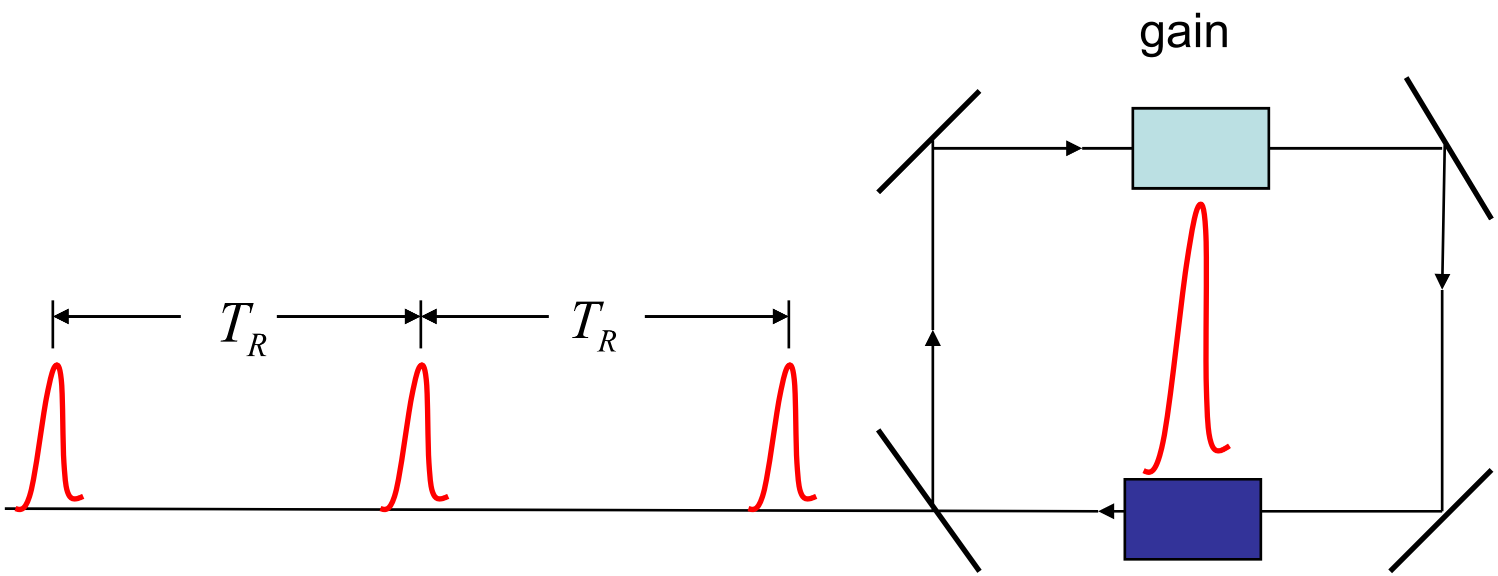
Ultrafast Optical Physics II (SoSe 2019)
Lecture 7, May 24

6 Passive Mode Locking

6.1 Slow Saturable Absorber Mode Locking
6.2 Fast Saturable Absorber Mode Locking

5. Active mode-locking using amplitude modulator

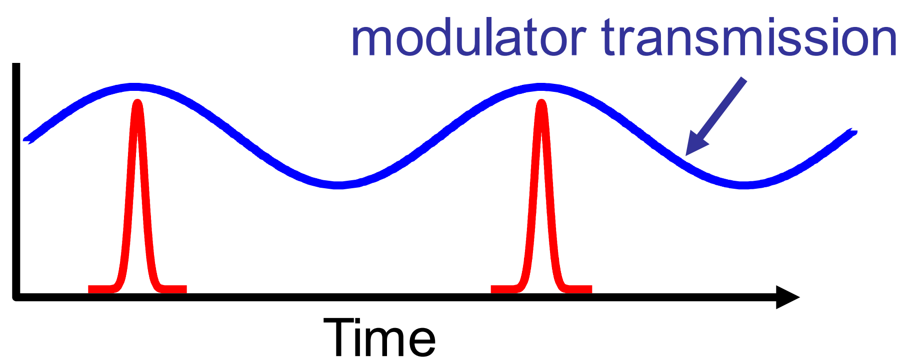


Loss coefficient $q(t) = M(1 \quad \cos \quad _m t)$

Transmission of the modulator

$$T_m = e^{\, M(1 \quad \cos \quad _m t)}$$

$$T_m \quad 1 \quad M(1 \quad \cos \quad _m t)$$

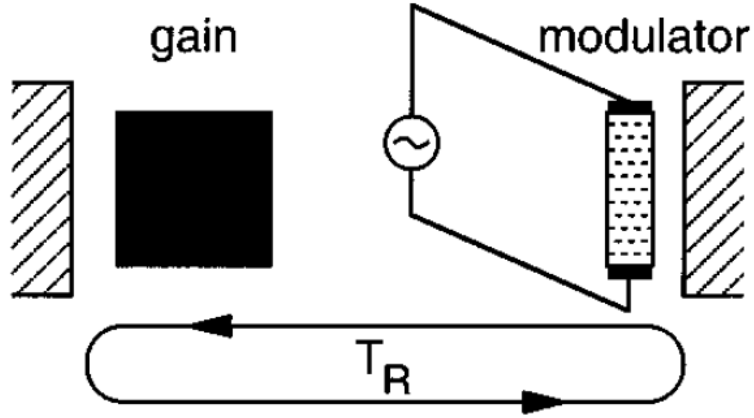


Active mode-locking using amplitude modulator

$$T_R \frac{\partial A}{\partial T} = \left[g(T) + D_g \frac{\partial^2}{\partial t^2} - l - M\left(1 - \cos(\omega_M t)\right) \right] A.$$

$$T_R \frac{\partial A}{\partial T} = \left[g - l + D_g \frac{\partial^2}{\partial t^2} - M_s t^2 \right] A.$$

$$D_g = \frac{g}{\Omega_g^2} \qquad M_s = \frac{M\omega_M^2}{2}$$



Hermite-Gaussian Solution

$$A_n(t) = \sqrt{\frac{W_n}{2^n \sqrt{\pi} n! \tau_a}} H_n(t/\tau_a) e^{-\frac{t^2}{2\tau_a^2}}$$

$$\tau_a = \sqrt[4]{D_g/M_s} \longrightarrow$$

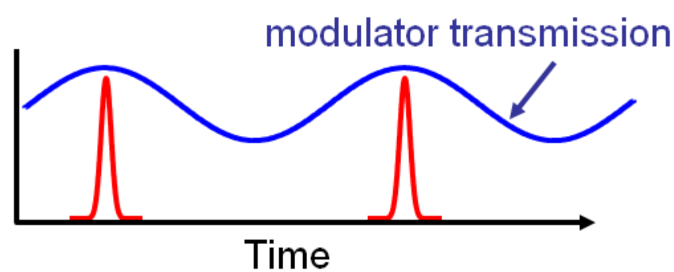
$$_a = \sqrt[4]{2} (g/M)^{1/4} / \sqrt{\quad g \quad M}$$

Comments on active mode-locking

Pulse duration: $_a = \sqrt[4]{2}(g/M)^{1/4}/\sqrt{\;_g \;\; _M}$

1) Larger modulation depth, M, and higher modulation frequency will give shorter pulses because the "low loss" window becomes narrower and shortens the pulse.
2) A broader gain bandwidth yields shorter pulses because the filtering effect of gain narrowing is lower and more modes are lasing.

Disadvantages of active mode-locking:



1) It requires an externally driven modulator. Its modulation frequency has to match precisely the cavity mode spacing.
1) A broader gain bandwidth yields shorter pulses because the filtering effect of gain narrowing is lower and more modes are lasing.

