Ultrafast Optical Physics II (SoSe 2019)
Lecture 9, June 7

9 Pulse Characterization

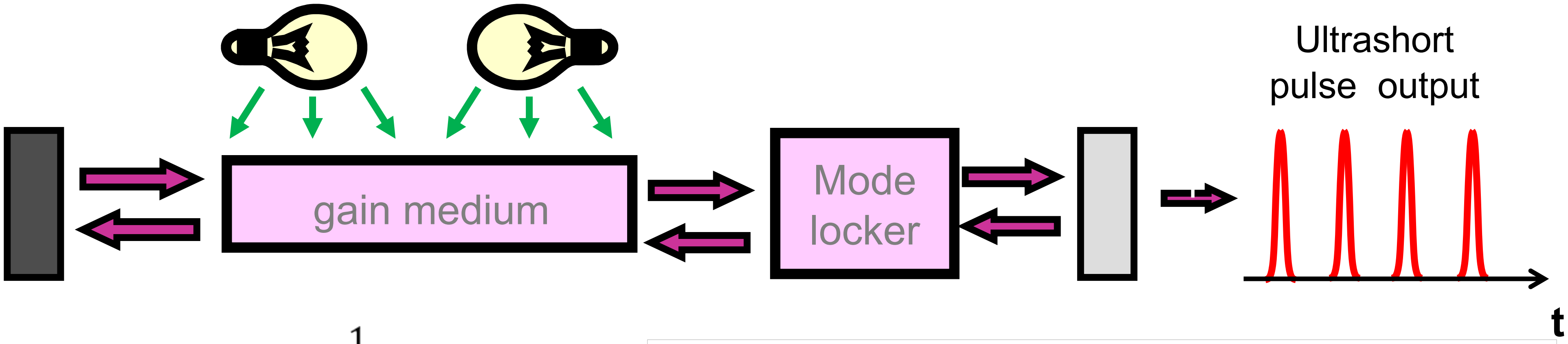
9.1 Intensity Autocorrelation

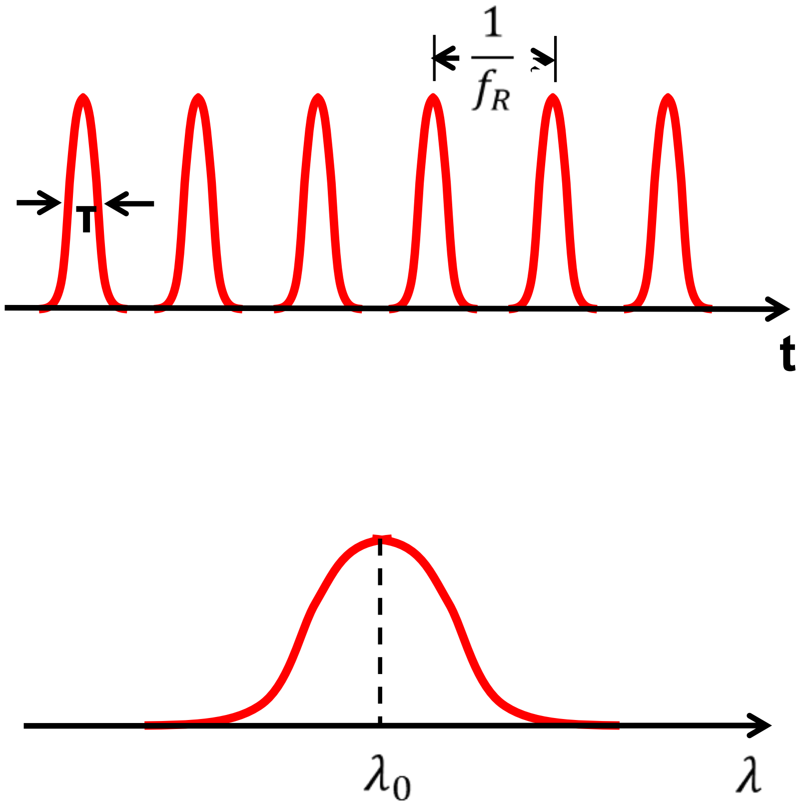
9.2 Interferometric Autocorrelation (IAC)

9.3 Frequency Resolved Optical Gating (FROG)

9.4 Spectral Shearing Interferometry for Direct Electric Field Reconstruction (SPIDER)

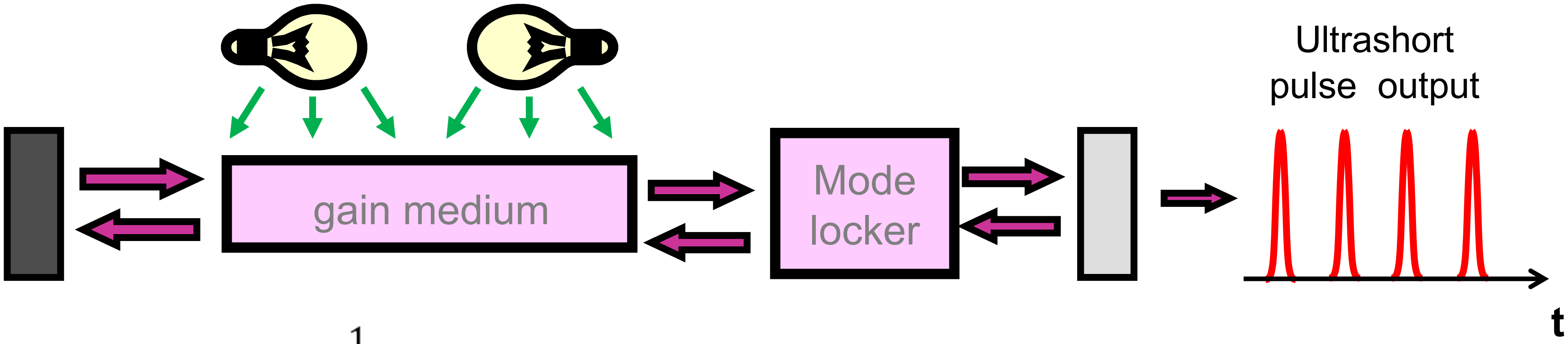
Ultrafast laser: the 4th element—mode locker

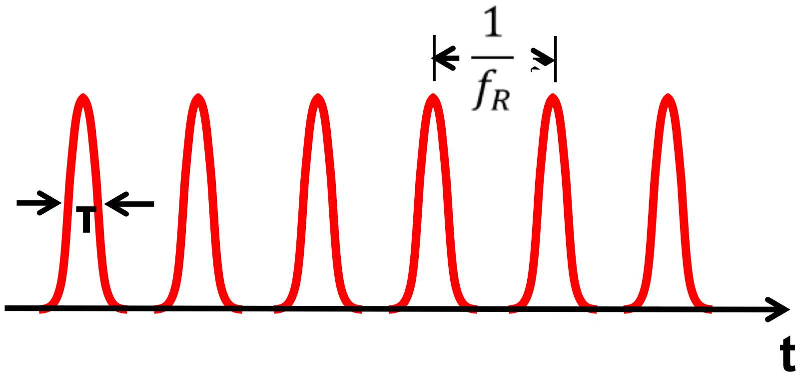


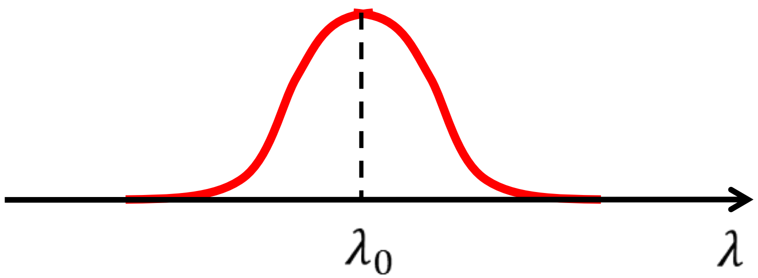


- Pulse duration T (fs – ps)
- Pulse energy E (pJ – mJ)
- Peak power P_p (1 kW – 1 PW)
  $P_p \approx E/T$ (e.g., 1 nJ, 100 fs pulse leads to 10 kW peak power.)
- Repetition rate f_R (10 MHz – 10 GHz)
- Average power P (10 mW – 100 W)
  $P = E \times f_R$ (e.g., 1 nJ, 100 MHz rep-rate laser produces 100 mW average power.
- Center wavelength λ_0 (700 nm – 2000 nm)

Measurement of pulse quantities using 'meters'





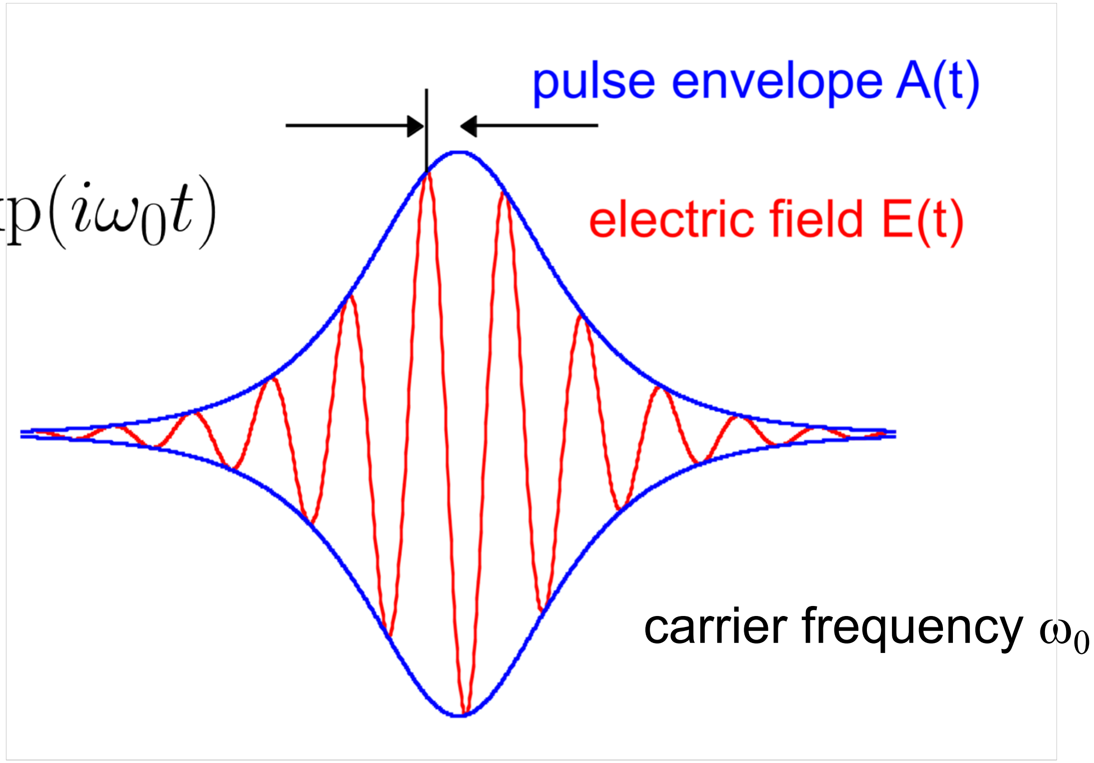


Physical quantity	Measuring device
Average power	Power meter
Repetition rate	RF spectrum analyzer
Optical spectrum	Optical spectrum analyzer (OSA)
Optical pulse	Pulse meter (?)

Optical Metrology: Autocorrelation methods

The laser pulse

$$E(t) = A(t)\exp(i\omega_0 t)$$



$$A(t) = \exp(-\Gamma t^2) = exp(-(\Gamma_1 + i\Gamma_2)t^2)$$ For a gaussian pulse

$$E(t) = exp(-\Gamma t^2)exp(i\omega_0 t) = exp(-\Gamma_1 t^2)exp(i(\omega_0 - \Gamma_2 t)t)$$

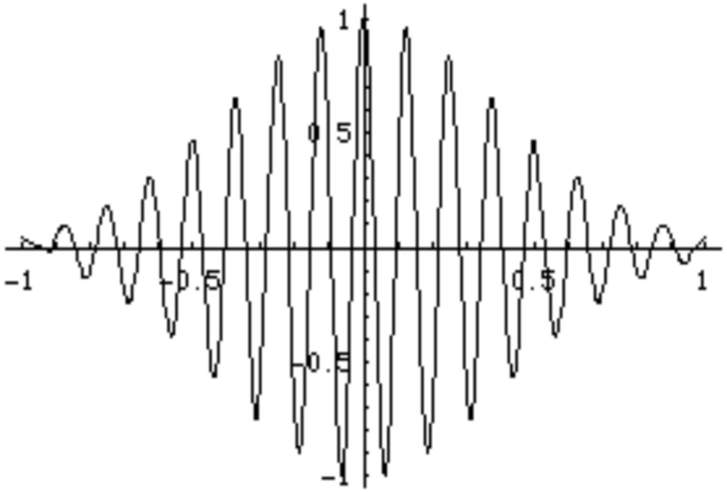
$$\phi_{tot}(t) = \omega_0 t - \Gamma_2 t^2 = \omega_0 t - \phi(t)$$ Temporal phase of the pulse

$$\omega(t) = \frac{\mathrm{d}\phi_{tot}(t)}{\mathrm{d}t} = \omega_0 - \frac{\mathrm{d}\phi(t)}{\mathrm{d}t}$$ Instantaneous frequency

Optical Metrology: Autocorrelation methods

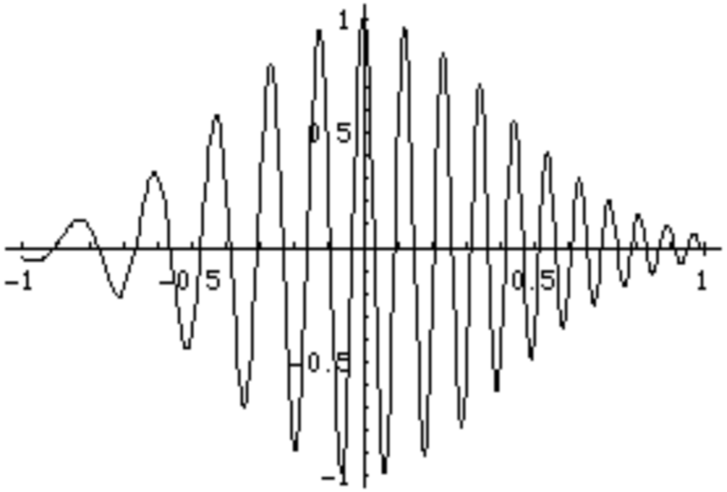
The laser pulse

Transform-limited pulse



$$\Gamma \equiv \Gamma_1 - i\Gamma_2$$
$$\Gamma_2 = 0$$

Chirped pulse



$$\Gamma \equiv \Gamma_1 - i\Gamma_2$$
$$\Gamma_2 \neq 0$$

$$\phi_{tot}(t) = \omega_0 t - \Gamma_2 t^2 = \omega_0 t - \phi(t)$$

Temporal phase of the pulse

$$\omega(t) = \frac{\mathrm{d}\phi_{tot}(t)}{\mathrm{d}t} = \omega_0 - \frac{\mathrm{d}\phi(t)}{\mathrm{d}t}$$

