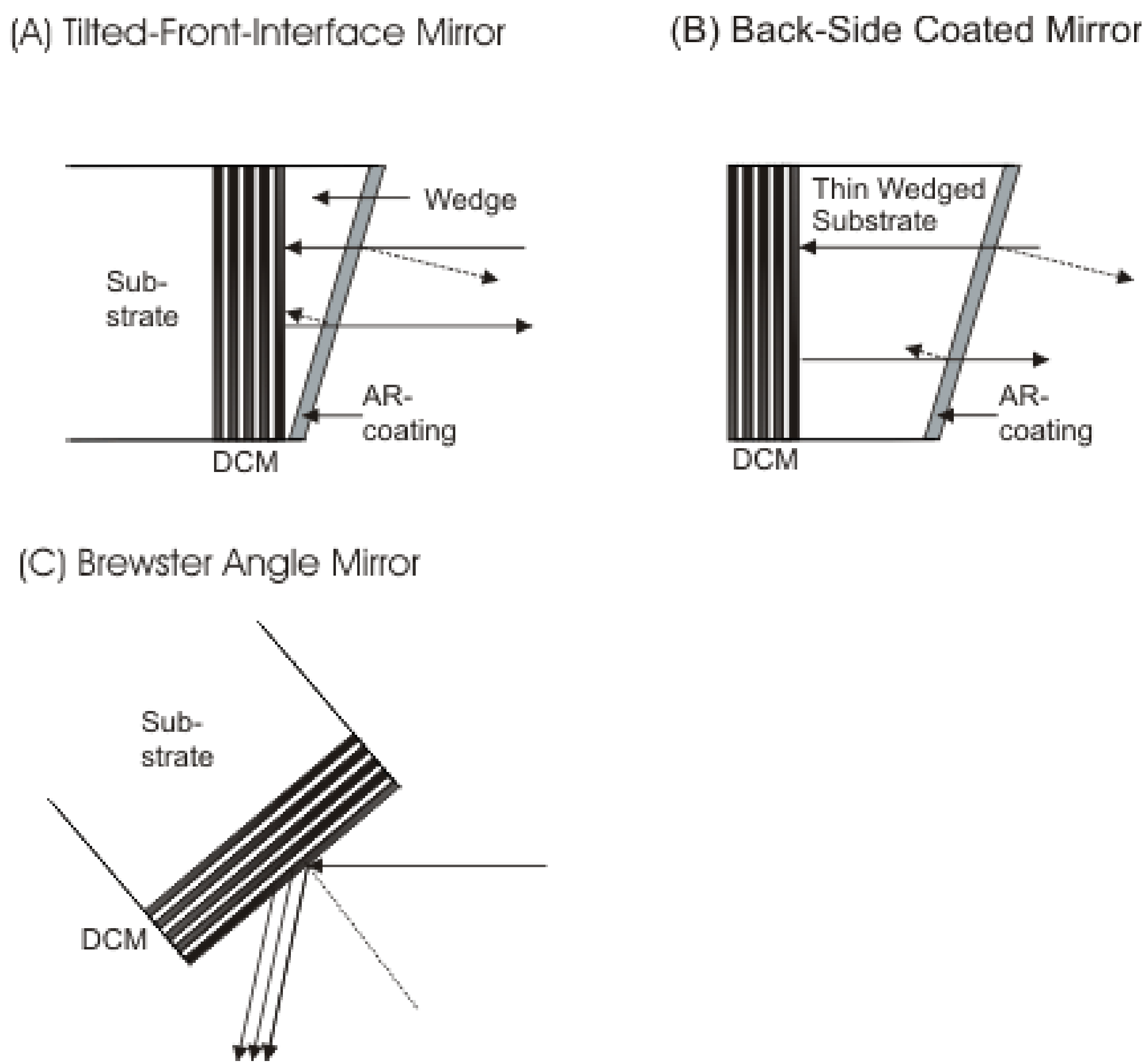
High reflecitvity bandwidth of Bragg mirror: $r_B = \frac{\Delta f}{f_c} = \frac{n_H - n_L}{n_H + n_L}$



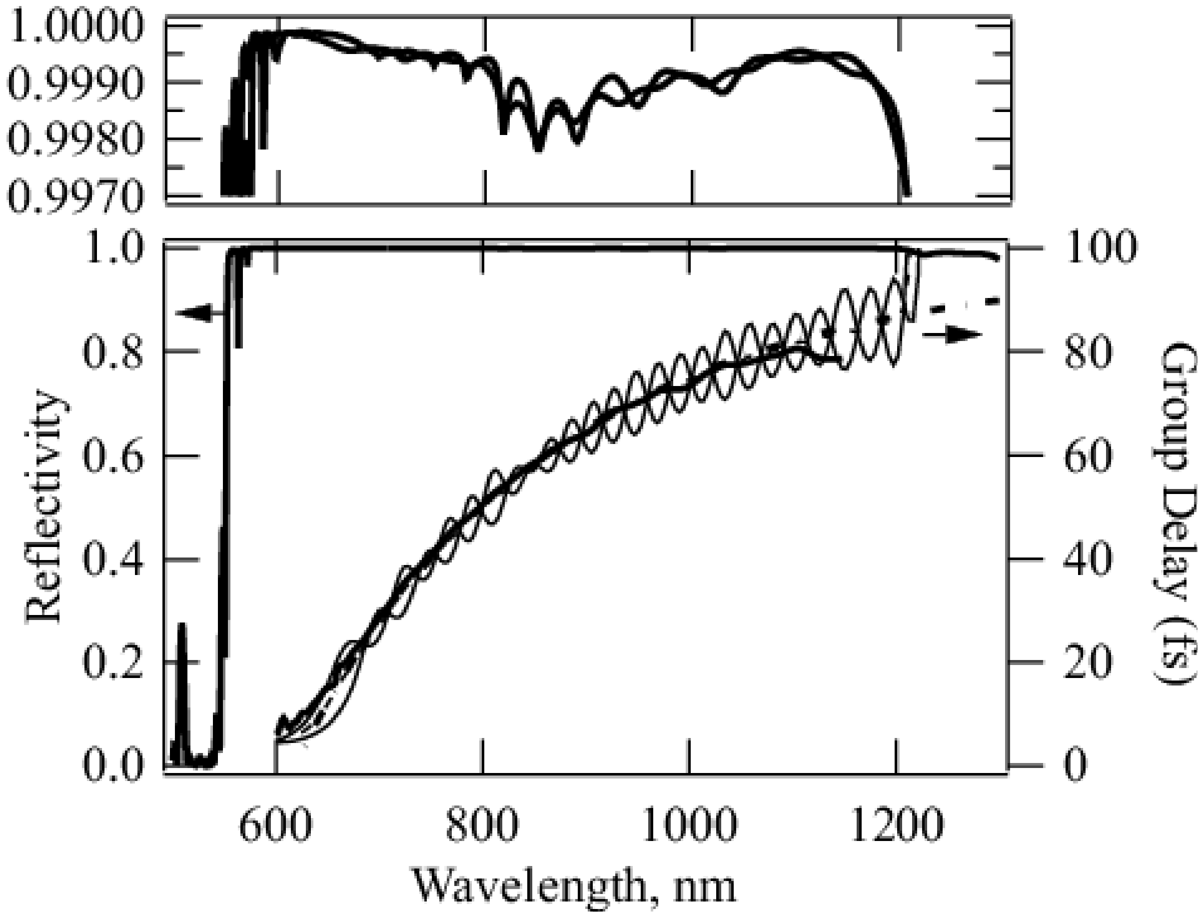
(a) Standard mirror, (b) simple chirped mirror,
(c) double-chirped mirror



Comparison for different chirping of the high-index layer

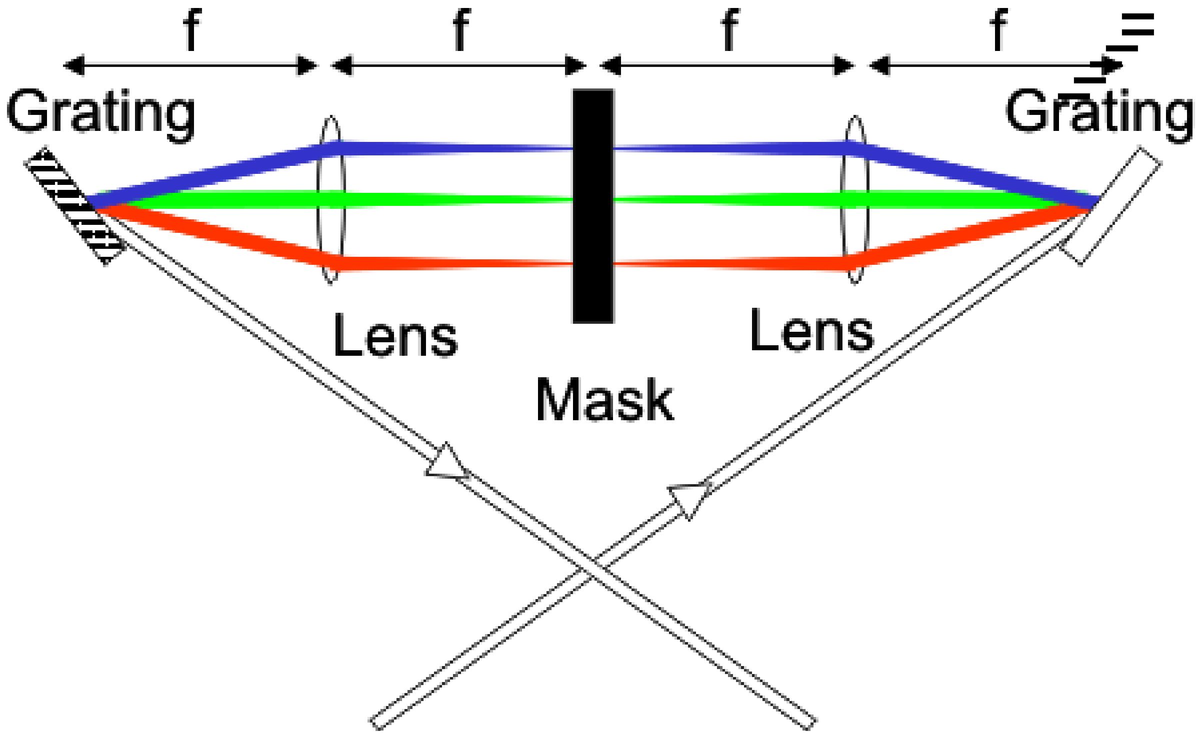


Proposed structures that avoid GTI-effects



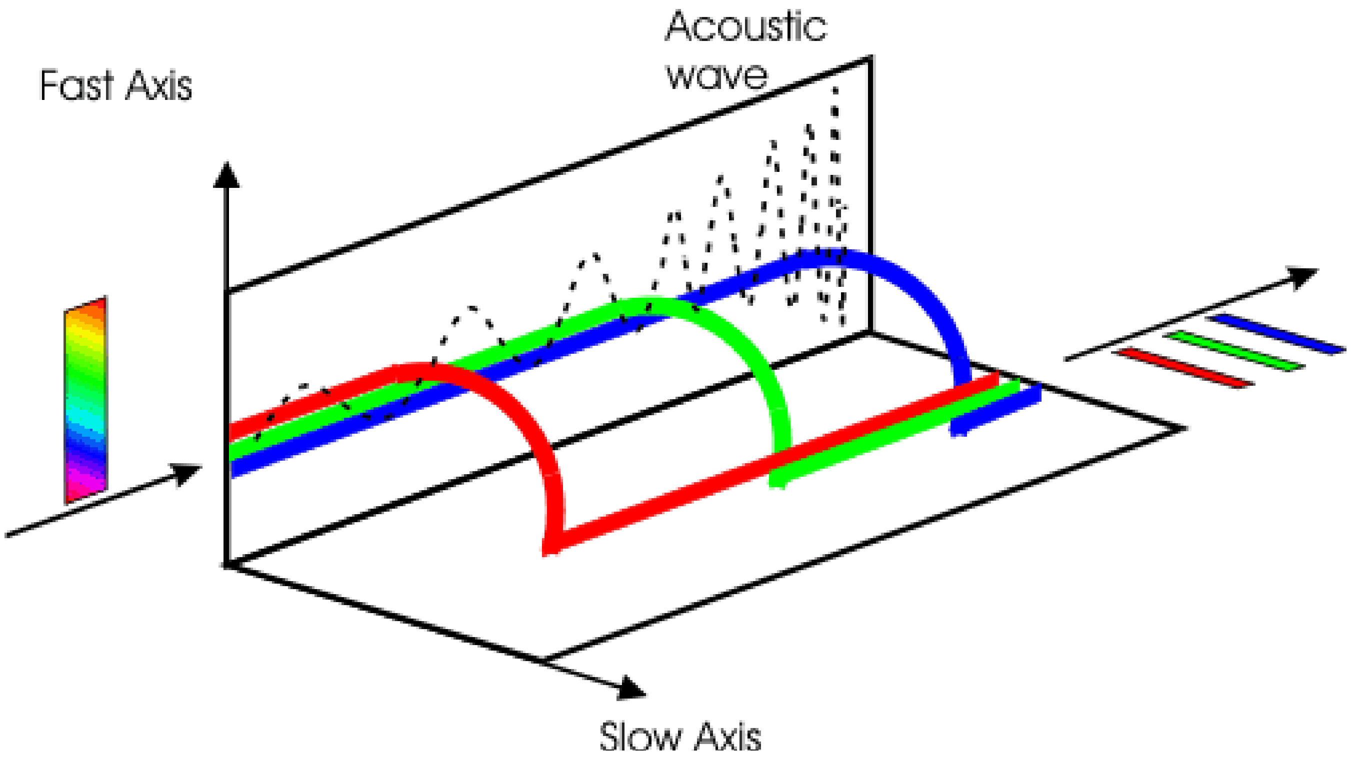
Double-chirped mirror pair

Dispersion Compensation with 4f-Pulse Shaper



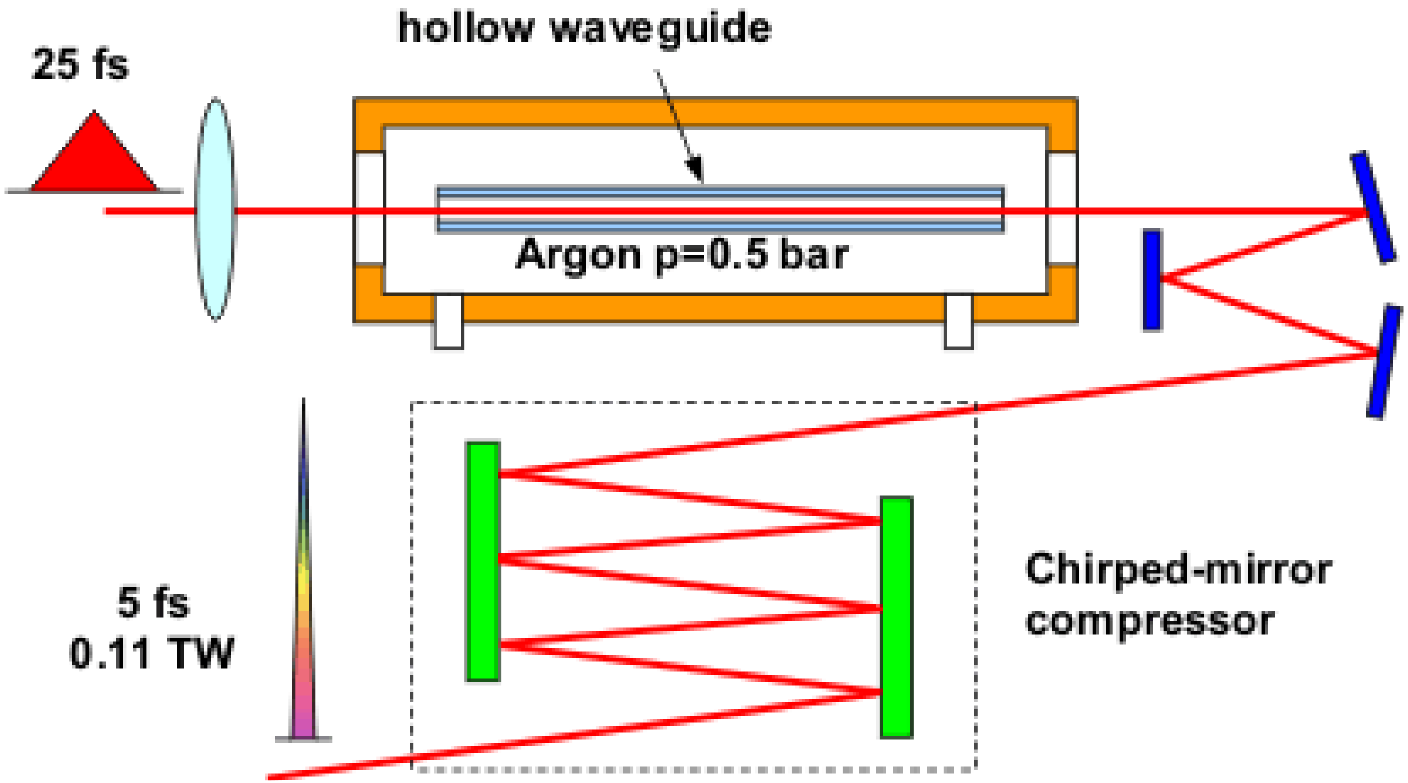
Grating pair and LCM pulse shaper

Dispersion Compensation with Acousto-Optic Programable Filter (DAZZLER)



Acousto-Optic Programable Dispersive Filter (AOPDF)

3.7.5 Hollow Fiber Compression Technique

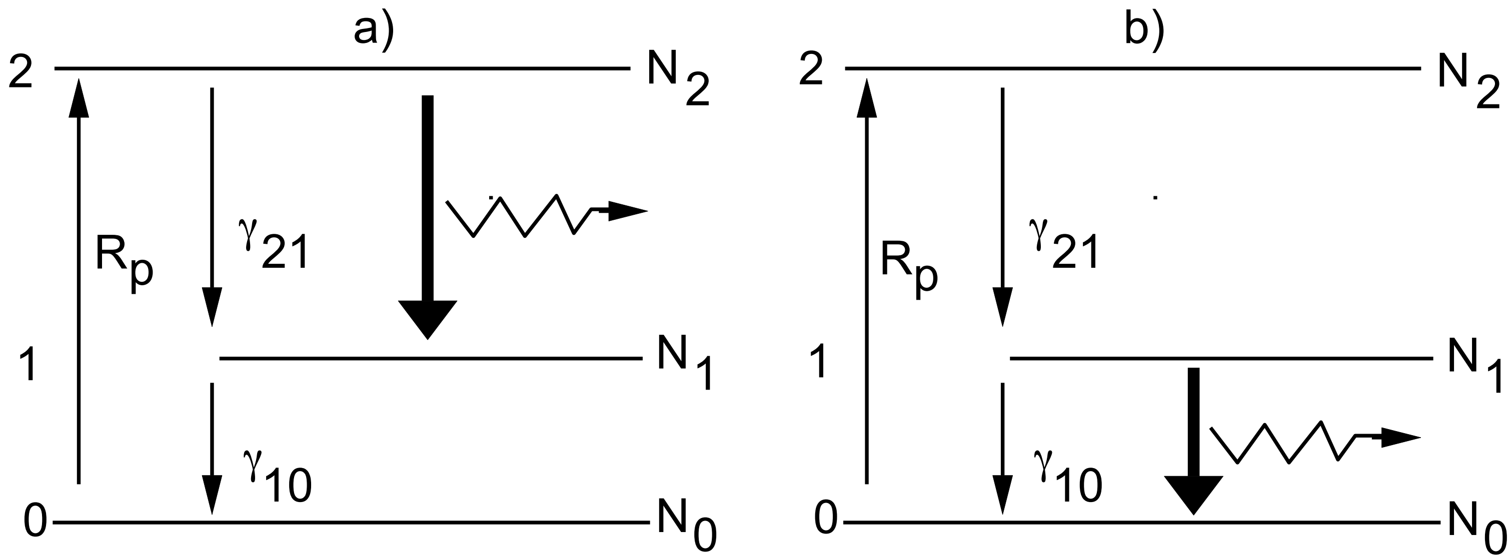


Hollow fiber compression technique

4 Laser Dynamics

4.1 Laser Rate Equations

How is inversion achieved? What is T_1, T_2 and σ of the laser transition? What does this mean for the laser dyanmics, i.e.for the light that can be generated with these media?



$\gamma_{10} \rightarrow \infty$

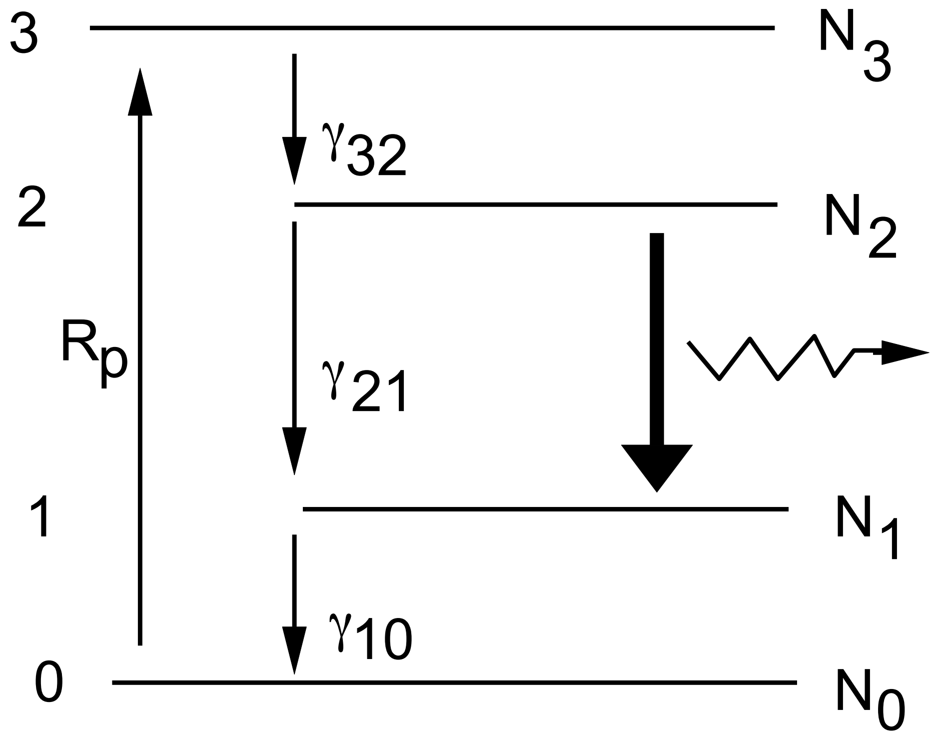
$w = N_2$

Three-level laser medium

$R_p \rightarrow \infty$

$N_0=0$

$w = N_1$



$\gamma_{10} \rightarrow \infty$

$\gamma_{32} \rightarrow \infty$

$w = N_2$

Four-level laser.

