Ultrafast Optical Physics II (SoSe 2019)
Lecture 5, May 10

Part I

(1) Kerr nonlinearity and self-phase modulation

(2) Nonlinear Schrödinger equation and soliton solution

(3) Soliton perturbation theory

Interaction between EM waves and materials

Light wave perturbs material $\longrightarrow$ $P = \varepsilon_0 \chi E$

Perturbed material alters the light wave $\longrightarrow$ $(\nabla^2 - \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2})E = \mu_0 \frac{\partial^2 P}{\partial t^2}$

Examples of changes to light wave: Frequency, amplitude, phase, polarization state, direction of propagation, transverse profile

Output of a linear optical system with multiple inputs is simply the field summation of the outputs for each individual input.

$E_{in}(\omega_1)$

$E_{in}(\omega_2)$

$E_{in}(\omega_1) + E_{in}(\omega_2)$

$\Rightarrow$ **Linear Optical System** $\Rightarrow$

$E_{out}(\omega_1)$

$E_{out}(\omega_2)$

$E_{out}(\omega_1) + E_{out}(\omega_2)$

Intensity dependent nonlinear refractive index

In general, a medium responses nonlinearly to an optical field. Here we are interested in intensity dependent nonlinear refractive index arising from the third-order nonlinearity:

$$P = \varepsilon_0 \left[\chi^{(1)} E + \chi^{(3)} |E|^2 E\right] = \varepsilon_0 \left[\chi^{(1)} + \chi^{(3)} |E|^2\right] E$$

Substituting the polarization into the wave equation

$$\frac{\partial^2 E}{\partial z^2} - \frac{1}{c_0^2} \frac{\partial^2 E}{\partial t^2} = \mu_0 \varepsilon_0 \left[\chi^{(1)} + \chi^{(3)} |E|^2\right] \frac{\partial^2 E}{\partial t^2}$$

$$\frac{\partial^2 E}{\partial z^2} - \frac{\left[1 + \chi^{(1)} + \chi^{(3)} |E|^2\right]}{c_0^2} \frac{\partial^2 E}{\partial t^2} = 0$$ since $\mu_0 \varepsilon_0 = 1/c_0^2$

So the refractive index is: $$n = \sqrt{1 + \chi^{(1)} + \chi^{(3)} |E|^2}$$

Intensity dependent nonlinear refractive index

The refractive index in the presence of linear and nonlinear polarizations:

$$n = \sqrt{1 + \chi^{(1)} + \chi^{(3)} |E|^2}$$

Now, the usual refractive index (which we'll call n_0) is: $n_0 = \sqrt{1 + \chi^{(1)}}$

So:

$$n = \sqrt{n_0^2 + \chi^{(3)} |E|^2} = n_0 \sqrt{1 + \chi^{(3)} |E|^2 / n_0^2}$$

Assume that the nonlinear term << n_0:

$$n \approx n_0 \left[1 + \chi^{(3)} |E|^2 / 2n_0^2 \right]$$

So:

$$n \approx n_0 + \chi^{(3)} |E|^2 / 2n_0$$

since: $I \propto |E|^2$

Usually we define a "nonlinear refractive index", $n_{2,L}$:

$$\boxed{n = n_0 + n_{2,L} I}$$

<u>Kerr effect: refractive index linearly dependent on light intensity.</u>

Who is Kerr?

John Kerr (1824-1907) was a Scottish physicist. He was a student in Glasgow from 1841 to 1846, and at the Theological College of the Free Church of Scotland, in Edinburgh, in 1849. Starting in 1857 he was mathematical lecturer at the Free Church Training College in Glasgow.

He is best known for the discovery in 1875 of what is now called Kerr effect—the first nonlinear optical effect to be observed. In the Kerr effect, a change in refractive index is proportional to the square of the electric field. The Kerr effect is exploited in the *Kerr cell*, which is used in applications such as shutters in high-speed photography, with shutter-speeds as fast as 100 ns.

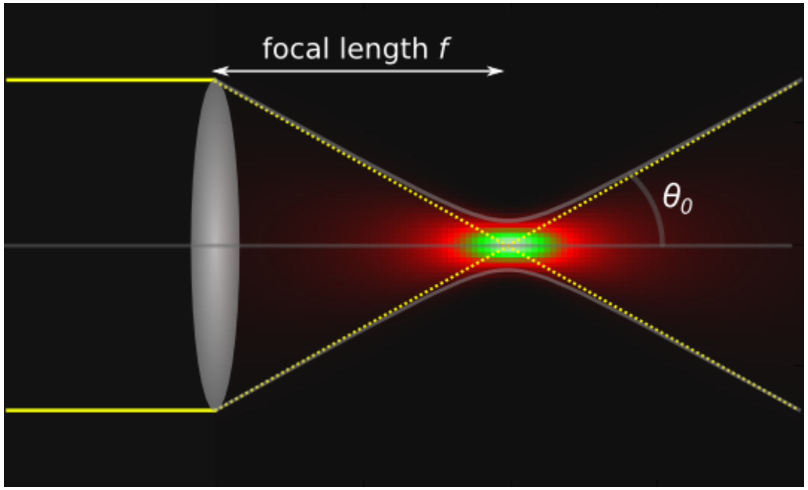
John Kerr, c. 1860,
photograph by Thomas Annan

Magnitude of nonlinear refractive index

Material	Refractive index n	$n_{2,L}[cm^2/W]$
Sapphire (Al_2O_3)	1.76 @ 850 nm	$3{\cdot}10^{-16}$
Fused Quarz	1.45 @ 1064 nm	$2.46{\cdot}10^{-16}$
Glass (LG-760)	1.5 @ 1064 nm	$2.9{\cdot}10^{-16}$
YAG ($Y_3Al_5O_{12}$)	1.82 @ 1064 nm	$6.2{\cdot}10^{-16}$
YLF ($LiYF_4$), n_e	1.47 @ 1047 nm	$1.72{\cdot}10^{-16}$
Si	3.3 @ 1550 nm	$4{\cdot}10^{-14}$

1) A variety of effects give rise to a nonlinear refractive index.
2) Those that yield a large n_2 typically have a slow response.
3) Nonlinear coefficient can be negative.

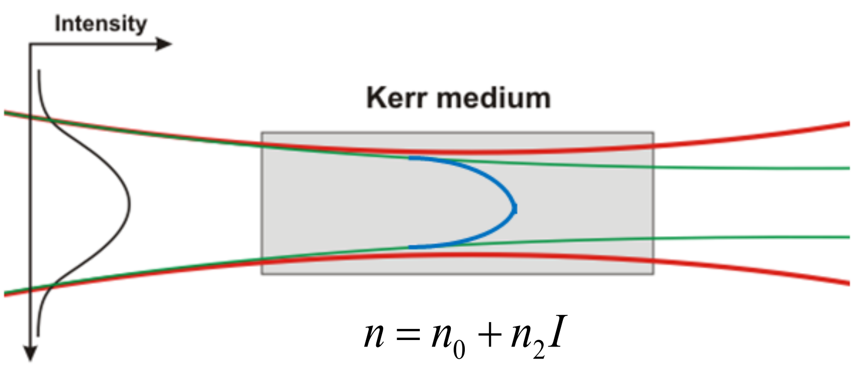
Kerr effect for an optical beam: self focusing



Due to diffraction, a lens can only focus a beam to a finite size

$$r = \frac{0.61}{\sin\theta_0}\lambda$$

For $n_2 > 0$, Kerr effect in a medium acts as a positive lens for a Gaussian beam—the beam's center experiences larger refractive index than the edge. If the peak power exceeds a critical power, self focusing overtakes diffraction and the beam converges rapidly leading to material damage.



Focusing an optical beam at three different powers
red: low power green: near critical power
blue: above critical power

$$P_{cr} = \frac{\pi(0.61)^2\lambda_0^2}{8n_0n_2}$$

Example: self focusing critical power in fused silica

$$\lambda_0 = 1.03\,\mu m \quad n_0 = 1.45 \quad n_2 = 2.7\times10^{-20}\,m^2/W$$

$$P_{cr} = 4MW$$

Kerr effect for an optical pulse: self-phase modulation

In a purely one dimensional propagation problem, the intensity dependent refractive index imposes an additional self-phase shift on the pulse envelope during propagation, which is proportional to the instantaneous intensity of the pulse:

$$\frac{\partial A(z,t)}{\partial z} = -jk_0 n_{2,L}|A(z,t)|^2 A(z,t) = -j\delta|A(z,t)|^2 A(z,t)$$

$$\delta = k_0 n_{2,L}$$

Note that here the pulse profile has been re-normalized so that its square gives intensity:

$$\boxed{\begin{aligned} A(z,t) &= A(0,t)e^{j\varphi_{NL}} \\ &= A(0,t)e^{-j\delta|A(z,t)|^2 z} \end{aligned}}$$

Pulse shape does not change, but the pulse acquires nonlinear phase:

$$|A(z,t)| = |A(0,t)|$$

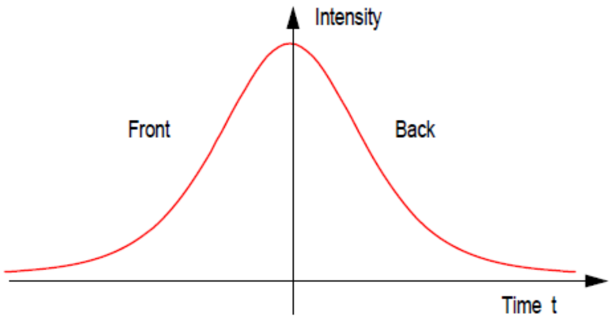
$$\varphi_{NL} = -\delta|A(z,t)|^2 z$$

<u>Self-phase modulation (SPM)</u>:

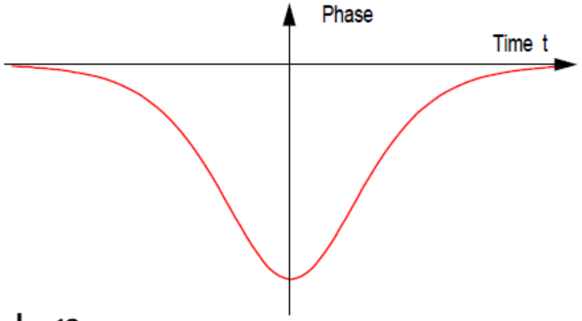
Nonlinear phase modulation of a pulse, caused by its own intensity via the Kerr effect.

SPM induces positive chirp

$$|A(z,t)|^2$$



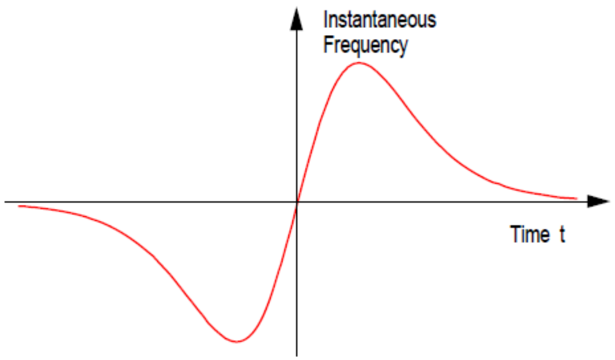
$$\varphi_{NL} = -\delta |A(z,t)|^2 z$$



$$n = n_0 + n_2 I(t)$$

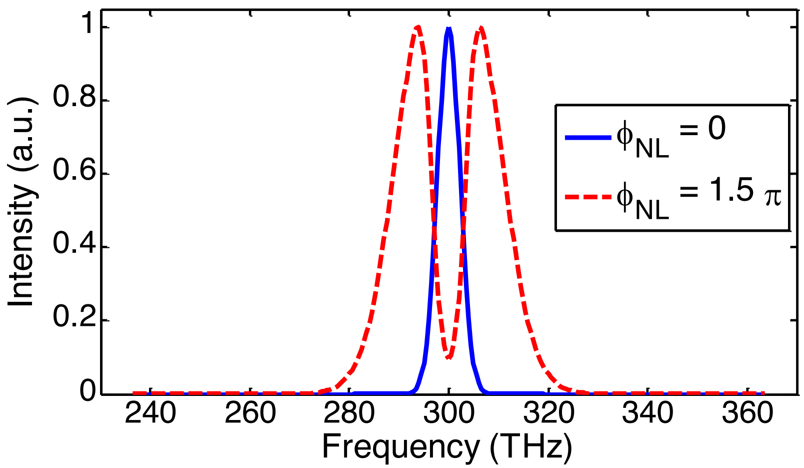
$$\varphi = \omega_0 t - k_0 z(n_0 + n_2 I(t)) = \varphi_0 + \varphi_{NL}$$

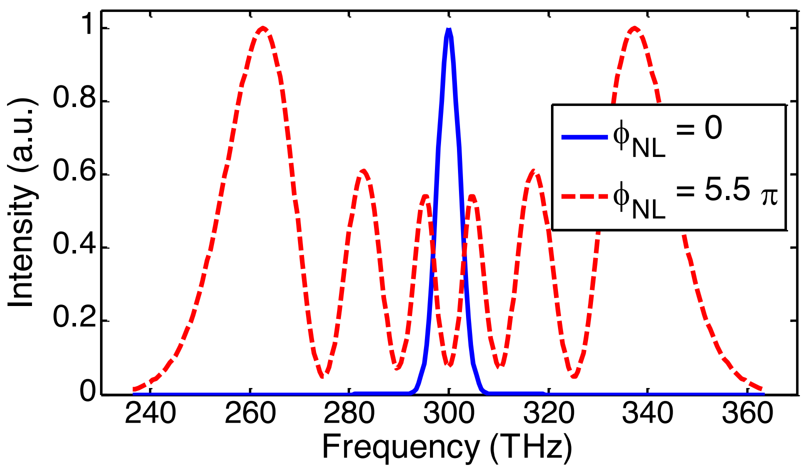
$$\varphi_{NL} = -k_0 z n_2 I(t) = -\delta z I(t)$$