6. Passive Mode Locking

Principles of Passive Mode Locking

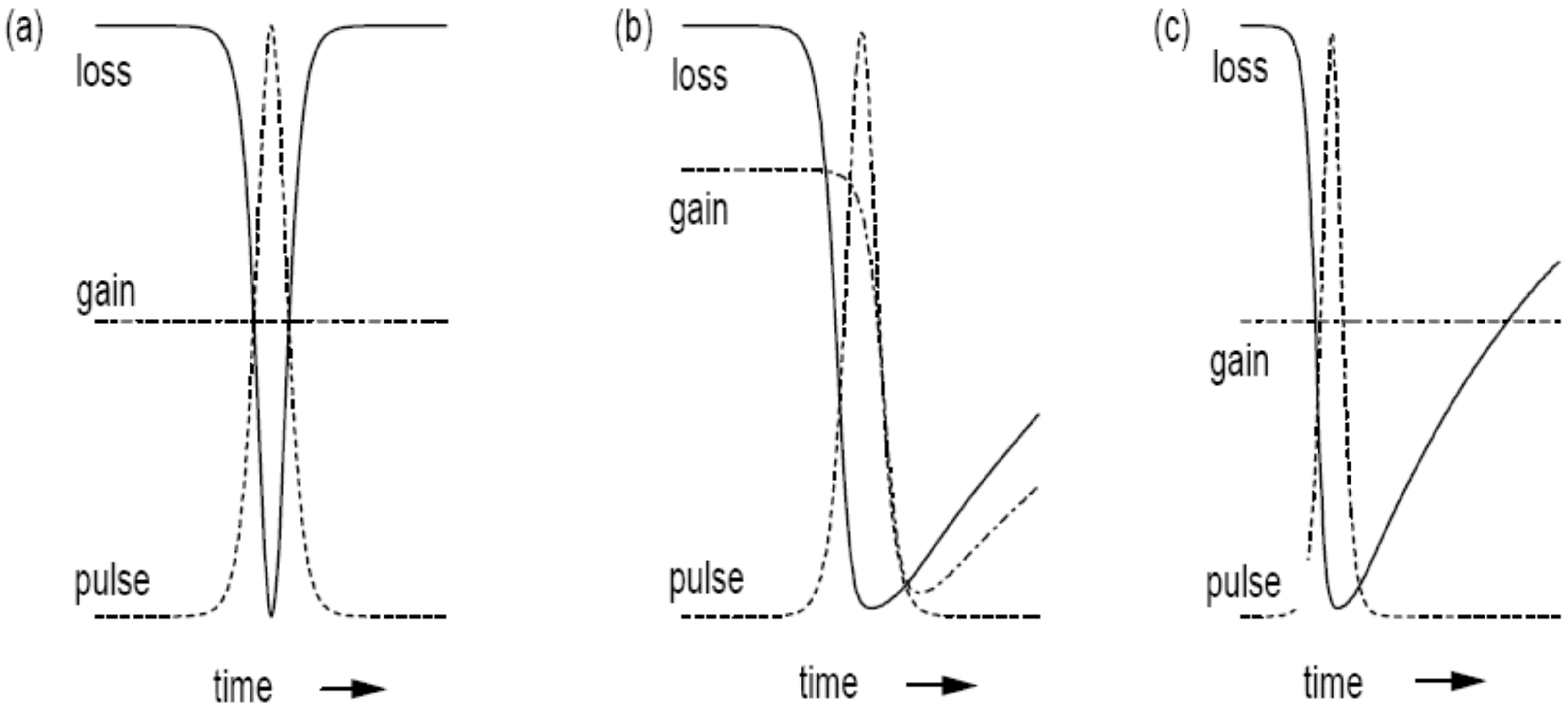
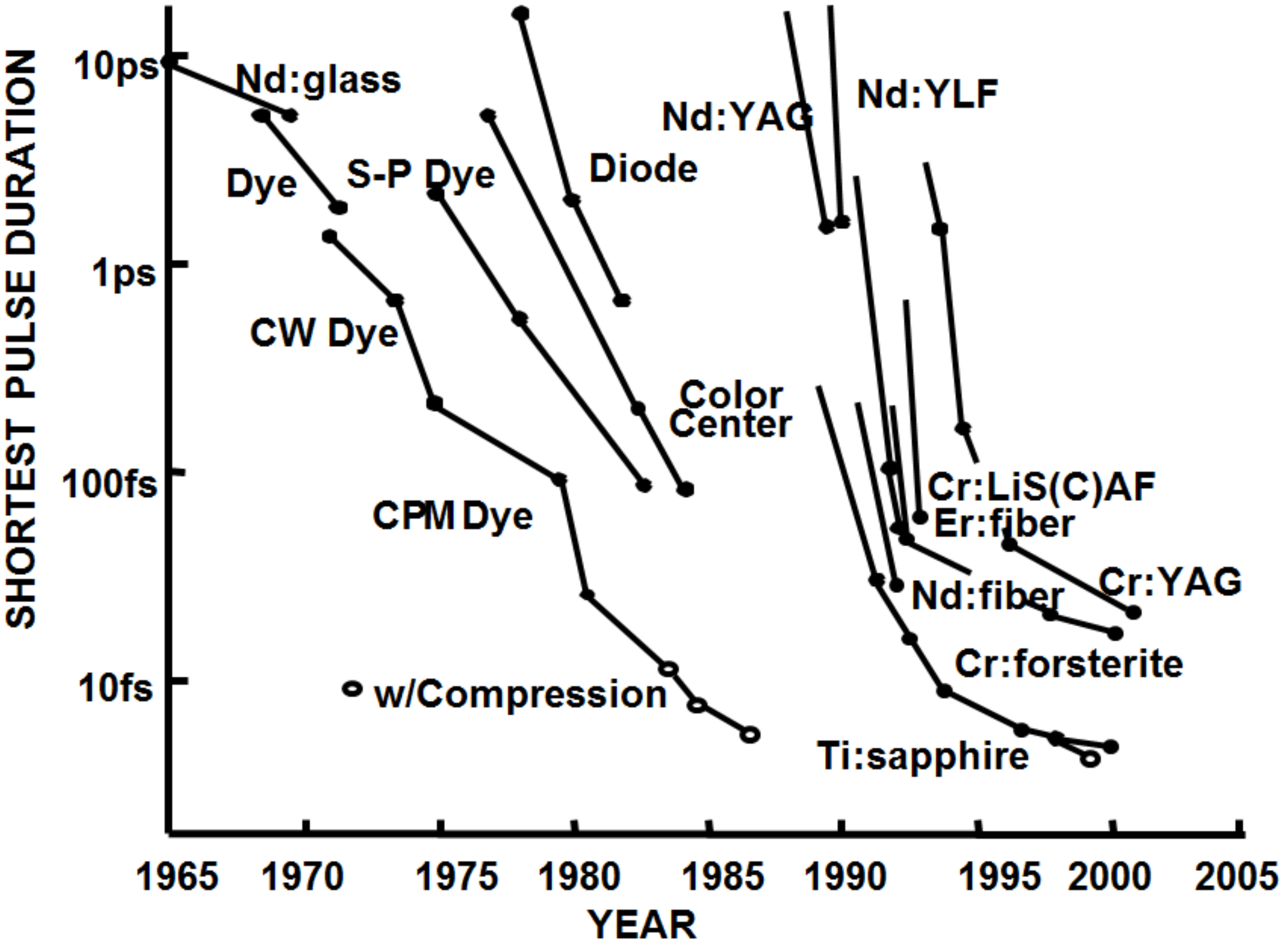
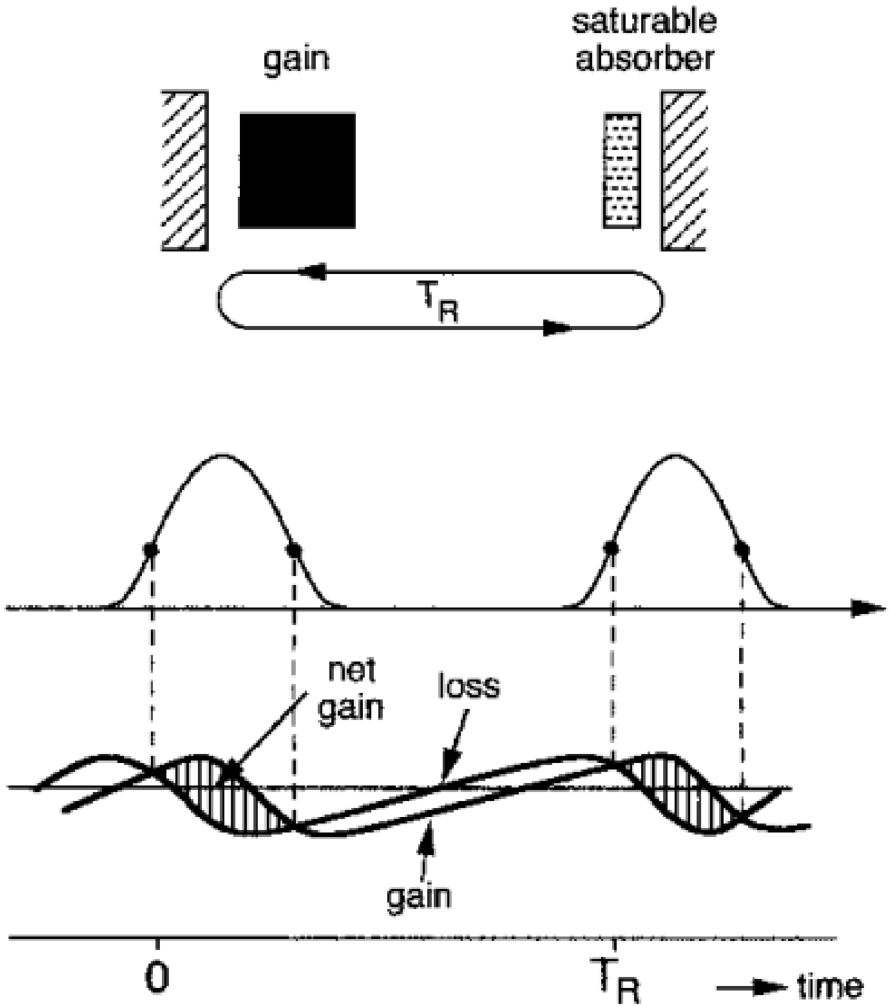


Fig. 6.1: Principles of mode locking

Evolution of shortest pulse duration



6.1 Slow Saturable Absorber Mode Locking



No fast element necessary:
Both absorber and gain
may recover on ns-time scale

Fig. 6.2: Slow saturable absorber modelocking

$$\frac{dg}{dt} = -g\frac{|A(t)|^2}{E_L}$$

Introduce pulse energy: $E(t) = \int_{-T_{R/2}}^{t} dt |A(t)|^2$

→ $$g(t) = g_i \exp\left[-E(t)/E_L\right]$$

$$q(t) = q_0 \exp\left[-E(t)/E_A\right]$$

Master Equation:

$$T_R\frac{\partial}{\partial T}A \; = \; [g_i\left(\exp\left(-E(t)/E_L\right)\right)A - lA - q_0\exp\left(-E(t)/E_A\right)]A + \frac{1}{\Omega_f^2}\frac{\partial^2}{\partial t^2}A$$

Fixed filtering / finite bandwidth

Approximate absorber response:

$$q_0\exp\left(-E(t)/E_A\right) \; \approx \; q_0\left[1 - \left(E(t)/E_A\right) + \frac{1}{2}\left(E(t)/E_A\right)^2\right]$$

$$T_R \frac{\partial A(T,t)}{\partial T} = \left[g(t) - q(t) - l + D_f \frac{\partial^2}{\partial t^2} \right] A(T,t)$$

Ansatz: $A(t) = A_o \,\mathrm{sech}(t/\tau)$

Stationary solution: *A(T+T_R,t)* reproduces itself up to a timing shift?

$$A(t,T) \;=\; A_o \;\mathrm{sech}(\tfrac{t}{\tau} + \alpha \tfrac{T}{T_R})$$

$$E(t) \;\;=\;\; \int_{-T_{R/2}}^{t} dt|A(t)|^2 = \frac{W}{2}\left(1 + \tanh(\frac{t}{\tau} + \alpha\frac{T}{T_R})\right)$$

Shortest pulse width possible: $\tau = \frac{2\sqrt{2}}{\sqrt{q_0}\Omega_f}\frac{E_A}{W} > \frac{\sqrt{2}}{\sqrt{q_0}\Omega_f}$

6.2 Fast Saturable Absorber Mode Locking

Saturable absorption responds to instantaneous power: $q(A) = \dfrac{q_0}{1+\frac{|A|^2}{P_A}}$

Approximately: $q(A) = q_0 - \gamma|A|^2$ with: $l_0 = l + q_0$ and $\gamma = q_0/P_A$

$$T_R\frac{\partial A(T,t)}{\partial T} = \left[g - l_0 + D_f\frac{\partial^2}{\partial t^2} + \gamma|A|^2 + jD_2\frac{\partial^2}{\partial t^2} - j\delta|A|^2\right]A(T,t).$$

Dispersion + SPM

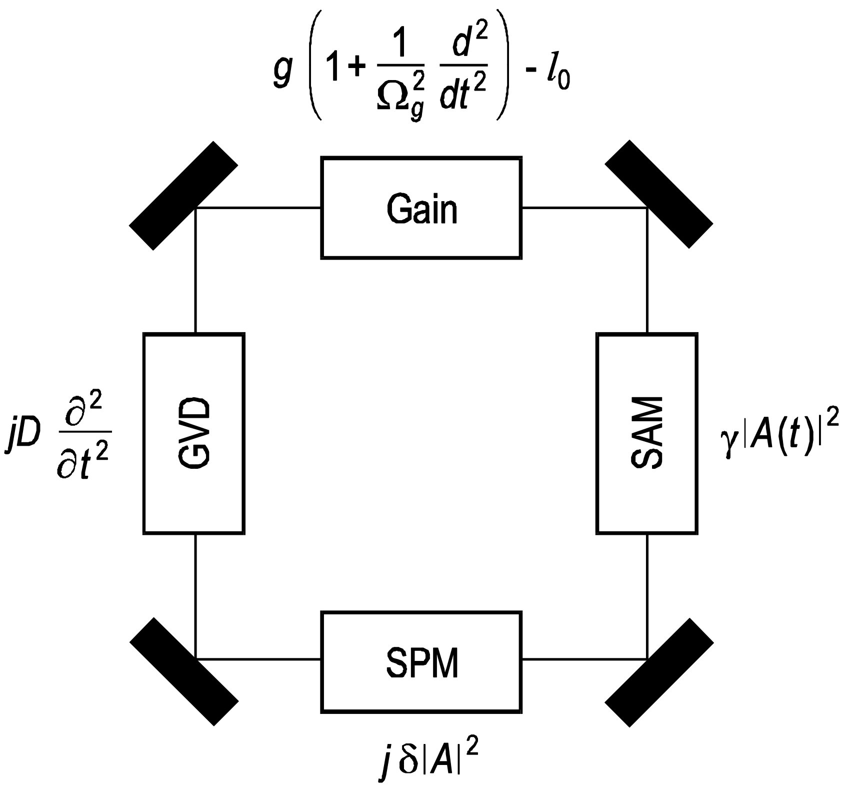


Fig. 6.3: Fast saturable absorber modelocking

Without GDD and SPM

$$T_R \frac{\partial A(T,t)}{\partial T} = \left[g - l_0 + D_f \frac{\partial^2}{\partial t^2} + \gamma |A|^2 \right] A(T,t)$$

$$T_R \frac{\partial A_s(T,t)}{\partial T} = 0. \longrightarrow A_s(T,t) = A_s(t) = A_0 \text{sech}\left(\frac{t}{\tau}\right)$$

$$\begin{aligned} 0 &= \left[(g - l_0) + \frac{D_f}{\tau^2}\left[1 - 2\text{sech}^2\left(\frac{t}{\tau}\right)\right] \right. \\ &\left. + \gamma |A_0|^2 \text{sech}^2\left(\frac{t}{\tau}\right) \right] \cdot A_0 \text{sech}\left(\frac{t}{\tau}\right) \end{aligned}$$

$$\frac{D_f}{\tau^2} = \frac{1}{2}\gamma |A_0|^2$$

Pulse Energy: $W = 2A_0^2 \longrightarrow \tau = \frac{4D_f}{\gamma W}.$

$$g = l_0 - \frac{D_f}{\tau^2}$$

Pulse Energy Evolution:

$$\begin{aligned} T_R \frac{\partial W(T)}{\partial T} &= T_R \frac{\partial}{\partial T} \int_{-\infty}^{\infty} |A(T,t)|^2 \, dt \\ &= T_R \int_{-\infty}^{\infty} \left[A(T,t)^* \frac{\partial}{\partial T} A(T,t) + c.c. \right] dt \\ &= 2G(g_s, W)W, \end{aligned}$$

$$\begin{aligned} \int_{-\infty}^{\infty} \left(\text{sech}^2 x\right) dx &= 2, \\ \int_{-\infty}^{\infty} \left(\text{sech}^4 x\right) dx &= \frac{4}{3}, \end{aligned}$$