Rate Equations and Cross Sections

$$T_2 \rightarrow 0$$ e.g. semiconductors: $T_2 \sim 50$fs

$$\dot{w} = -\frac{w(t) - w_0}{T_1} - \frac{w(t)}{T_1 I_s} L(\omega) I(t),$$

Can be used for time dependent intensity varying much slower than T_2:

$$I(t) = |E_0(t)|^2 / (2Z_F)$$

Interaction cross section: $I_{ph} = I / \hbar\omega_{eg}$

$$\dot{w}\big|_{induced} = -\sigma w I_{ph} = -\frac{w}{T_1 I_s} I$$

$$\begin{aligned} \sigma &= \frac{\hbar\omega_{eg}}{T_1 I_s} \\ &= \frac{2\omega_{eg} T_2 Z_F}{\hbar} |\vec{M}_{eg}^{*} \cdot \vec{e}|^2. \end{aligned}$$

Lorentzian line shape:

$$L(\omega) = \frac{(1/T_2)^2}{(1/T_2)^2 + (\omega_{eg} - \omega)^2};$$

Intensity:

$$I = \frac{1}{2Z_F} |\underline{E}_0|^2$$

Steady state inversion:

$$w_s = \frac{w_0}{1 + \frac{I}{I_s} L(\omega)}$$

Saturation intensity:

Dipole matrix element of transition

$$I_s = \left[\frac{2T_1 T_2 Z_F}{\hbar^2} |\vec{M}^* \cdot \vec{e}_p|^2 \right]^{-1},$$

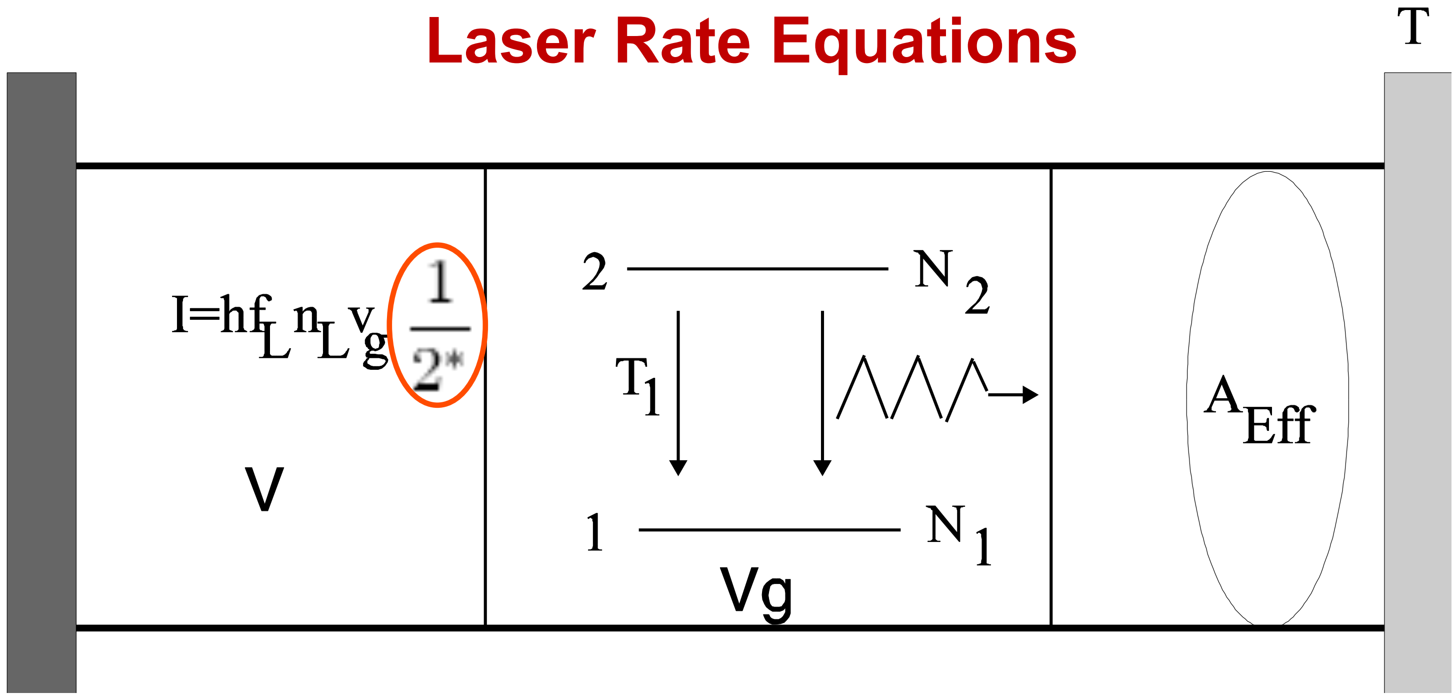
$$\dot{w} = -\frac{w(t) - w_0}{T_1} - \frac{w(t)}{T_1 I_s} L(\omega) I(t),$$

$$\frac{\partial g(z,t)}{\partial t} = -\frac{g - g_0}{\tau_L} - g\frac{\cancel{|}I(z,t)\cancel{|}^{\cancel{2}}}{E_L}.$$

Saturation Energy: $E_L = I_s \tau_L$

For laser media: $T_1 = \tau_L$

Laser Rate Equations



Rate equations for a laser with two-level atoms and a resonator.

***V*:= *A*$_{eff}$ *L* Mode volume**
***f*$_L$: laser frequency**
***I*: Intensity**
***V*$_g$: group velocity at laser frequency**
***N*$_L$: number of photons in mode**

$$I = hf_L \frac{N_L}{2^* V} v_g = \frac{1}{2^*} hf_L n_L v_g$$

σ: interaction cross section

$$\sigma = \frac{hf_L}{I_s \tau_L}$$

Laser Rate Equations:

Intracavity power: P
Round trip amplitude gain: g

$$P = I \cdot A_{eff} = hf_L \frac{N_L}{T_R}$$
$$g = \frac{\sigma v_g}{2V} N_2 T_R.$$

Output power: P_{out} $\quad P_{out} = T \cdot P.$

$$\frac{d}{dt} g = -\frac{g - g_0}{\tau_L} - \frac{gP}{E_{sat}}$$
$$\frac{d}{dt} P = -\frac{1}{\tau_p} P + \frac{2g}{T_R}(P + P_{vac})$$

$$E_{sat} = \frac{hf_L V}{\sigma v_g T_R} = \frac{1}{2^*} I_s A_{eff} \tau_L$$
$$P_{sat} = E_{sat}/\tau_L$$
$$P_{vac} = hf_L/T_R$$
$$g_0 = 2^* \frac{R_p}{2A_{eff}} \sigma \tau_L,$$

small signal gain ~ $\sigma\tau_L$ - product

4.2 Continuous Wave Operation

$P_{vac} = 0$

Steady State: d/dt = 0

Case 1:

$g_s = g_0$

$P_s = 0$

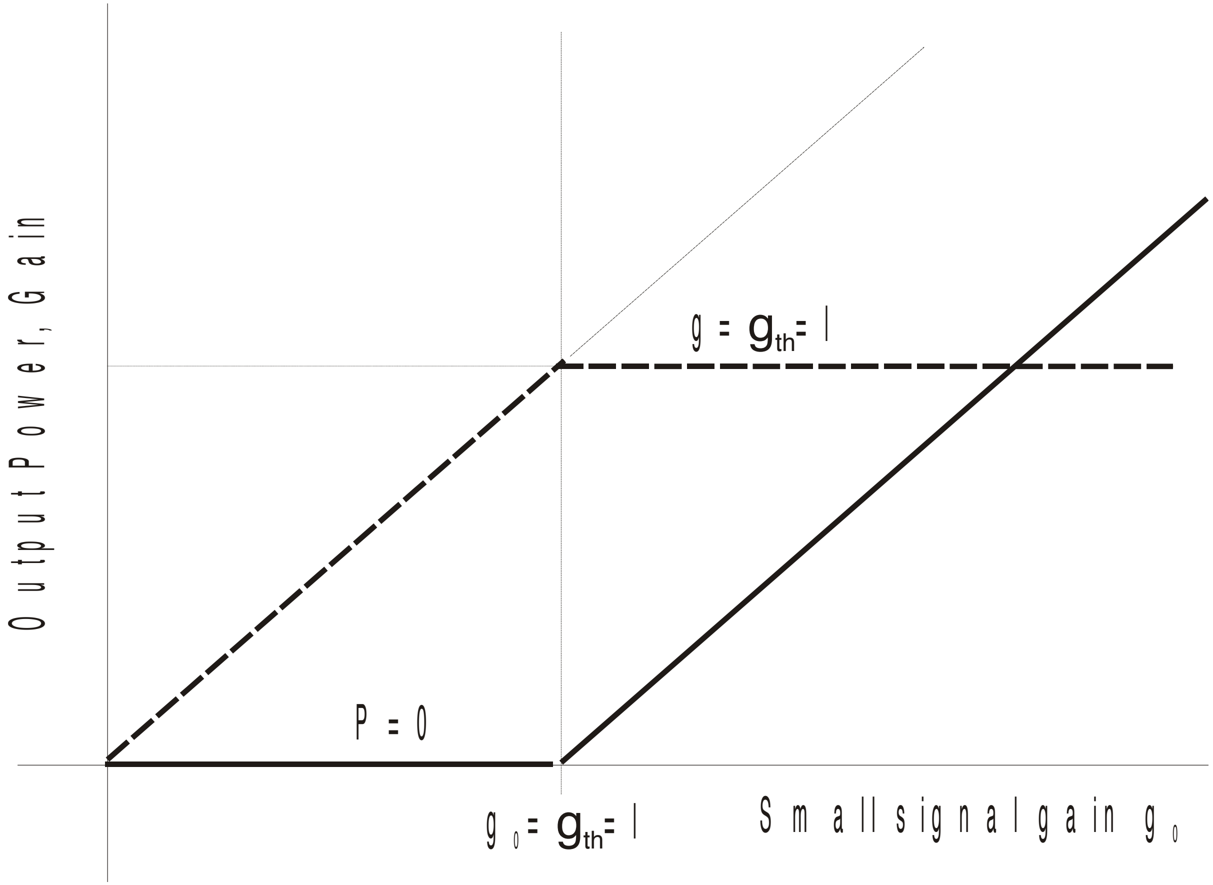
Case 2:

$$g_s = \frac{g_0}{1+\frac{P_s}{P_{sat}}} = l$$

$$P_s = P_{sat}\left(\frac{g_0}{l} - 1\right)$$

$$g_{th} = l,$$

$$R_{p,th} = \frac{2lA_{eff}}{2^* \sigma \tau_L}$$



Output power versus small signal gain or pump power

4.3 Stability and Relaxation Oscillations

Perturbations:

$$\begin{aligned} g &= g_s + \Delta g \\ P &= P_s + \Delta P \end{aligned}$$

$$\frac{1}{\tau_{stim}} = \frac{1}{\tau_L}\left(1 + \frac{P_s}{P_{sat}}\right)$$

$$\begin{pmatrix} \Delta P \\ \Delta g \end{pmatrix} = \begin{pmatrix} \Delta P_0 \\ \Delta g_0 \end{pmatrix} e^{st}$$

$$A\begin{pmatrix} \Delta P_0 \\ \Delta g_0 \end{pmatrix} = \begin{pmatrix} -s & 2\frac{P_s}{T_R} \\ -\frac{T_R}{E_{sat}2\tau_p} & -\frac{1}{\tau_{stim}} - s \end{pmatrix}\begin{pmatrix} \Delta P_0 \\ \Delta g_0 \end{pmatrix} = 0.$$

$$s_{1/2} = -\frac{1}{2\tau_{stim}}\left(1 \pm j\sqrt{\frac{4(r-1)}{r}\frac{\tau_{stim}}{\tau_p} - 1}\right)$$

$$\boxed{= -\frac{r}{2\tau_L} \pm j\sqrt{\frac{(r-1)}{\tau_L\tau_p} - \left(\frac{r}{2\tau_L}\right)^2}}$$

$$\boxed{\text{parameter } r = 1 + \frac{P_s}{P_{sat}}}$$

- (i): The stationary state $(0, g_0)$ for $g_0 < l$ and (P_s, g_s) for $g_0 > l$ are always stable, i.e. $\mathrm{Re}\{s_i\} < 0$.

- (ii): For lasers pumped above threshold, $r > 1$, and long upper state lifetimes, i.e. $\frac{r}{4\tau_L} < \frac{1}{\tau_p}$,

  the relaxation rate becomes complex, i.e. there are relaxation oscillations

$$s_{1/2} = -\frac{1}{2\tau_{stim}} \pm j\omega_R. \tag{6.47}$$

  with a frequency ω_R approximately equal to the geometric mean of inverse stimulated lifetime and photon life time

$$\omega_R \approx \sqrt{\frac{1}{\tau_{stim}\tau_p}}. \tag{6.48}$$

- If the laser can be pumped strong enough, i.e. r can be made large enough so that the stimulated lifetime becomes as short as the cavity decay time, relaxation oscillations vanish.

4.4 Lasers and Its Spectroscopic Parameters

Laser Medium	Wave-length $\lambda_0 (nm)$	Cross Section σ (cm^2)	Upper-St. Lifetime τ_L (μs)	Linewidth $\Delta f_{FWHM} = \frac{2}{T_2}$(THz)	Typ	Refr. index n
Nd^{3+}:YAG	1,064	$4.1 \cdot 10^{-19}$	1,200	0.210	H	1.82
Nd^{3+}:LSB	1,062	$1.3 \cdot 10^{-19}$	87	1.2	H	1.47 (ne)
Nd^{3+}:YLF	1,047	$1.8 \cdot 10^{-19}$	450	0.390	H	1.82 (ne)
Nd^{3+}:YVO_4	1,064	$2.5 \cdot 10^{-19}$	50	0.300	H	2.19 (ne)
Nd^{3+}:glass	1,054	$4 \cdot 10^{-20}$	350	3	H/I	1.5
Er^{3+}:glass	1,55	$6 \cdot 10^{-21}$	10,000	4	H/I	1.46
Ruby	694.3	$2 \cdot 10^{-20}$	1,000	0.06	H	1.76
Ti^{3+}:Al_2O_3	660-1180	$3 \cdot 10^{-19}$	3	100	H	1.76
Cr^{3+}:LiSAF	760-960	$4.8 \cdot 10^{-20}$	67	80	H	1.4
Cr^{3+}:LiCAF	710-840	$1.3 \cdot 10^{-20}$	170	65	H	1.4
Cr^{3+}:LiSGAF	740-930	$3.3 \cdot 10^{-20}$	88	80	H	1.4
He-Ne	632.8	$1 \cdot 10^{-13}$	0.7	0.0015	I	$\sim$1
Ar^{+}	515	$3 \cdot 10^{-12}$	0.07	0.0035	I	$\sim$1
CO_2	10,600	$3 \cdot 10^{-18}$	2,900,000	0.000060	H	$\sim$1
Rhodamin-6G	560-640	$3 \cdot 10^{-16}$	0.0033	5	H	1.33
semiconductors	450-30,000	$\sim 10^{-14}$	$\sim$ 0.002	25	H/I	3 - 4

Spectroscopic parameters of selected laser materials

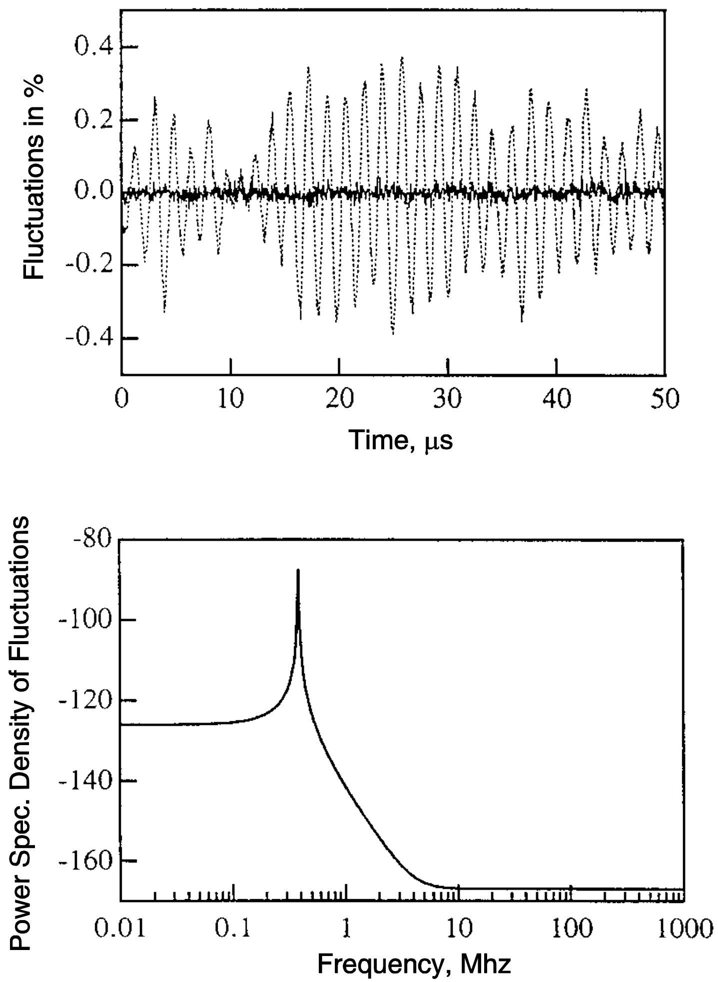
Example: diode-pumped Nd:YAG-Laser

$$\begin{aligned}\lambda_0 &= 1064 \text{ nm}, \sigma = 4 \cdot 10^{-19} cm^2,\; A_{eff} = \pi\left(100\mu m \times 150\mu m\right), r = 50 \\ \tau_L &= 1.2\; ms,\; l = 1\%,\; T_R = 10ns\end{aligned}$$

From Eq.(6.16) we obtain:

$$\begin{aligned}I_{sat} &= \frac{hf_L}{\sigma\tau_L} = 0.4\frac{kW}{cm^2},\; P_{sat} = I_{sat}A_{eff} = 0.18\; W,\;\; P_s = 9.2W \\ \tau_{stim} &= \frac{\tau_L}{r} = 24\mu s,\;\; \tau_p = 1\mu s, \omega_R = \sqrt{\frac{1}{\tau_{stim}\tau_p}} = 2 \cdot 10^5 s^{-1}.\end{aligned}$$

Quality factor of relaxation oscillations: $Q = \sqrt{\frac{4\tau_L}{\tau_p}\frac{(r-1)}{r^2}}$

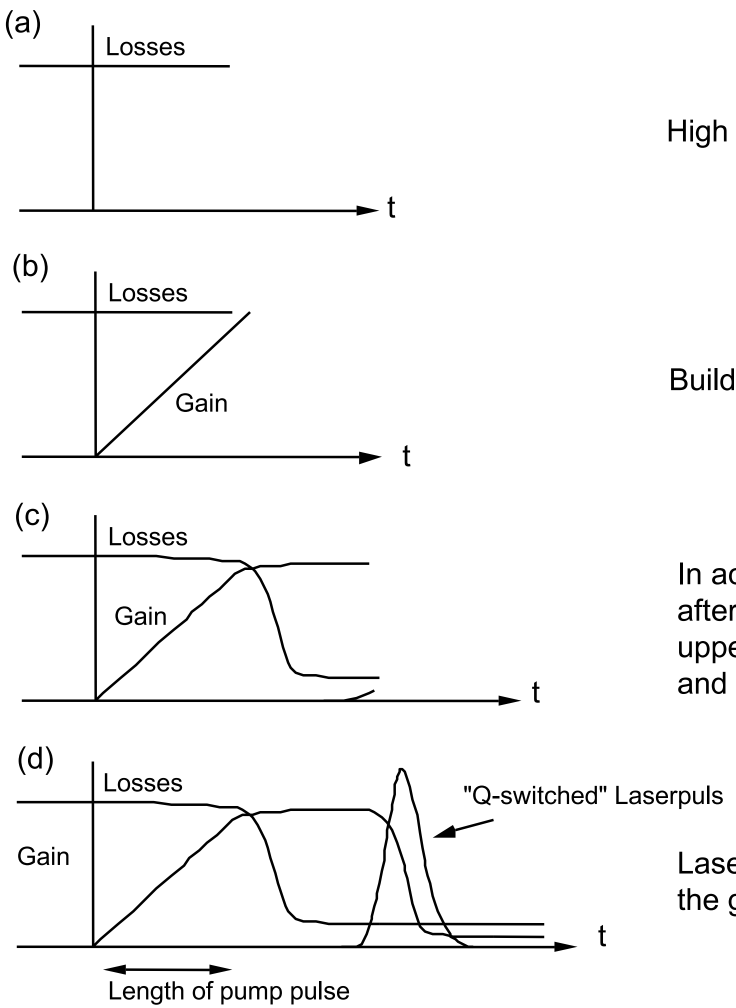


Relaxation Oscillations

4.5 Short pulse generation by Q-Switching

4.5.1 Active Q-Switching

$$\tau_L \gg T_R \gg \tau_p$$



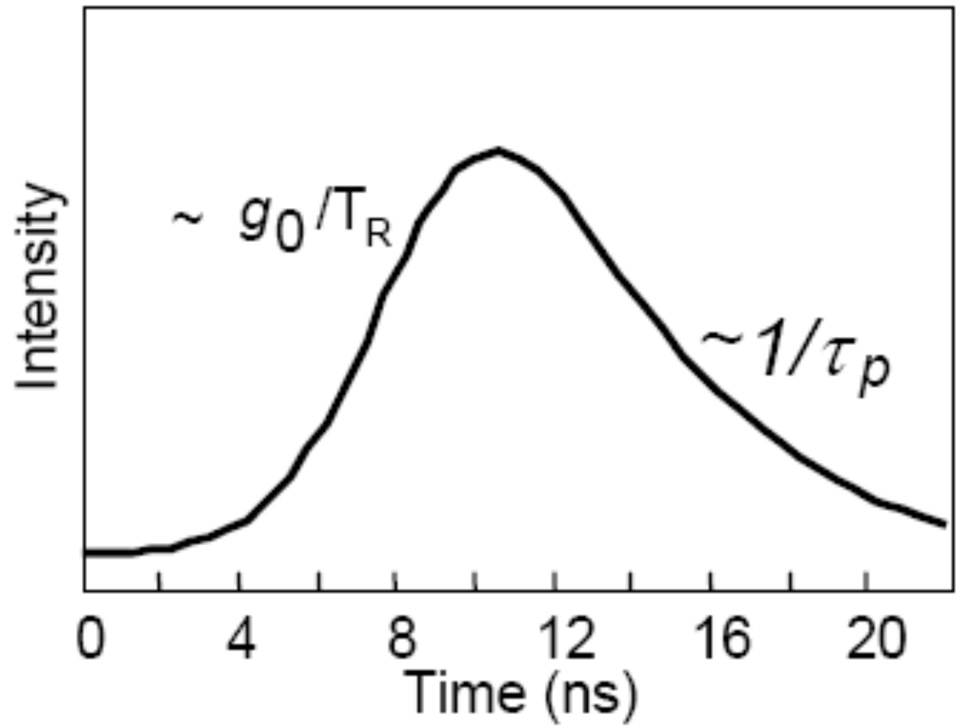
High losses, laser is below threshold

Build-up of inversion by pumping

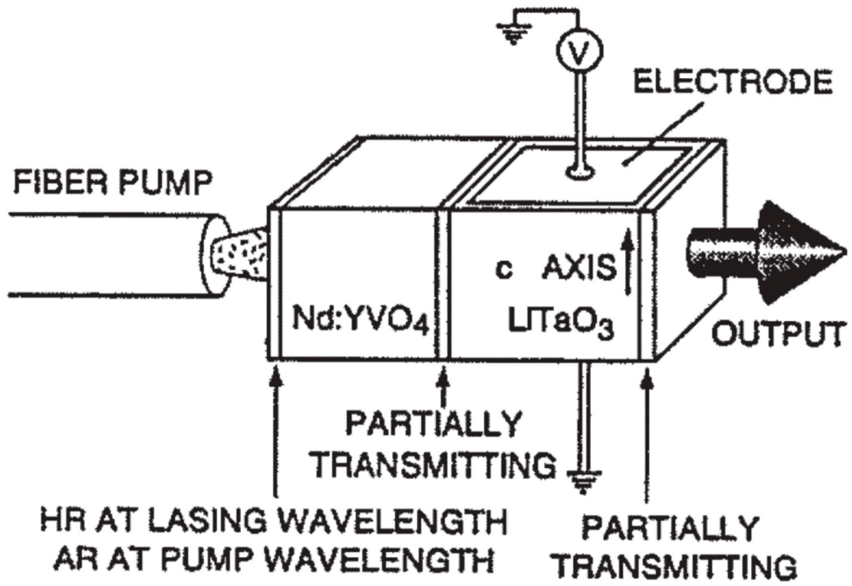
In active Q-switching, the losses are reduced, after the laser medium is pumped for as long as the upper state lifetime. Then the loss is reduced rapidly and laser oscillation starts.