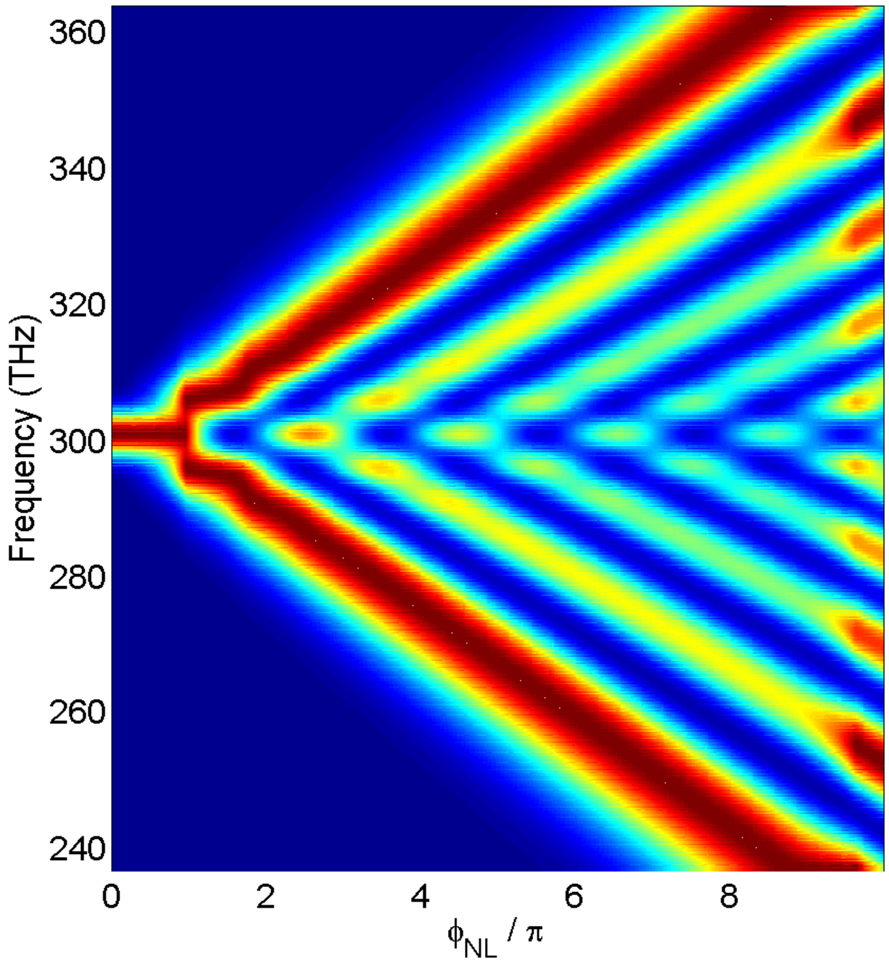


$$\Delta\omega = \frac{d\varphi_{NL}}{dt} = -\delta z \frac{d|A(z,t)|^2}{dt}$$

SPM modifies spectrum



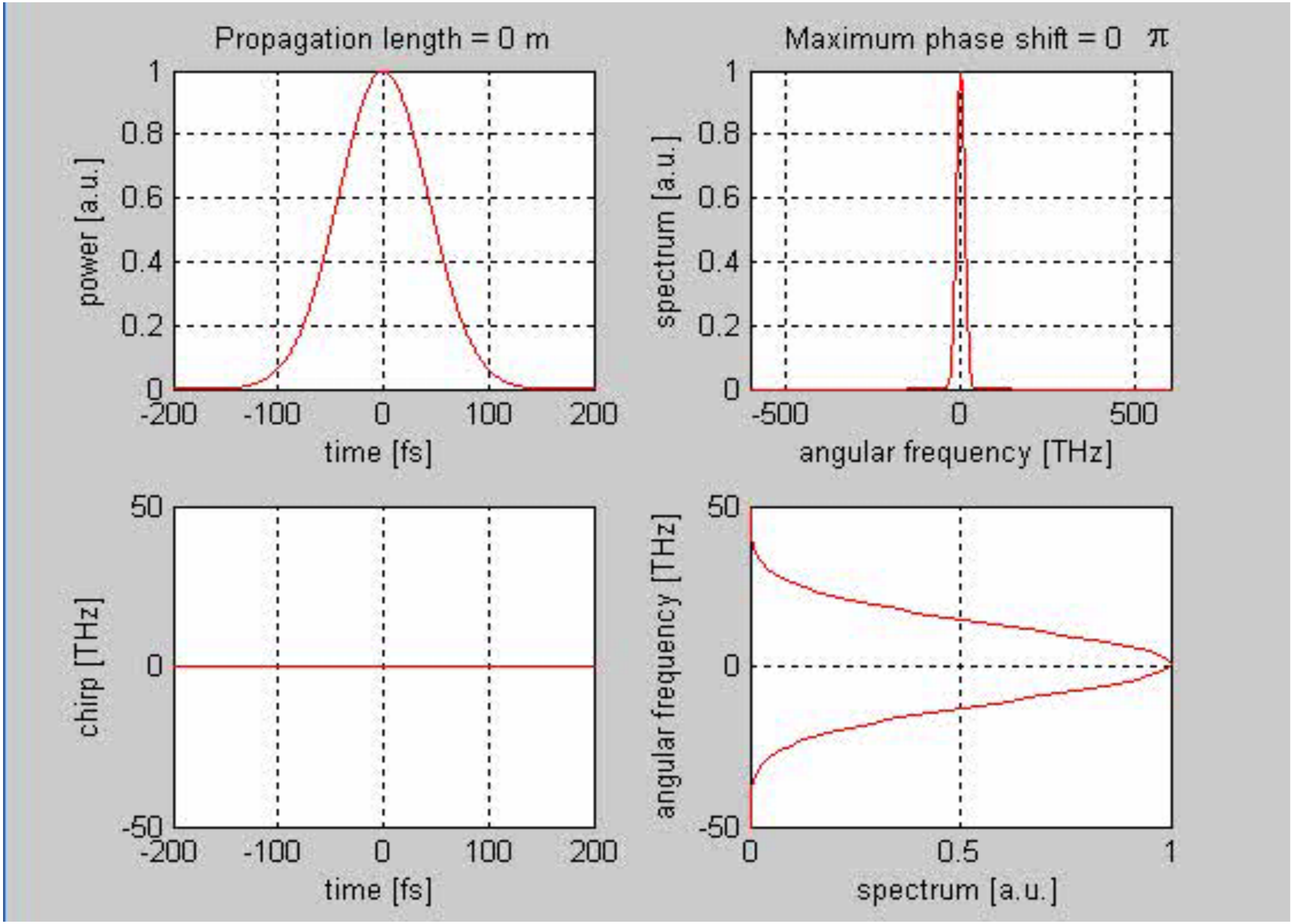




Spectral bandwidth is proportional to the amount of nonlinear phase accumulated inside the fiber.

$$\phi_{NL} \approx (M - \frac{1}{2}) \times \pi$$

M is the number of spectral peaks.



Input: Gaussian pulse, Pulse duration = 100 fs, Peak power = 1 kW

Pulse propagation: pure dispersion Vs pure SPM

- Pure dispersion $$j\frac{\partial A(z,t)}{\partial z} = -D_2\frac{\partial^2 A}{\partial t^2}$$ $$D_2 = \frac{\beta_2}{2}$$ GVD

  (1) Pulse's spectrum acquires phase.

  (2) Spectrum profile does not change.

  (3) In the time domain, pulse may be stretched or compressed depending on its initial chirp .

- Pure SPM $$j\frac{\partial A(z,t)}{\partial z} = \delta|A|^2 A$$

  (1) Pulse acquires phase in the time domain.

  (2) Pulse profile does not change.

  (3) In the frequency domain, pulse's spectrum may be broadened or narrowed depending on its initial chirp.

Nonlinear Schrödinger Equation (NLSE)

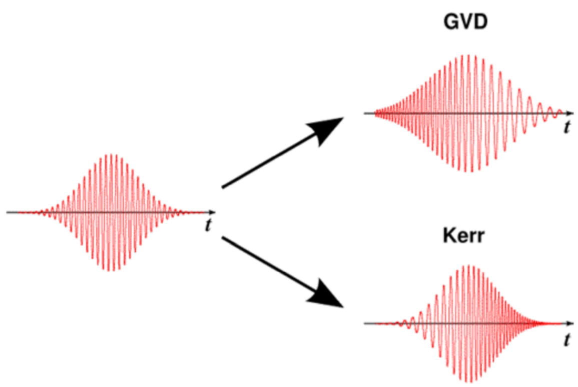
$$j\frac{\partial A(z,t)}{\partial z} = -D_2\frac{\partial^2 A}{\partial t^2} + \delta|A|^2 A \qquad D_2 = \frac{\beta_2}{2}$$

<u>Positive GVD (normal dispersion) + SPM:</u>

GVD and SPM both act to shift the red frequency to the front of the pulse. Therefore the pulse will spread faster than it would in the purely linear case.

<u>Negative GVD (anomalous dispersion) + SPM:</u>

GVD and SPM shift frequency in the opposite direction. At a certain condition, the dispersive spreading of the pulse is exactly balanced by the compression due to the opposite chirp induced by SPM. A steady-state pulse can propagate without changing its shape. (i.e. soliton regime)



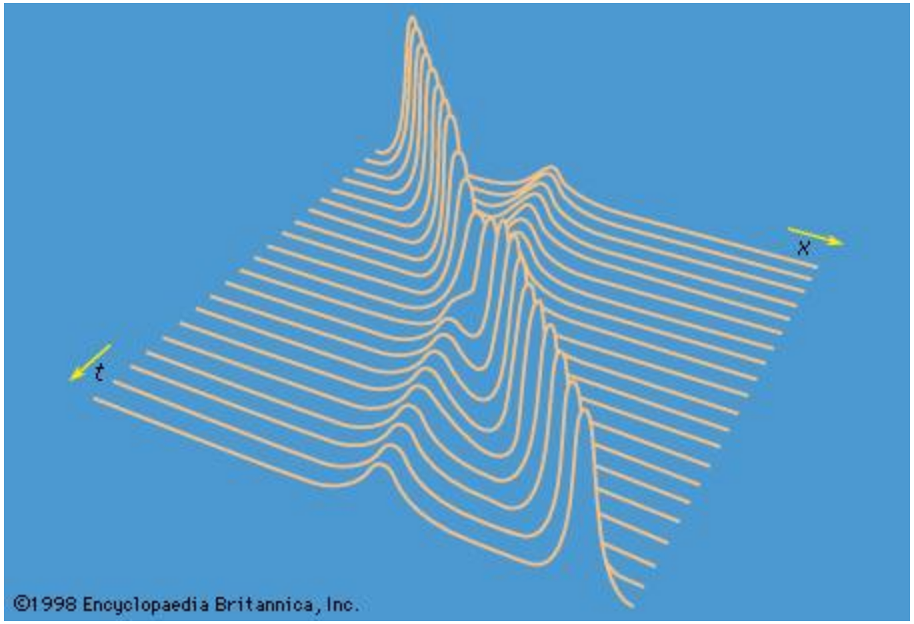
NLSE has soliton solution.

General properties of soliton

In mathematics and physics, a **soliton** is a self-reinforcing solitary wave (a wave packet or pulse) that maintains its shape while it travels at constant speed. Solitons are caused by a cancellation of nonlinear and dispersive effects in the medium. ---Wiki

- When two solitons get closer, they gradually collide and merge into a single wave packet.

- This packet soon splits into two solitons with the same shape and velocity before "collision".

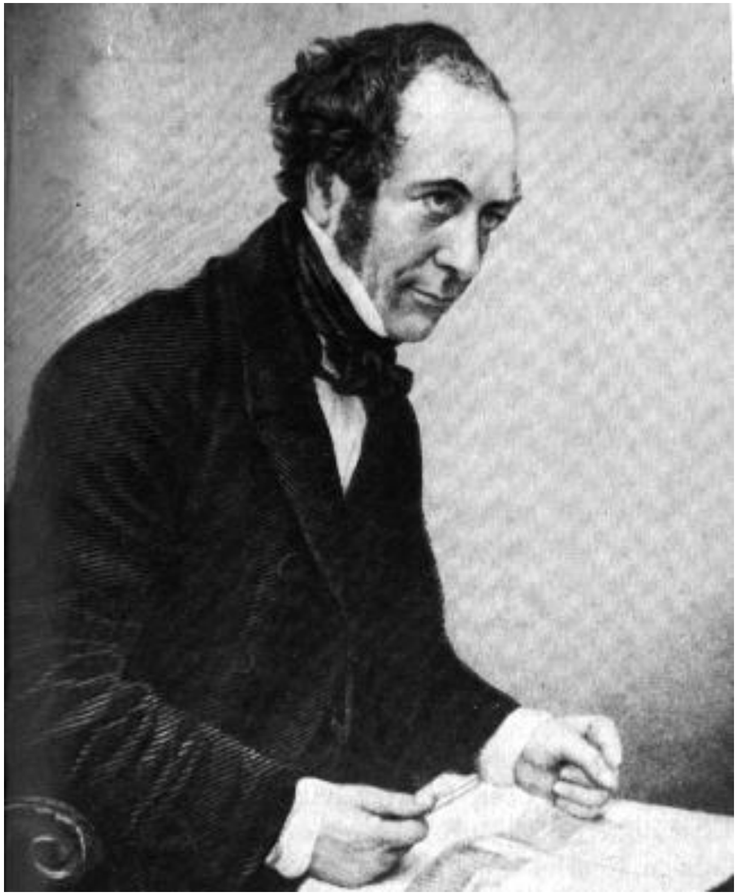


Who discovered soliton?

John Scott Russell (1808 – 1882) was a Scottish civil engineer, naval architect and shipbuilder.

In 1834, while conducting experiments to determine the most efficient design for canal boats, John Scott Russell made a remarkable scientific discovery, leading to a conference paper— Report on Waves.

Report of the fourteenth meeting of the British Association for the Advancement of Science, York, September 1844 (London 1845), pp 311-390, Plates XLVII-LVII).



John Scott Russell (1808-1882)

Russell's report

"I was observing the motion of a boat which was rapidly drawn along a narrow channel by a pair of horses, when the boat suddenly stopped - not so the mass of water in the channel which it had put in motion; it accumulated round the prow of the vessel in a state of violent agitation, then suddenly leaving it behind, rolled forward with great velocity, assuming the form of a large solitary elevation, a rounded, smooth and well-defined heap of water, which continued its course along the channel apparently without change of form or diminution of speed."

"I followed it on horseback, and overtook it still rolling on at a rate of some eight or nine miles an hour, preserving its original figure some thirty feet long and a foot to a foot and a half in height. Its height gradually diminished, and after a chase of one or two miles I lost it in the windings of the channel. Such, in the month of August 1834, was my first chance interview with that singular and beautiful phenomenon which I have called the Wave of Translation."

Report of the fourteenth meeting of the British Association for the Advancement of Science, York, September 1844 (London 1845), pp 311-390, Plates XLVII-LVII).

Water wave soliton in Scott Russell Aqueduct

89.3m long, 4.13m wide,1.52m deep, On the union Canal, Near Heroit-Watt Univ.

www.spsu.edu/math/txu/research/presentations/**soliton/talk.ppt**

Water wave soliton in Scott Russell Aqueduct

www.spsu.edu/math/txu/research/presentations/**soliton/talk.ppt**

A brief history (mainly for optical soliton)

- 1838 – soliton observed in water
- 1895 – KdV equation: mathematical description of waves on shallow water surfaces.
- 1972 – optical solitons arising from NLSE
- 1980 – experimental demonstration in optical fibers
- 1990's – development of techniques to control soliton
- 2000's – understanding soliton in the context of supercontinuum generation

Soliton solution of NLSE：fundamental soliton

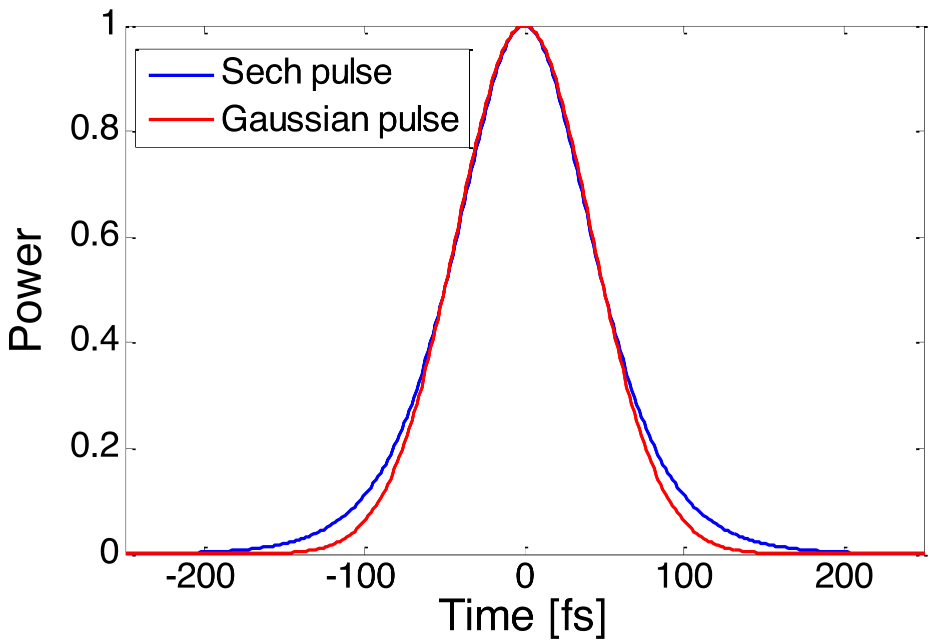
$$j\frac{\partial A(z,t)}{\partial z} = -D_2 \frac{\partial^2 A(z,t)}{\partial t^2} + \delta |A(z,t)|^2 A(z,t)$$

The NLSE possesses the following genereral fundamental soliton solution:

$$A_S(z,t) = A_0 \,\mathrm{sech}(x(z,t))\, e^{-j\theta(z,t)}$$

$$\mathrm{sech}(x) = \frac{2}{e^{x} + e^{-x}}$$

$$\boxed{A_S(z,t) = A_0 \,\mathrm{sech}\left(\frac{t}{\tau}\right) e^{-j\theta(z,t)}}$$



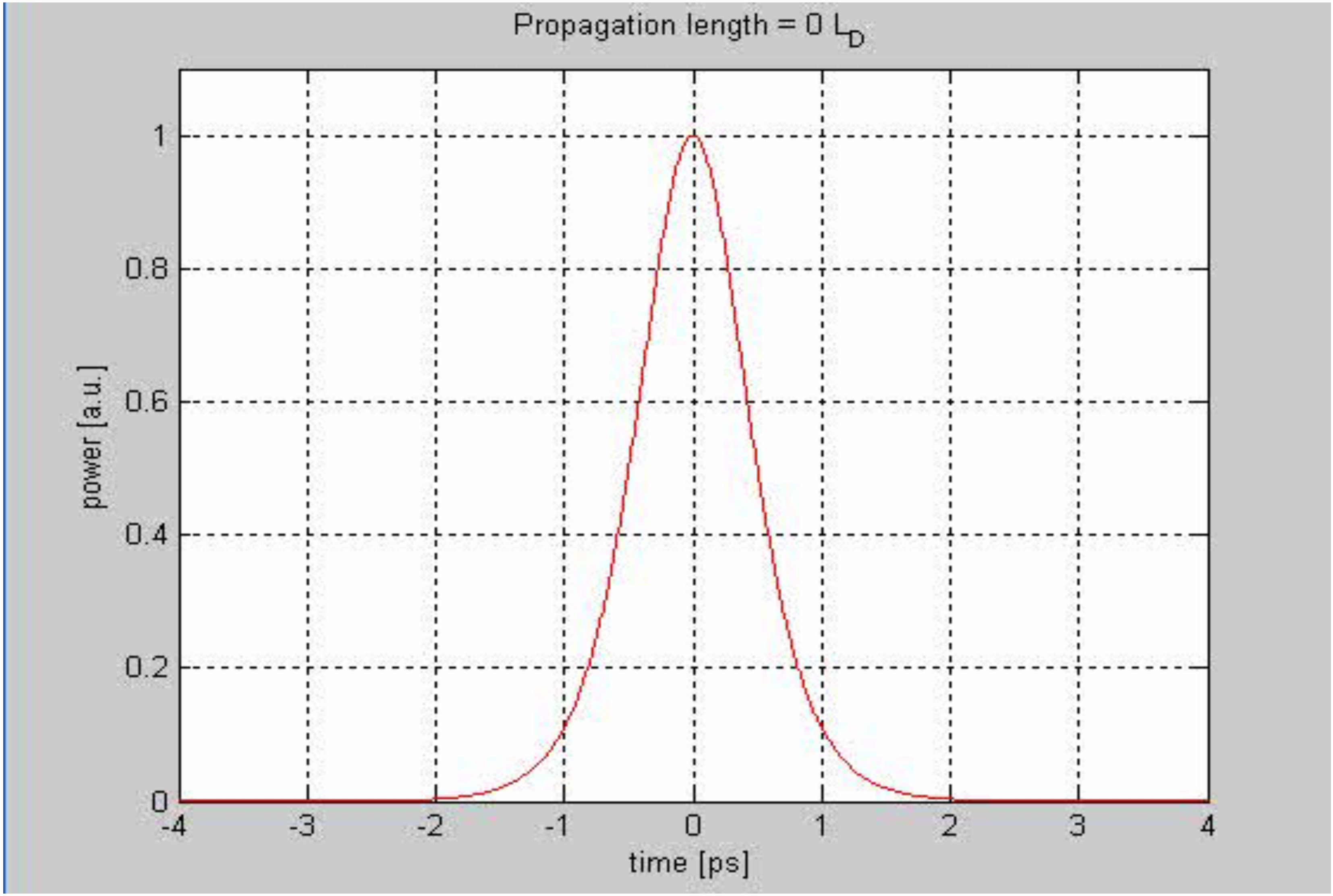
A phase linearly proportional to propagation distance:

$$\theta = \frac{1}{2}\delta A_0^2 z = \frac{|D_2|}{\tau^2} z$$

$$\underbrace{\frac{1}{2}\delta A_0^2}_{\text{nonlinearity}} = \underbrace{\frac{|D_2|}{\tau^2}}_{\text{dispersion}}$$

Soliton is the result of balance between nonlinearity and dispersion.

