Ultrafast Optical Physics II (SoSe 2019)
Lecture 6, May 17

5 Active Mode Locking

5. Laser Mode Locking

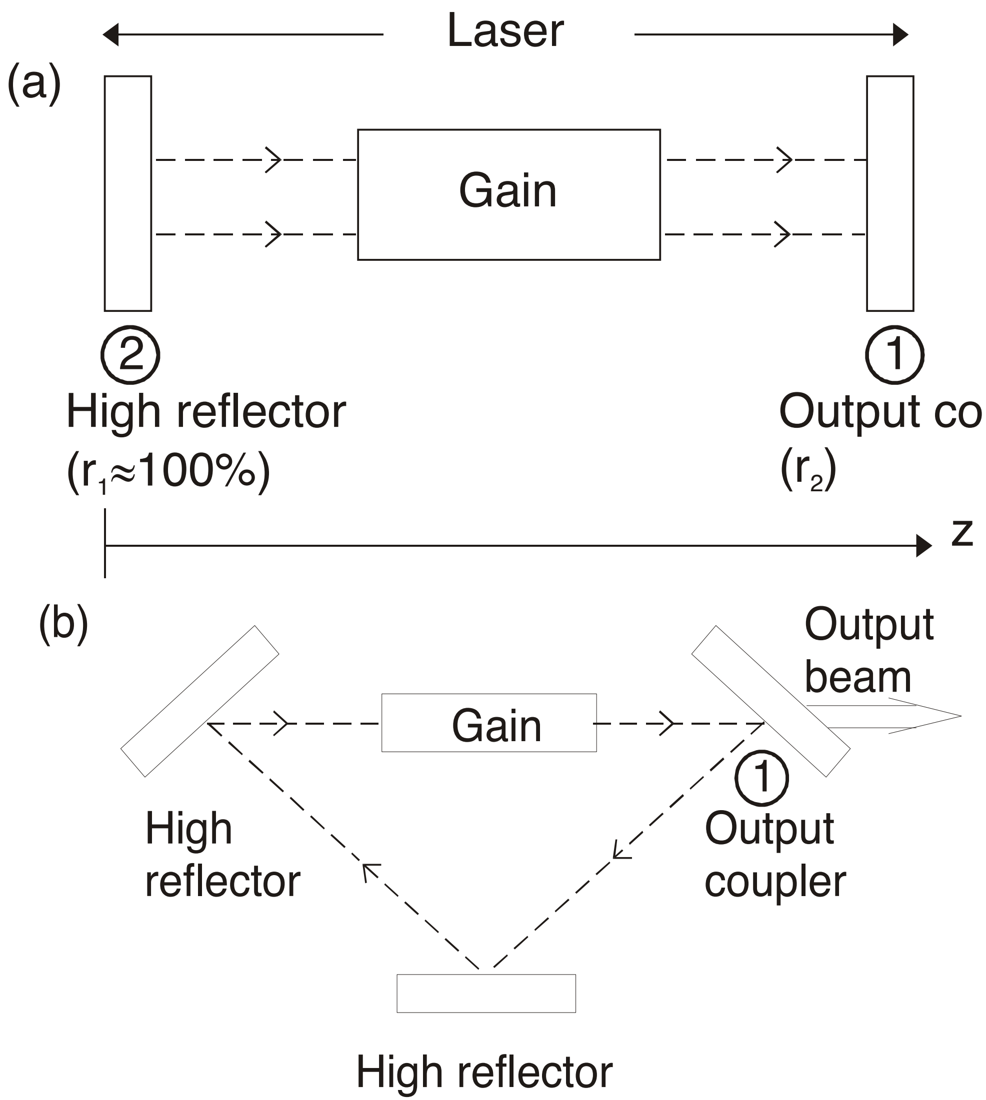


Figure 1.12: Possible cavity configurations

Steady State Lasing

$$E(z,t) = \Re\left\{E_0 e^{j(\omega t - kz)}\right\}$$

Gain and loss: $n = n' + jn''$

$$k = \frac{\omega}{c} n$$

After propagation through gain medium and air path: $\ell = n_g' \ell_g + \ell_a$

$$E = \Re\left\{E_0 e^{\frac{\omega}{c} n_g'' \ell_g} e^{j\omega t} e^{-j\frac{\omega}{c}(n_g' \ell_g + \ell_a)}\right\}$$

Steady-State Condition:

$$E = \Re\left\{r_1 r_2 e^{2\frac{\omega}{c} n_g'' \ell_g} E_0 e^{j\omega t - j2\frac{\omega}{c}\ell}\right\} \Rightarrow r_1 r_2 e^{2\frac{\omega}{c} n_g'' \ell_g} = 1$$

Mode Condition: $$\frac{2\omega\ell}{c} = 2m\pi$$

Resonance Frequencies: $$\omega_m = \frac{m\pi c}{\ell} \qquad f_m = \frac{mc}{2\ell}$$

$$\Delta f = f_m - f_{m-1} = \frac{c}{2\ell}$$

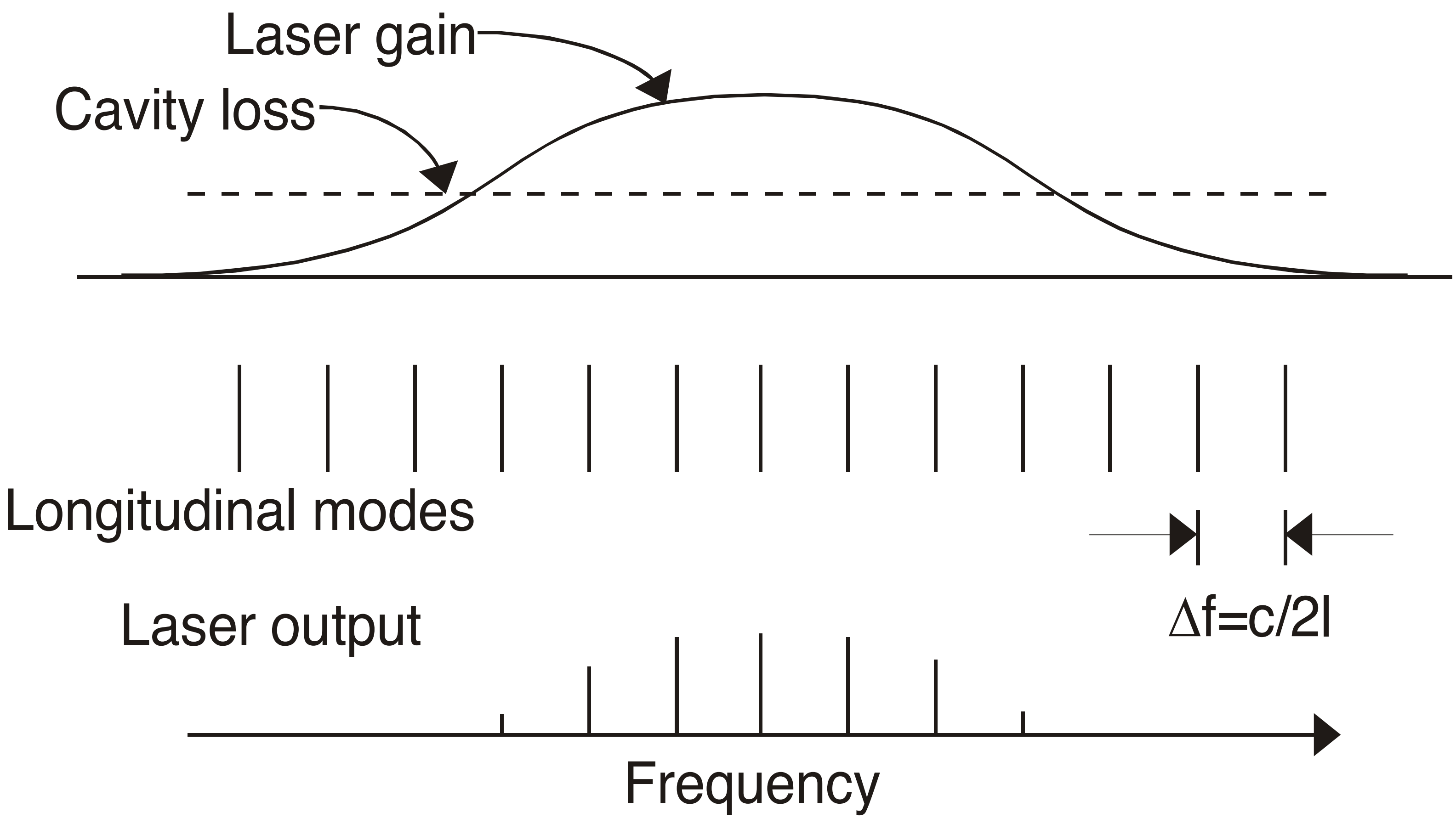


Figure 1.13: Laser gain and cavity loss spectrum

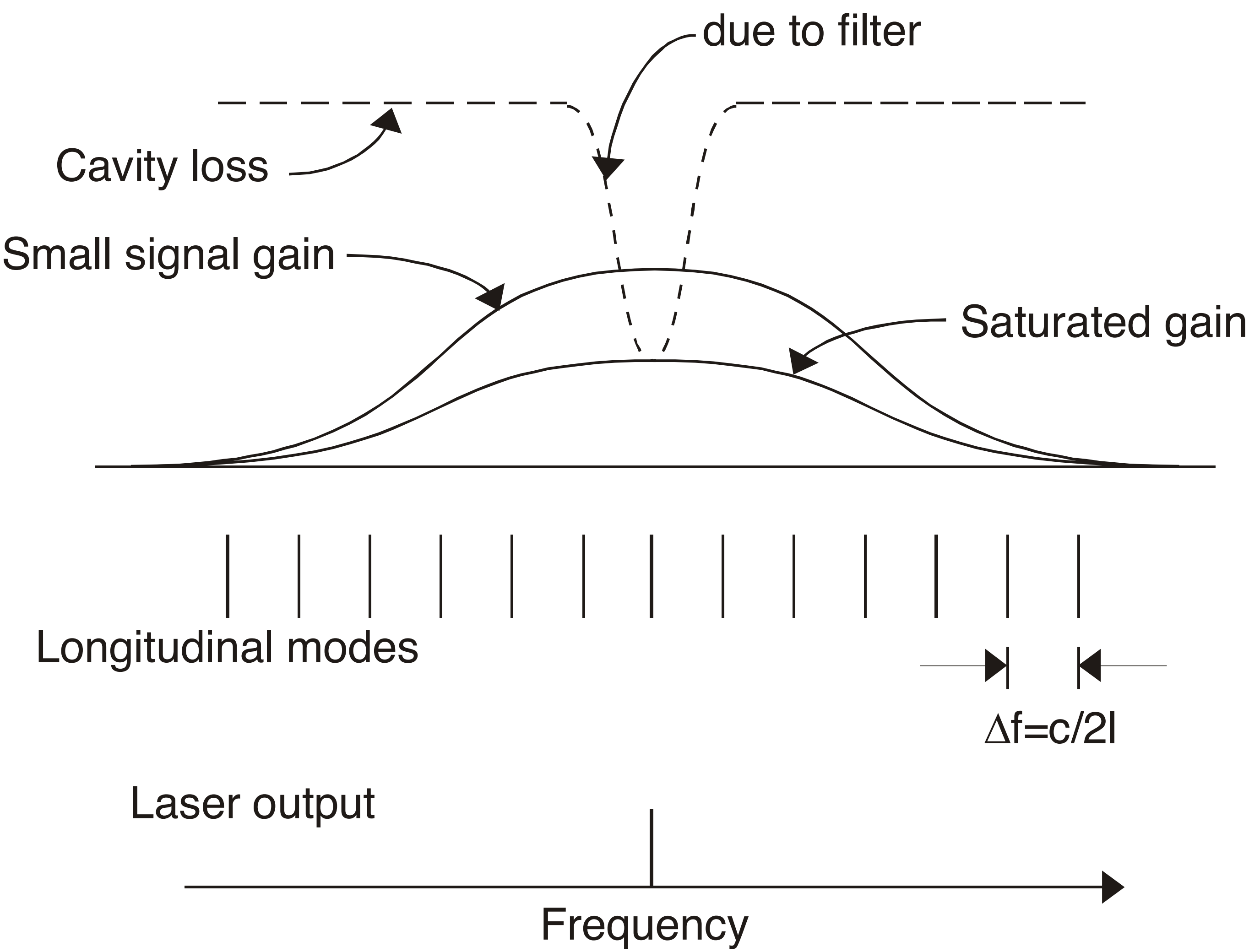


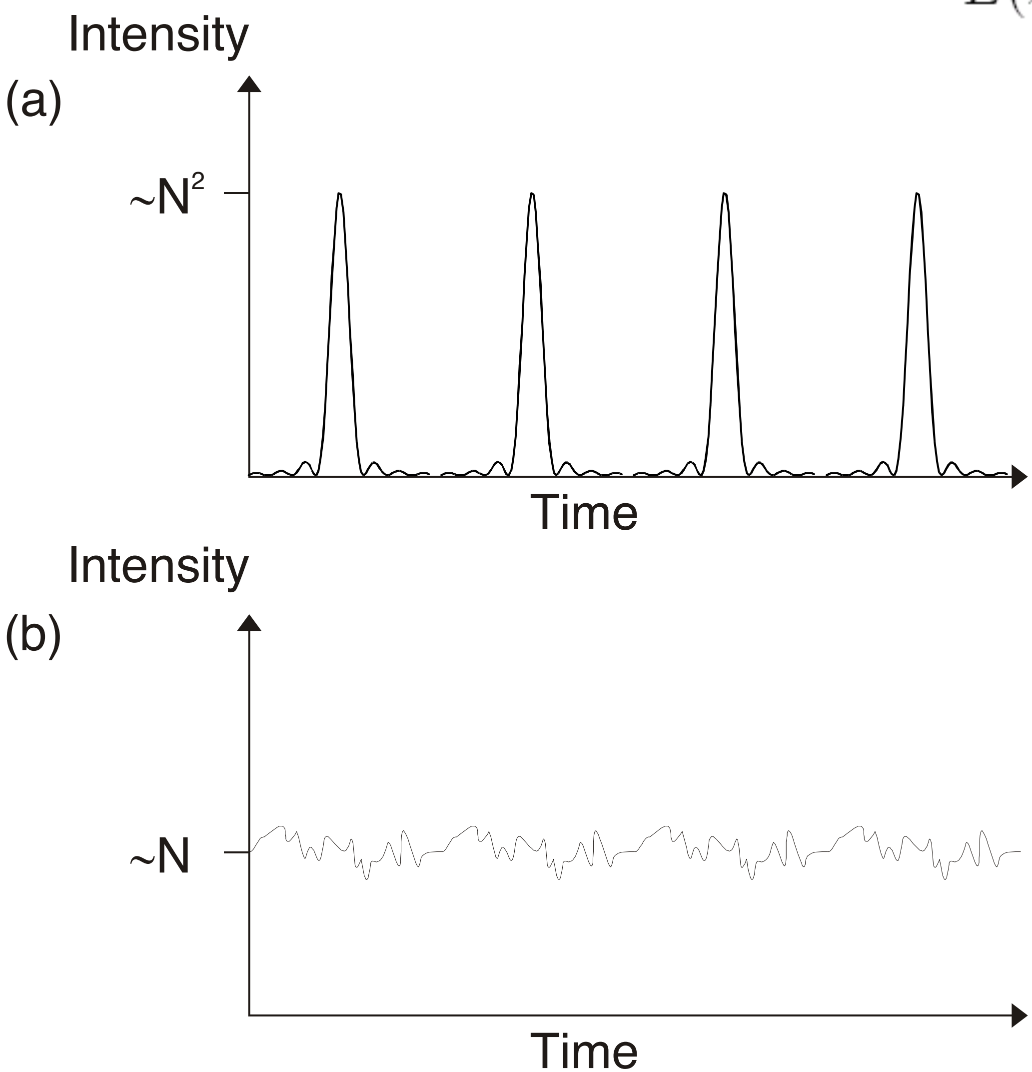
Figure 1.14: Gain and loss spectra.