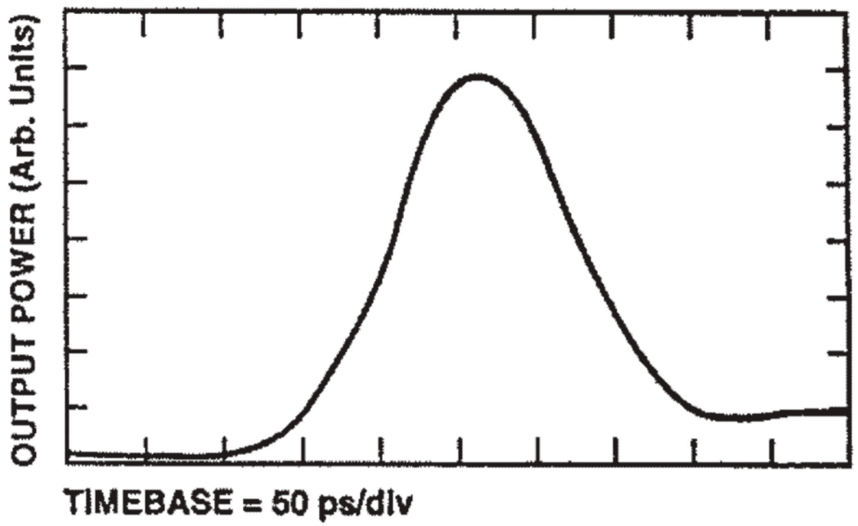
Laser emission stops after the energy stored in the gain medium is extracted.

Active Q-switching



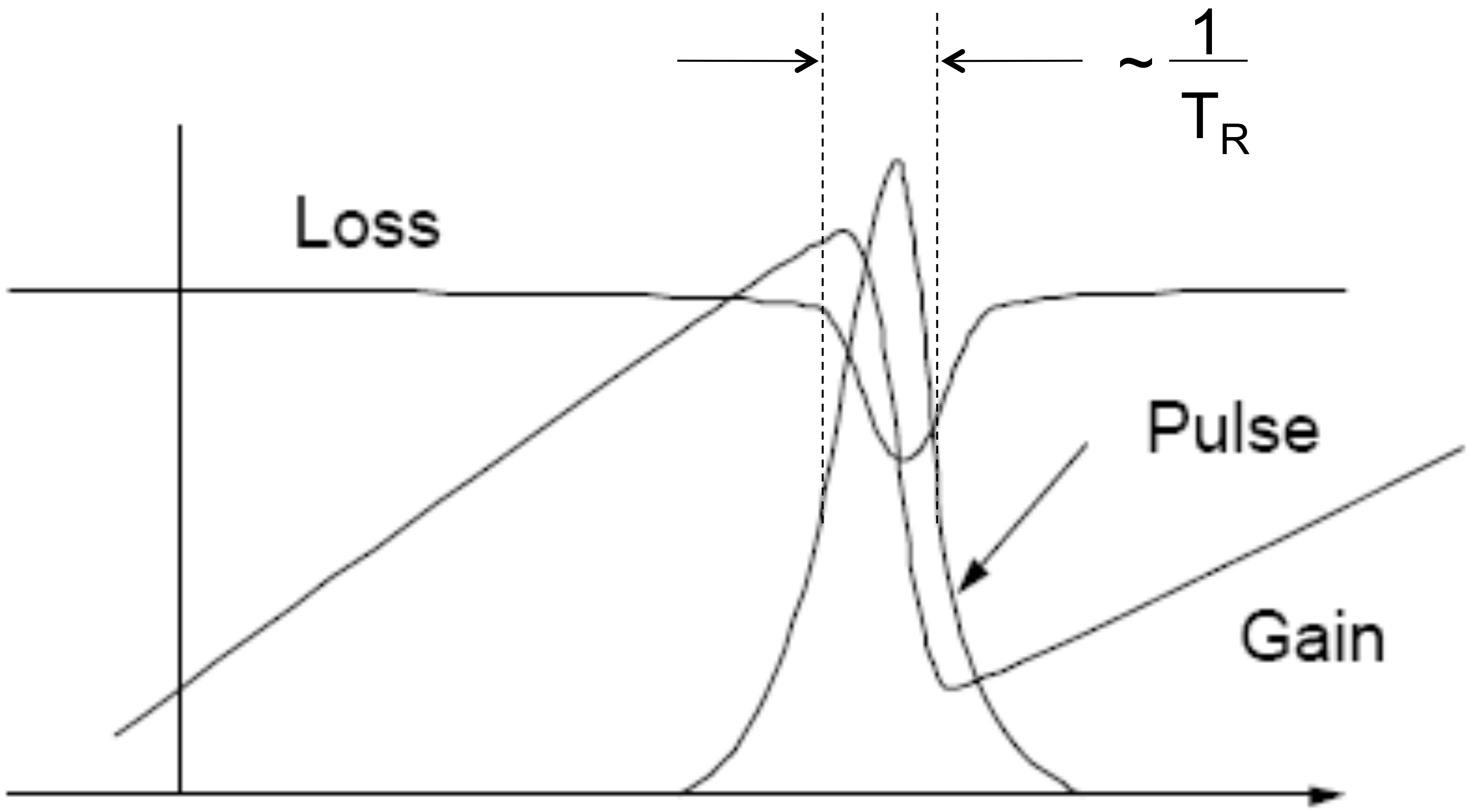
Asymmetric actively Q-switched pulse.





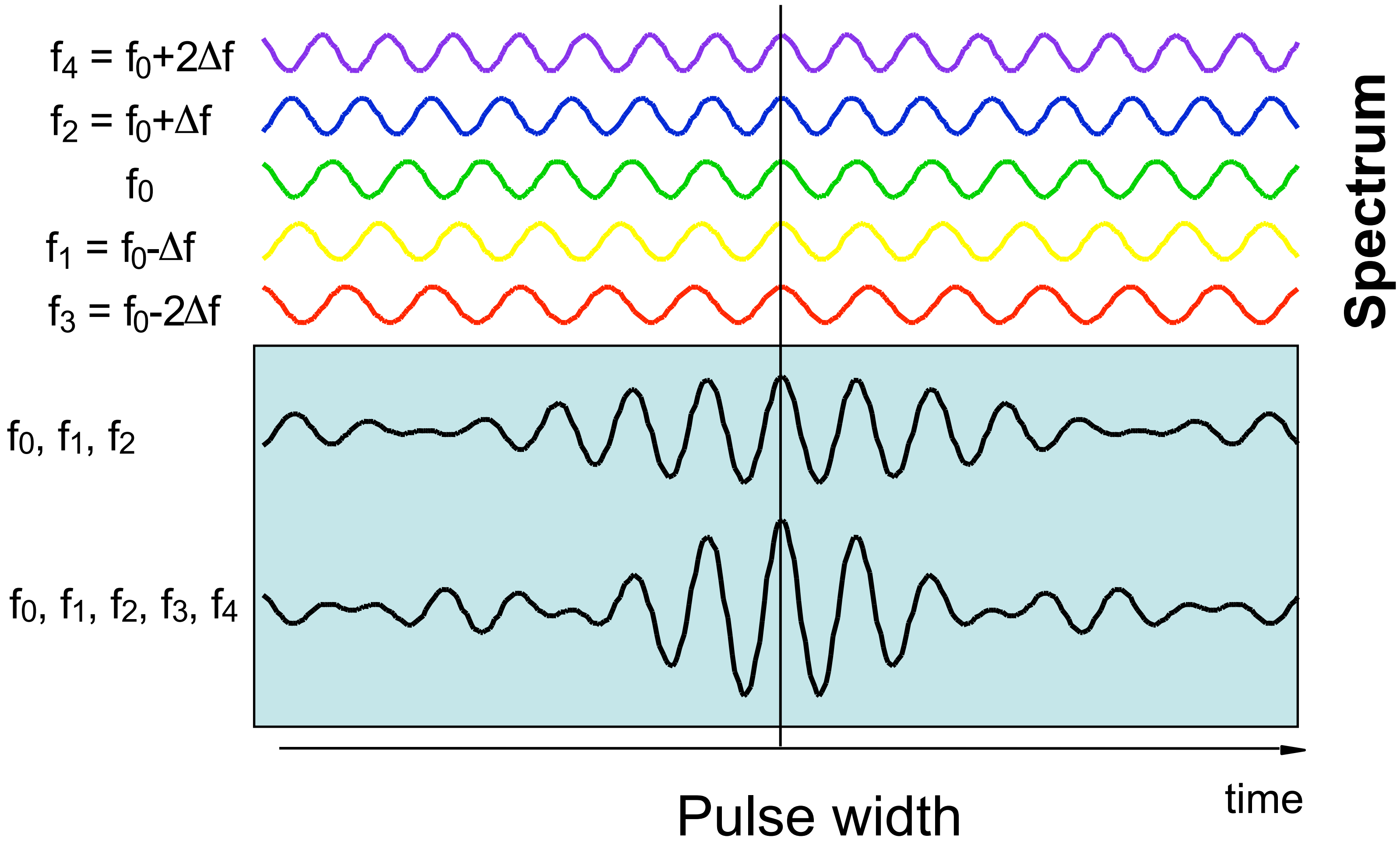
Q-switched microchip laser

4.5.2 Passive Q-Switching



Passively Q-switched Laser

5. Mode Locking



Superposition of longitudinal modes:

$$
\begin{aligned}
E(z,t) &= \Re\left[\sum_m \hat{E}_m e^{\mathrm{j}(\omega_m t - k_m z + \phi_m)}\right] \\
\omega_m &= \omega_0 + m\Delta\omega = \omega_0 + \frac{m\pi c}{\ell}, \\
k_m &= \frac{\omega_m}{c}.
\end{aligned}
$$

$$
\begin{aligned}
E(z,t) &= \Re\left\{e^{\mathrm{j}\omega_0(t-z/c)} \sum_m \hat{E}_m e^{\mathrm{j}(m\Delta\omega(t-z/c)+\phi_m)}\right\} \\
&= \Re\left[A(t-z/c)e^{\mathrm{j}\omega_0(t-z/c)}\right]
\end{aligned}
$$

Slowly varying complex pulse amplitude

$$
A\left(t - \frac{z}{c}\right) = \sum_m E_m e^{\mathrm{j}(m\Delta\omega(t-z/c)+\phi_m)} \qquad T = \frac{2\pi}{\Delta\omega} = \frac{2\ell}{c} = \frac{L}{c}
$$

N modes with equal amplitude E_0 and phase $\phi_m = 0$

$$A(z,t) = E_0 \sum_{m=-(N-1)/2}^{(N-1)/2} e^{j(m\Delta\omega(t-z/c))} \qquad \sum_{m=0}^{q-1} a^m = \frac{1-a^q}{1-a}$$

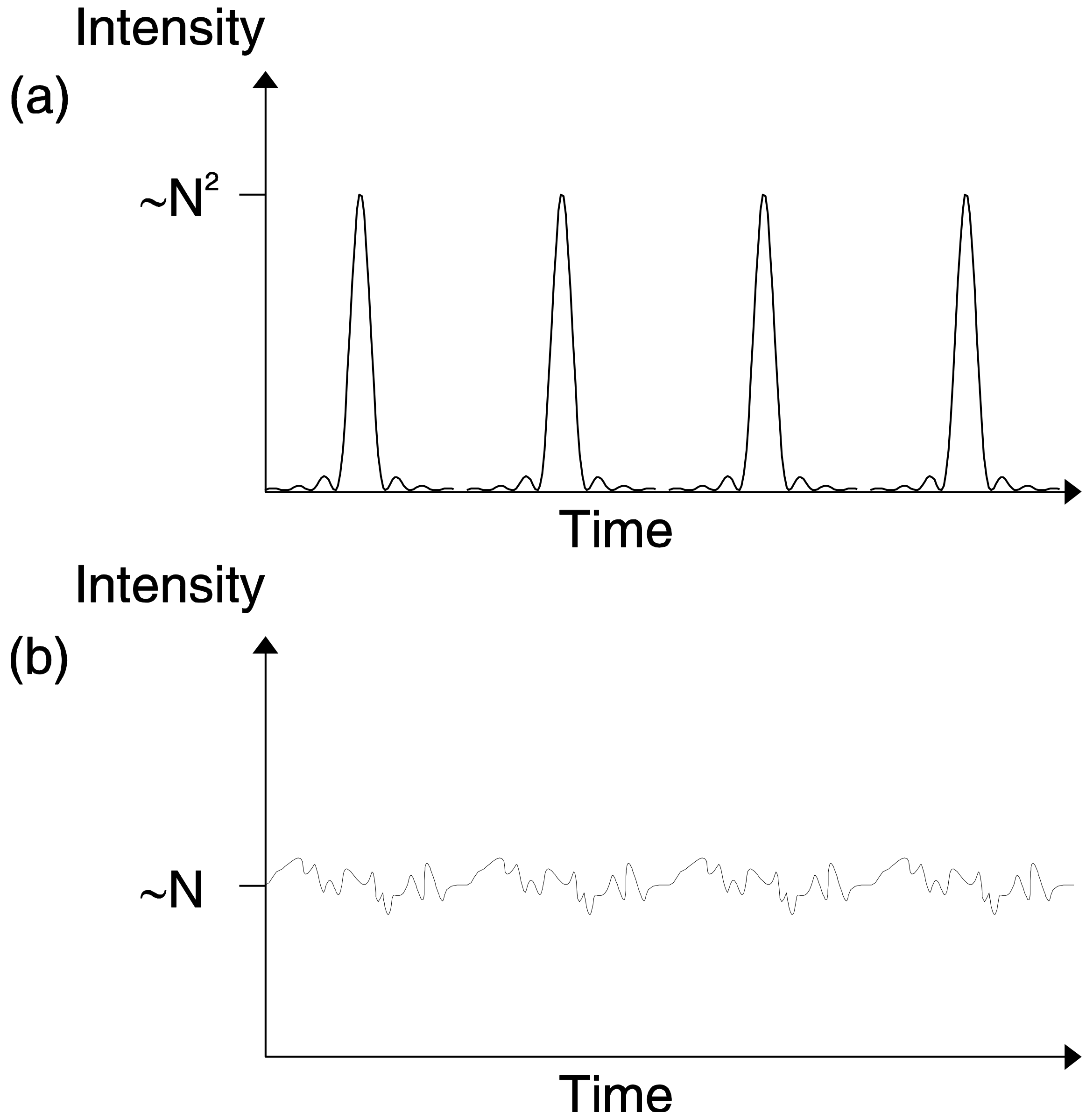
$$A(z,t) = E_0 \frac{\sin\left[\frac{N\Delta\omega}{2}\left(t-\frac{z}{c}\right)\right]}{\sin\left[\frac{\Delta\omega}{2}\left(t-\frac{z}{c}\right)\right]}.$$

$$I \sim |A(z,t)|^2 \qquad I(t) \sim |E_0|^2 \frac{\sin^2\left(\frac{N\Delta\omega t}{2}\right)}{\sin^2\left(\frac{\Delta\omega t}{2}\right)}$$

Scaling of pulse train with number of modes N:

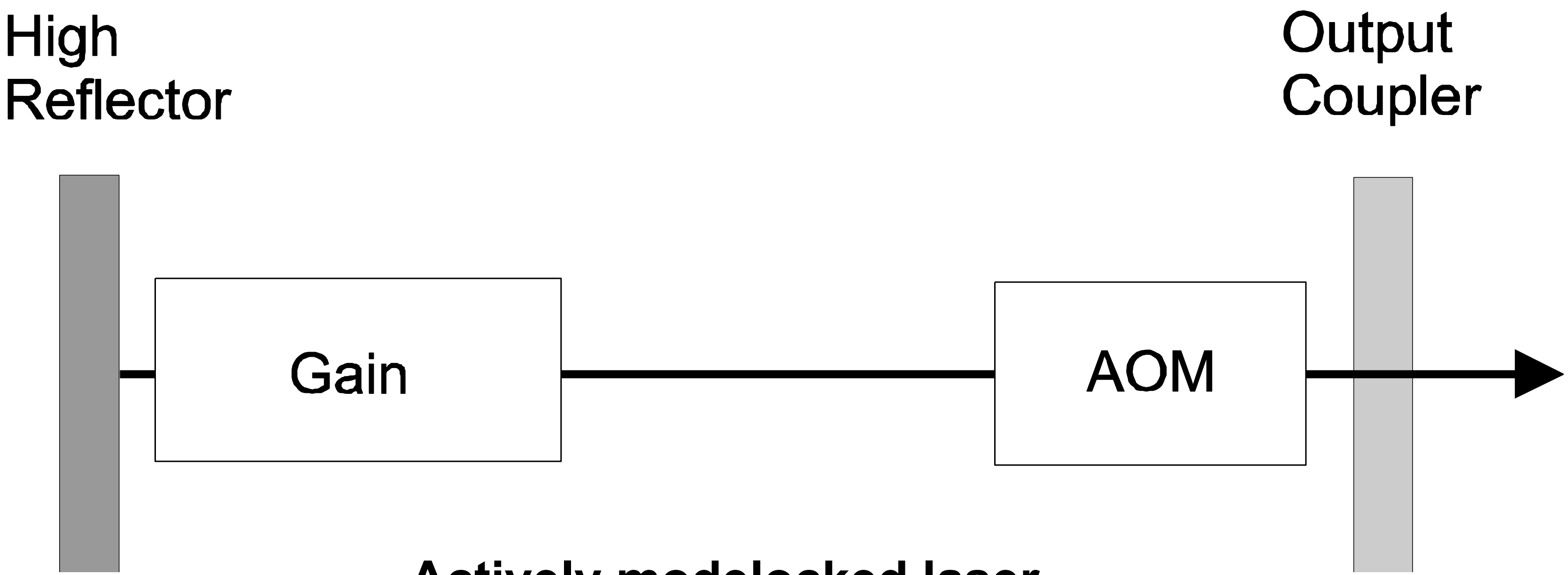
- the period: $T = 1/\Delta f = L/c$
- pulse duration: $\Delta t = \frac{2\pi}{N\Delta\omega} = \frac{1}{N\Delta f}$
- peak intensity $\sim N^2|E_0|^2$

- average intensity $\sim N|E_0|^2 \Rightarrow$ peak intensity is enhanced by a factor N.



Laser intensity for mode-locked operation and multimode lasing with random phase.

5.1 Active Mode Locking



Actively modelocked laser.

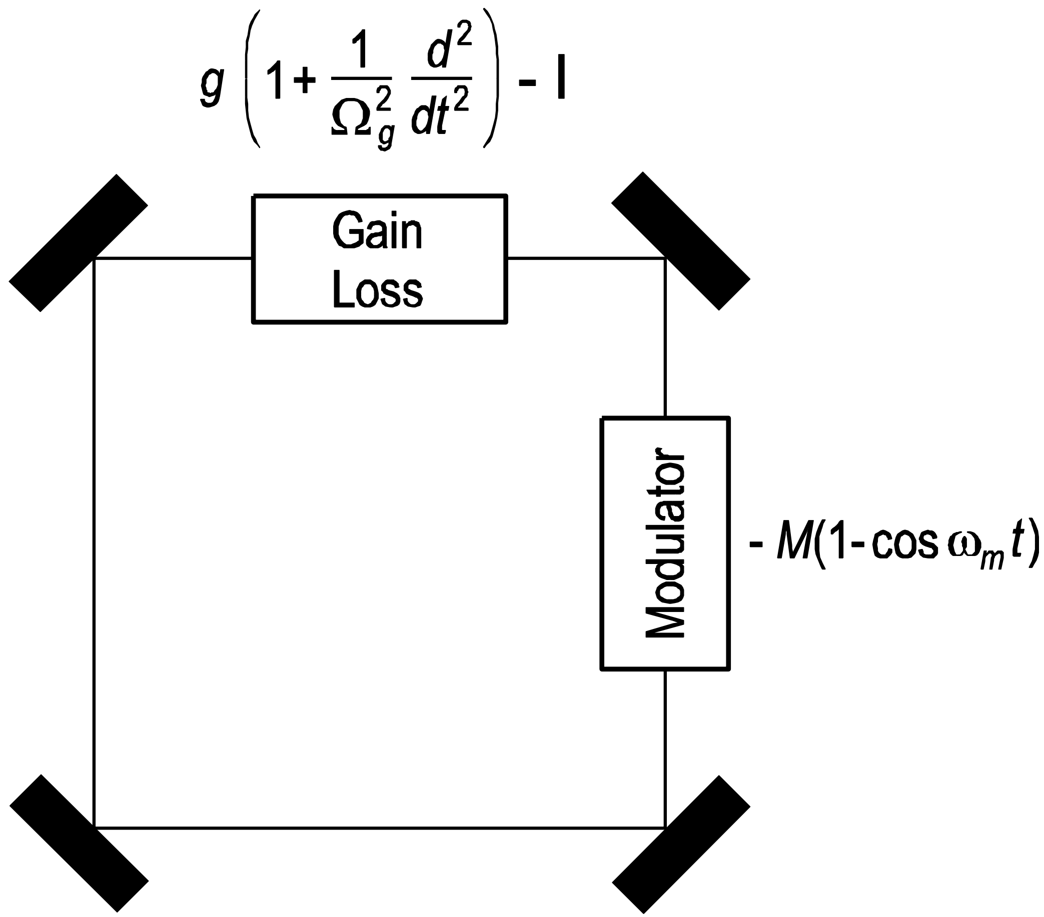
Master Equation:

$$T_R \frac{\partial A}{\partial T} = \left[g(T) + D_g \frac{\partial^2}{\partial t^2} - l - M\left(1 - \cos(\omega_M t)\right) \right] A$$

loss modulation

Parabolic approximation at position where pulse will form;

$$T_R \frac{\partial A}{\partial T} = \left[g - l + D_g \frac{\partial^2}{\partial t^2} - M_s t^2 \right] A$$



Schematic representation of actively modelocked laser.

$$D_g = \frac{g}{\Omega_g^2},$$

$$M_s = \frac{M\omega_M^2}{2}$$

Compare with Schroedinger Equation for harmonic oscillator

$$
\begin{aligned}
A_n(T,t) &= A_n(t)e^{\lambda_n T/T_R}, \\
A_n(t) &= \sqrt{\frac{W_n}{2^n\sqrt{\pi}n!\tau_a}} H_n(t/\tau_a)e^{-\frac{t^2}{2\tau_a^2}}
\end{aligned}
$$

with

$$
\tau_a = \sqrt[4]{D_g/M_s}.
$$

Eigen value determines roundtrip gain of n=th pulse shape

$$
\lambda_n = g_n - l - 2M_s\tau_a^2(n + \frac{1}{2}).
$$

Pulse shape with n=0, lowest order mode, has highest gain.