Propagation of fundamental soliton



Input: 1ps soliton centered at 1.55 um; medium: single-mode fiber

Important Relations

Balance between dispersion and nonlinearity

$$\frac{1}{2}\delta A_0^2 = \frac{|D_2|}{\tau^2} \longrightarrow A_0 = \frac{1}{\tau}\sqrt{\frac{2|D_2|}{\delta}}$$

Phase acquired during soliton propagation

$$\theta = \frac{1}{2}\delta A_0^2 z = \frac{|D_2|}{\tau^2} z$$

Soliton pulse area

$$\text{Pulse Area} = \int_{-\infty}^{\infty} |A_S(z,t)|\, dt = \int_{-\infty}^{\infty} A_0 \operatorname{sech}\left(\frac{t}{\tau}\right) dt = \pi A_0 \tau = \pi\sqrt{\frac{2|D_2|}{\delta}}$$

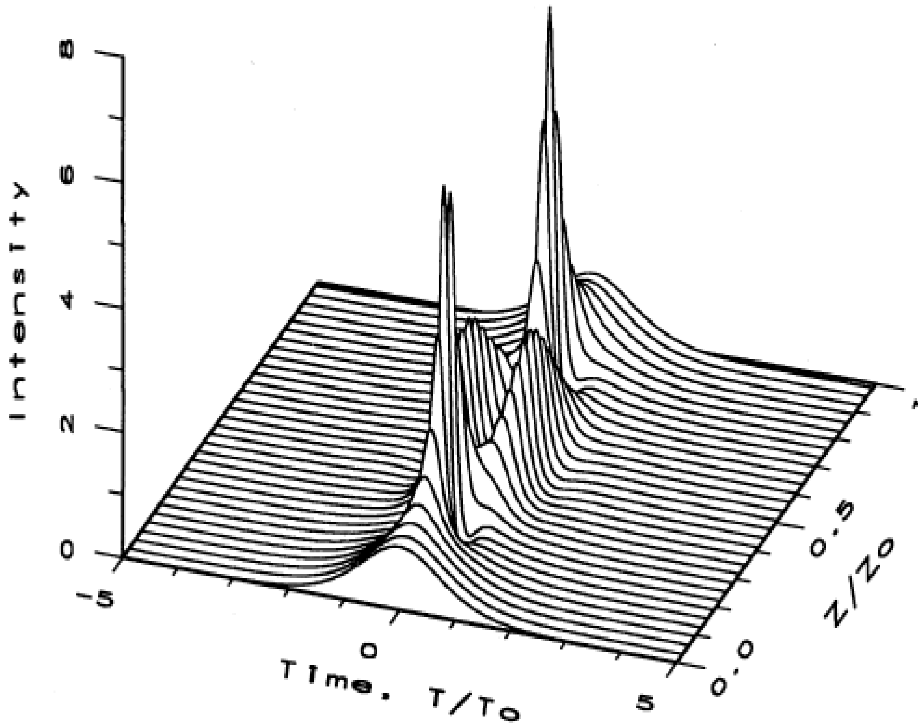
Soliton energy

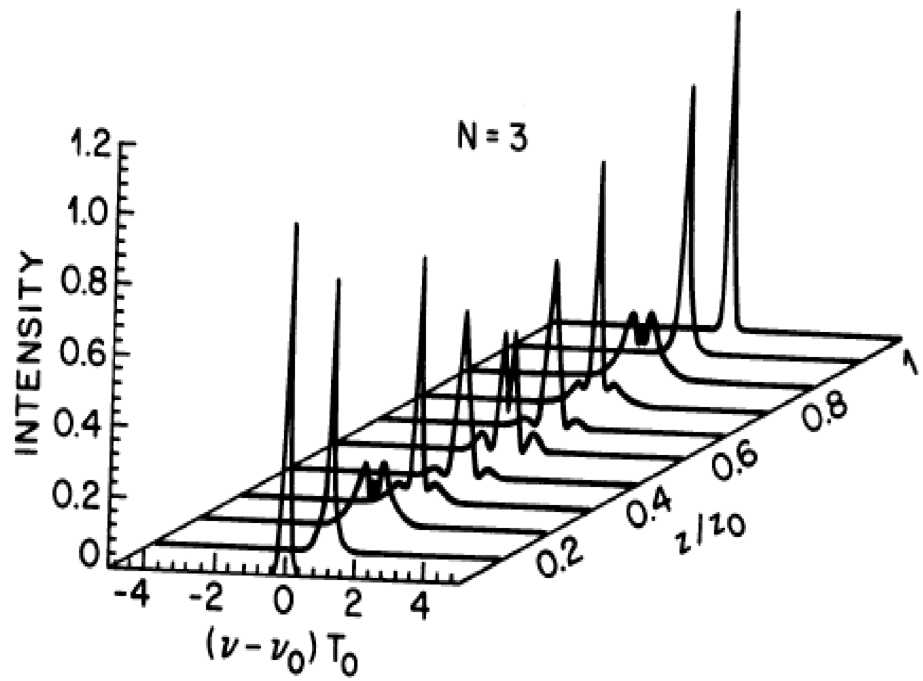
$$w = \int_{-\infty}^{\infty} |A_S(z,t)|^2\, dt = \int_{-\infty}^{\infty} A_0^2 \operatorname{sech}^2\left(\frac{t}{\tau}\right) dt = 2A_0^2\tau$$

Pulse width $\tau = \frac{4|D_2|}{\delta w}$

Higher-order Solitons: periodical evolution in both the time and the frequency domain

$$A_0\tau = N\sqrt{\frac{2|D_2|}{\delta}}, N = 1,2,3\ldots \qquad A_s(z,t) = NA_0\mathrm{sech}(\frac{t}{\tau})e^{-j\theta}$$





G. P. Agrawal, *Nonlinear fiber optics* (2001)

Interaction between solitons (soliton collision)

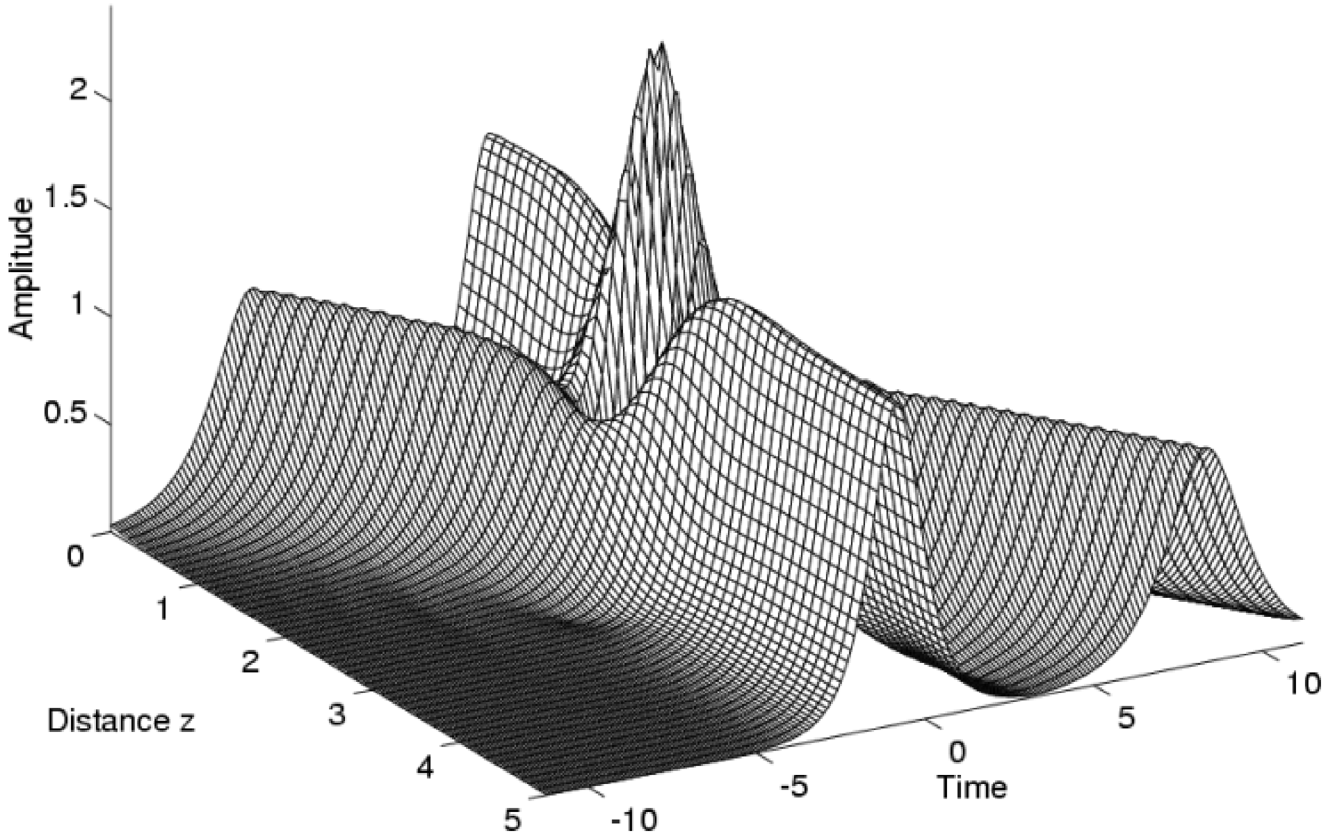
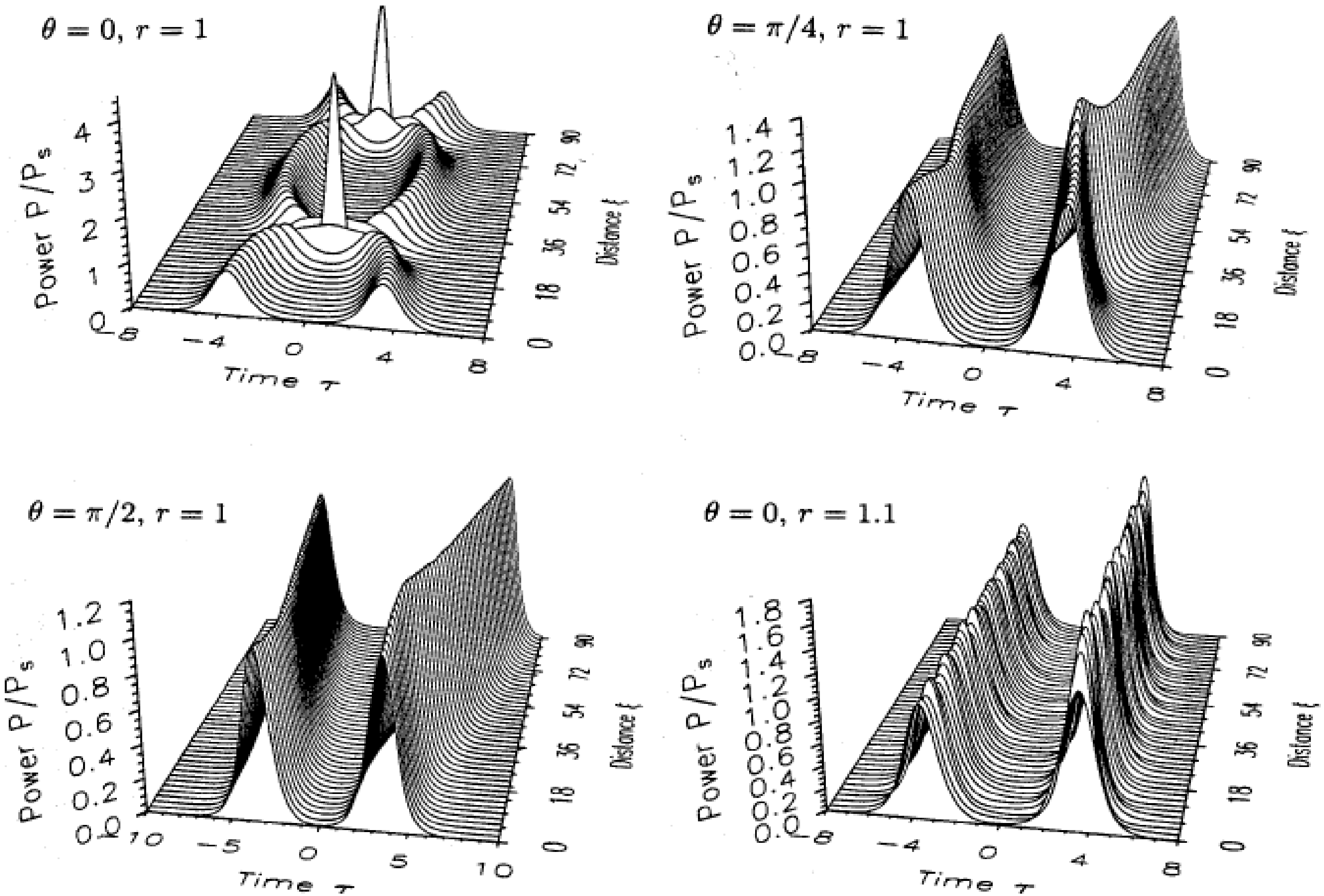


Figure 3.4: A soliton with high carrier frequency collides with a soliton of lower carrier frequency.