Small signal gain

$$g = \frac{g_0}{1 + \frac{I}{I_{sat}}}$$

Saturation Intensity

Mode Locking:



$$E(z,t) = \Re\left[\sum_m \tilde{E}_m e^{\mathrm{j}(\omega_m t - k_m z + \phi_m)}\right]$$

$$\omega_m = \omega_0 + m\Delta\omega = \omega_0 + \frac{m\pi c}{\ell},$$

$$k_m = \frac{\omega_m}{c},$$

$$= \Re\left[A(t - z/c)e^{\mathrm{j}\omega_0(t-z/c)}\right]$$

Envelope   Carrier

$$\Delta t = \frac{2\pi}{N\Delta\omega} = \frac{1}{N\Delta f}$$

Figure 1.15: (a) mode-locked laser output with constant pulse
(b): with random phase.

Self-consistent Model for Mode-Locked Laser

Medium modeled by pumped two level atoms
(Polarization and Inversion, at each point in space and time)

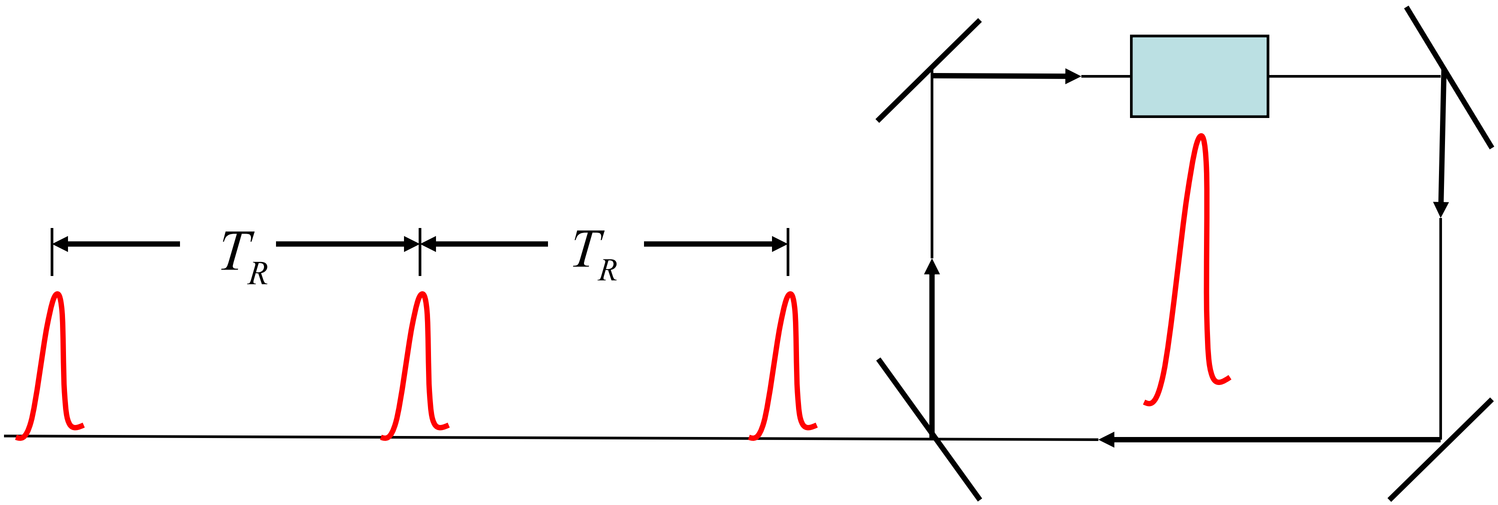
Medium Polarization is source in the Maxwell's Equations

Maxwell Equations describe Field in Resonator

Resonator included by proper boundary conditions for fields at the cavity mirrors.

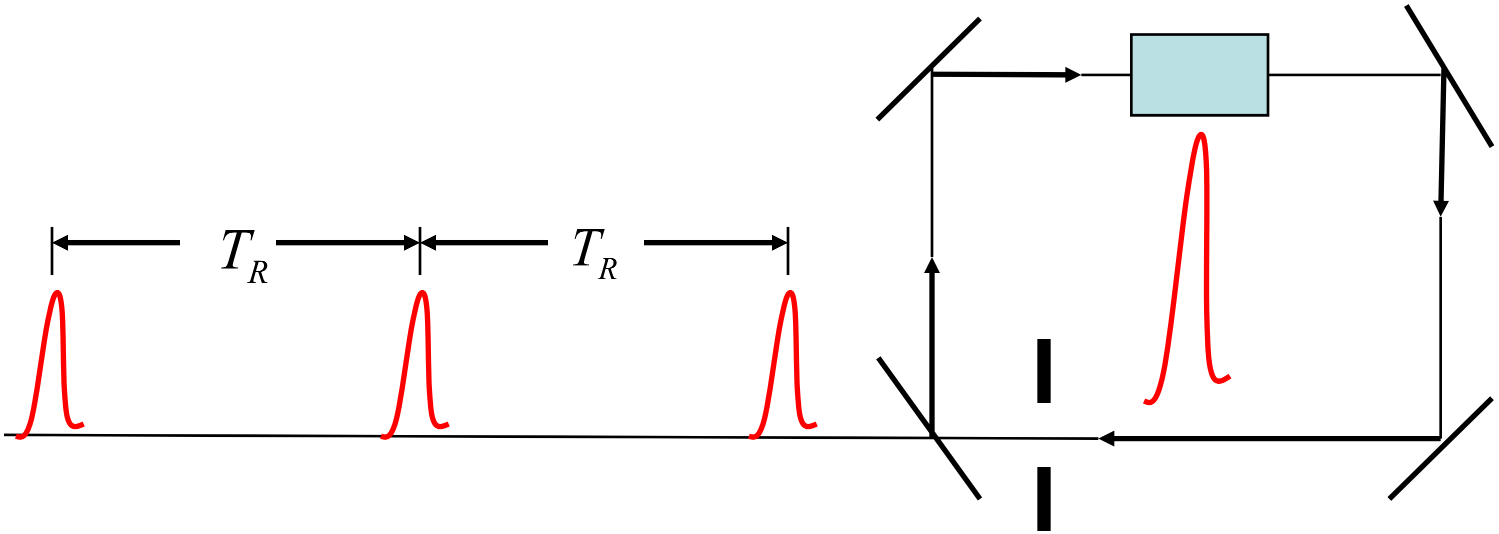
Directly write field as a sum of modes whose amplitudes change slowly with time due to coupling to the gain medium, dispersion, ….

Time-domain picture of mode-locking



Each time the pulse hits the output coupler, a small fraction of the power is transmitted out of the cavity. The output is a pulse train with repetition rate $1/T_{R.}$

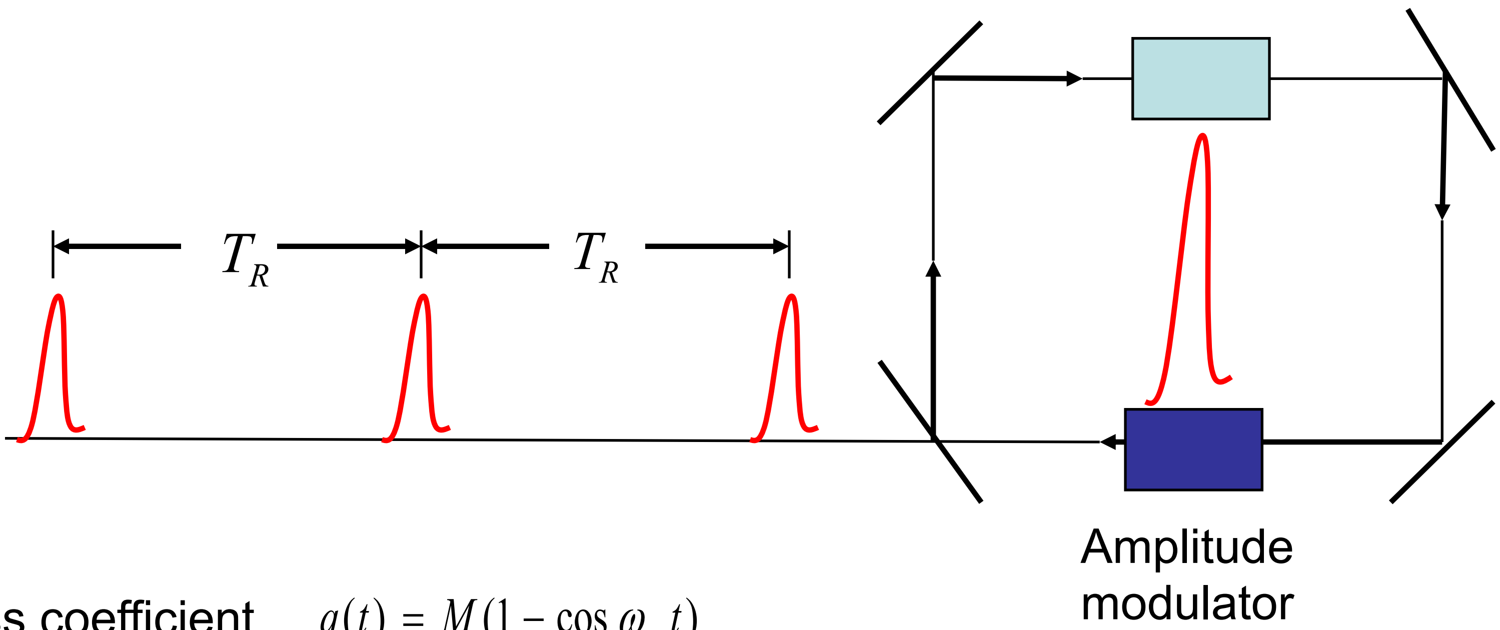
Force laser to generate short-pulse train using a "shutter" to modulate cavity loss



Transient process:

•Shutter is opened, loss is low → laser is above threshold → peak builds up
•Shutter is closed, loss is high → laser is blow threshold → wings developed

Active mode-locking

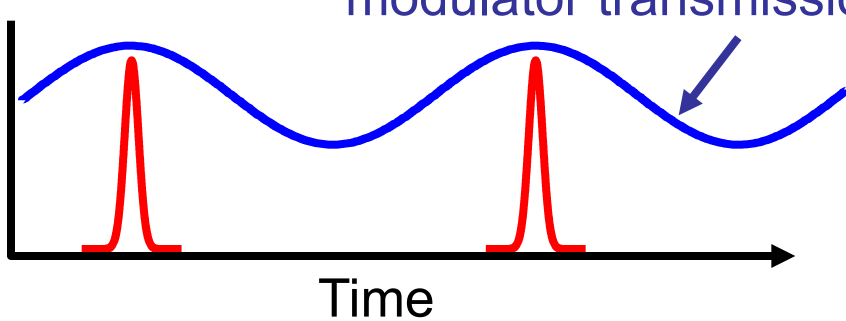


Loss coefficient $q(t) = M(1 - \cos\omega_m t)$

Transmission of the modulator

$$T_m = e^{-M(1-\cos\omega_m t)}$$

$$T_m \approx 1 - M(1-\cos\omega_m t)$$

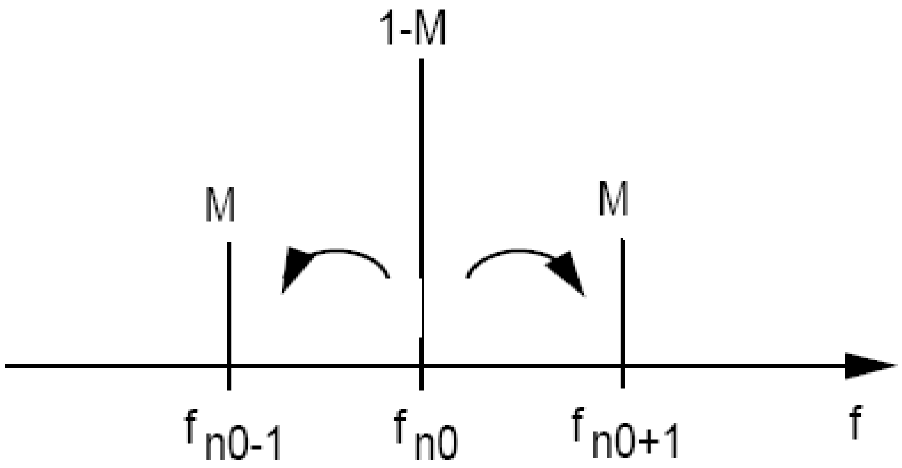


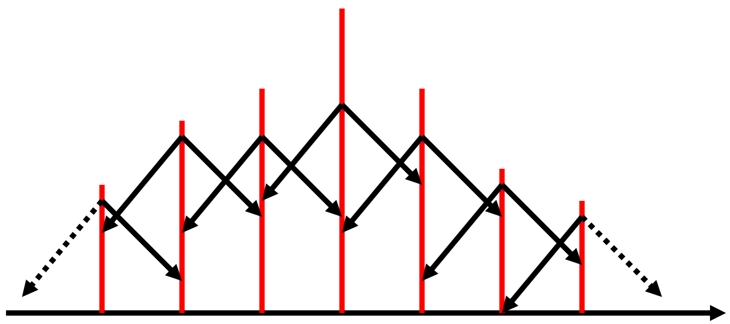
Talking among modes

$$e^{j\omega_{n0}t}T_m \approx e^{j\omega_{n0}t}(1 - M + M\cos\omega_m t)$$

$$= (1-M)e^{j\omega_{n0}t} + \frac{M}{2}(e^{j\omega_m t} + e^{-j\omega_m t})e^{j\omega_{n0}t}$$

$$= (1-M)e^{j\omega_{n0}t} + \frac{M}{2}e^{j(\omega_{n0}+\omega_m)t} + \frac{M}{2}e^{j(\omega_{n0}-\omega_m)t}$$





The modulator actually takes power out of the central mode and redistributes it to the other modes. This is how mode-locking can occur in a homogeneously broadened laser.

Decompose wave into oscillating modes

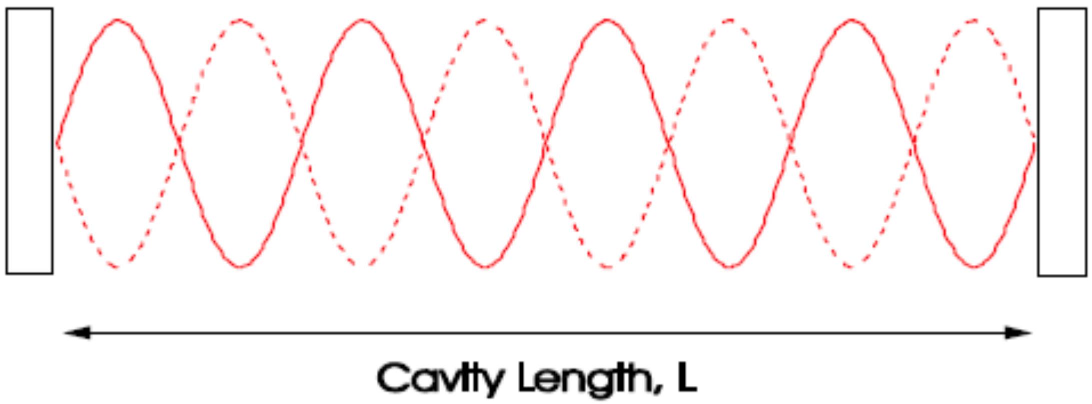


Figure 5.1: Fabry-Perot resonator

$$E^{(left)}(z,t) = Re\left\{\sum_{n=0}^{\infty} \hat{E}_n e^{j(\Omega_n t + K_n z)}\right\},$$

$$K_n = n\pi/L$$

$$E^{(right)}(z,t) = Re\left\{\sum_{n=0}^{\infty} \hat{E}_n e^{j(\Omega_n t - K_n z)}\right\}.$$

