Ultrafast Optical Physics II (SoSe 2019)
Lecture 10, June 21

1) Second-order nonlinear optical effects

2) Optical Parametric Amplifiers and Oscillators

- Optical Parametric Generation (OPG)
- Nonlinear Optical Susceptibilities
- Continuous Wave OPA
- Theory of Optical Parametric Amplification
- Phase Matching
- Quasi Phase Matching
- Ultrashort Pulse Parametric Amplifiers (OPA)
- Optical Parametric Amplifier Designs
- Ultrabroadband Optical Parametric Amplifiers
- Using Noncollinear Phase Matching
- Optical Parametric Chirped Pulse Amplification (OPCPA)

[5] Largely follows the review paper of Cerullo et al., "Ultrafast Optical Parametric Amplifiers" Rev. Sci. Instr. 74, pp 1-17 (2003)

Second-order nonlinear optical effects

Wave equation: $$\left(\Delta - \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\right)\vec{E} = \underline{\mu_0 \frac{\partial^2}{\partial t^2}\vec{P}}.$$ **Source term describing light-matter interaction**

$$P = \varepsilon_0[\chi^{(1)}E + \chi^{(2)}E^2]$$

$\chi^{(2)} \longrightarrow$ 2nd order susceptibility

Example: Pockels Effect

$$E = A_0 + A_1 \cos(\omega t)$$ $\longrightarrow$ Input electric field

The total polarization at frequency ω is:

$$P^{(\omega)} = \varepsilon_0[\chi^{(1)} + 2\chi^{(2)}A_0]A_1 \cos(\omega t)$$

New refractive index:

$$n = \sqrt{1 + \chi^{(1)} + 2\chi^{(2)}A_0}$$

The Pockels effect is used to make optical switch (or modulator) using an electrical field to control the interaction between an optical crystal and the optical field propagating in it.

Mixing of two sine waves

$$E = \frac{1}{2}\left[\tilde{E}(\omega_1)e^{j\omega_1 t} + \tilde{E}(\omega_2)e^{j\omega_2 t} + \text{c.c.}\right] \longrightarrow$$ Input electric field

$$P_{\text{NL}}^{(2)} = \frac{\varepsilon_0}{4}\{\underline{\chi^{(2)}(2\omega_1 : \omega_1, \omega_1)\tilde{E}^2(\omega_1)e^{j2\omega_1 t}} + \underline{\chi^{(2)}(2\omega_2 : \omega_2, \omega_2)\tilde{E}^2(\omega_2)e^{j2\omega_2 t}}$$

Second-harmonic generation (SHG)

$$+\underline{2\chi^{(2)}(\omega_1 + \omega_2 : \omega_1, \omega_2)\tilde{E}(\omega_1)\tilde{E}(\omega_2)e^{j(\omega_1+\omega_2)t}}$$

Sum-frequency generation (SFG)

$$+\underline{2\chi^{(2)}(\omega_1 - \omega_2 : \omega_1, -\omega_2)\tilde{E}(\omega_1)\tilde{E}^*(\omega_2)e^{j(\omega_1-\omega_2)t}}$$

Difference-frequency generation (DFG)

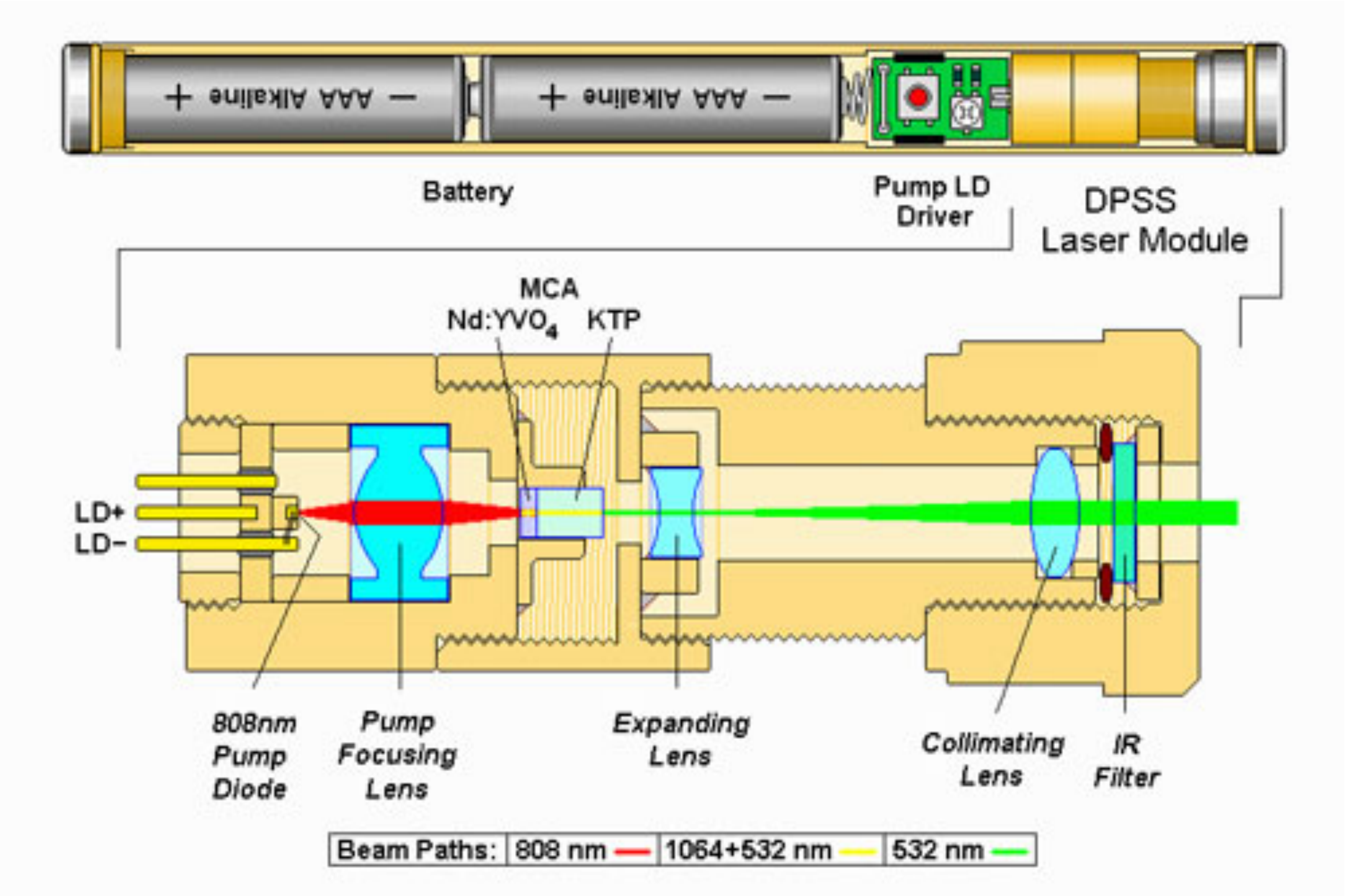
$$+\underline{\chi^{(2)}(0 : \omega_1, -\omega_1)\tilde{E}(\omega_1)\tilde{E}^*(\omega_1) + \chi^{(2)}(0 : \omega_2, -\omega_2)\tilde{E}(\omega_2)\tilde{E}^*(\omega_2)}\}$$

Optical rectification

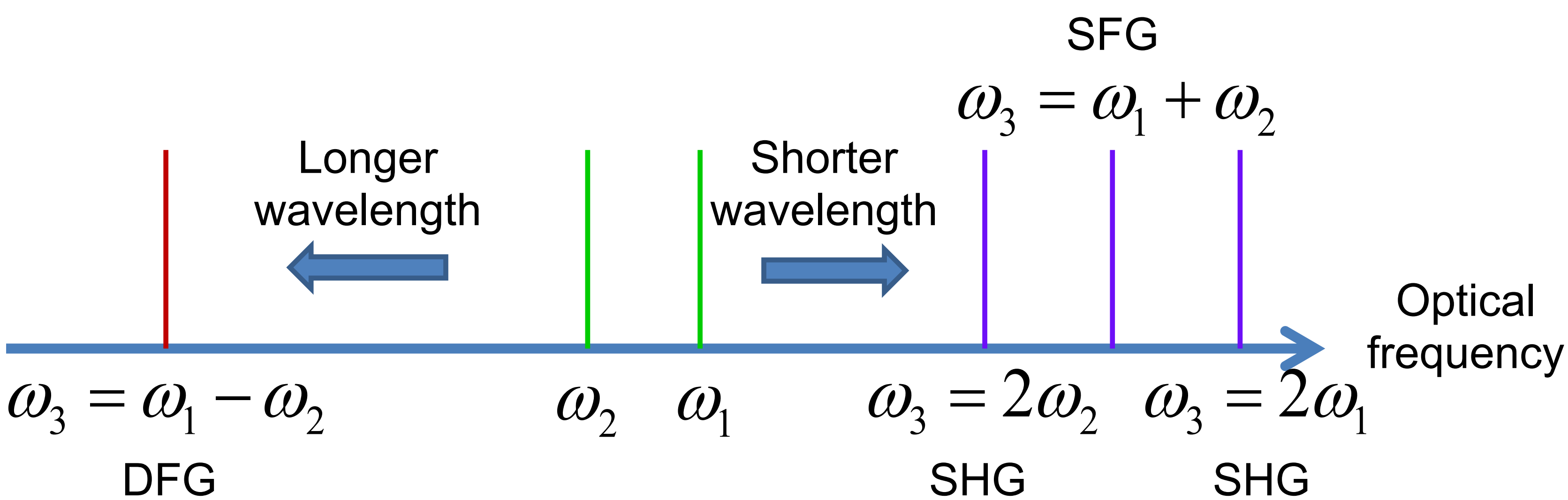
$\chi^{(2)}$ may be both complex and frequency dependent.

$\chi^{(2)}(\omega_a + \omega_b : \omega_a, \omega_b)$ keeps track of the input and output frequencies involved in a particular interaction.

SHG in daily life: green laser pointer

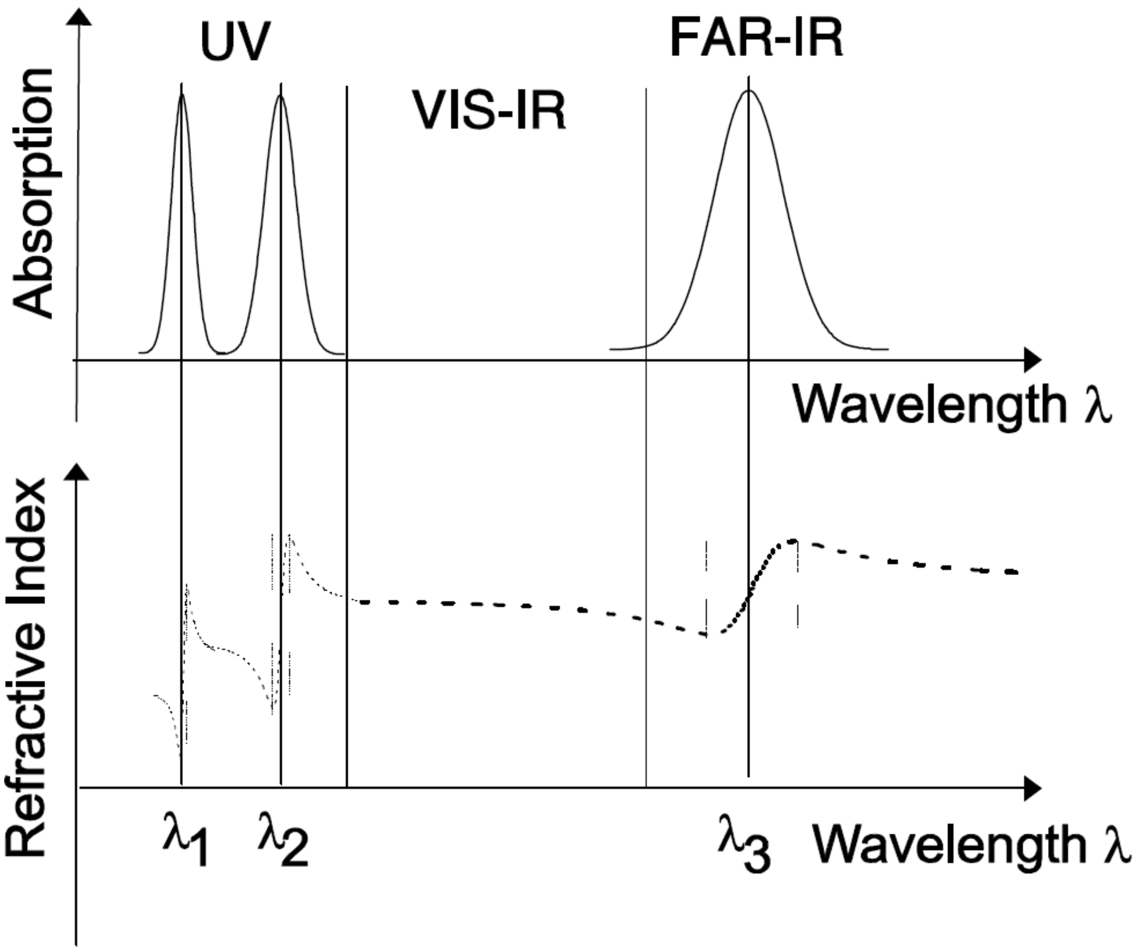


Wavelength conversion using 2nd order nonlinear optics



	Energy conservation	Momentum conservation	Phase matching condition
SHG	$\omega_3 = 2\omega_1$	$k_3 = 2k_1$	$\frac{\omega_3}{c} n_3 = 2\frac{\omega_1}{c} n_1 \quad n_3 = n_1$
SFG	$\omega_3 = \omega_1 + \omega_2$	$k_3 = k_1 + k_2$	$\omega_3 n_3 = \omega_1 n_1 + \omega_2 n_2$
DFG	$\omega_3 = \omega_1 - \omega_2$	$k_3 = k_1 - k_2$	$\omega_3 n_3 = \omega_1 n_1 - \omega_2 n_2$

How to achieve phase matching?

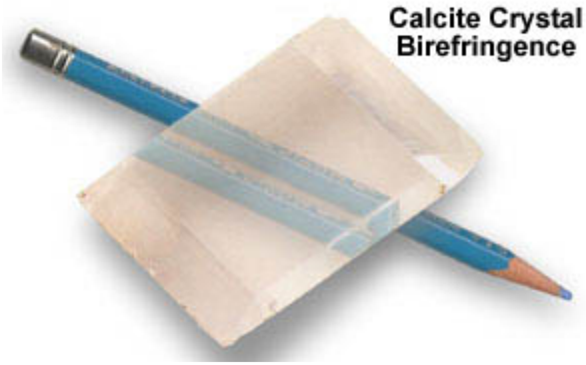


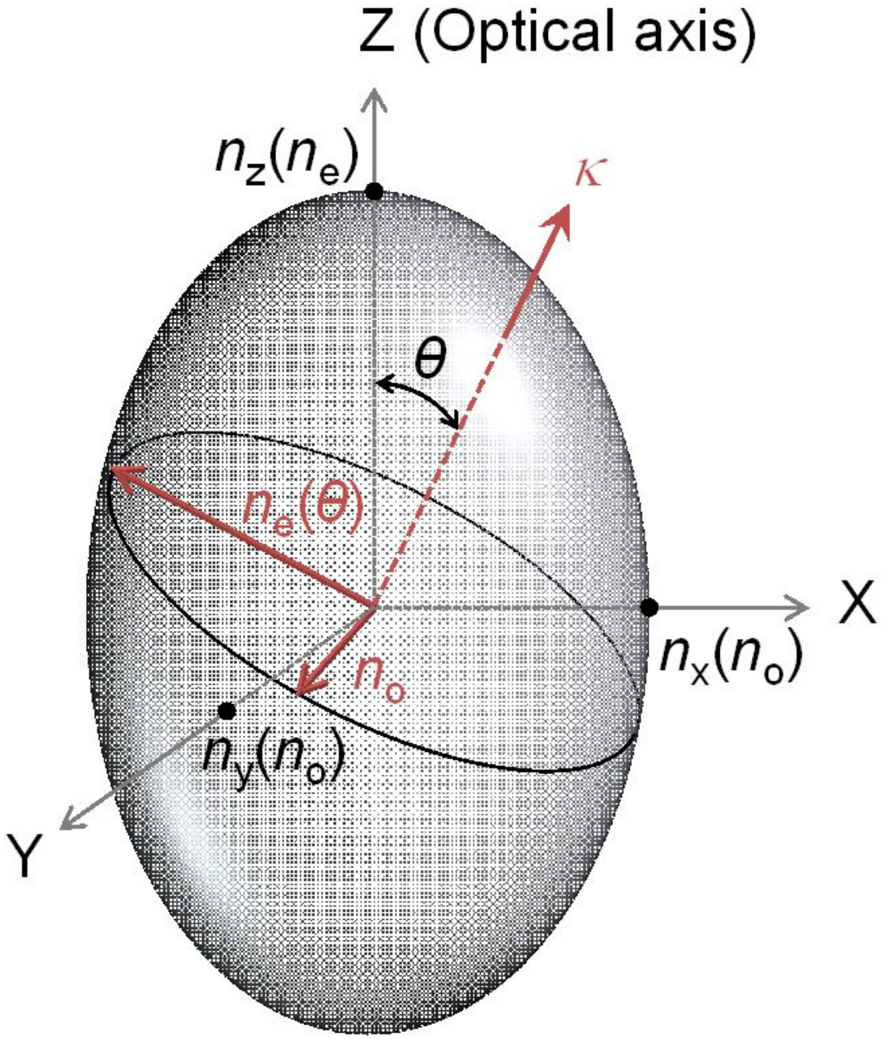
For the frequency (wavelength) far away from absorption resonance, refractive index increases with increasing frequency, which leads to

$$\omega_3 n_3 > \omega_1 n_1 + \omega_2 n_2$$

Dispersion prevents phase matching.