$$-\int_{-\infty}^{\infty} \text{sech}x \frac{d^2}{dx^2} \left(\text{sech}x\right) dx = \int_{-\infty}^{\infty} \left(\frac{d}{dx} \text{sech}x \right)^2 dx = \frac{2}{3}$$

$$\begin{aligned} G(g_s, W) &= g_s - l_0 - \frac{D_f}{3\tau^2} + \frac{2}{3}\gamma |A_0|^2 \\ &= g_s - l_0 + \frac{1}{2}\gamma |A_0|^2 = g_s - l_0 + \frac{D_f}{\tau^2} = 0 \end{aligned}$$

$$g_s(W) = \frac{g_0}{1 + \frac{W}{P_L T_R}}$$

Steady State Pulse Energy:

$$g_s(W) = \frac{g_0}{1 + \frac{W}{P_L T_R}} = l_0 - \frac{D_f}{\tau^2}$$

$$= l_0 - \frac{(\gamma W)^2}{16 D_{\cancel{g}}}$$

Replace by *f*

With $q_0 = \gamma A_0^2.$

$$\frac{D_f}{\tau^2} = \frac{q_0}{2},$$

$$\tau = \sqrt{\frac{2}{q_0} \frac{1}{\Omega_f}}.$$

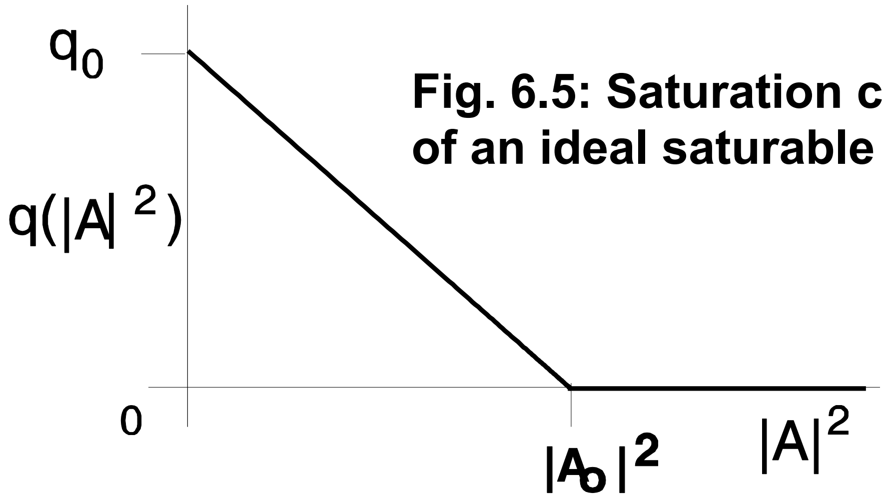


Fig. 6.5: Saturation characteristic of an ideal saturable absorber

Minimum Pulse Width: $g_s = l \cancel{+} - \frac{1}{2} q_0$ $\quad D_f = D_g = \frac{g}{\Omega_g^2}$

$$\tau_{\min} = \frac{1}{\Omega_g} \longrightarrow \Delta f_{FWHM} = \frac{0.315}{1.76 \cdot \tau_{\min}} = \frac{\Omega_g}{1.76 \cdot \pi}$$

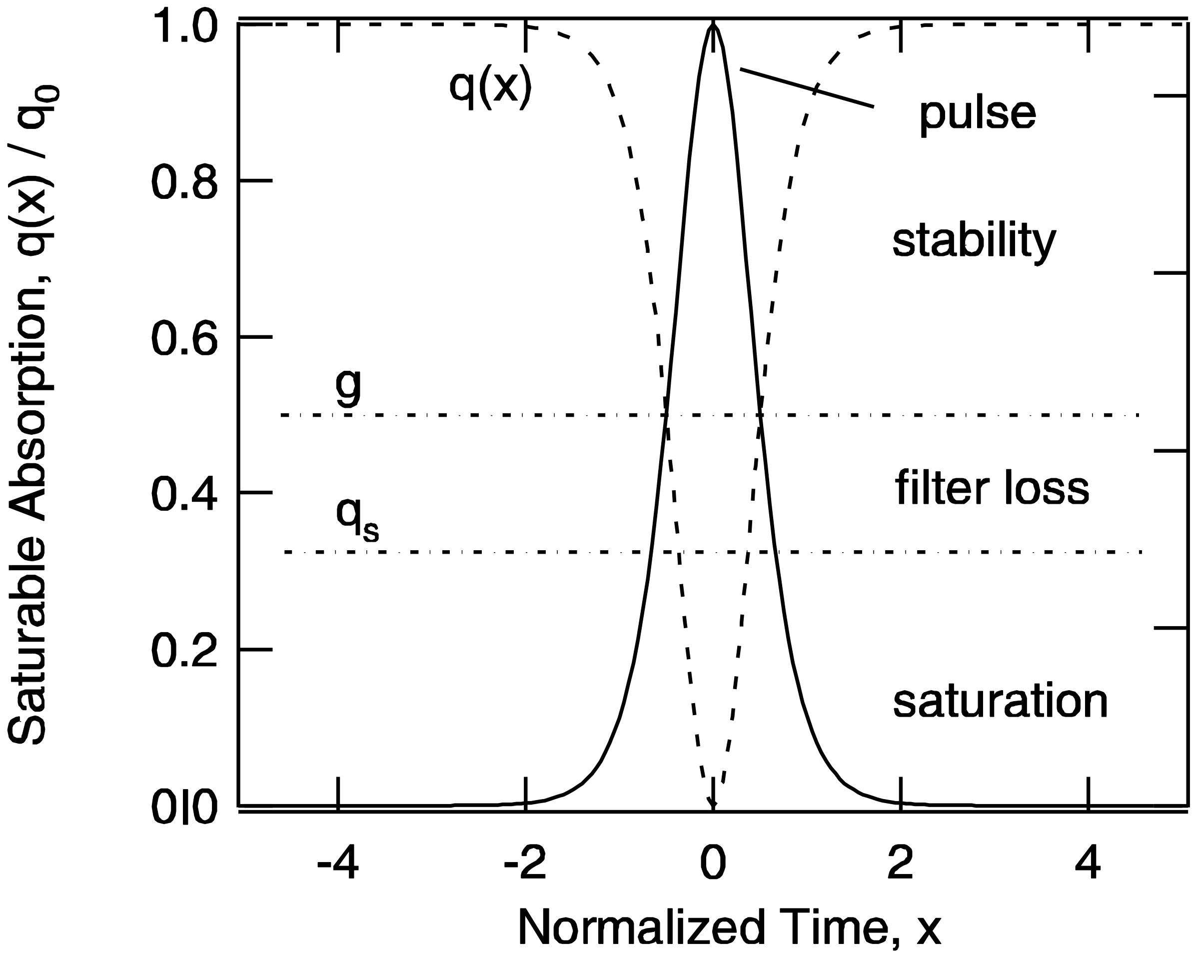


Fig. 6.4: Gain and loss in a fast saturable absorber (FSA) modelocked laser

Fast SA mode locking with GDD and SPM

$$T_R \frac{\partial A(T,t)}{\partial T} = \left[g - l_0 + D_f \frac{\partial^2}{\partial t^2} + \gamma |A|^2 + jD_2 \frac{\partial^2}{\partial t^2} - j\delta |A|^2 \right] A(T,t)$$

Steady-state solution is chirped sech-shaped pulse with 4 free parameters:

$$A_s(T,t) = A_0 \left(\mathrm{sech}\left(\frac{t}{\tau} \right) \right)^{(1+j\beta)} e^{j\psi T/T_R}$$

Pulse amplitude: A_0 or Energy: $W = 2 A_0^2 \tau$

Pulse width: τ

Chirp parameter : β

Carrier-Envelope phase shift : ψ

Substitute above trial solution into the master equation and comparing the coefficients to the same functions leads to two complex equations:

$$\frac{1}{\tau^2} (D_f + jD_2) \left(2 + 3j\beta - \beta^2 \right) = (\gamma - j\delta) |A_0|^2 \quad (6.49)$$

$$l_0 - \frac{(1+j\beta)^2}{\tau^2} (D_f + jD_2) = g - j\psi \quad (6.50)$$

Fast SA mode locking with GDD and SPM

The real part and imaginary part of Eq.(6.49) give

$$\frac{1}{\tau^2}\left[D_f\left(2-\beta^2\right)-3\beta D_2\right] = \gamma|A_0|^2 \qquad (6.52)$$

$$\frac{1}{\tau^2}\left[D_2\left(2-\beta^2\right)+3\beta D_f\right] = -\delta|A_0|^2 \qquad (6.53)$$

Normalized parameters:

Normalized nonlinearity

$$\delta_n = \delta/\gamma$$

Normalized dispersion

$$D_n = D_2/D_f$$

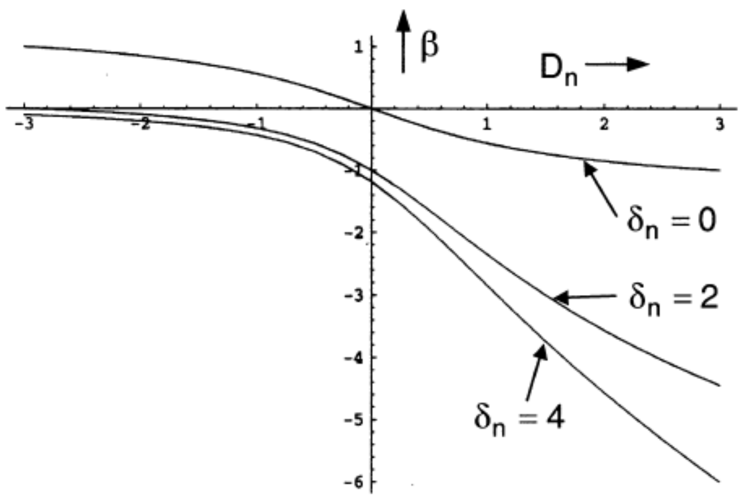
Dividing Eq.(6.53) by (6.52) leads to a quadratic equation for the chirp:

$$\frac{D_n\left(2-\beta^2\right)+3\beta}{\left(2-\beta^2\right)-3\beta D_n} = -\delta_n \longrightarrow \frac{3\beta}{2-\beta^2} = \frac{\delta_n + D_n}{-1+\delta_n D_n} \equiv \frac{1}{\chi} \qquad (6.54)$$

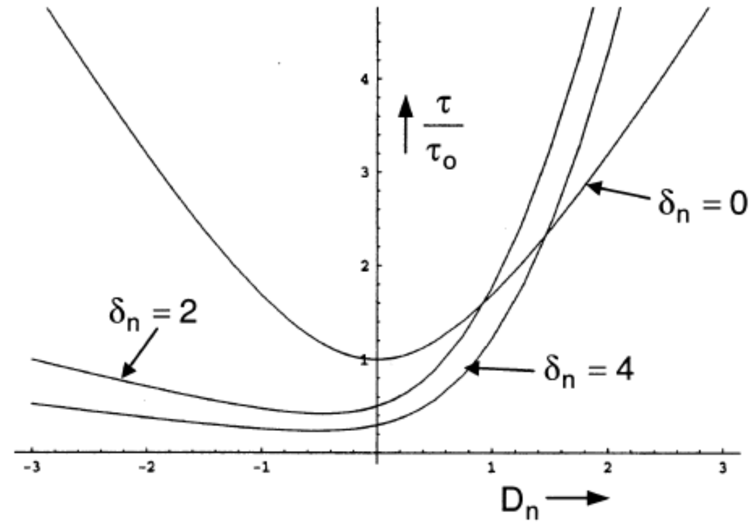
χ depends only on the system parameters

Fast SA mode locking with GDD and SPM

Chirp $$\beta = -\frac{3}{2}\chi \pm \sqrt{\left(\frac{3}{2}\chi\right)^2 + 2}$$



Pulse width $$\tau = \frac{3\tau_0}{2}\beta\left(\chi - D_n\right)$$



- strong soliton-like pulse shaping if $\delta_n \gg 1$ and $-D_n \gg 1$ the chirp is always much smaller than for positive dispersion and the pulses are solitonlike.
- pulses are even chirp free if $\delta_n = -D_n$, with the shortest with directly from the laser, which can be a factor 2-3 shorter than by pure SA modelocking.
- Without SPM and GDD, SA has to shape the pulse. When SPM and GDD included, they can shape the pulse via soliton formation; SA only has to stabilize the pulse.