This pulse shape will saturate the gain and keep all other pulse shapes below threshold.

Pulse width: $\Delta t_{FWHM} = 2\ln 2\tau_a = 1.66\tau_a$

Gaussian pulse with spectrum:

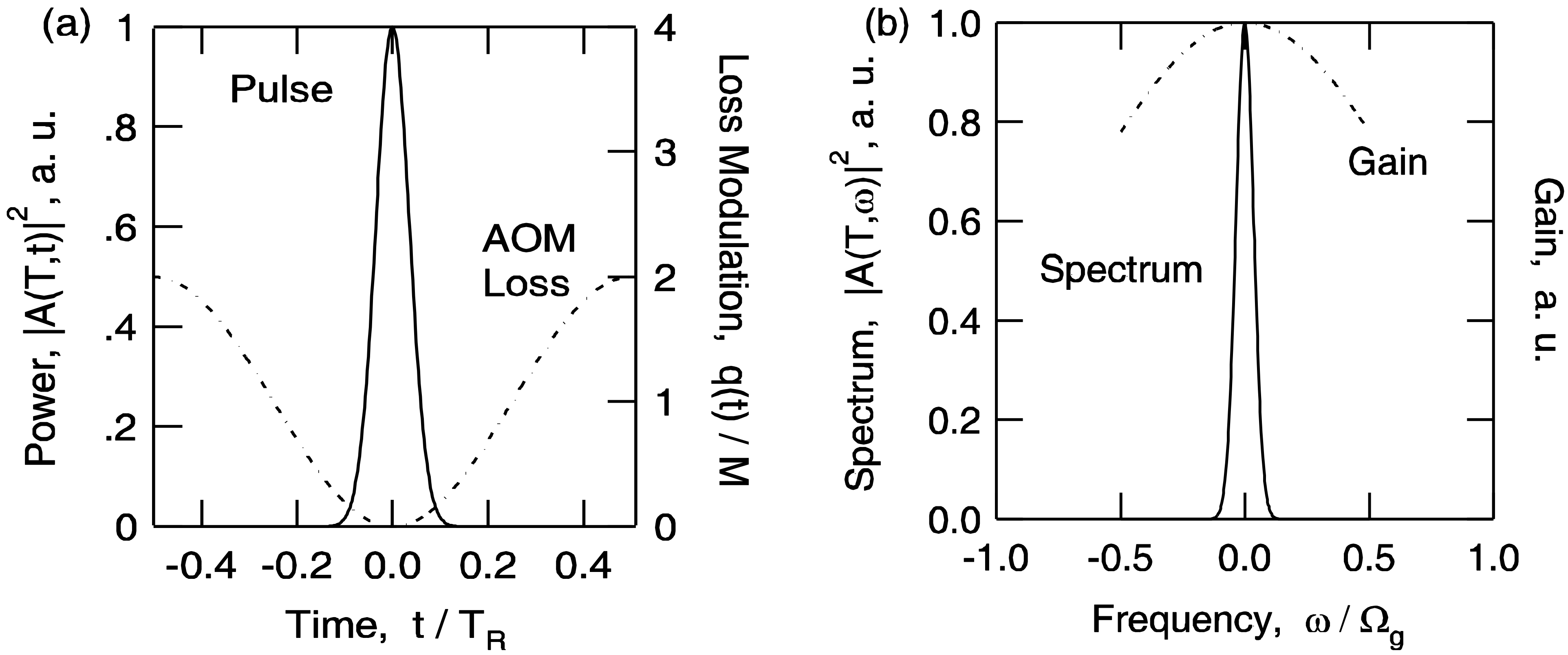
$$\tilde{A}_0(\omega) = \int_{-\infty}^{\infty} A_0(t)e^{i\omega t}dt = \sqrt{\sqrt{\pi}W_n\tau_a}e^{-\frac{(\omega\tau_a)^2}{2}},$$

FWHM spectral width:

$$\Delta f_{FWHM} = \frac{1.66}{2\pi\tau_a}$$

Time bandwidth product:

$$\Delta t_{FWHM} \cdot \Delta f_{FWHM} = 0.44$$



Pulse shaping in time and frequency domain.

For example: Nd:YAG; $2l = 2g = 10\%$, $\Omega_g = \pi \Delta f_{FWHM} = 0.65$ THz $M = 0.2$, $f_m = 100$ MHz, $D_g = 0.24$ ps^2, $M_s = 4 \cdot 10^{16} s^{-1}$, $\tau_p \approx 99$ ps.

Pulse width depends only weak on gain bandwidth.

10-100 ps pulses typical for active mode locking:

$$g_s = l + M_s \tau_a^2 = l + \frac{D_g}{\tau_a^2} = l + \frac{1}{2} M_s \tau_a^2 + \frac{1}{2} \frac{D_g}{\tau_a^2}.$$

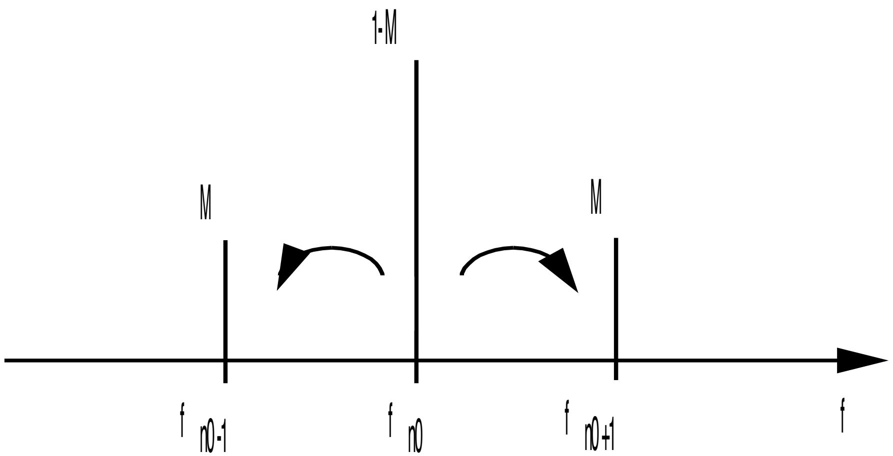
saturated gain = linear losses + losses in mdoulator + losses due to gain filtering

Additional gain for multimode operation: $$\frac{g_s - l}{l} = \frac{M_s \tau_a^2}{l} \ll 1$$

Saturated gain is approximately: $$g_s = \frac{1}{1 + \frac{W_s}{P_L T_R}} = l$$

Active mode locking can be understood as injection seeding of neighboring modes by those already present.

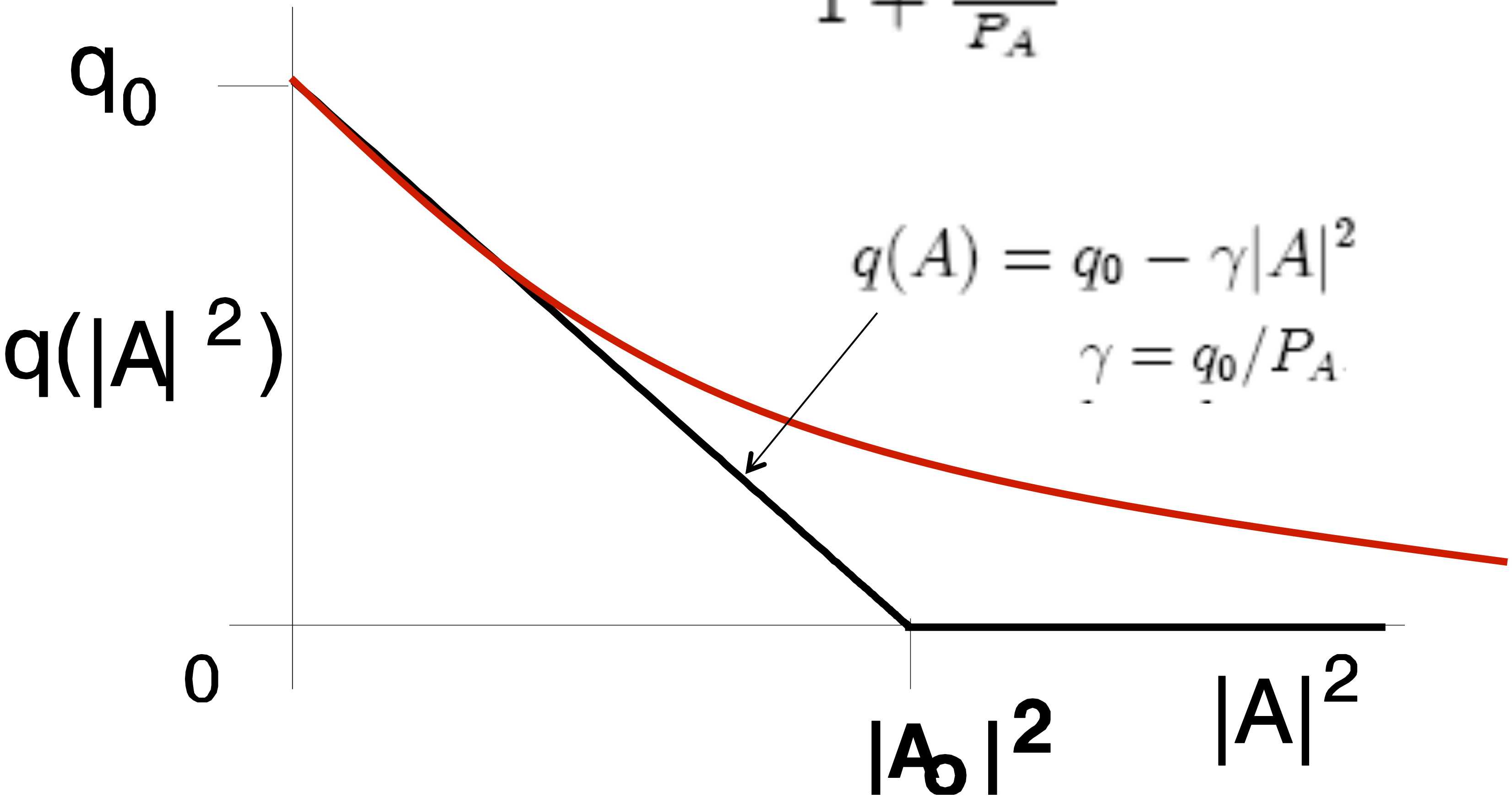
$$
\begin{aligned}
& -M\left[1-\cos(\omega_M t)\right]\exp(j\omega_{n_0}t) \\
= \; & -M\left[\exp(j\omega_{n_0}t)-\frac{1}{2}\exp(j(\omega_{n_0}t-\omega_M t))-\frac{1}{2}\exp(j(\omega_{n_0}t+\omega_M t))\right] \\
= \; & M\left[-\exp(j\omega_{n_0}t)+\frac{1}{2}\exp(j\omega_{n_0-1}t)+\frac{1}{2}\exp(j\omega_{n_0+1}t)\right]
\end{aligned}
$$



Mode Locking

5.2 Passive Mode Locking

$$q(A) = \frac{q_0}{1 + \frac{|A|^2}{P_A}}$$



Saturation characteristic of an ideal saturable absorber and linear approximation.

$$T_R \frac{\partial A(T,t)}{\partial T} = \left[g - l_0 + D_g \frac{\partial^2}{\partial t^2} + \gamma |A|^2 \right] A(T,t)$$

$$l_0 = l + q_0$$

Saturable absorber provides gain for the pulse

There is a stationary solution:

$$A_s(T,t) = A_s(t) = A_0 \mathrm{sech}\left(\frac{t}{\tau}\right)$$

Easy to check with:

$$\begin{aligned} \frac{d}{dx}\mathrm{sech}x &= -\tanh x \ \mathrm{sech}x, \\ \frac{d^2}{dx^2}\mathrm{sech}x &= \tanh^2 x \ \mathrm{sech}x - \mathrm{sech}^3 x \\ &= \left(\mathrm{sech}x - 2\ \mathrm{sech}^3 x\right). \end{aligned}$$

Leads to:

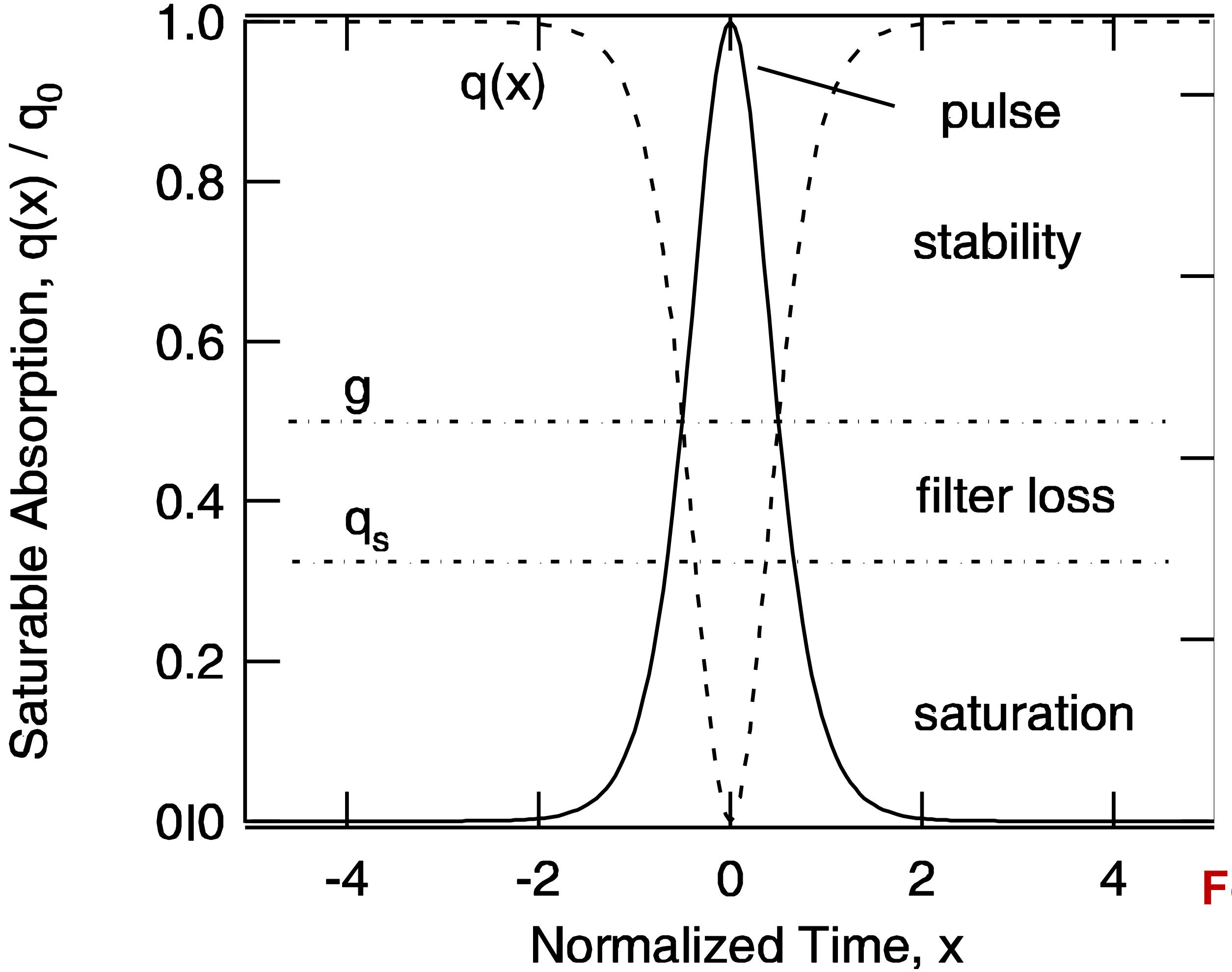
$$
\begin{aligned}
0 \;=\; & \left[(g-l_0)+\frac{D_g}{\tau^2}\left[1-2\operatorname{sech}^2\left(\frac{t}{\tau}\right)\right]\right. \\
& \left.+\gamma|A_0|^2\operatorname{sech}^2\left(\frac{t}{\tau}\right)\right]\cdot A_0\operatorname{sech}\left(\frac{t}{\tau}\right)
\end{aligned}
\quad \Rightarrow \quad
\begin{aligned}
\frac{D_g}{\tau^2} &= \frac{1}{2}\gamma|A_0|^2; \\
g &= l_0-\frac{D_f}{\tau^2}.
\end{aligned}
$$

Pulse energy, pulse width and saturated gain:

$$W=\int_{-\infty}^{+\infty}\cancel{2}|A_s(t)|^2dt=2|A_0|^2\tau$$

$$\tau=\frac{4D_g}{\gamma W}. \qquad\qquad g_s(W)=\frac{g_0}{1+\frac{W}{P_LT_R}}$$

$$
\begin{aligned}
g_s(W) &= \frac{g_0}{1+\frac{W}{P_LT_R}}=l_0-\frac{D_g}{\tau^2} \\
&= l_0-\frac{(\gamma W)^2}{16D_g}
\end{aligned}
$$



For shortest pulse half of absorption depth can be used to overcome gain filtering losses, only marginally stable!

$$\frac{D_g}{\tau^2} = \frac{q_0}{2},$$

Shortest pulse:

$$\tau = \sqrt{\frac{2g_s}{q_0}\frac{1}{\Omega g}}.$$

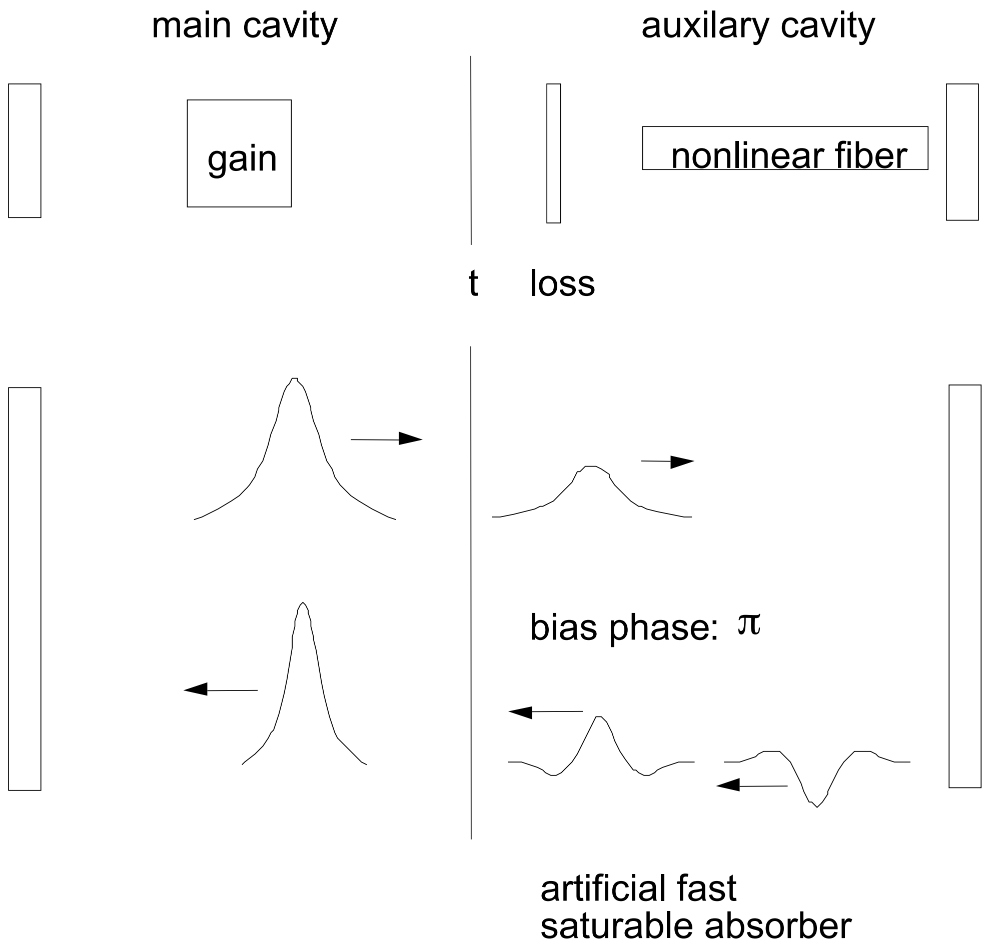
$$\tau_{\min} = \frac{1}{\Omega_g}$$

For Ti:sapphire

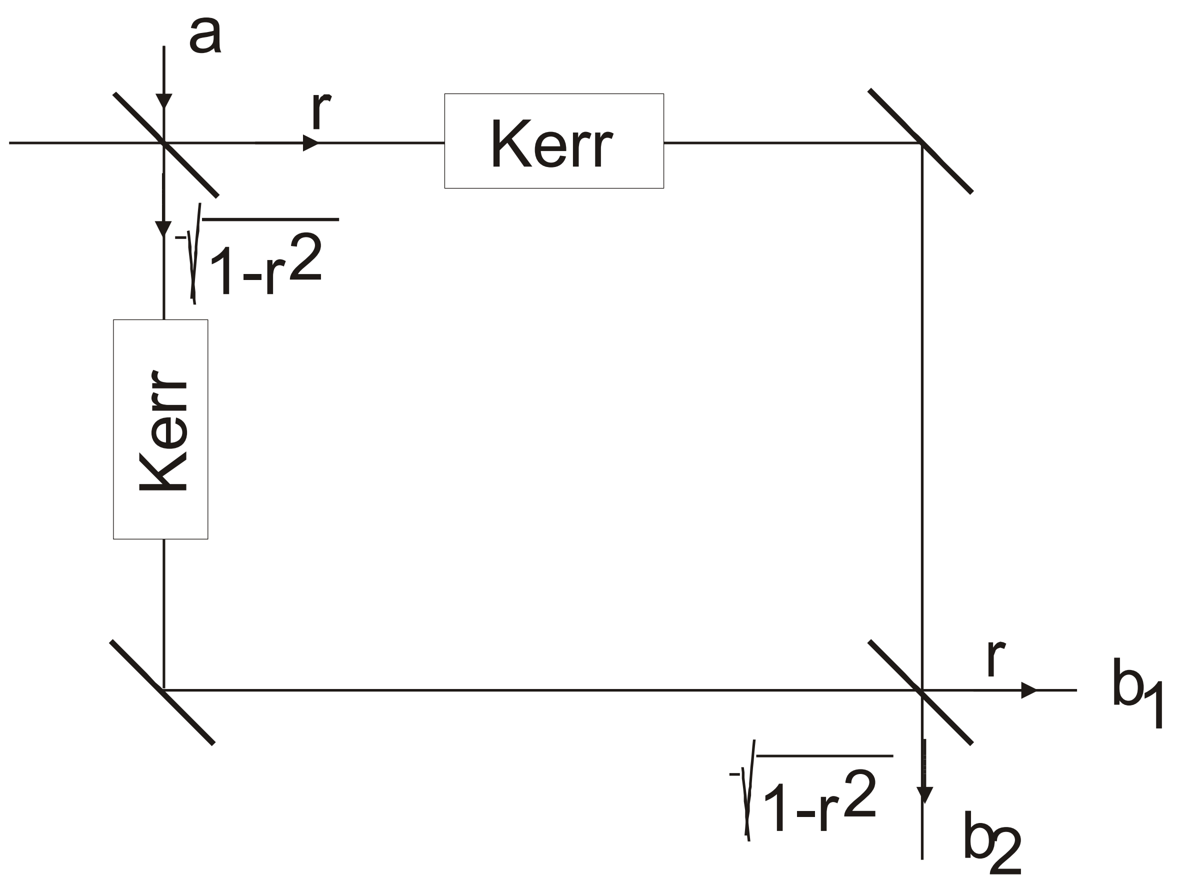
$$\tau_{FWHM} = 6.5 \text{ fs}$$

Schematic representation of gain and loss dynamics in passive mode locking.