We consider the modes in Eq.(5.2) as a continuum and replace the sum by an integral

$$E^{(right)}(z,t) = \frac{1}{2\pi} Re \left\{ \int_{K=0}^{\infty} \hat{E}(K) e^{j(\Omega(K)t - Kz)} dK \right\} \tag{5.3}$$

with

$$\hat{E}(K_m) = \hat{E}_m 2L. \tag{5.4}$$

Eq.(5.3) is similar to the pulse propagation discussed in chapter 2 and describes the pulse propagation in the resonator. However, here it is rather an initial value problem, rather than a boundary value problem. Note, the wavenumbers of the modes are fixed, not the frequencies. To emphasize this even more, we introduce a new time variable $T = t$ and a local time frame $t' = t - z/\upsilon_{g,0}$, instead of the propagation distance z, where $\upsilon_{g,0}$ is the group velocity at the central wave number K_{n_0} of the pulse

$$\upsilon_{g,0} = \left.\frac{\partial \omega}{\partial k}\right|_{k=0} = \left.\left(\frac{\partial k}{\partial \omega}\right)^{-1}\right|_{\omega=0}. \tag{5.5}$$

$$
\begin{aligned}
k &= K - K_{n_0}, &(5.6)\\
\omega(k) &= \Omega(K_{n_0} + k) - \Omega_{n_0}, &(5.7)\\
\hat{E}(k) &= \hat{E}(K_{n_0} + k), &(5.8)
\end{aligned}
$$

The temporal evolution of the pulse is then determined by

$$
E^{(right)}(z,t) = \frac{1}{2\pi} Re \left\{ \int_{-K_{n_0} \to -\infty}^{\infty} \hat{E}(k) e^{j(\omega(k)t - kz)} dk \right\} e^{j(\omega_0 t - k_0 z)}. \tag{5.9}
$$

Analogous to chapter 2, we define a slowly varying field envelope, that is already normalized to the total power flow in the beam

$$
A(z,t) = \sqrt{\frac{A_{eff}}{2Z_0}} \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{E}(k) e^{j(\omega(k)t - kz)} dk. \tag{5.10}
$$

$$A(T,t') = \sqrt{\frac{A_{eff}}{2Z_0}} \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{E}(k) e^{j((\omega(k) - \upsilon_{g,0} k)T + k\upsilon_{g,0} t'} dk. \tag{5.11}$$

which can be written as

$$T_R \frac{\partial A(T,t')}{\partial T}\bigg|_{(GDD)} = j \sum_{n=2}^{\infty} D_n \left(-j \frac{\partial}{\partial t'} \right)^n A(T,t'), \tag{5.12}$$

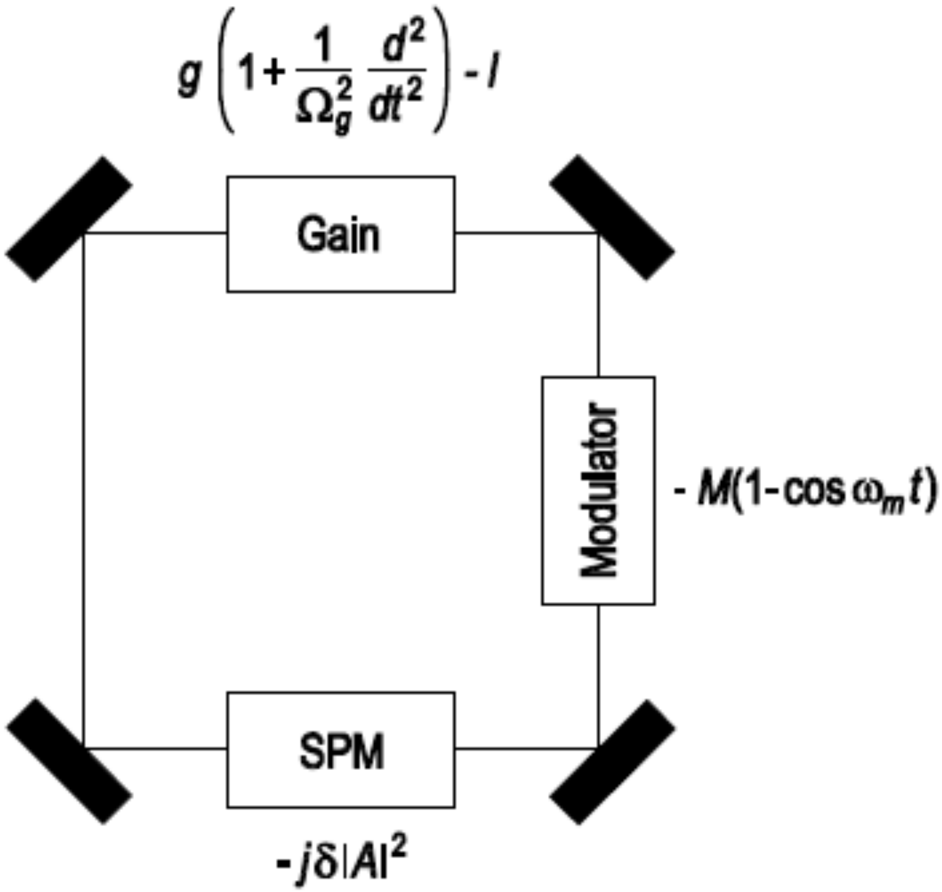
with the dispersion coefficients per resonator round-trip $T_R = \frac{2L}{\upsilon_{g,0}}$

$$D_n = \frac{2L}{n! \upsilon_{g,0}^{n+1}} \frac{\partial^{n-1} \upsilon_g(k)}{\partial k^{n-1}}\bigg|_{k=0}. \tag{5.13}$$

The dispersion coefficients (5.13) look somewhat suspicious, however, it is not difficult to show, that they are equivalent to derivatives of the roundtrip phase $\phi_R(\Omega) = \frac{\Omega}{c} n(\Omega) 2L$ in the resonator at the center frequency

$$D_n = -\frac{1}{n!} \frac{\partial^n \phi_R^{(n)}(\Omega)}{\partial \Omega^n}\bigg|_{\Omega=\omega_0}, \tag{5.14}$$

5.1 Master equation of mode-locking



Assume in steady state, the change in the pulse caused by each element in the cavity small.

$$T_R \frac{\partial A(T,t)}{\partial T} = \sum_i \Delta A_i = 0$$

A: the pulse envelope
T_R: the cavity round-trip time
T: the time that develops on a time scale of the order of T_R
t: the fast time of the order of the pulse duration
ΔA_i: the changes of the pulse envelope due to different elements in the cavity.

Loss:

$$T_R \frac{\partial A(T,t')}{\partial T}\bigg|_{(loss)} = -lA(T,t') \qquad (5.15)$$

Gain:

$$T_R \frac{\partial A(T,t')}{\partial T}\bigg|_{(gain)} = \left(g(T) + D_g \frac{\partial^2}{\partial t'^2}\right) A(T,t'), \qquad D_g = \frac{g(T)}{\Omega_g^2}$$

loss

dispersion

$$\begin{aligned} T_R \frac{\partial A(T,t')}{\partial T} &= -lA(T,t') + j\sum_{n=2}^{\infty} D_n \left(j\frac{\partial}{\partial t'}\right)^n A(T,t') \\ &+ g(T)\left(1 + \frac{1}{\Omega_g^2}\frac{\partial^2}{\partial t'^2}\right) A(T,t') \\ &- q(T,t')A(T,t') - j\delta|A(T,t')|^2 A(T,t'). \end{aligned} \qquad (5.21)$$

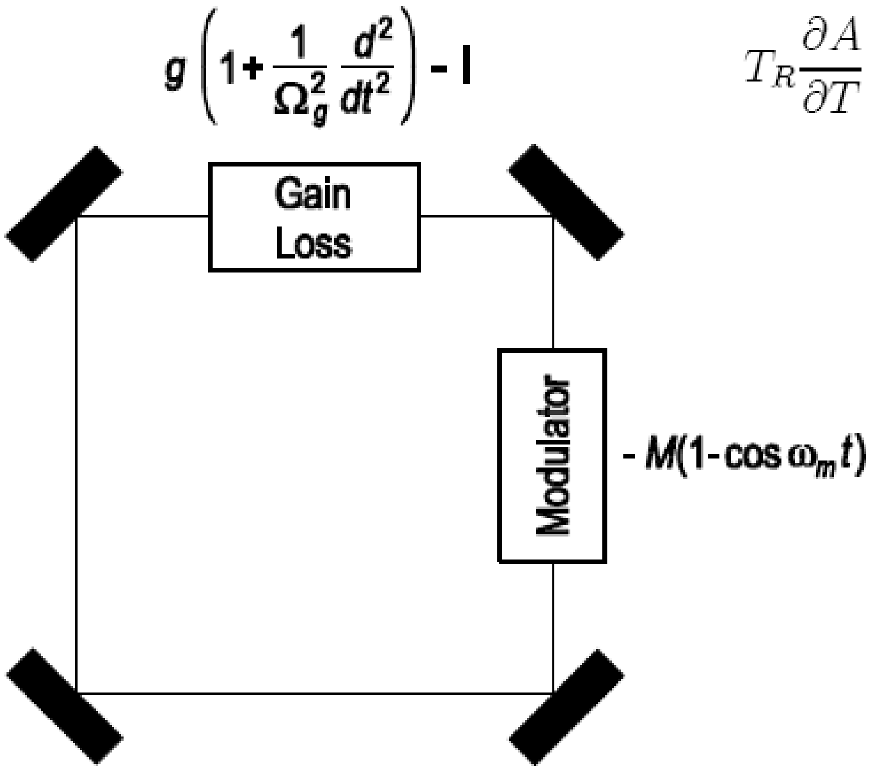
gain

Mode-locking element

Gain dispersion

Self-phase modulation

5.2 Active mode-locking by loss modulation



$$T_R\frac{\partial A}{\partial T}=\left[g(T)+D_g\frac{\partial^2}{\partial t^2}-l-M\left(1-\cos(\omega_M t)\right)\right]A.$$

↓

$$T_R\frac{\partial A}{\partial T}=\left[g-l+D_g\frac{\partial^2}{\partial t^2}-M_s t^2\right]A.$$

$$D_g=\frac{g}{\Omega_g^2},$$

$$M_s=\frac{M\omega_M^2}{2}.$$

Gain tends to narrow the spectrum, spectrum broadened by modulator.

Hermite-Gaussian Solutions

$$A_n(T,t) = A_n(t)e^{\lambda_n T/T_R} \qquad A_n(t) = \sqrt{\frac{W_n}{2^n\sqrt{\pi}n!\tau_a}}H_n(t/\tau_a)e^{-\frac{t^2}{2\tau_a^2}}$$

$$\tau_a = \sqrt[4]{D_g/M_s}. \longrightarrow \tau_a = \sqrt[4]{2}\left(\frac{g}{M}\right)^{\frac{1}{4}}\frac{1}{\sqrt{\Omega_g\omega_M}}$$

1) Larger modulation depth, M, and higher modulation frequency will give shorter pulses because the “low loss” window becomes narrower, thus shortening the pulses.
2) A broader gain bandwidth yields shorter pulses because the filtering effect of gain narrowing is lower and more modes are lasing.

$$\lambda_n = g_n - l - 2M_s\tau_a^2(n+\frac{1}{2}).$$

For given g, the eigen solution with n=0 has the largest gain per round-trip and saturate the gain to

$$g_s = l + M_s\tau_a^2$$

All other modes will decay.

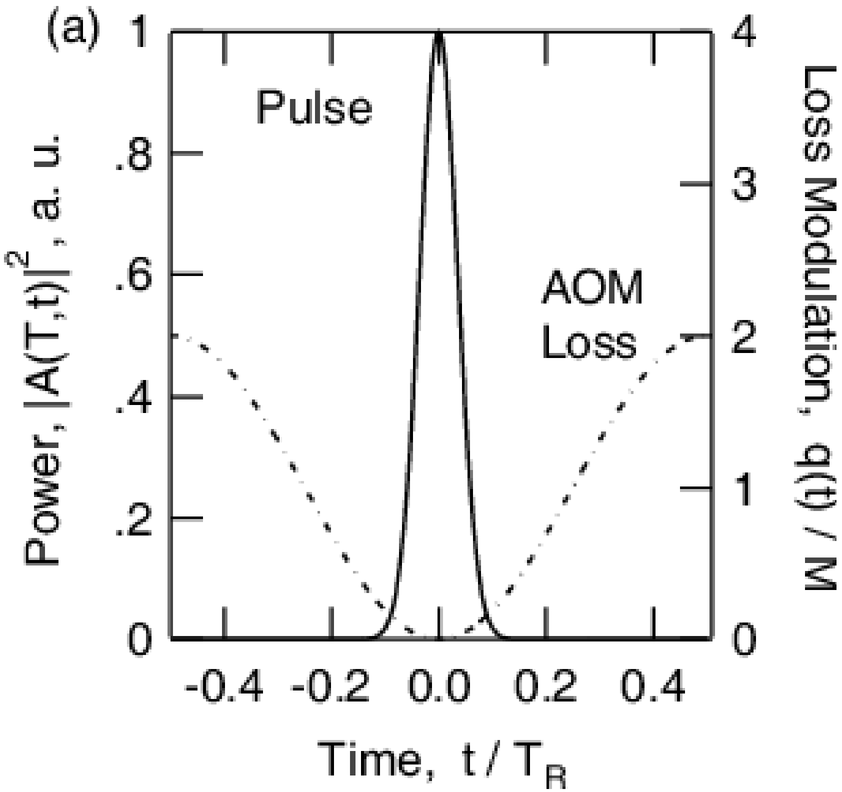


Fig. 5.4: Loss modulation results in pulse shortening in each roudntrip

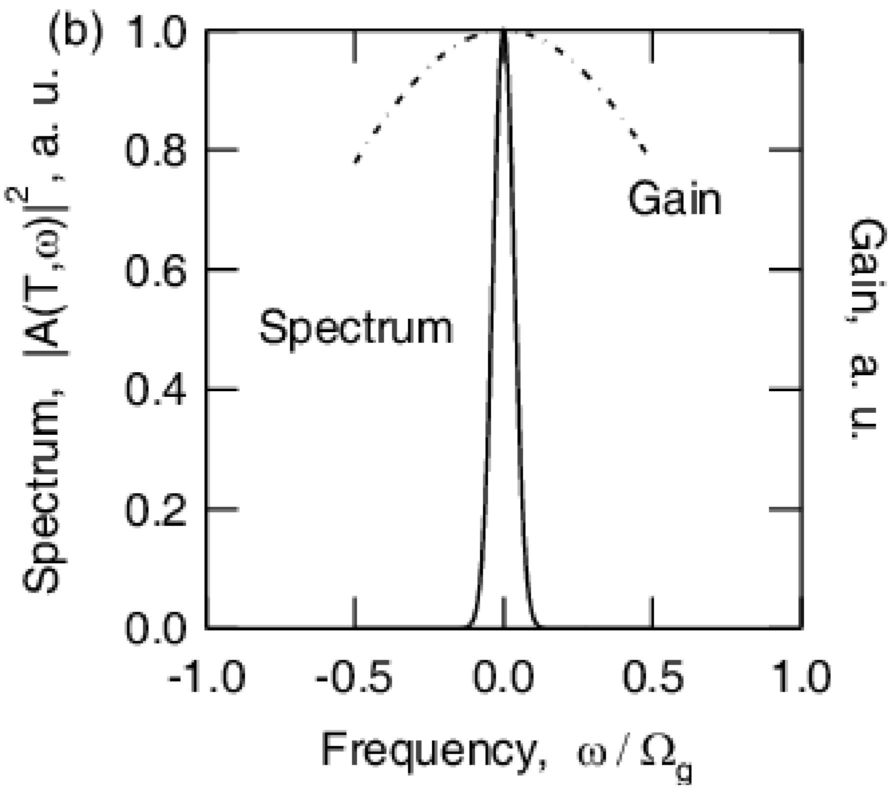


Fig. 5.5: Gain filtering broadens the pulse in each roundtrip

Example:

Nd:YAG; 2l=2g=10%,

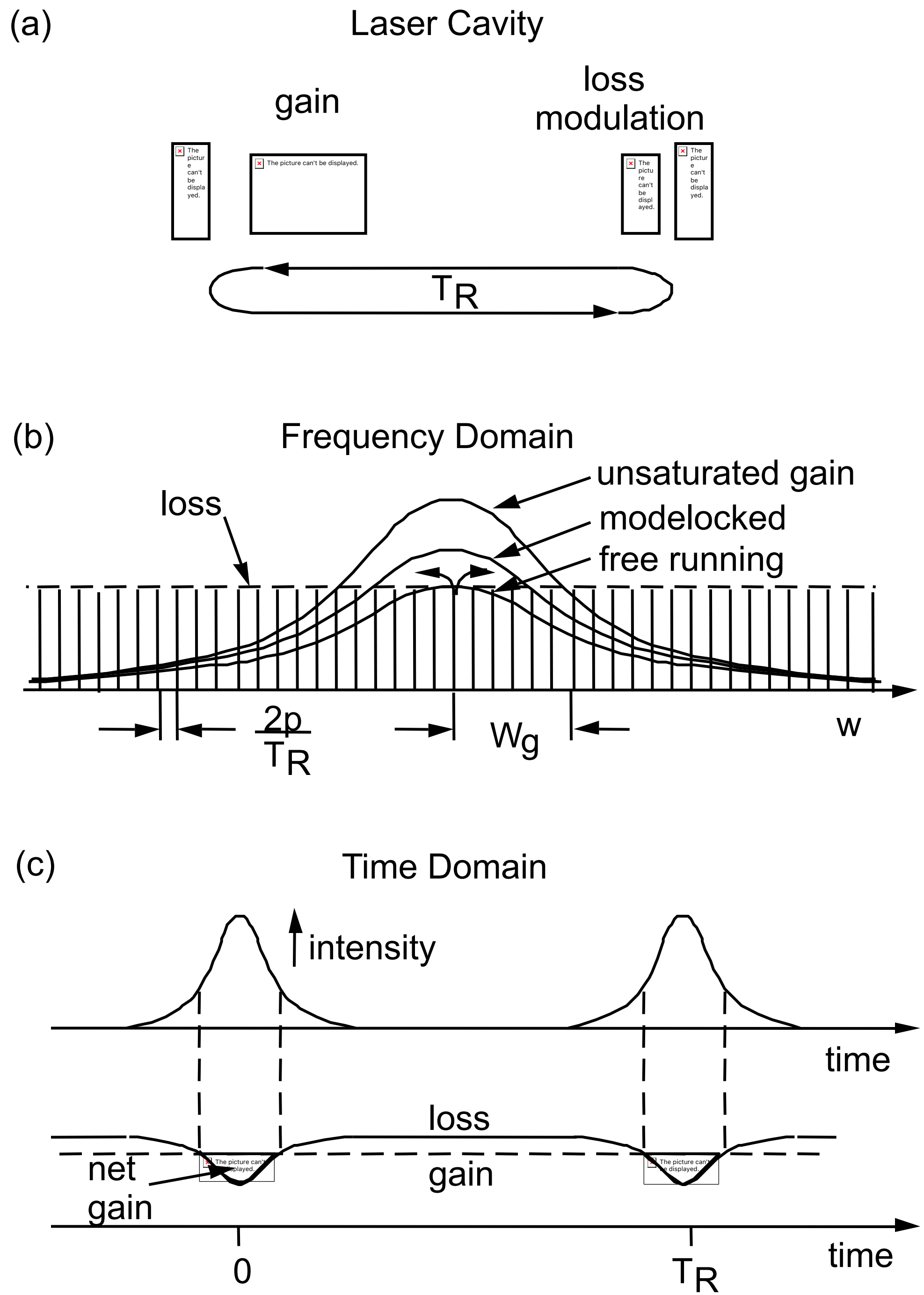
$\Omega_g = \pi \, \Delta f_{FWHM}$=0.65 THz,

M = 0.2, f_R =100 MHz,

D_g = 0.24 ps²,

M_s =4e16 /s².

$\tau = 99$ ps.


(a)
Laser Cavity
gain
loss
modulation
TR
(b)
Frequency Domain
unsaturated gain
loss
modelocked
free running
2p
TR
Wg
w
(c)
Time Domain
intensity
time
loss
net
gain
gain
0
TR
time

5.2 Active mode-locking by phase modulation

It can be modeled using master equation by replacing *M* by *jM*

$$T_R \frac{\partial A}{\partial T} = \left[g(T) + D_g \frac{\partial^2}{\partial t^2} - l - jM\left(1 - \cos(\omega_M t)\right) \right] A$$

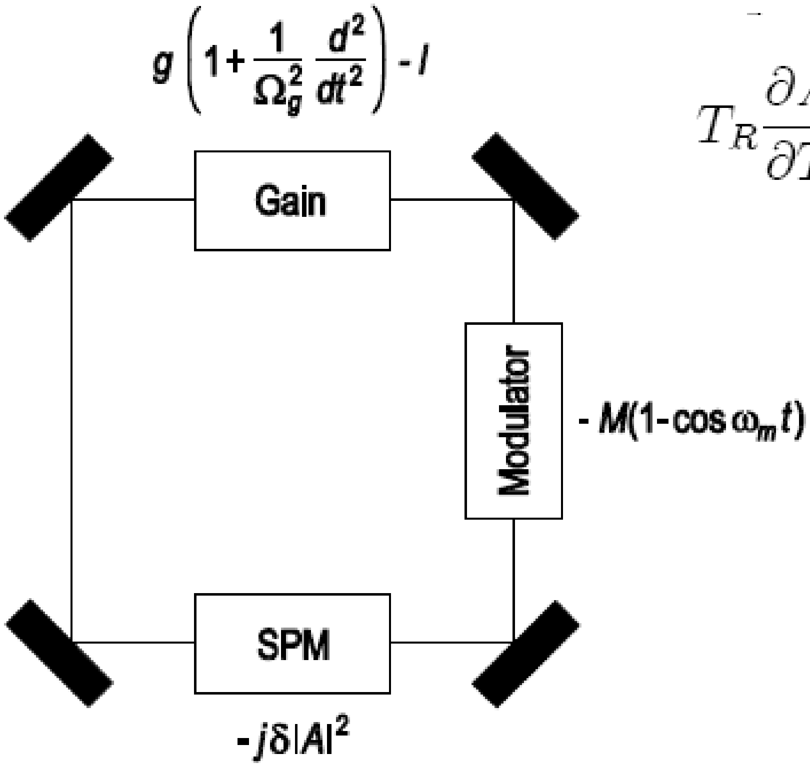
$$\tau'_a = \sqrt[4]{-j}\,\sqrt[4]{D_g/M_s}.$$

$$A_0(t) = \sqrt{\frac{W_s}{2^n \sqrt{\pi} n! \tau'_a}} e^{-\frac{t^2}{2\tau_a^2} \frac{1}{\sqrt{2}}(1+j)}$$

Chirp pulse

Pulse duration $\tau_a = \sqrt[4]{D_g/M_s}$

5.4 Active mode-locking with additional SPM



$$T_R\frac{\partial A}{\partial T}=\left[g(T)+D_g\frac{\partial^2}{\partial t^2}-l-M_s t^2-j\delta|A|^2\right]A.$$

$$A_0(t)=Ae^{-\frac{t^2}{2\tau_a^2}(1+j\beta)+j\Psi T/T_R}$$

$$\begin{aligned}j\Psi A_0(t) &= \left\{g-l+D_g\left[\frac{t^2}{\tau_a^4}(1+j\beta)^2-\frac{1}{\tau_a^2}(1+j\beta)\right]\right.\\ &\quad\left.-M_s t^2-j\delta|A|^2 e^{-\frac{t^2}{\tau_a^2}}\right\}A_0(t).\end{aligned}$$

$$
\begin{aligned}
j\Psi A_0(t) \;&=\; \left\{g - l + D_g\left[\frac{t^2}{\tau_a^4}(1+j\beta)^2 - \frac{1}{\tau_a^2}(1+j\beta)\right]\right. \\
&\left. -M_s t^2 - j\delta\,|A|^2 e^{-\frac{t^2}{\tau_a^2}}\right\} A_0(t).
\end{aligned}
$$

$$
j\Psi = g - l + D_g\left[\frac{t^2}{\tau_a^4}(1+j\beta)^2 - \frac{1}{\tau_a^2}(1+j\beta)\right] - M_s t^2 - j\delta\,|A|^2\left(1 - \frac{t^2}{\tau_a^2}\right)
$$

$$
\begin{aligned}
0 \;&=\; \frac{D_g}{\tau_a^4}\left(1-\beta^2\right) - M_s \\
0 \;&=\; 2\beta\frac{D_g}{\tau_a^4} + \frac{\phi_0}{\tau_a^2},
\end{aligned}
$$

$$
\begin{aligned}
g - l \;&=\; \frac{D_g}{\tau_a^2}, \\
\Psi \;&=\; D_g\left[-\frac{1}{\tau_a^2}\beta\right] - \phi_0
\end{aligned}
$$

$$0 = \frac{D_g}{\tau_a^4}\left(1-\beta^2\right) - M_s$$
$$0 = 2\beta\frac{D_g}{\tau_a^4} + \frac{\phi_0}{\tau_a^2},$$

$\longrightarrow$

$$\beta = -\frac{\phi_0 \tau_a^2}{2D_g}$$
$$\tau_a^4 = \frac{D_g}{M_s + \frac{\phi_0^2}{4D_g}}.$$

The resulting pulse is shorter due to additional SPM. However, as the pulse shortens by a factor of 2, SPM drives the mode-locking unstable. It is why broadband laser media can not simply generate femtosecond pulses via active mode-locking, since SPM drives them unstable.

5.5 Active Modelocking with Soliton Formation

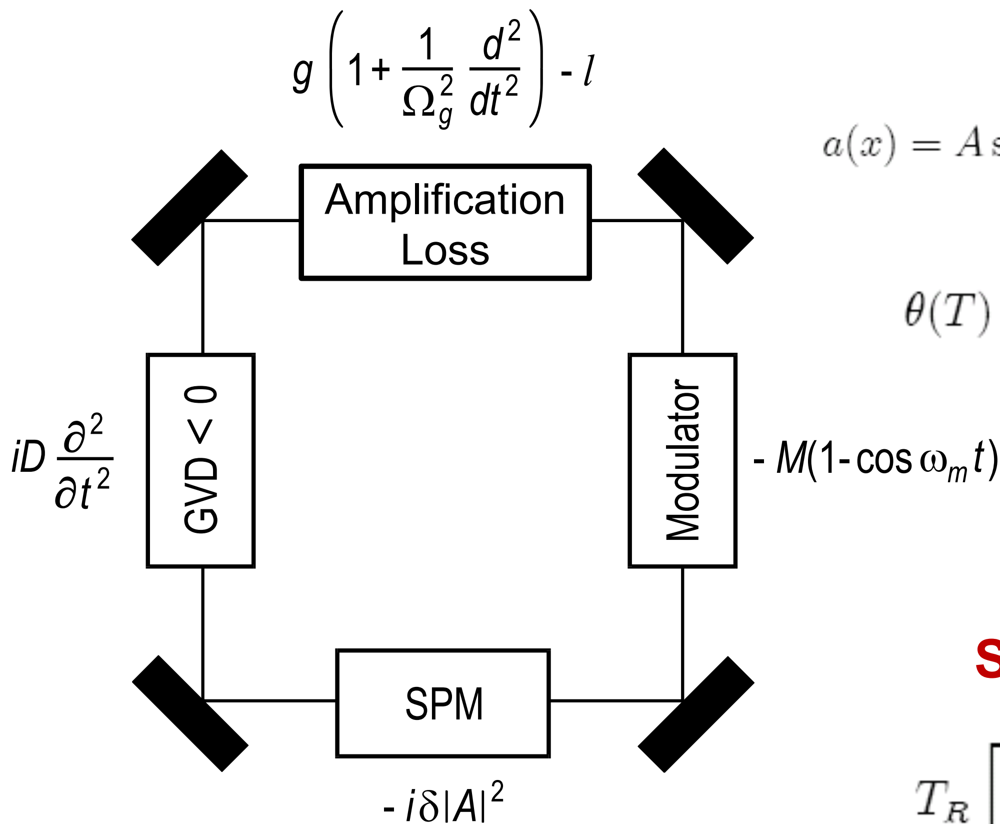


Fig. 5.8: Active Modelocking with Soliton Formation

$$A(T,t) = \left(a(x)e^{jpt} + a_c(T,t)\right)e^{-j\theta}$$

$$a(x) = A\,\mathrm{sech}(x), \quad \text{and} \quad x = \frac{1}{\tau}\left(t + 2D\int_0^T p(T')dT' - t_0\right)$$

$$\theta(T) = \theta_0(T) - \frac{D}{T_R}\int_0^T \left(\frac{1}{\tau(T')^2} - p(T')^2\right)dT'$$

$$\frac{|D|}{\tau(T)^2} = \frac{\delta A(T)^2}{2}.$$

Soliton Perturbation Theory

$$T_R\left[\frac{\partial a_c}{\partial T} + \frac{\partial W}{\partial T}\mathbf{f}_w + \frac{\partial \Delta\theta}{\partial T}\mathbf{f}_\theta + \frac{\partial \Delta p}{\partial T}\mathbf{f}_p + \frac{\partial \Delta t}{\partial T}\mathbf{f}_t\right]$$
$$= \phi_0\mathbf{L}\left(\mathbf{a}_c + \Delta p\mathbf{f}_p\right) + \mathbf{R}(\mathbf{a} + \Delta p\mathbf{f}_p + \mathbf{a}_c)$$
$$-M\omega_M\sin(\omega_M\tau x)\Delta t\,\mathbf{a}(x)$$

$$\mathbf{R} = g\left(1 + \frac{1}{\Omega_g^2\tau^2}\frac{\partial^2}{\partial x^2}\right) - l - M\left(1 - \cos(\omega_M\tau x)\right),$$

5.5.1 Steady State and Stability Conditions

Energy:

$$T_R \frac{\partial W}{\partial T} = 2\left(g - l - \frac{g}{3\Omega_g^2\tau^2} - \frac{\pi^2}{24} M\omega_M^2\tau^2\right) W + < \mathbf{f}_w^{(+)} | \mathbf{R}\mathbf{a}_c > .$$

Steady State:

$$g - l = \frac{\pi^2}{24} M\omega_M^2\tau^2 + \frac{g}{3\Omega_g^2\tau^2} \longrightarrow W_0 = 2A_0^2\tau$$

$$g = \frac{g_0}{1 + W_0/E_L}$$

Linearization:

$$\begin{aligned} T_R \frac{\partial \Delta W}{\partial T} &= 2\left(-\frac{g}{(1 + W_0/E_L)}\left(\frac{W_0}{E_L} + \frac{1}{3\Omega_g^2\tau^2}\right)\right. \\ &+ \left.\frac{\pi^2}{12} M\omega_M^2\tau^2\right)\Delta W + < \mathbf{f}_w^{(+)} | \mathbf{R}\mathbf{a}_c > \\ T_R \frac{\partial \Delta\theta}{\partial T} &= < \mathbf{f}_\theta^{(+)} | \mathbf{R}\mathbf{a}_c > \\ T_R \frac{\partial \Delta p}{\partial T} &= -\frac{4g}{3\Omega_g^2\tau^2}\Delta p + < \mathbf{f}_p^{(+)} | \mathbf{R}\mathbf{a}_c > \\ T_R \frac{\partial \Delta t}{\partial T} &= -\frac{\pi^2}{6} M\omega_M^2\tau^2\Delta t + 2|D|\Delta p \\ &+ < \mathbf{f}_t^{(+)} | \mathbf{R}\mathbf{a}_c > \end{aligned}$$

$$\begin{aligned} \frac{g(k)}{\partial T} &= j\Phi_0(k^2 + 1)g(k) + < \mathbf{f}_k^{(+)} | \mathbf{R}\mathbf{a}_c > \\ &+ < \mathbf{f}_k^{(+)} | \mathbf{R}\left(\mathbf{a}_0(x) + \Delta w\, \mathbf{f}_w + \Delta p\, \mathbf{f}_p\right) > \\ &- < \mathbf{f}_k^{(+)} | M\omega_M \sin(\omega_M\tau x)\mathbf{a}_0(x) > .\Delta t \end{aligned}$$

5.5.1 Steady State and Stability Conditions

Pulse width of stable solitons will satisfy: $\omega_M \tau \ll 1 \ll \Omega_g \tau$

Soliton sheds energy into continuum, but we can neglect coupling back of the continuum to the soliton for large normalized dispersion

Normalized Dispersion: $D_n = |D|\Omega_g^2/g.$

Generated continuum:

$$T_R \frac{\partial G}{\partial T} = \left[g - l + j\Phi_0 + \frac{g}{\Omega_g^2}(1 - jD_n)\frac{\partial^2}{\partial t^2} - M\left(1 - \cos(\omega_M t)\right) \right] G + \mathcal{F}^{-1}\Big\{ < \mathbf{f}_k^{(+)} | \mathbf{R}\mathbf{a}_0(x) > - < \mathbf{f}_k^{(+)} | M\omega_M \sin(\omega_M \tau x)\mathbf{a}_0(x) > \Delta t \Big\}$$