Instantaneous frequency

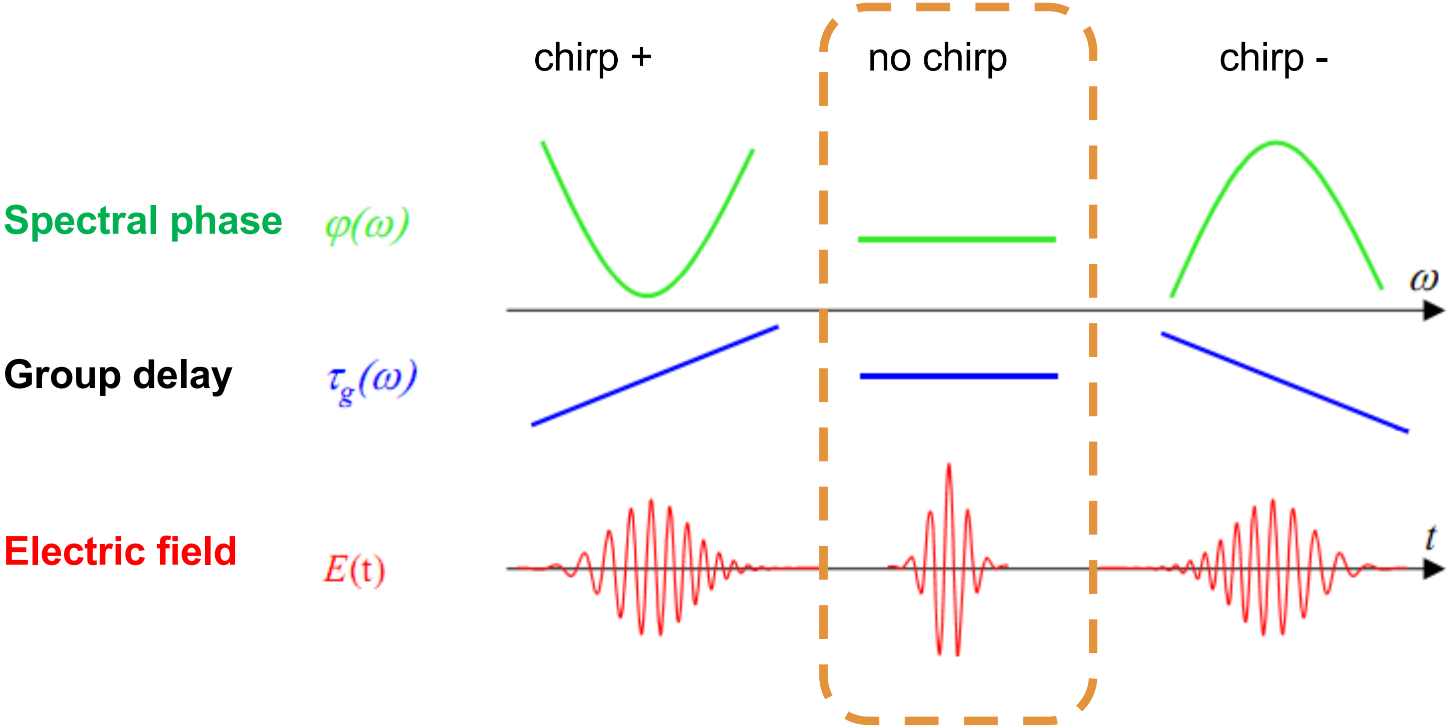
Optical Metrology: Autocorrelation methods

The laser pulse

$$E(t) = \sqrt{I(t)}exp(i\omega_0 t - i\phi(t)) \xleftrightarrow{\text{Fourier Transform}} \tilde{E}(\omega) = \sqrt{\overbrace{I(\omega-\omega_0)}^{\text{Intensity}}}exp(-i\overbrace{\varphi(\omega-\omega_0)}^{\text{Spectral phase}})$$

Instantaneous frequency: $\omega(t) = \dfrac{\mathrm{d}\phi_{tot}(t)}{\mathrm{d}t} = \omega_0 - \dfrac{\mathrm{d}\phi(t)}{\mathrm{d}t}$

Group delay: $\tau_g = \dfrac{\mathrm{d}\varphi}{\mathrm{d}\omega}$



No chirp:

All the *ω* *in phase*

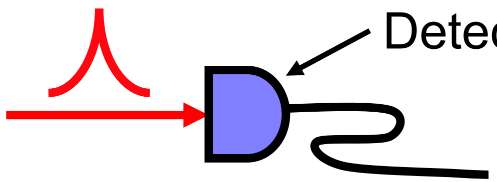
↓

minimum pulse duration

Measure pulse in time domain using photo-detectors

Photo-detectors are devices that emit electrons in response to photons.

Examples: Photo-diodes, Photo-multipliers



Detectors have very **slow** rise and fall times: ~ 1 nanosecond.

As far as we're concerned, detectors have **infinitely slow** responses.
They measure the time integral of the pulse intensity from $-\infty$ to $+\infty$:

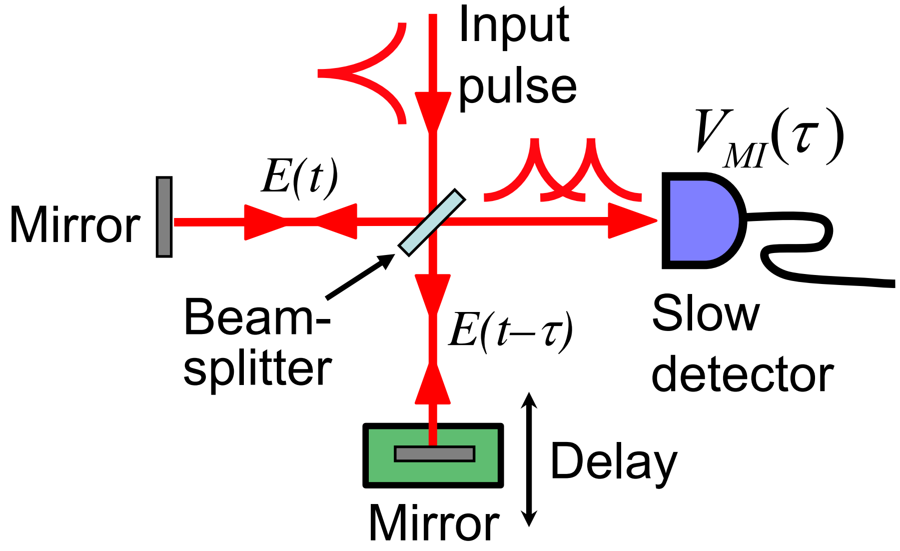
$$V_{detector} \propto \int_{-\infty}^{\infty} |E(t)|^2 \, dt$$

The detector output voltage is proportional to the pulse energy.
By themselves, detectors tell us little about a pulse.

Pulse measurement by field autocorrelation

$$V_{MI}(\tau) \propto \int_{-\infty}^{\infty} \left|E(t) - E(t-\tau)\right|^2 dt$$

$$= \int_{-\infty}^{\infty} \left|E(t)\right|^2 + \left|E(t-\tau)\right|^2 - 2\,\mathrm{Re}[E(t)E^*(t-\tau)]\; dt$$



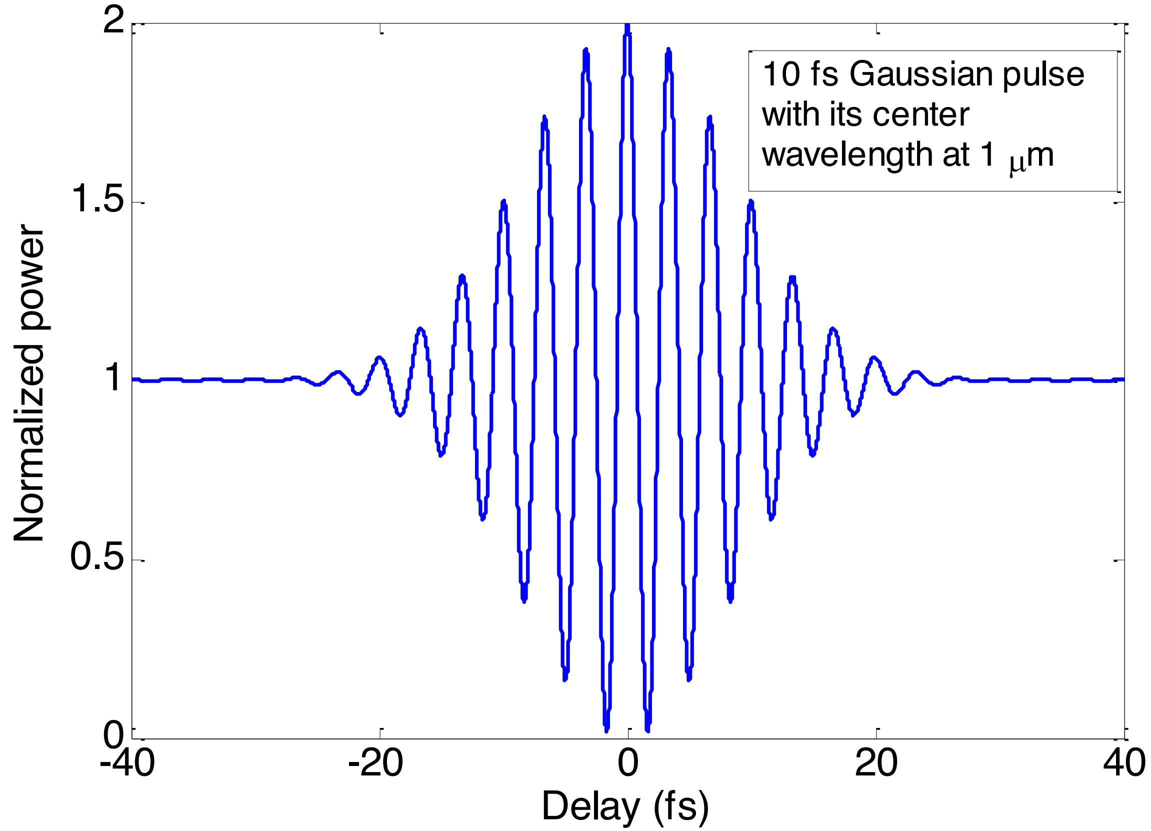
$$\Rightarrow \quad V_{MI}(\tau) \propto \underbrace{2\int_{-\infty}^{\infty} \left|E(t)\right|^2 dt}_{\propto \text{ Pulse energy}} - \underbrace{2\,\mathrm{Re}\int_{-\infty}^{\infty} E(t)E^*(t-\tau)\; dt}_{\text{Field autocorrelation}}$$

$$\mathrm{Re}\int_{-\infty}^{\infty} E(t)E^*(t-\tau)dt = \mathrm{Re}\,F^{-1}[E(\omega)E^*(\omega)] = \mathrm{Re}\,F^{-1}[I(\omega)]$$

<u>Field autocorrelation measurement is equivalent to measuring the spectrum.</u>

Comments on field correlation measurement

The information obtained from measuring electric field correlation and measuring the optical power spectrum is identical.



The correlation time is roughly the inverse of the optical bandwidth.

Field correlation measurement gives no information about the spectral phase.

Field correlation measurement cannot distinguish a transform-limited pulse from a longer chirped pulse with the same bandwidth.

Coherent ultrashort pulse and continuous-wave incoherent light (i.e., noise) with the same optical spectra give the same result.

How to measure both pulse intensity profile and the phase?

Result: Using only time-independent, linear filters, complete characterization of a pulse is **NOT** possible with a slow detector.

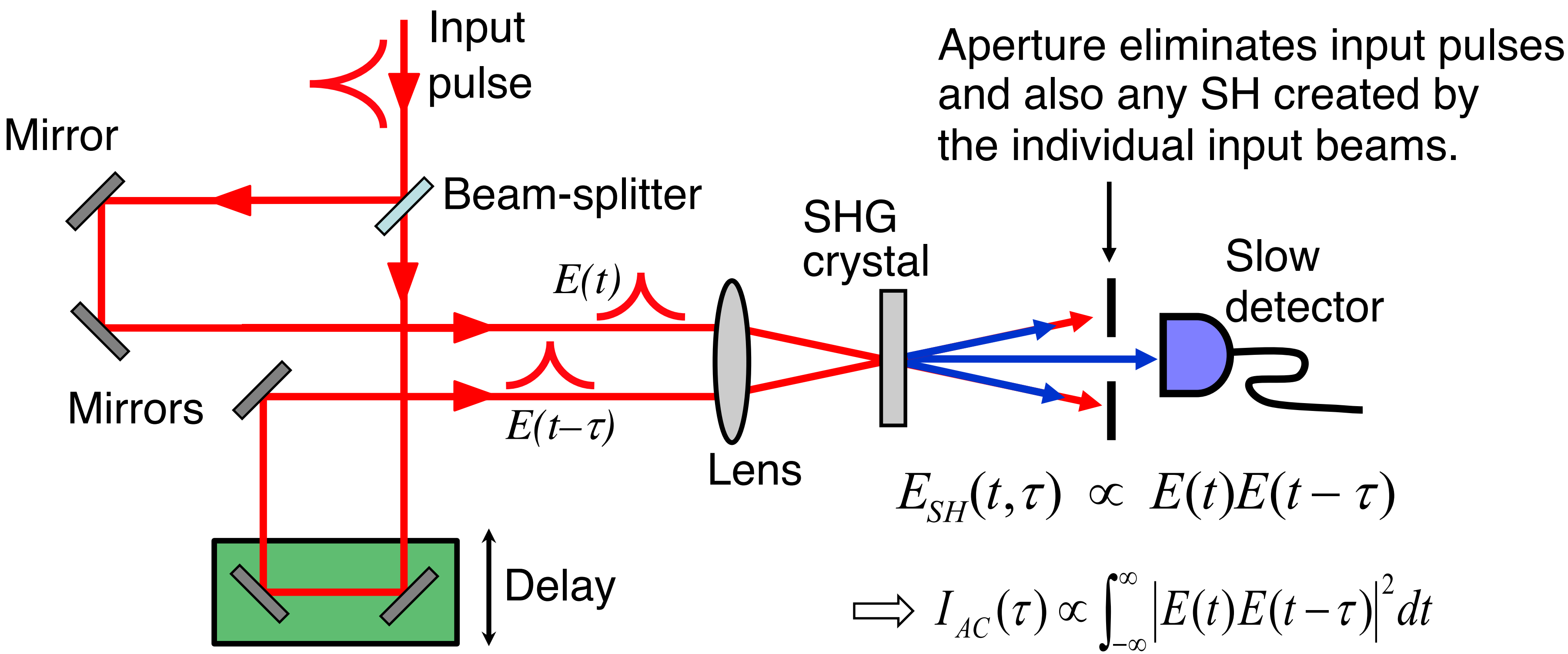
Translation: If you don't have a detector or modulator that is fast compared to the pulse width, you **CANNOT** measure the pulse intensity and phase with only linear measurements, such as a detector, interferometer, or a spectrometer.

V. Wong & I. A. Walmsley, Opt. Lett. **19**, 287-289 (1994)
I. A. Walmsley & V. Wong, J. Opt. Soc. Am B, **13**, 2453-2463 (1996)

We need a shorter event, and we don’t have one.
But we do have the pulse itself, which is a start.
And we can devise methods for the pulse to gate itself using optical nonlinearities.