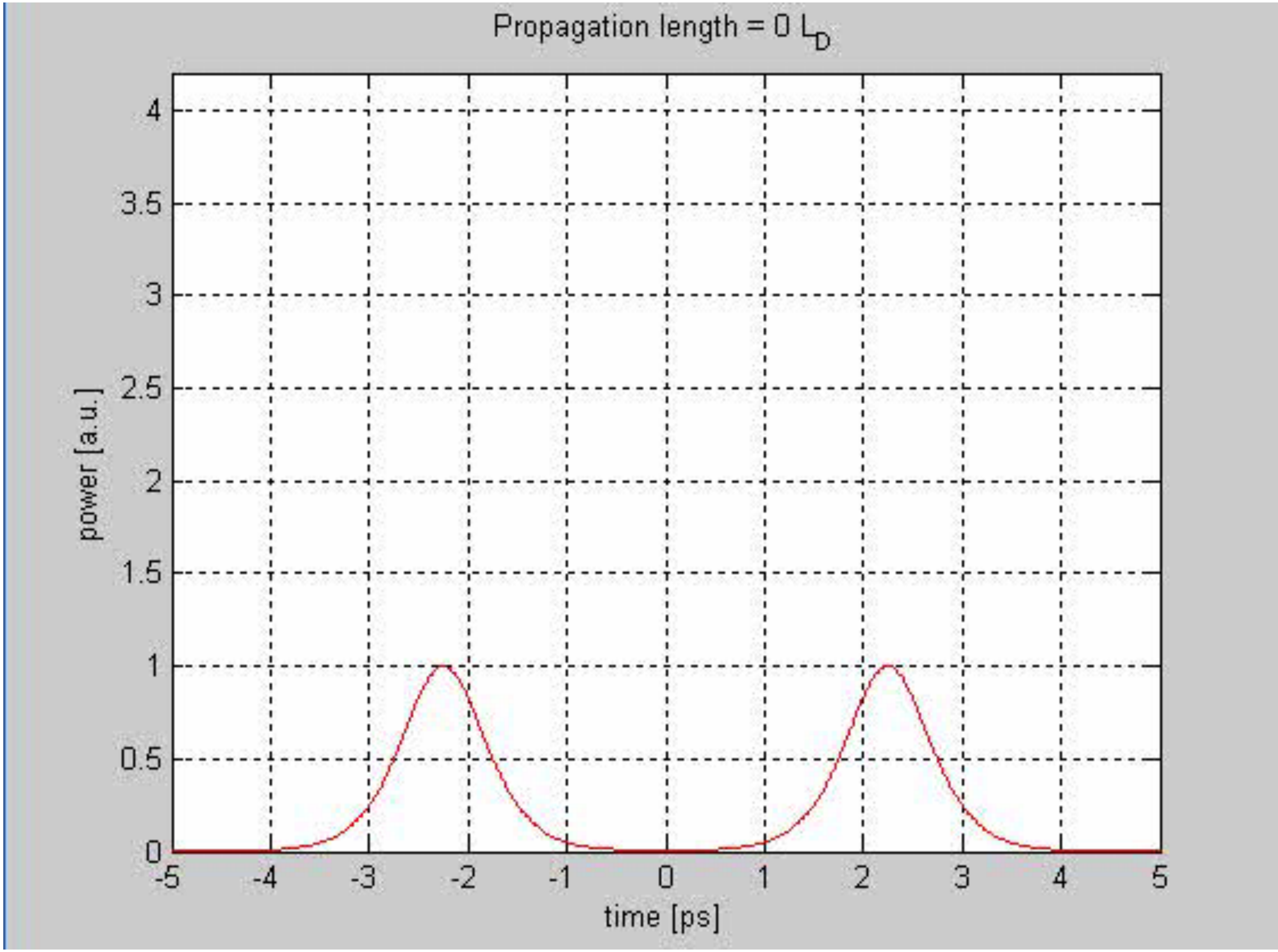
Interaction of two solitons at the same center frequency

Input to NLSE: $u(0,\tau) = \mathrm{sech}(\tau - q_0) + r\,\mathrm{sech}[r(\tau + q_0)]e^{i\theta}$

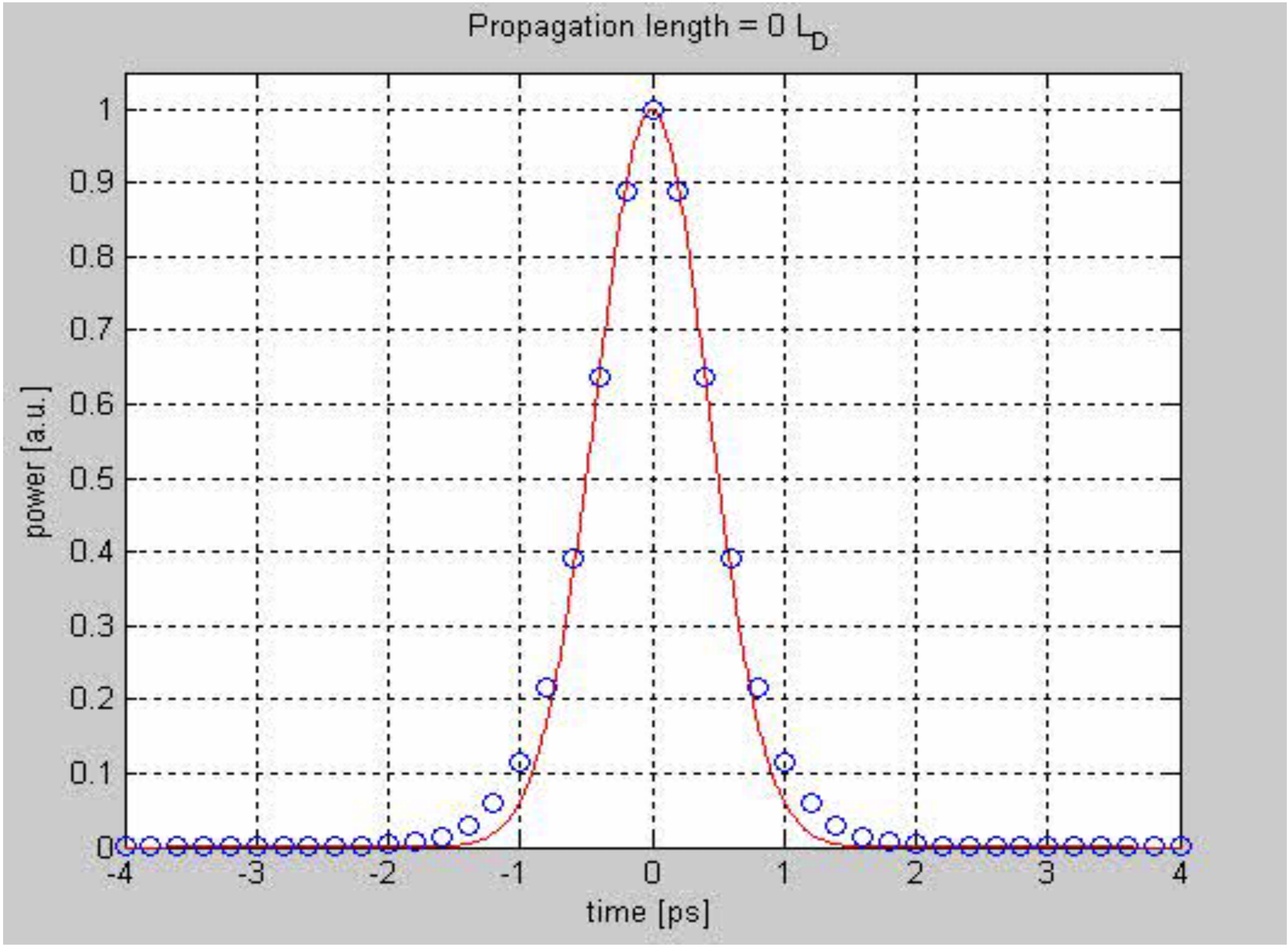


G. P. Agrawal, *Nonlinear fiber optics* (2001)

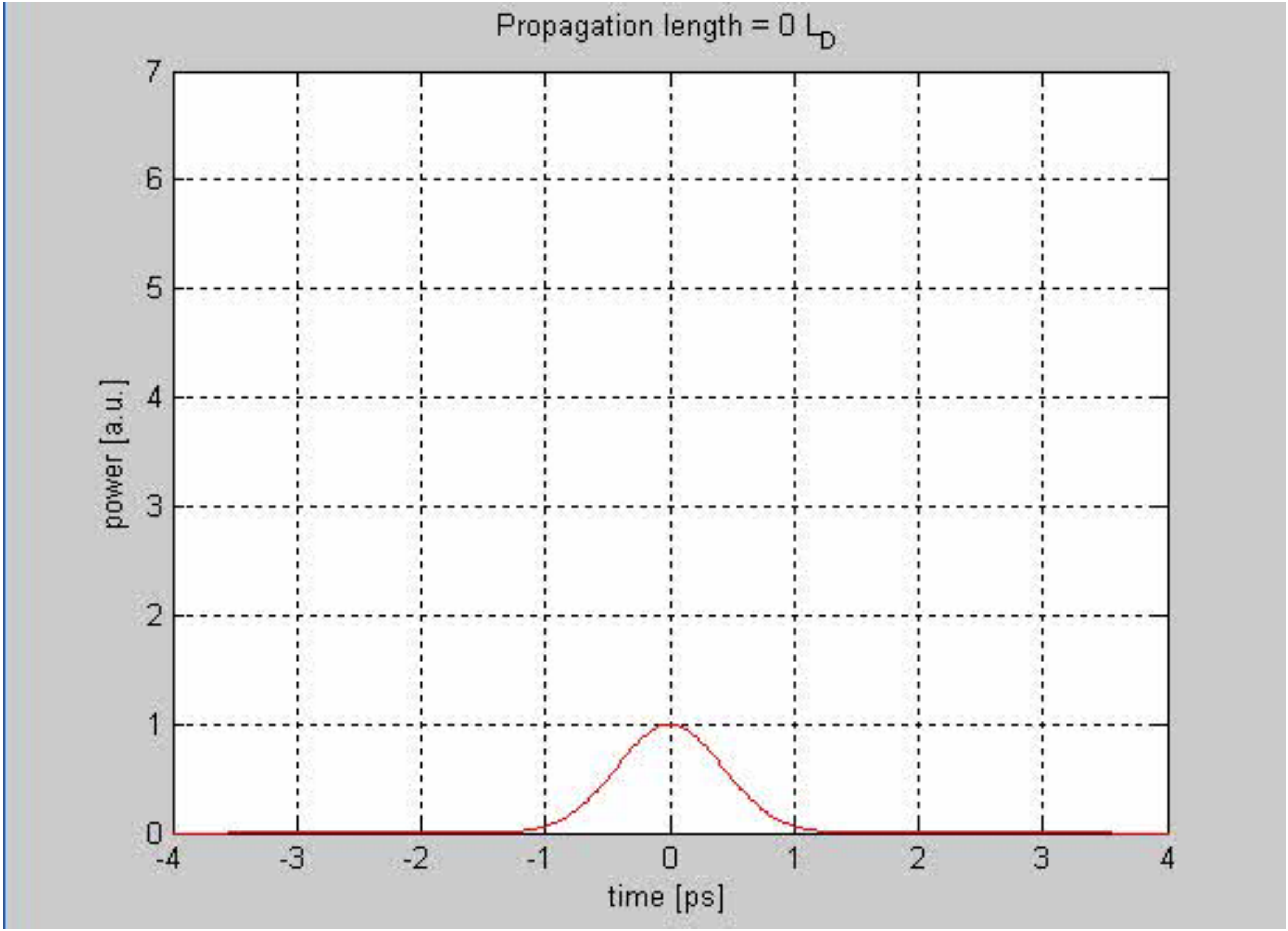
Interactions of two fundamental solitons



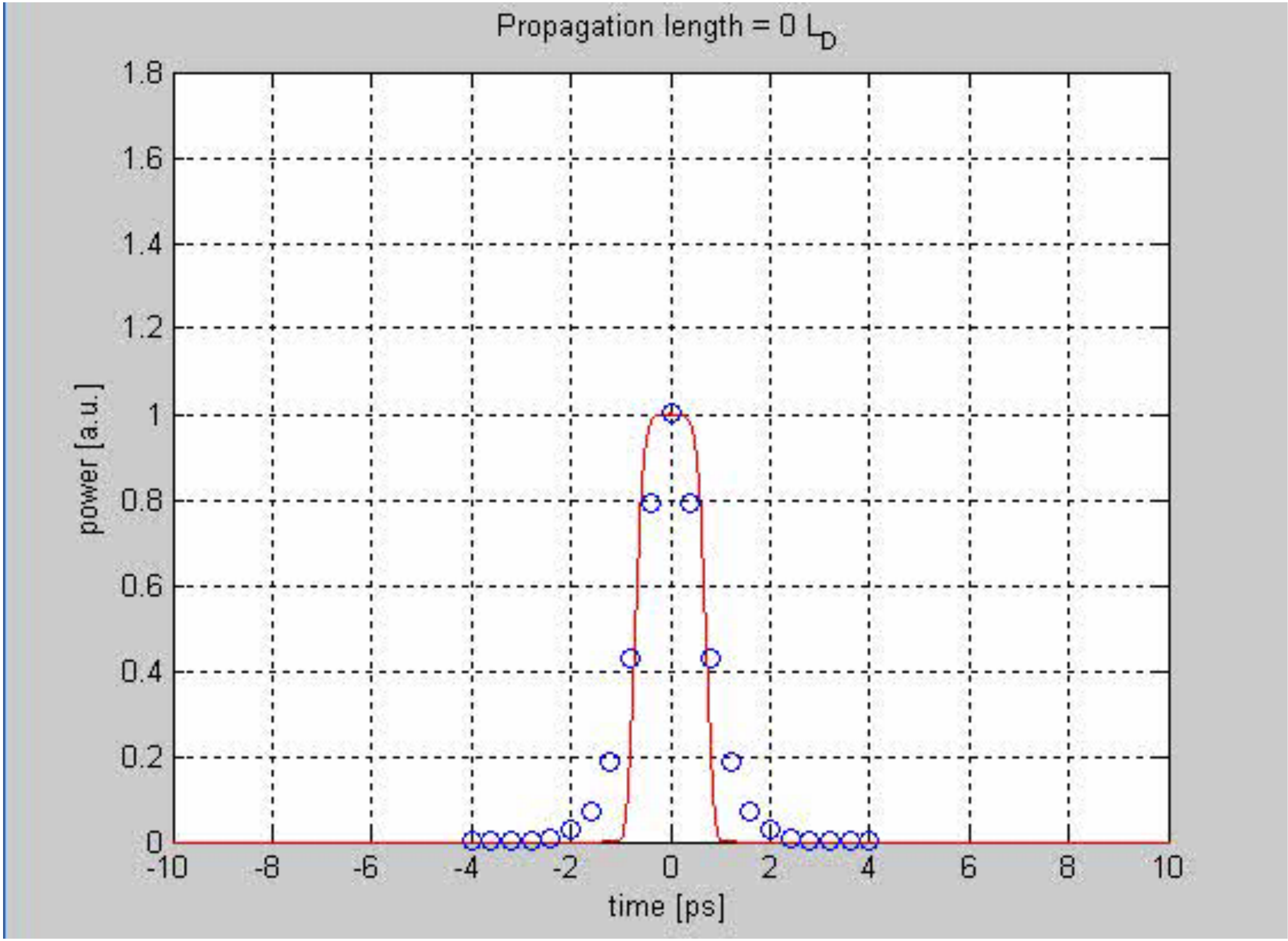
From Gaussian pulse to fundamental soliton



Gaussian pulse to 3-order soliton



Evolution of a super-Gaussian pulse to soliton



Soliton perturbation theory: a very brief introduction

$$\underbrace{\frac{\partial A(z,t)}{\partial z} = -j\left[|D_2|\frac{\partial^2 A}{\partial t^2} + \delta|A|^2 A\right]}_{\text{Perfect World}} + \underbrace{F(A, A^*, z)}_{\text{Reality: Perturbations}}$$

Without perturbations

$$A_s(z,t) = A_0 \mathrm{sech}(x(z,t))e^{-j\theta(z,t)}$$

Galilei transformation to a moving reference frame

$$x = \frac{1}{\tau}(t - 2|D_2|p_0 z - t_0)$$

$$\theta = p_0(t - t_0) + |D_2|\left(\frac{1}{\tau^2} - p_0^2\right)z + \theta_0$$

$$\frac{\delta A_0^2}{2} = \frac{|D_2|}{\tau^2}$$

Four degrees of freedom:

energy fluence w or amplitude A_0

carrier frequency p_0

phase θ_0

origin t_0

What happens to the soliton in the presence of perturbations? Will it fall apart?

Is it just kicked around? If yes, can we understand how it is kicked around?

