Universität Hamburg
Physics Department
Ultrafast Optical Physics II
SoSe 2019

Problem Set 4

Issued: May 10, 2019 Due: May 17, 2019

Instruction: Please write your answer to each problem on separate paper sheet. If you are using programming language to do numerical simulations, attach the original code with your answers.

Problem 4.1: the Nonlinear Schrödinger Equation (NSE) and optical soliton (15 points in total)

The Nonlinear Schrödinger Equation is written as follows, (here we assume $D_2 < 0$)

$$\frac{\partial A(z,t)}{\partial z} = jD_2 \frac{\partial^2 A(z,t)}{\partial t^2} - j\delta|A(z,t)|^2 A(z,t)$$

(a) Show by using the following transform

$$\xi = \frac{z}{L_D}, \tau = \frac{t}{\tau_0},$$

$$L_D = \frac{\tau_0^2}{|D_2|}, u = \sqrt{\frac{\delta}{|D_2|}}\,\tau_0 A$$

the NSE can be rewritten in to the normalized form

$$\frac{\partial u(\xi,\tau)}{\partial \xi} = -j\frac{\partial^2 u(\xi,\tau)}{\partial \tau^2} - j|u(\xi,\tau)|^2 u(\xi,\tau).$$ **(5 points)**

(b) Prove that $u(\xi,\tau) = \mathrm{sech}\left(\frac{\tau}{\sqrt{2}}\right) \cdot \exp\left(-j\frac{\xi}{2}\right)$ is the solution to the normalized NSE. **(5 points)**

Hint: $\frac{\cosh(\sqrt{2}\tau)+1}{4} \cdot \mathrm{sech}^3\left(\frac{\tau}{\sqrt{2}}\right) = \frac{1}{2} \cdot \mathrm{sech}\left(\frac{\tau}{\sqrt{2}}\right)$

(c) Ultrafast optical solitons have been generated in optical fiber SMF-28 at $\lambda = 1.55\ \mu\mathrm{m}$. Using pulses from mode-locked lasers, hyperbolic-secant pulses with $\tau_0 = 4\ \mathrm{ps}$ were obtained after 700 m propagation in the optical fiber. The nonlinear index coefficient of the fiber is $n_2 = 3 \times 10^{-20}\mathrm{m}^2/\mathrm{W}$. Calculate the corresponding peak intensity and the corresponding peak power of the soliton. **(5 points)**

Hint: the relationship between fiber dispersion $D(\lambda)$ characterized by fiber communication community and group velocity dispersion β_2 or D_2 can be found at RP-Photonics website, http://www.rp-photonics.com/group_velocity_dispersion.html .

Problem 4.2: The Split-Step Fourier method (15 points in total)

The normalized Nonlinear Schrödinger Equation (NSE) can be numerically solved using the Split-Step Fourier transform. Firstly the NSE can be understood in the following way

$$\frac{\partial u(\xi,\tau)}{\partial \xi} = (\hat{D} + \hat{N})u(\xi,\tau)$$

as the simultaneous action of a dispersion operator $\hat{D} = -j\frac{\partial^2 u(\xi,\tau)}{\partial \tau^2}$ (assume the dispersion is negative), and a nonlinear operator $\hat{N} = -j|u(\xi,\tau)|^2$. If the linear and nonlinear changes for the pulse evolution are small within a short distance of propagation $\Delta\xi$, the solution of the NSE can be symbolically written as

$$u(\Delta\xi,\tau) = e^{(\hat{D}+\hat{N})\Delta\xi}u(0,\tau)$$

and approximated by

$$u(\Delta\xi,\tau) = e^{\frac{1}{2}\hat{D}\Delta\xi}e^{\hat{N}\Delta\xi}e^{\frac{1}{2}\hat{D}\Delta\xi}u(0,\tau).$$

One can show that iterative application of this propagation step only leads to an error of order $\Delta\xi^3$. Since the linear operator can be easily applied in the Fourier domain and the nonlinear operator (self-phase modulation only) in the time domain, one can simulate the NSE over one propagation step $\Delta\xi$ by the following algorithm

$$u(\xi+\Delta\xi,\tau) = F^{-1}[e^{\frac{1}{2}j\Omega^2\Delta\xi}F[e^{-j|u(\xi,\tau)|^2\Delta\xi}F^{-1}[e^{\frac{1}{2}j\Omega^2\Delta\xi}F[u(\xi,\tau)]]]].$$

where $\Omega = \omega - \omega_0$ is the difference between the real frequency and the carrier frequency.