Background-free intensity autocorrelation

Crossing beams in an second-harmonic generation (SHG) crystal, varying the delay between them, and measuring the second-harmonic (SH) pulse energy vs. delay yields the **Intensity Autocorrelation**:



$$E_{SH}(t,\tau) \propto E(t)E(t-\tau)$$

$$\Rightarrow I_{AC}(\tau) \propto \int_{-\infty}^{\infty} \left|E(t)E(t-\tau)\right|^2 dt$$

The Intensity Autocorrelation:

$$\boxed{I_{AC}(\tau) \propto \int_{-\infty}^{\infty} I(t)I(t-\tau)dt}$$

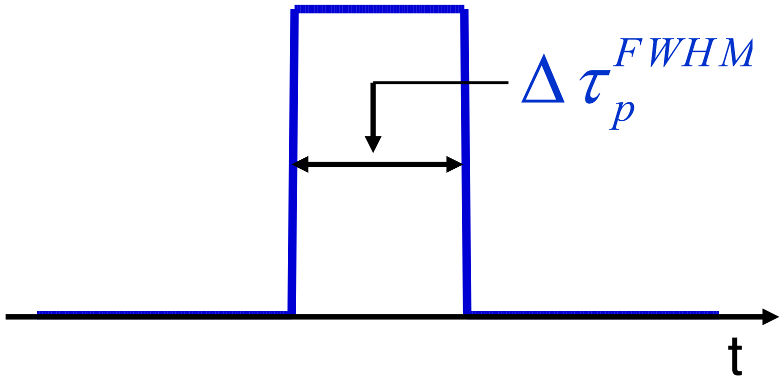
Square pulse and its autocorrelation

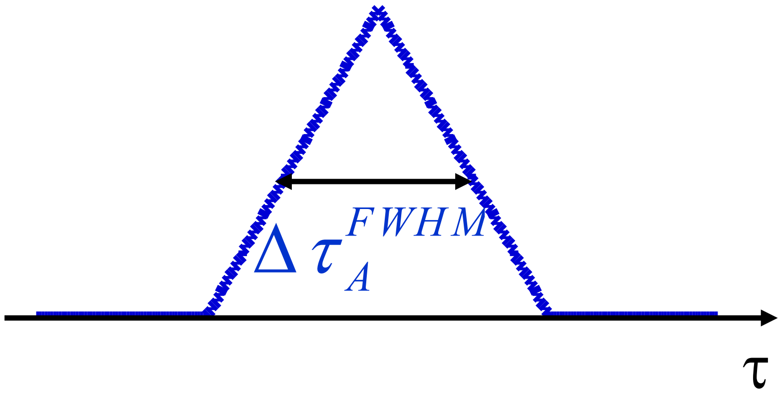
Pulse

Autocorrelation

$$I(t) = \begin{cases} 1; & |t| \leq \Delta\tau_p^{FWHM}/2 \\ 0; & |t| > \Delta\tau_p^{FWHM}/2 \end{cases}$$

$$A^{(2)}(\tau) = \begin{cases} 1 - \left|\dfrac{\tau}{\Delta\tau_A^{FWHM}}\right|; & |\tau| \leq \Delta\tau_A^{FWHM} \\ 0; & |\tau| > \Delta\tau_A^{FWHM} \end{cases}$$





$$\Delta\tau_A^{FWHM} = \Delta\tau_p^{FWHM}$$

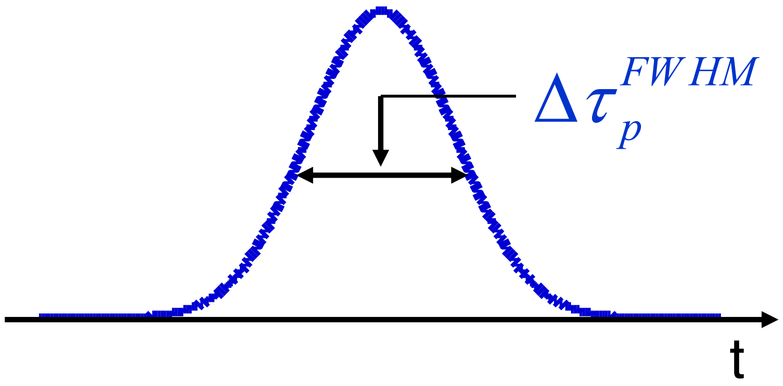
Gaussian pulse and its autocorrelation

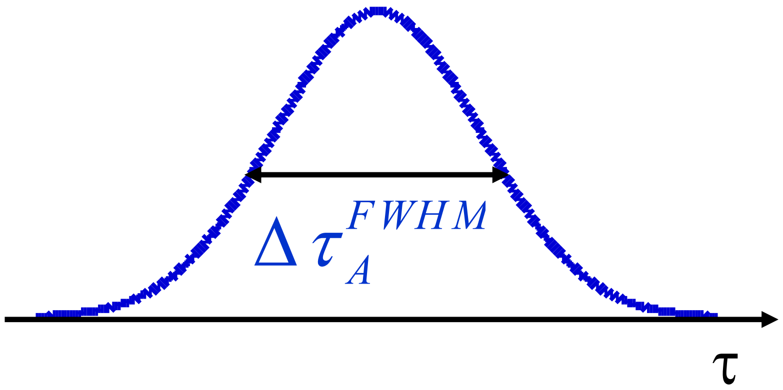
Pulse

Autocorrelation

$$I(t) = \exp\left[-\left(\frac{2\sqrt{\ln 2}\,t}{\Delta\tau_p^{FWHM}}\right)^2\right]$$

$$A^{(2)}(\tau) = \exp\left[-\left(\frac{2\sqrt{\ln 2}\,\tau}{\Delta\tau_A^{FWHM}}\right)^2\right]$$





$$\Delta\tau_A^{FWHM} = 1.41\,\Delta\tau_p^{FWHM}$$

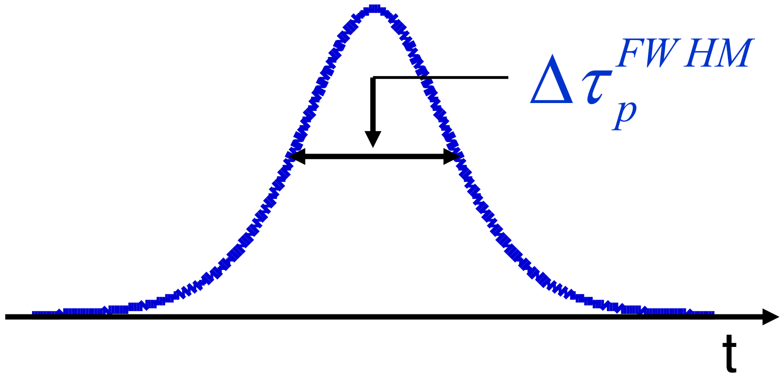
Sech^2 pulse and its autocorrelation

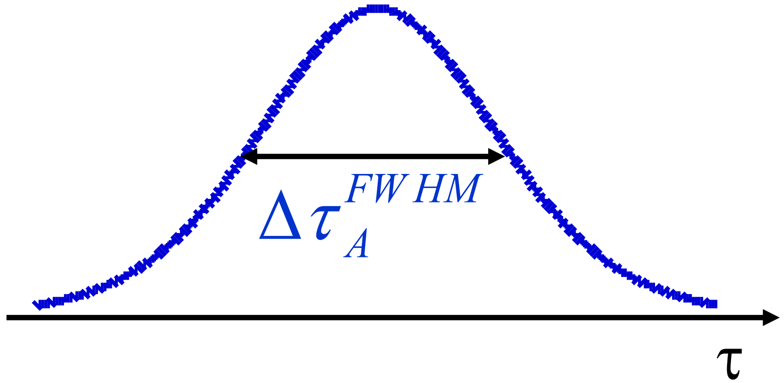
Pulse

$$I(t) = \operatorname{sech}^2\left[\frac{1.7627\,t}{\Delta t_p^{FWHM}}\right]$$

Autocorrelation

$$A^{(2)}(\tau) = \frac{3}{\sinh^2\left(\dfrac{2.7196\tau}{\Delta\tau_A^{FWHM}}\right)}\left[\frac{2.7196\tau}{\Delta\tau_A^{FWHM}}\coth\left(\frac{2.7196\tau}{\Delta\tau_A^{FWHM}}\right)-1\right]$$





$$\Delta\tau_A^{FWHM} = 1.54\,\Delta\tau_p^{FWHM}$$

Theoretical models for passively mode-locked lasers often predict sech^2 pulse shapes.

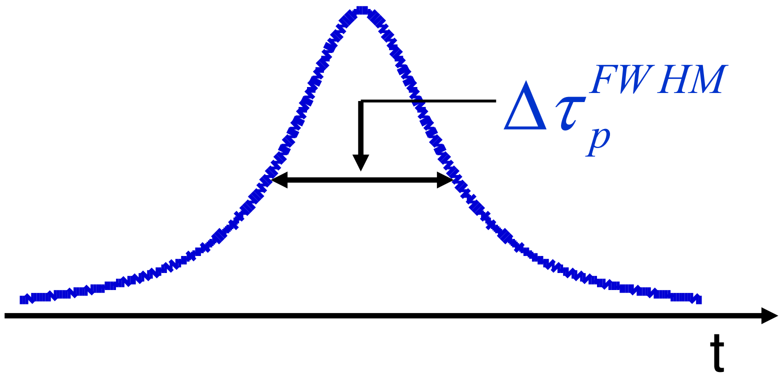
Lorentzian Pulse and Its Autocorrelation

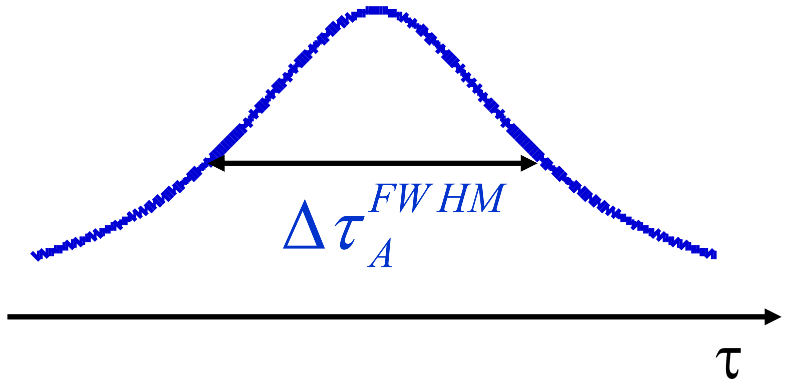
Pulse

Autocorrelation

$$I(t) = \frac{1}{1+\left(2t/\Delta\tau_p^{FWHM}\right)^2}$$

$$A^{(2)}(\tau) = \frac{1}{1+\left(2\tau/\Delta\tau_A^{FWHM}\right)^2}$$



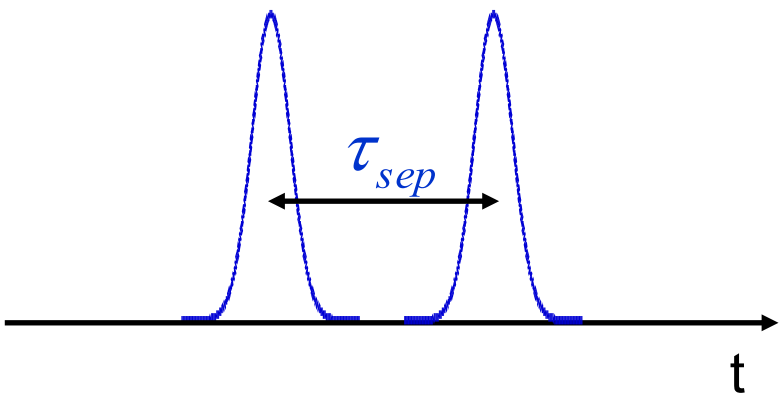


$$\Delta\tau_A^{FWHM} = 2.0\,\Delta\tau_p^{FWHM}$$

Double pulse and Its Autocorrelation

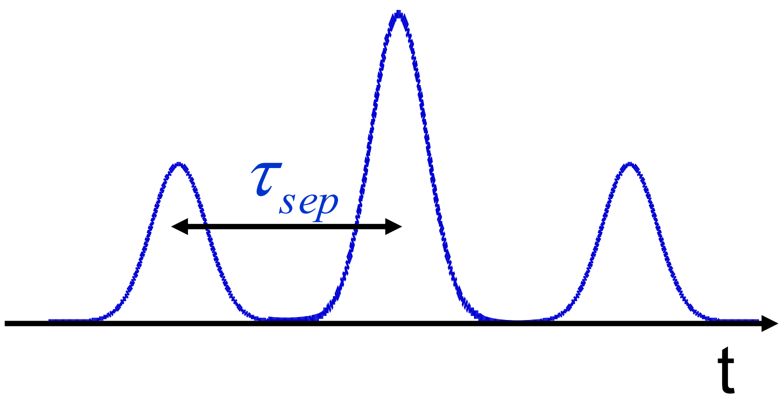
Pulse

$$I(t) = I_0(t) + I_0(t+\tau_{sep})$$



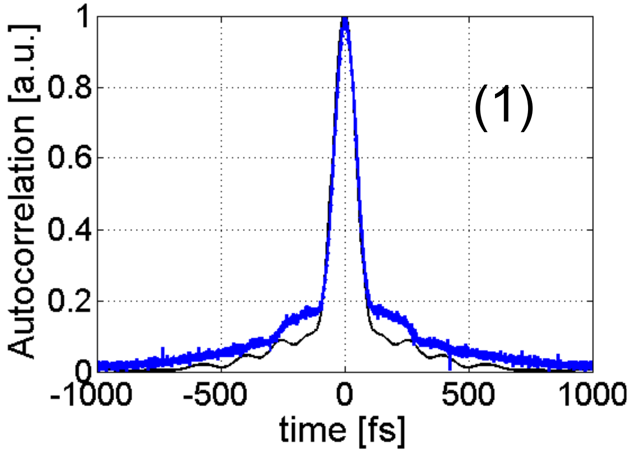
Autocorrelation

$$A^{(2)}(\tau) = A_0^{(2)}(\tau+\tau_{sep}) + 2A_0^{(2)}(\tau) + A_0^{(2)}(\tau-\tau_{sep})$$

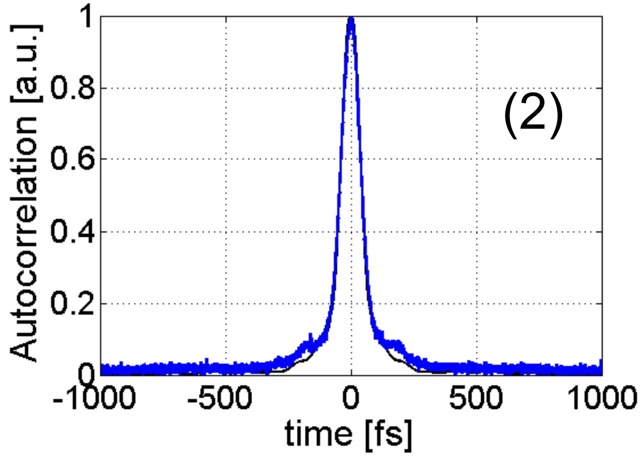


where: $$A_0^{(2)}(\tau) = \int I_0(t)\, I_0(t-\tau)\, dt$$

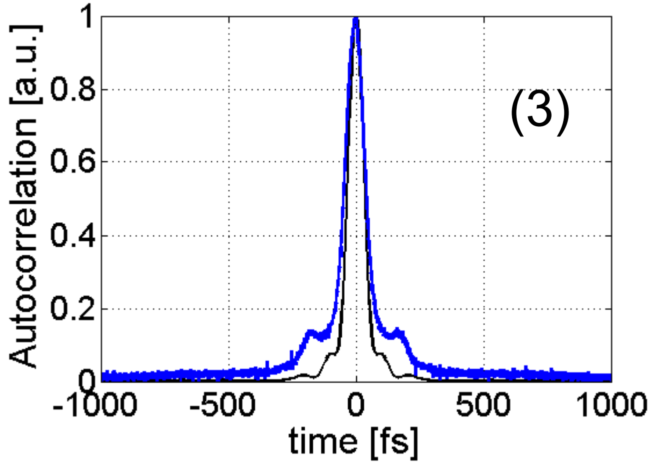
Optimizing the amplifier system using intensity autocorrelation measurement