Drive terms

Continuum

$$T_R \frac{\partial G}{\partial T} = \left[g - l + j\Phi_0 + \frac{g}{\Omega_g^2}(1 - jD_n)\frac{\partial^2}{\partial t^2} \right.$$
$$\left. - M\left(1 - \cos(\omega_M t)\right) \right] G$$

In parabolic approx. of the modulation function, Hermite Gaussians are again the solutions with the complex width:

$$\tau_c = \tau_a \sqrt[4]{(1 - jD_n)}$$

and associated eigenvalues:

$$\lambda_m = j\Phi_0 + g - l - M\omega_M^2 \tau_a^2 \sqrt{(1 - jD_n)}(m + \frac{1}{2})$$

$$\lambda_m = +j\Phi_0 + \frac{1}{3}\sqrt{D_g M_s}\left[\left(\frac{\tau_a}{\tau}\right)^2 + \frac{\pi^2}{4}\left(\frac{\tau_a}{\tau}\right)^{-2} \right.$$
$$\left. -6\sqrt{(1 - jD_n)}(m + \frac{1}{2}) \right].$$

Stability of Continuum

Condition for stability: net loss per roundtrip for all continuum modes:

$$Re\{\lambda_m\} < 0 \text{ for } m \geq 0$$

with: $\xi = (\tau_a/\tau)^2$

$$\xi^2 - 3Re\{\sqrt{(1-jD_n)}\}\xi + \frac{\pi^2}{4} < 0.$$

Minimum amount of negative dispersion:

$$D_{n,crit} = 0.652.$$

For even larger amounts of dispersion,we define the pulse shortening factor in terms of FWHM pulse width

$$R = \frac{1.66}{1.76}\sqrt{\xi}.$$

$$R_{crit} \approx 1.2$$

Pulse width reduction

$$\xi < \sqrt{\frac{9D_n}{2}} \quad \text{or} \quad R < \frac{1.66}{1.76}\sqrt[4]{\frac{9D_n}{2}}.$$

$$\tau^2 = \frac{|D|}{\Phi_0}.$$

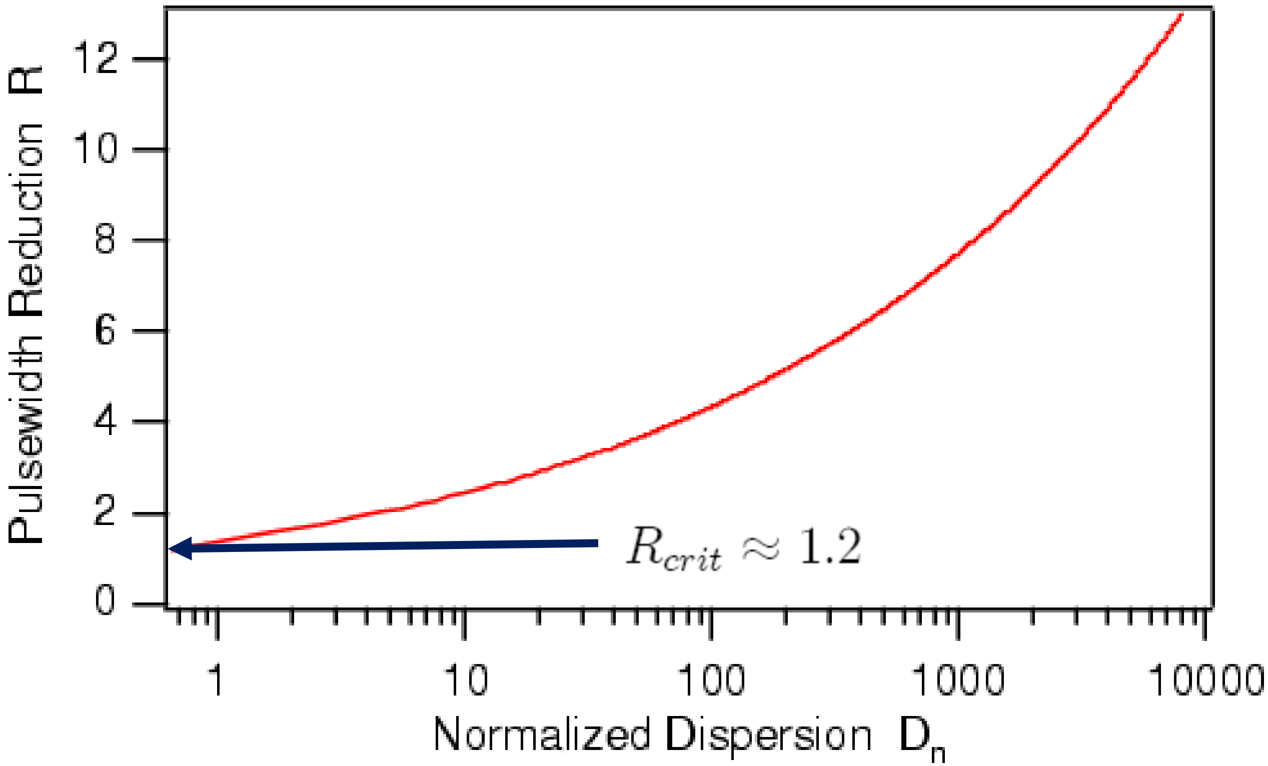


Fig. 5.9: Pulse width reduction

Nonlinear phase shift per roundtrip is limited.

$$R_{max} = \frac{1.66}{1.76}\sqrt[12]{\frac{(9\Phi_0/2)^2}{D_g\, M_s}}$$

$$\tau_{min} = \sqrt[6]{\frac{2D_g^2}{9\Phi_0\, M_s}}.$$

$$D_n = \frac{2}{9}\sqrt[3]{\frac{(9\Phi_0/2)^2}{D_g\, M_s}}.$$

gain material	$\frac{\Omega_g}{2\pi\,(THz)}$	M	$\frac{\omega_M}{2\pi\,(MHz)}$	$\frac{D_g}{(ps^2)}$	$M_s \cdot (ps^2)$	$\frac{\tau_{a,FWHM}}{(ps)}$
Nd:YAG	0.06	0.2	250	0.7	$2.5 \cdot 10^{-7}$	68
Nd:glass	4	0.2	250	$158 \cdot 10^{-6}$	$2.5 \cdot 10^{-7}$	8.35
Cr:LiSAF	32	0.2	250	$2.4 \cdot 10^{-6}$	$2.5 \cdot 10^{-7}$	3
Ti:sapphire	43	0.01	100	$1.4 \cdot 10^{-6}$	$2 \cdot 10^{-9}$	8.5

gain material	R_{max}	$\frac{\tau_{min,FWHM}}{(ps)}$	D_n	$\frac{\tau_{trans}}{T_R}$	
Nd:YAG	3	22.7	23.4	702	
Nd:glass	6	1.4	385	11,538	
Cr:LiSAF	8.6	0.35	1563	46,600	
Ti:sapphire	13.5	0.63	9367	281,000	

Table 5.1: Maximum pulsewidth reduction and necessary normalized GVD for different laser systems. In all cases we used for the saturated gain $g = 0.1$ and the soliton phase shift per roundtrip $\Phi_0 = 0.1$. For the broadband gain materials the last column indicates rather long transient times which calls for regenerative mode locking.

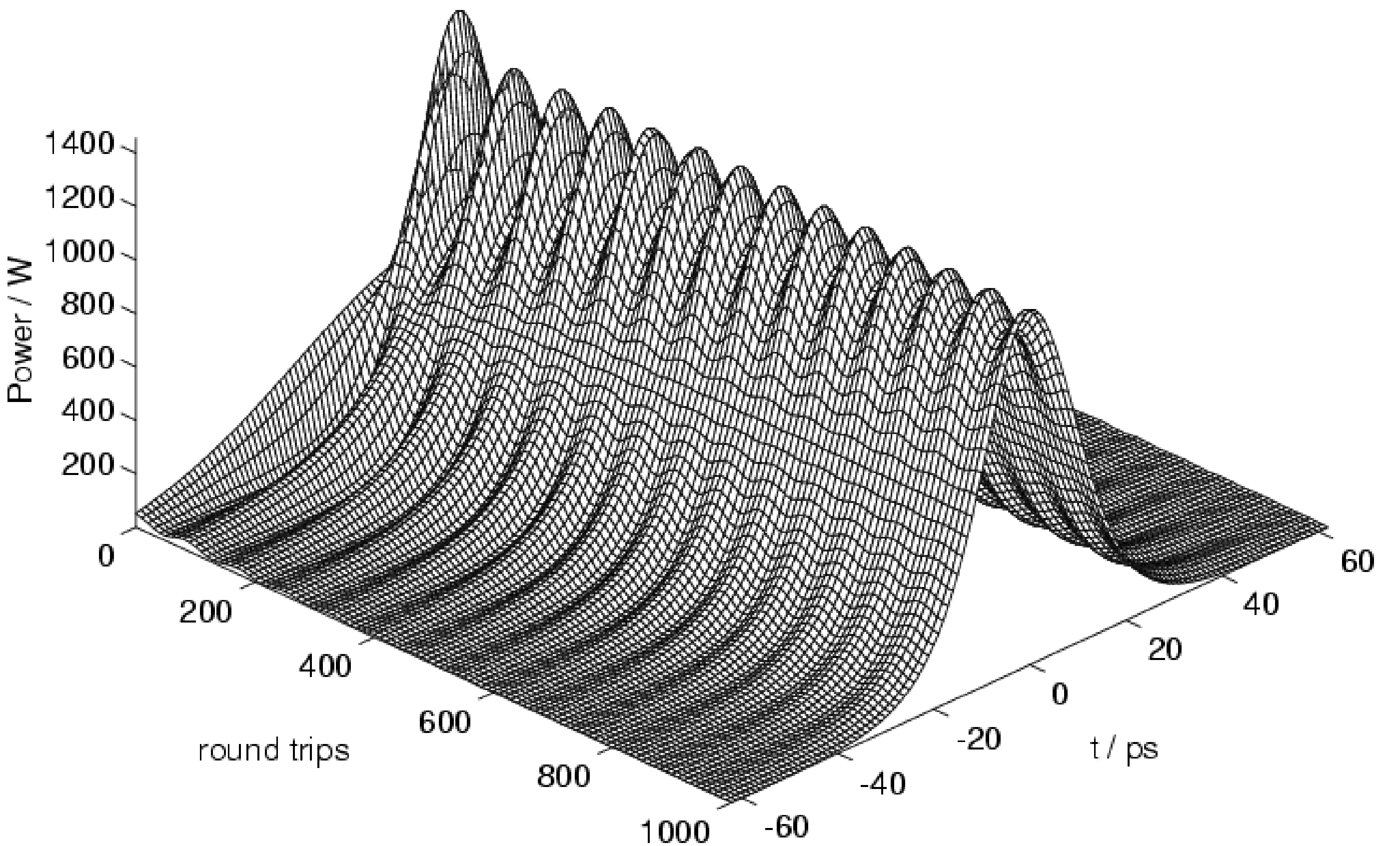


Fig. 5.10: Time evolution in a Nd:YAG laser D = -17ps^2 with initial pulsewidth 68 ps.