Phase matching in birefringent media





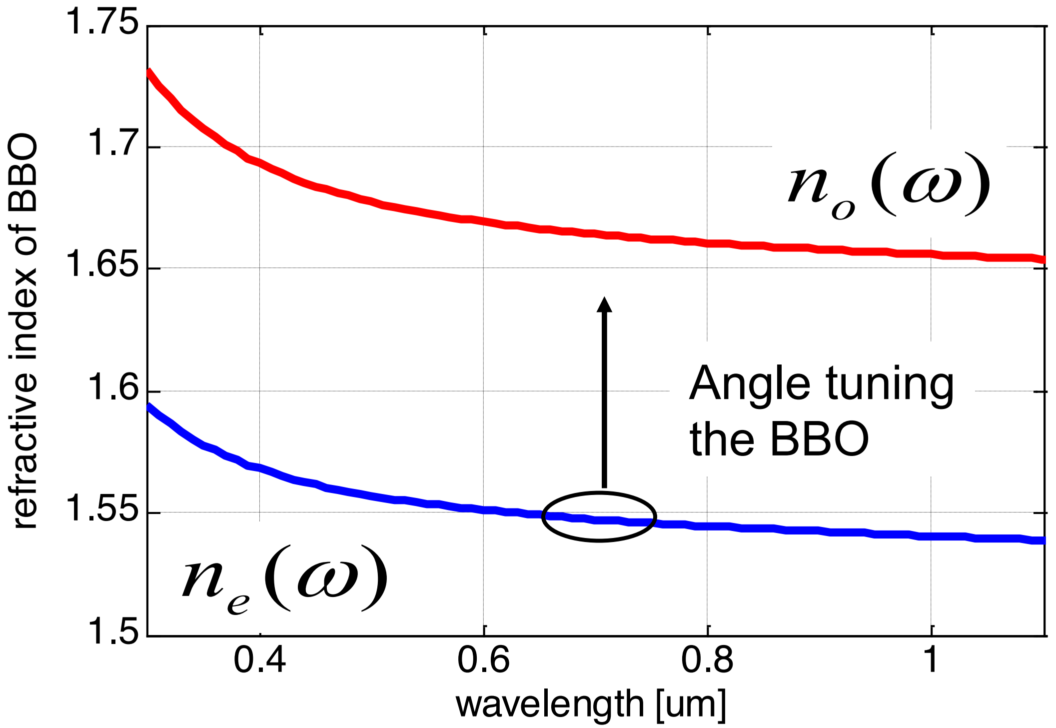
Birefringent materials have different refractive indices for different polarizations. **Ordinary** (o-wave) light has its polarization perpendicular to the optical axis and its refractive index, n_o , does not depend on propagation direction, θ. **Extraordinary** (e-wave) light has its polarization in the plane containing optical axis and propagation vector, and its refractive index, n_o , depends on propagation direction, θ.

Phase matching in birefringent media

- In an isotropic medium, normal dispersion always results in

$$n(\omega) < n(2\omega)$$

- In birefringent uniaxial crystal there are ordinary wave and extraordinary wave.



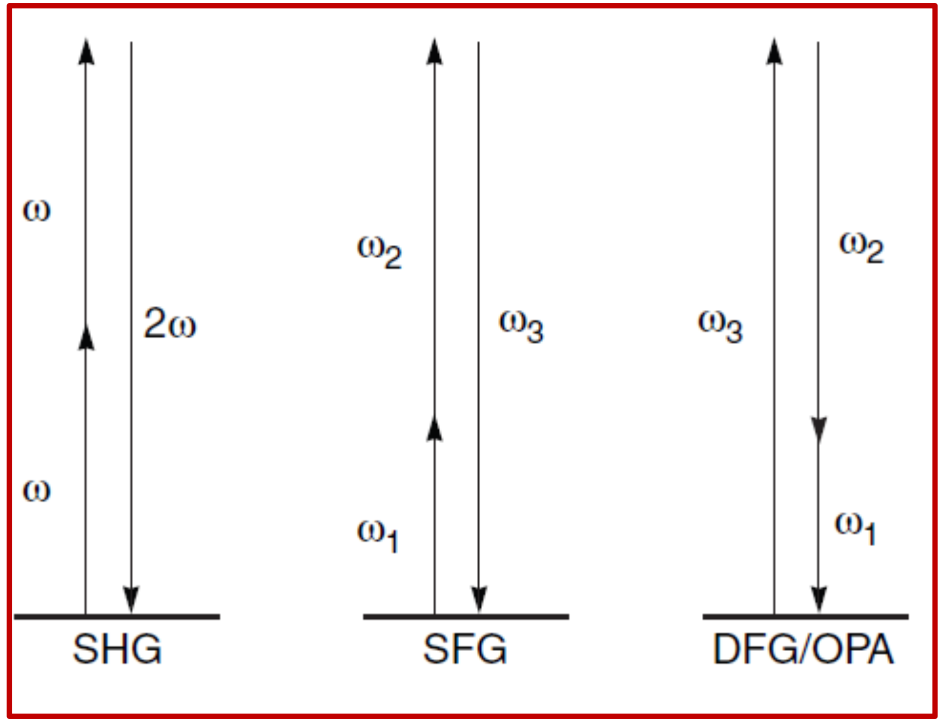
BBO crystal is a typical negative uniaxial crystal with $n_o > n_e$. If red light is set as the ordinary beam and the SHG the extraordinary one, angle tuning the BBO crystal permits achieving phase matching condition.

Phase matching: type I Vs. type II

In general, second-order nonlinear effects involve three waves with frequencies linked by the equation

$$\omega_1 + \omega_2 = \omega_3$$

Here ω_3 is the highest frequency of the three.



<u>Type I phase matching:</u>

ω_1 wave and ω_2 wave have the same polarization; that is, they are both ordinary waves or extraordinary waves: $o + o \rightarrow e$ or $e + e \rightarrow o$

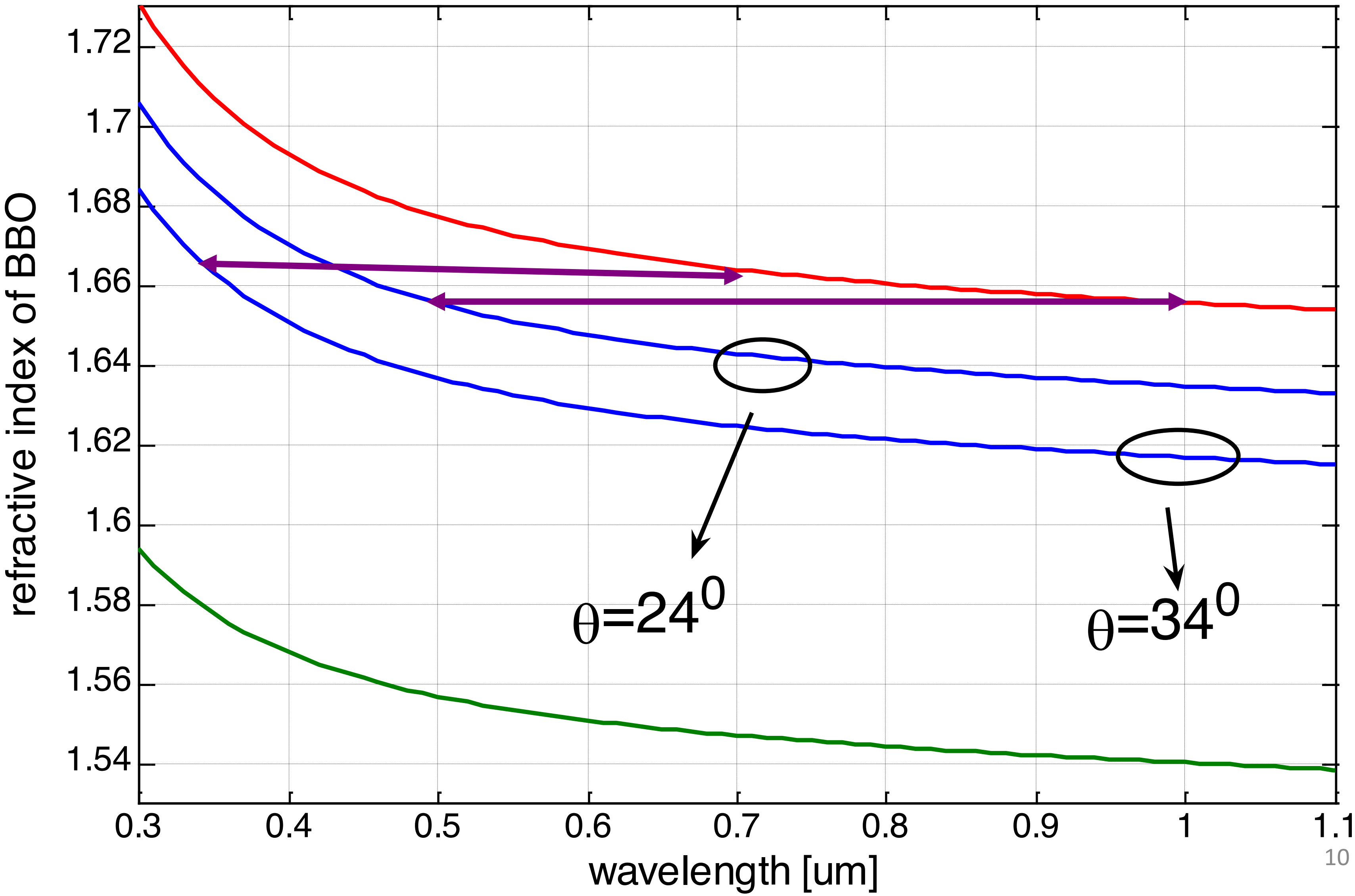
<u>Type II phase matching:</u>

ω_1 wave and ω_2 wave have different polarization: $o + e \rightarrow e$

$$o + e \rightarrow o \qquad e + o \rightarrow e \qquad e + o \rightarrow o$$

Type I phase matching SHG

$$n_e(34^0, 350nm) = n_o(700nm) \qquad n_e(24^0, 500nm) = n_o(1000nm)$$



Linear susceptibility is a matrix for optically anisotropic media

$$P = \varepsilon_0 \chi^{(1)} E \quad \begin{cases} P_x = \varepsilon_0 \chi^{(1)} E_x \\ P_y = \varepsilon_0 \chi^{(1)} E_y \\ P_z = \varepsilon_0 \chi^{(1)} E_z \end{cases}$$

$\Rightarrow$ Only true for optically isotropic media

For optically anisotropic media, linear susceptibility is a 3X3 matrix (a second-rank tensor):

$$\left.\begin{aligned} P_x &= \varepsilon_0 [\chi_{xx}^{(1)} E_x + \chi_{xy}^{(1)} E_y + \chi_{xz}^{(1)} E_z] \\ P_y &= \varepsilon_0 [\chi_{yx}^{(1)} E_x + \chi_{yy}^{(1)} E_y + \chi_{yz}^{(1)} E_z] \\ P_z &= \varepsilon_0 [\chi_{zx}^{(1)} E_x + \chi_{zy}^{(1)} E_y + \chi_{zz}^{(1)} E_z] \end{aligned}\right\} \quad P_i = \varepsilon_0 \sum_j \chi_{ij}^{(1)} E_j$$

$$(i, j) = (x, y, z)$$

2^{nd}-order susceptibility is a 3^{rd}-rank tensor

Take sum frequency generation(SFG) $\omega_1 + \omega_2 = \omega_3$ as an example:

$$P_x(\omega_3) = \varepsilon_0[\chi^{(2)}_{xxx}E_x(\omega_1)E_x(\omega_2) + \chi^{(2)}_{xxy}E_x(\omega_1)E_y(\omega_2) + \chi^{(2)}_{xxz}E_x(\omega_1)E_z(\omega_2)$$

$$+ \chi^{(2)}_{xyx}E_y(\omega_1)E_x(\omega_2) + \chi^{(2)}_{xyy}E_y(\omega_1)E_y(\omega_2) + \chi^{(2)}_{xyz}E_y(\omega_1)E_z(\omega_2)$$

$$+ \chi^{(2)}_{xzx}E_z(\omega_1)E_x(\omega_2) + \chi^{(2)}_{xzy}E_z(\omega_1)E_y(\omega_2) + \chi^{(2)}_{xzz}E_z(\omega_1)E_z(\omega_2)]$$

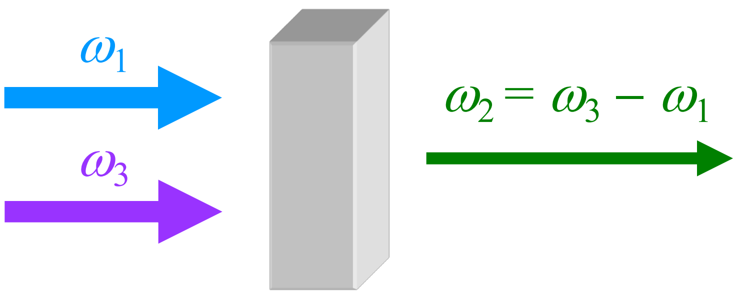
We can represent the lengthy expression using tensor notation:

$$P_i^{(2)}(\omega_3) = \varepsilon_0 \sum_{j,k} \chi^{(2)}_{ijk}(\omega_3 : \omega_1, \omega_2)E_j(\omega_1)E_k(\omega_2) \quad (i, j, k) = (x, y, z)$$

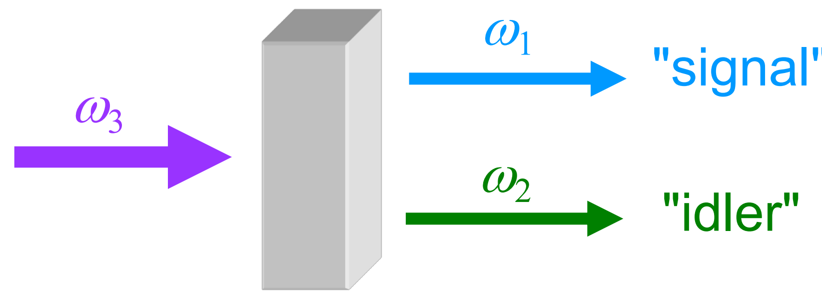
$\chi^{(2)}_{ijk}(\omega_3 : \omega_1, \omega_2)$ is a 3^{rd}-order tensor with 27 (3X3X3) elements. According to the crystal symmetry, most of them are zeros.

Difference-frequency generation: optical parametric generation, amplification, oscillation

Difference-frequency generation takes many useful forms.

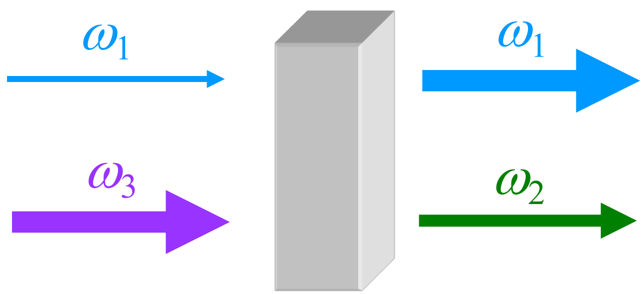


Parametric Down-Conversion
(Difference-frequency generation)

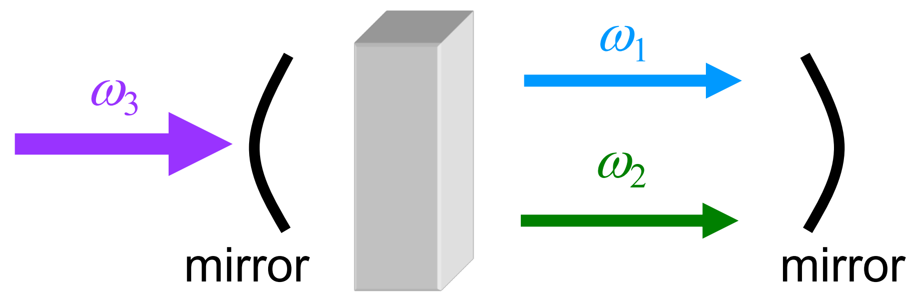


Optical Parametric Generation (OPG)

By convention:
$\omega_{signal} > \omega_{idler}$



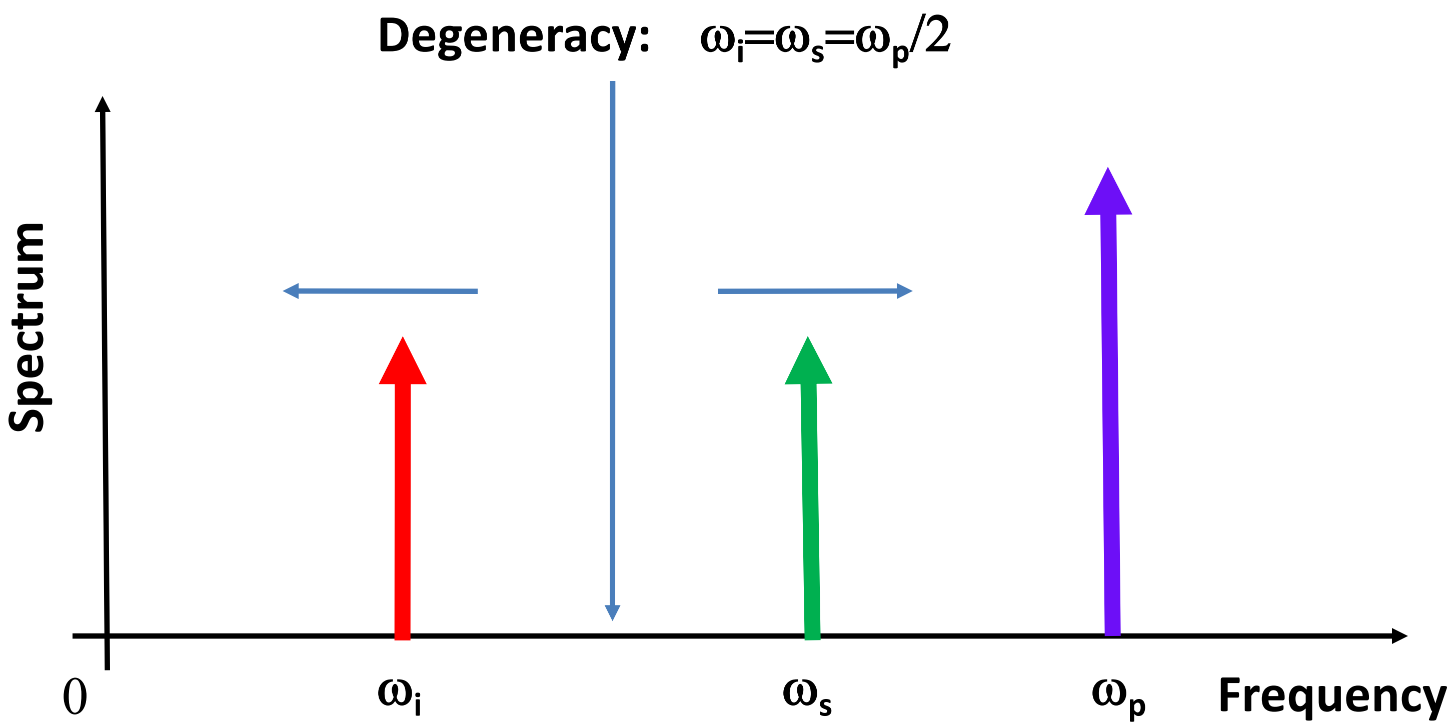
Optical Parametric Amplification (OPA)