Fast SA mode locking with GDD and SPM

$$l_0 - \frac{(1+j\beta)^2}{\tau^2}(D_f + jD_2) \;=\; g - j\psi \qquad (6.50)$$

The real part of Eq.(6.50) gives the saturated gain:

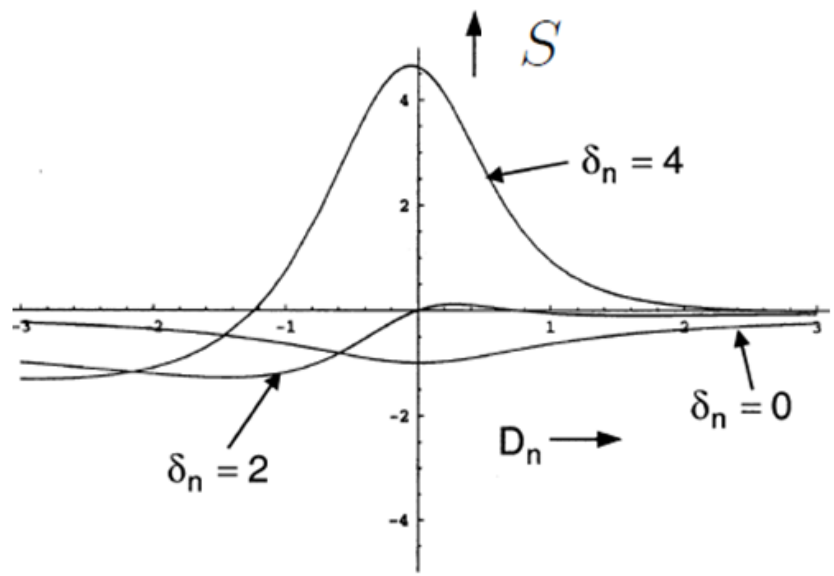
$$g = l_0 - \frac{1-\beta^2}{\tau^2}D_f + \frac{2\beta D_2}{\tau^2}$$

A necessary but not sufficient criterion for the pulse stability is that there must be net loss leading and following the pulse:

$$g - l_0 = -\frac{1-\beta^2}{\tau^2}D_f + \frac{2\beta D_2}{\tau^2} < 0$$

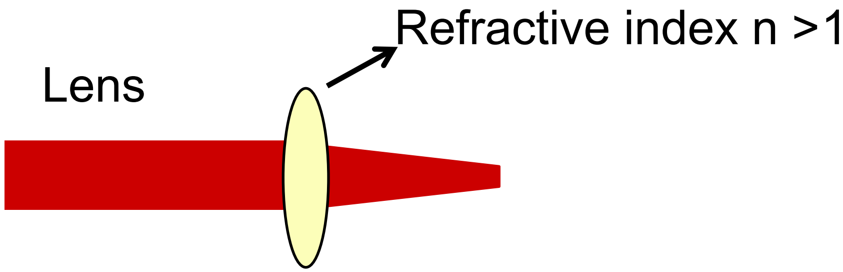
If we define the stability parameter S

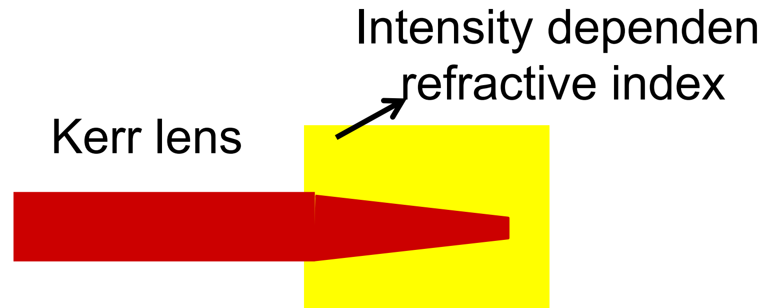
$$S = 1 - \beta^2 - 2\beta D_n < 0$$



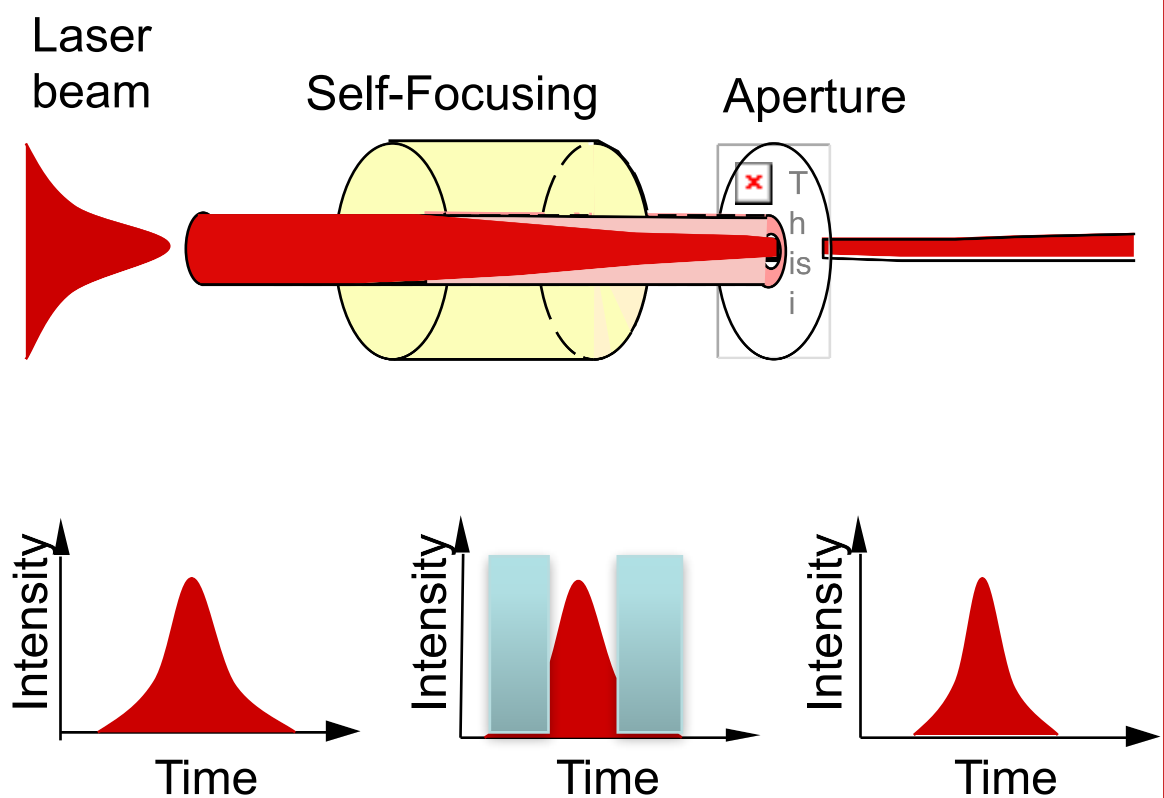
- Without SPM, the pulses are always stable.
- Excessive SPM can lead to instability near zero dispersion and for positive dispersion.

7. Mode locking using artificial fast SA
7.1 Kerr-lens mode locking



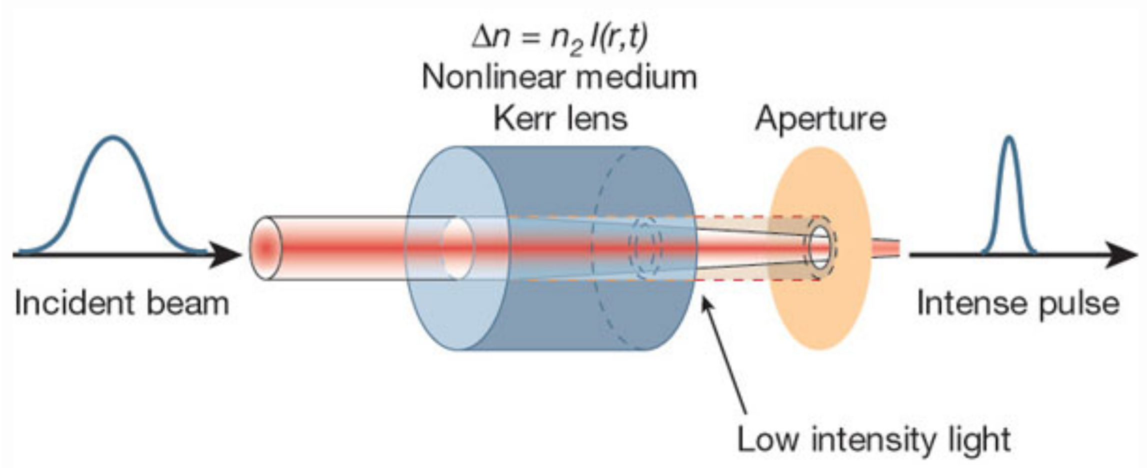


- A spatial-temporal laser pulse propagating through the Kerr medium has a time dependent mode size: pulse peak corresponds to smaller beam size than the wings.
- A hard aperture placed at the right position in the cavity strips of the wings of the pulse, shortening the pulse.
- The combined mechanism is equivalent to a fast saturable absorber.

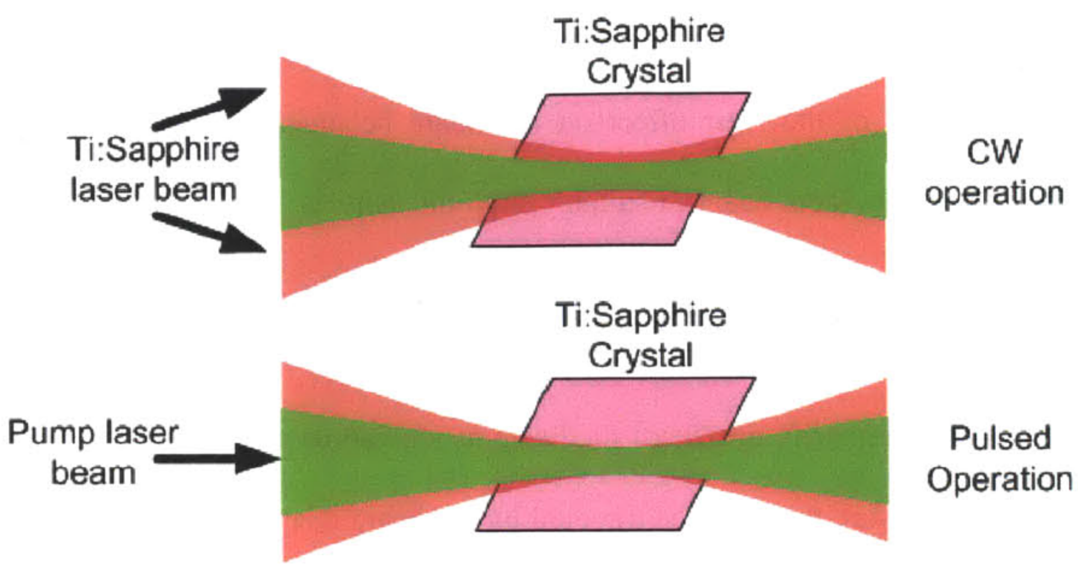


Kerr-lens mode locking: hard aperture versus soft aperture

Hard-aperture Kerr-lens mode-locking: a hard aperture placed at the right position in the cavity attenuates the wings of the pulse, shortening the pulse.



Soft-aperture Kerr-lens mode-locking: gain medium can act both as a Kerr medium and as a soft aperture (i.e. increased gain instead of saturable absorption). In the CW case the overlap between the pump beam and laser beam is poor, and the mode intensity is not high enough to utilize all of the available gain. The additional focusing provided by the high intensity pulse improves the overlap, utilizing more of the available gain.



Mode locking using artificial fast SA: additive pulse mode locking

- A small fraction of the light emitted from the main laser cavity is injected externally into a nonlinear fiber. In the fiber strong SPM occurs and introduces a significant phase shift between the peak and the wings of the pulse. In the case shown the phase shift is π
- A part of the modified and heavily distorted pulse is reinjected into the main cavity in an interferometrically stable way, such that the injected pulse interferes constructively with the next cavity pulse in the center and destructively in the wings.
- This superposition leads to a shorter intracavity pulse and the pulse shaping generated by this process is identical to the one obtained from a fast saturable absorber.