(a) Dispersion effect on pulse evolution. The electric field of an unchirped Gaussian pulse is written as $E(0,t) = \mathrm{A}_0 \cdot \exp(-t^2/2\tau_0^2) \cdot \exp(i\omega_0 t)$, where $\tau_0 = 100\ fs$ and the center wavelength of such pulse locates at 1550 nm. We let the pulse propagate inside optical fiber SMF-28 and we do NOT consider the nonlinear effect. Moreover, we ONLY consider second order dispersion effect. What is the characteristic dispersion length L_D (based on the definition in Problem 2.1)? Plot the intensity distribution of such pulse at 0, $1L_D$, $2L_D$, $3L_D$ and $4L_D$ (You can use normalized scale. Indicate proper axis label, i.e. t/τ_0). Is the pulse positively chirped or negatively chirped after propagation? **(5 points)**

(b) Nonlinear effect on the evolution of the pulse spectrum. Ignore the dispersion effect, the nonlinear pulse evolution can be written as $u(\Delta\xi,\tau) = e^{\hat{N}\Delta\xi}u(0,\tau)$, where $\hat{N} = -j|u(\xi,\tau)|^2$ and we have changed the NSE to its normalized form. Plot the spectrum $S(\Omega) = |\tilde{u}(\Omega)|^2$

as the pulse accumulates different nonlinear phase $\phi_{NL} = |u(\xi,0)|^2 = 0, \frac{\pi}{2}, \pi, \frac{3}{2}\pi$ and 2π assuming $u(0,\tau) = \exp(-\tau^2)$. **(5 points)**

(c) Write a program and simulate the normalized NSE for the following initial pulse

$$u(0,\tau) = N \cdot \mathrm{sech}\,(\tau/\sqrt{2})$$

for N = 1,2 and 3. Make use of the Fast Fourier Transform (FFT) and use at least 1024 points. Plot the pulse shape (in the time domain) and corresponding amplitude spectra (in the frequency domain) as a function of propagation distance. **(5 points)**

Problem 4.3: Pulse compression (15 points in total)

A pulse propagating through an optical fiber becomes frequency modulated due to self-phase modulation (SPM). The pulse is chirped and can be represented by

$$A(t) = e^{-\Gamma_1 t^2} \cdot e^{-i\Gamma_2 t^2} \cdot e^{i\omega_0 t} = e^{-\Gamma t^2} \cdot e^{i\omega_0 t}$$

with $\Gamma = \Gamma_1 + i\Gamma_2$, where Γ_1 determines the initial pulse width and Γ_2 determines the corresponding chirp due to SPM (in the following context, we assume Γ_2 is positive). The pulse can be compressed by the addition of dispersion generated by a grating pair or prism pair.

$$A'(\omega) = A(\omega) \cdot e^{iD_2(\omega-\omega_0)^2}.$$

(a) We assume the input pulse has a Gaussian shape (i.e. $\Gamma_1 = \frac{1}{2\tau^2}$) and we take the action of self-phase modulation into account by expanding the nonlinear phase shift up to second order in time, so that the first equation in this problem can be applied. What would be the new value for Γ' of the new pulse in terms of Γ_1, Γ_2 and D_2? **(5 points)**

(b) If we reduce the amount of the nonlinear phase shift introduced by self-phase modulation, we can compress the initial pulse. What is the peak nonlinear phase shift, $\phi_0 = -\Gamma_2\tau^2$, necessary to shorten the pulse by a factor of 2 (in terms of FWHM of the intensity)? **(5 points)**

(c) What is the necessary dispersion to compress the pulse? Express the final answer in terms of τ. Can you find the reason why making negative dispersion with an artificial structure, such as prism-pair, grating-pair or chirped mirror (which will be shown later in this course), is important in pulse compression? **(5 points)**

Problem 4.4: Diffraction Grating Pair (30 points in total)