Optical Parametric Oscillation (OPO)

12. 8 Optical Parametric Amplifiers and Oscillators

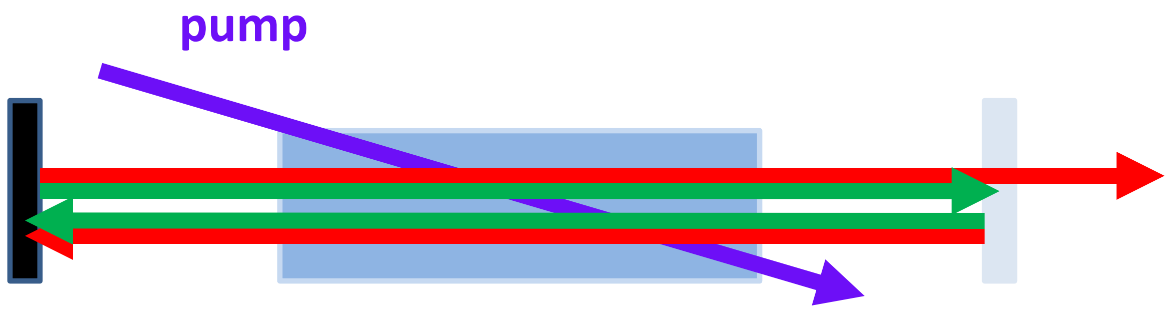
12.8.1 Optical Parametric Generation (OPG)



Energy Conservation: $$\hbar\omega_p = \hbar\omega_s + \hbar\omega_i.$$

Momentum Conservation: $$\hbar\vec{k}_p = \hbar\vec{k}_s + \hbar\vec{k}_i$$

Optical Parametric Oscillator (OPO)



Double resonant: Signal and idler resonant

Single resonant: Only Signal resonant

Advantage: Widely tunable, both signal and idler can be used!

For OPO to operate, less gain is necessary in contrast to an OPA.

Nonlinear Optical Susceptibilities

Total field: Pump, signal and idler:

$$\vec{E}(\vec{r},t) = \sum_{\omega_a>0} \sum_{i=1}^{3} \frac{1}{2} \left\{ \hat{E}_i(\omega_a) e^{j(\omega_a t - \vec{k}_a \vec{r})} + c.c. \right\} \vec{e}_i.$$

Drives polarization in medium:

$$\vec{P}(\vec{r},t) = \sum_n \vec{P}^{(n)}(\vec{r},t)$$

Polarization can be expanded in power series of the electric field:

$$\vec{P}^{(n)}(\vec{r},t) = \sum_{\omega_b>0} \sum_{i=1}^{3} \frac{1}{2} \left\{ P_i^{(n)}(\omega_b) e^{j(\omega_b t - \vec{k}_b' \vec{r})} + c.c. \right\} \vec{e}_i.$$

Defines susceptibility tensor:

$$P_i^{(n)}(\omega_b) = \frac{\varepsilon_0}{2^{m-1}} \sum_P \sum_{j \ldots k} \chi_{ij \ldots k}^{(n)}(\omega_b : \omega_1, \ldots., \omega_n) E_j(\omega_1) \cdots E_k(\omega_n)$$

$$\omega_b = \sum_{i=1}^{n} \omega_i \text{ and } \mathbf{k}_b' = \sum_{i=1}^{n} \mathbf{k}_i$$

Special Cases

$$\begin{aligned}\hat{P}_i^{(2)}(\omega_3) &= \varepsilon_0 \sum_{jk} \chi_{ijk}^{(2)}(\omega_3 : \omega_1, -\omega_2)\hat{E}_j(\omega_1)\hat{E}_k^*(\omega_2),\\ \omega_3 &= \omega_1 - \omega_2 \text{ und } \mathbf{k}_3' = \mathbf{k}_1 - \mathbf{k}_2.\end{aligned}$$

($\longrightarrow$ Difference Frequency Generation (DFG))

$$\begin{aligned}\hat{P}_i^{(2)}(\omega_2) &= \varepsilon_0 \sum_{jk} \chi_{ijk}^{(2)}(\omega_2 : \omega_3, -\omega_1)\hat{E}_j(\omega_3)\hat{E}_k^*(\omega_1),\\ \omega_2 &= \omega_3 - \omega_1 \text{ und } \mathbf{k}_2' = \mathbf{k}_3 - \mathbf{k}_1.\end{aligned}$$

($\longrightarrow$ Parametric Generation (OPG))

$$\begin{aligned}\hat{P}_i^{(3)}(\omega_4) &= \frac{6\varepsilon_0}{4} \sum_{jkl} \chi_{ijkl}^{(3)}(\omega_4 : \omega_1, \omega_2, -\omega_3)\hat{E}_j(\omega_1)\hat{E}_k(\omega_2)\hat{E}_l^*(\omega_3)\\ \omega_4 &= \omega_1 + \omega_2 - \omega_3 \text{ und } \mathbf{k}_4' = \mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}_3.\end{aligned}$$

($\longrightarrow$ Four Wave Mixing (FWM))

Continuous Wave OPA

Wave equation (2.7) : $$\left(\Delta - \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\right)\vec{E} = \mu_0 \frac{\partial^2}{\partial t^2}\vec{P}$$

Include linear and second order terms:

$$\left(\Delta - \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\right)\vec{E} = \mu_0 \frac{\partial^2}{\partial t^2}\left(\vec{P}^{(l)}(\vec{r},t) + \vec{P}^{(2)}(\vec{r},t)\right)$$

Changes group and phase velocities of waves

$k(\omega)$

Nonlinear interaction of waves

z-propagation only:

$$\vec{E}_{p,s,i}(z,t) = \mathrm{Re}\left\{E_{p,s,i}(z)\ e^{j(\omega_{p,s,i}t - k_{p,s,i}\ z)}\vec{e}_{p,s,i}\right\}$$

Wave amplitudes

$$\vec{P}^{(2)}_{p,s,i}(z,t) = \mathrm{Re}\left\{ P^{(2)}_{p,s,i}(z)\ e^{j\left(\omega_{p,s,i}t - k'_{p,s,i}z\right)} \vec{e}_{p,s,i} \right\}$$

Separate into three equations for each frequency component:

Slowly varying amplitude approximation:

$$d^2_{p,s,i}E(z)\ /dz^2 << \ 2k\ dE_{p,s,i}(z)\ /dz,$$

$$\frac{\partial E_{p,s,i}(z)}{\partial z} = -\frac{jc_0^2\omega_{p,s,i}}{2n(\omega_{p,s,i})} P^{(2)}_{p,s,i}(z)\ e^{-j\left(k'_{p,s,i} - k_{p,s,i}\right)z}$$

Introduce phase mismatch: $\Delta k = k(\omega_p) - k(\omega_s) - k(\omega_i)$

and eff. nonlinearity and coupling coefficients:

$$d_{eff} = \frac{1}{2}\chi^{(2)}_{ijk}(\omega_p : \omega_s, \omega_i), \quad \kappa_{p,s,i} = \omega_{p,s,i}\ d_{eff}/(n_{p,s,i}c_0)$$

Coupled wave equations:

$$\begin{aligned}\frac{\partial E_p(z)}{\partial z} &= -j\kappa_p\ E_s(z)E_i(z)\ e^{j\Delta kz}\ ,\\ \frac{\partial E_s(z)}{\partial z} &= -j\kappa_s\ E_p(z)E_i^*(z)\ e^{-j\Delta kz},\\ \frac{\partial E_i(z)}{\partial z} &= -j\kappa_i\ E_p(z)E_s^*(z)\ e^{-j\Delta kz}.\end{aligned}$$

X $n_{p,s,i}c_0\varepsilon_0 E^*_{p,s,i}/2$

Intensity of waves: $I_{p,s,i} = \frac{n_{p,s,i}}{2Z_{F_0}}\left|E_{p,s,i}\right|^2$

Manley-Rowe Relations: $-\frac{1}{\omega_p}\frac{dI_p}{dz} = \frac{1}{\omega_s}\frac{dI_s}{dz} = \frac{1}{\omega_i}\frac{dI_i}{dz}$

Theory of Optical Parametric Amplification

Undepleted pump approximation: $E_p = const.$

$$\frac{\partial E_s(z)}{\partial z} = -j\kappa_s\ E_p E_i^*(z)\ e^{-j\Delta k z},$$
$$\frac{\partial E_i(z)}{\partial z} = -j\kappa_i\ E_p E_s^*(z)\ e^{-j\Delta k z}.$$

with:

$$E_s(z=0) = E_s(0) \qquad E_i(z=0) = 0$$

$$E_s(z)\tilde{\ }E_s(0)\ e^{gz-j\Delta kz/2} \text{ and } E_i(z)\tilde{\ }E_i(0)\ e^{gz-j\Delta kz/2}$$

$$\longrightarrow \quad \begin{vmatrix} g - j\frac{\Delta k}{2} & j\kappa_s\ E_p \\ j\kappa_i\ E_p^* & g + j\frac{\Delta k}{2} \end{vmatrix} = 0$$

$$g = \sqrt{\Gamma^2 - \left(\frac{\Delta k}{2}\right)^2}, \text{ with } \Gamma = \sqrt{\kappa_i\ \kappa_s\ \left|E_p\right|^2}.$$

Gain

Max. gain, when phase matched

Maximum Gain

$$\boxed{\Gamma^2 = \frac{\omega_s\omega_i}{n_s n_i c_0^2} d_{eff}^2 \ |E_p|^2 = \frac{2Z_{F0}\omega_s\omega_i}{n_p n_s n_i c_0^2} d_{eff}^2 \ \cancel{|I_p|^2}}$$

$$\boxed{FOM = \frac{d_{eff}}{\sqrt{\lambda_s\lambda_i n_p n_s n_i}}}$$

General solutions:

$$E_s(z) = \{E_s(0)\cosh gz + B\sinh gz\}\, e^{-j\Delta kz/2} \qquad B = -j\frac{\Delta k}{2g}E_s(0) - j\frac{\kappa_1}{g}E_p E_i^*(0)$$

$$E_i(z) = \{E_i(0)\cosh gz + D\sinh gz\}\, e^{-j\Delta kz/2} \qquad D = -j\frac{\Delta k}{2g}E_i(0) - j\frac{\kappa_2}{g}E_p^* E_s^*(0)$$

Here:

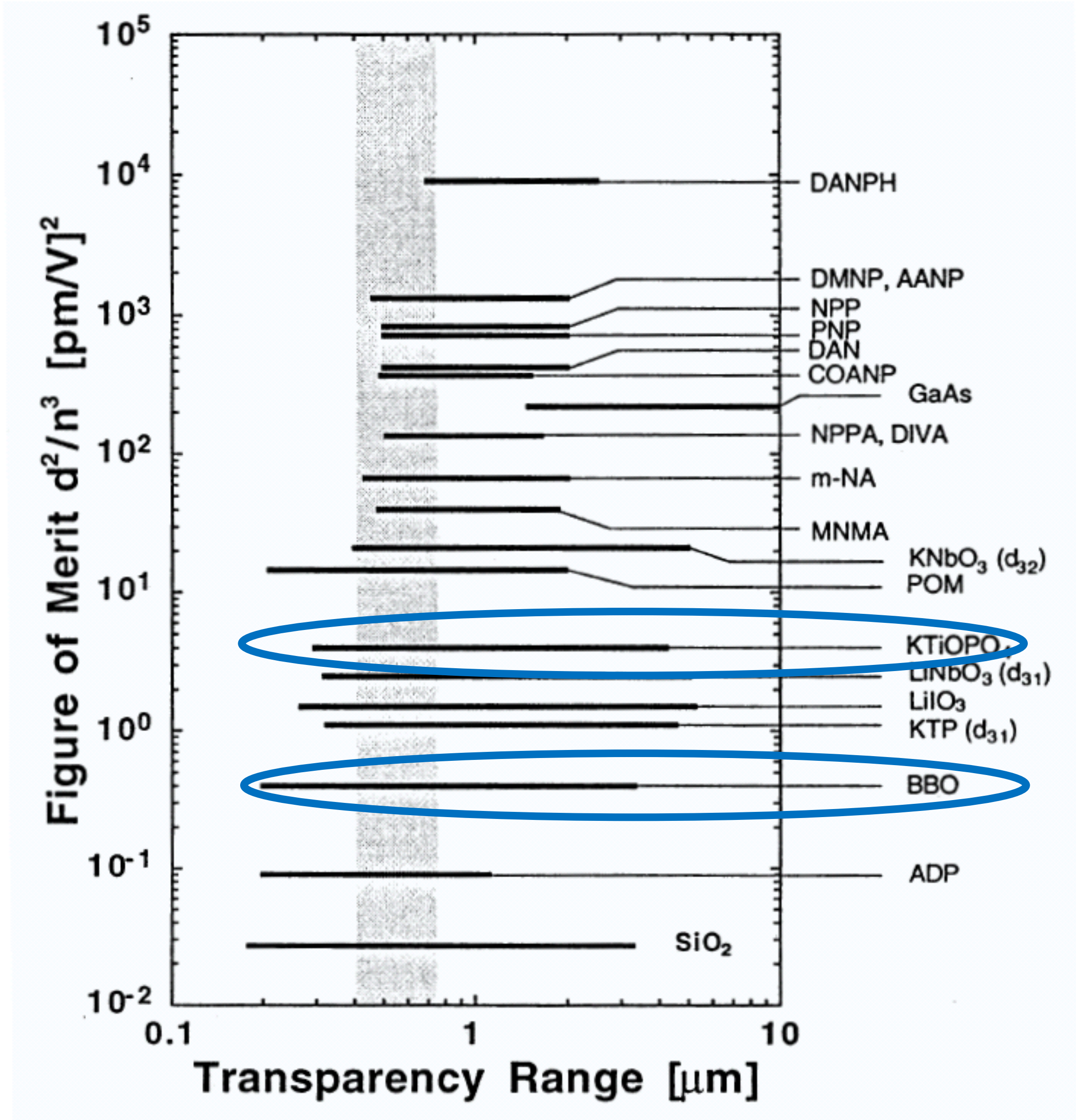
$$\begin{aligned} I_s(L) &= I_s(0)\left[1 + \frac{\Gamma^2}{g^2}\sinh^2 gL\right] \\ I_i(L) &= I_s(0)\,\frac{\omega_i}{\omega_s}\frac{\Gamma^2}{g^2}\sinh^2 gL. \end{aligned}$$

For large gain: $\Gamma L >> 1$

$$\begin{aligned} I_s(L) &= \frac{1}{4}I_s(0)\ e^{2\Gamma L}, \\ I_i(L) &= \frac{1}{4}I_s(0)\ \frac{\omega_i}{\omega_s}\ e^{2\Gamma L} \end{aligned} \longrightarrow G = \frac{I_s(L)}{I_s(0)} = \frac{1}{4}\ e^{2\Gamma L}$$

Figure of merit:

$$\boxed{FOM = \frac{d_{eff}}{\sqrt{\lambda_s \lambda_i n_p n_s n_i}}}$$



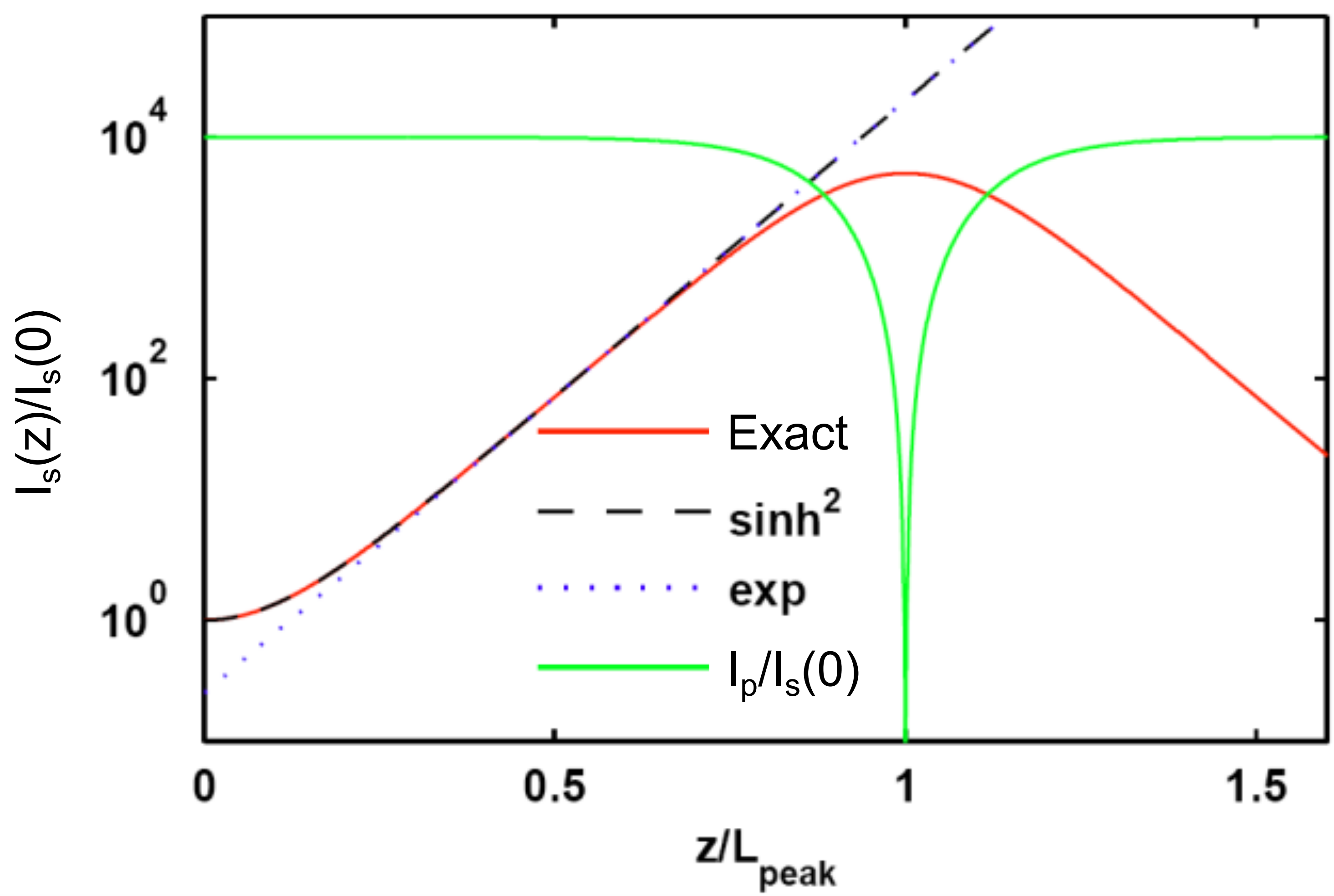


Fig. Exact solution for signal gain, plotted together with hyperbolic secant and exponential function solutions, approximate solutions derived by assuming the pump is undepleted. The exact solution for the pump intensity is also shown.