Soliton perturbation theory: a very brief introduction

$$\frac{\partial A(z,t)}{\partial z} = -j\left[|D_2|\frac{\partial^2 A}{\partial t^2} + \delta |A|^2 A\right] + F(A, A^*, z)$$

Ansatz: Solution of perturbed equation is a soliton + a small component:

$$A(z,t) = \left[a(\frac{t}{\tau}) + \Delta A(z,t)\right] e^{-jk_s z}$$ with: $$a(\frac{t}{\tau}) = A_0 \,\mathrm{sech}(\frac{t}{\tau}) \quad k_s = \frac{1}{2}\delta A_0^2$$

Any deviation ΔA can be decomposed into a contribution that leads to a soliton with a shift in the four soliton parameters and a continuum contribution:

$$\Delta A(z) = \underbrace{\Delta w(z)}_{\text{Energy fluctuation}} f_w + \underbrace{\Delta \theta(z)}_{\text{Optical phase fluctuation}} f_\theta + \underbrace{\Delta p(z)}_{\text{Center frequency fluctuation}} f_p + \underbrace{\Delta t(z)}_{\text{Timing fluctuation}} f_t + \underbrace{a_c(z)}_{\text{Continuum background}}$$

$$f_w = \frac{\partial A}{\partial w}$$

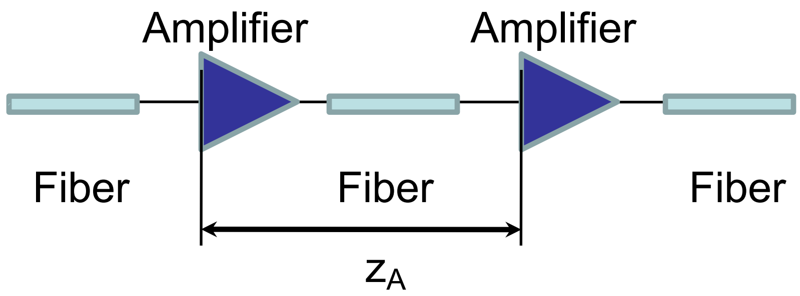
$$f_\theta = \frac{\partial A}{\partial \theta}$$

$$f_p = \frac{\partial A}{\partial p}$$

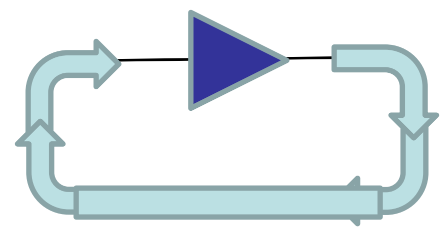
$$f_t = \frac{\partial A}{\partial t}$$

Soliton instabilities by periodic perturbations

Long haul opt. communication link

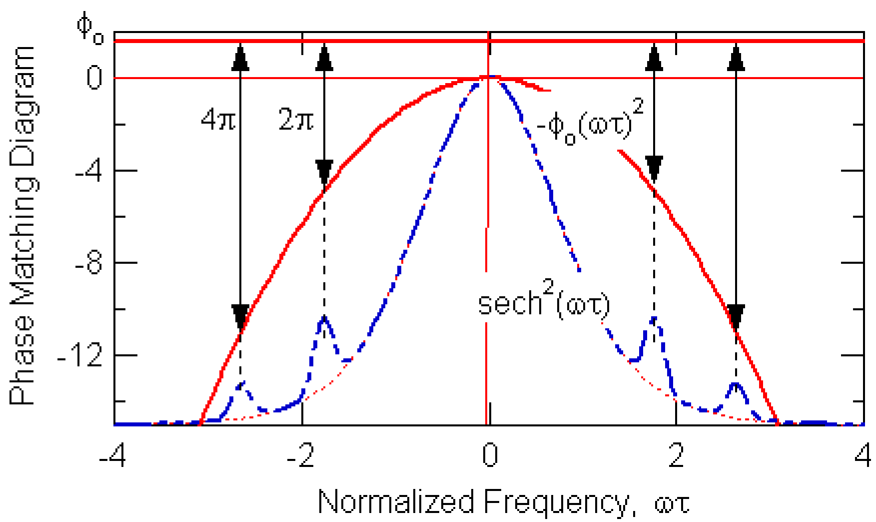


Modelocked fiber laser



Amplification every roundtrip in the oscillator results in a periodic perturbation leading to the appearance of sidebands in the soliton spectrum

$$F(A, A^*, z) = \xi \sum_{n=-\infty}^{\infty} \delta(z - n z_A) A(z, t).$$



Rogue wave

Find more information from New York times:
http://www.nytimes.com/2006/07/11/science/11wave.html

One more Rogue wave

Ultrafast Optical Physics II (SoSe 2019)
Lecture 5, May 10

Part II

(1) Pulse compression: general idea

(2) Dispersion compensation

Examples of ultrafast solid-state laser media

Broader gain bandwidth produces shorter laser pulses.

Laser Materials	Absorption Wavelength	Average Emission λ	Band Width	Pulse Width
Nd:YAG	808 nm	1064 nm	0.45 nm	$\sim$ 6 ps
Nd:YLF	797 nm	1047 nm	1.3 nm	$\sim$ 3 ps
Nd:LSB	808 nm	1062 nm	4 nm	$\sim$ 1.6 ps
$Nd{:}YVO_4$	808 nm	1064 nm	2 nm	$\sim$ 4.6 ps
Nd:fiber	804 nm	1053 nm	22-28 nm	$\sim$ 33 fs
Nd:glass	804 nm	1053 nm	22-28 nm	$\sim$ 60 fs
Yb:YAG	940, 968 nm	1030 nm	6 nm	$\sim$ 300 fs
Yb:glass	975 nm	1030 nm	30 nm	$\sim$ 90 fs
$Ti{:}Al_2O_3$	480-540 nm	796 nm	200 nm	$\sim$ 5 fs
$Cr^{4+}{:}Mg_2SiO_4$:	900-1100 nm	1260 nm	200 nm	$\sim$ 14 fs
Cr^{4+}:YAG	900-1100 nm	1430 nm	180 nm	$\sim$ 19 fs

Transform-limited pulse

$$\tilde{E}(\omega) = \int_{-\infty}^{\infty} E(t)\exp(-j\omega t)dt \qquad E(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \tilde{E}(\omega)\exp(j\omega t)d\omega$$

$\left|\tilde{E}(\omega)\right|^2$ has a spectrum bandwidth of $\Delta\nu$

$\left|E(t)\right|^2$ has a pulse duration of Δt

Both are measured at full-width at half-maximum (FWHM).

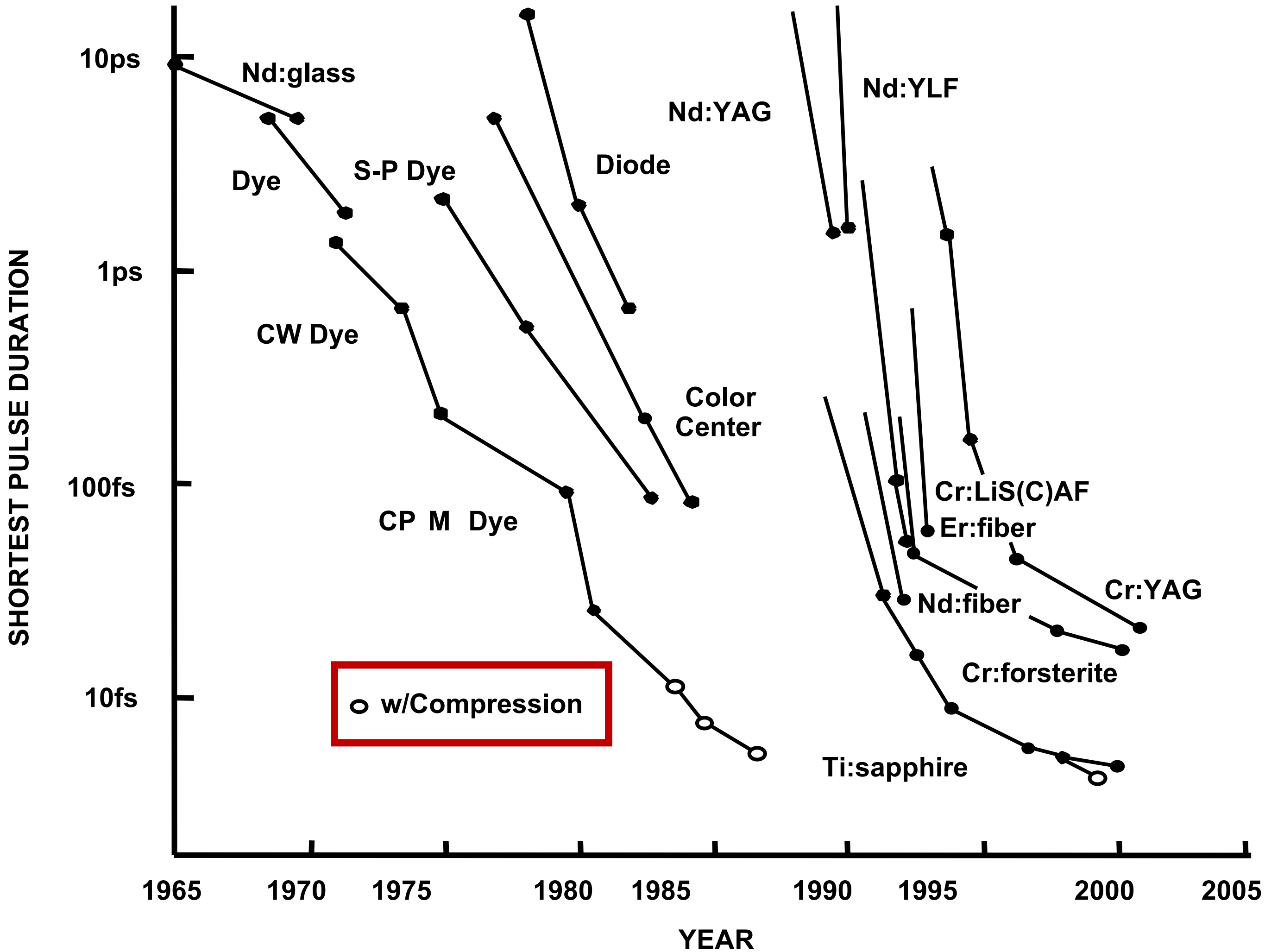
Uncertainty principle:

$$\underbrace{\Delta\nu\Delta t}_{\text{Time Bandwidth Product (TBP)}} \geq \underbrace{K}_{\text{A number depending only on pulse shape}}$$

For a given optical spectrum, there exist a lower limit for the pulse duration.
If the equality is reached, we say the pulse is a transform-limited pulse.

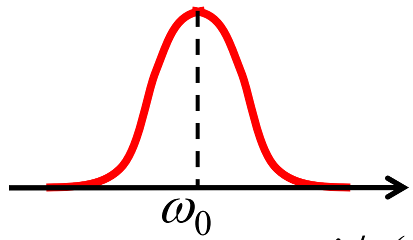
<u>To get a shorter transform-limited pulse, one needs a broader optical spectrum.</u>

How to achieve ultrashort pulse?
To compress or not to compress



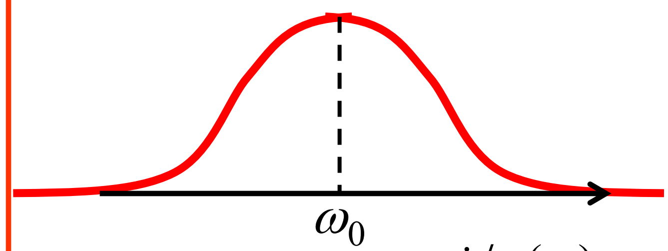
General idea of pulse compression

Step 1: nonlinear spectral broadening



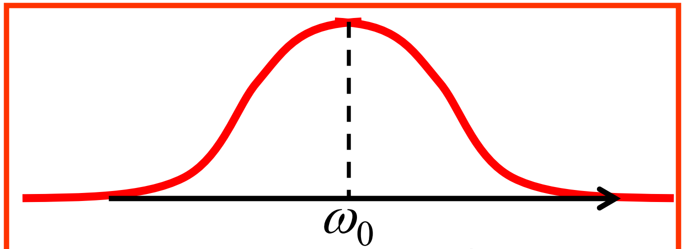
$$A_I(\omega-\omega_0)e^{j\phi_I(\omega)}$$

spectral broadening by light-matter nonlinear interaction



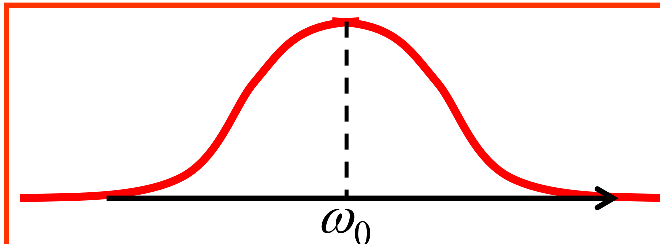
$$A_O(\omega-\omega_0)e^{j\phi_O(\omega)}$$

Step 2: pulse compression by a linear dispersive device



$$A_O(\omega-\omega_0)e^{j\phi_O(\omega)}$$

pick up an extra spectral phase



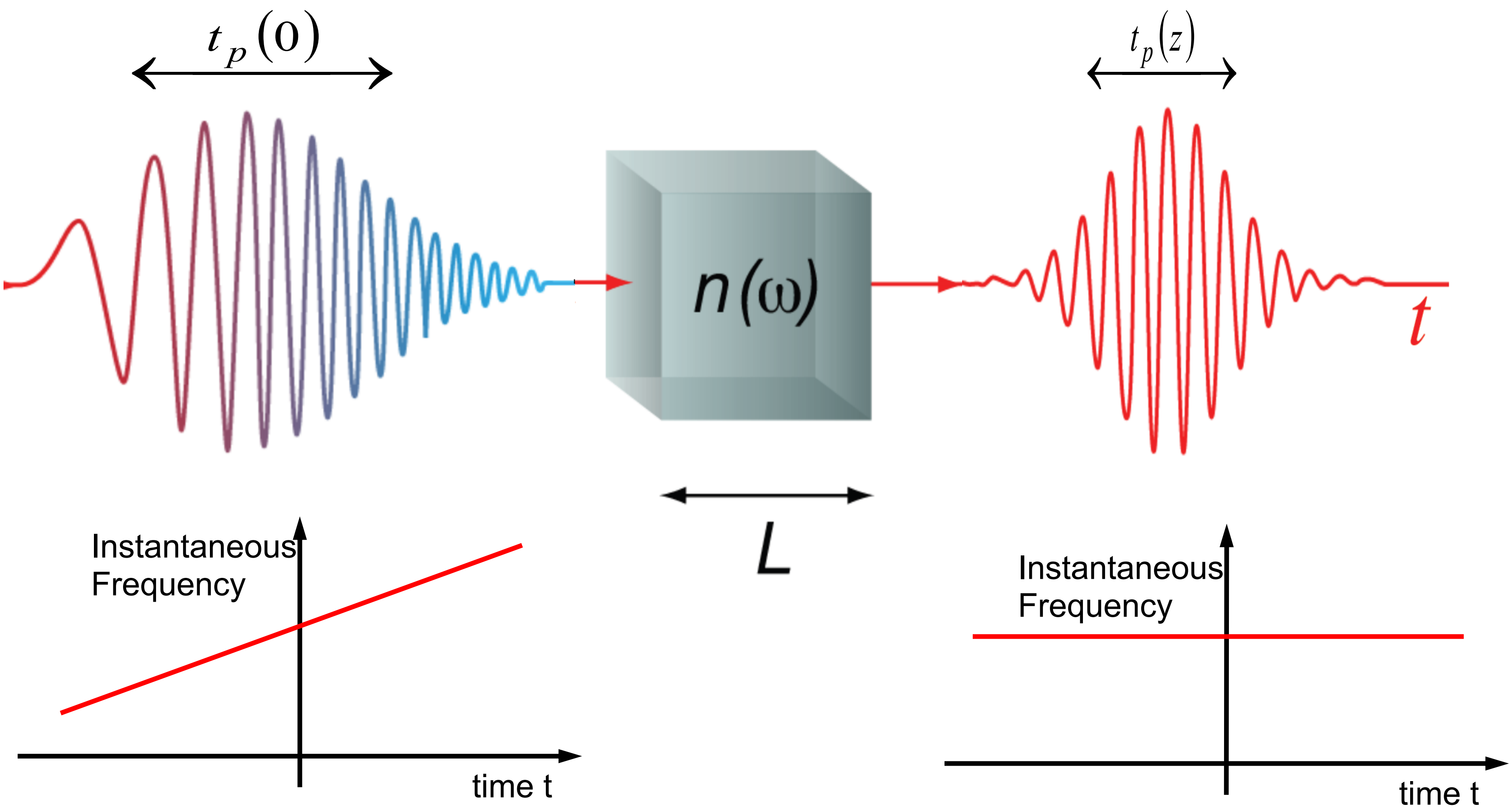
$$A_O(\omega-\omega_0)e^{j[\phi_O(\omega)+\phi_d(\omega)]}$$

Ideal scenario:

$$\phi_O(\omega)+\phi_d(\omega)=\phi_0+\phi_1\times(\omega-\omega_0)$$

This condition guarantees a transform-limited pulse—the shortest pulse allowed by the spectrum.

Pulse travels through a dispersive bulk medium



Positive chirp

Transform-limited pulse

A dispersion compensating device can compensate for the spectral phase and then compress the stretched pulse to its transform-limited duration.

General idea of pulse compression

$$\phi_O(\omega) = \phi_{O,0} + \phi_{O,1} \times (\omega - \omega_0) + \frac{1}{2}\phi_{O,2} \times (\omega - \omega_0)^2 + \frac{1}{6}\phi_{O,3} \times (\omega - \omega_0)^3 + \ldots$$

$$\phi_d(\omega) = \phi_{d,0} + \underbrace{\phi_{d,1}}_{\text{Group delay}} \times (\omega - \omega_0) + \frac{1}{2}\underbrace{\phi_{d,2}}_{\text{Group delay dispersion}} \times (\omega - \omega_0)^2 + \frac{1}{6}\underbrace{\phi_{d,3}}_{\text{3rd-order dispersion}} \times (\omega - \omega_0)^3 + \ldots$$

<u>Ideal scenario:</u>

$$\phi_O(\omega) + \phi_d(\omega) = \phi_0 + \phi_1 \times (\omega - \omega_0) \quad \Rightarrow \quad \begin{aligned} \phi_{d,2} &= -\phi_{O,2} \\ \phi_{d,3} &= -\phi_{O,3} \end{aligned}$$

The broader the spectrum, the more higher-order dispersion should be matched.