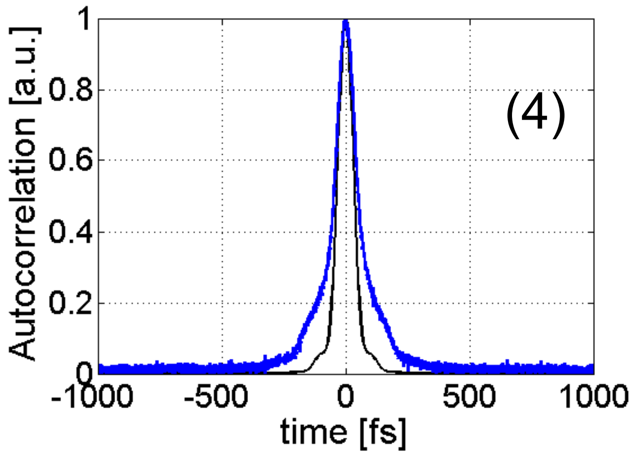
20740 fs^2



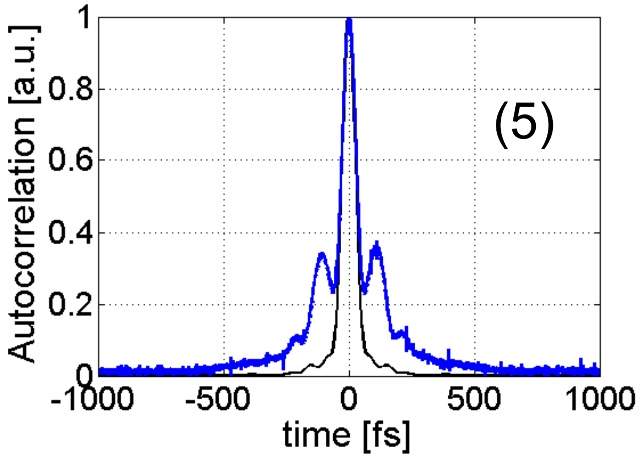
16130 fs^2



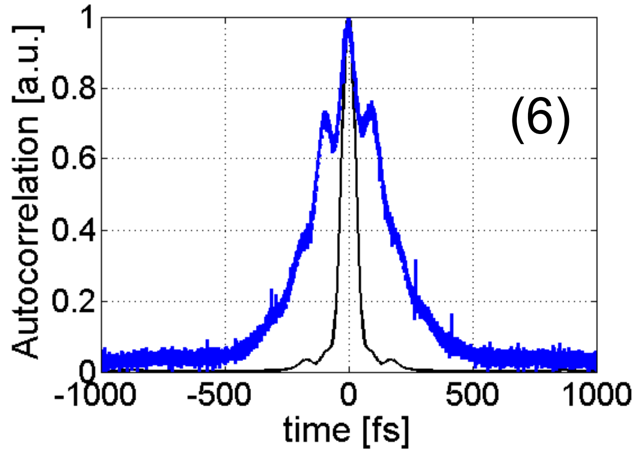
8725 fs^2



4550 fs^2



0



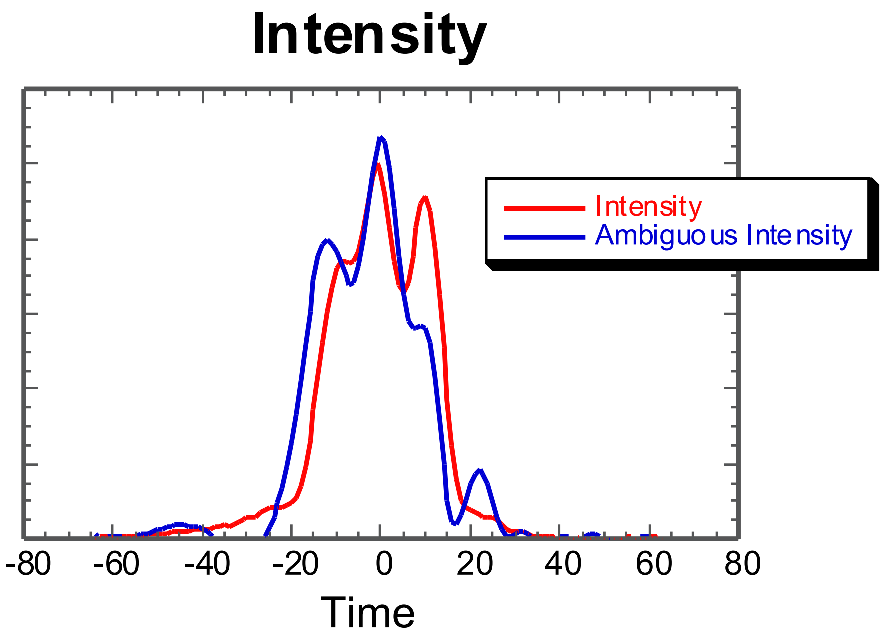
-3840 fs^2

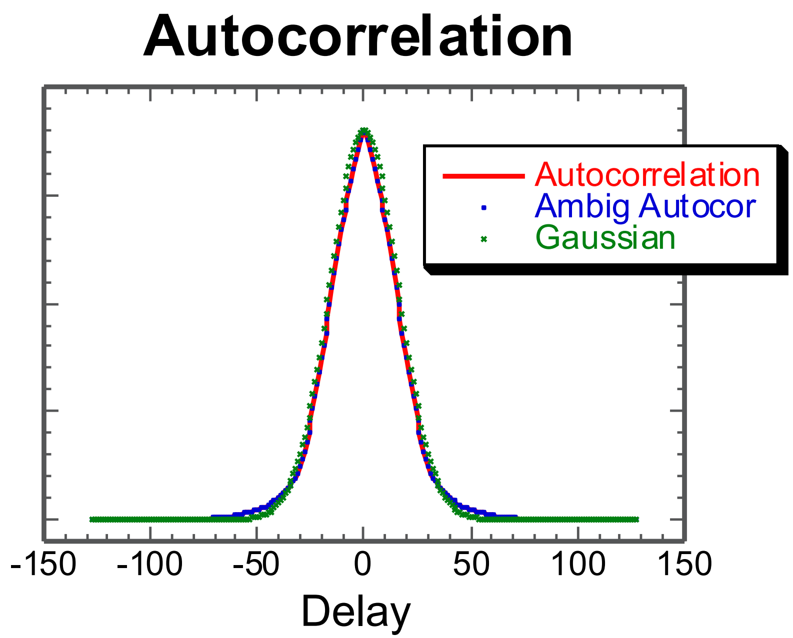
W. Liu, et al. “Pre-chirp managed nonlinear amplification in fibers delivering 100 W, 60 fs pulse” Opt. Lett. 40, 151 (2015).

Properties of intensity autocorrelation

Caution: Autocorrelation is not un-ambiguous!

These complex intensities have nearly Gaussian autocorrelations !





Autocorrelation contains only partial phase information,

- must be applied carefully
- fails for complex pulses containing higher order phase terms...

Properties of intensity autocorrelation

1) It is always symmetric, and assumes its maximum value at $\tau = 0$.

$$I_{AC}(\tau) \propto \int_{-\infty}^{\infty} I(t) I(t-\tau) dt \qquad I_{AC}(\tau) = I_{AC}(-\tau)$$

2) Width of the correlation peak gives information about the pulse width.

3) Pulse phase information is missing from the background-free Intensity autocorrelation.

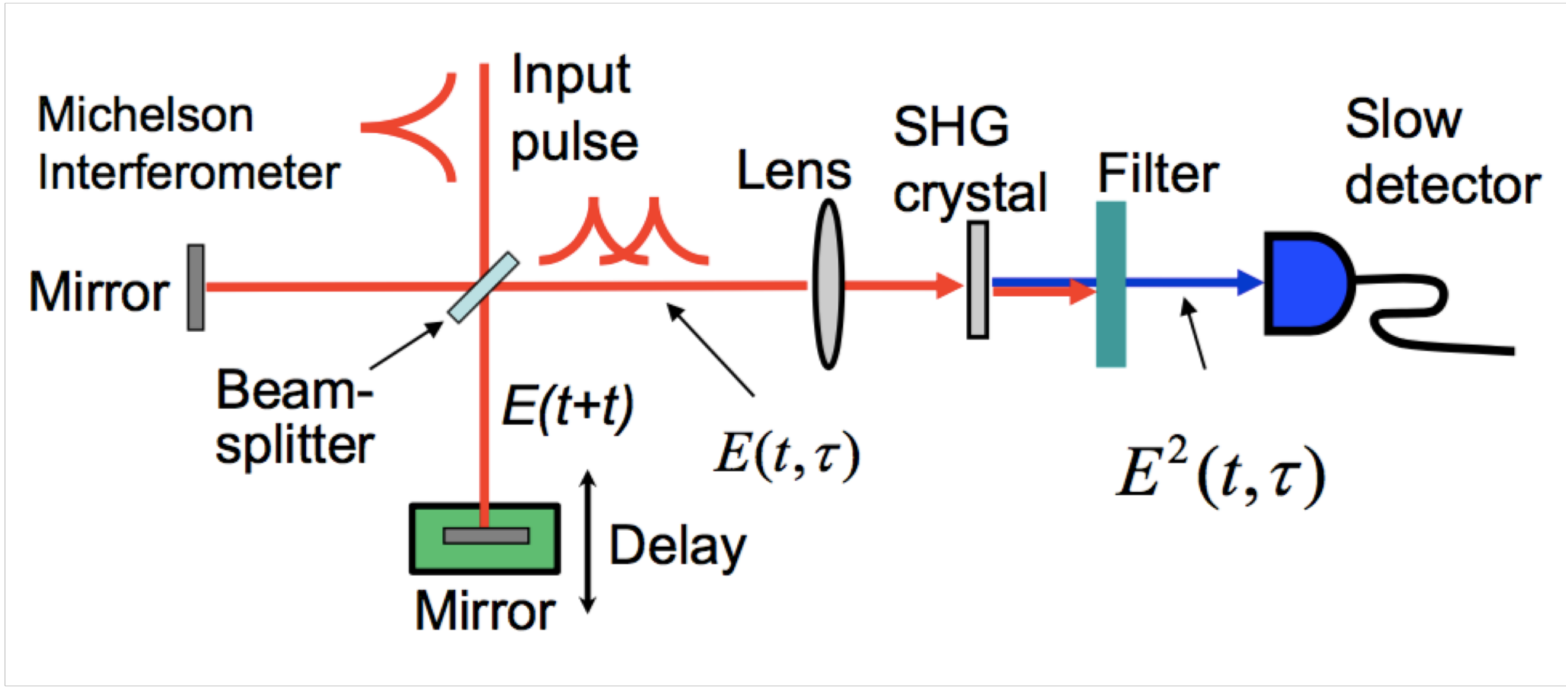
4) Intensity autocorrelation trace is broader than the pulse itself. To get the pulse duration, it is necessary to assume a pulse shape, and to use the corresponding deconvolution factor.

4) For short pulses, very thin crystals must be used to guarantee enough phase-matching bandwidth. This reduces the efficiency and hence the sensitivity of the device.

5) Conversion efficiency must be kept low, or distortions due to "depletion" of input light fields will occur.

6) The Intensity autocorrelation is **not** sufficient to determine the intensity profile.

Interferometric autocorrelation



An alternative approach is to use a collinear beam geometry, and allow the autocorrelator signal light to interfere with the SHG from each individual beam

$$IA^{(2)}(\tau) \equiv \int_{-\infty}^{\infty} \left| [E(t) - E(t-\tau)]^2 \right|^2 dt$$

$$IA^{(2)}(\tau) \equiv \int_{-\infty}^{\infty} \left| \underbrace{E^2(t) + E^2(t-\tau)}_{\text{New terms}} - 2\underbrace{E(t)E(t-\tau)}_{\text{Autocorrelation term}} \right|^2 dt$$

Interferometric autocorrelation

$$IA^{(2)}(\tau) \equiv \int_{-\infty}^{\infty} \left[E^2(t) + E^2(t-\tau) - 2E(t)E(t-\tau)\right]\left[E^{*2}(t) + E^{*2}(t-\tau) - 2E^*(t)E^*(t-\tau)\right] dt$$

$$\begin{aligned} IA^{(2)}(\tau) = \int_{-\infty}^{\infty} \Big\{ & \left|E^2(t)\right|^2 + E^2(t)E^{*2}(t-\tau) - 2E^2(t)E^*(t)E^*(t-\tau) + \\ & E^2(t-\tau)E^{*2}(t) + \left|E^2(t-\tau)\right|^2 - 2E^2(t-\tau)E^*(t)E^*(t-\tau) + \\ & -2E(t)E(t-\tau)E^{*2}(t) - 2E(t)E(t-\tau)E^{*2}(t-\tau) + 4\left|E(t)\right|^2\left|E(t-\tau)\right|^2 \Big\}\, dt \\ = \int_{-\infty}^{\infty} \Big\{ & I^2(t) + E^2(t)E^{*2}(t-\tau) - 2I(t)E(t)E^*(t-\tau) + \\ & E^2(t-\tau)E^{*2}(t) + I^2(t-\tau) - 2I(t-\tau)E^*(t)E(t-\tau) + \\ & -2I(t)E(t-\tau)E^*(t) - 2I(t-\tau)E(t)E^*(t-\tau) + 4I(t)I(t-\tau) \Big\}\, dt \end{aligned}$$

Where: $I(t) \equiv \left|E(t)\right|^2$

Interferometric autocorrelation

From the math we can extract 4 terms:

$$= \int_{-\infty}^{\infty} I^2(t) + I^2(t-\tau)\, dt \quad = I_{back}$$

Background

$$+\ 4\int_{-\infty}^{\infty} I(t)I(t-\tau)\, dt \quad = I_{int}$$

Intensity autocorrelation

$$-\ 2\int_{-\infty}^{\infty} \left[I(t) + I(t-\tau)\right] E(t)E^*(t-\tau)\, dt \ +\ c.c \quad = I_{\omega}$$

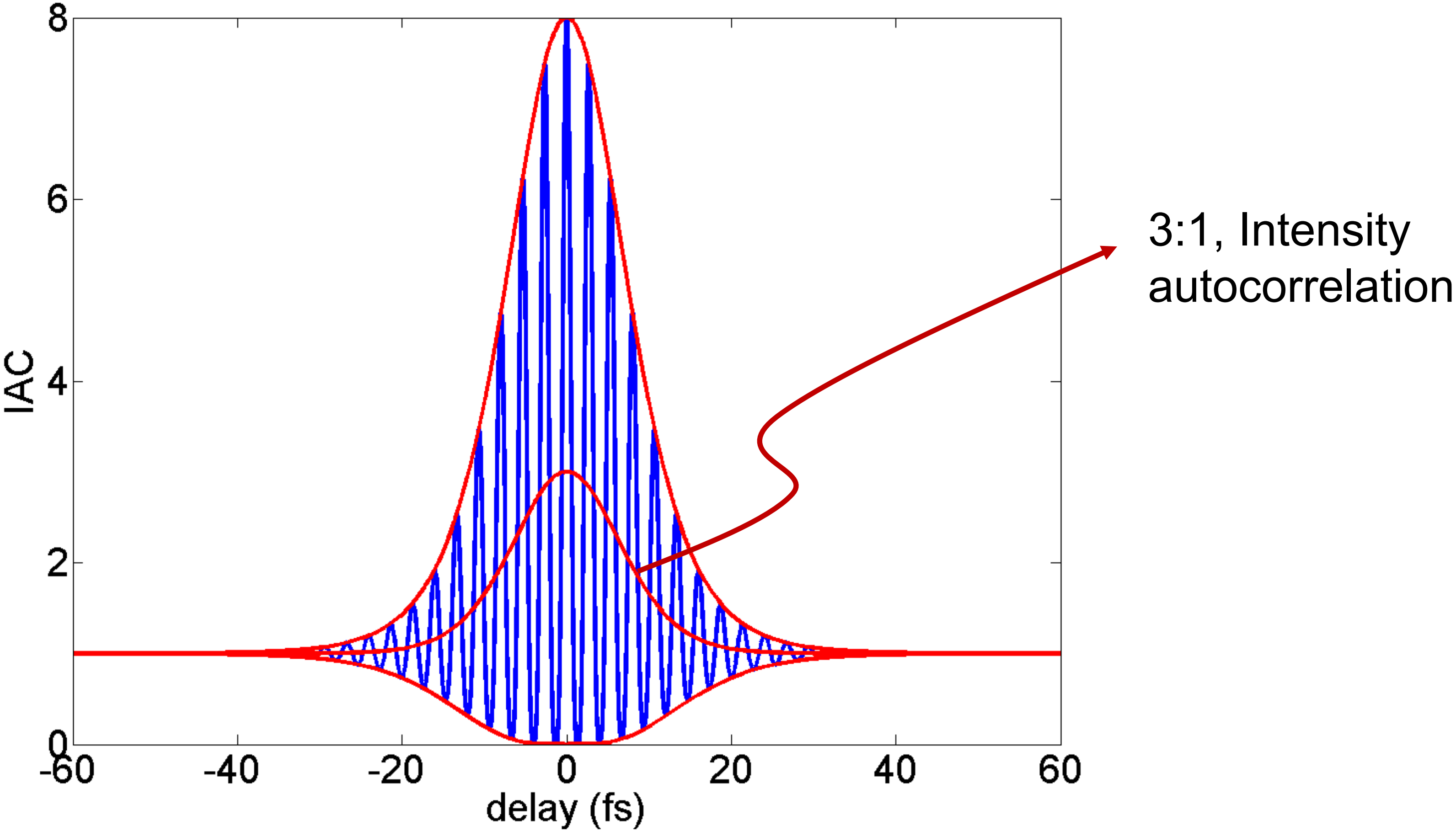
Interferogram of E(t), oscillating at ω

$$+\int_{-\infty}^{\infty} E^2(t)E^{2*}(t-\tau)\, dt \ +\ c.c. \quad = I_{2\omega}$$