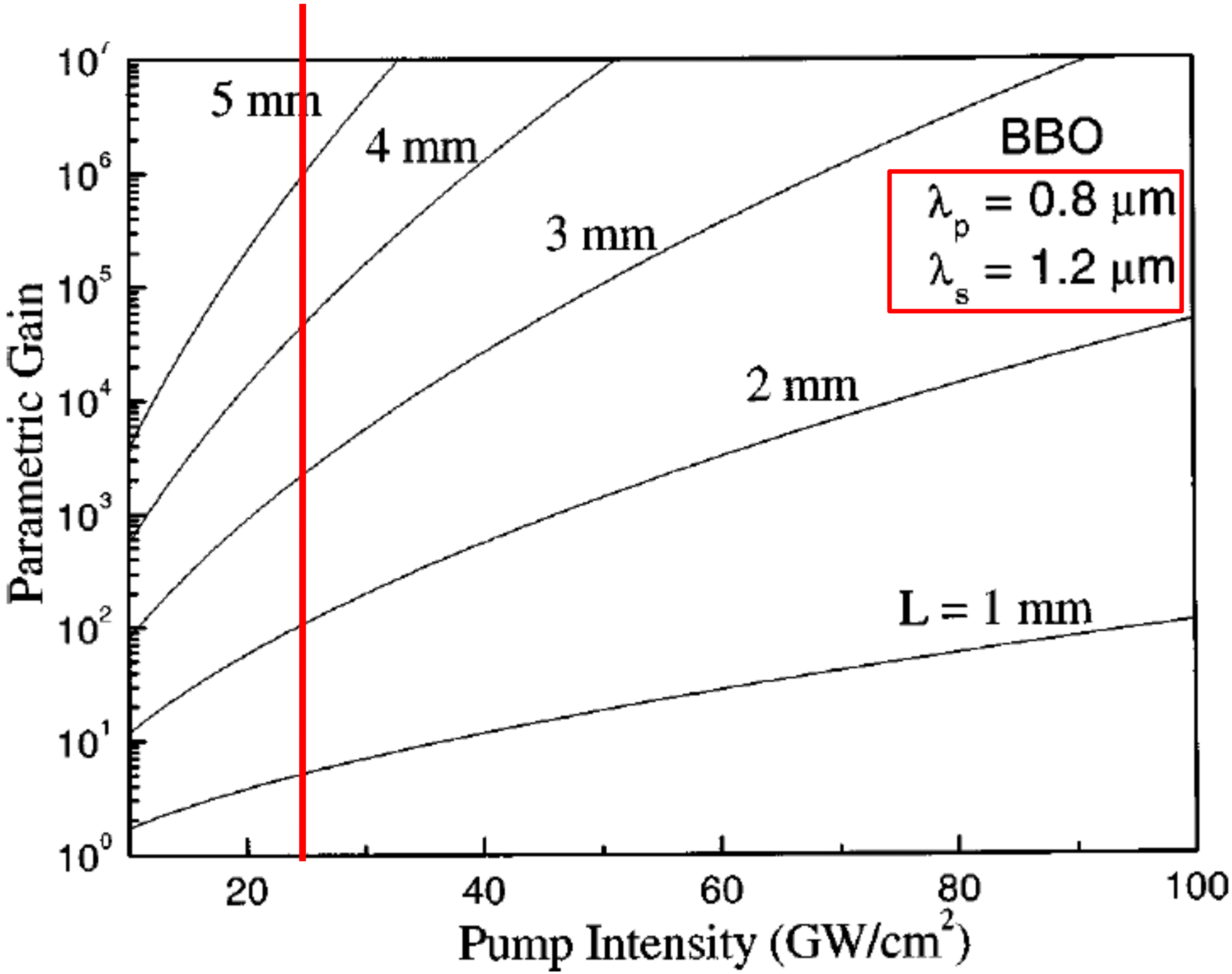


Fig. 12.24 Parametric gain for an OPA at the pump wavelength λ_p *= 0.8 μm* and the signal wavelength λ_s *= 1.2 μm, using type I phase matching in BBO* (d_{eff} *= 2 pm/V).*

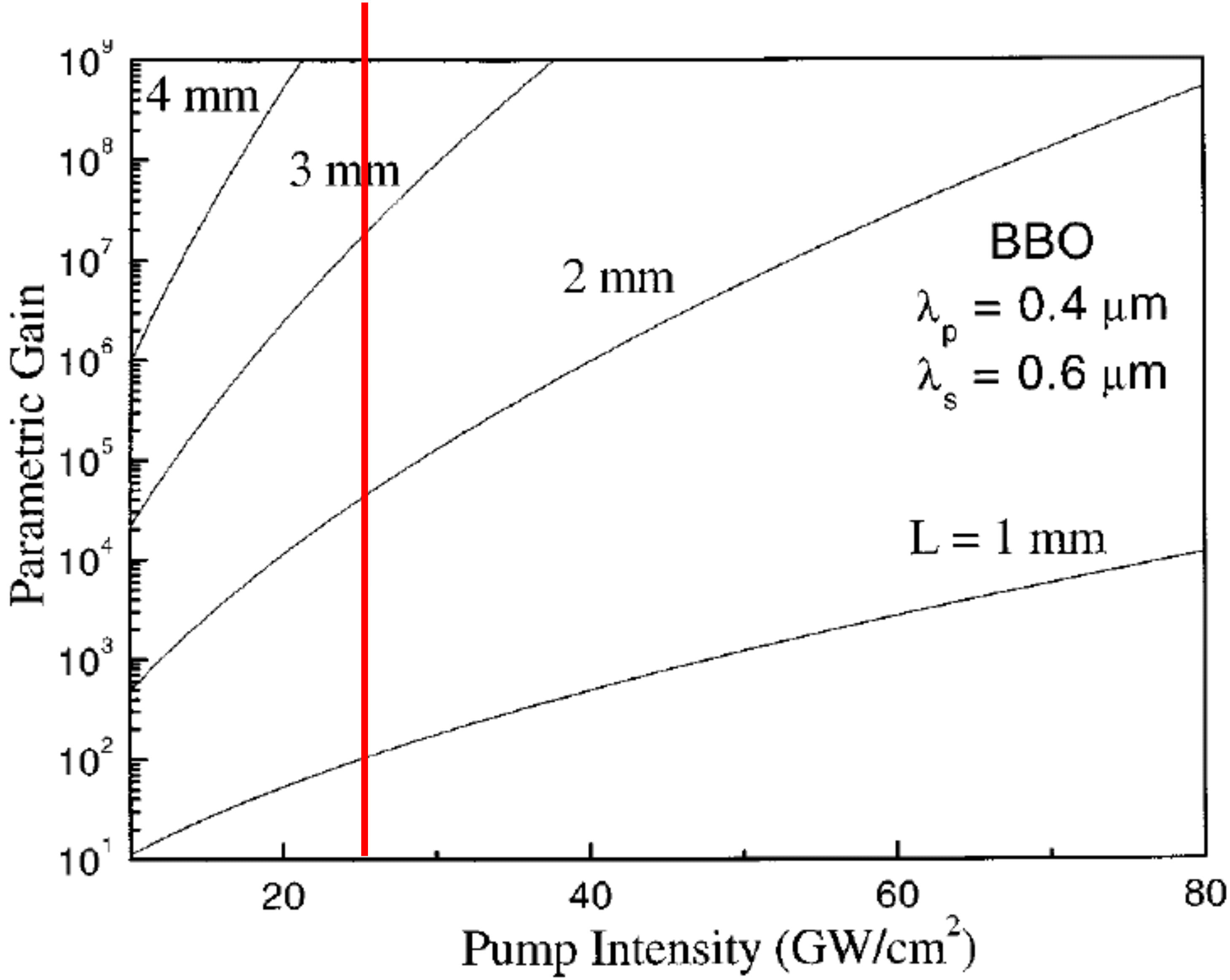


Fig. 12.25 Parametric gain for an OPA at the pump wavelength λ_p *= 0.4* μ*m* and the signal wavelength λ_s *= 0.6* μ*m, using type I phase matching in BBO* (d_{eff} *= 2 pm/V).*

Phase Matching

$$\Delta k = 0 \longrightarrow n_p = \frac{n_s\omega_s + n_i\omega_i}{\omega_p}$$

Uniaxial Crystal: $n_e < n_o$

Type I: noncritical

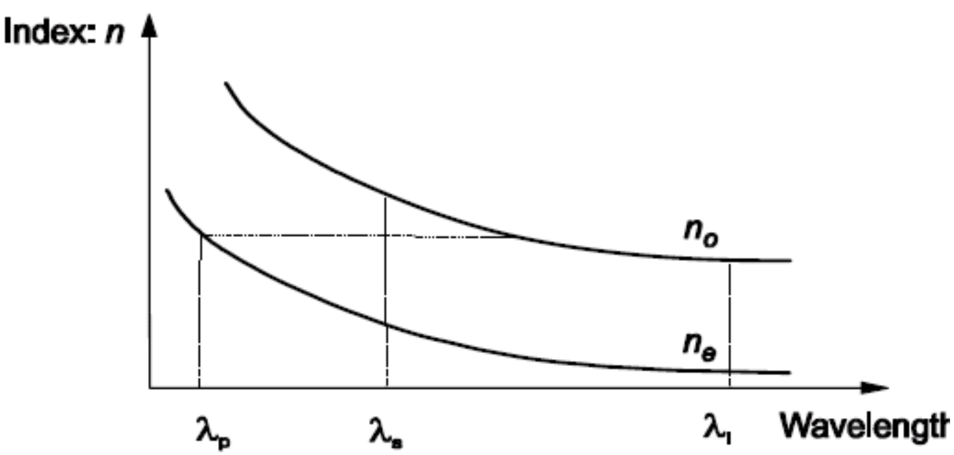


Fig. 12.26 Type I noncritical phase matching.

Type I: critical

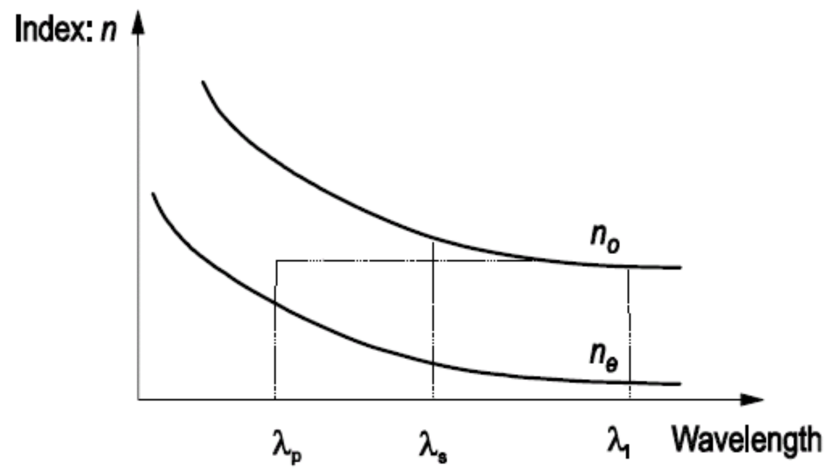
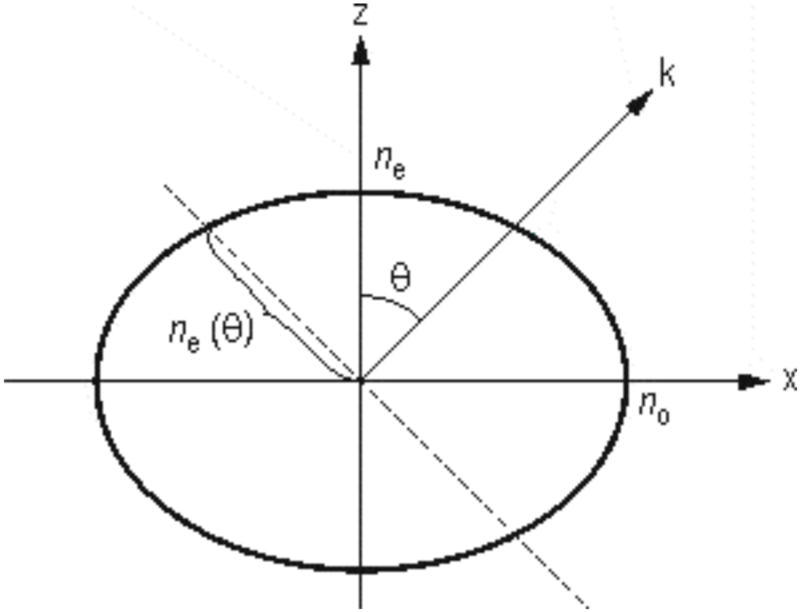


Fig. 12.27 Type I critical phase matching by adjusting the angle θ between wave vector of the propagating beam and the optical axis.

Phase Matching



Critical Phase Matching

$$n_{ep}(\theta)\omega_p = n_{os}\omega_s + n_{oi}\omega_i$$

$$\frac{1}{n_{ep}(\theta)^2} = \frac{\sin^2\theta}{n_{ep}^2} + \frac{\cos^2\theta}{n_{op}^2}$$

$$\theta = \arcsin\left[\frac{n_{ep}}{n_{ep}(\theta)}\sqrt{\frac{n_{op}^2 - n_{ep}^2(\theta)}{n_{op}^2 - n_{ep}^2}}\right]$$

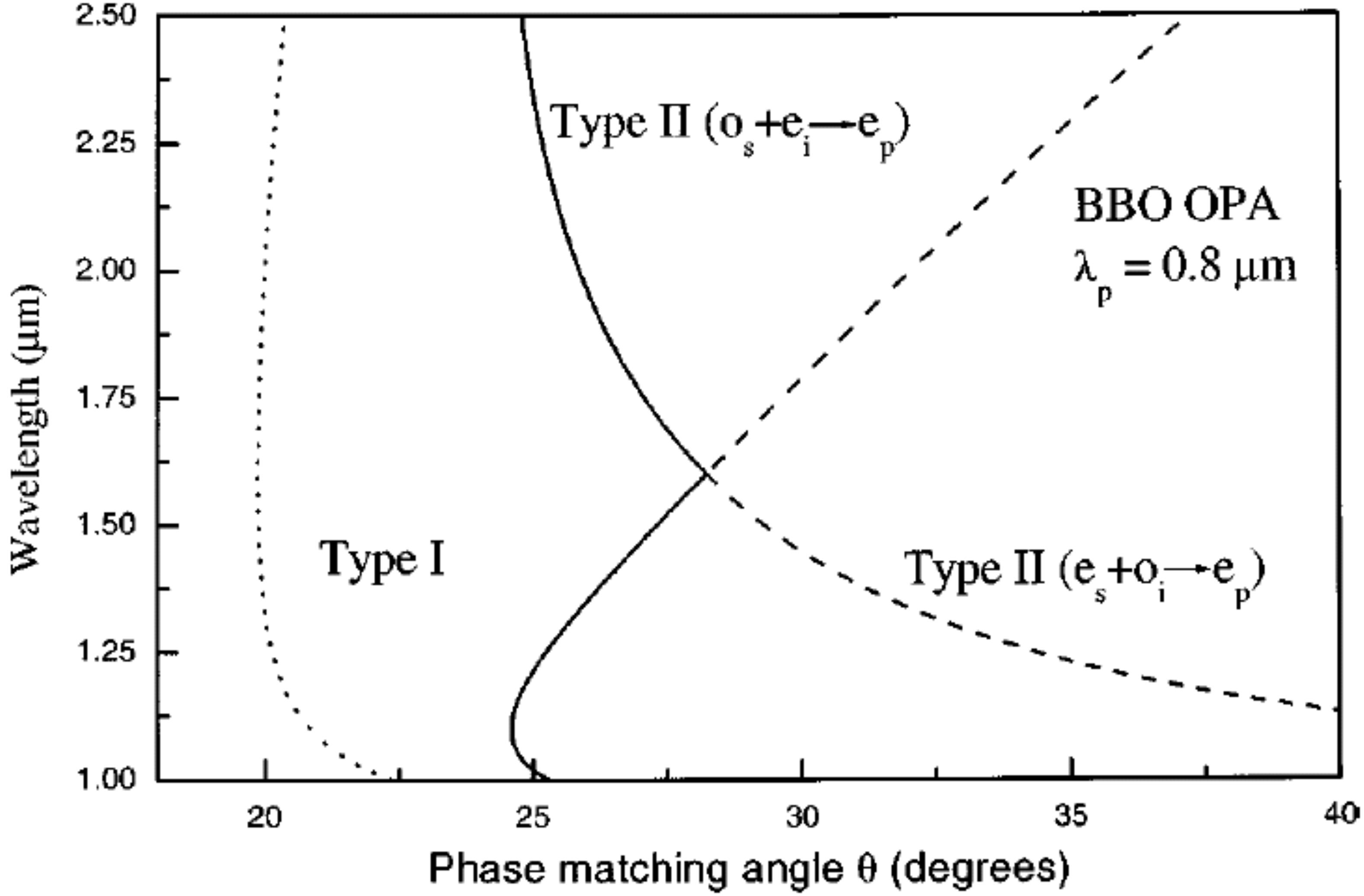


Fig. 12.28 Angle tuning curves for a BBO OPA at the pump wavelength λ_p=*0.8* μm for type I phase matching (dotted line), type II ($o_s + e_i \rightarrow e_p$) phase matching (solid line), and type II ($e_s + o_i \rightarrow e_p$) phase matching (dashed line).