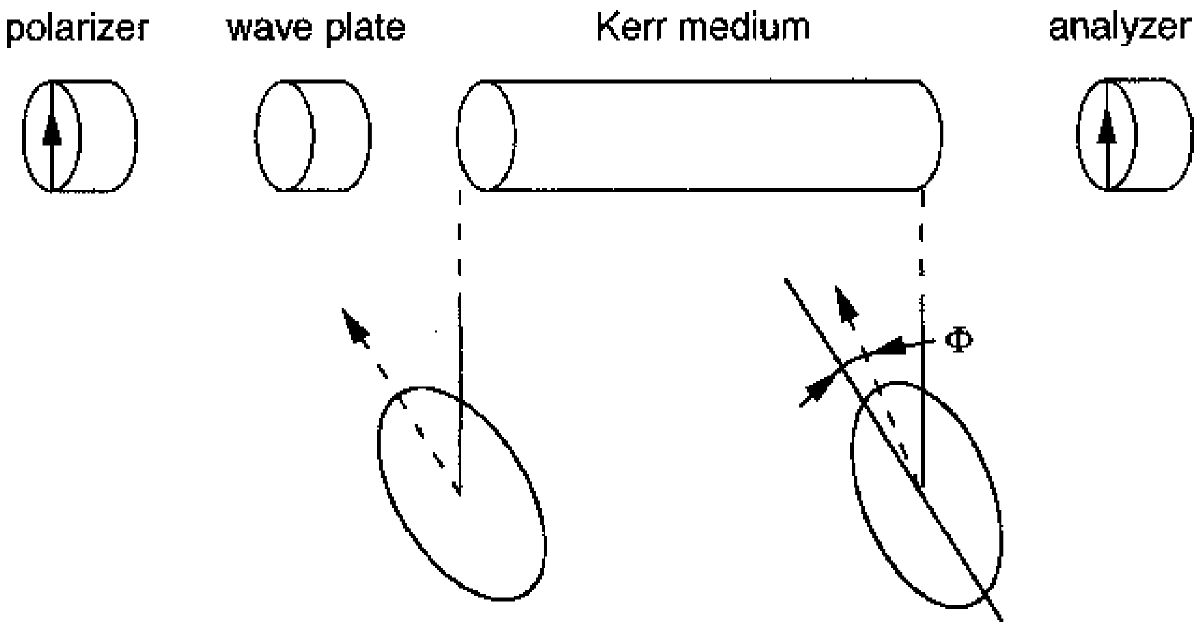
5.3 Kerr-Lens and Additive Pulse Mode Locking
5.3.1 Additive Pulse Mode Locking



Principle mechanism of APM

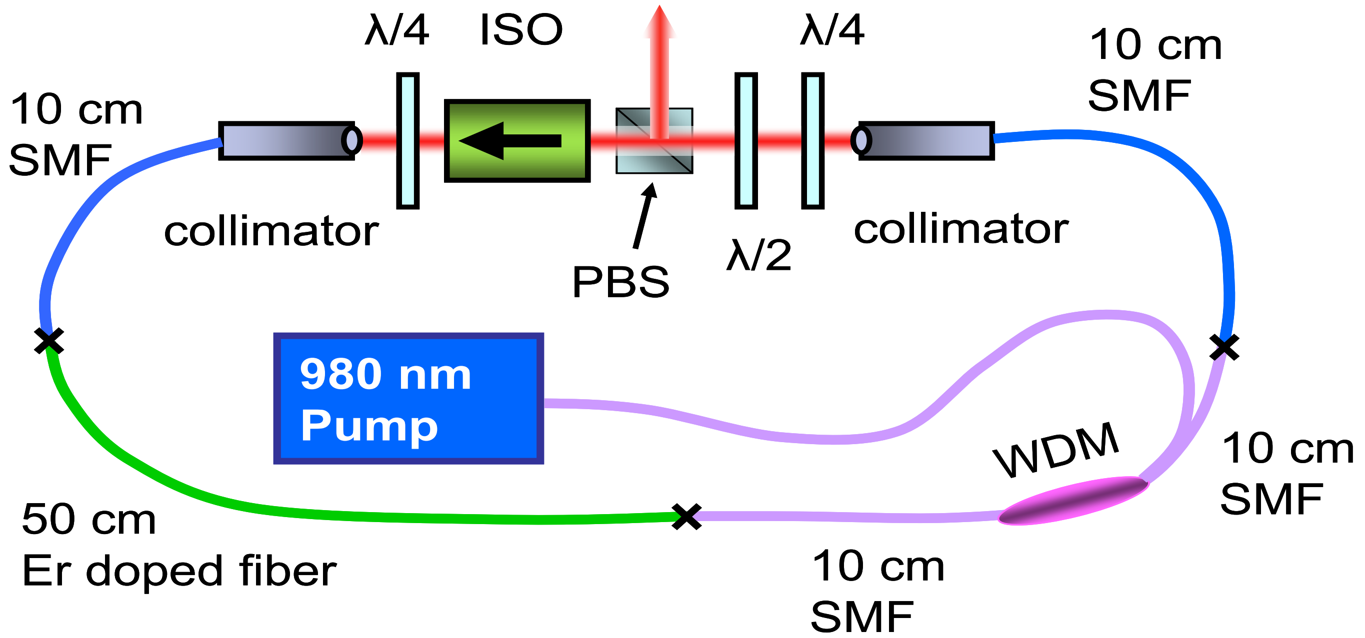


Noninear Mach-Zehnder Interferometer



NLMZ using nonlinear polarization rotation in a fiber

200 MHz Soliton Er-fiber Laser modelocked by APM

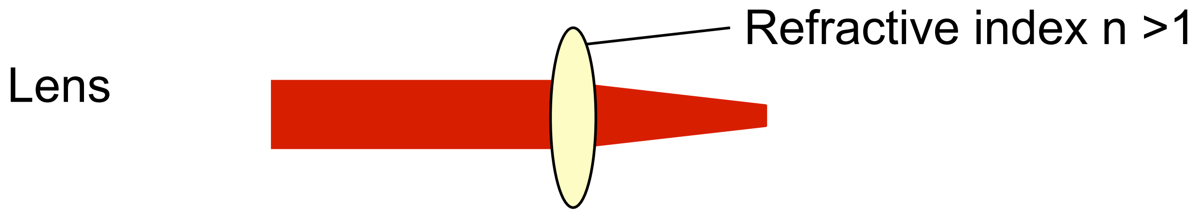


- 167 fs pulses
- 200 pJ intracavity pulse energy
- 200 pJ output pulse energy

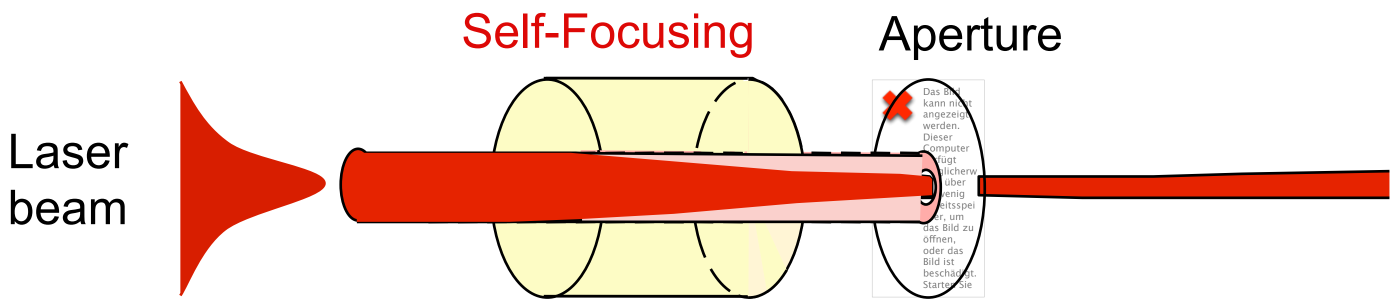
K. Tamura et al. Opt. Lett. 18, 1080 (199

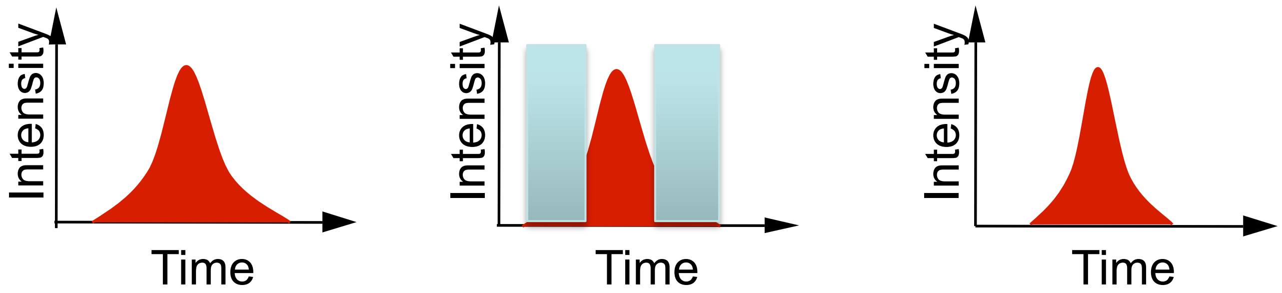
J. Chen et al, Opt. Lett. 32, 1566 (2007).

5.3.2 Kerr Lens Modelocking

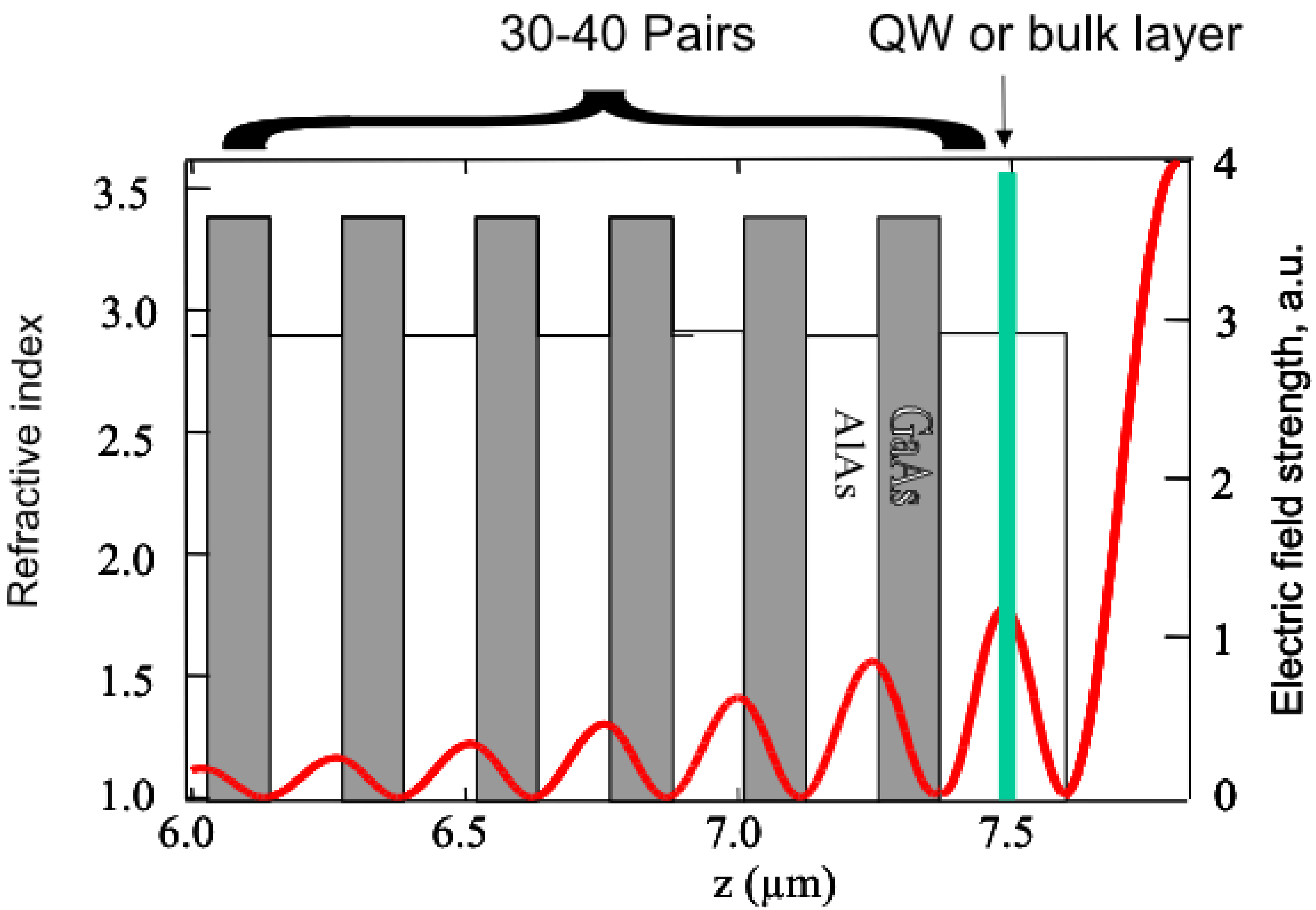


Intensity dependent refractive index: "Kerr-Lens"



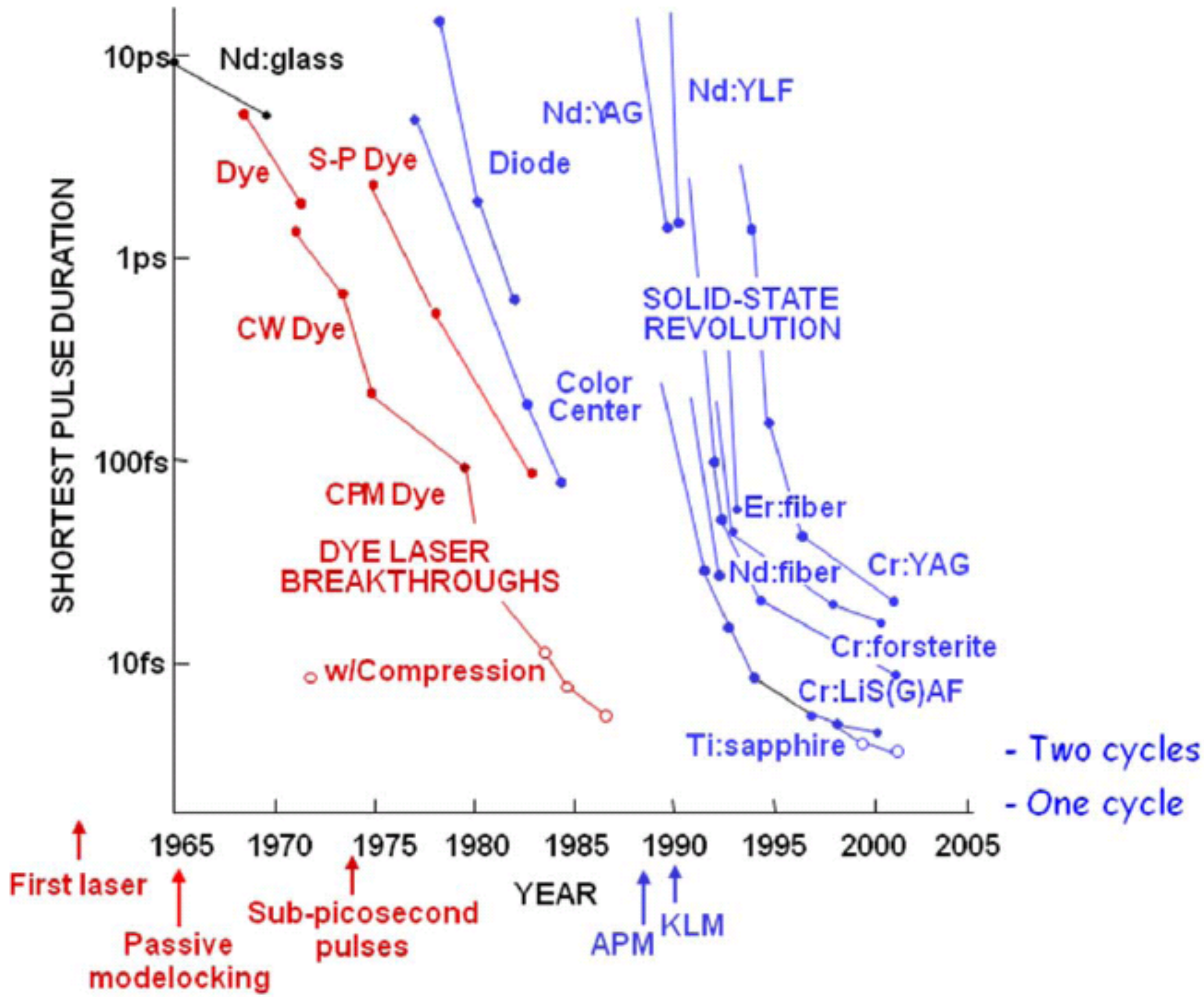


5.4 Semiconductor Saturable Absorbers



Semiconductor saturable absorber mirror (SESAM)
or Semiconductor Bragg mirror (SBR)

5.5 Oscillators: Historical Development

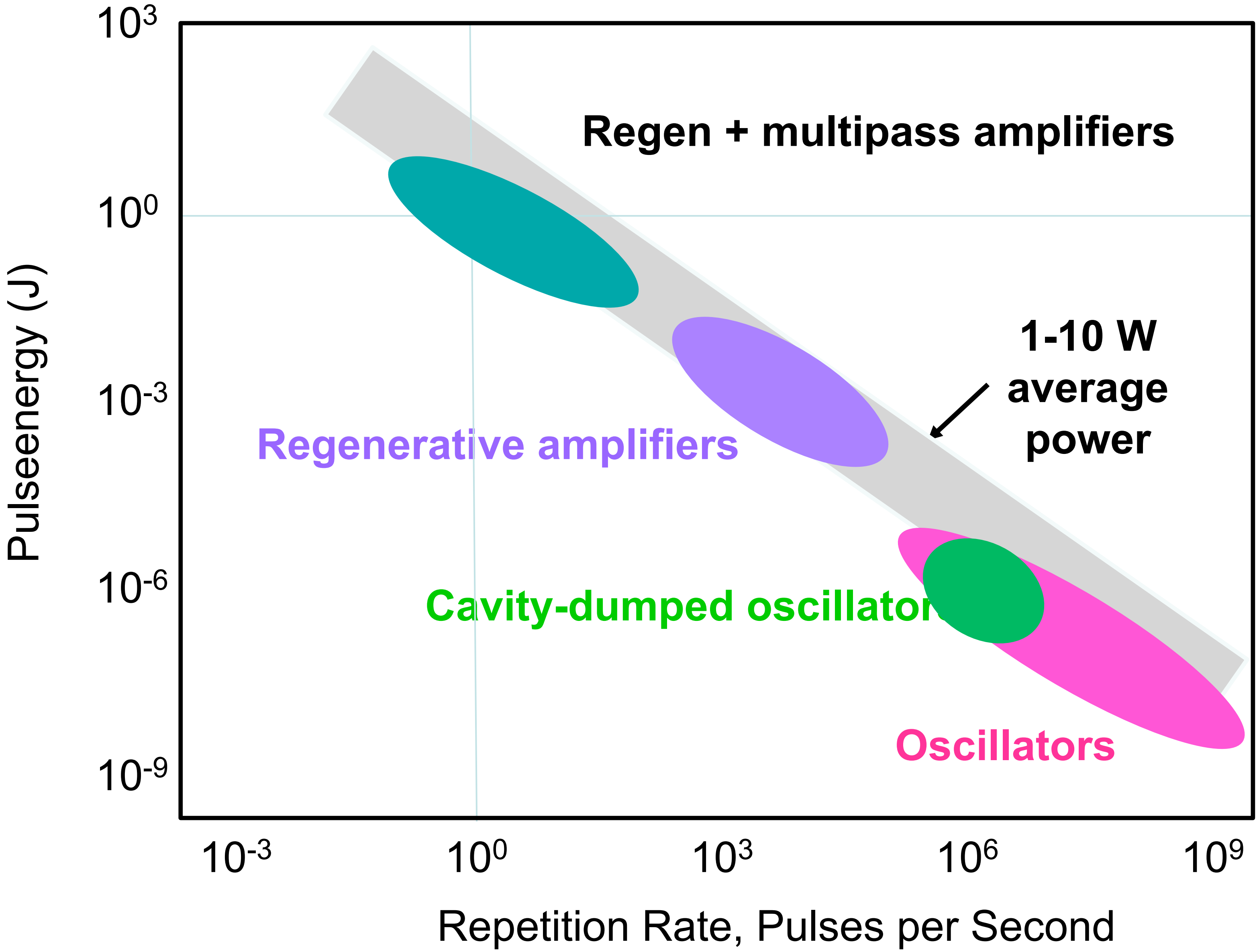


Pulse width of different laser systems by year.

6. Short Pulse Amplification

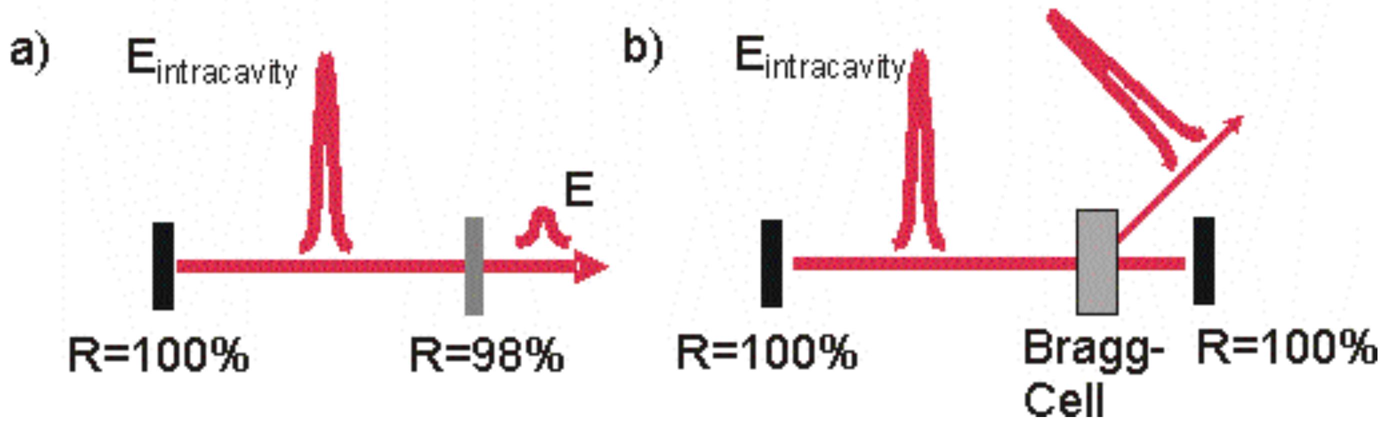
[1] Largely follows lecture on Ultrafast Amplifiers by Francois Salin, http://www.physics.gatech.edu/gcuo/lectures/index.html.