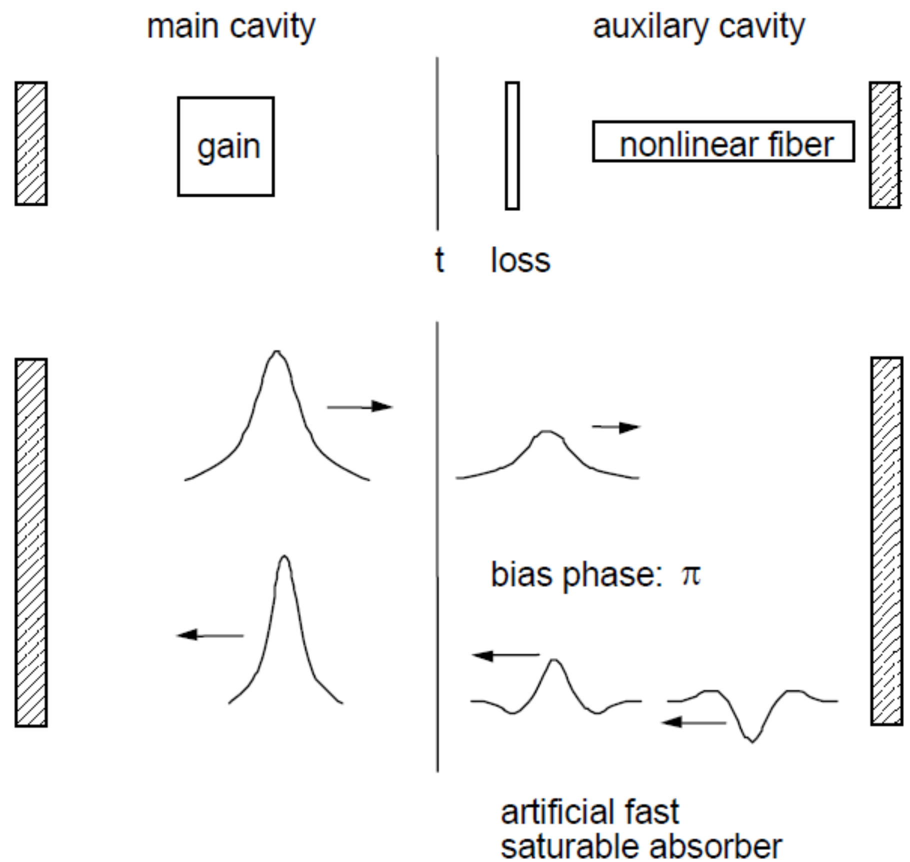
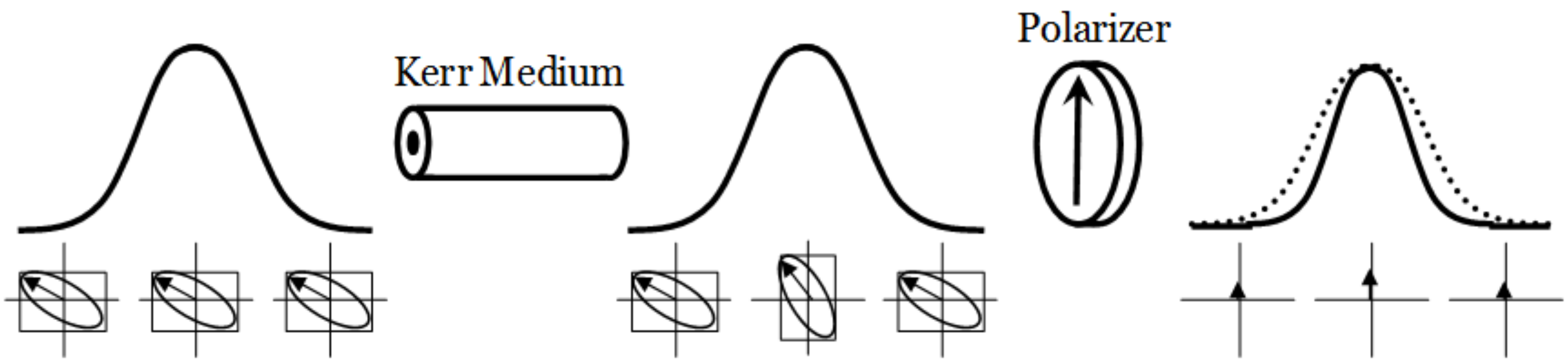


Fig. 7.17: Principle mechanism of additive pulse mode locking

7.2 Additive pulse mode locking using nonlinear polarization rotation in a fiber



- When an intense optical pulse travels in an isotropic optical fiber, intensity-dependent change of the polarization state can happen.
- The polarization state of the pulse peak differs from that of the pulse wings after the fiber section due to Kerr effect.
- If a polarizer is placed after the fiber section and is aligned with the polarization state of the pulse peak, the pulse wings are attenuated more by the polarizer and the pulse becomes shorter.

8. Semiconductor Saturable Absorbers

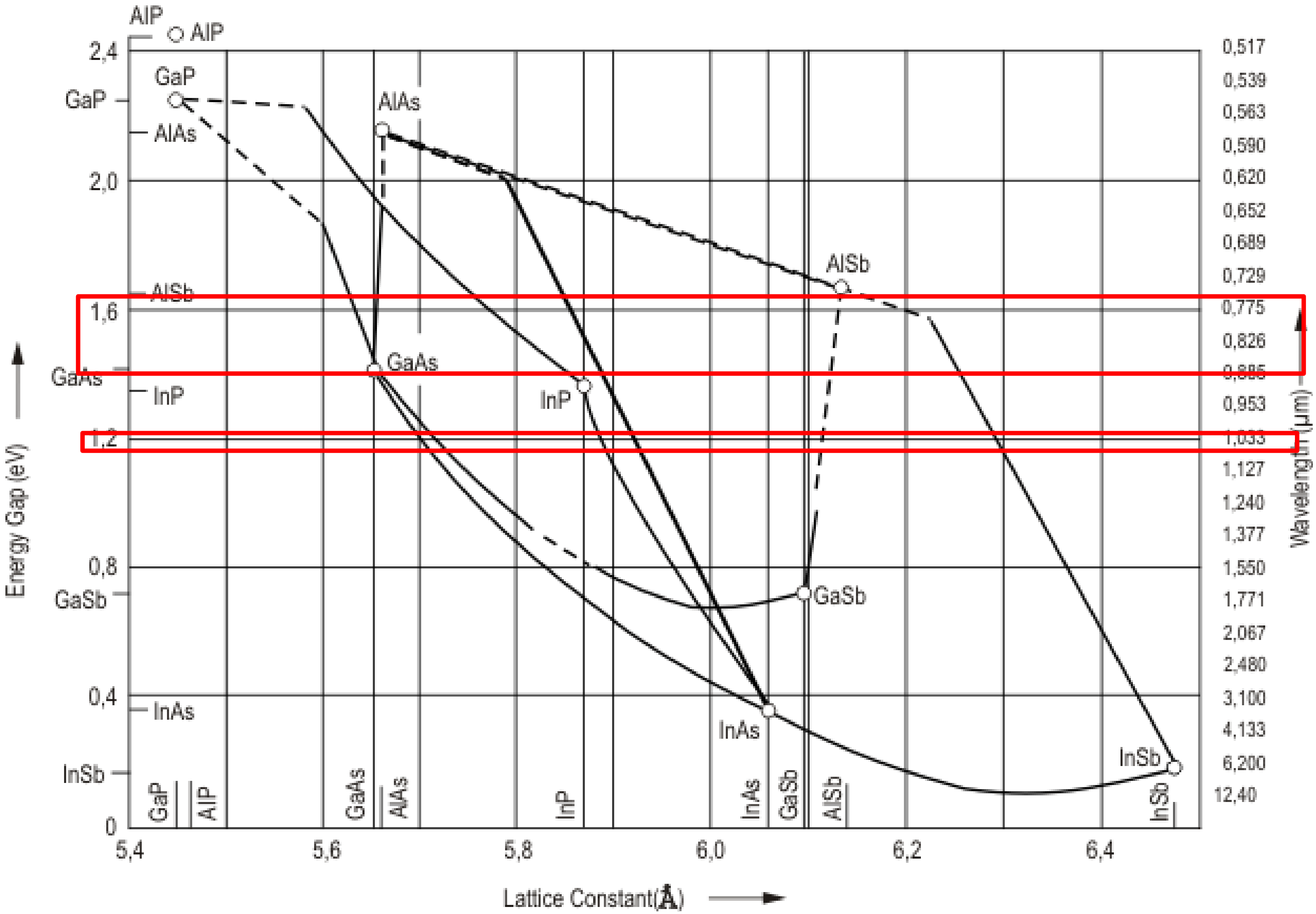


Fig. 8.1: Band Gap and lattice constant for various compound semiconductors. Dashed lines indicate ind. transitions.

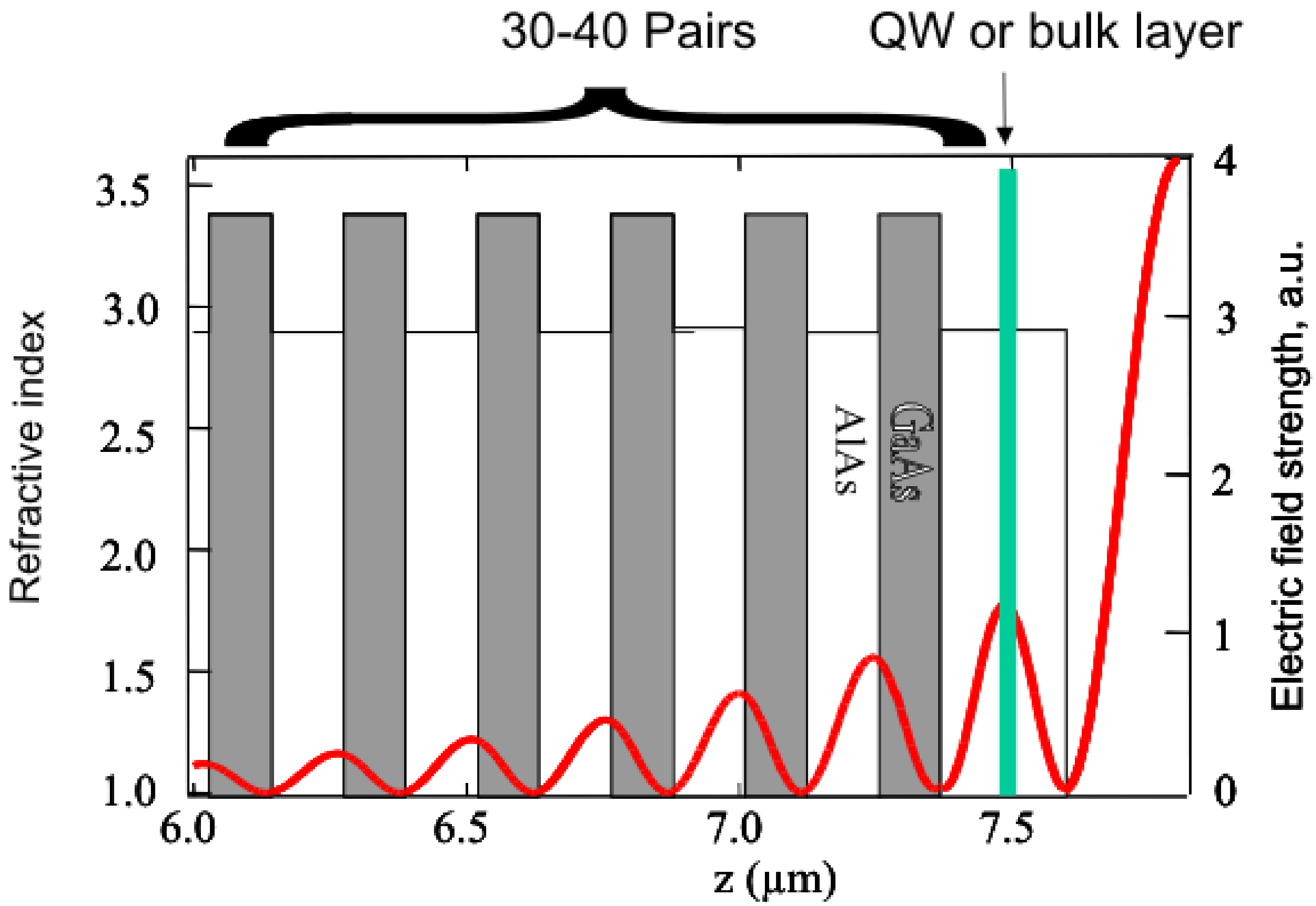


Fig. 8.2: Semiconductor saturable absorber mirror (SESAM) or Semiconductor Bragg mirror (SBR)