Interferogram of the SH oscillating at 2ω

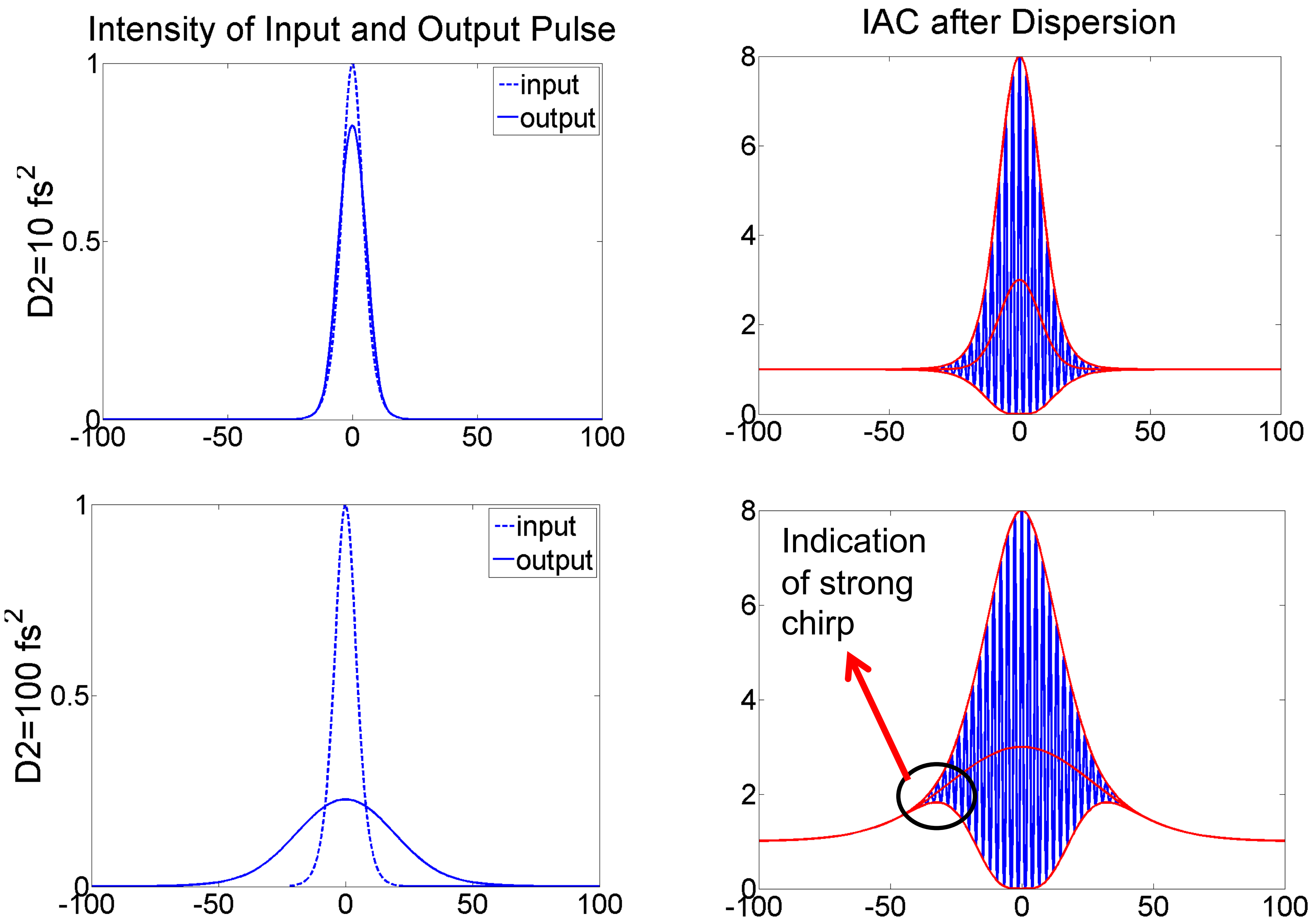
$IA^{(2)}(\tau = 0) = 8$ $\qquad$ $IA^{(2)}(\tau \rightarrow \infty) = 1$

IAC of 10 fs Sech-shaped pulse

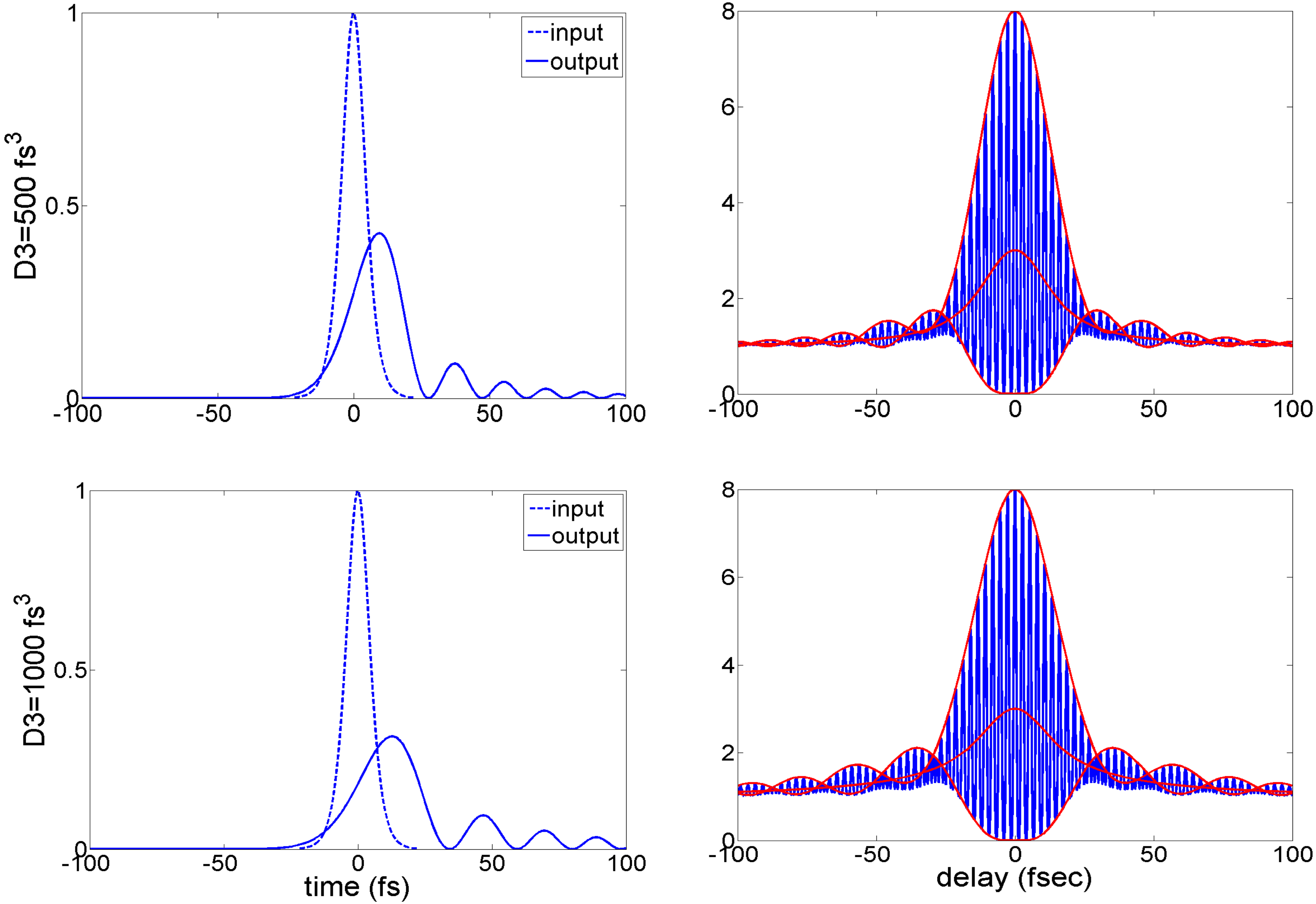


The interferometric autocorrelation simply combines several measures of the pulse into one (admittedly complex) trace. Conveniently, however, they occur with different oscillation frequencies: 0, ω, and 2ω.

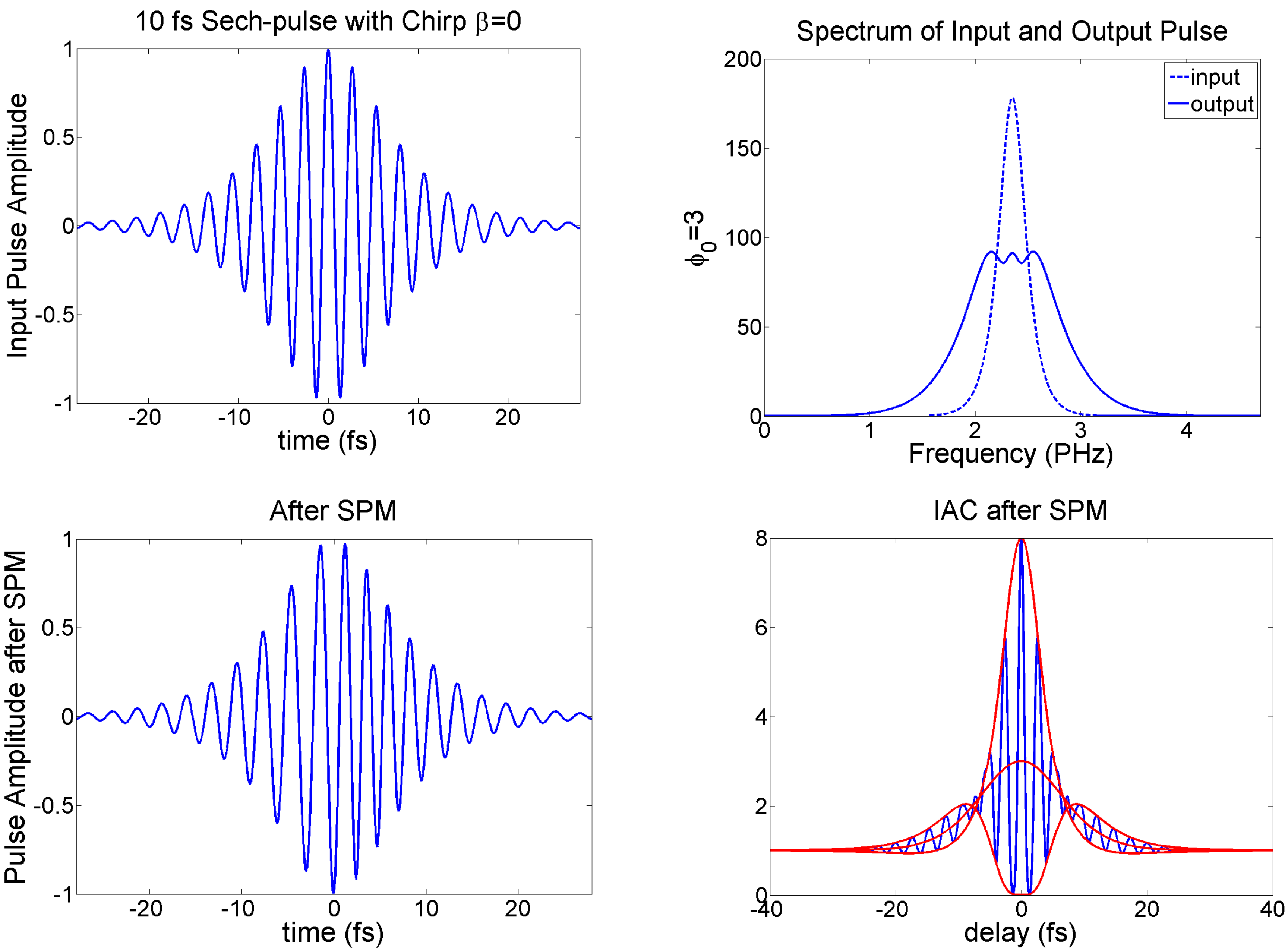
Effects of second-order dispersion



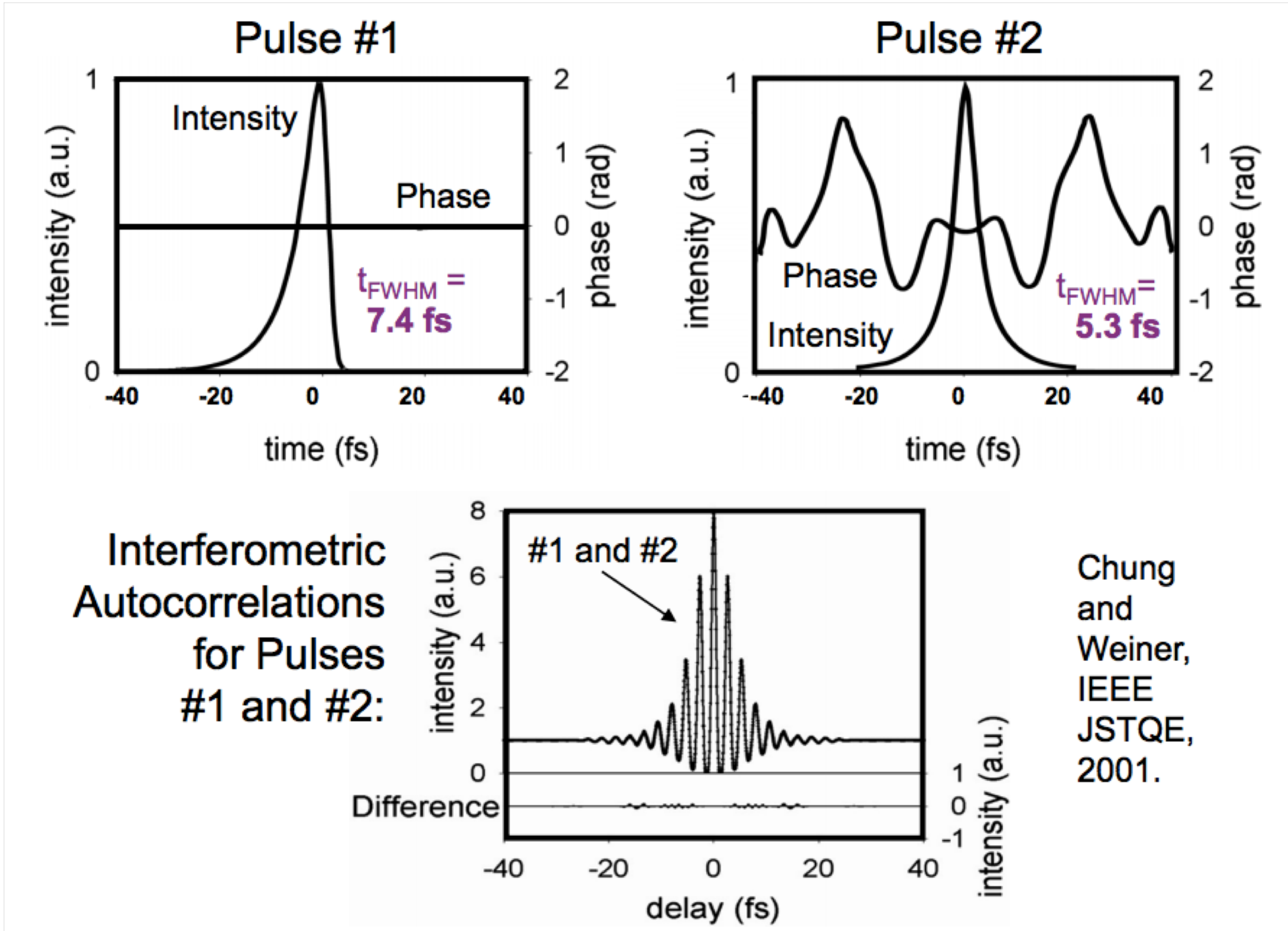
Effects of third-order dispersion



Effects of self-phase modulation



Interferometric autocorrelation



Chung and Weiner, IEEE JSTQE, 2001.