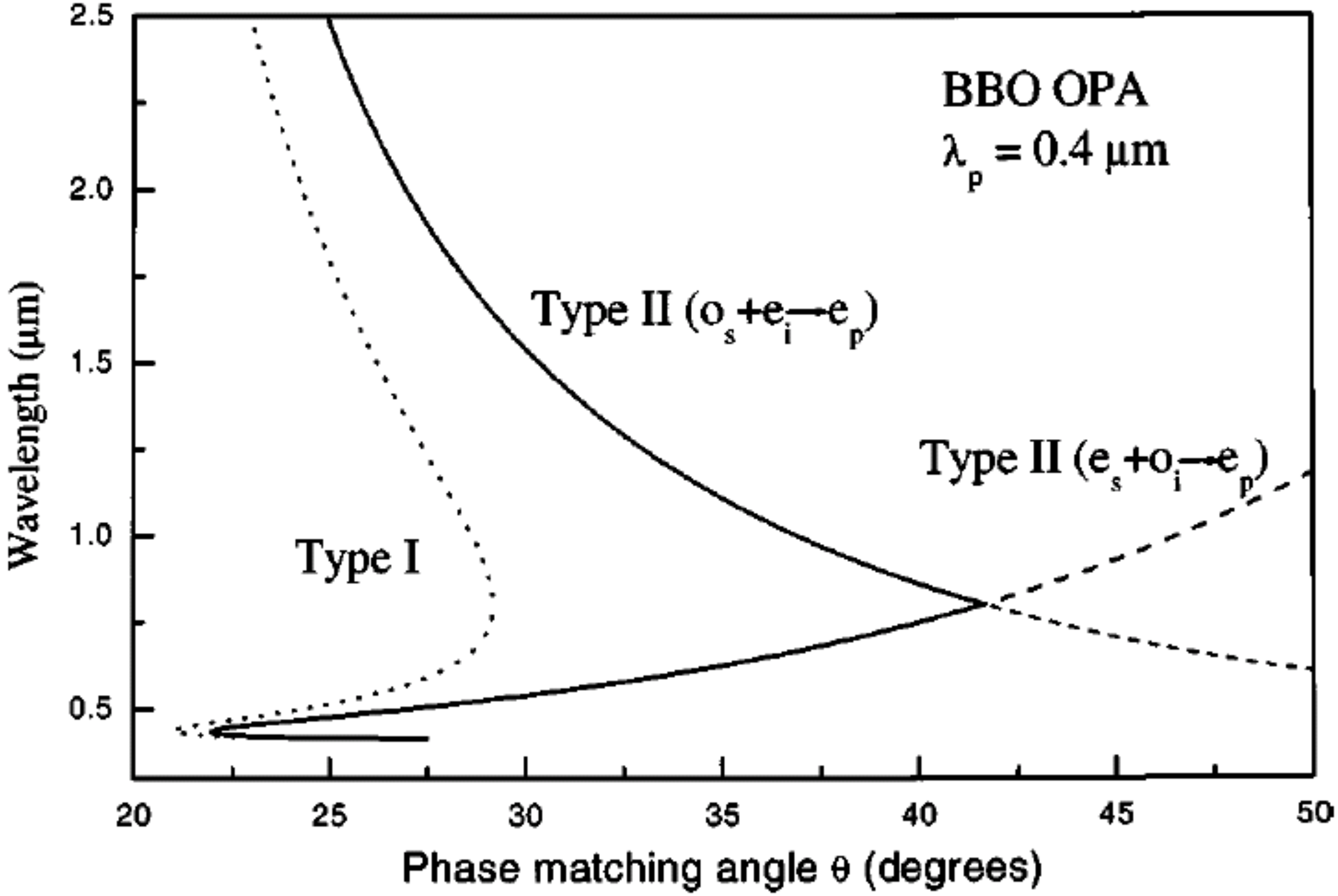
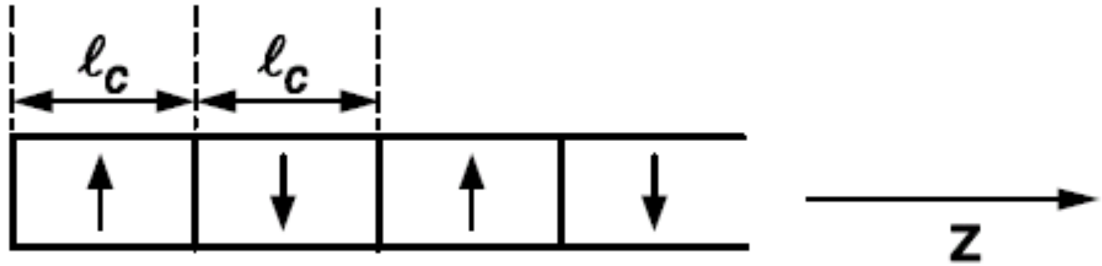


Fig. 12.28 Angle tuning curves for a BBO OPA at the pump wavelength $\lambda_p=0.4$ µm for type I phase matching (dotted line), type II ($o_s + e_i \rightarrow e_p$) phase matching (solid line), and type II ($e_s + o_i \rightarrow e_p$) phase matching (dashed line).

Quasi Phase Matching



Periodically poled crystal

Fig.12.30: Variation of def f in a quasi phase matched material as a function of propagation distance.

$$d_{eff}(z) = \sum_{m=-\infty}^{+\infty} d_m e^{jm\kappa z}$$

$$\frac{\partial E_p(z)}{\partial z} = -j\kappa_p \, E_s(z)E_i(z) \, e^{j\Delta k z}$$

Ultrashort Pulse Optical Parametric Amplification

$$\vec{E}_{p,s,i}(z,t) = \mathrm{Re}\left\{E_{p,s,i}(z,t)\; e^{j(\omega_{p,s,i}t - k_{p,s,i}\; z)}\vec{e}_{p,s,i}\right\}$$

Pulse envelopes

$$\begin{aligned}
\frac{\partial E_p}{\partial z} + \frac{1}{v_p}\frac{\partial E_p}{\partial t} &= -j\kappa_p\; E_s E_i\; e^{j\Delta k z}\;, \\
\frac{\partial E_s}{\partial z} + \frac{1}{v_s}\frac{\partial E_s}{\partial t} &= -j\kappa_s\; E_p E_i^*\; e^{-j\Delta k z}, \\
\frac{\partial E_i}{\partial z} + \frac{1}{v_i}\frac{\partial E_s}{\partial t} &= -j\kappa_i\; E_p E_s^*\; e^{-j\Delta k z},
\end{aligned}$$

$v_{p,s,i} = dk/d\omega|_{\omega_{p,s,i}}$ are the corresponding group velocities

$t' = t - z/v_p$

$$\begin{aligned}
\frac{\partial E_p}{\partial z} &= -j\kappa_p\; E_s E_i\; e^{j\Delta k z}\;, \\
\frac{\partial E_s}{\partial z} + \left(\frac{1}{v_s} - \frac{1}{v_p}\right)\frac{\partial E_s}{\partial t} &= -j\kappa_s\; E_p E_i^*\; e^{-j\Delta k z}, \\
\frac{\partial E_i}{\partial z} + \left(\frac{1}{v_i} - \frac{1}{v_p}\right)\frac{\partial E_s}{\partial t} &= -j\kappa_i\; E_p E_s^*\; e^{-j\Delta k z}.
\end{aligned}$$

Temporal walkoff
Group Velocity Mismatch (GVM)

Pump pulse width

$$\ell_{jp} = \frac{\tau}{\delta_{jp}}, \text{ with } \delta_{jp} = \left(\frac{1}{v_j} - \frac{1}{v_p}\right)$$

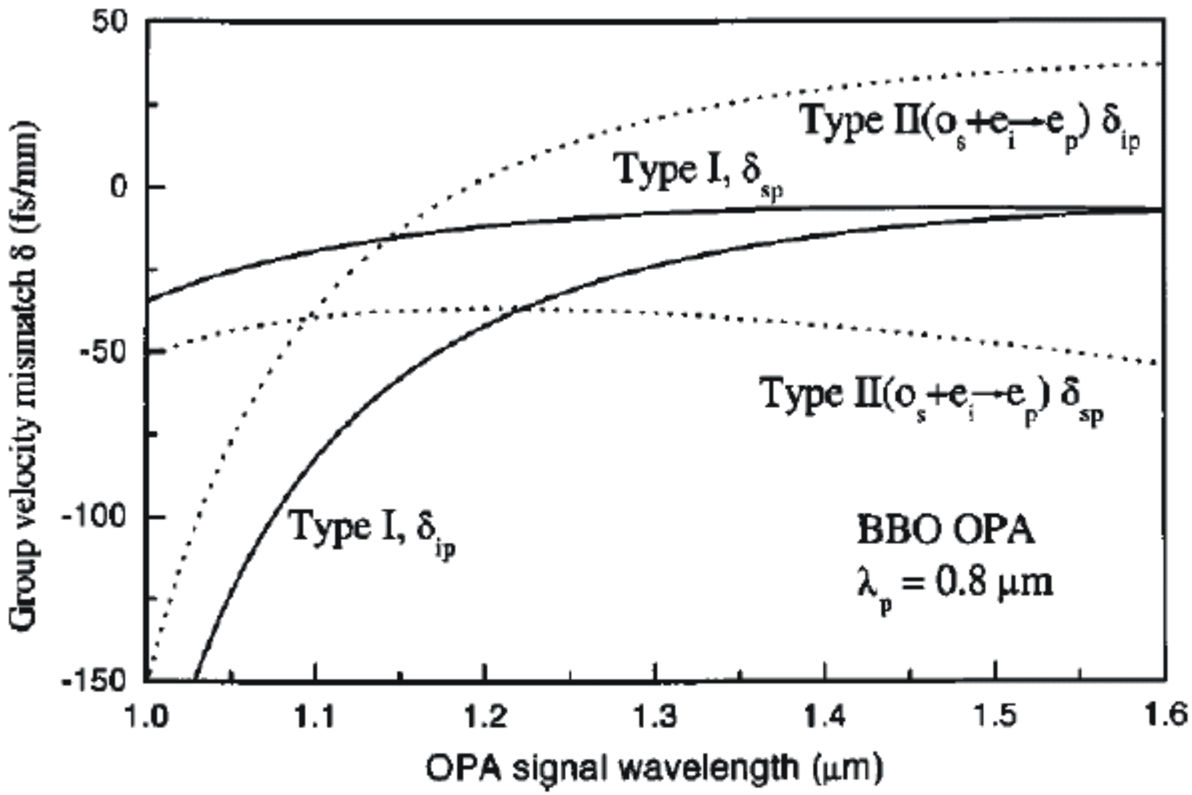


Fig. 12.31: Pump-signal (δ_{sp}) and pump-idler (δ_{ip}) group velocity mismatch curves for a BBO OPA at the pump wavelength λ_p=0.8 µm for type I phase matching (solid line) and type II ($o_s + e_i \rightarrow e_p$) phase matching (dashed line).

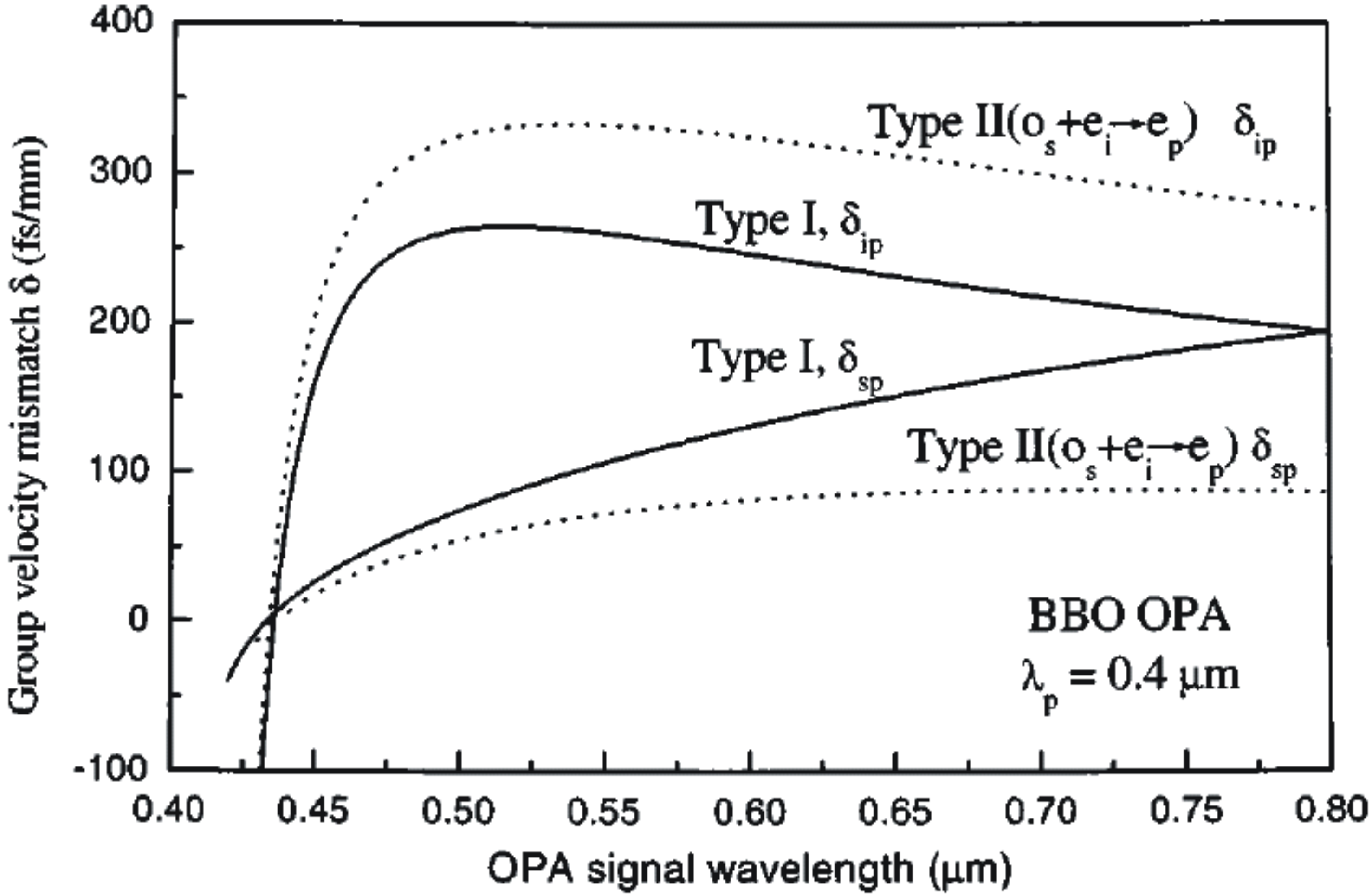


Fig. 12.32: Pump-signal (δ_{sp}) and pump-idler (δ_{ip}) group velocity mismatch curves for a BBO OPA at the pump wavelength λ_p=0.4 μm for type I phase matching (solid line) and type II ($o_s + e_i \rightarrow e_p$) phase matching (dashed line).