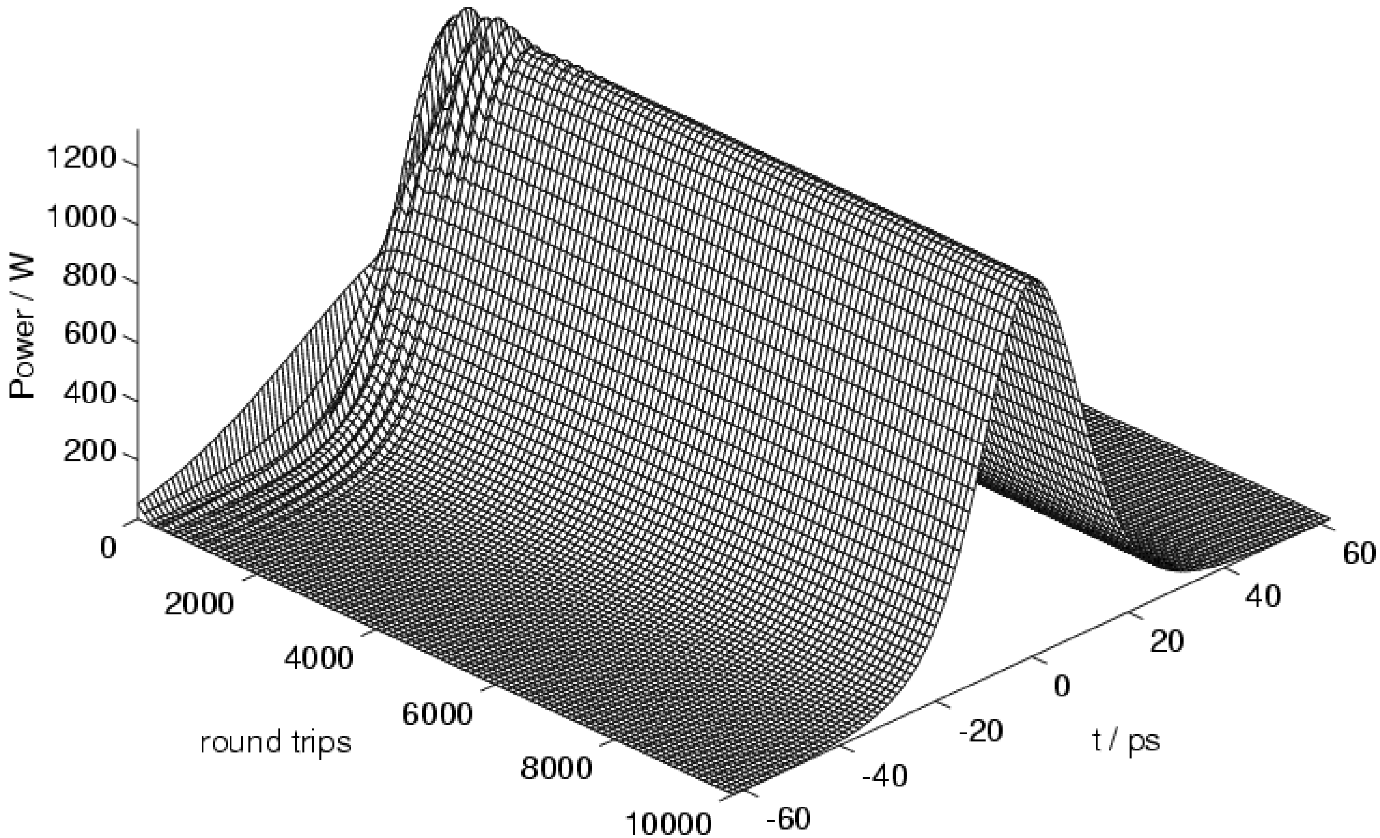


Fig. 5.11: 10000 roundtrips

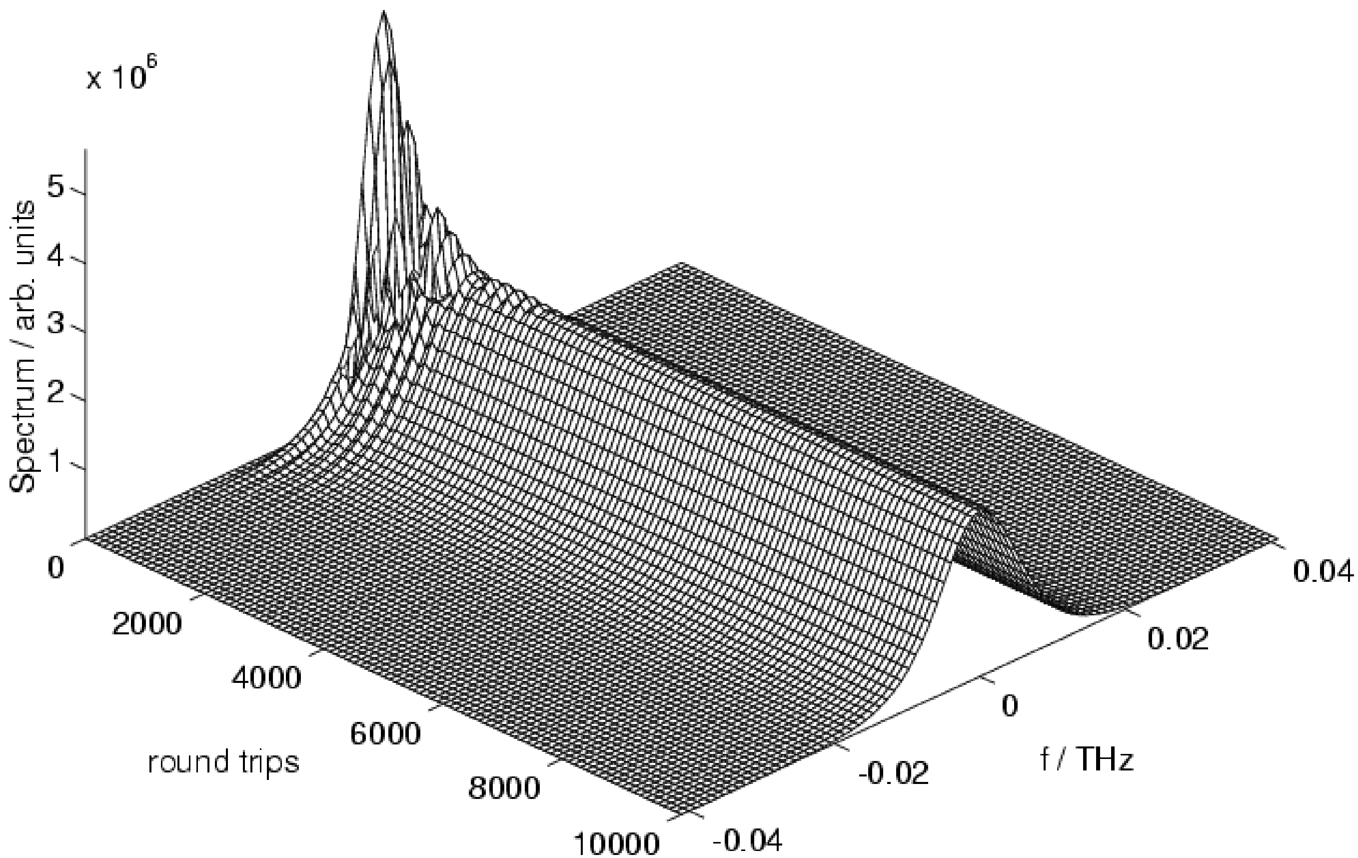


Fig. 5.11: 10000 roundtrips

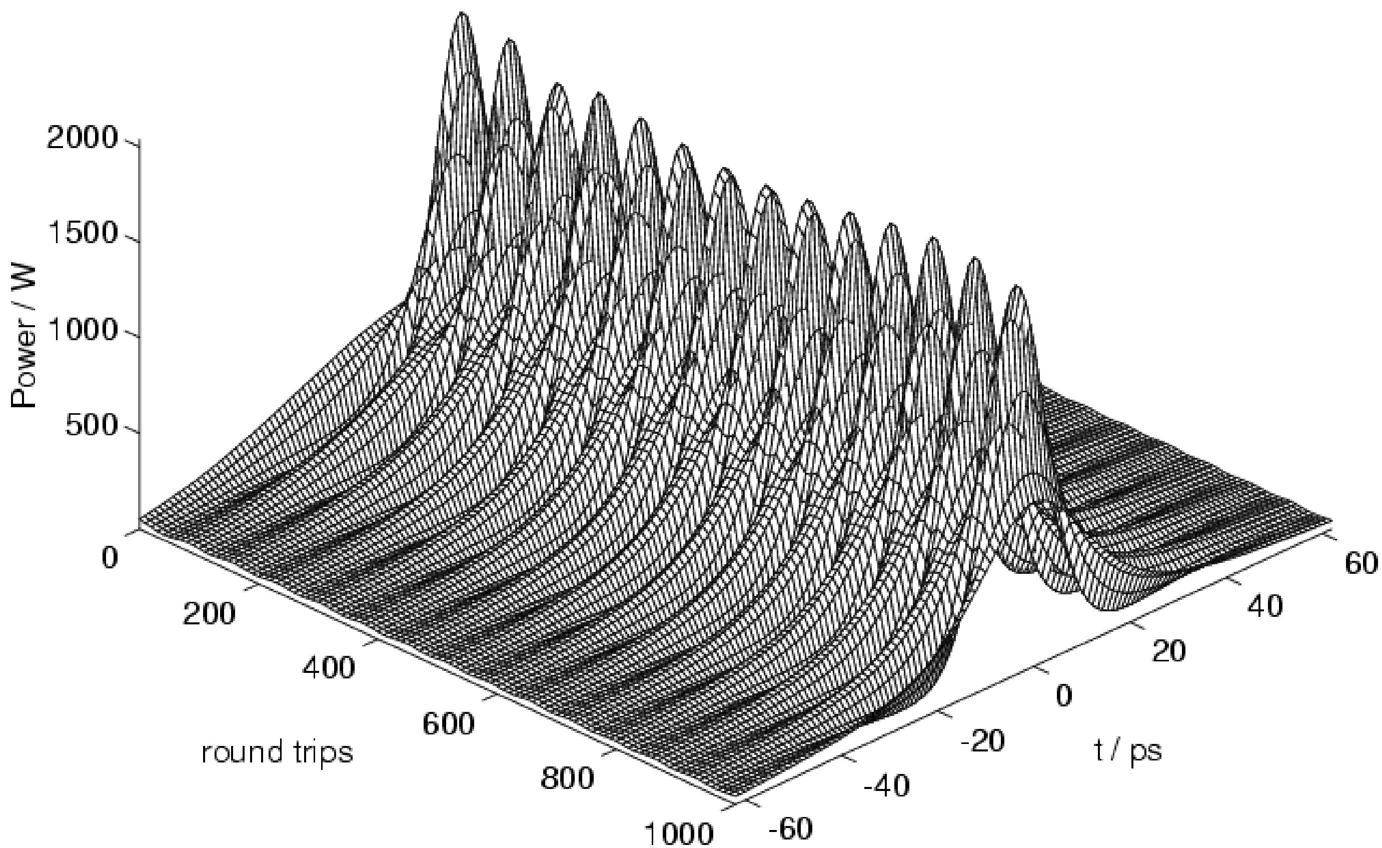


Fig. 5.12a: D = -10ps^2

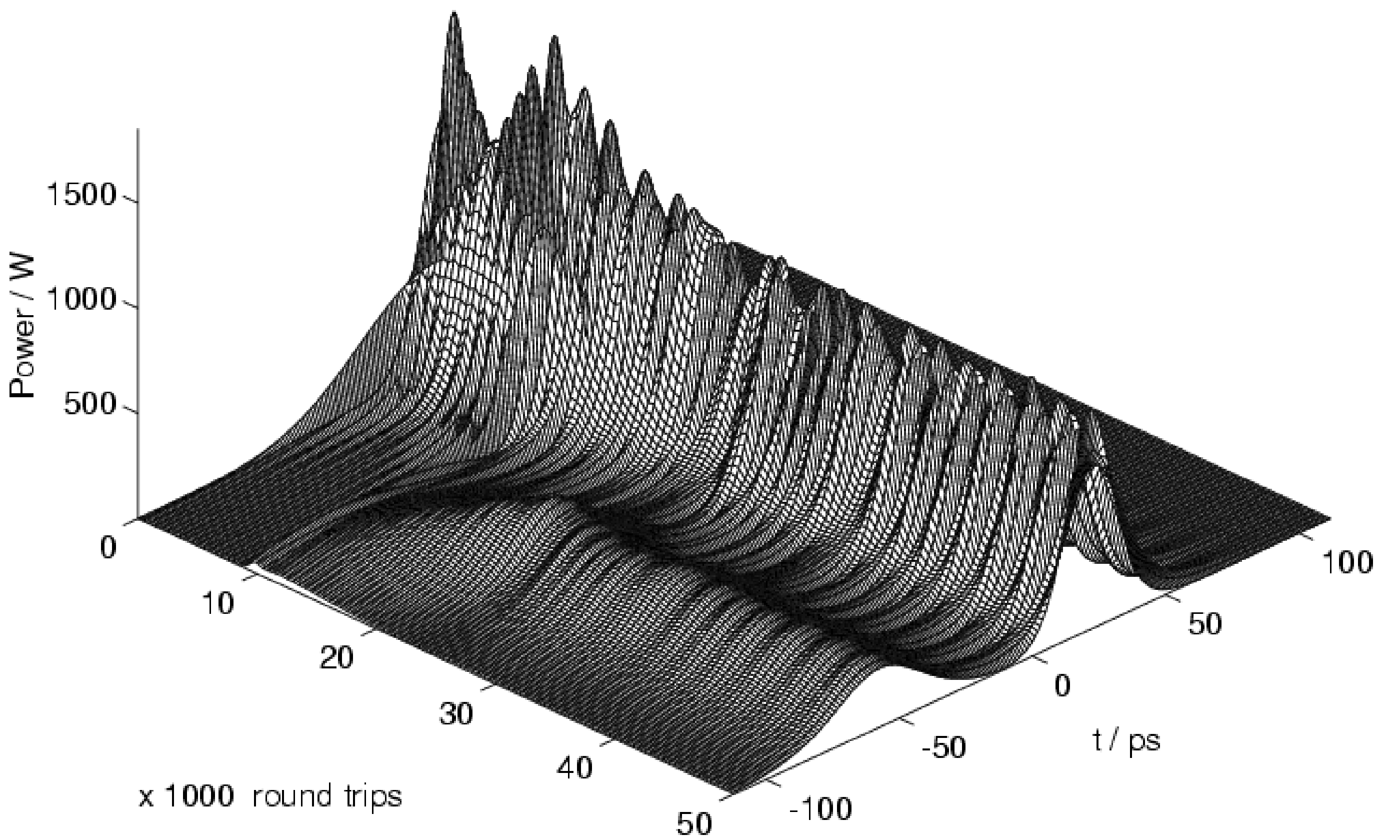


Fig. 5.12 b: 50000 roundtrips

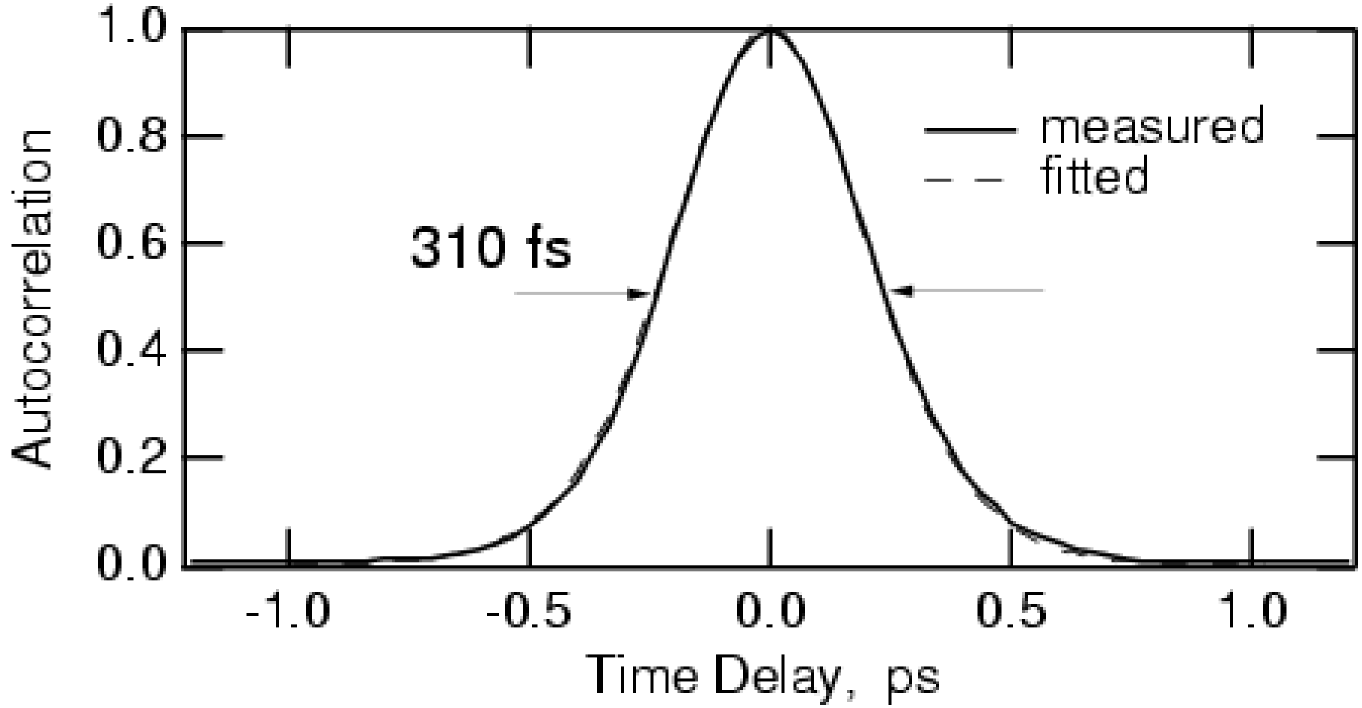


Fig. 5.13: Autocorrelation of the actively mode-locked laser

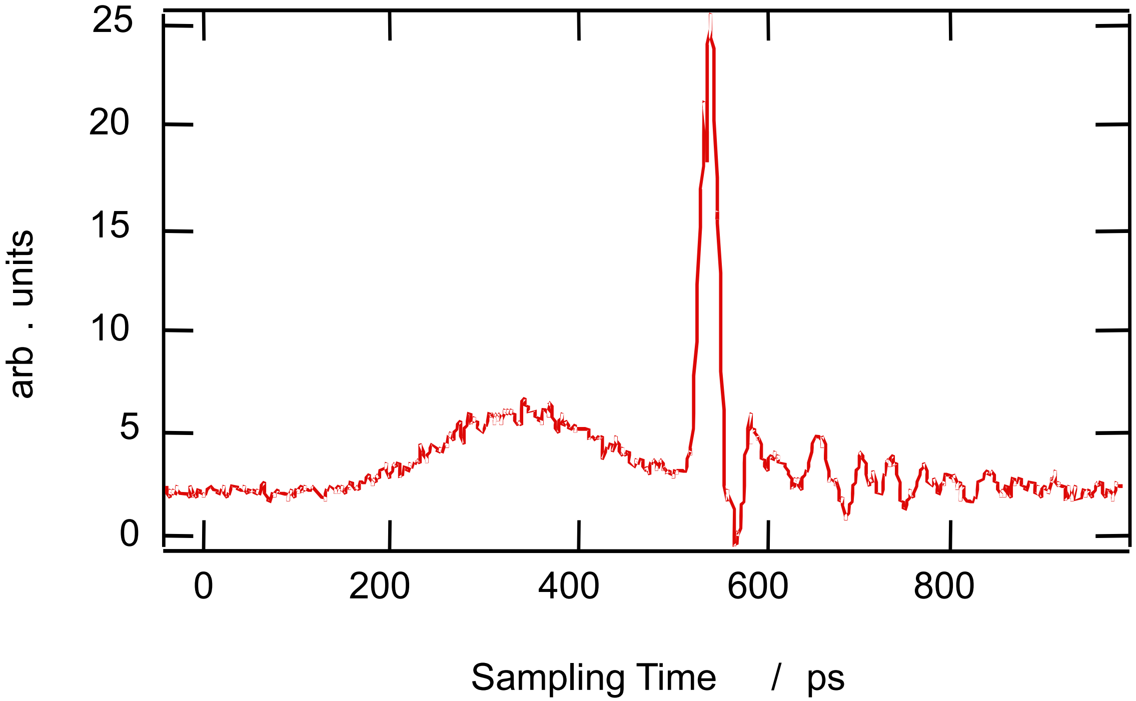


Fig. 5.14: Sampling oscilloscope trace

5.6 Active Modelocking with Detuning

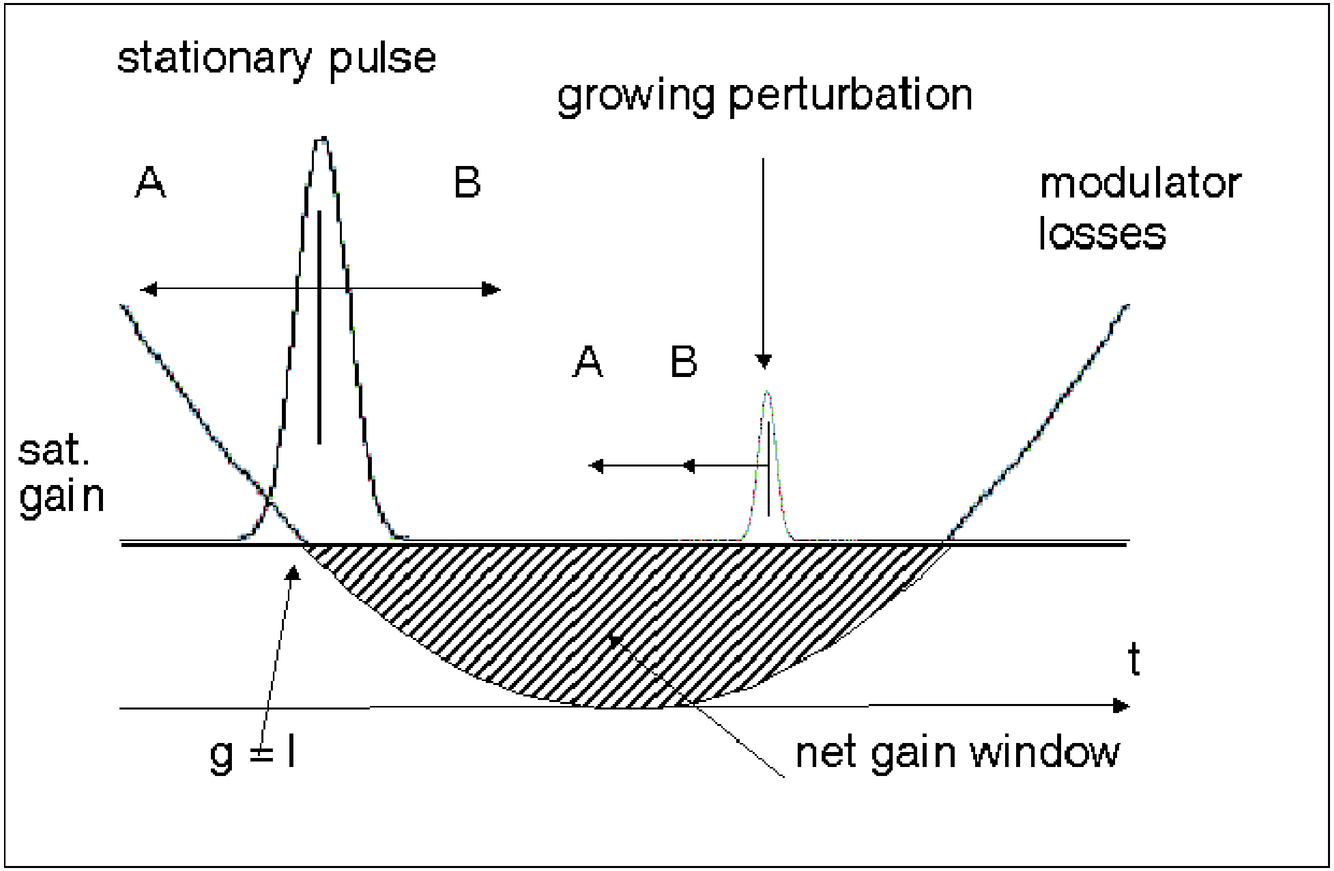


Fig. 5.15: Drifting pulse dynamics in a detuned laser