Interferometric autocorrelation also have ambiguities

Properties of interferometric autocorrelation

- It is always symmetric and the peak-to-background ratio should be 8.
- This device is difficult to align; there are several sensitive degrees of freedom in aligning two collinear pulses.
- Dispersion in each arm must be the same, so it is necessary to insert a compensator plate in one arm.
- Using optical spectrum and background-free intensity autocorrelator can determine the presence or absence of strong chirp. The interferometric autocorrelation serves as a clear visual indication of moderate to large chirp.
- It is difficult to distinguish between different pulse shapes and, especially, different phases from interferometric autocorrelations.
- Like the intensity autocorrelation, it must be curve-fit to an assumed pulse shape and so should only be used for rough estimates.

How to measure both pulse intensity profile and the phase?

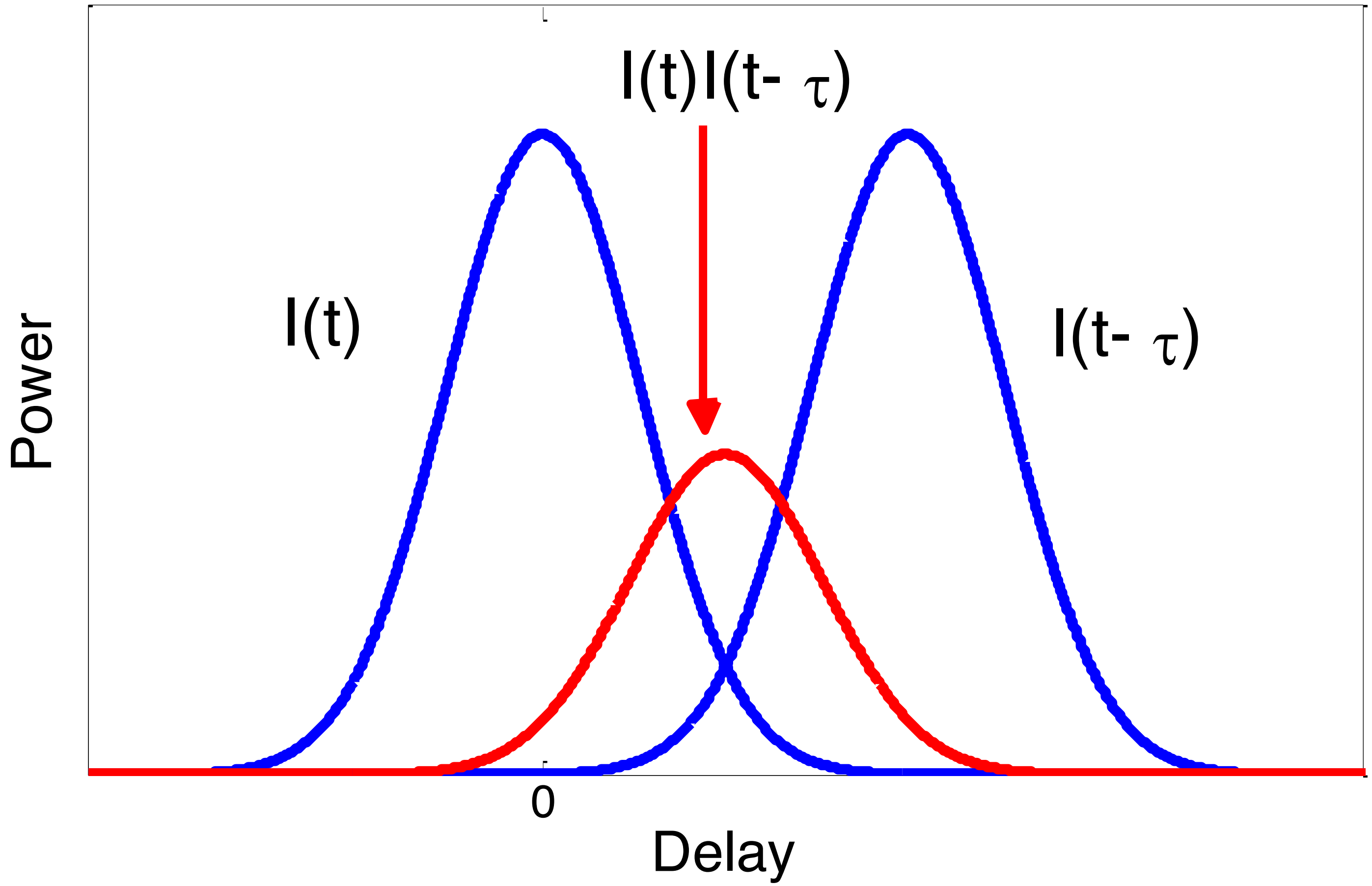
1) A pulse can be represented by two arrays of data with length N, one for the amplitude/intensity and the other for the phase. Totally we have 2N degrees of freedom (corresponding to the real and imaginary parts for the electric field).

2) Intensity autocorrelator provides only one array of data with length N. Optical spectrum measurement can provide another array of data with length N. However some information, especially about phase, is missing from both measurements.

3) Need to have more data, providing enough redundancy to recover the both the amplitude and phase.

How to generate more data (information) from intensity autocorrelation measurement?

Pulse gating in background-free intensity autocorrelation



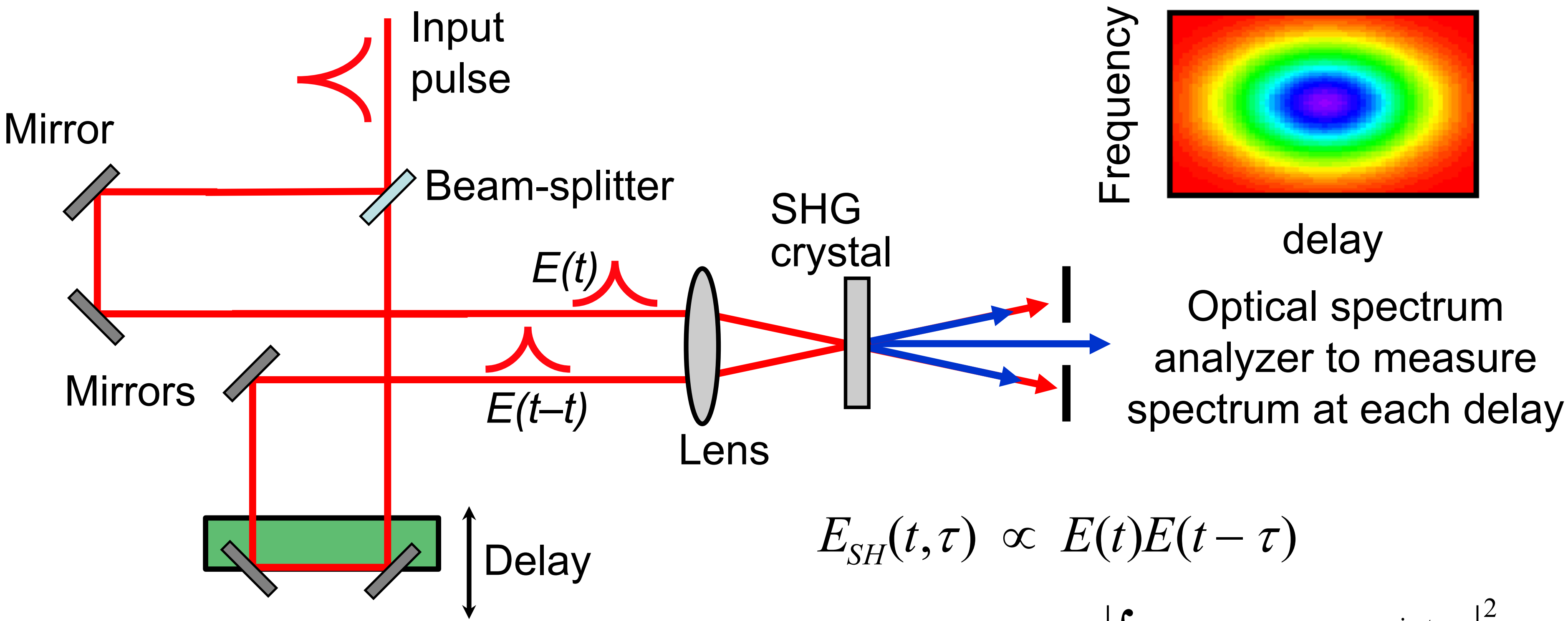
Varying the delay yields varying overlap between the two replicas of the pulse.

The intensity autocorrelation is only nonzero when the pulses overlap.

<u>How about measuring the spectrum of the autocorrelation pulse at each delay?</u>

Frequency-Resolved Optical Gating (FROG): SHG-FROG

Background-free intensity autocorrelator + optical spectrum analyzer

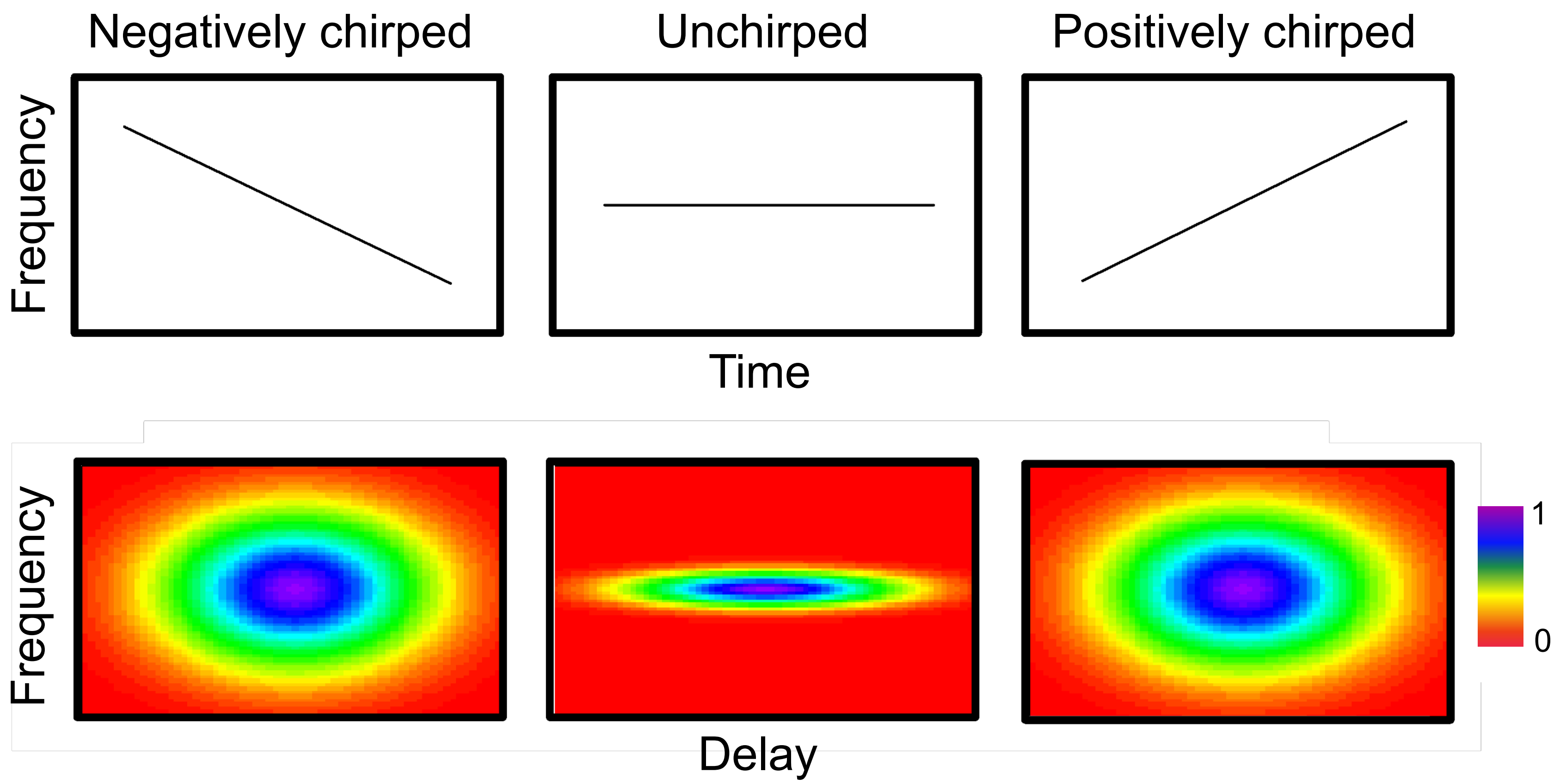


$$E_{SH}(t,\tau) \propto E(t)E(t-\tau)$$

$$\Rightarrow \quad I_{FROG}(\omega,\tau) \propto \left| \int E(t)E(t-\tau)e^{j\omega t}dt \right|^2$$

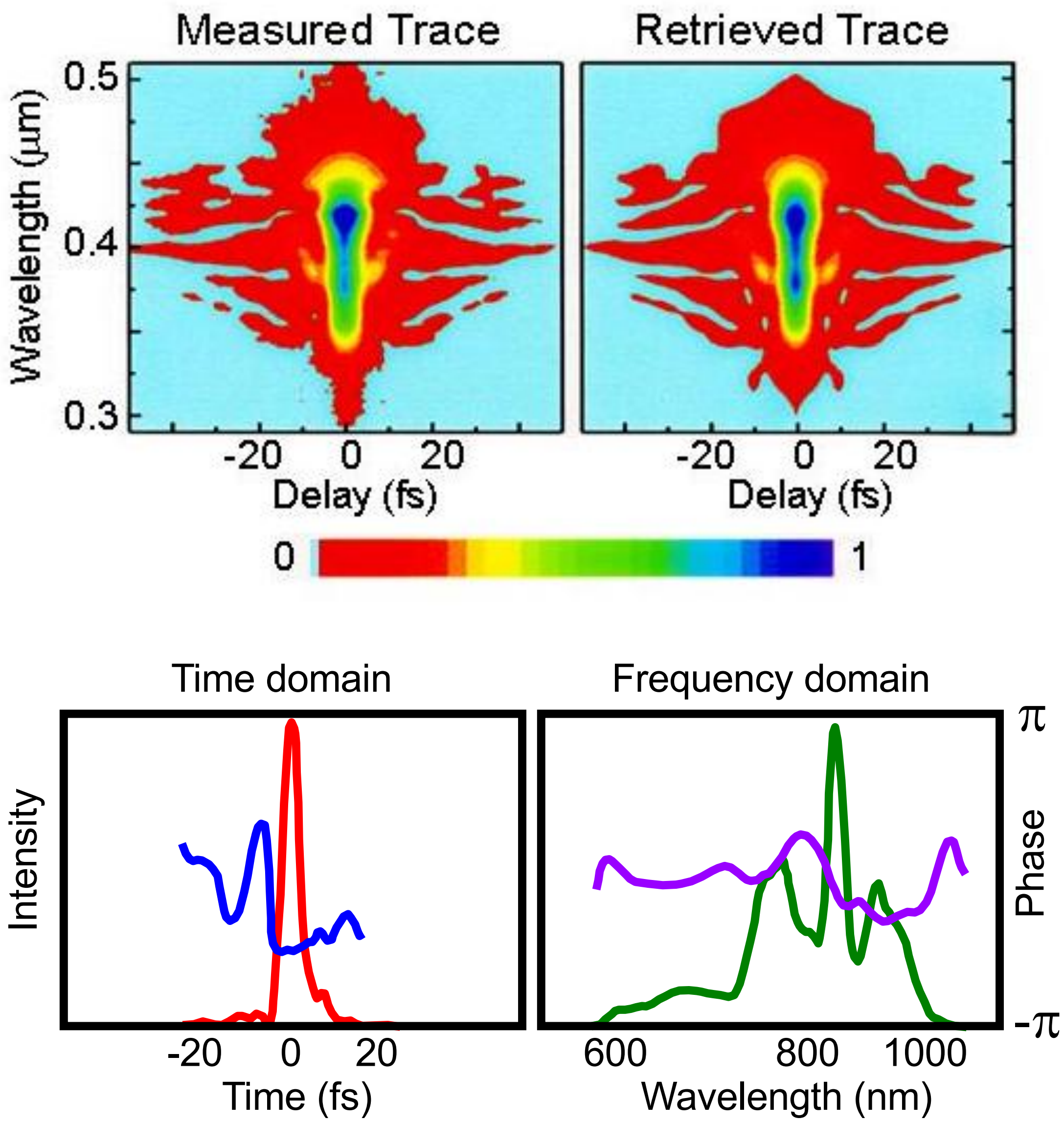
Now we have N X N data points. Iterative algorithm can retrieve both the amplitude and phase of the measured optical pulse.

