Universität Hamburg
Physics Department
Ultrafast Optical Physics II
SoSe 2019

Problem Set 5

Issued: **May 17, 2019** Due: **May 24, 2019**

Instruction: Please write your answer to each problem on separate paper sheet. If you are using programming language to do numerical simulations, attach the original code with your answers.

Problem 5.1: Active Mode-Locking and Gaussian Pulse Analysis (20 points in total)

In this problem, the steady-state pulse width in an actively mode-locked laser using Gaussian pulse analysis will be obtained.

The master equation for active mode-locking by pure loss modulation is given by the following expression

$$T_R \frac{\partial A}{\partial T} = \left[g - l + D_g \frac{\partial^2}{\partial t^2} - M_s t^2 \right] A \tag{1}$$

where the loss modulator is already described in parabolic approximation. In the same way as the Split-Step Fourier transform method to simulate the NSE, we can use it to simulate the dynamics of the actively mode-locked laser. We start from a Gaussian pulse

$$A_1(t) = e^{-\Gamma_1 t^2/2} \tag{2}$$

with the complex Γ-parameter, $\Gamma_1 = a + jb$. Propagation through the gain and loss part of the system within one round trip can be described in the frequency domain by the operation

$$\hat{A}_2(\omega) = e^{\left(g - l - D_g \omega^2\right)} \hat{A}_1(\omega) \tag{3}$$

which is again a Gaussian pulse in the frequency and time domain

$$A_2(t) = e^{-\Gamma_2 t^2/2} \tag{4}$$

Propagation through the modulator is most simply described in the time domain by

$$A_3(t) = e^{-M_s t^2} A_2(t) = e^{-\Gamma_3 t^2/2} \tag{5}$$

(a) Calculate the complex Gamma coefficients Γ_2 and Γ_3 as a function of Γ_1, D_g and M_s. Assume $D_g \Gamma_1 \ll 1$. **(7 points)**

(b) What is the condition for steady-state pulse after one round-trip? **(7 points)**

(c) Derive the Γ-parameter for the steady-state pulse. How do the stationary pulse width and chirp depend on the system parameters? **(6 points)**

Problem 5.2: Fast Saturable Absorber Mode-Locked Ti:Sapphire Laser (30 points)

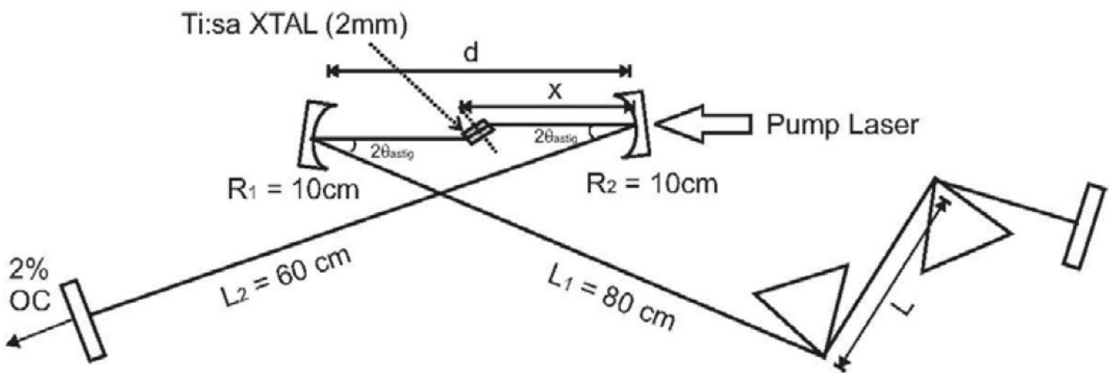

Figure 1: Standard femtosecond Ti:Sapphire laser with a prism pair for dispersion compensation.