$$T_M \frac{\partial A(T,t)}{\partial T} = \Big[g(T) - l + D_f \frac{\partial^2}{\partial t^2} \\ - M\left(1 - \cos(\omega_M t)\right) + T_d \frac{\partial}{\partial t} \Big] A(T,t)$$

Normalized detuning: $\Delta = \frac{1}{2\sqrt{2 D_f M_s}} \frac{T_d}{\tau_a}.$

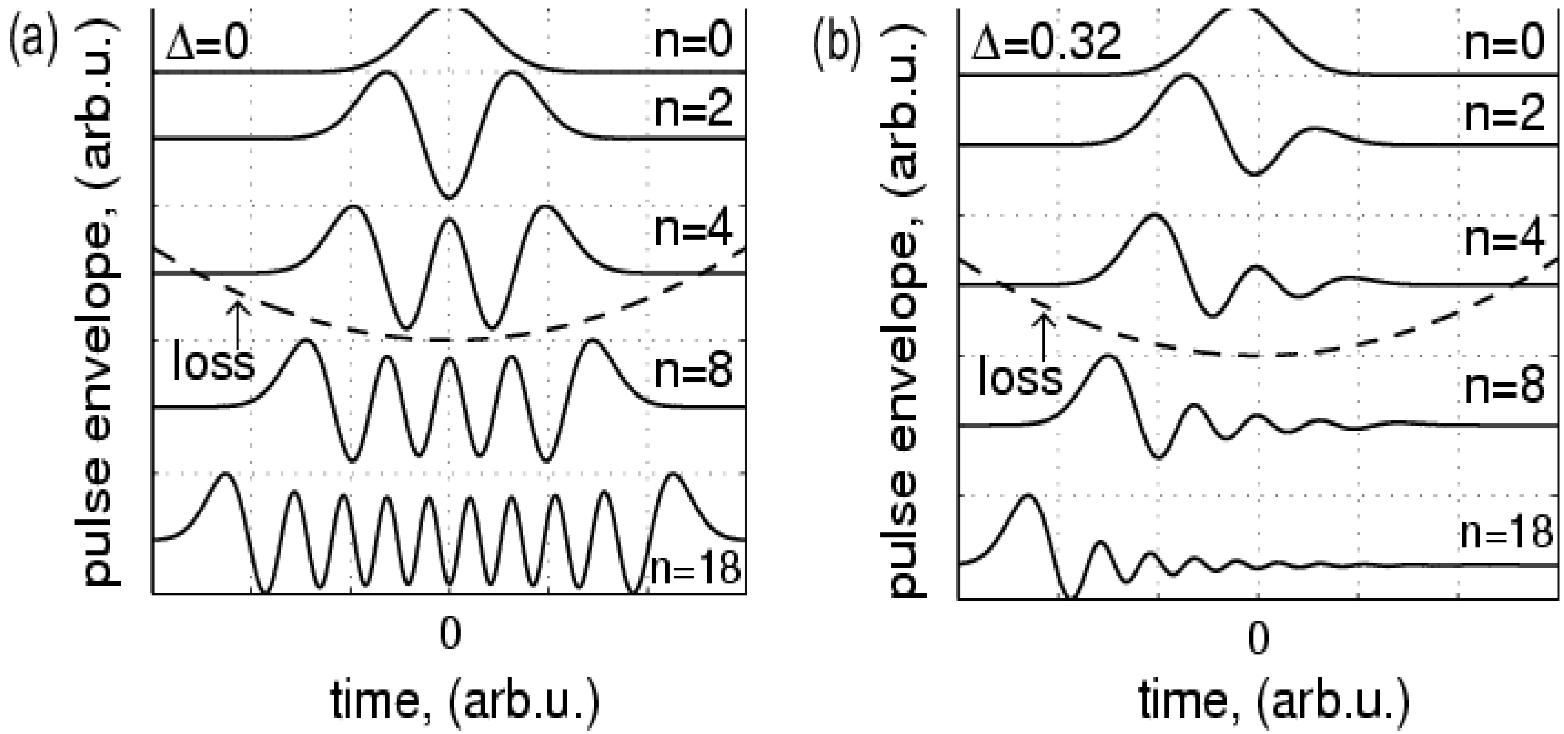


Fig. 5.16: Lower order eigenmodes

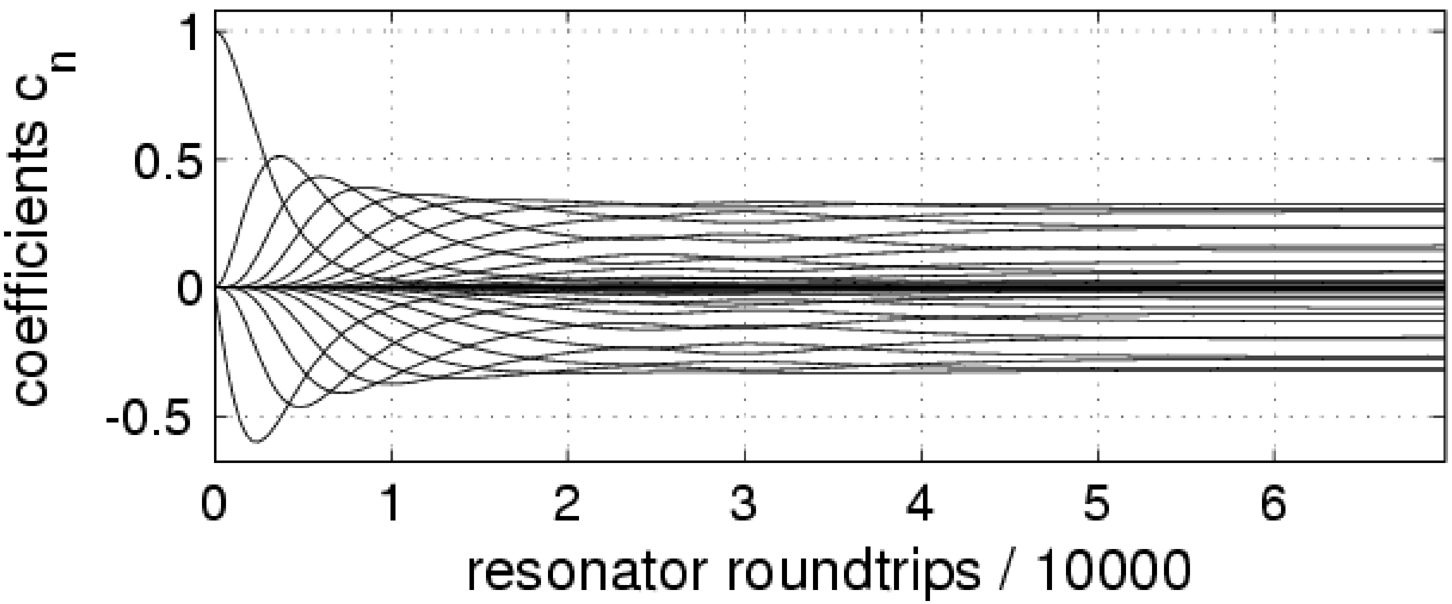


Fig. 5.17: Coefficients of envelope in a Hermite-Gaussian Basis centered at t=0 for $\Delta = 3.5$

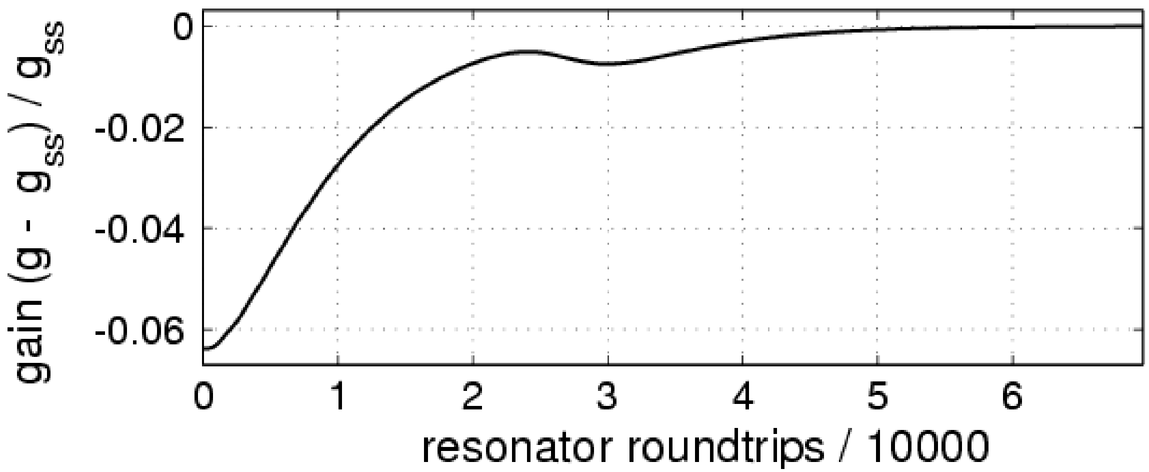


Fig. 5.18: Normalized deviation from steady state gain for $\Delta = 3.5$

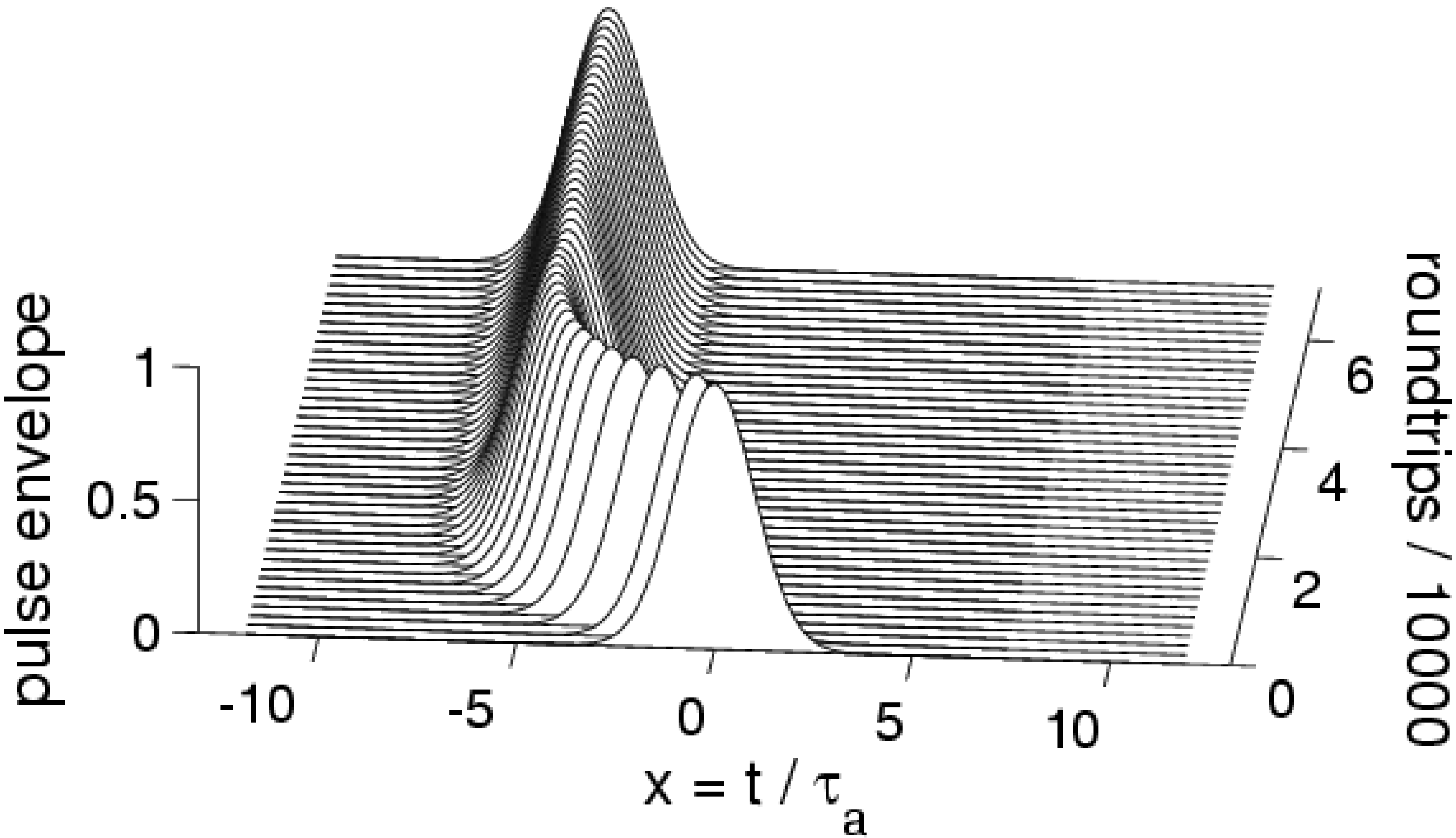


Fig. 5.19: Temporal evolution to steady state position at to $2^{1/2}\ \Delta = 4.9$