SHG FROG traces are symmetrical with respect to delay



SHG FROG has an ambiguity in the direction of time, but it can be removed.

SHG FROG measurements of a 4.5-fs pulse

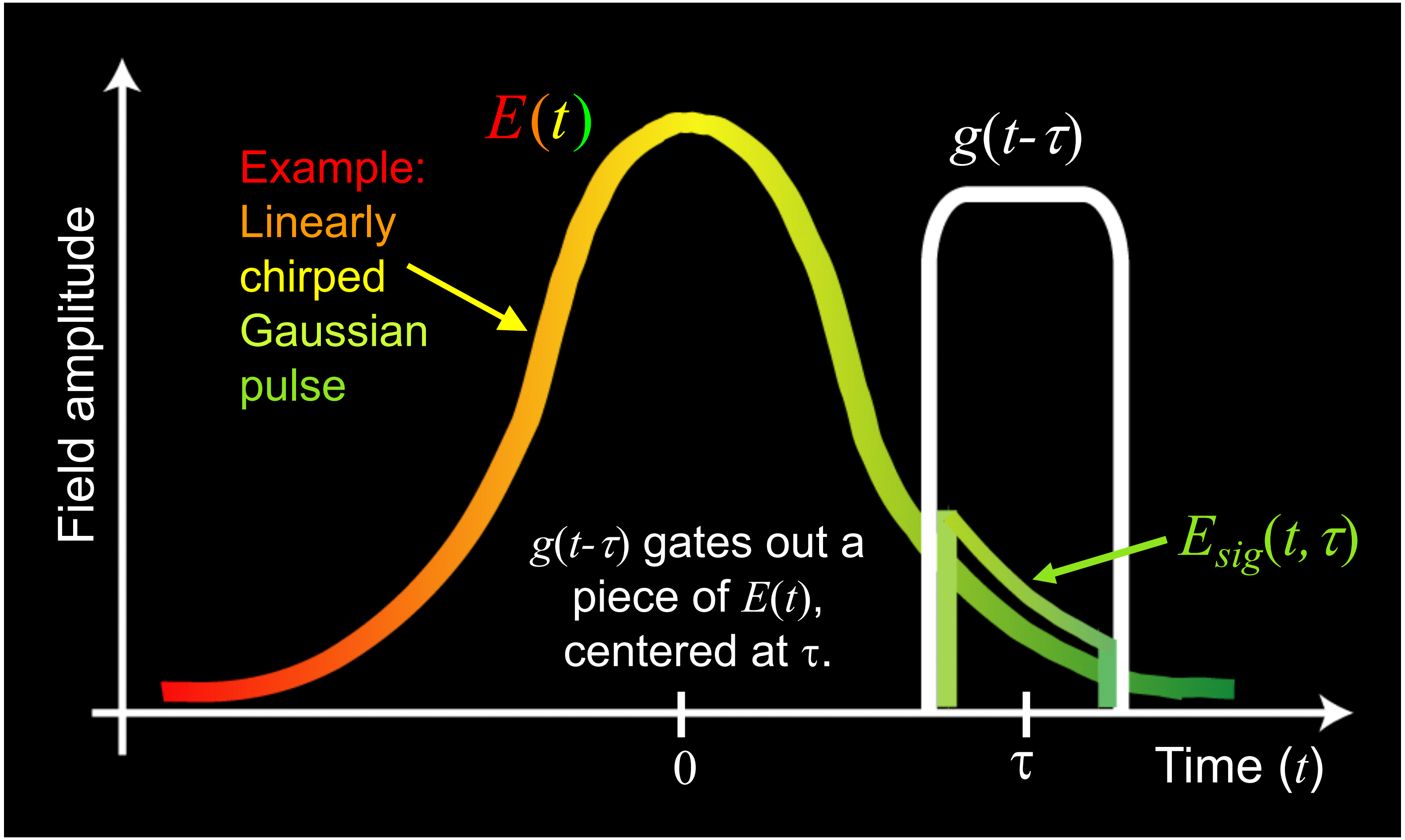


Agreement between the experimental and reconstructed FROG traces provides a nice check on the measurement.

Baltuska, Pshenichnikov, and Weirsma, J. Quant. Electron., 35, 459 (1999).

Spectrogram of a pulse in general

We must compute the spectrum of the product: $E(t)\ g(t-\tau)$



The spectrogram tells the color and intensity of $E(t)$ at the time, τ.

Mathematical form of a spectrogram

If $E(t)$ is the waveform of interest, its spectrogram is:

$$\boxed{\Sigma_E(\omega,\tau) \equiv \left| \int_{-\infty}^{\infty} E(t)\, g(t-\tau)\, \exp(-i\omega t)\, dt \right|^2}$$

where $g(t\text{-}\tau)$ is a variable-delay gate function and τ is the delay.

Without $g(t\text{-}\tau)$, $\Sigma_E(\omega,\tau)$ would simply be the spectrum.

The spectrogram is a function of ω and τ.
It is the set of spectra of all temporal slices of $E(t)$.

Properties of spectrogram

1) Algorithms exist to retrieve $E(t)$ from its spectrogram.

2) The spectrogram essentially uniquely determines the waveform intensity, $I(t)$, and phase, $\phi(t)$.

There are a few ambiguities, but they're "trivial."

3) The gate need not be—and should not be—much shorter than $E(t)$.

Suppose we use a delta-function gate pulse:

$$\left| \int_{-\infty}^{\infty} E(t)\, \delta(t-\tau) \exp(-i\omega t)\, dt \right|^2 = \left| E(\tau) \exp(-i\omega\tau) \right|^2$$

$$= \left| E(\tau) \right|^2$$ **= The Intensity.**

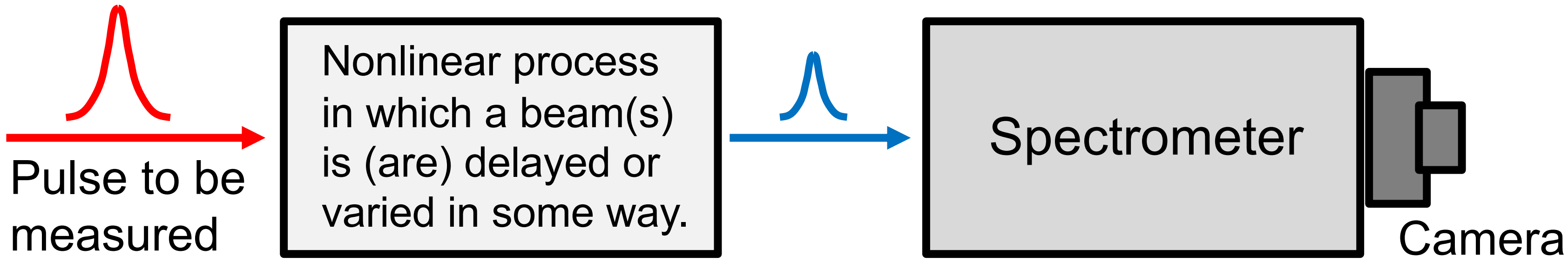
No phase information!

<u>The spectrogram resolves the dilemma! It doesn't need the shorter event! It temporally resolves the slow components and spectrally resolves the fast components.</u>

Frequency-Resolved Optical Gating (FROG)

FROG using arbitrary nonlinear-optical interactions

FROG is simply a frequency-resolved nonlinear-optical signal that's a function of time and delay (or another variable)



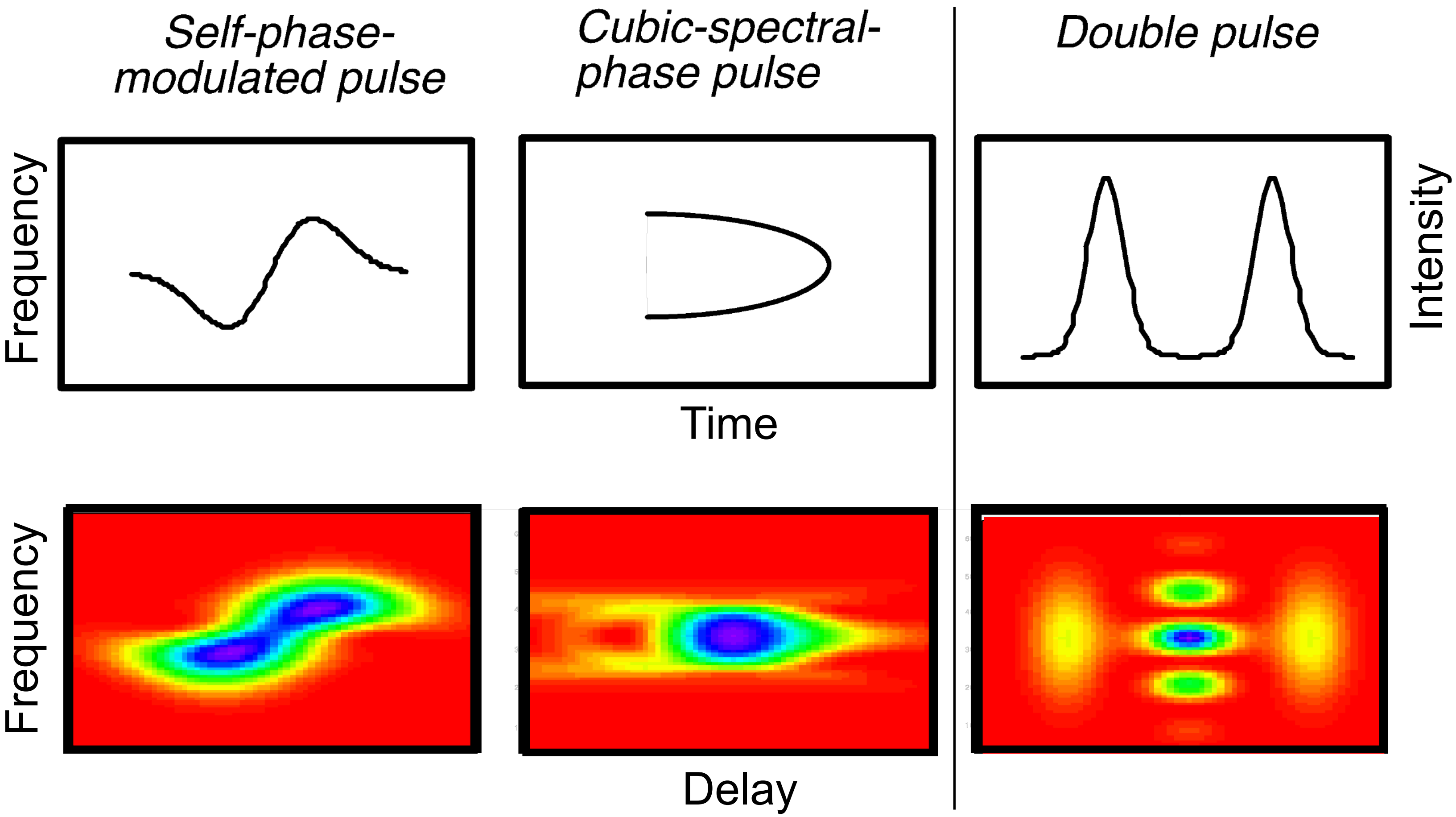
$$E_{sig}(t,\tau) = \begin{cases} E(t)E(t-\tau) & \text{SHG} \\ E(t)\left|E(t-\tau)\right|^2 & \text{PG} \\ E(t)^2 E^*(t-\tau) & \text{SD} \\ E(t)^2 E(t-\tau) & \text{THG} \end{cases}$$

Use any nonlinear-optical process that is fast enough

$$I_{FROG}(\omega,\tau) = \left| \int_{-\infty}^{\infty} E_{sig}(t,\tau)\exp(-i\omega t)\,dt \right|^2$$

FROG provides N X N data points. With an **iterative multi-dimensional algorithm** it is possible to retrieve both the amplitude and phase of the measured optical pulse.