Pulse energies from different laser systems

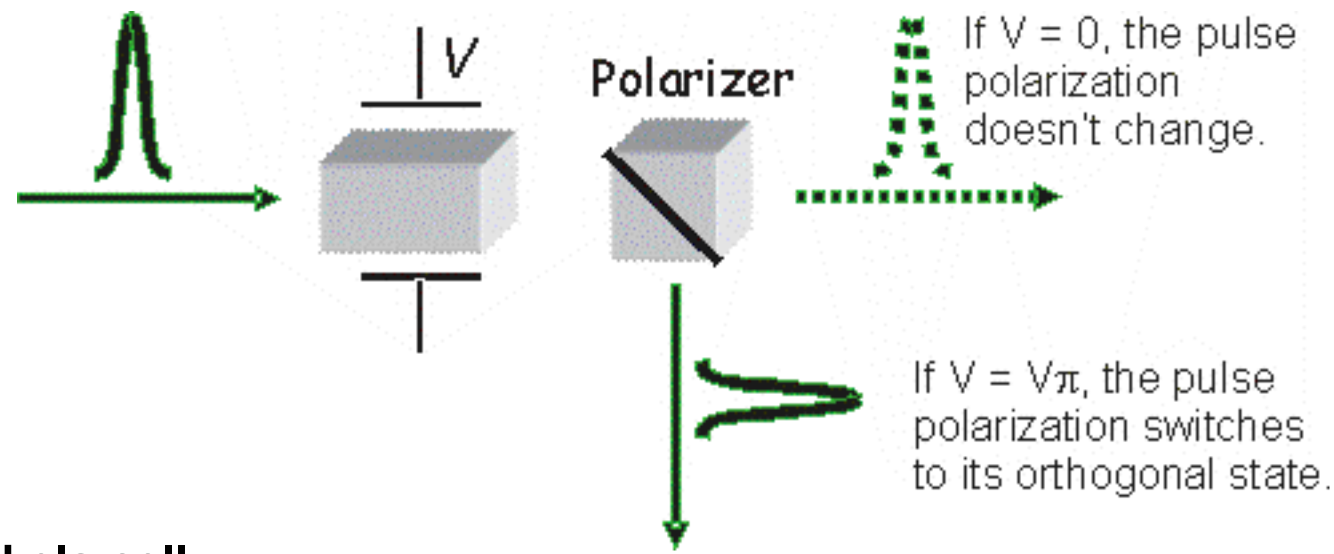


Schemes for generating high energy laser pulses.

6.1 Cavity Dumping

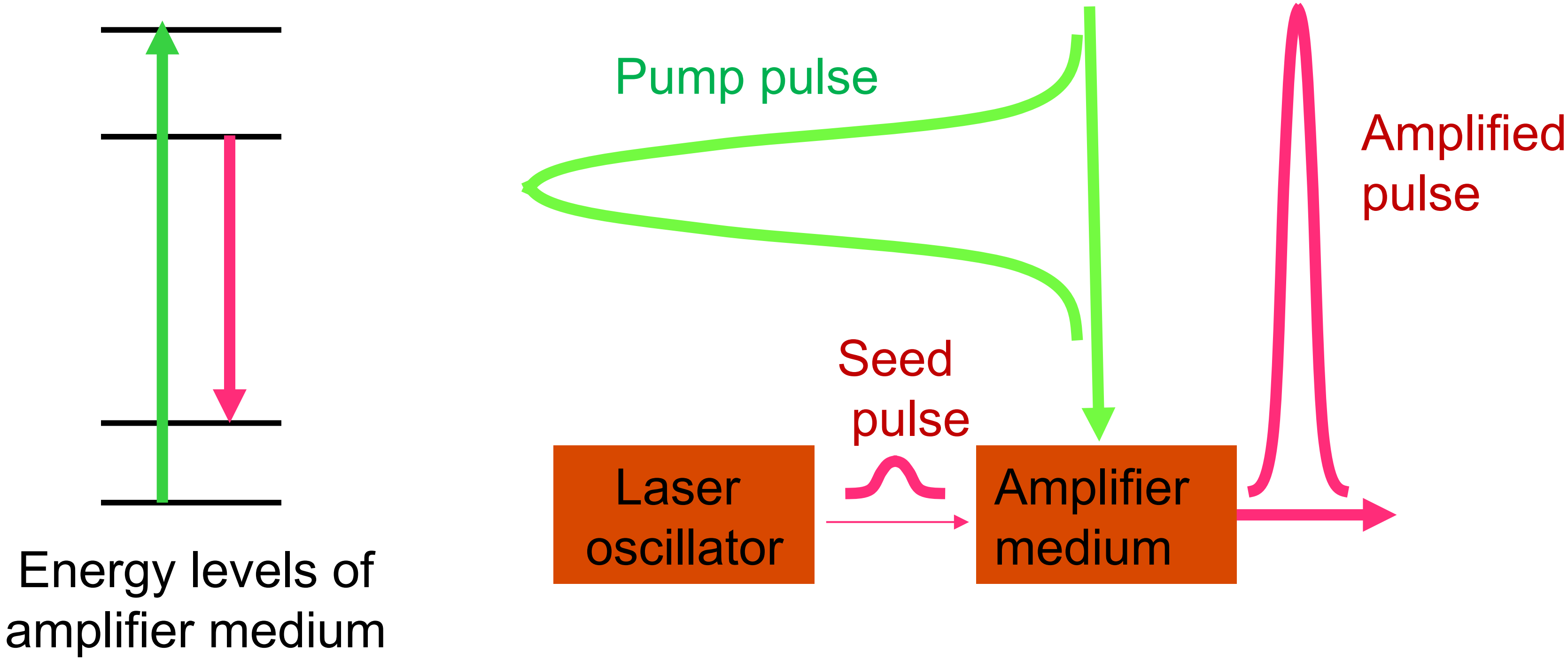


With Bragg cell



With Pockels cell

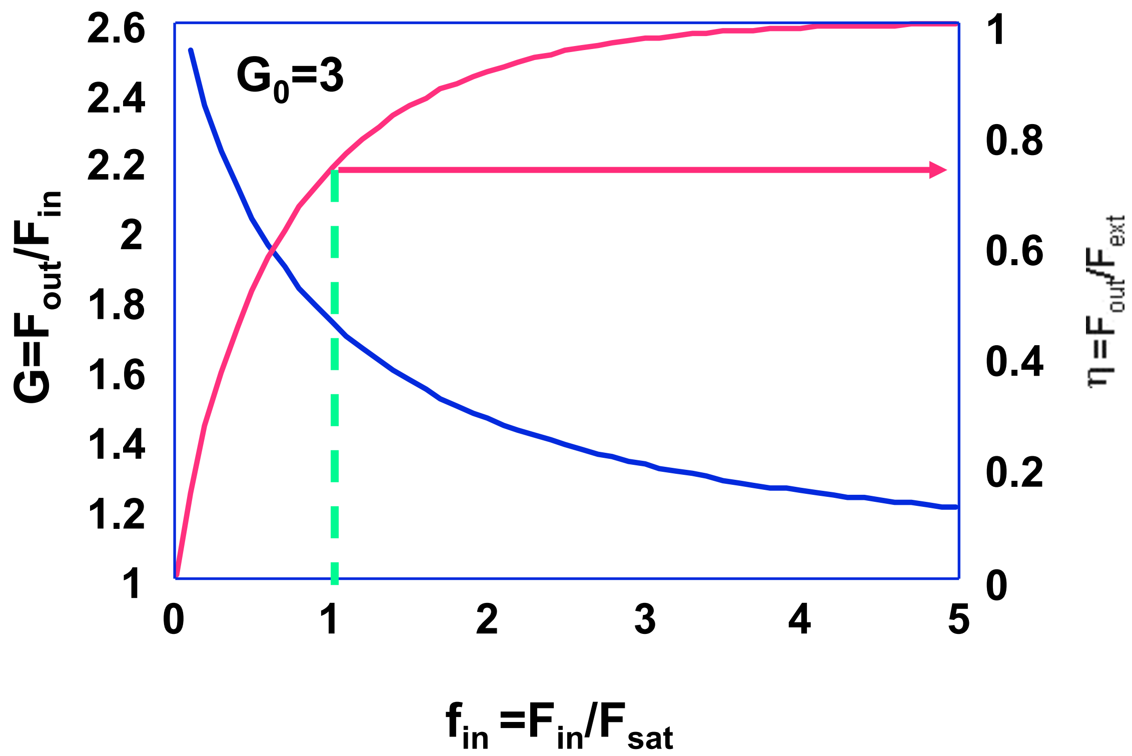
6.2 Laser Amplifiers



Laser amplifier: Pump pulse should be shorter than upper state lifetime. Signal pulse arrives at medium after pumping and well within the upper state lifetime to extract the energy stored in the medium, before it is lost due to energy relaxation.

6.2.1 Frantz-Nodvik Equation

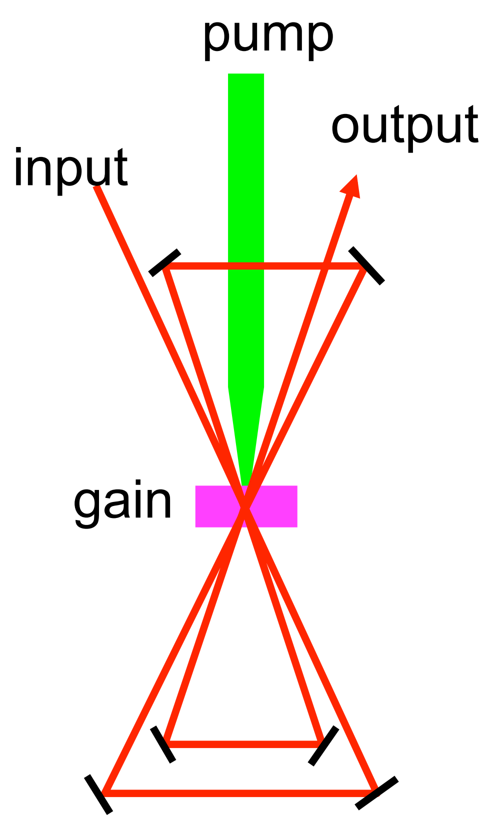
$$\begin{aligned}\eta_{eff} &= \frac{f_{out}}{f_{ext}} = \frac{1}{f_{ext}} \ln\left[1 + \left(e^{f_{in}} - 1\right)\ e^{f_{ext}}\right] = \frac{1}{\ln G_0} \ln\left[1 + \left(e^{f_{in}} - 1\right)\ G_0\right] \\ &\approx \frac{1}{f_{ext}} \ln\left[1 + f_{in} e^{f_{ext}}\right] \begin{cases} \frac{f_{in}}{f_{ext}} e^{f_{ext}} = \frac{f_{in}}{f_{ext}} G_0, & \text{for } f_{in} e^{f_{ext}} << 1 \\ 1, & \text{for } f_{in} e^{f_{ext}} >> 1 \end{cases} \end{aligned} \quad (12.16)$$



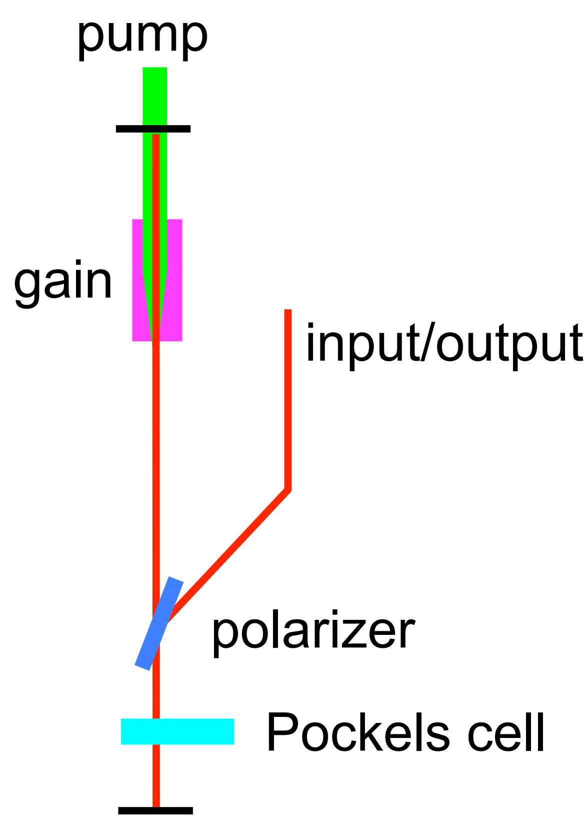
Gain and extraction efficiency for a small signal gain of $G_0 = 3$.

Basic Amplifier Schemes

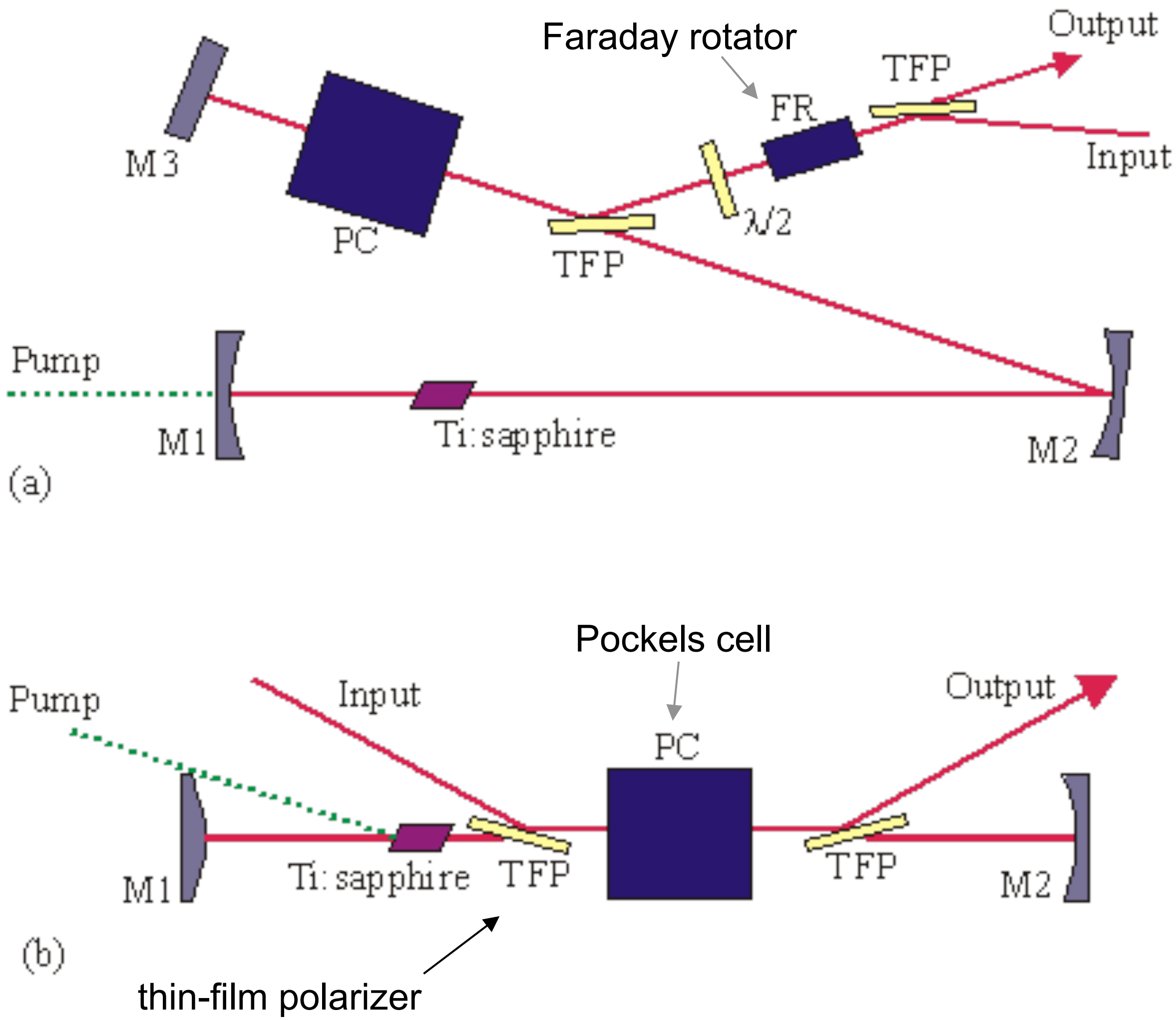
a) Multi-pass amplifier



b) Regenerative amplifier



6.2.2 Regenerative Amplifier Geometries

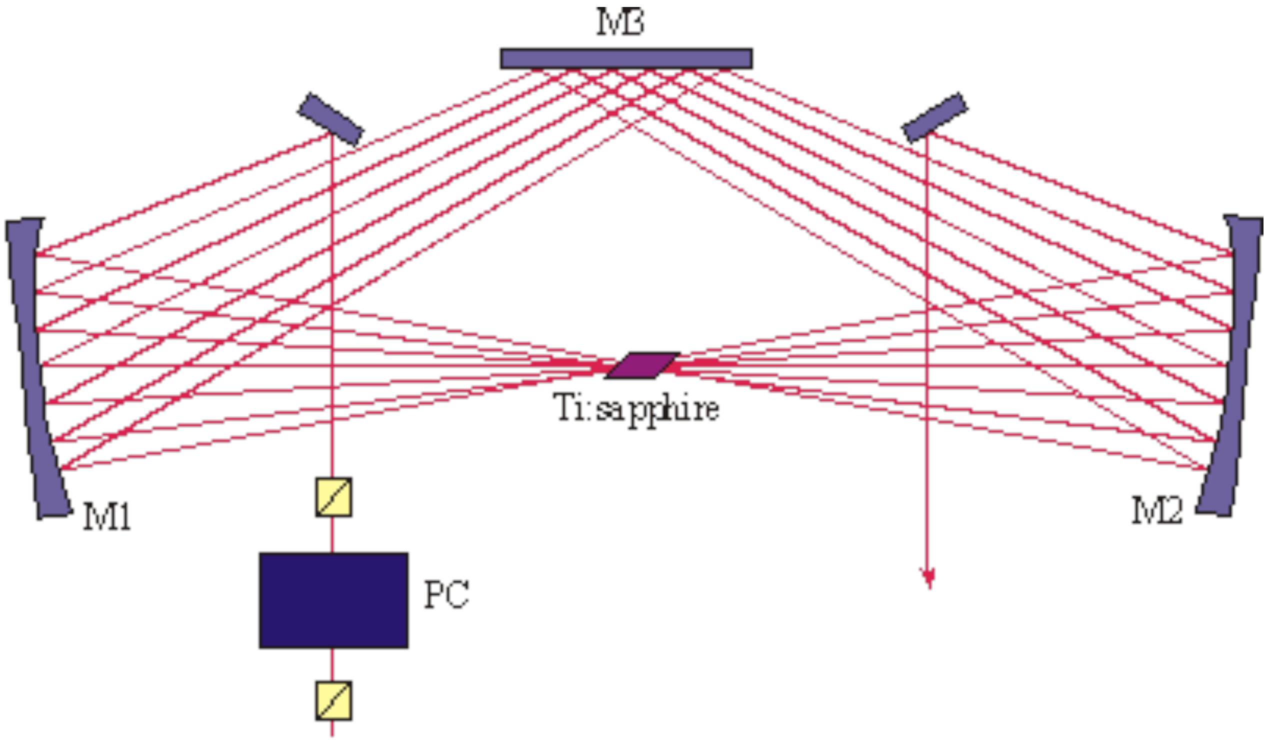


Two regens.

The design in (a) is often used for kHz-repetition-rate amplifiers and the lower (b) at a 10-20-Hz repetition rate. The lower design has a larger spot size in the Ti:sapphire rod.

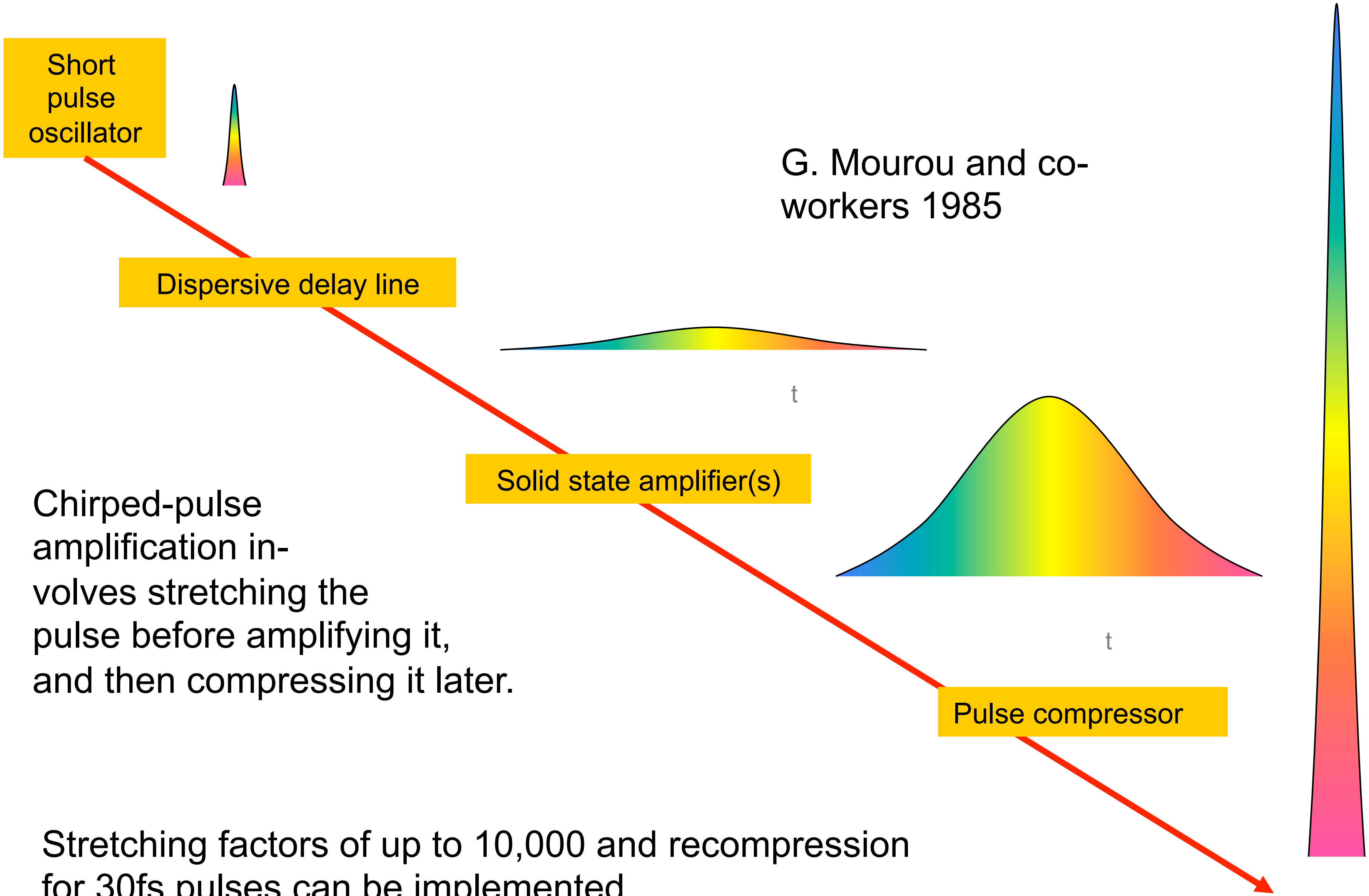
The Ti:sapphire rod is usually ~20-mm long and doped for 90% absorption.