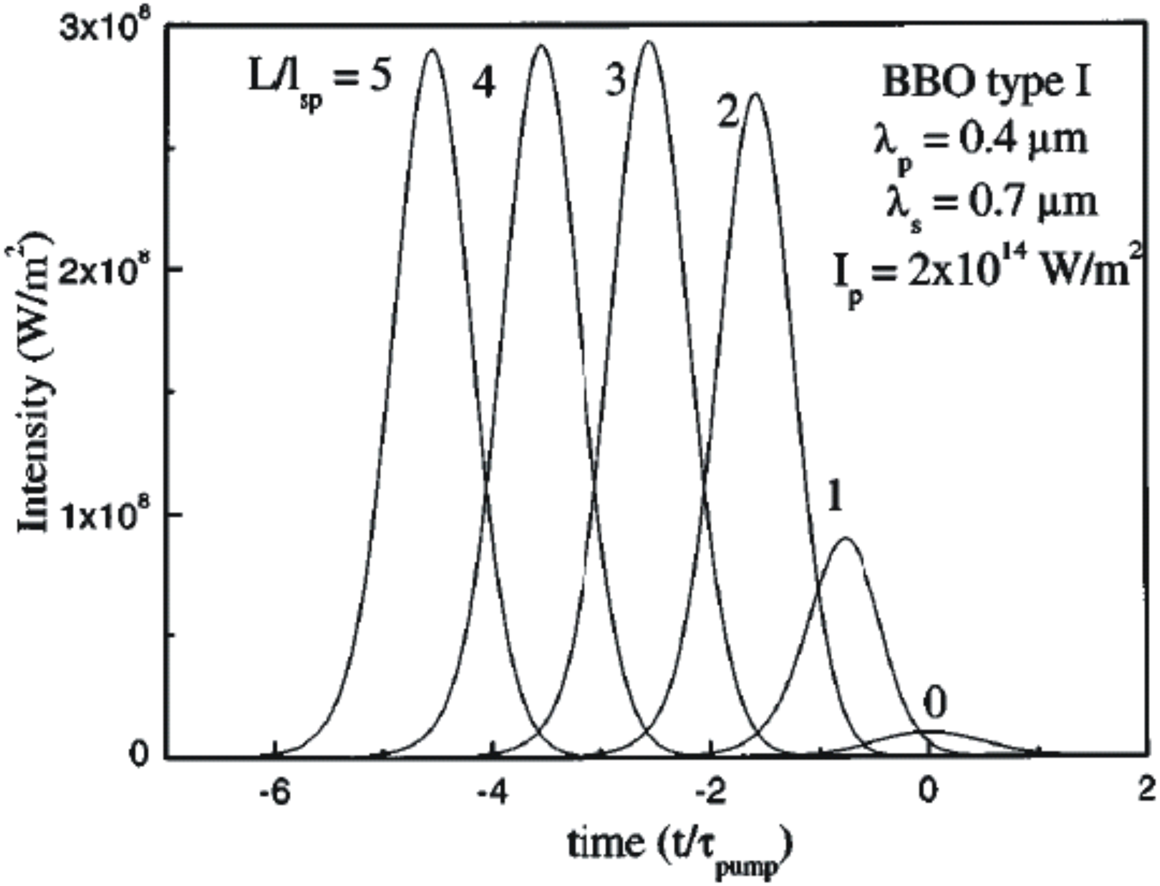


Figure 12.34: Signal pulse evolution for a BBO type I OPA with λ_p = 0.4 µm, λ_s = 0.7 µm, for different lengths L of the nonlinear crystal. Pump intensity is 20 GW/cm^2. Time is normalized to the pump pulse duration and the crystal length to the pump-signal pulse splitting length. [5]

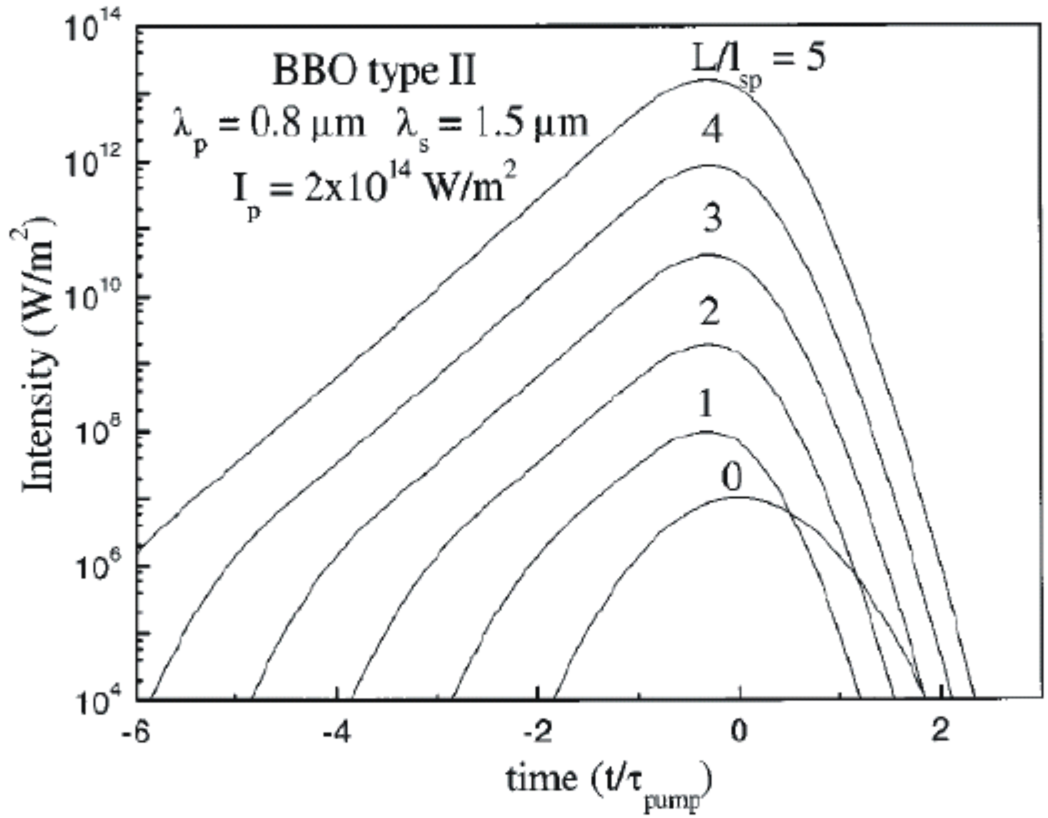


Figure 12.35: Signal pulse evolution for a BBO type II OPA with λ_p = 0.8 µm, λ_s = 1.5 µm, for different lengths L of the nonlinear crystal. Pump intensity is 20 GW/cm^2. Time is normalized to the pump pulse duration and the crystal length to the pump-signal pulse splitting length. [5]

OPA Bandwidth

$$\omega_s \rightarrow \omega_s + \Delta\omega \qquad \omega_i \rightarrow \omega_i - \Delta\omega$$

$$\Delta k = -\frac{dk_s}{d\omega}\Delta\omega + \frac{dk_i}{d\omega}\Delta\omega = \left(\frac{1}{v_i} - \frac{1}{v_s}\right)\Delta\omega$$

Bandwidth limitation due to GVM

$$\Delta f = -\frac{2\sqrt{\ln 2}}{\pi}\sqrt{\frac{\Gamma}{L}}\frac{1}{\left|\frac{1}{v_i} - \frac{1}{v_s}\right|}$$

For vanishing dispersion:

$$\Delta f = -\frac{2\sqrt[4]{\ln 2}}{\pi}\sqrt[4]{\frac{\Gamma}{L}}\frac{1}{\left|\frac{d^2k_s}{d\omega^2} + \frac{d^2k_s}{d\omega^2}\right|}.$$

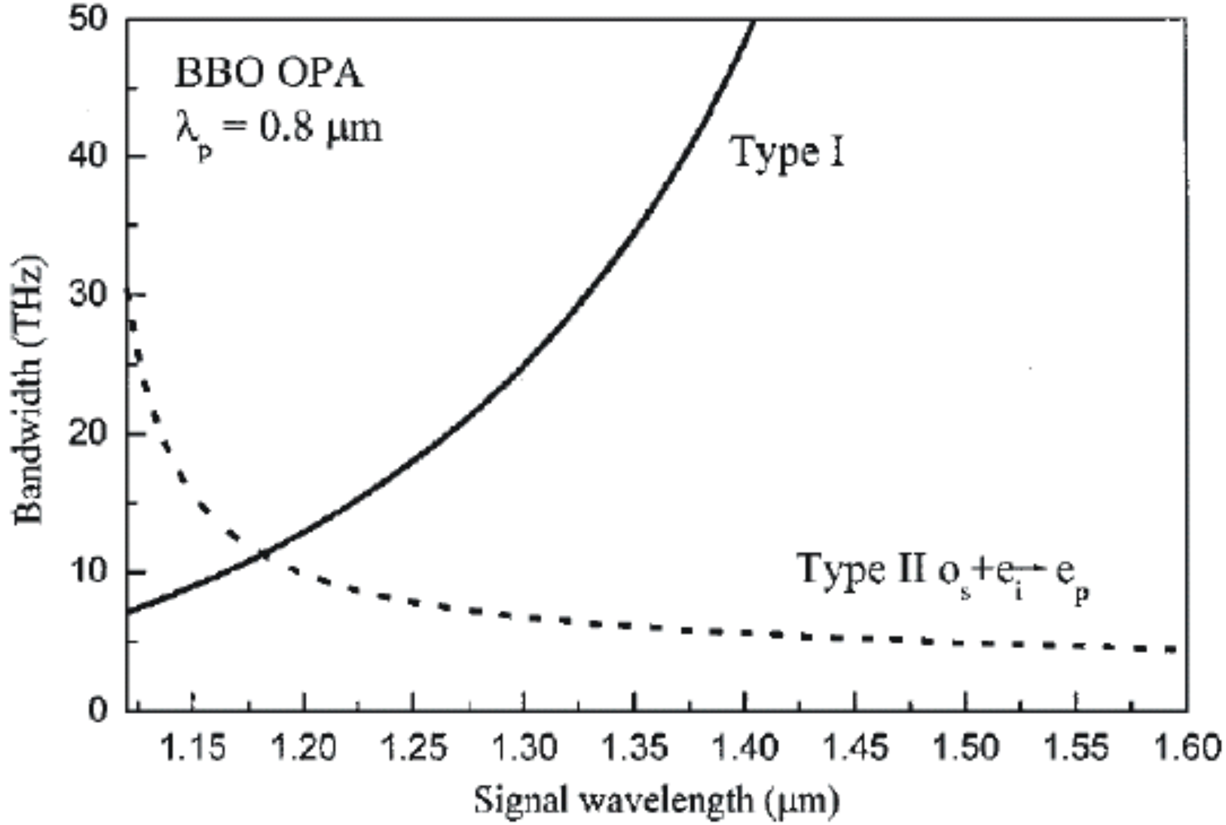


Figure 12.35: Phase matching bandwidth for a BBO OPA at the pump wavelength λ_p=0.8 µm for type I phase matching (solid line) and type II ($o_s + e_i \rightarrow e_p$) phase matching (dashed line). Crystal length is 4 mm and pump intensity 50 GW/cm^2.

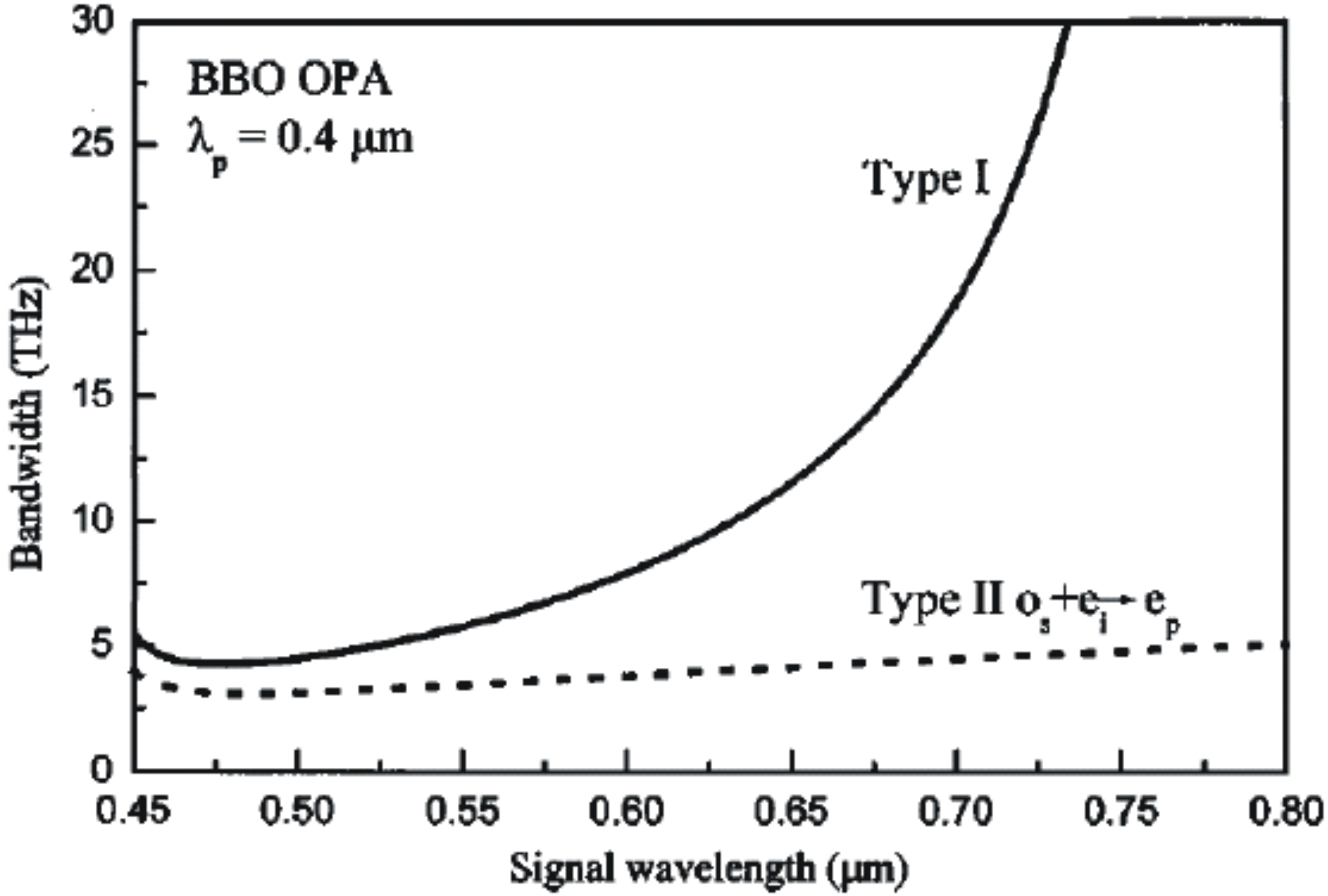


Figure 12.36: Phase matching bandwidth for a BBO OPA at the pump wavelength λ_p=0.4 μm for type I phase matching (solid line) and type II ($o_s + e_i \rightarrow e_p$) phase matching (dashed line). Crystal length is 2 mm and pump intensity 100 GW/cm^2.

Optical Parametric Amplifier Designs

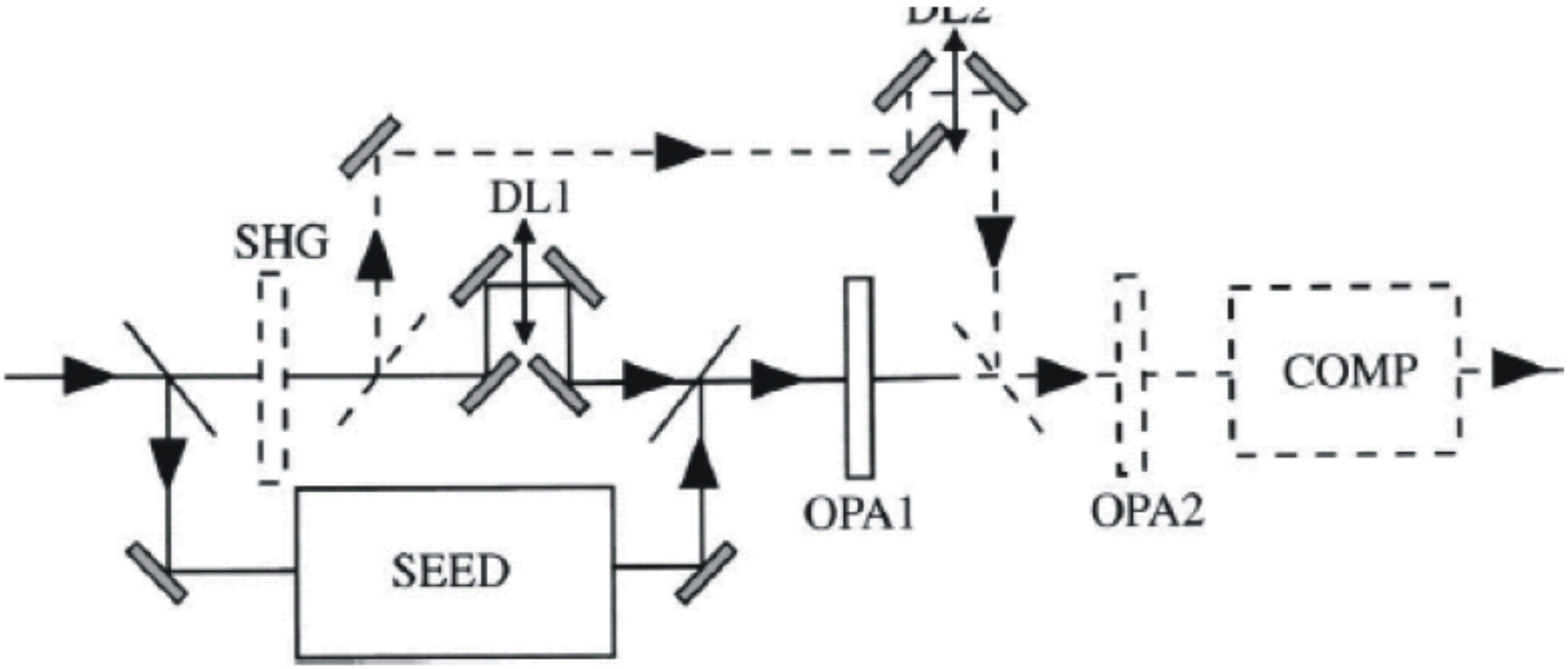


Figure 12.37: Scheme of an ultrafast optical parametric amplifier. SEED: seed generation stage; DL1, DL2: delay lines; OPA1, OPA2 parametric amplification stages; COMP: ompressor.