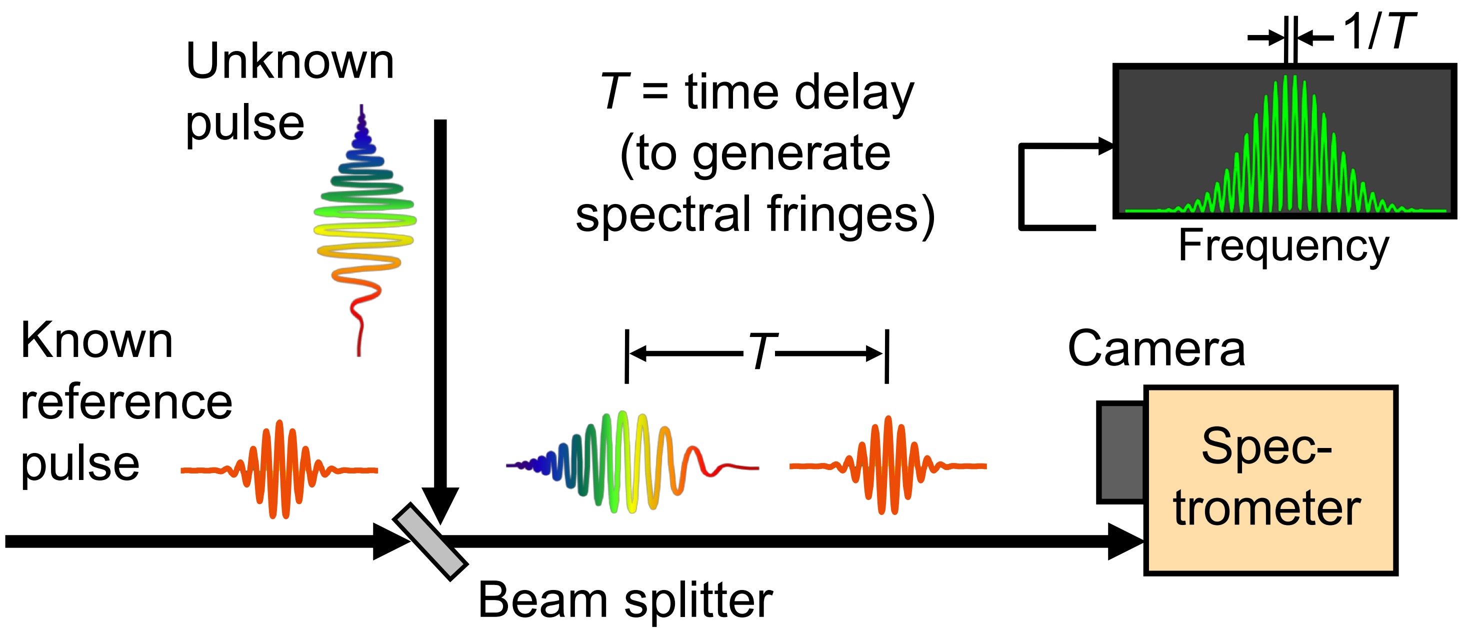
FROG traces for more complex pulses



General concept of spectral interferometry

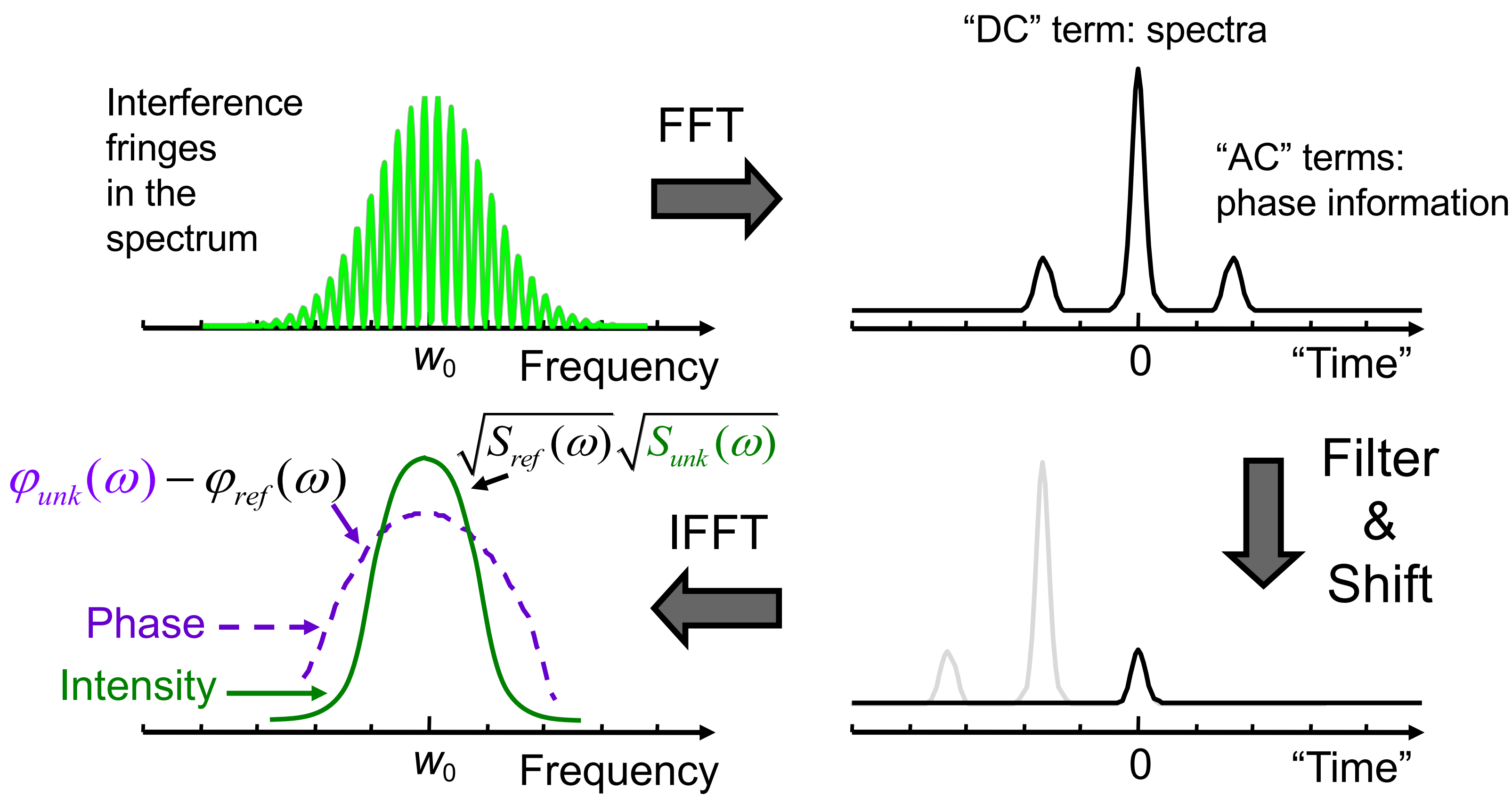
Measure the spectrum of the sum of a known and unknown pulse

Retrieve the unknown pulse from the spectral fringes



$$S_{SI}(\omega) = S_{ref}(\omega) + S_{unk}(\omega) + 2\sqrt{S_{ref}(\omega)}\sqrt{S_{unk}(\omega)}\cos[\varphi_{unk}(\omega) - \varphi_{ref}(\omega) + \omega T]$$

General concept of spectral interferometry



This retrieval algorithm is quick, direct, and reliable

A reference pulse is usually not available!

Spectral interferometry

Spectral Phase Interferometry for Direct Electric-Field Reconstruction (SPIDER)

If we perform spectral interferometry between a pulse and itself, the spectral phase cancels out. Perfect sinusoidal fringes always occur:

$$S_{SI}(\omega) = S_{unk}(\omega) + S_{unk}(\omega) + 2\sqrt{S_{unk}(\omega)}\sqrt{S_{unk}(\omega)}\cos[\cancel{\varphi_{unk}(\omega) - \varphi_{unk}(\omega)} + \omega T]$$

SPIDER approach:

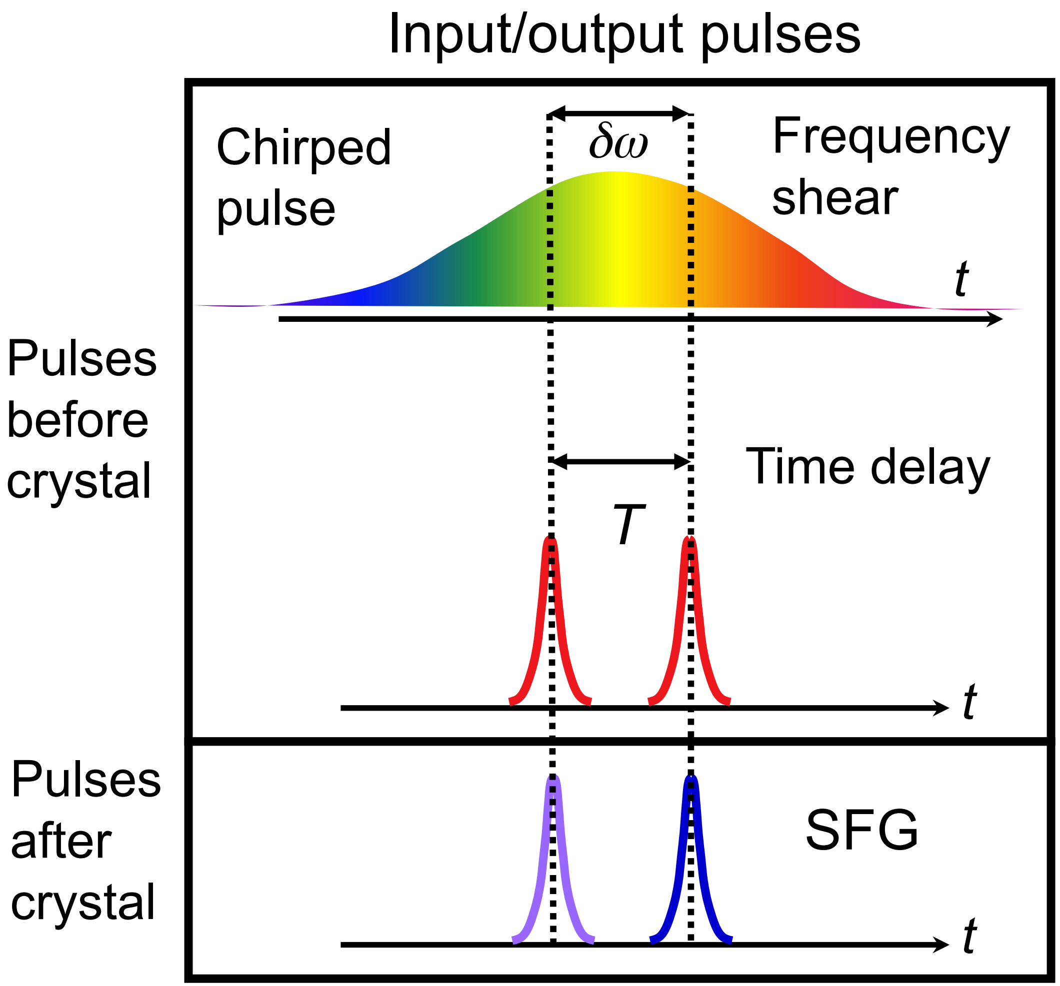
$$S_{SI}(\omega) = S(\omega) + S(\omega+\delta\omega) + 2\sqrt{S(\omega)}\sqrt{S(\omega+\delta\omega)}\cos[\varphi(\omega+\delta\omega) - \varphi(\omega) + \omega T]$$

$$\phi_{SPIDER} = \varphi(\omega+\delta\omega) - \varphi(\omega) + \omega T = \underbrace{\delta\omega}_{\text{frequency shear}} \overbrace{\frac{d\varphi}{d\omega}}^{\text{group delay vs. } \omega} + \underbrace{\omega T}_{\text{Time delay}}$$

This measures the derivative of the spectral phase (the group delay)

Spectral interferometry

Spectral Phase Interferometry for Direct Electric-Field Reconstruction (SPIDER)



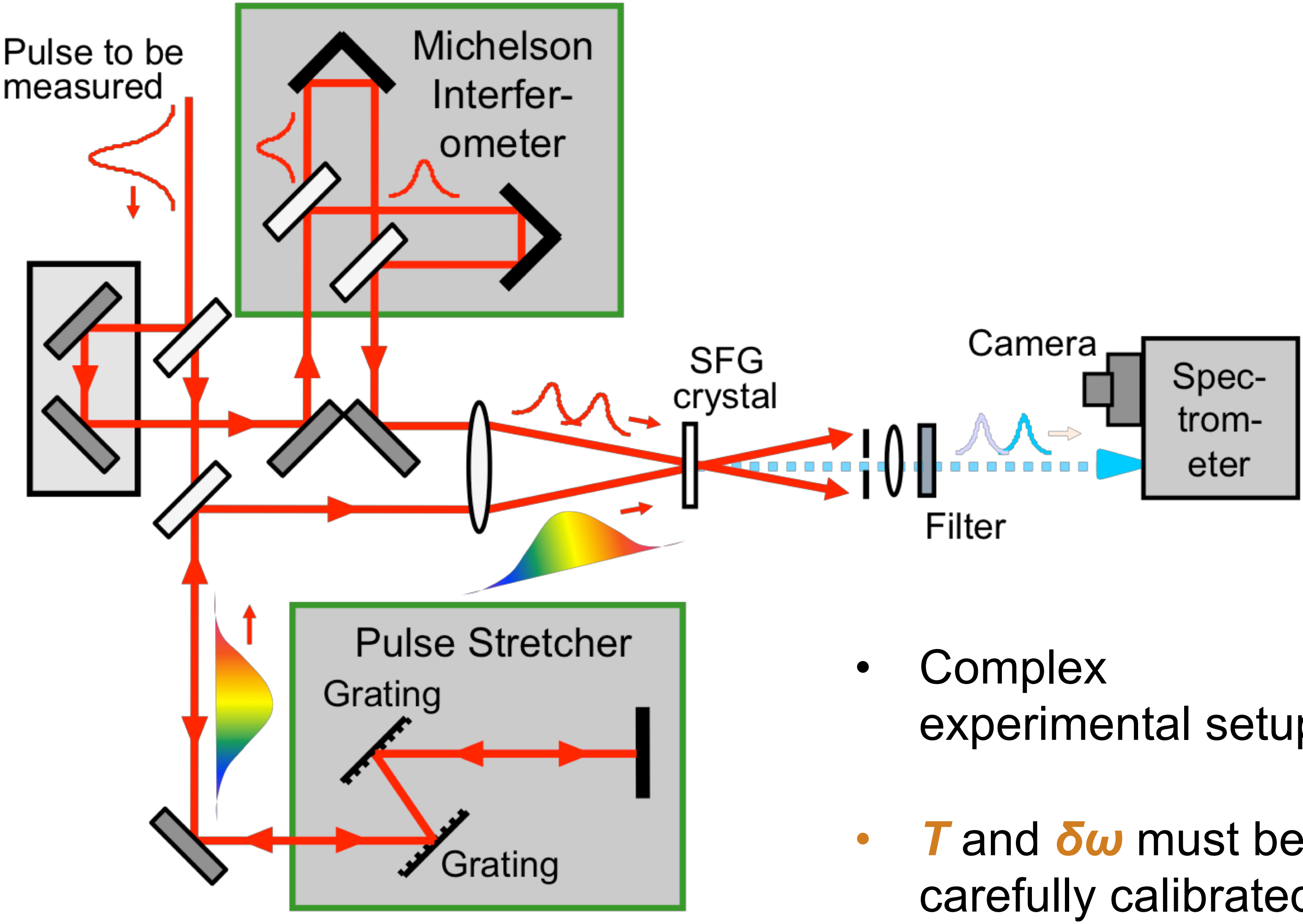
1) Make a very chirped pulse

2) Create two replicas of the pulse

3) Frequency shift the 2 replicas by SFG with the broadband pulse and perform SI

Spectral interferometry

Spectral Phase Interferometry for Direct Electric-Field Reconstruction (SPIDER)

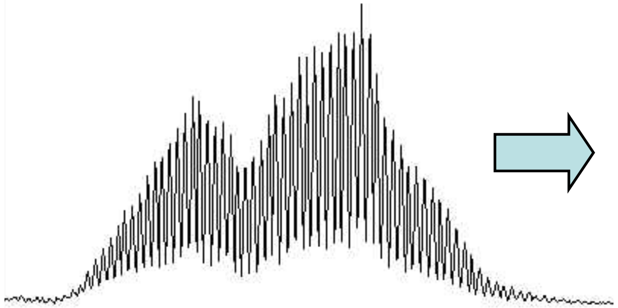


- Complex experimental setup
- *T* and *δω* must be carefully calibrated!

Spectral interferometry

Extraction of the spectral phase

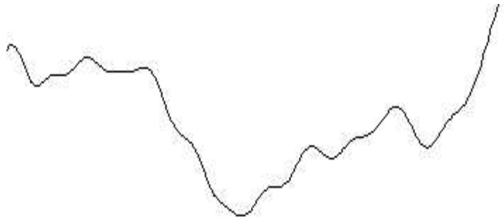
Measurement of the interferogram



Extraction of their spectral phase difference using spectral interferometry

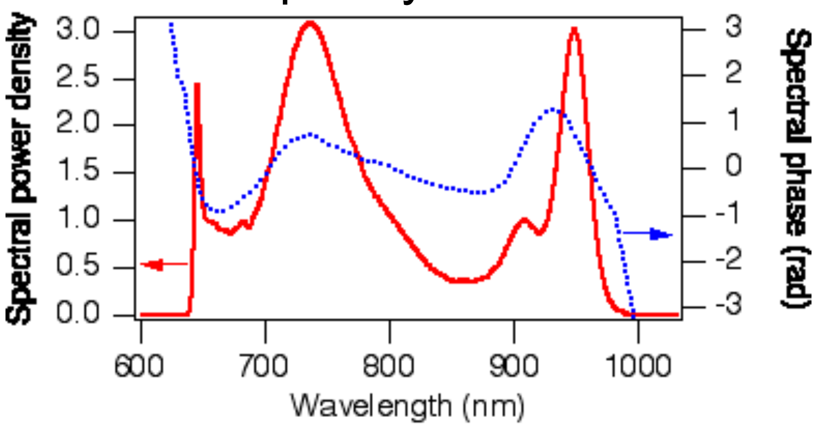
$$\varphi(\omega+\delta\omega)-\varphi(\omega)$$

Integration of the phase



$$\varphi(\omega)$$

Frequency domain



Time domain