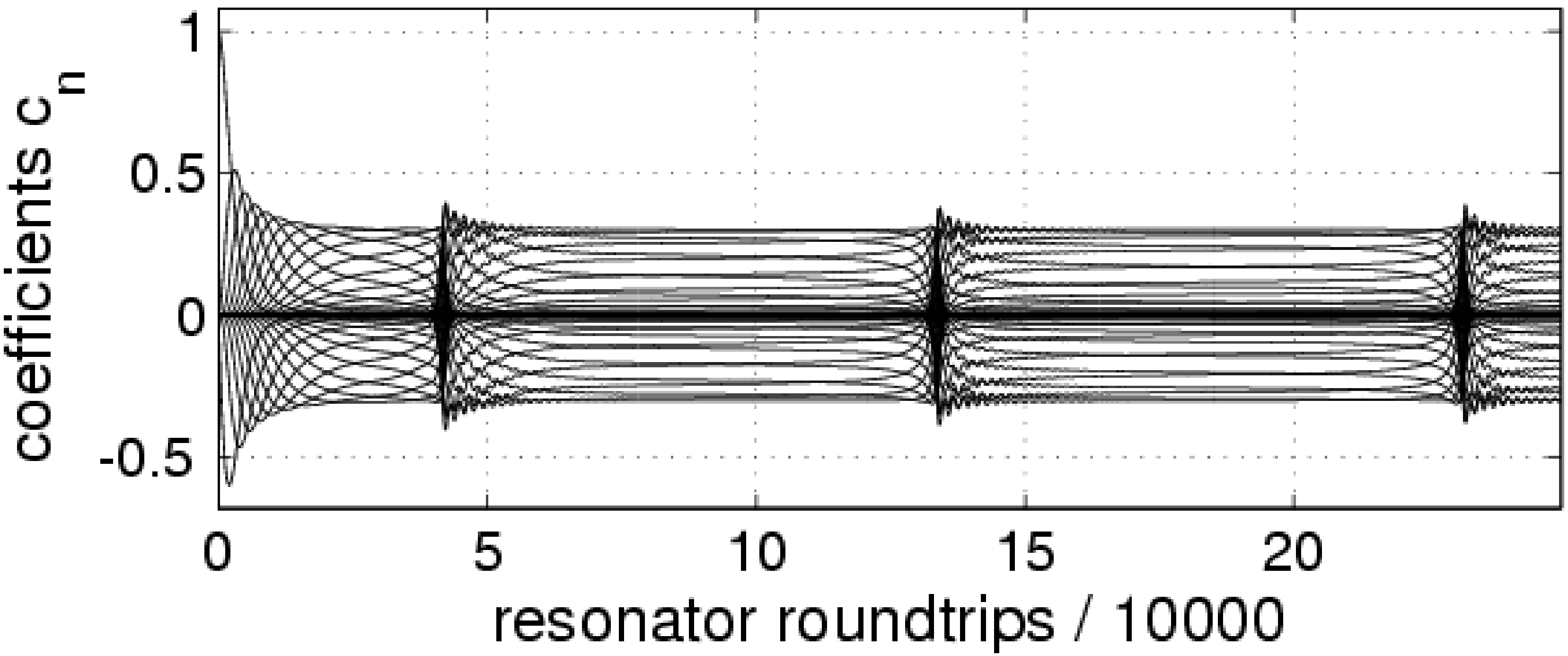


Fig. 5.20: Coefficients of envelope in a Hermite-Gaussian Basis centered at t=0 for $\Delta = 4$

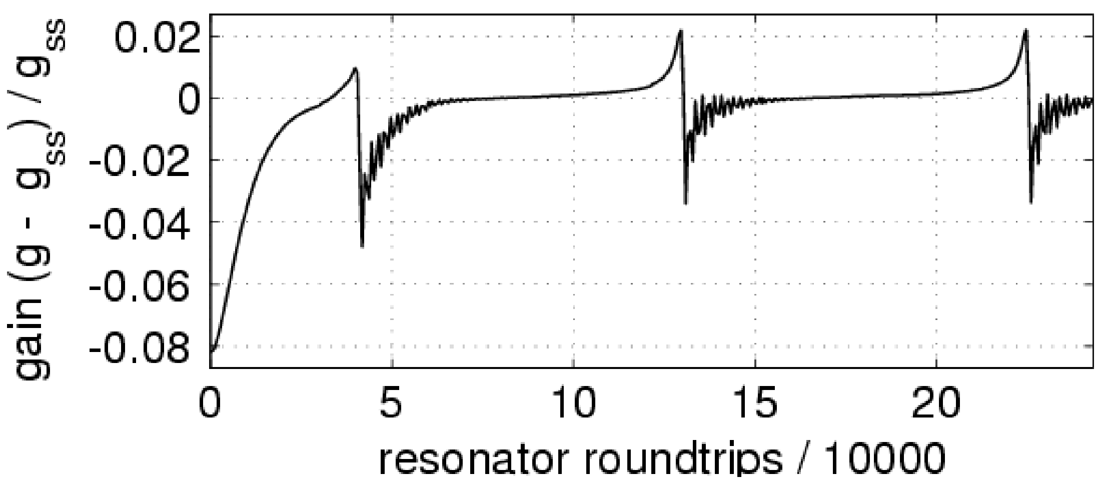


Fig. 5.21: Temporal evolution of gain deviation from quasi steady state

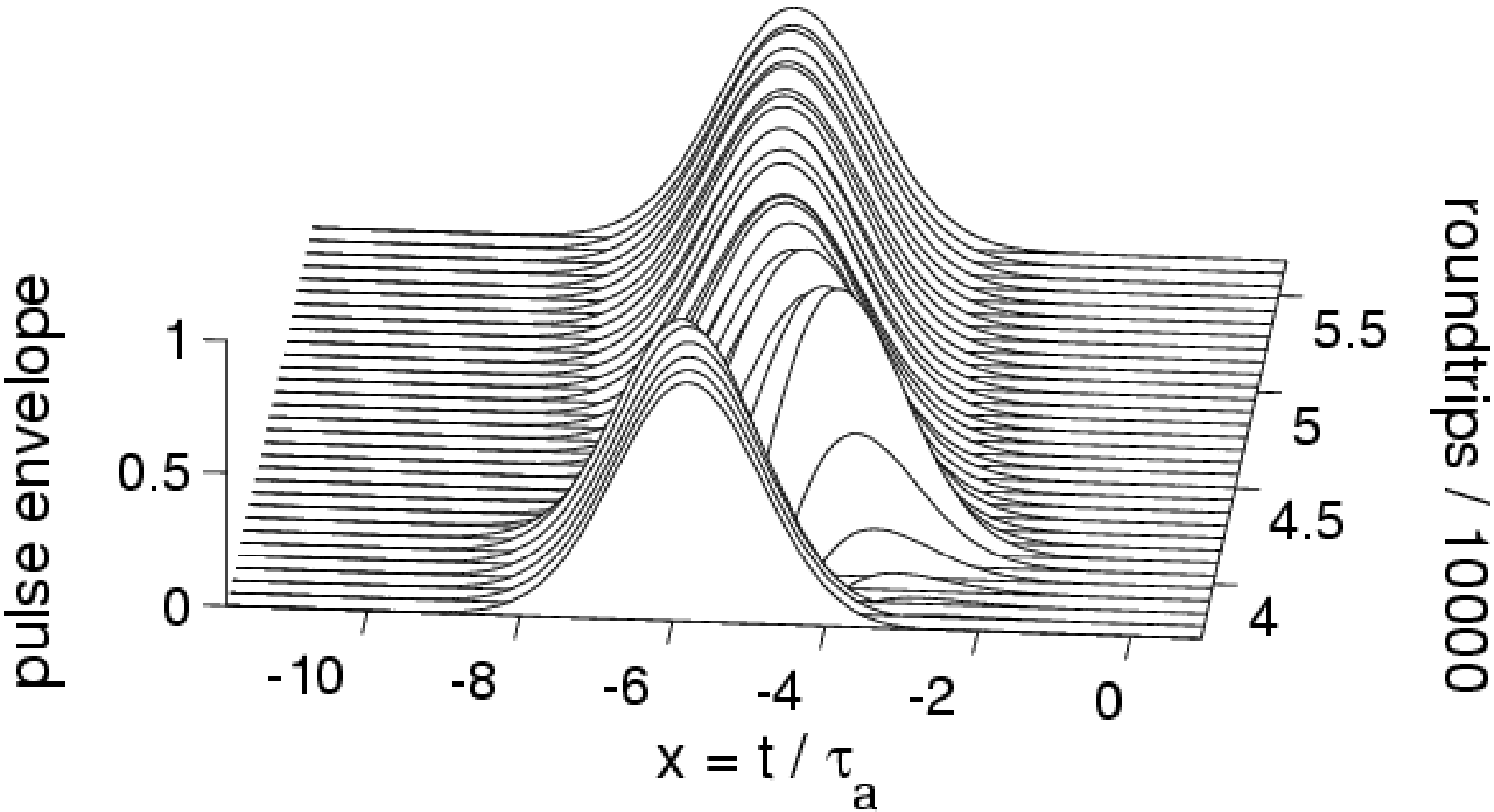


Fig. 5.22: Time evolution of pulse envelope for Δ = 4

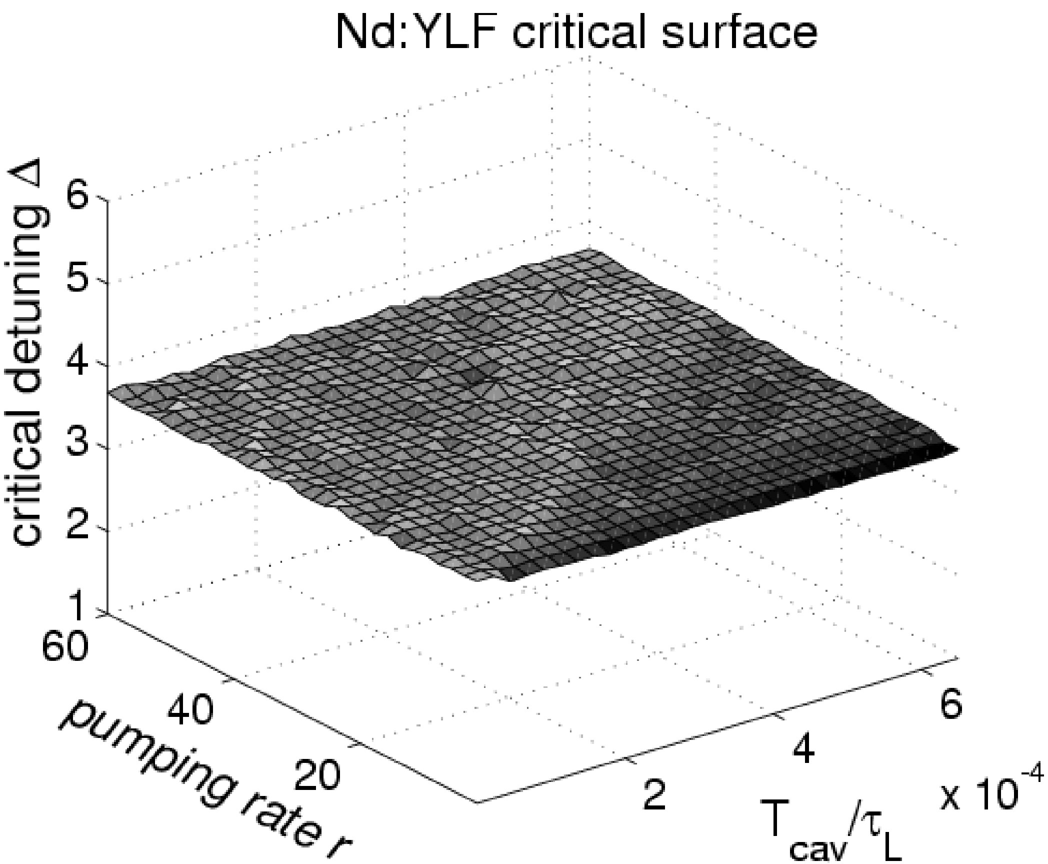


Fig. 5.25: Critical detuning ~ 3.65

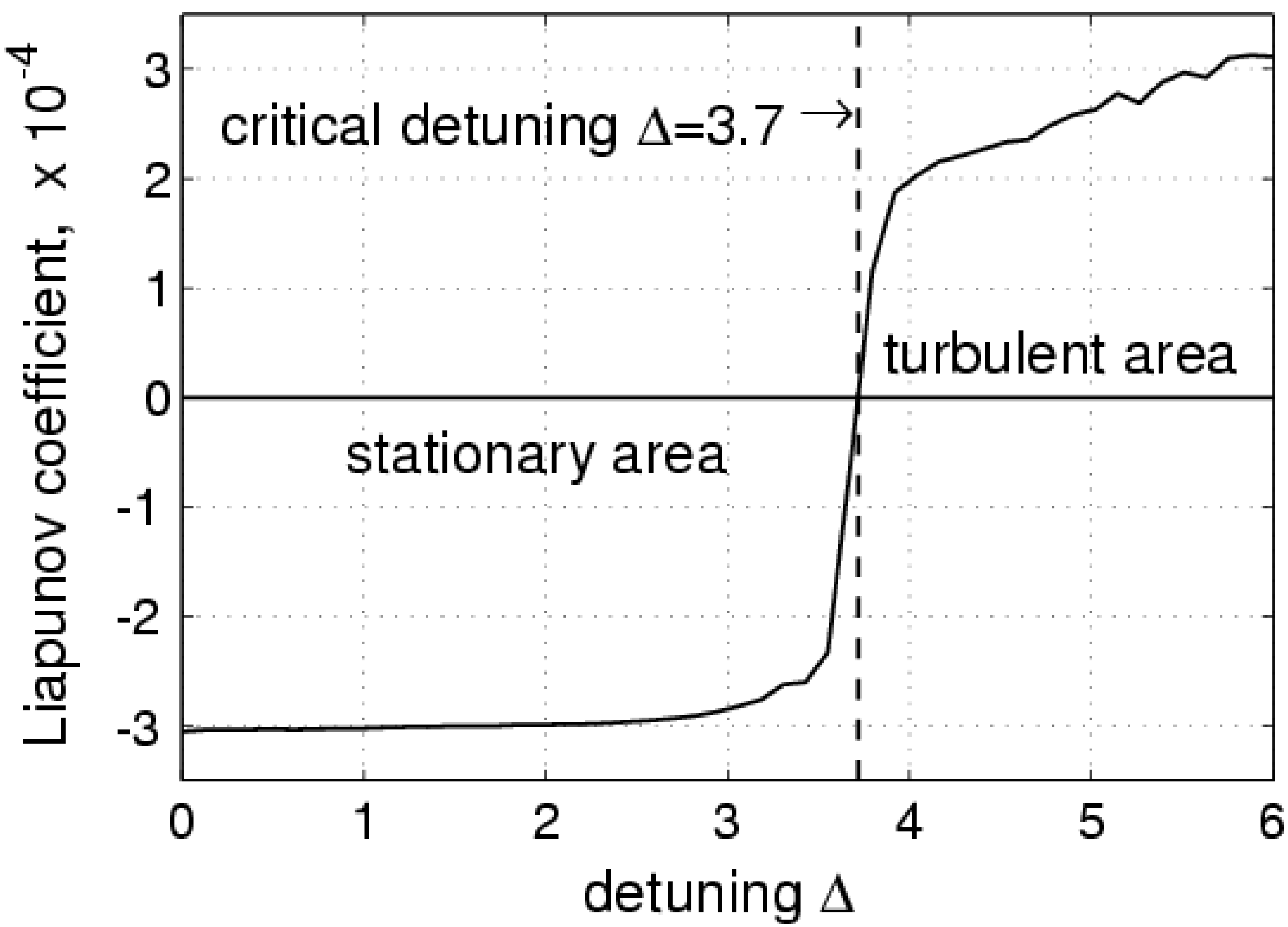


Fig. 5.26: Liapunov coefficient over normalized detuning