Near-IR OPA

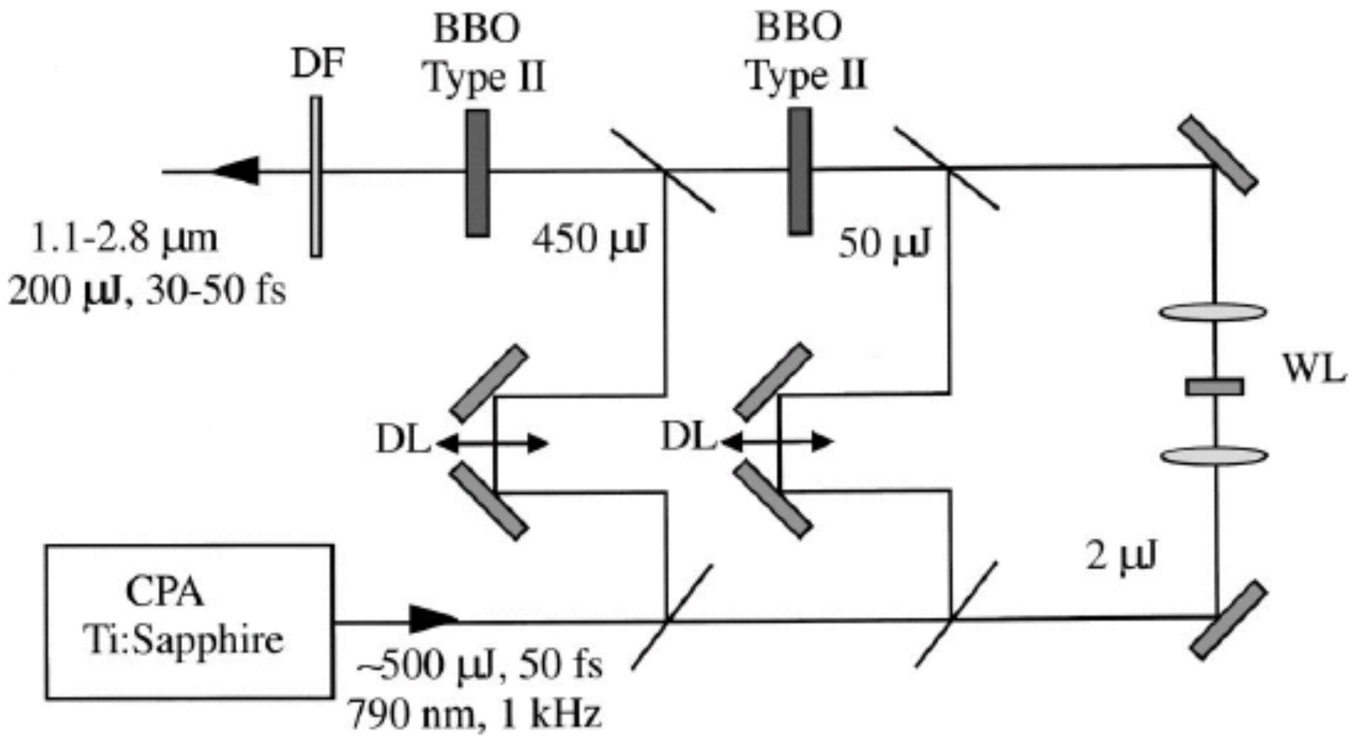


Figure 12.38: Scheme of a near-IR OPA DL: delay lines; WL: white light generation stage; DF: dichroic filter. [5]

Noncollinear Optical Parametric Amplifier (NOPA)

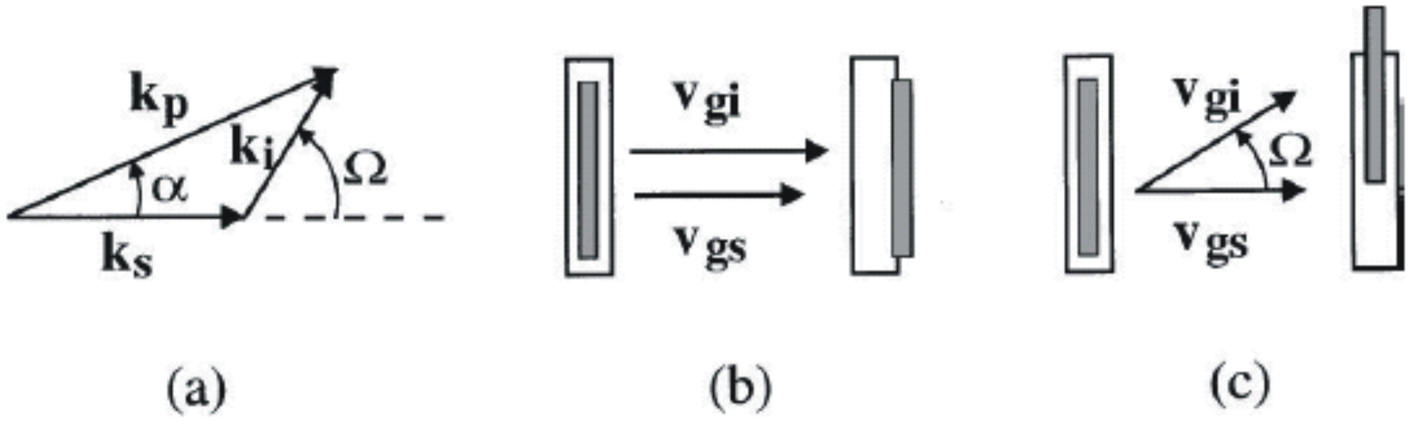


Figure 12.39: a) Schematic of a noncollinear interaction geometry; b) representation of signal and idler pulses in the case of collinear interaction; and c) same as b) for noncollinear interaction.

Phase Matching Condition: Vector Condition:

$$\begin{aligned} \Delta k_{par} &= k_p \cos\alpha - k_s - k_i \cos\Omega = 0 \\ \Delta k_{perp} &= k_p \sin\alpha - k_i \sin\Omega = 0 \end{aligned}$$

Variation on phase matching condition by $\Delta\omega$

$$\Delta k_{par} = -\frac{dk_s}{d\omega_s}\Delta\omega + \frac{dk_i}{d\omega_i}\cos\Omega\ \Delta\omega - k_i \sin\Omega \frac{d\Omega}{d\omega_i}\Delta\omega = 0 \qquad \mathbf{X} \quad \cos(\Omega)$$

$$\Delta k_{perp} = \frac{dk_i}{d\omega_i}\sin\Omega\ \Delta\omega + k_i \cos\Omega \frac{d\Omega}{d\omega_i}\Delta\omega = 0 \qquad \mathbf{X} \quad \sin(\Omega)$$

And addition

$$\frac{dk_i}{d\omega_i} - \cos\Omega \frac{dk_s}{d\omega_s} = 0$$

Correct index

$$\boxed{v_{gs} - v_{gi}\ \cos\Omega = 0}$$

Only possible if:

$$v_{gi} > v_{gs}$$

$$\alpha = \arcsin\left[\frac{1 - \frac{v_s^2}{v_i^2}}{1 + 2v_s n_s \lambda_i / v_i n_i \lambda_s + (n_s\lambda_i / n_i\lambda_s)^2}\right]$$

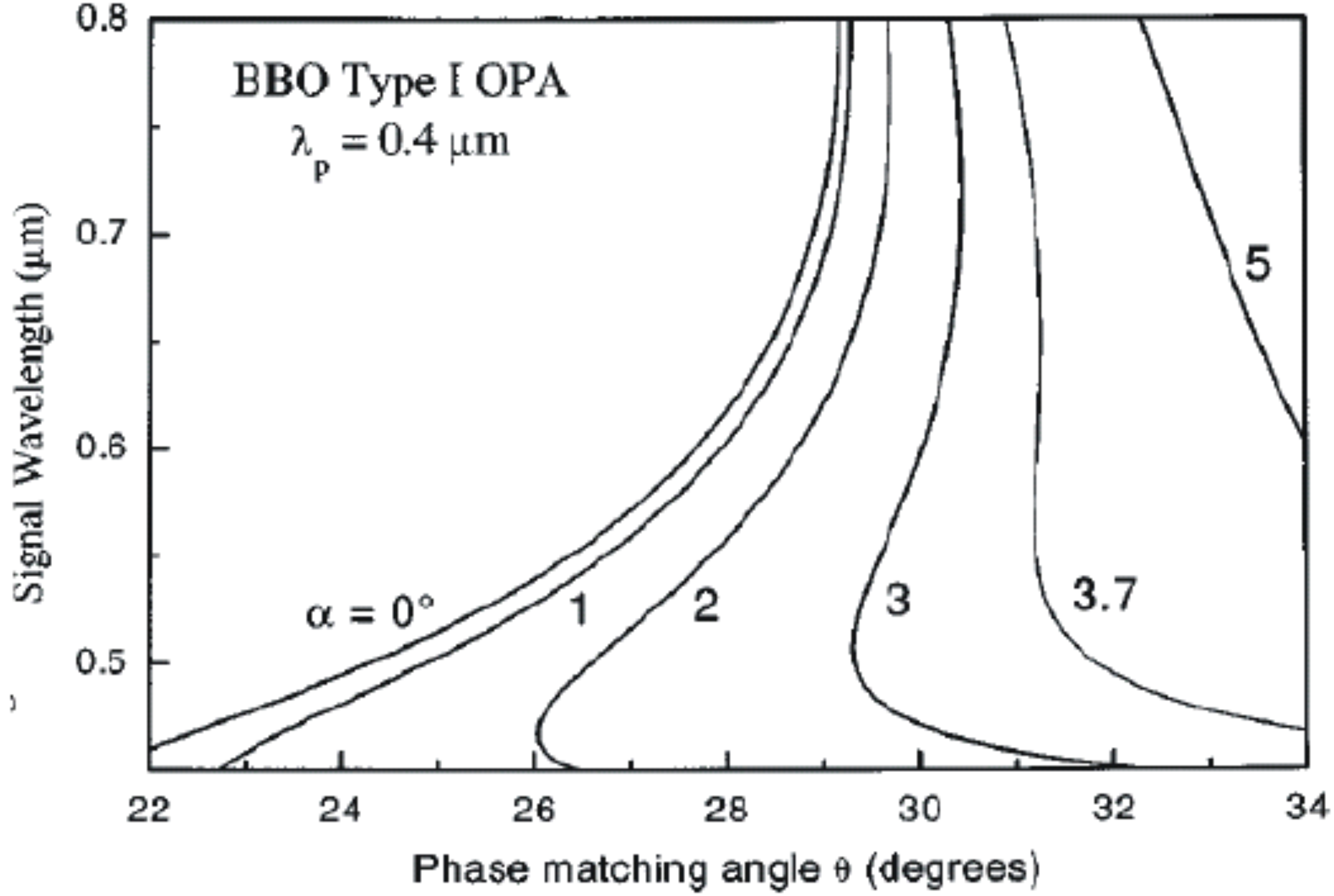


Figure 12.40: Phase-matching curves for a noncollinear type I BBO OPA pumped atpumped at λp=0.4 μm, as a function of the pump-signal angle a. [5]

NOPA Layout

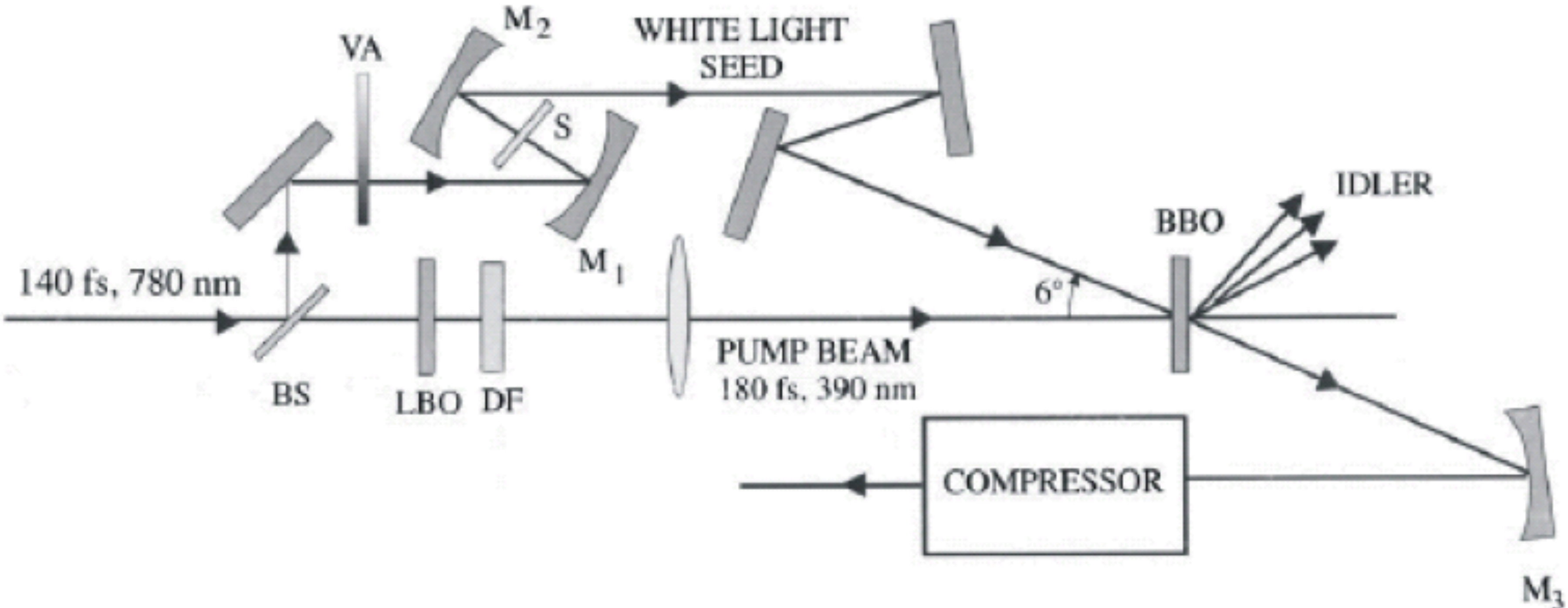


Figure 12.41: Scheme of a noncollinear visible OPA. BS: beam splitter; VA: variable attenuator; S: 1-mm-thick sapphire plate; DF: dichroic filter; M1 ,M2 , M3 , spherical mirrors.[5]

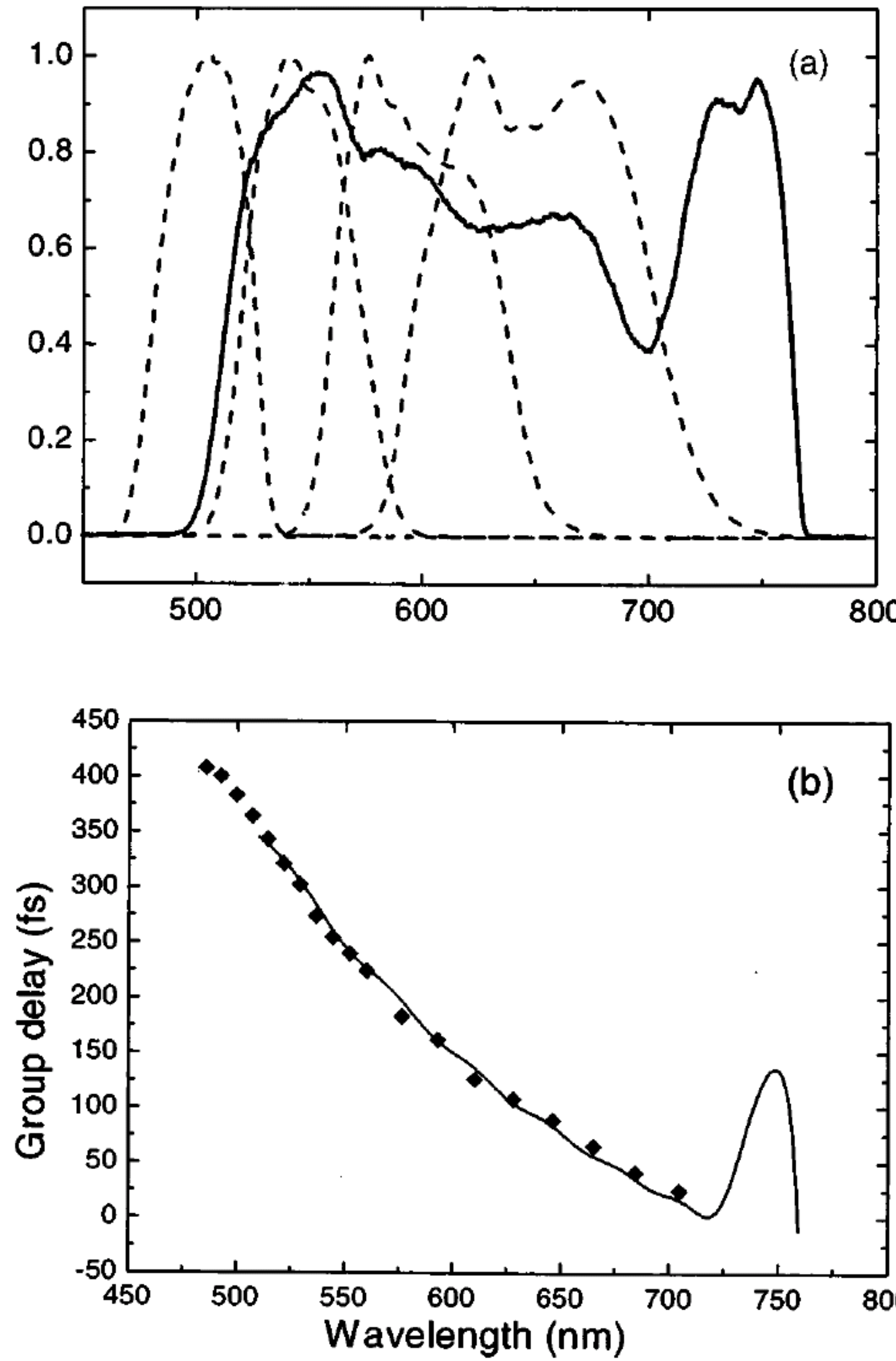


Figure 12.42: a) Solid line: NOPA spectrum under optimum alignment conditions; dashed line: sequence of spectra obtained by increasing the white light chirp;
b) points: measured GD of the NOPA pulses; dashed line: GD after ten bounces on the ultrabroadband chirped mirrors.