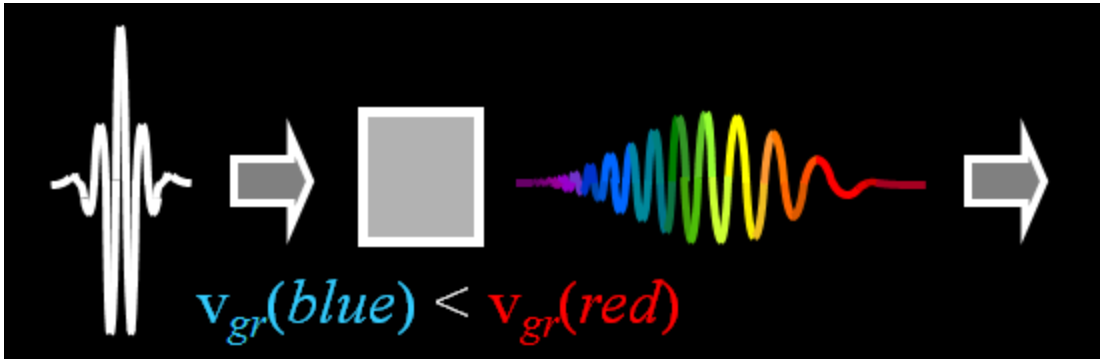
Dispersion parameters for various materials

material	λ [nm]	$n(\lambda)$	$\frac{dn}{d\lambda} \cdot 10^{-2} \left[\frac{1}{\mu m}\right]$	$\frac{d^2n}{d\lambda^2} \cdot 10^{-1} \left[\frac{1}{\mu m^2}\right]$	$\frac{dn^3}{d\lambda^3} \left[\frac{1}{\mu m^3}\right]$	$T_g \left[\frac{fs}{mm}\right]$	$GDD \left[\frac{fs^2}{mm}\right]$	$TOD \left[\frac{fs^3}{mm}\right]$
BK7	400	1,5308	-13,17	10,66	-12,21	5282	120,79	40,57
	500	1,5214	-6,58	3,92	-3,46	5185	86,87	32,34
	600	1,5163	-3,91	1,77	-1,29	5136	67,52	29,70
	800	1,5108	-1,97	0,48	-0,29	5092	43,96	31,90
	1000	1,5075	-1,40	0,15	-0,09	5075	26,93	42,88
	1200	1,5049	-1,23	0,03	-0,04	5069	10,43	66,12
SF10	400	1,7783	-52,02	59,44	-101,56	6626	673,68	548,50
	500	1,7432	-20,89	15,55	-16,81	6163	344,19	219,81
	600	1,7267	-11,00	6,12	-4,98	5980	233,91	140,82
	800	1,7112	-4,55	1,58	-0,91	5830	143,38	97,26
	1000	1,7038	-2,62	0,56	-0,27	5771	99,42	92,79
	1200	1,6992	-1,88	0,22	-0,10	5743	68,59	107,51
Sapphire	400	1,7866	-17,20	13,55	-15,05	6189	153,62	47,03
	500	1,7743	-8,72	5,10	-4,42	6064	112,98	39,98
	600	1,7676	-5,23	2,32	-1,68	6001	88,65	37,97
	800	1,7602	-2,68	0,64	-0,38	5943	58,00	42,19
	1000	1,7557	-1,92	0,20	-0,12	5921	35,33	57,22
	1200	1,7522	-1,70	0,04	-0,05	5913	13,40	87,30
Quartz	300	1,4878	-30,04	34,31	-54,66	5263	164,06	46,49
	400	1,4701	-11,70	9,20	-10,17	5060	104,31	31,49
	500	1,4623	-5,93	3,48	-3,00	4977	77,04	26,88
	600	1,4580	-3,55	1,59	-1,14	4934	60,66	25,59
	800	1,4533	-1,80	0,44	-0,26	4896	40,00	28,43
	1000	1,4504	-1,27	0,14	-0,08	4880	24,71	38,73
	1200	1,4481	-1,12	0,03	-0,03	4875	9,76	60,05

Negative GDD using angular dispersion

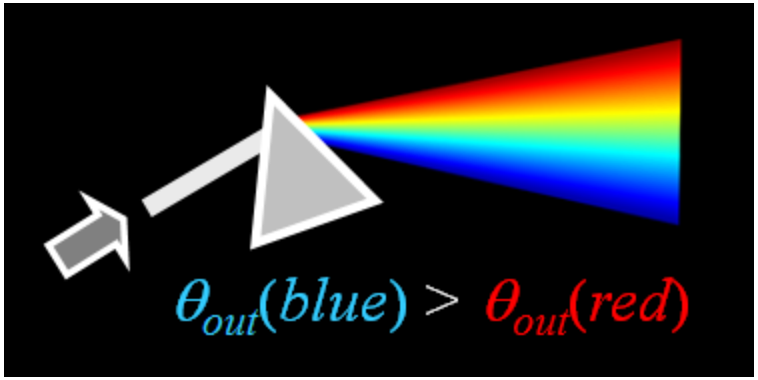
The dependence of the refractive index on wavelength has two effects on a pulse, one in time and the other in space.

Dispersion also disperses a pulse in time:



Group delay dispersion or Chirp
$d^2n/d\lambda^2$

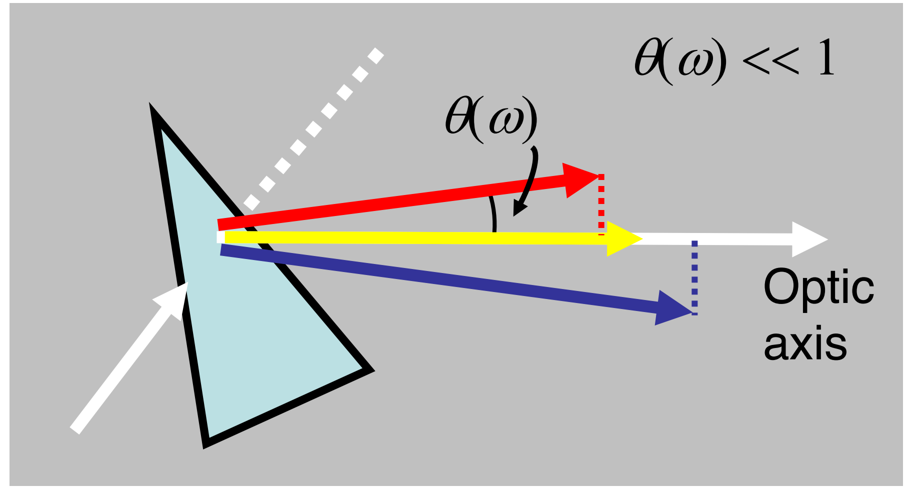
Dispersion disperses a pulse in space (angle):



Angular dispersion
$dn/d\lambda$

Negative GDD using angular dispersion

Taking the projection of $\vec{k}(\omega)$ onto the optic axis, a given frequency ω sees a phase delay of $\varphi(\omega)$:



z

$$\varphi(\omega) = \vec{k}(\omega) \cdot \vec{r}_{optic\ axis}$$
$$= k(\omega)\, z \cos[\theta(\omega)]$$
$$= (\omega / c)\, z \cos[\theta(\omega)]$$

We're considering only the GDD due to the angular dispersion $\theta(\omega)$ and not that of the prism material. Also $n = 1$ (that of the air after the prism).

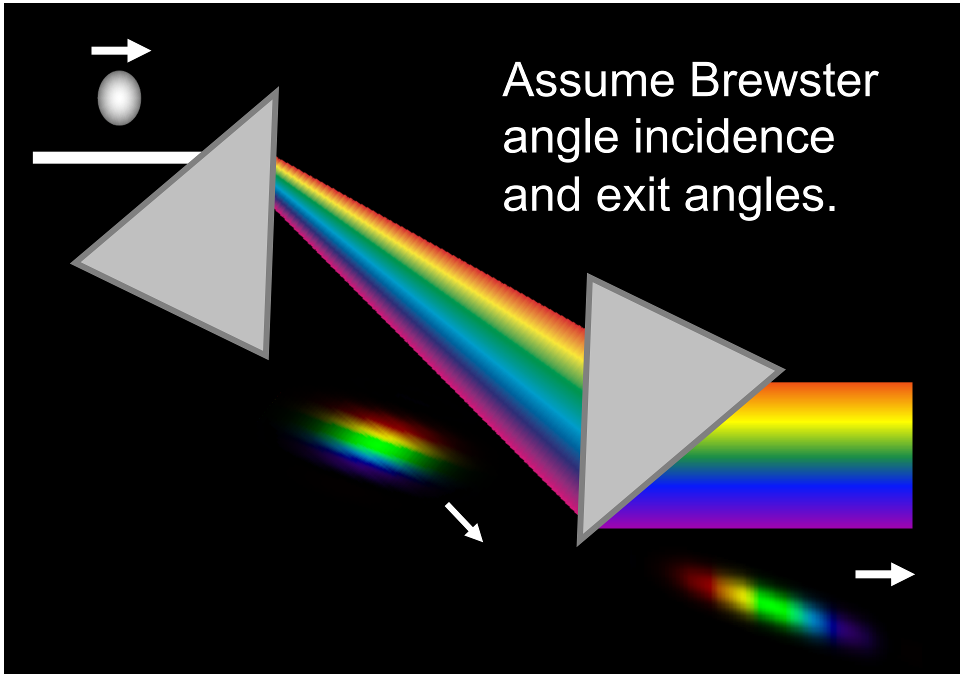
$$d\varphi / d\omega = (z / c) \cos(\theta) - (\omega / c)\, z \sin(\theta)\, d\theta / d\omega$$

$$\frac{d^2\varphi}{d\omega^2} = -\cancel{\frac{z}{c}\sin(\theta)\frac{d\theta}{d\omega}} - \cancel{\frac{z}{c}\sin(\theta)\frac{d\theta}{d\omega}} - \omega\frac{z}{c}\cos(\theta)\left(\frac{d\theta}{d\omega}\right)^2 - \omega\cancel{\frac{z}{c}\sin(\theta)\frac{d^2\theta}{d\omega^2}}$$

But $\theta << 1$, so the sine terms can be neglected, and $\cos(\theta) \sim 1$.

Slide from Rick Trebino's Ultrafast Optics course

A prism pair has negative GDD.



How can we use dispersion to introduce negative chirp conveniently?

Let L_{prism} be the path through each prism and L_{sep} ($z = L_{sep}$) be the prism separation.

$$\left.\frac{d^2\varphi}{d\omega^2}\right|_{\omega_0} \approx -4L_{sep}\frac{\lambda_0^3}{2\pi c^2}\left(\left.\frac{dn}{d\lambda}\right|_{\lambda_0}\right)^2 + L_{prism}\frac{\lambda_0^3}{2\pi c^2}\left.\frac{d^2n}{d\lambda^2}\right|_{\lambda_0}$$

Always negative!

This term assumes that the beam grazes the tip of each prism

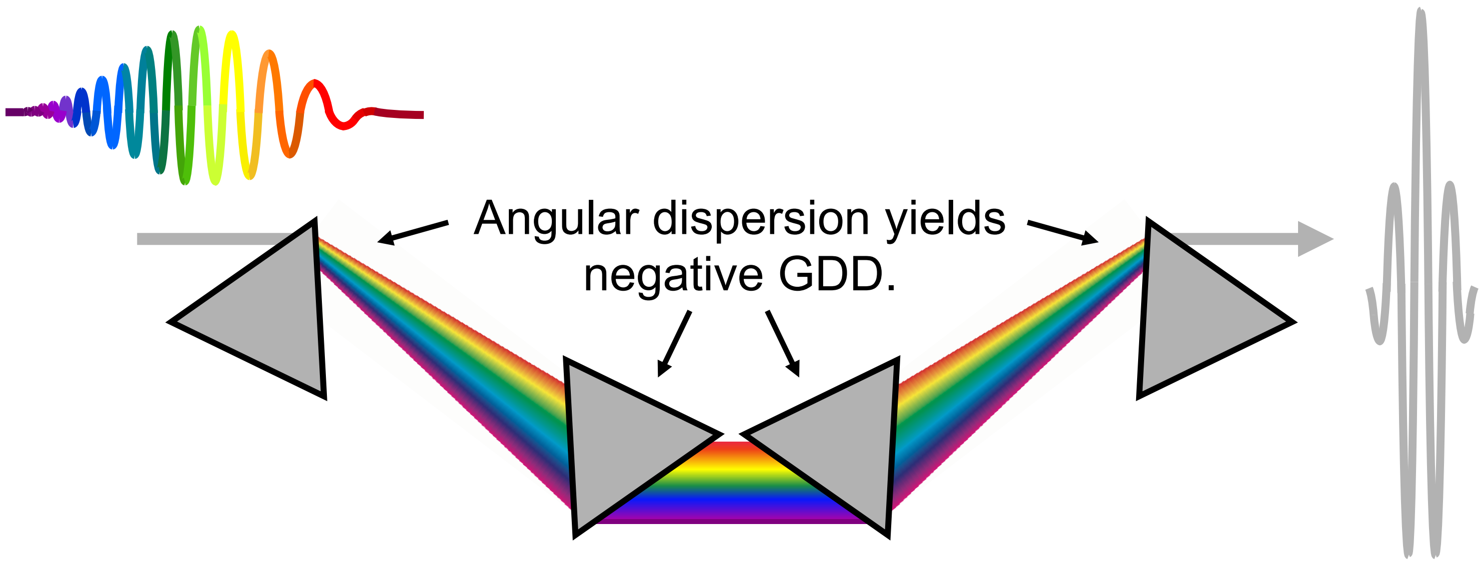
This term allows the beam to pass through an additional length, L_{prism}, of prism material.

Always positive (in visible and near-IR)

Vary L_{sep} or L_{prism} to tune the GDD!

Pulse compressor using 4 prisms

This device, which also puts the pulse back together, has **negative** group-delay dispersion and hence can compensate for propagation through materials (i.e., for positive chirp).

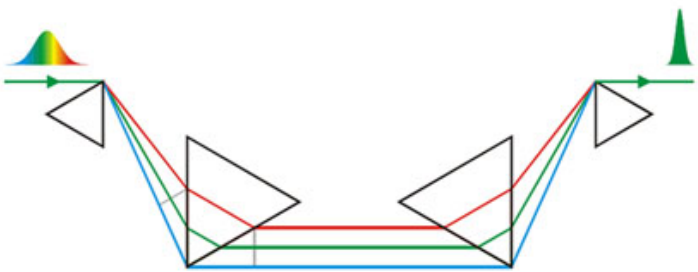


It's routine to stretch and then compress ultrashort pulses by factors of >1000.

Slide from Rick Trebino's Ultrafast Optics course

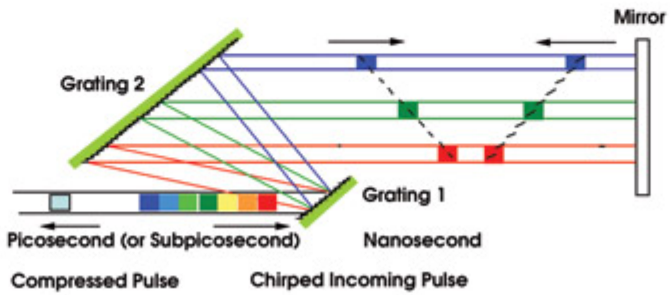
Dispersion compensation using angular dispersion

Prism pair



(1) Small dispersion
(2) Negligible loss at Brewster angle

Diffraction grating pair

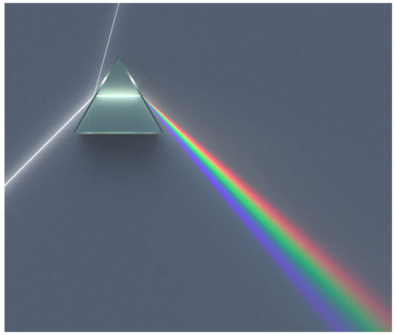


(1) Large dispersion
(2) Losses ~ 25%

Typical dispersion signs for material, grating pair, and prism pair

	Material	gratings	Prisms
2^{nd} order dispersion	+	-	-
3^{rd} order dispersion	+	+	-

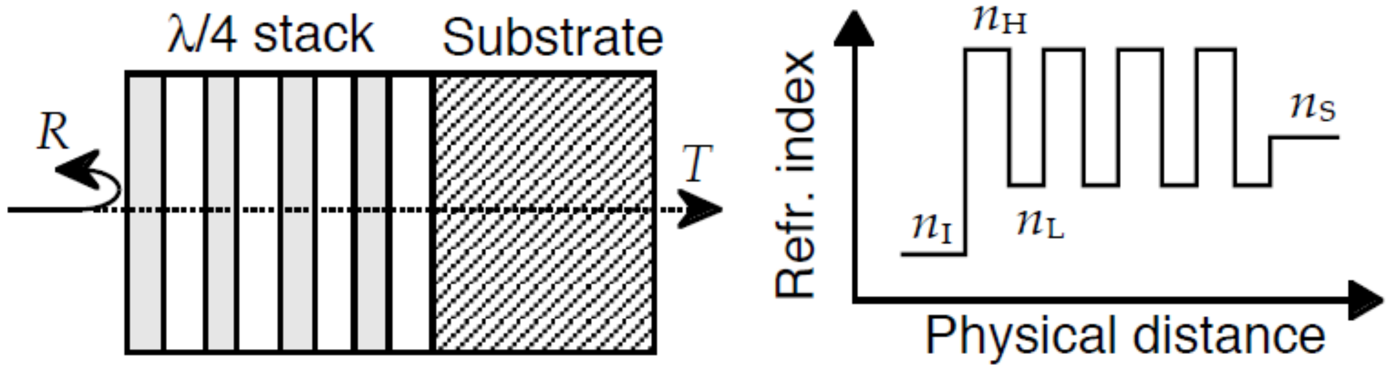
Grating pair versus prism pair



Device	λ_ℓ [nm]	φ'' [fs^{+2}]	φ''' [fs^{+3}]
SQ1 ($L = 1$ cm)	620	550	240
Piece of glass	800	362	280
Brewster prism pair, SQ1	620	-760	-1300
$\ell = 50$ cm	800	-523	-612
grating pair $b = 20$ cm; $\beta = 0^\circ$	620	$-8.2\ 10^4$	$1.1\ 10^5$
$d = 1.2\ \mu$m	800	$-3\ 10^6$	$6.8\ 10^6$

Dispersion of mirror structures: quarter-wave stack

High reflecting mirrors can be realized using a stack of thin dielectric films of different refractive indices.