We want to analyze a Ti:Sapphire laser mode-locked by an artificial fast saturable absorber (Kerr-Lens mode-locking). In this problem, we will design the linear resonator and analyze the pulse dynamics in the soliton pulse shaping regime, i.e. negative dispersion regime. One could follow up on this by also considering the Kerr-lens effect in the laser crystal that changes the resonator condition and provides an artificial saturable absorption effect. For brevity we will not consider the latter.

Gain Medium: The Ti:Sapphire crystal will be used for the gain medium. The HWHM-gain bandwidth of Ti:Sapphire is $\Omega_g = 2\pi \cdot 43\ THz$. The crystal length is $2\ mm$. The refractive index of the crystal is 1.76 at $800nm$. For a rough estimation of the pulsed operation, assume the fast saturable absorber coefficient γ and the self-phase modulation coefficient δ are: $\gamma = 10^{-7}/W$ and $\delta = 10^{-6}/W$.

Dispersion Compensation: To obtain short pulses from this laser the dispersion of the Ti:Sapphire crystal has to be compensated. For dispersion compensation we will put a prism pair (refer to Problem 3.1 again) in the longer arm (L_1). The prisms are cut at Brewster's angle for the center wavelength of $800\ nm$. The beam at center wavelength $800\ nm$ also defines the prism angle $\beta = 0$.

(a) What prism separation, L, would you choose for three different prism materials, (i) quartz (ii) SF10 and (iii) CaF_2, to compensate the second-order dispersion of the Ti:Sapphire crystal? The material parameters at $0.8\ \mu m$ are: Ti:Sapphire: $\frac{\partial^2 n}{\partial \lambda^2} = 0.064 \frac{1}{\mu m^2}$, quartz: $\frac{\partial n}{\partial \lambda} = -0.017 \frac{1}{\mu m}$, SF10: $\frac{\partial n}{\partial \lambda} = -0.05 \frac{1}{\mu m}$, CaF_2: $\frac{\partial n}{\partial \lambda} = -0.01 \frac{1}{\mu m}$. **(5 points)**

(b) How large is the remaining third-order dispersion for the different prism materials? Use the following material parameters for $0.8\ \mu m$: Ti:Sapphire: $\frac{\partial^3 n}{\partial \lambda^3} = -0.377 \frac{1}{\mu m^3}$, Quartz: $\frac{\partial^2 n}{\partial \lambda^2} = 0.04 \frac{1}{\mu m^2}$, SF10: $\frac{\partial^2 n}{\partial \lambda^2} = 0.18 \frac{1}{\mu m^2}$, CaF_2: $\frac{\partial^2 n}{\partial \lambda^2} = 0.031 \frac{1}{\mu m^2}$. **(5 points)**
Note, for computation of the third-order dispersion use the result for $\partial^2 P/\partial\lambda^2$, where P is the optical path length through the prism pair from Problem 3.1. The term proportional to $sin(\beta)$ can be neglected and, therefore, the coefficient $\partial^3 n/\partial\lambda^3$ occurring for the prism pair is not necessary.

(c) The lengthening of the pulse due to third-order dispersion in the absence of second-order dispersion can be approximated by

$$\frac{\tau_{out}}{\tau_{in}} = \sqrt{1 + \left(\frac{8\sqrt{2}ln(2)}{\tau_{in}^3}\frac{\partial^3\Phi}{\partial\omega^3}\right)^2} \tag{6}$$

Which prism material would you use to minimize the effects of third-order dispersion? **(5 points)**

(d) How much would a $15\ fs$ pulse be stretched within one round-trip in the cavity due to the remaining third-order dispersion? **(5 points)**

In the following we neglect third and higher order dispersion. Assume the average output power is $100\ mW$ and the repetition rate is $100\ MHz$.

(e) Assume that the pulses are soliton like. What is the necessary net intracavity dispersion for generating a $10\ fs$ pulse from this laser? How large is then the normalized dispersion, $D_n = D_2/D_g$? **(5 points)**

(f) How large is the chirp on the steady-state pulse without assuming a soliton-like pulse? **(5 points)**