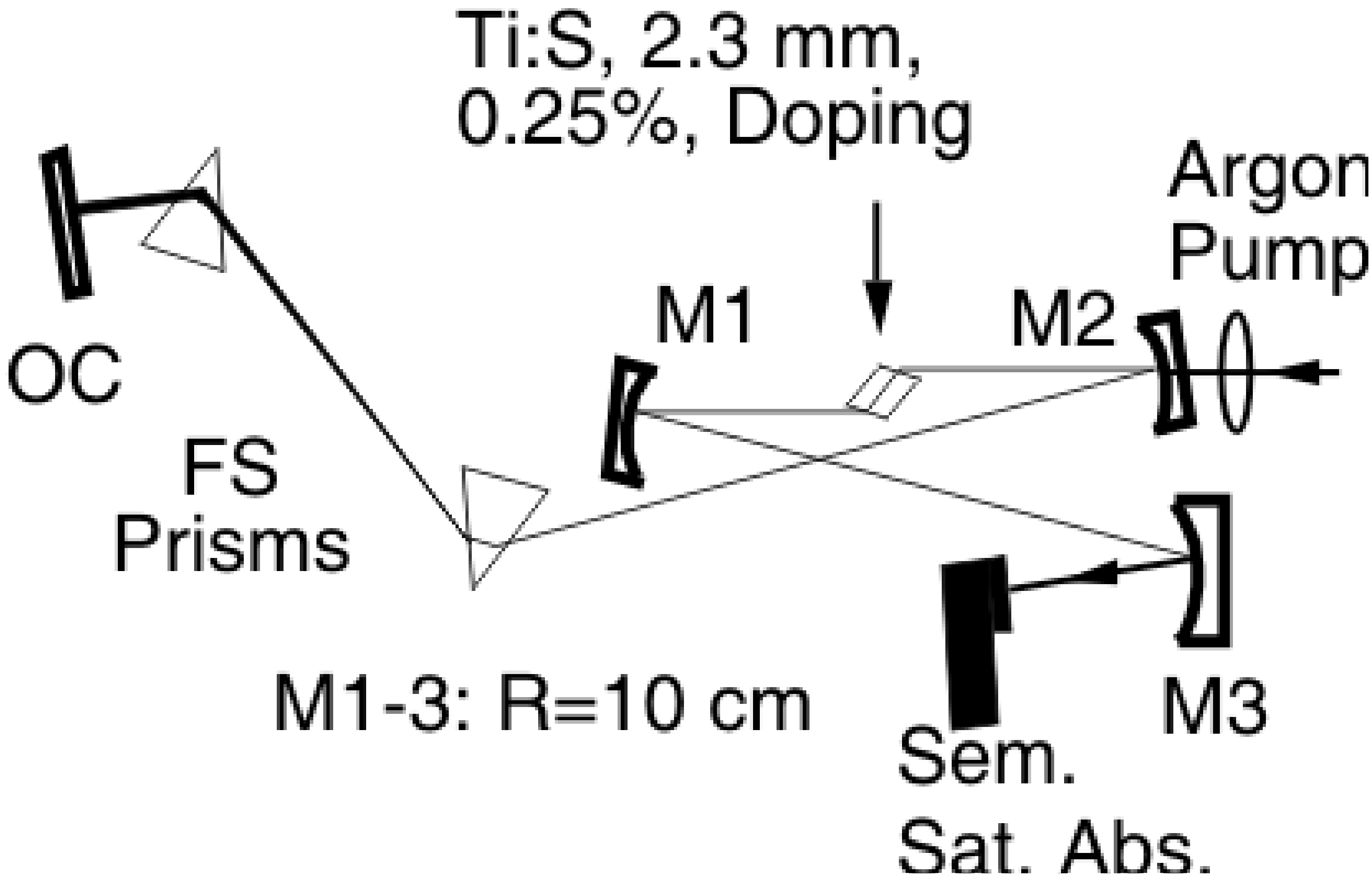


Fig. 8.3: Ti:sapphire laser modelocked by SBR

8.1 Carrier dynamics in semiconductors

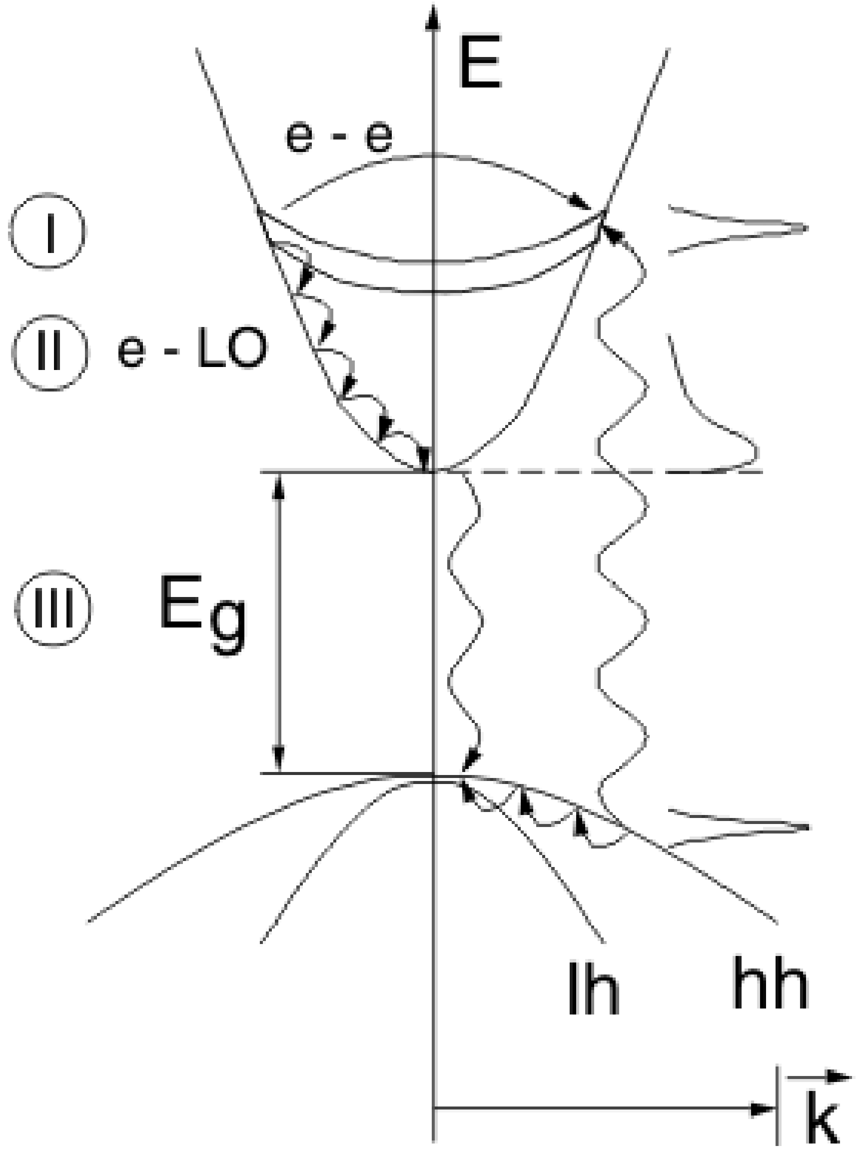


Table 8.4: Carrier dynamics in semiconductors

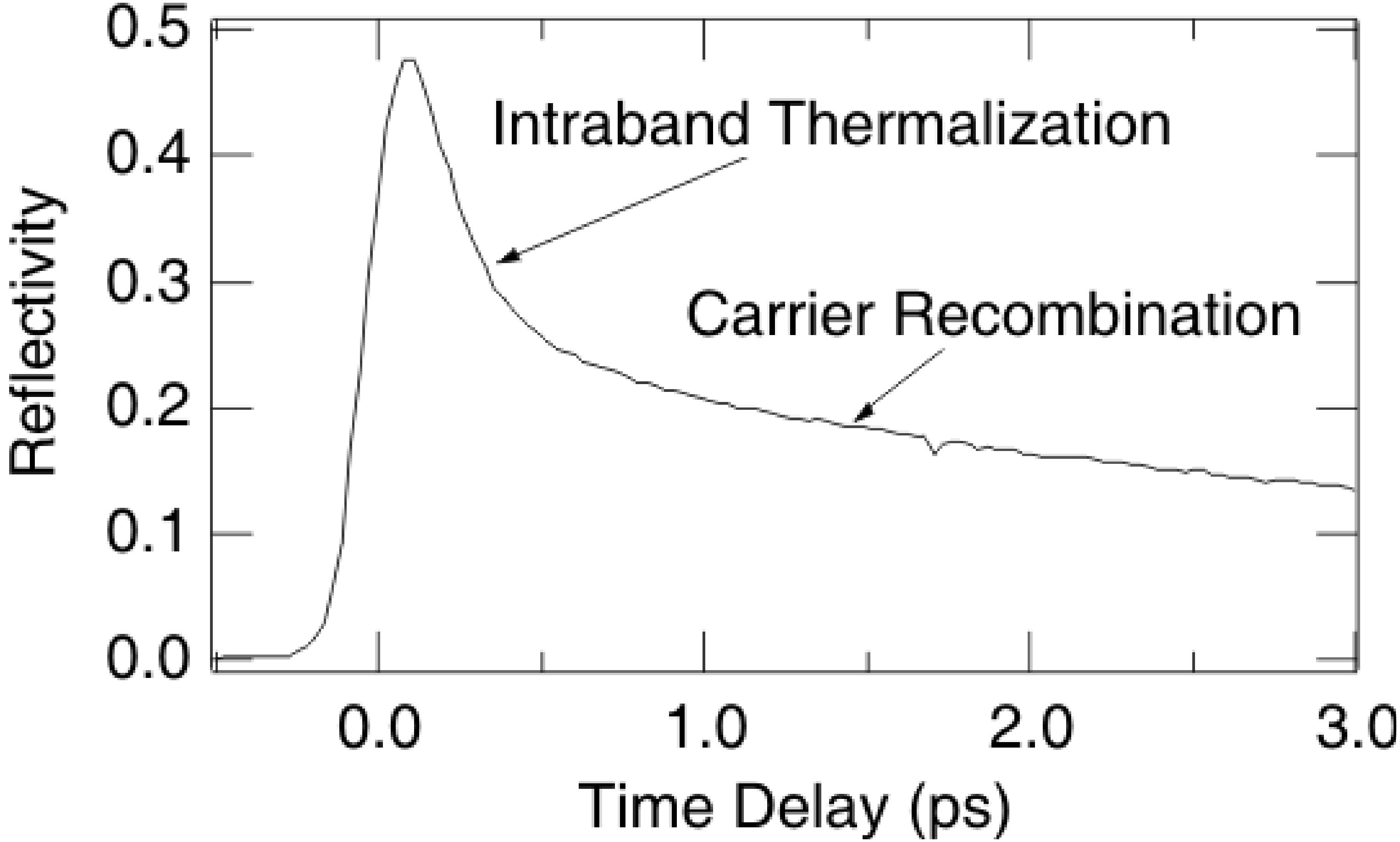


Fig. 8.5: Pump probe of a InGaAs multiple QW absorber

Elements of statistics: random process

Mean value: $$P_0 = \overline{P(t)} = \lim_{T} \frac{1}{T} \int_{T/2}^{T/2} P(t)dt$$

Variance: $$\overline{\left|P(t) \quad P_0\right|^2} = \lim_{T} \frac{1}{T} \int_{T/2}^{T/2} \left|P(t) \quad P_0\right|^2 dt$$