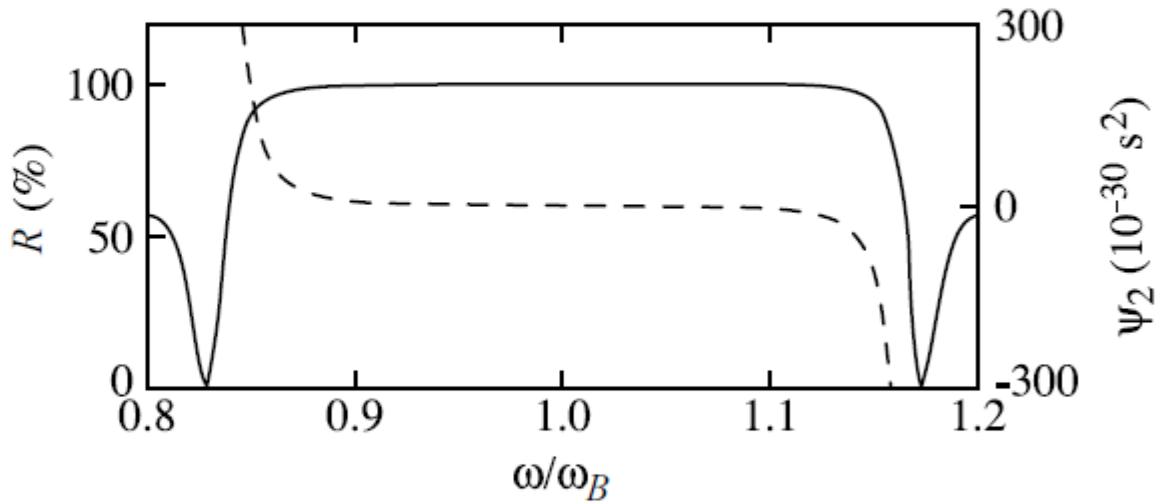
Bragg wavelength:

$$\lambda_B = 2(n_H d_H + n_L d_L)$$

$$n_H d_H = n_L d_L$$

Bandwidth of Bragg mirror:

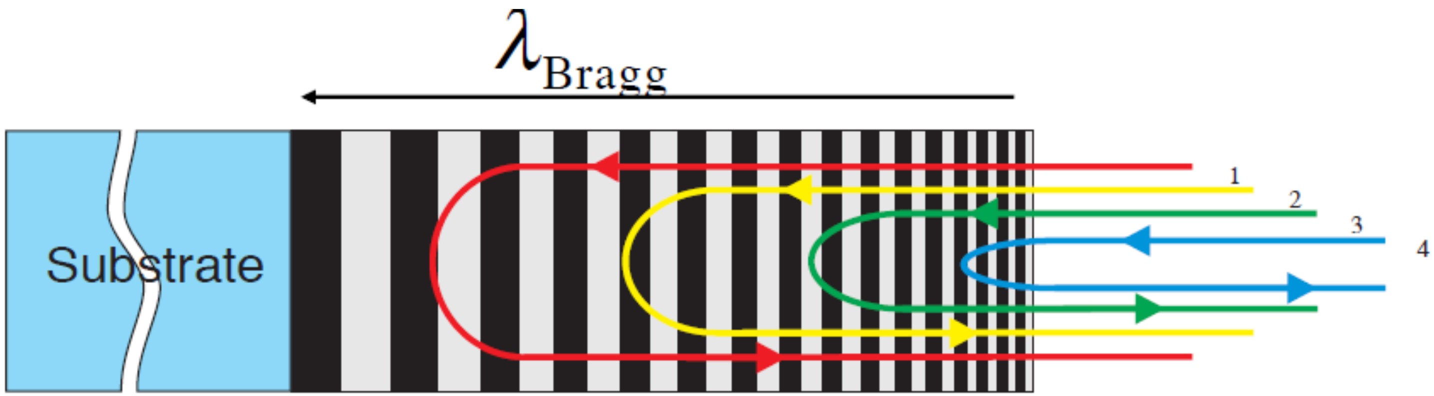
$$r_B = \frac{\Delta f}{f_c} = \frac{n_H - n_L}{n_H + n_L}$$



Typical coating example:

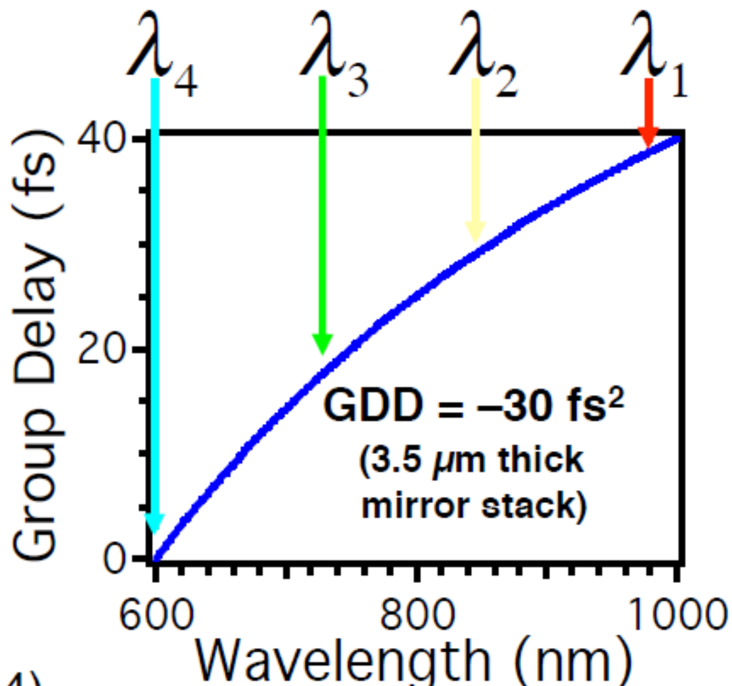
$$n_{SiO_2} = 1.48 \qquad n_{TiO_2} = 2.4 \longrightarrow \Delta f / f_c = 0.23$$

Chirped mirror by chirping the Bragg wavelength



Chirp Bragg wavelength

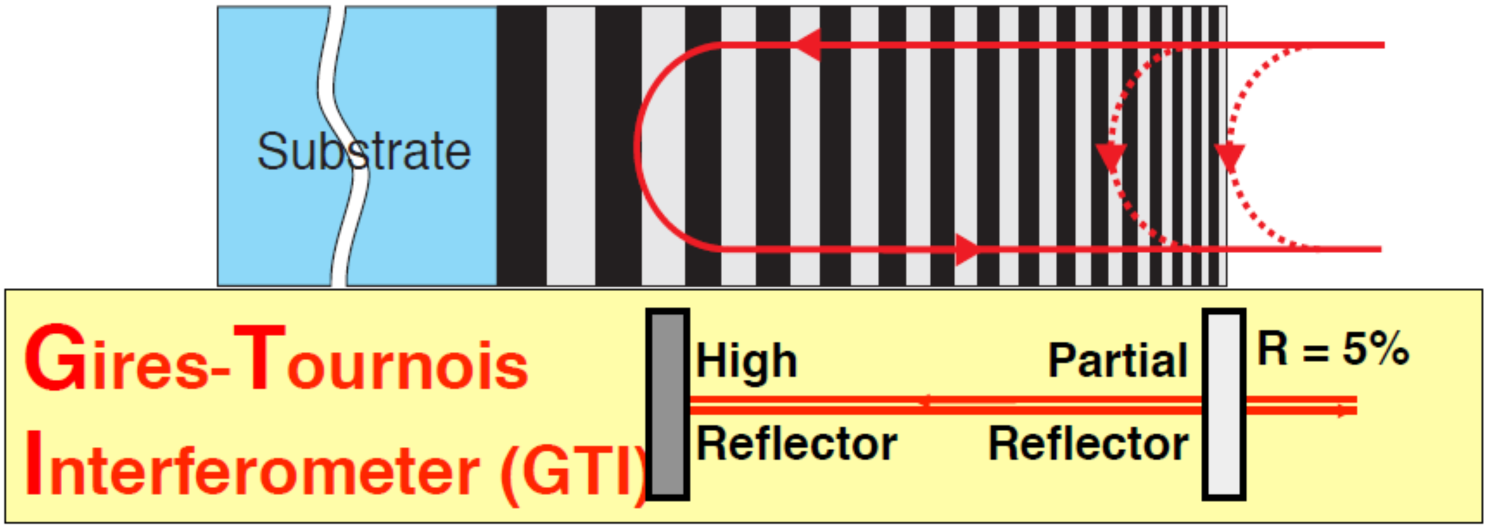
- ⇒ wavelength-dependent penetration depth
- ⇒ engineerable dispersion
- ⇒ compensation of arbitrary material dispersion
- ⇒ increased high-reflection bandwidth



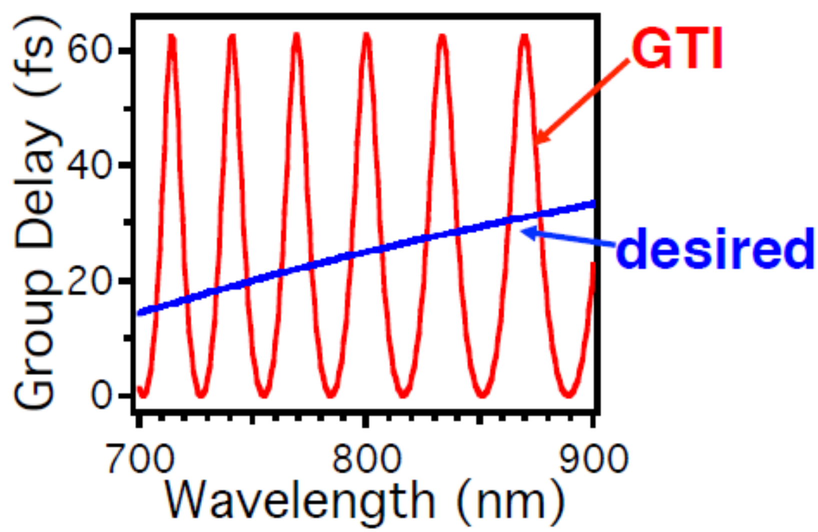
Szipöcs et al., *Opt. Lett.* **19**, 201 (1994)

Adapted from U. Keller's Ultrafast Laser Physics course

Interference causes ripples on group delay

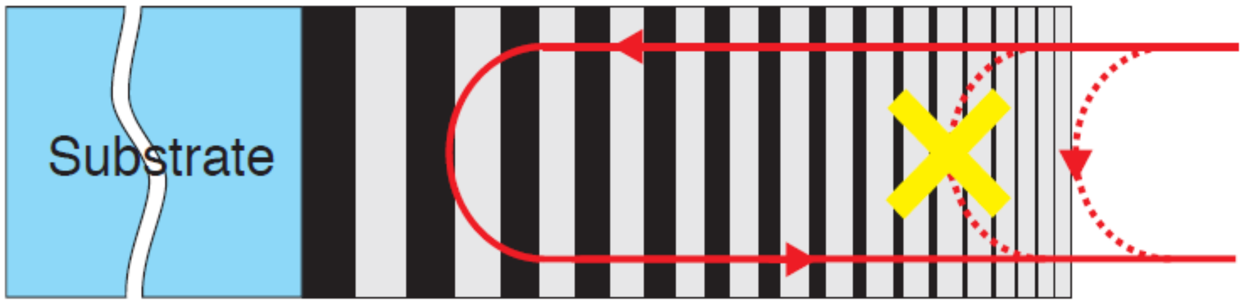


- front interface + high reflector form an *effective GTI*
- *dispersion oscillations*
- magnitude comparable to net mirror dispersion

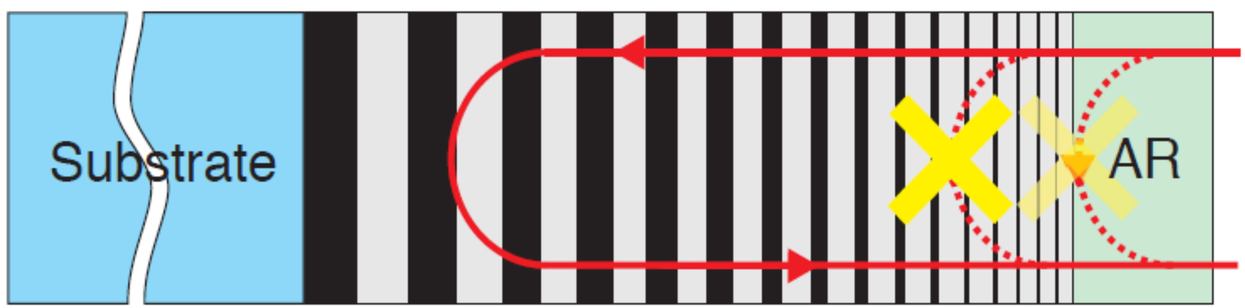


Adapted from U. Keller's Ultrafast Laser Physics course

Double chirped mirrors: eliminate dispersion oscillation



Additional chirp in the coupling between incident and reflected wave
⇔ chirp in the duty-cycle of high and low refractive index material
⇔ apodization of mirror impedance to first-layer impedance



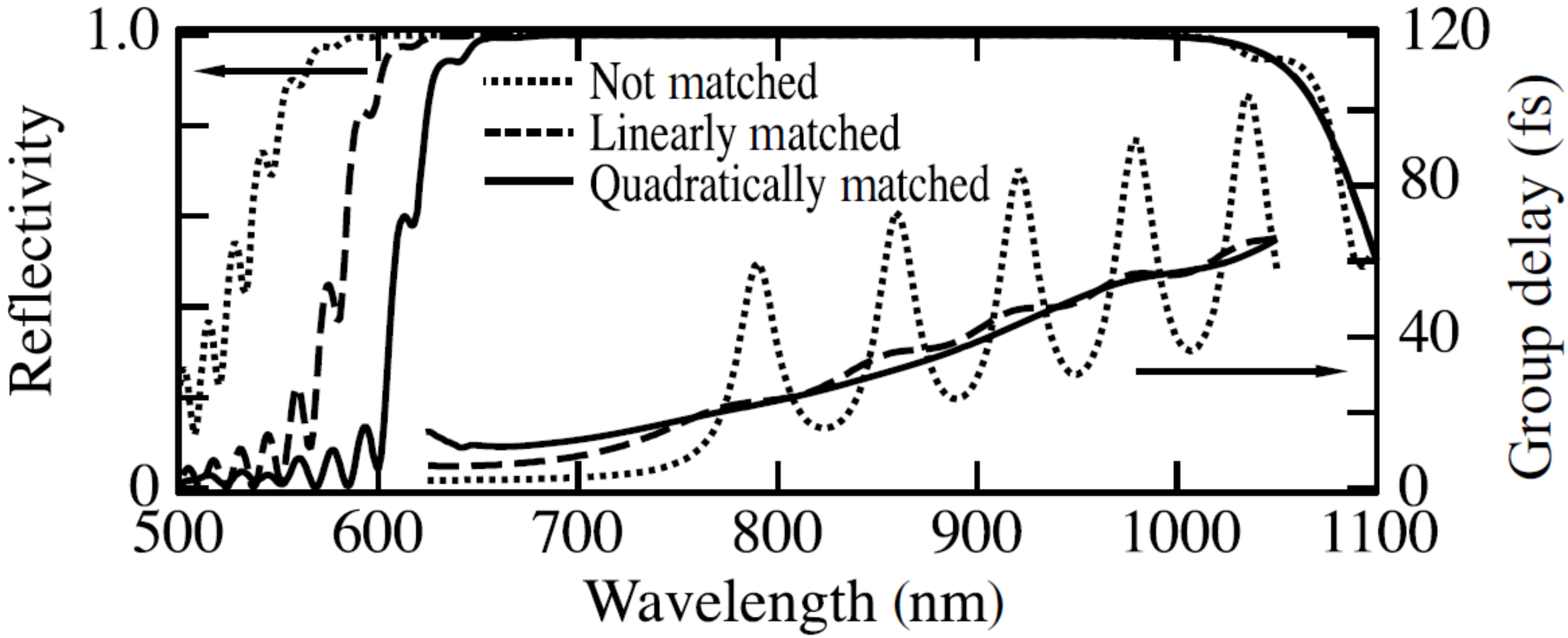
Anti-reflection coating matches first-layer impedance to air