6.2.3 A Multi-Pass Amplifier



A Pockels cell (PC) and a pair of polarizers are used to inject a single pulse into the amplifier

6.3 Chirped-Pulse Amplification

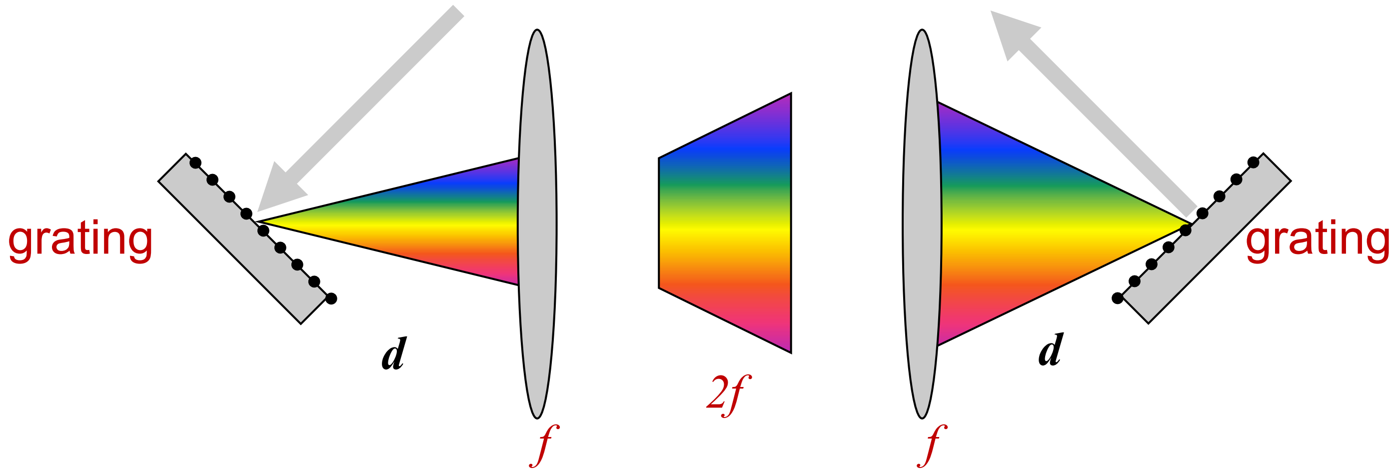


G. Mourou and co-workers 1985

Chirped-pulse amplification involves stretching the pulse before amplifying it, and then compressing it later.

Stretching factors of up to 10,000 and recompression for 30fs pulses can be implemented.

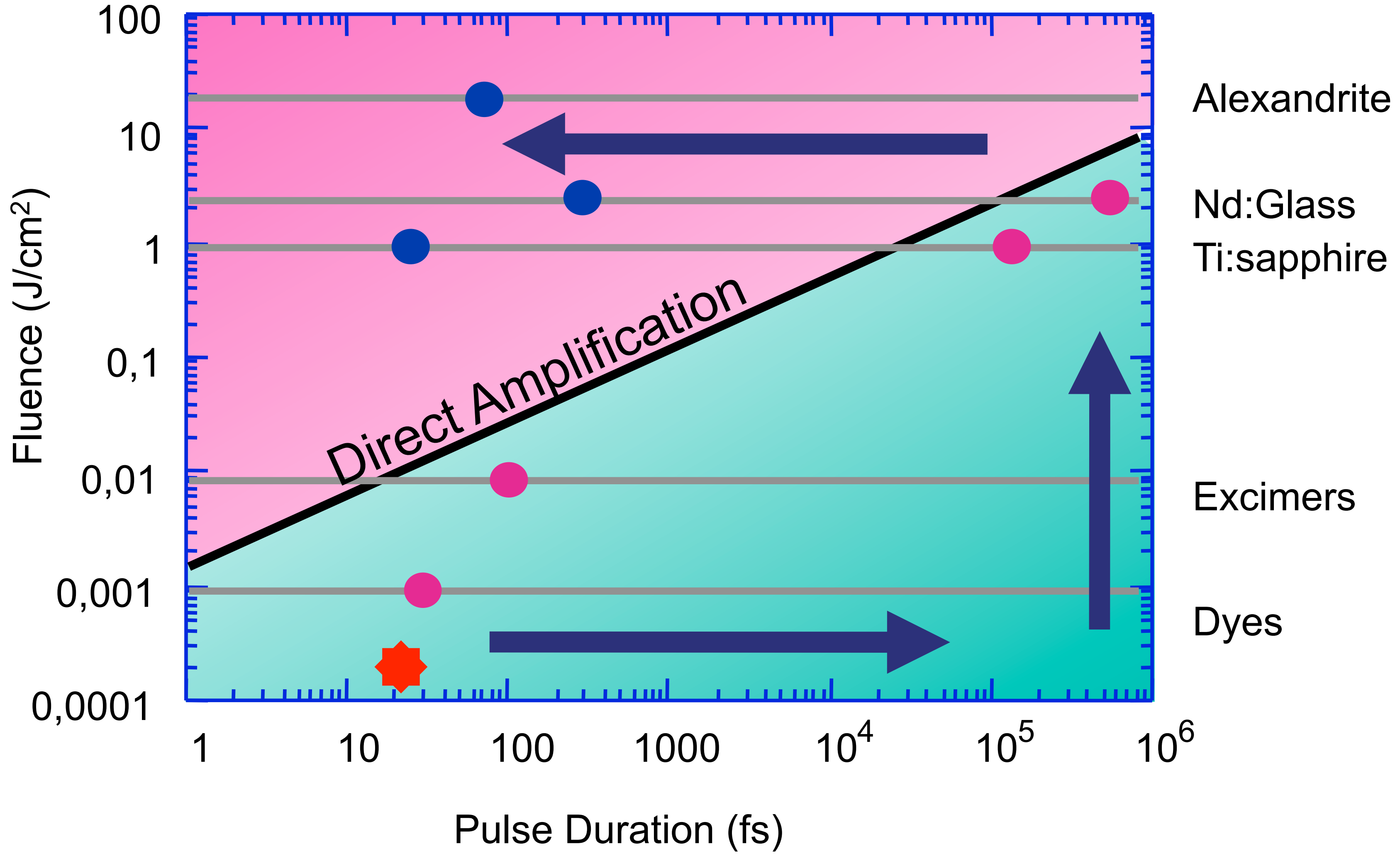
6.4 Stretchers and Compressors



Okay, this looks just like a “zero-dispersion stretcher” used in pulse shaping. But when $d \neq f$, it’s a dispersive stretcher and can stretch fs pulses by a factor of 10,000!

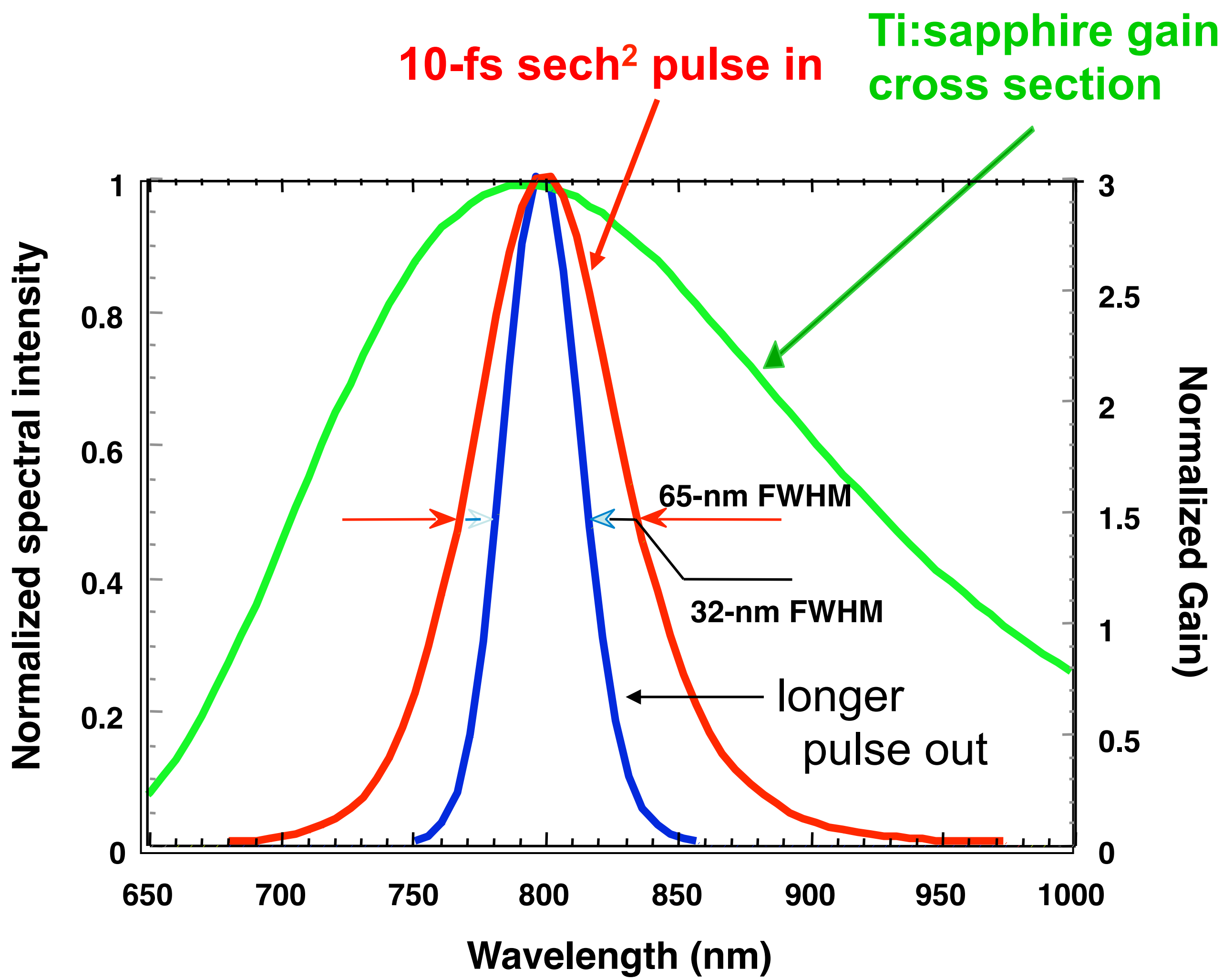
With the opposite sign of d-f, we can compress the pulse.

Achieveable fluences



Achieveable fluences using chirped pulse amplification for various stretching ratios. Compression of the pulses enables femtosecond pulses.

6.5 Gain Narrowing



Influence of gain narrowing in a Ti:sapphire amplifier on a 10 fs seed pulse

6.6 Contrast Ratio

If a pulse of 10^{18} W/cm^2 peak power has a "little" satellite pulse one millionth as strong, that's still 1 TW/cm^2! This can do some serious damage!

Ionization occurs at 10^{11} W/cm^2 : so at 10^{21} W/cm^2 we need a 10^{10} contrast ratio!

Major sources of poor contrast

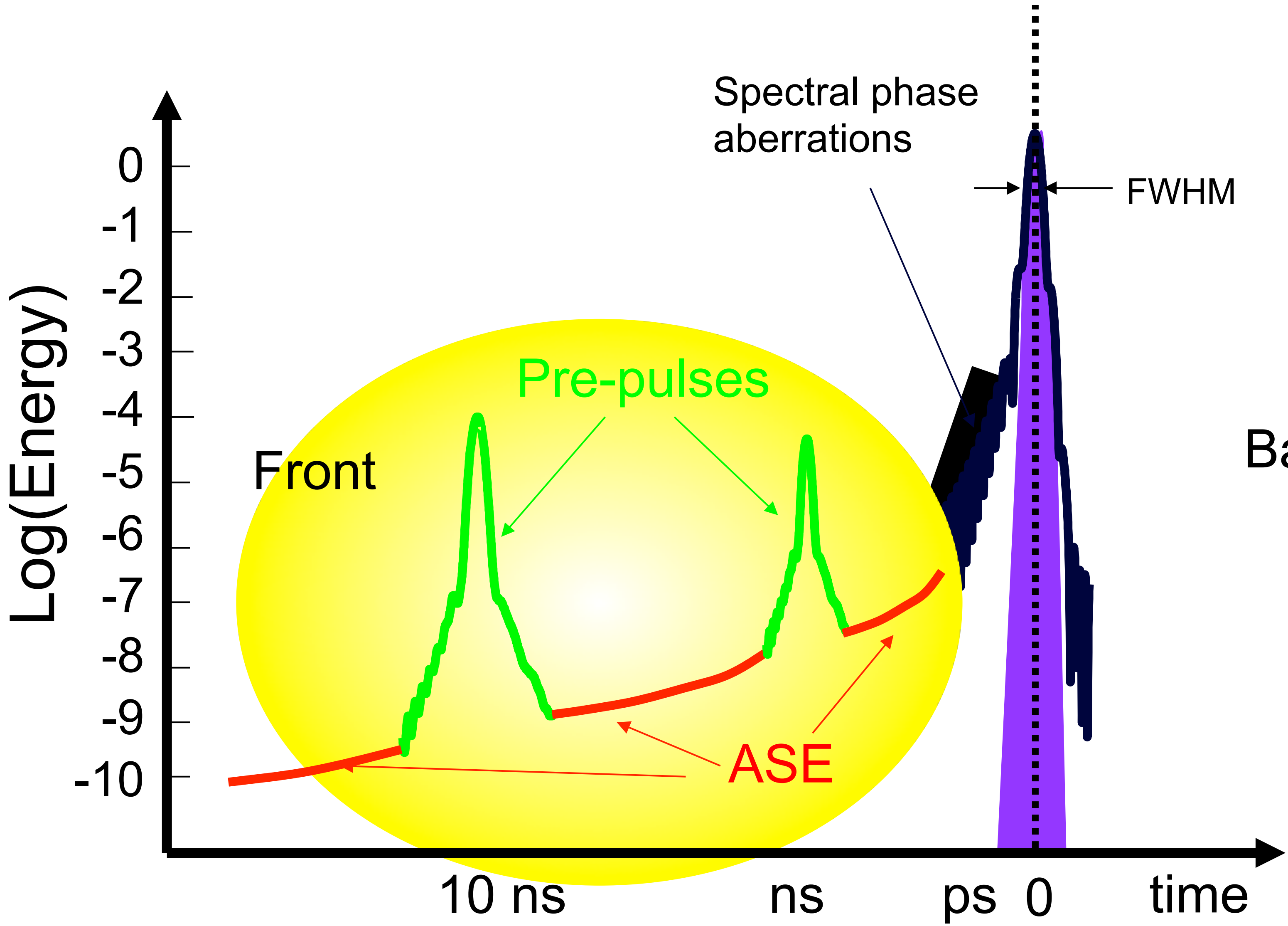
Nanosecond scale:

- pre-pulses from oscillator
- pre-pulses from amplifier
- ASE from amplifier

Picosecond scale:

- reflections in the amplifier
- spectral phase or amplitude distortions

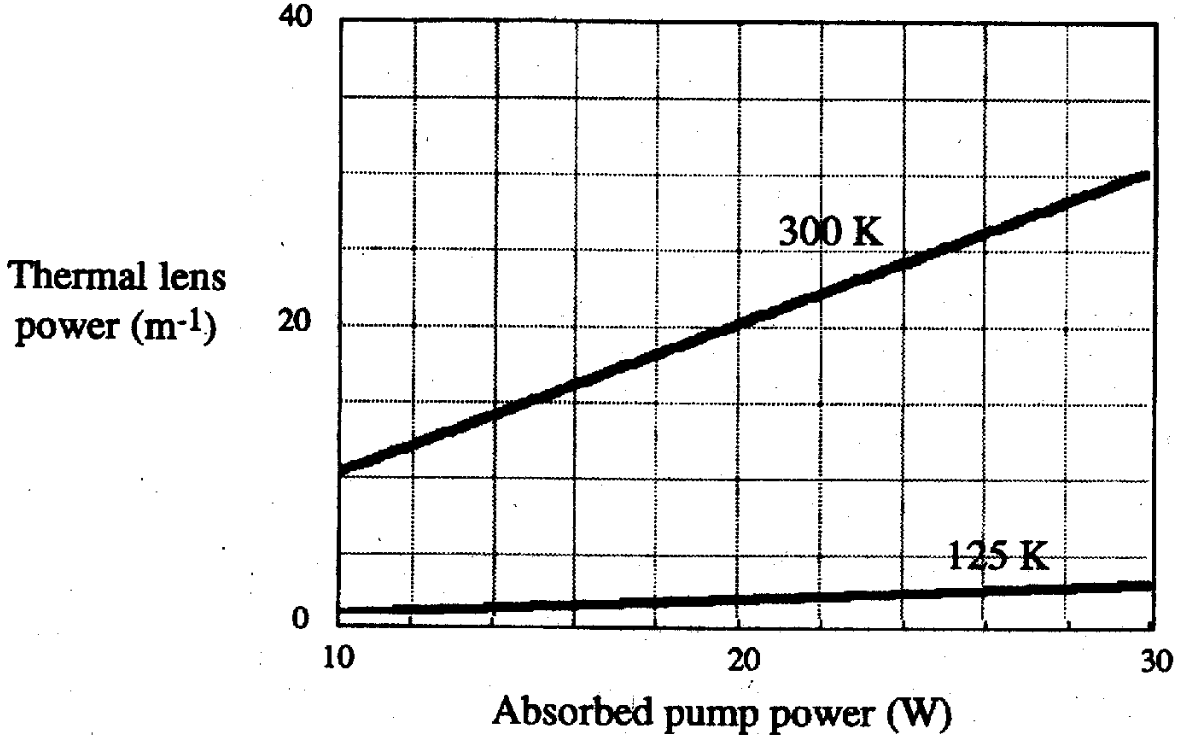
Amplified pulses often have poor contrast.



Pre-pulses do the most damage, messing up a medium beforehand.

6.7 Large Average Power: Cryogenic Cooling

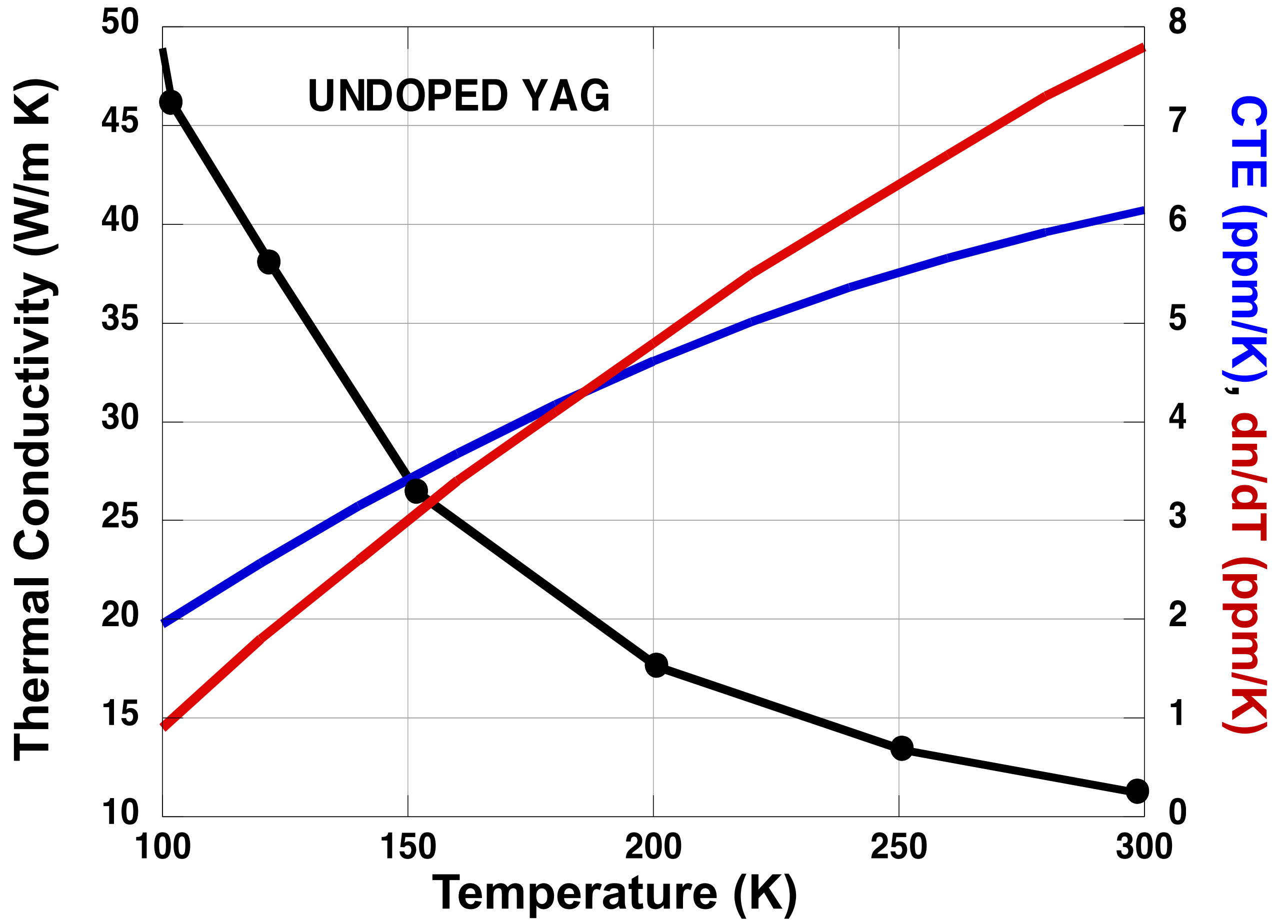
In sapphire, conductivity increases and dn/dt decreases with T