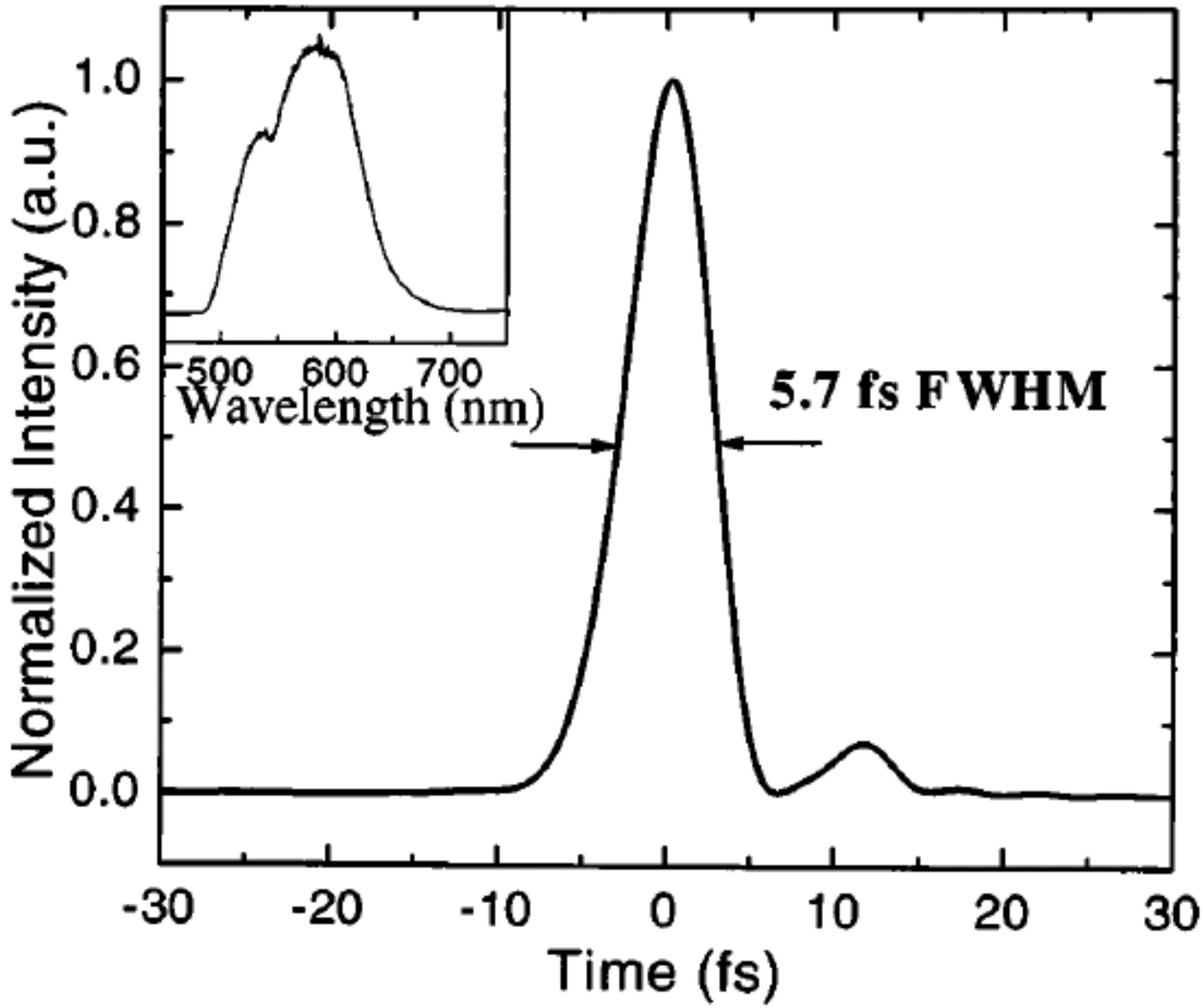
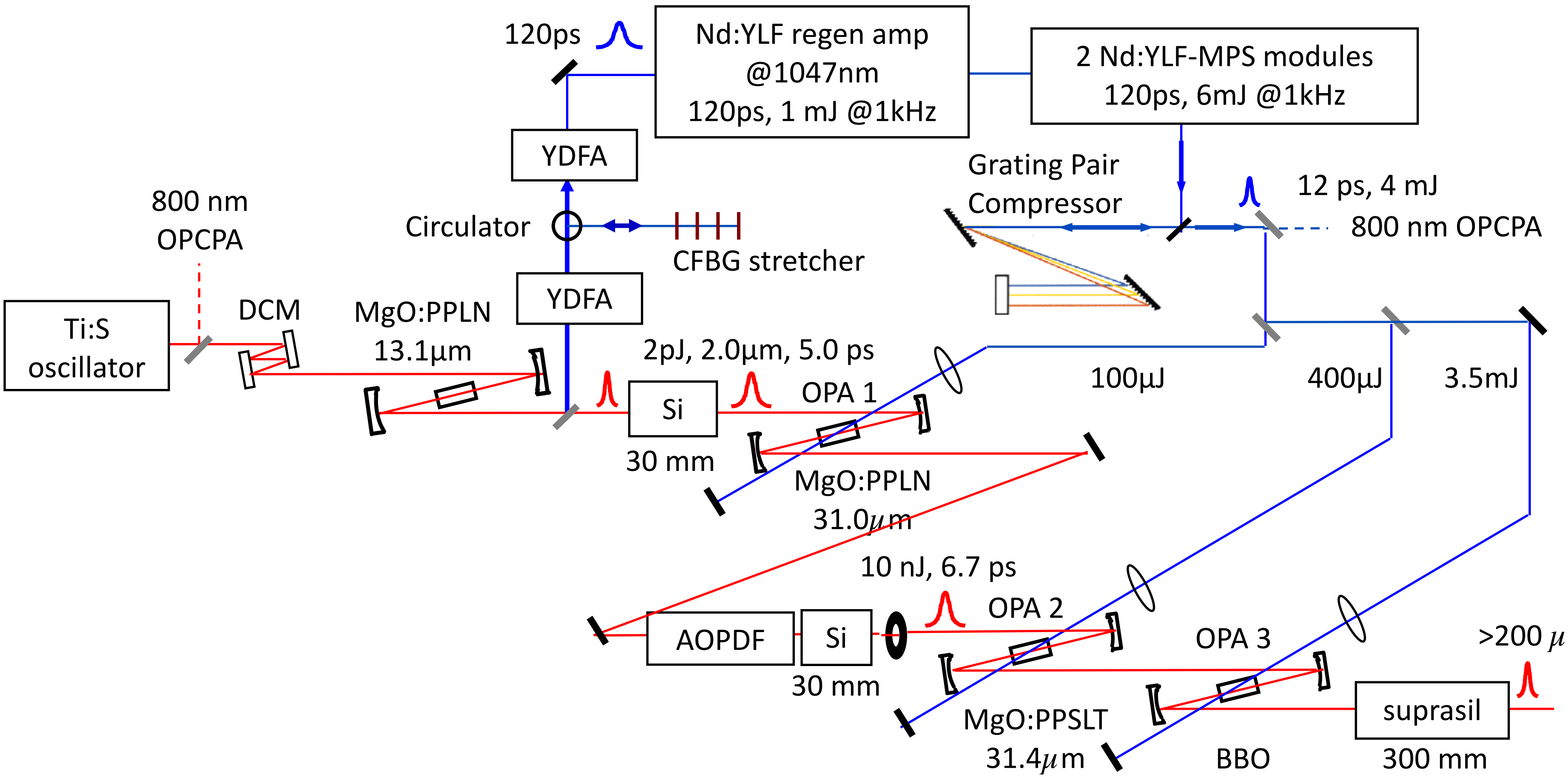


Figure 12.43: Reconstructed temporal intensity of the compressed NOPA pulse measured by the SPIDER technique. The inset shows the corresponding pulse pectrum.[5]

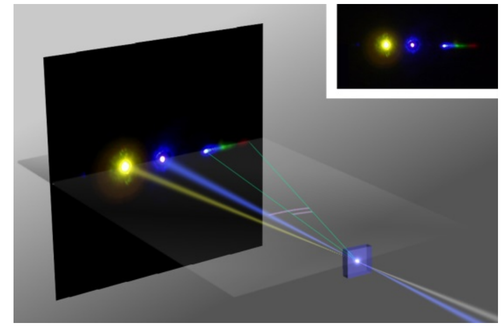
Optical Parametric Chirped Pulse Amplifier (OPCPA)

2-μm OPCPA

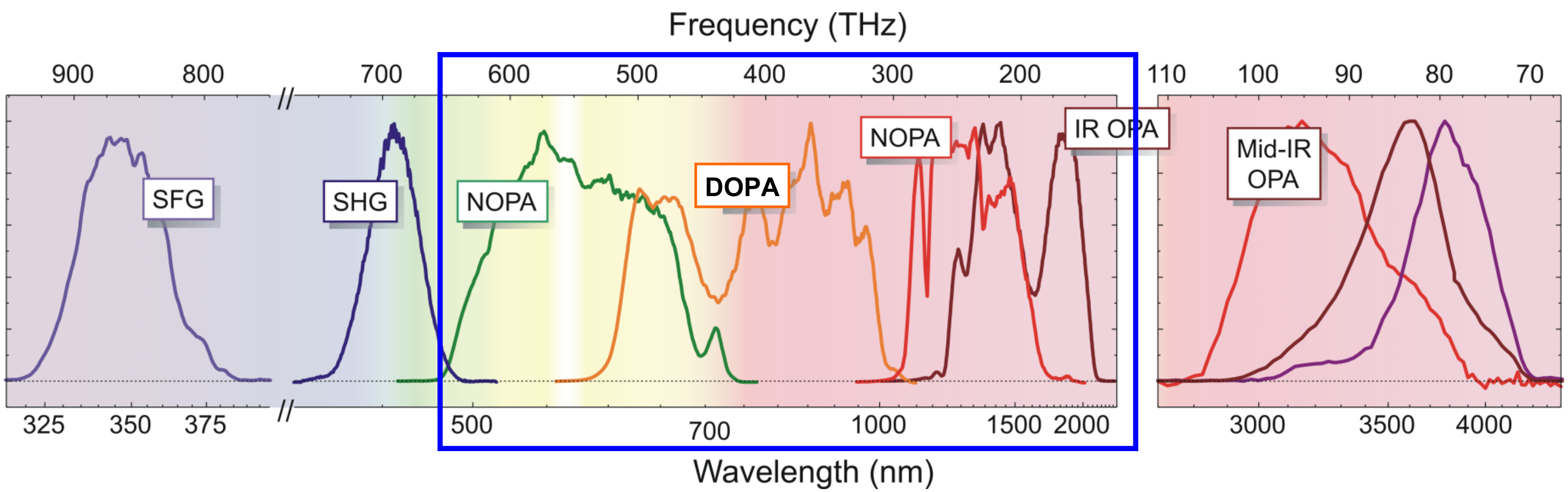


Optical Synthesis from OPAs

Combination of light from broadband Optical Parametric Amplifiers.



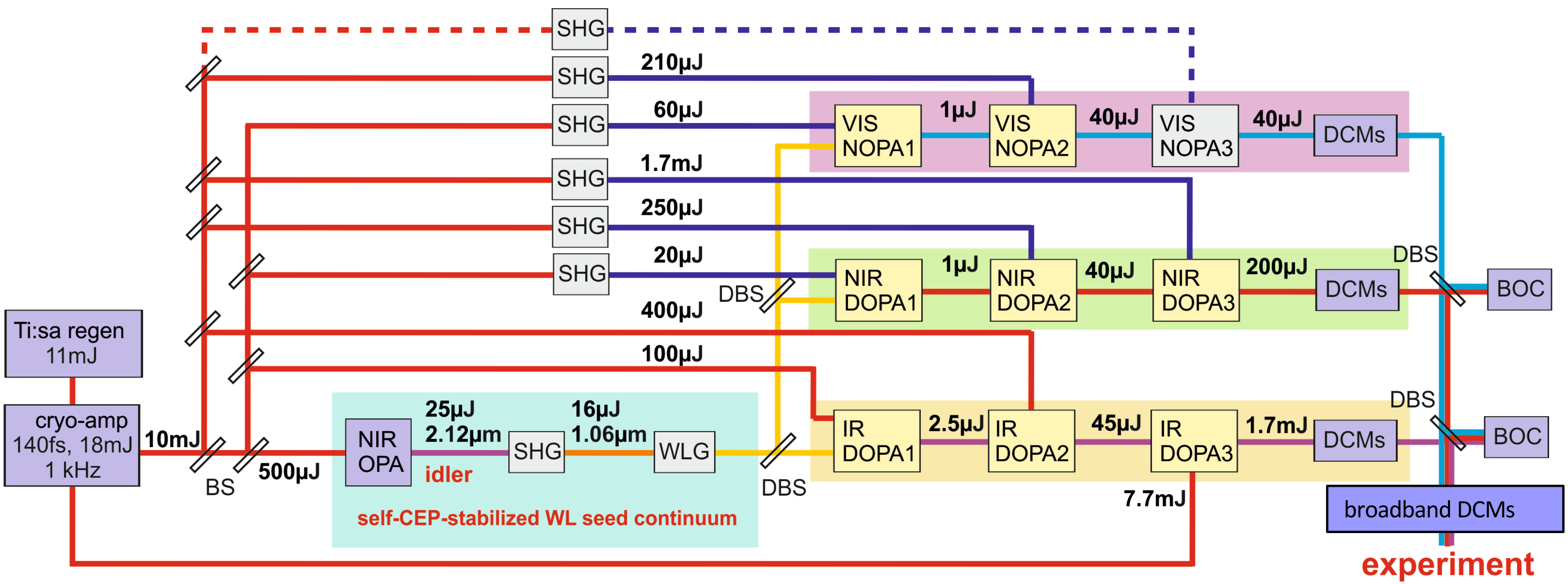
$\chi^{(2)}$ optical process in nonlinear crystals
Broadband phase-matching



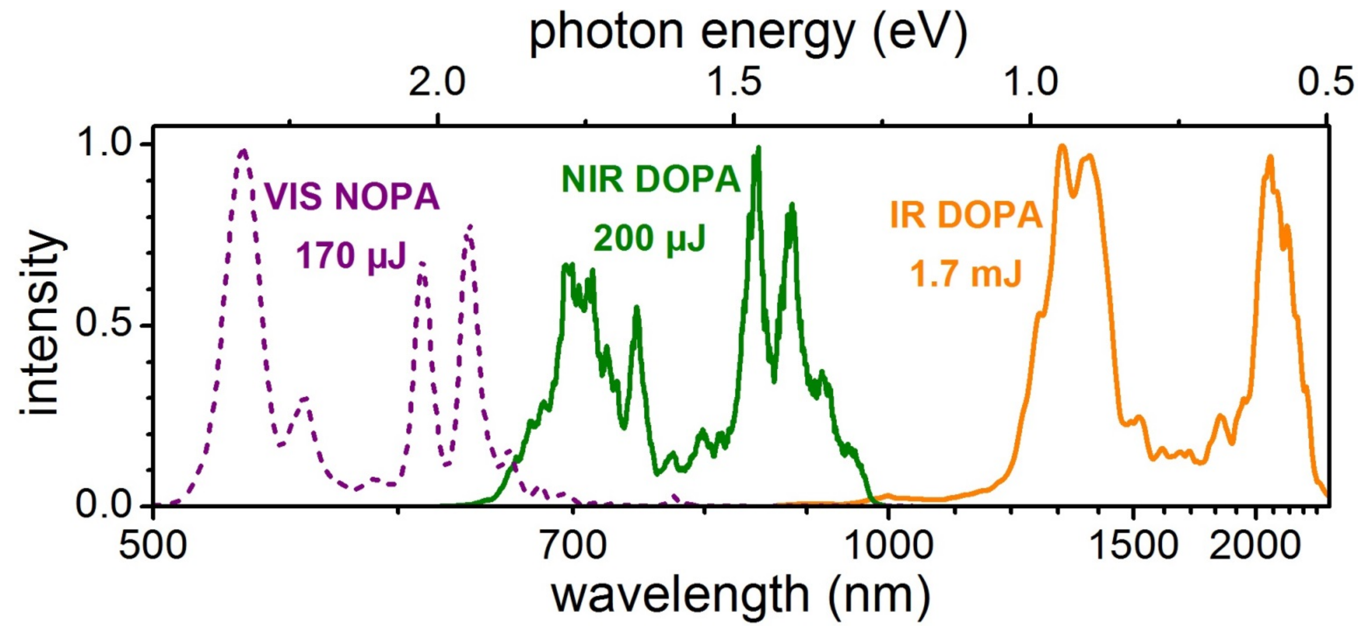
D. Brida et al., Journal of Optics A 12, 013001 (2010)

Multi-millijoule Pulse Synthesis with OPAs

- **Two-octave-wide** waveform synthesis from OPAs at the **multi-mJ energy level**
- **passively CEP-stable** WLG seed [G. Cerullo *et al.*, Laser Photonics Rev. **5**, 323 (2010)]
- WLG seed split into 3 wavelength channels and amplified in 3 OPA stages each
- Three channels are <u>individually</u> compressed and coherently recombined
- relative timing is tightly locked using balanced optical cross-correlators (BOCs)



Optical Pulse Synthesizer



VIS NOPA	NIR DOPA	IR DOPA
0.17 mJ signal	**0.20-0.25 mJ signal**	**1.7 mJ octave-spanning signal**
20% (0.8 mJ pump) pump-signal conversion efficiency	**12-15%** (1.7 mJ pump) pump-signal conversion efficiency	**22%** (7.7 mJ pump) pump-signal conversion efficiency
TL 5.6 fs	TL 5.2 fs	TL 5.2 fs
2.9 optical cycles @ λ_c=573nm	2.1 optical cycles @ λ_c=750nm	**1.1** optical cycle @ λ_c=1.4um