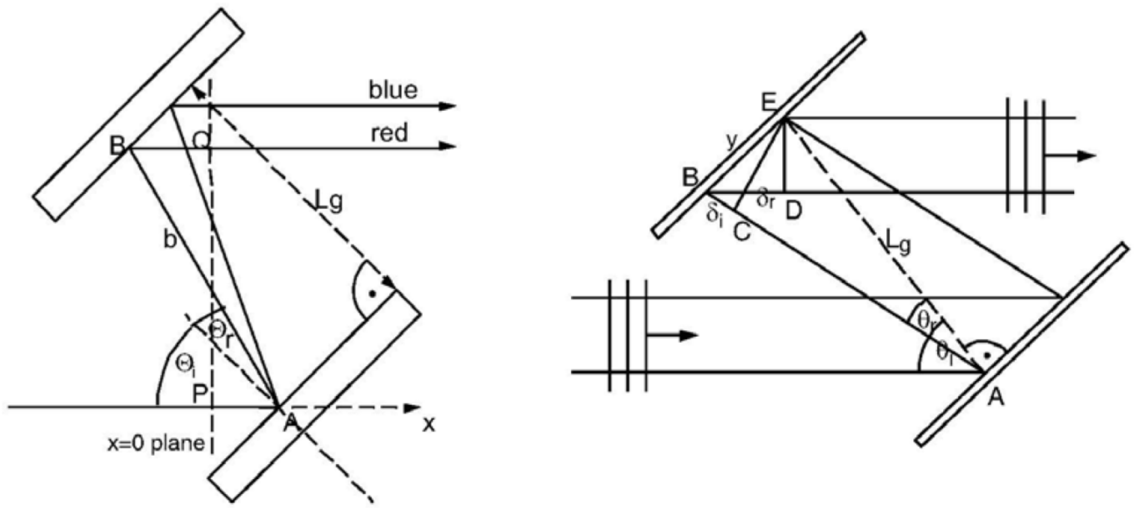

Figure 1: (Left): Schematic 1, (Right): Schematic 2.

(a) Derive from simple geometric principles a condition for the angle θ_r of maximum diffraction, as a function of wavelength λ, the incident angle θ_i, the grating period Λ, and the diffraction order m. Based on your answer, for which diffraction orders can the grating pair be used for generation of negative dispersion? **(4 points)**

(b) In Fig. 1 the x-plane is defined by beginning and end of the optical path $p_{opt} = \overline{PAQB}$ of an infinitely thin laser beam that is normal to the plane. Calculate this optical path as a function of λ, θ_r, θ_i, the diffraction order m and the grating distance L_g. What is the phase delay Φ_{opt} corresponding to this optical path? **(4 points)**

(c) Considering the finite beam cross-section, as indicated in schematic 2, an additional phase shift Φ_c must be added to account for the phase-matching provided by the grating. Schematic 2 shows that this adjustment is necessary, because the planes (CE) and (DE) represent planes with constant phase, and the grating must compensate the extra paths $\delta_i = \overline{CB}$ and $\delta_r = \overline{BD}$ travelled by the lower part of the beam with an appropriate phase shift. Deduce, again from geometric considerations, similar to part (a), this phase shift Φ_c as a result of phase matching! What is the total phase shift $\Phi = \Phi_{opt} + \Phi_c$? That is, calculate the phase shift from the total path difference. (It is essential here to observe the correct choice of signs!) (For more details consult the book by H. A. Haus: Waves and Fields in Optoelectronics, Chapter 2.6). **(6 points)**

(d) Calculate the group delay $T_g = \frac{\partial \Phi}{\partial \omega}$ and the second-order dispersion $D_2 = \frac{\partial T_g}{\partial \omega}$. What is the dispersion relation for a four-grating compressor? For what reason is a second grating pair used? **(8 points)**

(e) Example: Laser pulses with $200\, fs$ duration at a center wavelength of $800\, nm$ shall be delivered to a medical imaging system through a $1\, m$ long quartz fiber. To avoid broadening of the pulse the second order dispersion of the fiber link shall be compensated with grating pairs. Two grating pairs with 600 grooves per mm are given and shall be used at an angle of incidence of 50 degrees. What grating distance is necessary to compensate the second-order dispersion of the fiber when the grating is optimized for the diffraction order $m = -1$ (Blazed grating)? **(8 points)**