Calculations for kHz systems
Cryogenic cooling results in almost no focal power

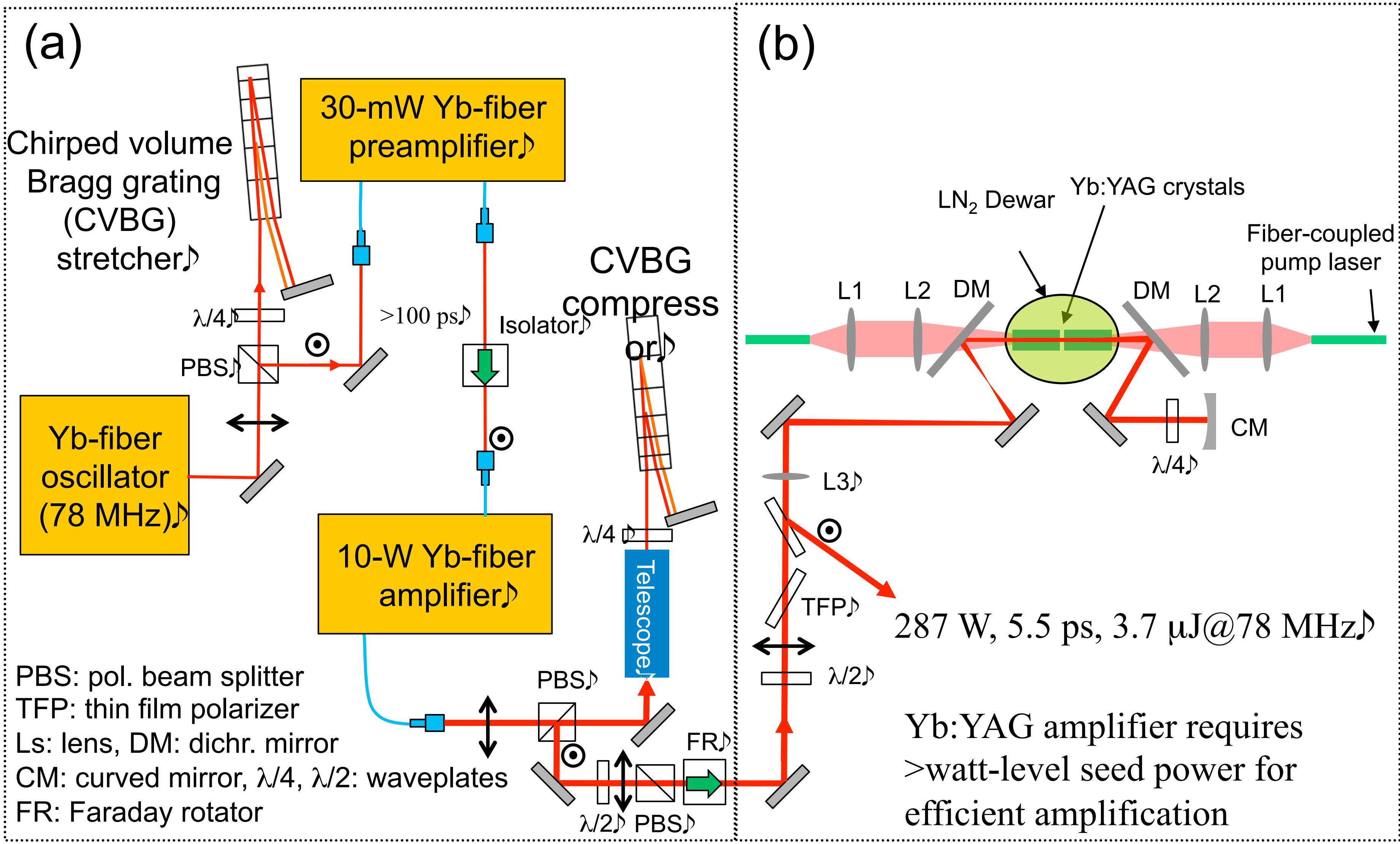
Murnane, Kapteyn, and coworkers

Thermal Properties of YAG



T. Y. Fan, and coworkers at Lincoln Laboratory

287-W Picosecond Amplifier

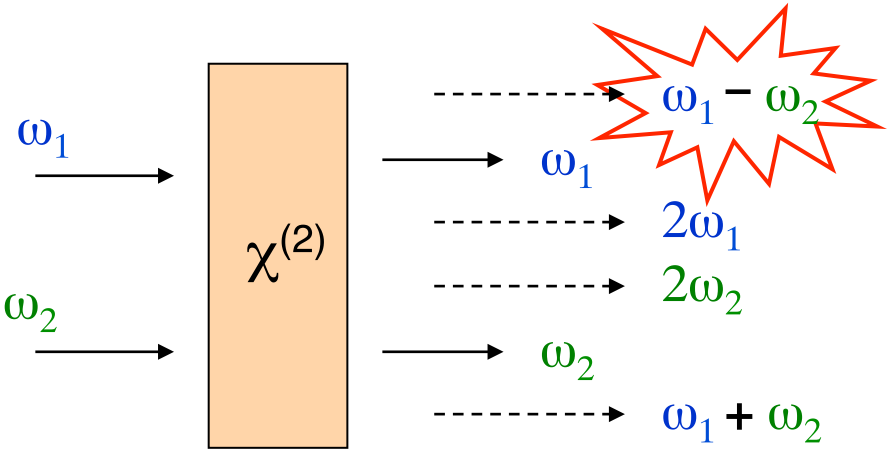


Yb:YAG amplifier requires >watt-level seed power for efficient amplification

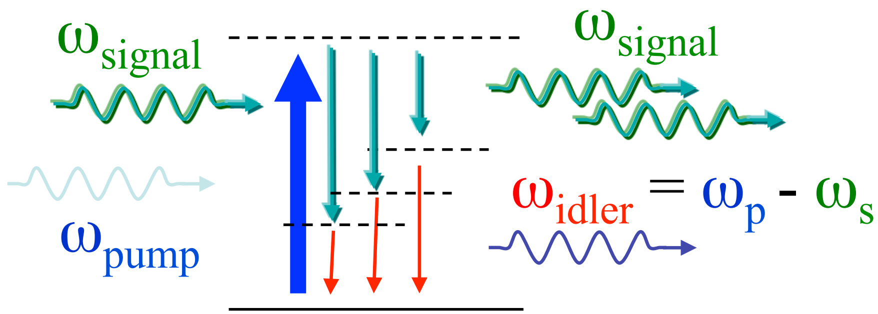
6. 8 Optical Parametric Amplifiers

Non-linear polarization effects

$$P = \varepsilon_0 \chi^{(1)} E + \varepsilon_0 \chi^{(2)} E^2 + \varepsilon_0 \chi^{(3)} E^3 + \ldots$$

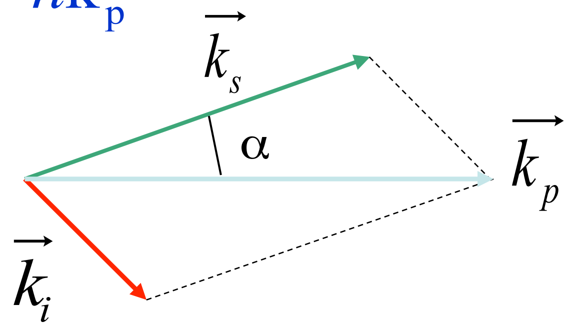


Optical Parametric Amplification (OPA)



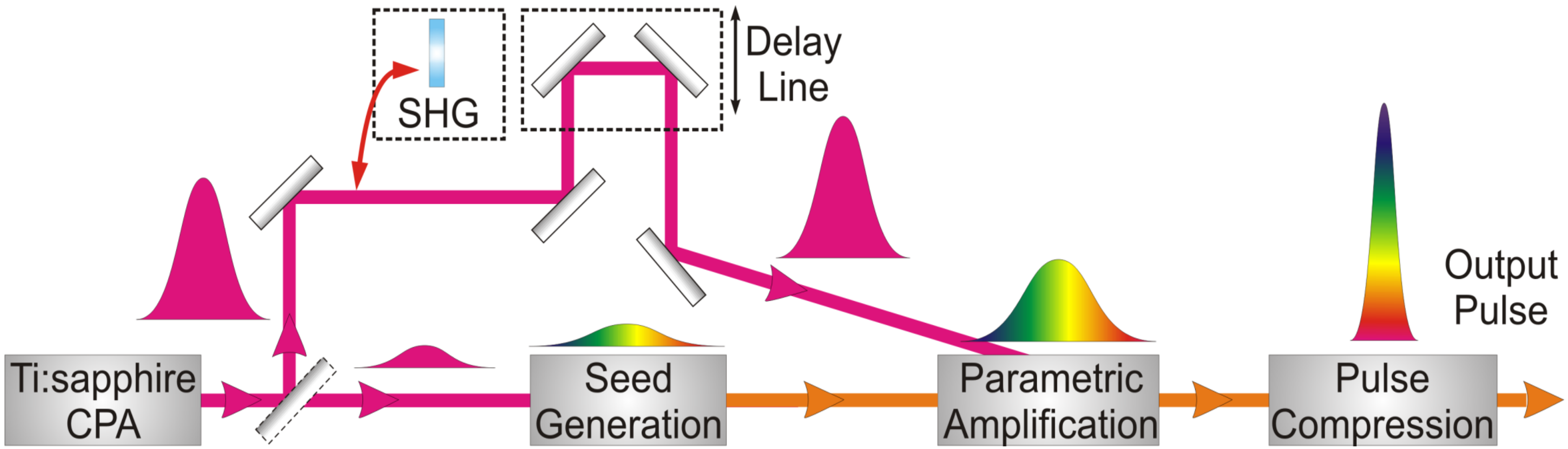
Energy conservation: $\hbar\omega_s + \hbar\omega_i = \hbar\omega_p$

Momentum conservation (vectorial): $\hbar\vec{k}_s + \hbar\vec{k}_i = \hbar\vec{k}_p$
(also known as **phase matching**)



⇒ **Broadband gain medium!**

Ultrabroadband Optical Parametric Amplifier



- Broadband seed pulses can be obtained by white light generation

- Broadband amplification requires phase matching over a wide range of signal wavelengths

G. Cerullo and S. De Silvestri, Rev. Sci. Instrum. **74**, 1 (2003).

Phase matching bandwidth in an OPA

If the signal frequency ω_s increases to $\omega_s+\Delta\omega$, by energy conservation the idler frequency decreases to $\omega_i-\Delta\omega$. The wave vector mismatch is

$$\Delta k = -\frac{\partial k_s}{\partial \omega}\Delta\omega + \frac{\partial k_i}{\partial \omega}\Delta\omega = \left(\frac{1}{v_{gs}} - \frac{1}{v_{gi}}\right)\Delta\omega$$

The phase matching bandwidth, corresponding to a 50% gain reduction, is

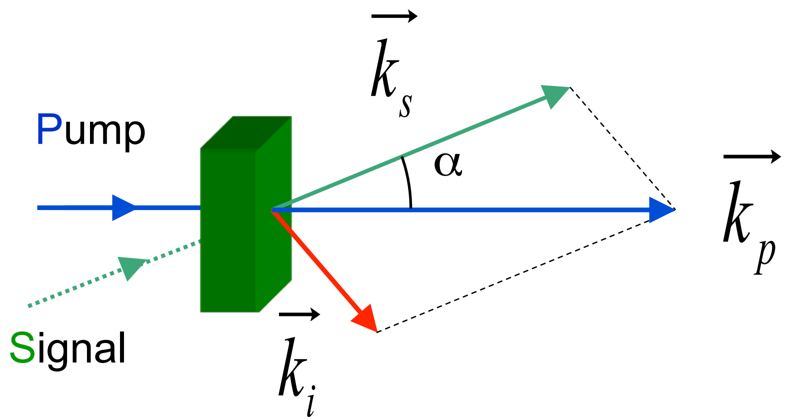
$$\Delta\nu \cong \frac{2(\ln 2)^{1/2}}{\pi}\left(\frac{\gamma}{L}\right)^{1/2}\frac{1}{\left|\frac{1}{v_{gs}} - \frac{1}{v_{gi}}\right|}$$

$\Rightarrow$ the achievement of broad gain bandwidths requires group velocity matching between signal and idler beams

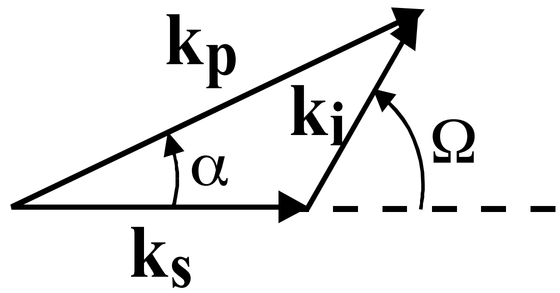
Broadband OPA configurations

- $v_{gi} = v_{gs}$: **Operation around degeneracy** $\omega_i = \omega_s = \omega_p/2$
  - ✓ **Type I, collinear configuration**
  - ✓ **Signal and idler have same refractive index**

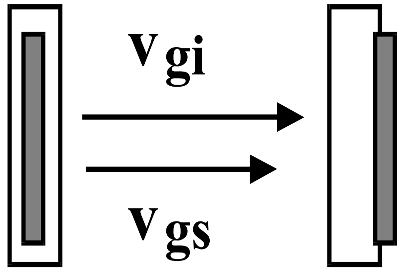
- $v_{gi} \neq v_{gs}$: **Non-collinear parametric amplifier (NOPA):**
  - ✓ **Pump and Signal at angle α**



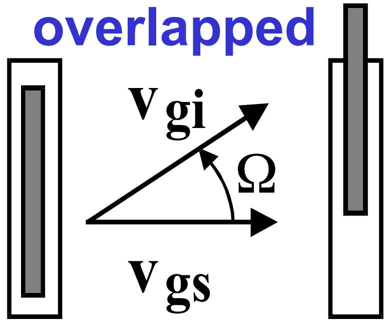
Noncollinear phase matching: geometrical interpretation



In a collinear geometry, signal and idler move with different velocities and get quickly separated



In the non-collinear case, the two pulses stay temporally overlapped



$$v_{gs} = v_{gi} \cos\Omega$$

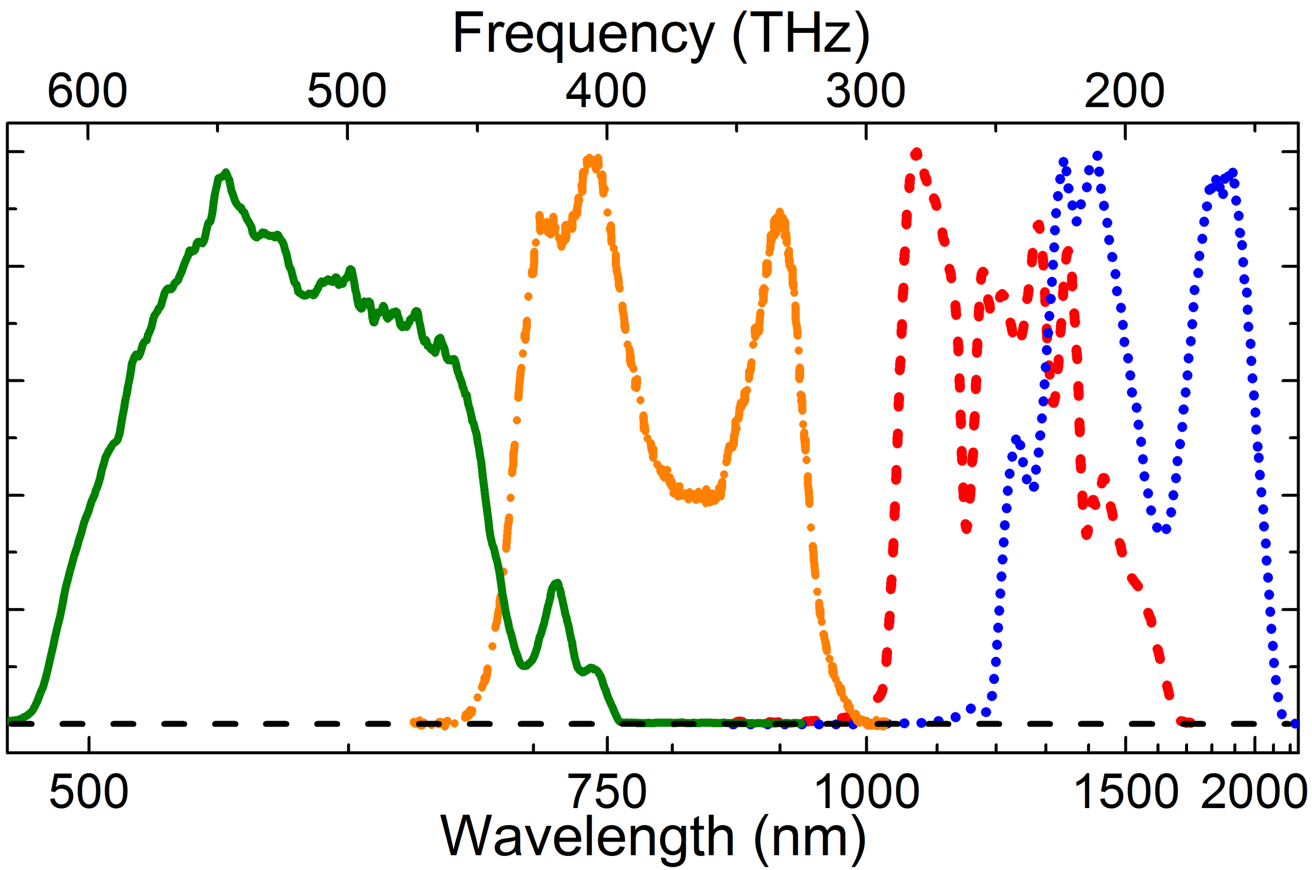
Note: this requires $v_{gi} > v_{gs}$ (not always true!)

Broadband OPA configurations

Pump wavelength	NOPA	Degenerate OPA
400 nm (SH Ti:sapphire)	500-750 nm	700-1000 nm
800 nm (Ti:sapphire)	1-1.6 μm	1.2-2 μm

OPAs should allow to cover nearly continuously the wavelength range from 500 to 2000 nm (two octaves!) with few-optical-cycle pulses

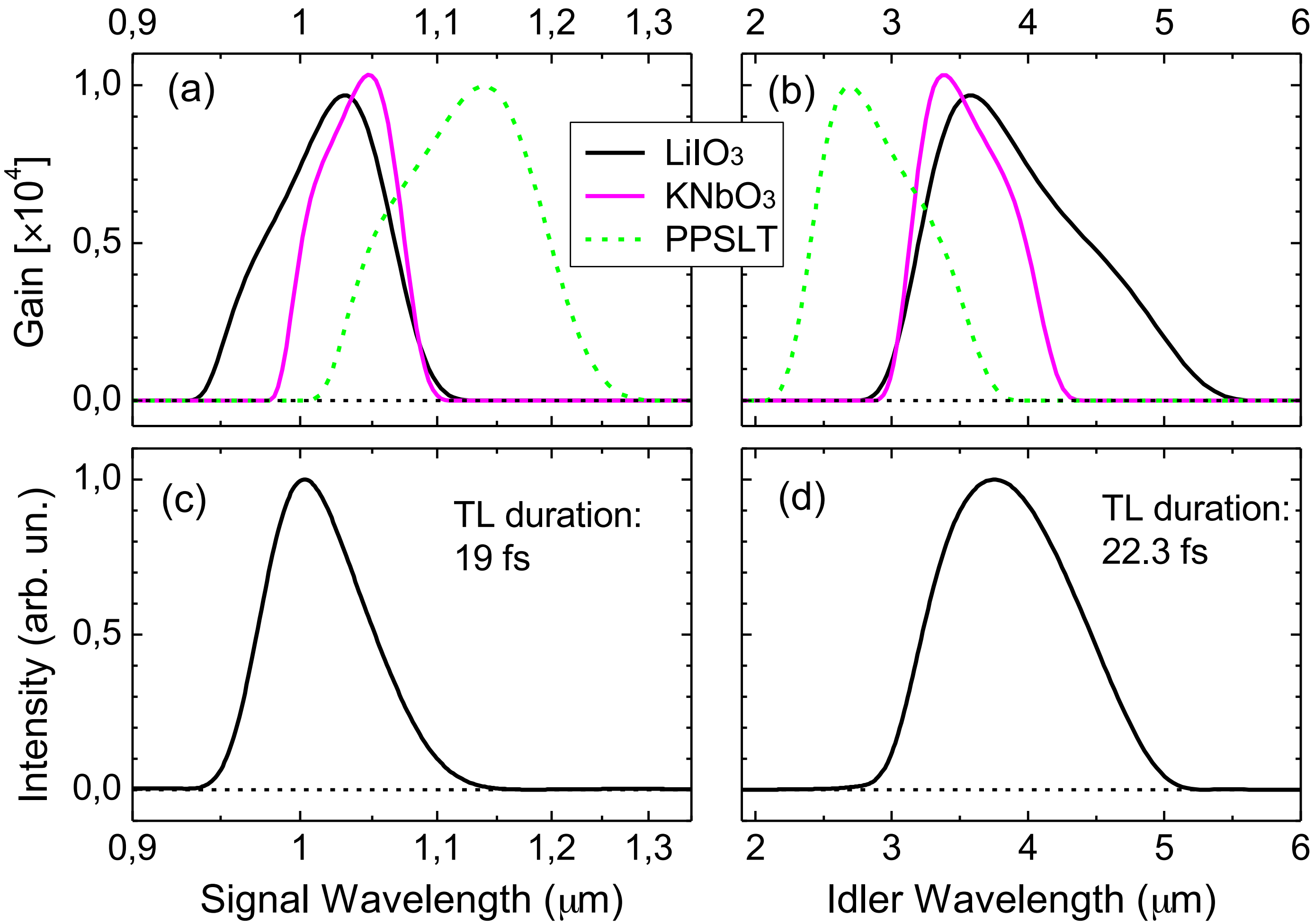
Tunable few-optical-cycle pulse generation



D Brida *et al.*, J. Opt. **12**, 013001 (2010).

Can we tune our pulses even more to the mid-IR? Yes, using the idler!

Broadband pulses in the mid-IR



- **Simulations confirm the generation of broadband idler pulses, with 20-fs duration (≈2 optical cycles) at 3 μm**

References:

B.E.A. Saleh and M.C. Teich, "Fundamentals of Photonics," John Wiley and Sons, Inc., 1991.
P. G. Drazin, and R. S. Johnson, ”Solitons: An Introduction,”Cambridge University Press, New York (1990).

R.L. Fork, O.E. Martinez, J.P. Gordon: Negative dispersion using pairs of prisms, Opt. Lett. 9 150-152 (1984).
M. Nisoli, S. Stagira, S. De Silvestri, O. Svelto, S. Sartania, Z. Cheng, M. Lenzner, Ch. Spielmann, F. Krausz: A novel high energy pulse compression system: generation of multigigawatt sub-5-fs pulses, Appl. Phys. B 65 189-196 (1997).

J. Kuizenga, A. E. Siegman: "FM und AM mode locking of the homoge- neous laser - Part II: Experimental results, IEEE J. Quantum Electron. 6, 709-715 (1970).
H.A. Haus: Theory of mode locking with a slow saturable absorber, IEEE J. Quantum Electron. QE 11, 736-746 (1975).
K. J. Blow and D. Wood: "Modelocked Lasers with nonlinear external cavity," J. Opt. Soc. Am. B 5, 629-632 (1988).
J. Mark, L.Y. Liu, K.L. Hall, H.A. Haus, E.P. Ippen: Femtosecond pulse generation in a laser with a nonlinear external resonator, Opt. Lett. 14, 48-50 (1989)·
E.P. Ippen, H.A. Haus, L.Y. Liu: Additive pulse modelocking, J. Opt. Soc. Am. B 6, 1736-1745 (1989).
D.E. Spence, P.N. Kean, W. Sibbett: 60-fsec pulse generation from a self-mode-locked Ti:Sapphire laser, Opt. Lett. 16, 42-44 (1991).
H. A. Haus, J. G. Fujimoto and E. P. Ippen, ”Structures of Additive Pulse Mode Locking,” J. Opt. Soc. Am. 8, pp. 2068 — 2076 (1991).
U. Keller, ”Semiconductor nonlinearities for solid-state laser modelock- ing and Q-switching,” in Semiconductors and Semimetals, Vol. 59A, edited by A. Kost and E. Garmire, Academic Press, Boston 1999.

Lecture on Ultrafast Amplifiers by Francois Salin, http://www.physics.gatech.edu/gcuo/lectures/index.html.
D. Strickland and G. Morou: "Compression of amplified chirped optical pulses," Opt. Comm. 56,219-221,(1985).

R. Boyd, Nonlinear Optics ~Academic, New York, 1992.
G. Cerullo and S. De Silvestri, "Ultrafast Optical Parametric Amplifiers," Review of Scientific Instruments 74, pp. 1-17 (2003).