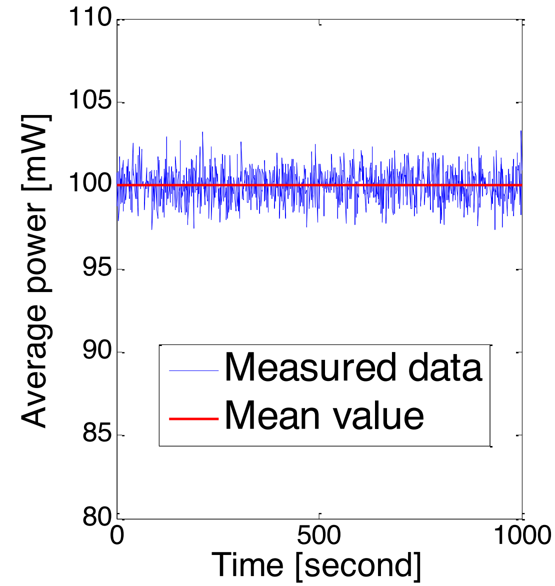
Autocorrelation function:

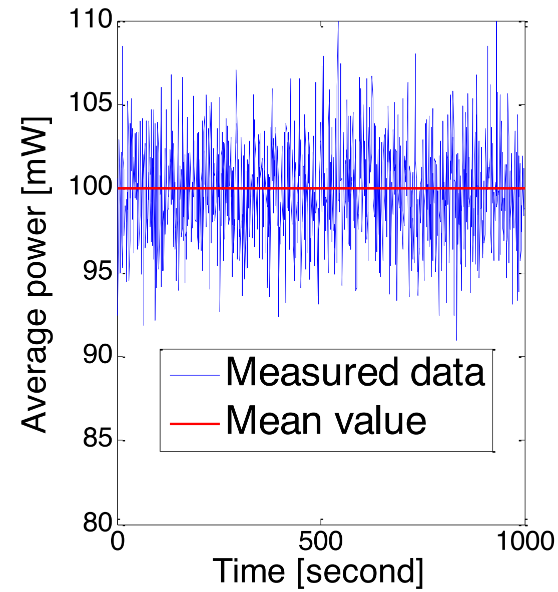
$$R(\quad) = \overline{P(t)P(t+\quad)} = \lim_{T} \frac{1}{T} \int_{T/2}^{T/2} P(t)P(t+\quad)dt$$

Spectrum: $$P_T(\quad) = \int_{T/2}^{T/2} P(t)e^{\ j\ \ t}dt$$

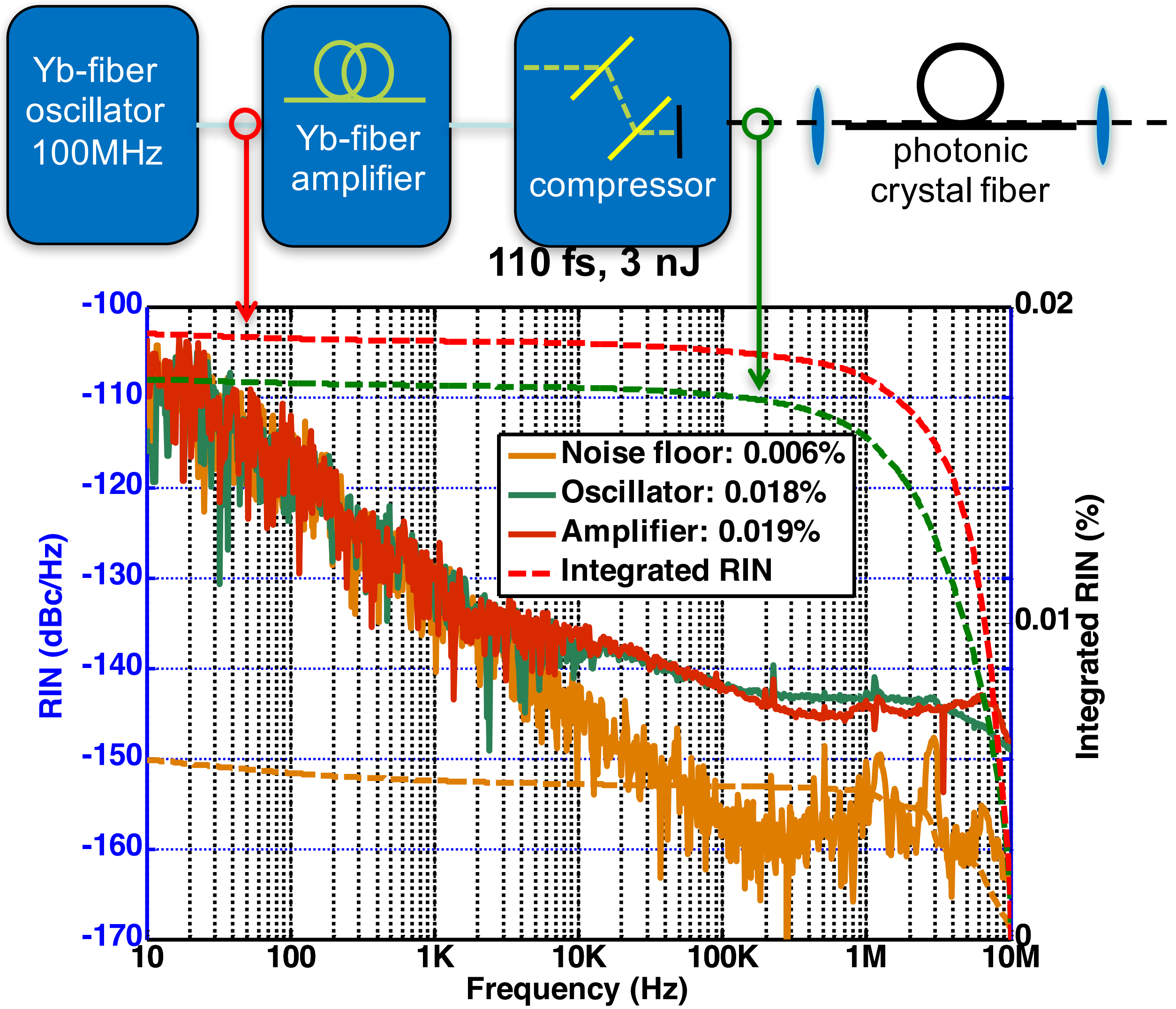
Power spectral density:

$$S_P(\quad) = \quad R(\quad)e^{\ j} \quad d \quad = \lim_{T} \frac{1}{T} \left|P_T(\quad)\right|^2$$





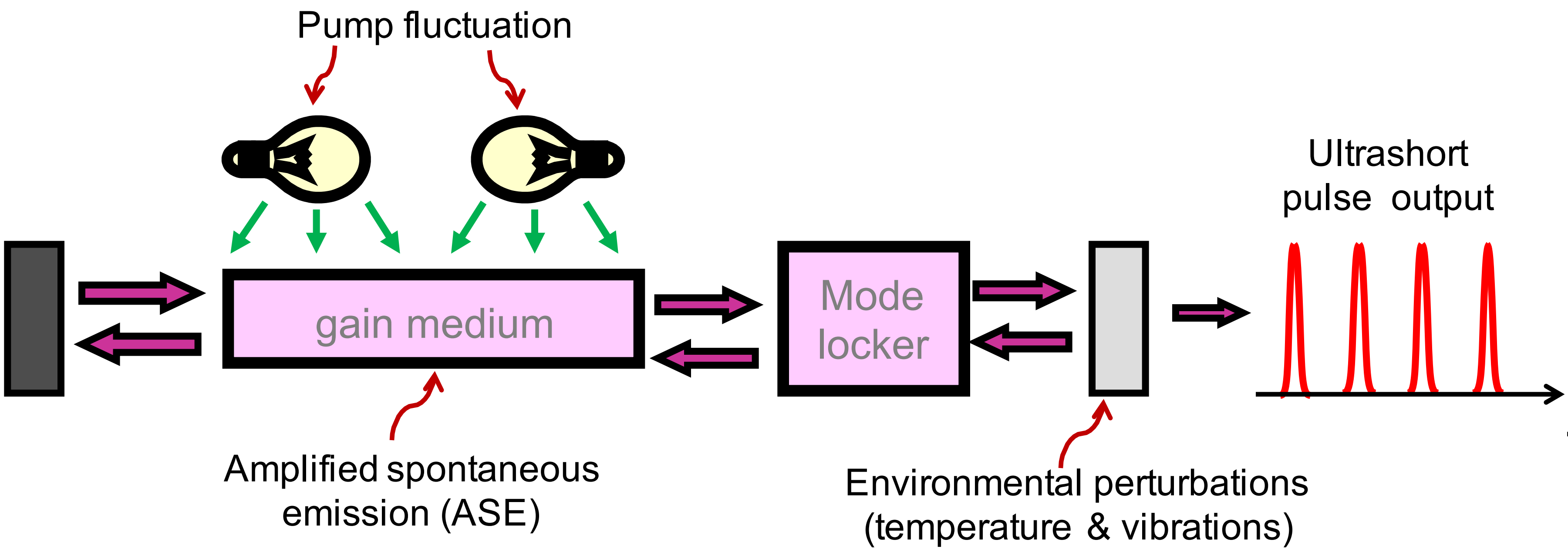
Relative intensity noise (RIN) of a laser



$$\frac{S_P(\quad)}{P_0^2}$$

dBc (decibels relative to the carrier) is the power ratio of a signal to a carrier signal, expressed in decibels.

Laser suffers from both technical and quantum noise

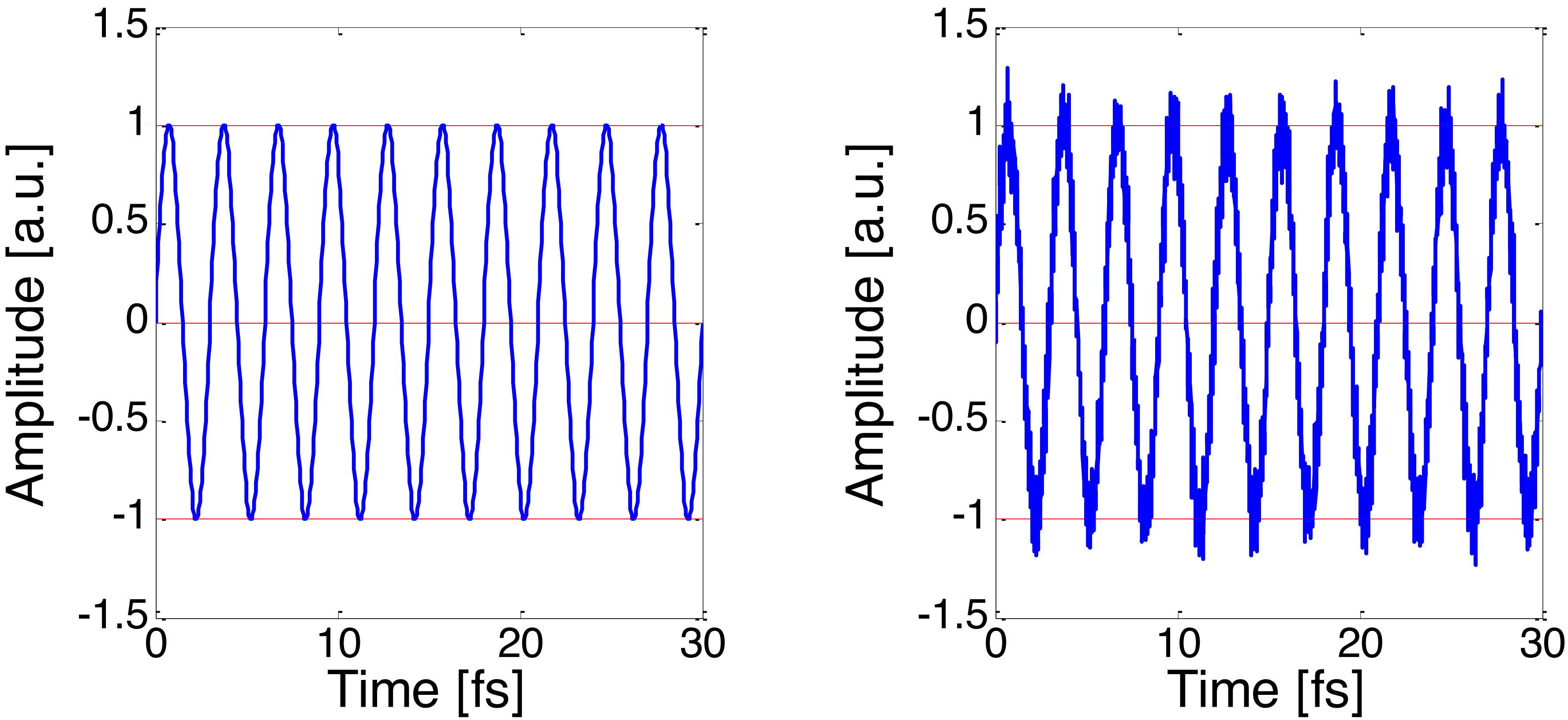


Many noise sources:

Fluctuations of the pump power Air currents and pressure fluctuations Temperature fluctuations Vibration	Technical noise
ASE in gain medium →	Quantum noise

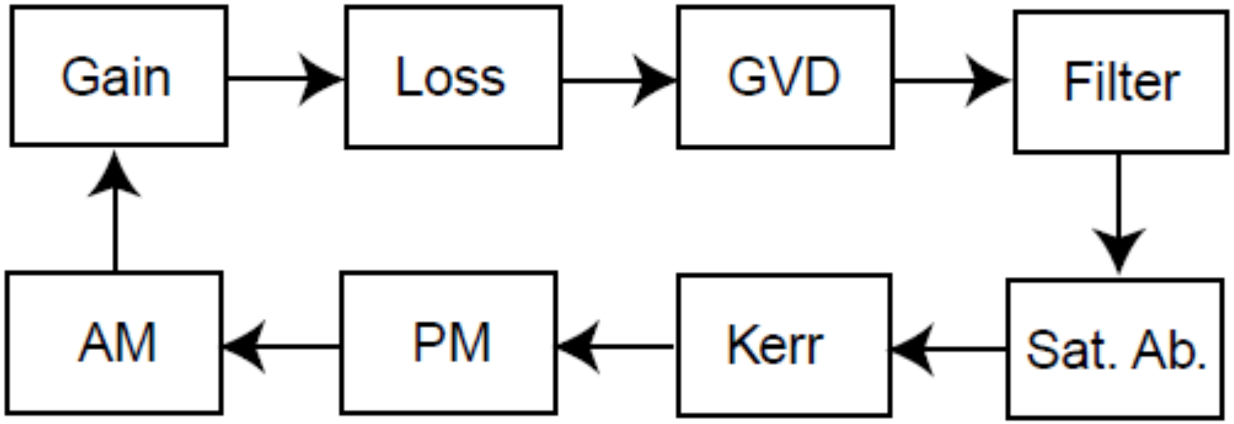
These noise sources cause the random fluctuation of the pulse's physical quantities, such as duration, energy, peak power, center wavelength, carrier-envelope phase shift, arrival time…

Example: amplitude noise and phase noise on the carrier



Pure sine wave has zero linewidth in the frequency domain. Existence of noise may broaden the line and lead to finite linewidth.

lumped element model of a mode-locked laser



GVD: group-velocity dispersion, Sat. Ab.: Saturable absorption, Kerr: Kerr nonlinearity, PM: phase modulation, AM: amplitude modulation

$$\hat{O}_{gain}(t) = g\left(1 + \frac{1}{\Omega_g^2}\frac{\partial^2}{\partial t^2}\right) \quad \text{(A.9)}$$

$$\hat{O}_{L,mirror}(t) = \frac{\ln R_1 + \ln R_2}{2} \quad \text{(A.10)}$$

$$\hat{O}_{L,material}(t) = -\alpha_i L \quad \text{(A.11)}$$

$$\hat{O}_{GVD}(t) = jD\frac{\partial^2}{\partial t^2} \quad \text{(A.12)}$$

$$\hat{O}_{filter}(t) = \frac{1}{\Omega_f^2}\frac{\partial^2}{\partial t^2} \quad \text{(A.13)}$$

$$\hat{O}_{L,filter} = 1 + \ln(1/2) \approx 0.3068528 \quad \text{(A.14)}$$

$$\hat{O}_{AM}(t) = \frac{M_{AM}}{2}(1 - \cos(\omega_M t)) \approx \frac{M_{AM}\omega_M^2 t^2}{4} \quad \text{(A.15)}$$

$$\hat{O}_{L,AM}(t) = \ln\left[\sqrt{\frac{1}{2} + M_{AM}}\right] \quad \text{(A.16)}$$

$$\hat{O}_{PM}(t) = j\frac{M_{PM}}{2}(1 - \cos(\omega_M t)) \approx j\frac{M_{PM}\omega_M^2 t^2}{4} \quad \text{(A.17)}$$

$$\hat{O}_{Kerr}(t) = -j\delta|a(t)|^2 \quad \text{(A.18)}$$

$$\hat{O}_{L,SA,slow} = -\frac{L_A}{2} \quad \text{(A.19)}$$

$$\hat{O}_{SA,slow} = -\frac{L_A w(t)}{2w_A} \quad \text{(A.20)}$$

$$\hat{O}_{L,SA,fast} = -\frac{L_A}{2} \quad \text{(A.21)}$$

$$\hat{O}_{SA,fast} = \gamma|a(t)|^2, \quad \text{(A.22)}$$

Leaf Alden Jiang, ultralow-noise modelocked lasers, MIT PhD thesis

Use master equation to calculate noise

$$
\begin{aligned}
T_R \frac{\partial a(t,T)}{\partial T} = & \left\{ -l + g\left(1 + \frac{1}{\Omega_g^2}\frac{\partial^2}{\partial t^2}\right) \right. \\
& + \frac{1}{\Omega_f^2}\frac{\partial^2}{\partial t^2} + jD\frac{\partial^2}{\partial t^2} \\
& - \frac{M_{AM} + jM_{PM}}{4}(\omega_M t)^2 \\
& \left. + (\gamma - j\delta)|a(t,T)|^2 \right\} a(t,T)
\end{aligned}
$$

t: short-term variable T: time variable on the scale of many cavity round-trip times.

1. Find the right master equation describing the laser cavity.
2. Find solution to master equation in absence of noise
3. Include noise terms as perturbations to the trial solution
4. Derive equations for the different noise fluctuations: amplitude, timing, frequency, phase, etc.
5. calculate power spectral density and correlations of the different noise fluctuation.

Example: perturbation theory for passive mode-locking with a fast saturable absorber

$$T_R \frac{\partial}{\partial T} a = jD \frac{\partial^2}{\partial t^2} a - j\delta |a|^2 a + (g-l)a + D_f \frac{\partial^2}{\partial t^2} a + \gamma |a|^2 a + L_{\text{pert}}$$

The perturbations cause fluctuations in amplitude, phase, center frequency and timing of the soliton and generate background radiation, i.e.

$$a_s(t,T) \to a_s(t,T) + \Delta A(t,T)$$

$$\Delta A(t,T) = \underline{\Delta E_p(T)} f_{E_p}(t) + \underline{\Delta\varphi(T)} f_{\varphi}(t) + \underline{\Delta f_c(T)} f_{f_c}(t) + \underline{\Delta t(T)} f_t(t) + \underline{a_c(T,t)}$$

- $\Delta E_p(T)$ → Energy fluctuation → Relative intensity noise (RIN)
- $\Delta\varphi(T)$ → Optical phase fluctuation
- $\Delta f_c(T)$ → Center frequency fluctuation
- $\Delta t(T)$ → Timing fluctuation → Timing jitter
- $a_c(T,t)$ → Continuum background

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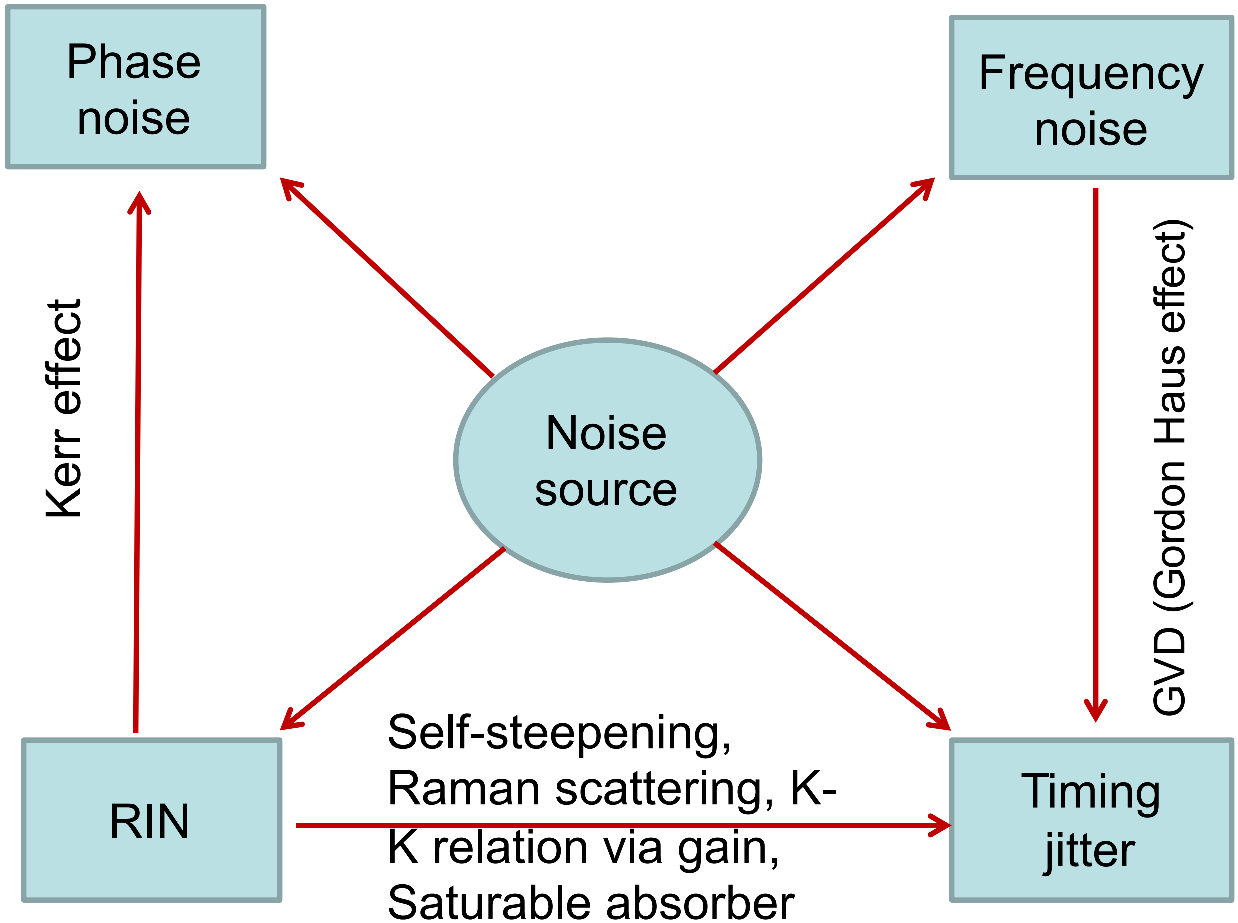
These noises are coupled by laser dynamics



Power spectral density for all four quantities

$$S_{E_p}(f) = \left|\frac{\Delta E(f)}{E_p}\right|^2 = \frac{4}{1/\tau^2{}_\omega + f^2}\frac{P_n}{E_p}$$

$$P_n \propto \frac{g}{T_R} = f_{rep} \times g$$

τ_ω: damping constant for energ

Relative intensity noise flattens out at low frequency due to gain saturation.

$$S_\varphi(f) = |\Delta\varphi(f)|^2 = \frac{1}{f^2}\left[\frac{4}{3}\left(1+\frac{\pi^2}{12}\right)\frac{P_n}{E_p} + \frac{16}{(1/\tau_\omega{}^2 + f^2)}\frac{\emptyset_0{}^2}{T_R{}^2}\frac{P_n}{E_p}\right]$$

An energy change couples to the phase evolution, because the change affects the Kerr phase shift.

$$S_{f_c}(f) = \frac{4}{3}\frac{1}{(1/\tau_p{}^2 + f^2)}\frac{P_n}{E_p}$$

τ_p: damping constant for center frequenc

Frequency deviations damp out, because the gain spectrum pushes the spectrum back to line center.

$$S_{\Delta t}(f) = \frac{1}{f^2}\frac{\pi^2}{3}\frac{P_n}{E_p}\tau^2 + \frac{1}{3}\frac{1}{f^2}\frac{16}{(1/\tau_p{}^2 + f^2)}\frac{|D|^2}{\tau^2 T_R{}^2}\frac{P_n}{E_p}$$

Gordon-Haus effect

1. A change of frequency leads to a change of timing jitter.
2. Shorter (longer) pulse reduce the effect of the first (second) term.

Haus, Noise of mode-locked lasers, IEEE JQE 29, 983 (1993)