Kärtner et al., Opt. Lett. **22**, 831 (1997)

Matuschek et al., IEEE J. Sel. Top. Quantum Electron. **4**, 197 (1998)

Adapted from U. Keller's Ultrafast Laser Physics course

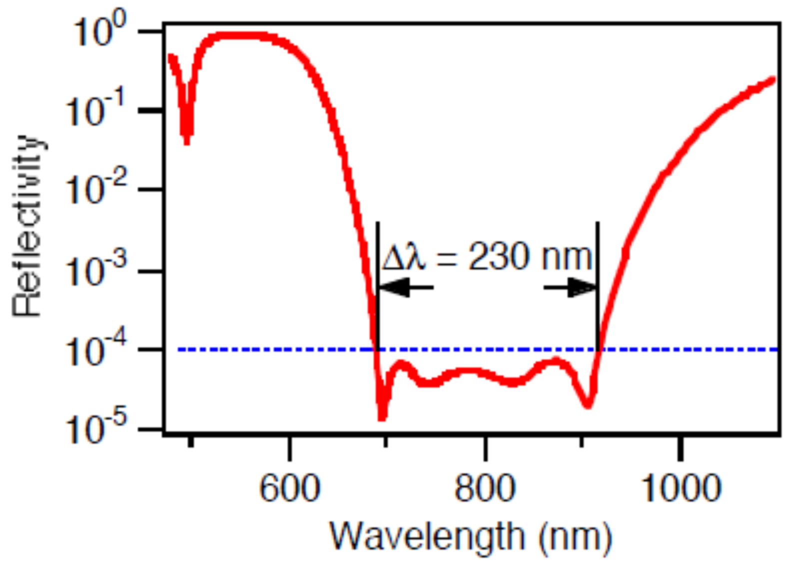
Comparison for different chirping of the high-index layer

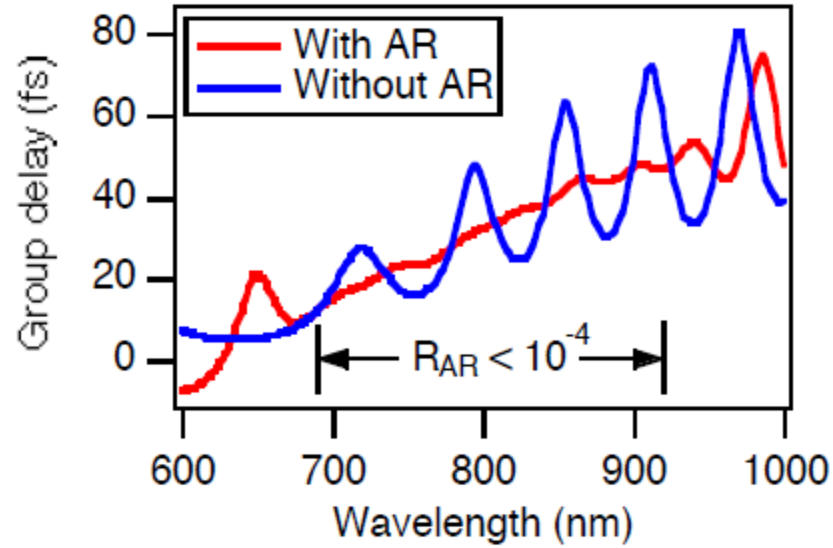


Calculated frequency-dependent reflectivity and group delay for 25-layer pair chirped mirrors with $n_H = 2.5$ and $n_L = 1.5$. The Bragg wavenumber $2\pi/\lambda_B$ is linearly chirped from $2\pi/(600\text{ nm})$ to $2\pi/(900\text{ nm})$ over the first 20 layer pairs, then held constant.

-- Dotted curve: a standard chirped mirror with d_H equal to a quarter-wave for all layers
-- Dashed curve: DCM with d_H linearly chirped over the first six layer pairs
-- Solid curve: DCM with d_H quadratically chirped over the first six layer pairs

Limitations of conventional DCMs





reduction of dispersion oscillations requires $R_{AR} < 10^{-4}$
$\Rightarrow$ limits bandwidth of direct approach to ≈ 250 nm

larger bandwidths by computer optimization of whole mirror structure, but...
$\Rightarrow$ dispersion oscillations strongly increase with bandwidth
$\Rightarrow$ increasing number of layers does not solve problem

"cannot make arbitrarily low reflectivity and arbitrarily broad bandwidth at the same time" J.A. Dobrowolski et al., *Appl. Opt.* **35**, 644 (1996)

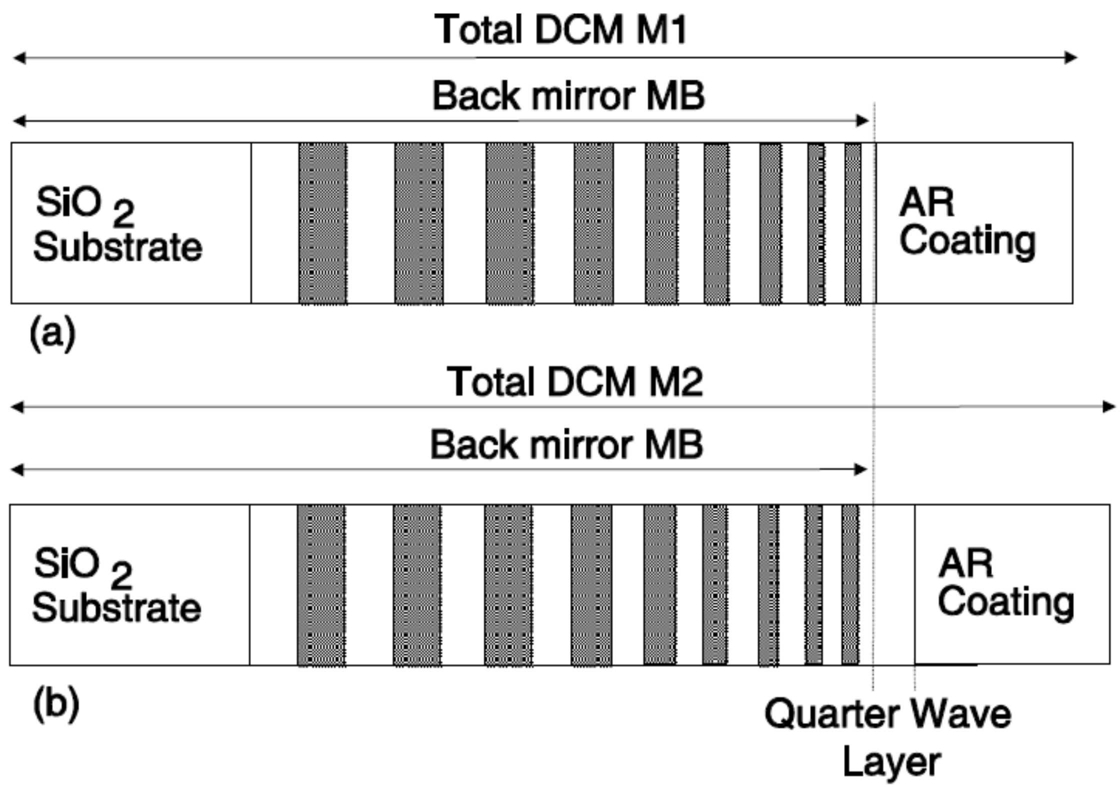
Adapted from U. Keller's Ultrafast Laser Physics course

Broadband DCM: DCM pair

The DCM M1 can be decomposed in a double-chirped back-mirror matched to a medium with the index of the top most layer.

In M2 a layer with a quarter wave thickness at the center frequency of the mirror and an index equivalent to the top most layer of the back-mirror MB is inserted between the back-mirror and the AR-coating.

The new back-mirror comprising the quarter wave layer can be re-optimized to achieve the same phase as MB with an additional π-phase shift over the whole octave of bandwidth.



DCM pair designed for Ti:Sapphire oscillator

Thick dash-dotted line with scale to the right: group delay design goal for perfect dispersion compensation of a prismless Ti:sapphire laser.

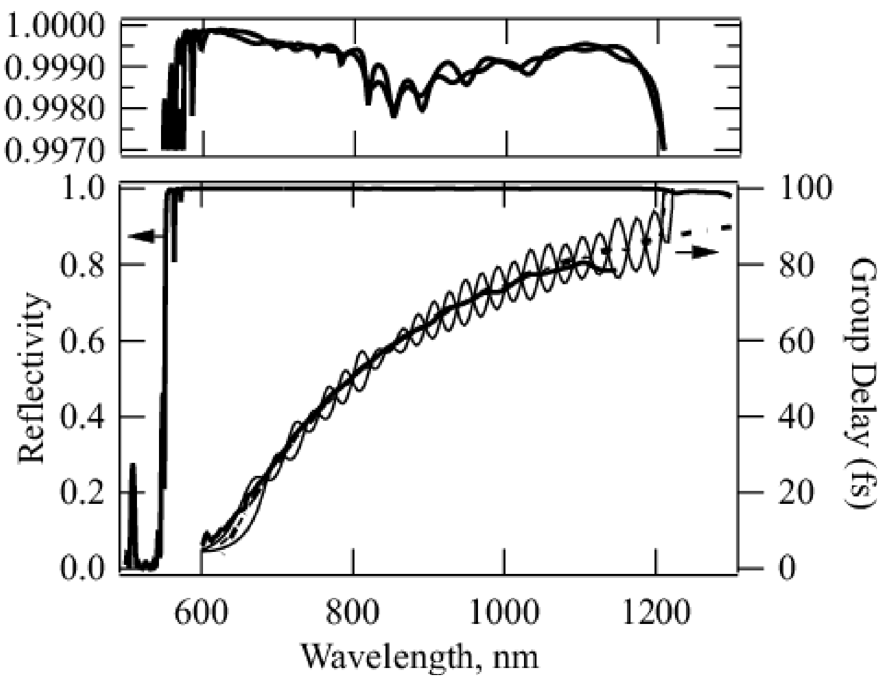
Thin line: individual group delay of the designed mirrors

Dashed line: average group delay of the two DCMs

Thick line: measured group delay from 600-1100 nm using white light interferometry

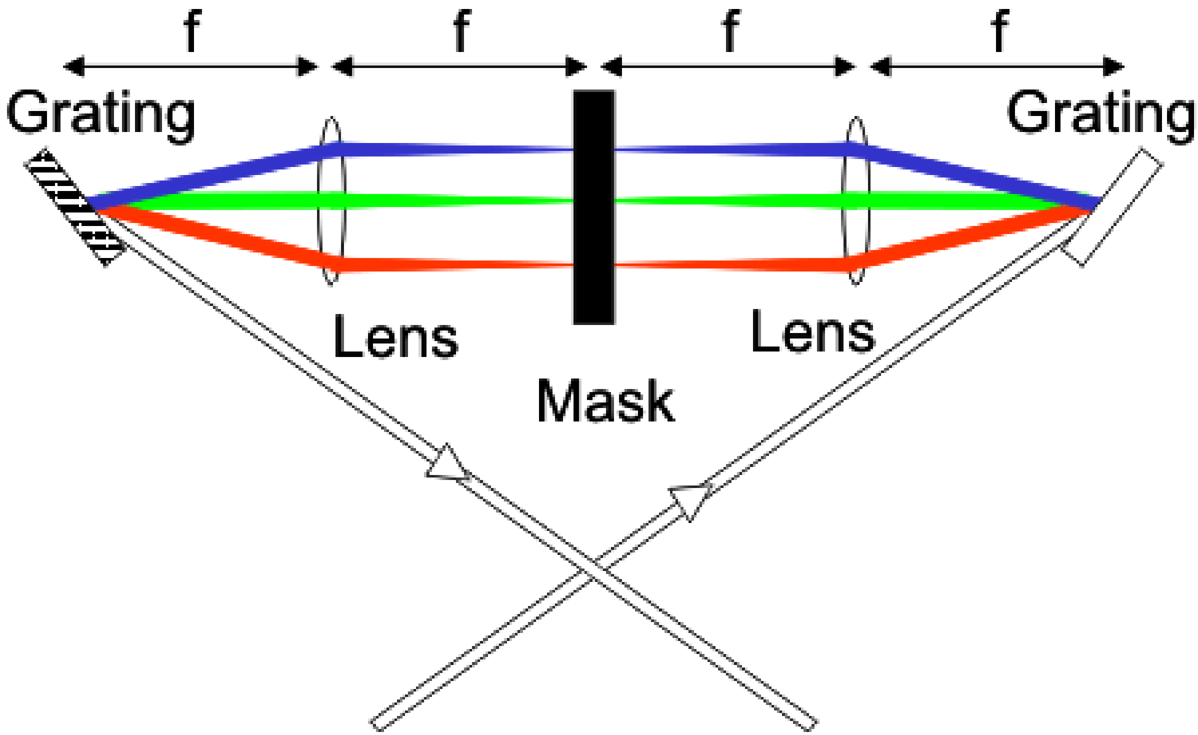
The average is almost identical with the design goal over the wavelength range form 650-1200 nm.

Beyond 1100nm the sensitivity of Si detector used prevented further measurements.



F. X. Kärtner et. al., "Ultrabroadband double-chirped mirror pairs for generation of octave spectra"J. of the Opt. Soc. of Am. 18, 882-885 (2001).

Active dispersion compensation: spatial light modulator

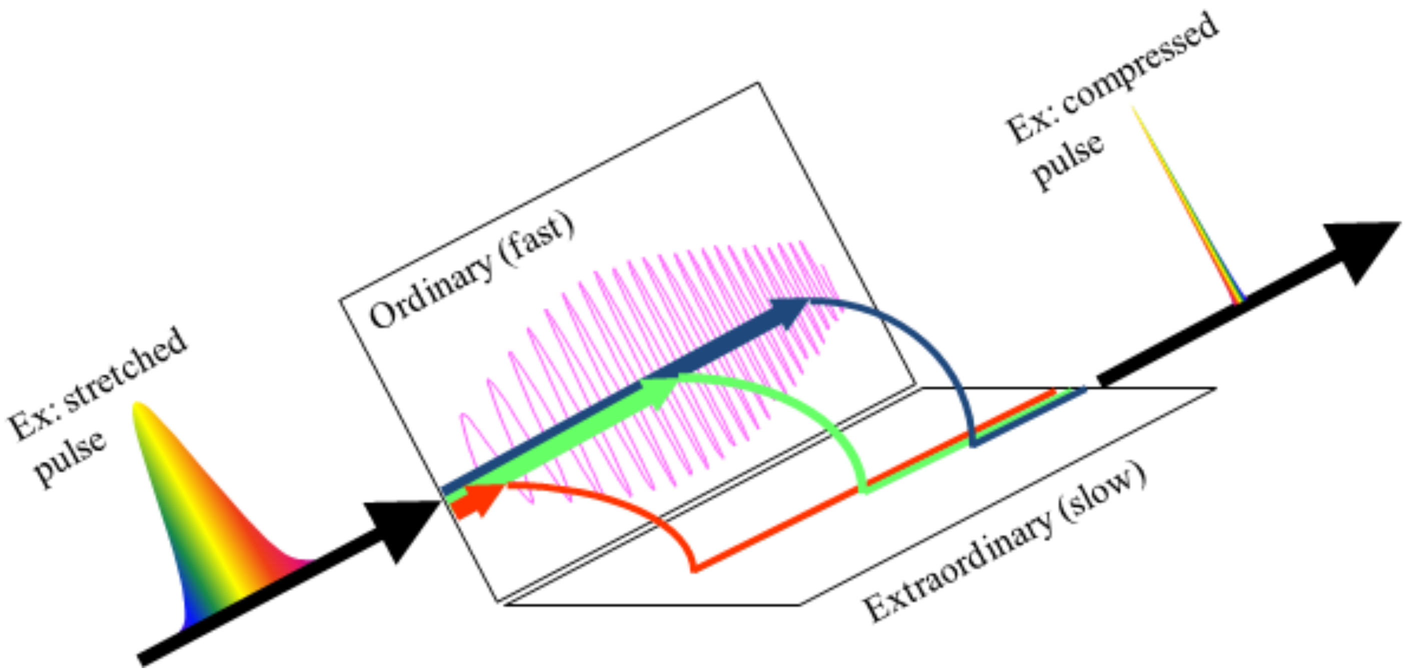


Dispersion Compensation with 4f-Pulse Shaper

Liquid crystal spatial light modulator (LCSLM) can be electronically controlled allowing programmable shaping of the pulse on a millisecond time scale.

A. M. Weiner, "Femtosecond pulse shaping using spatial light modulators" Rev. Sci. Instrum. 71, 1929 (2000).

Active dispersion compensation: AOPDF



Acousto-Optic Programmable Dispersive Filter (AOPDF), also known as Dazzler.

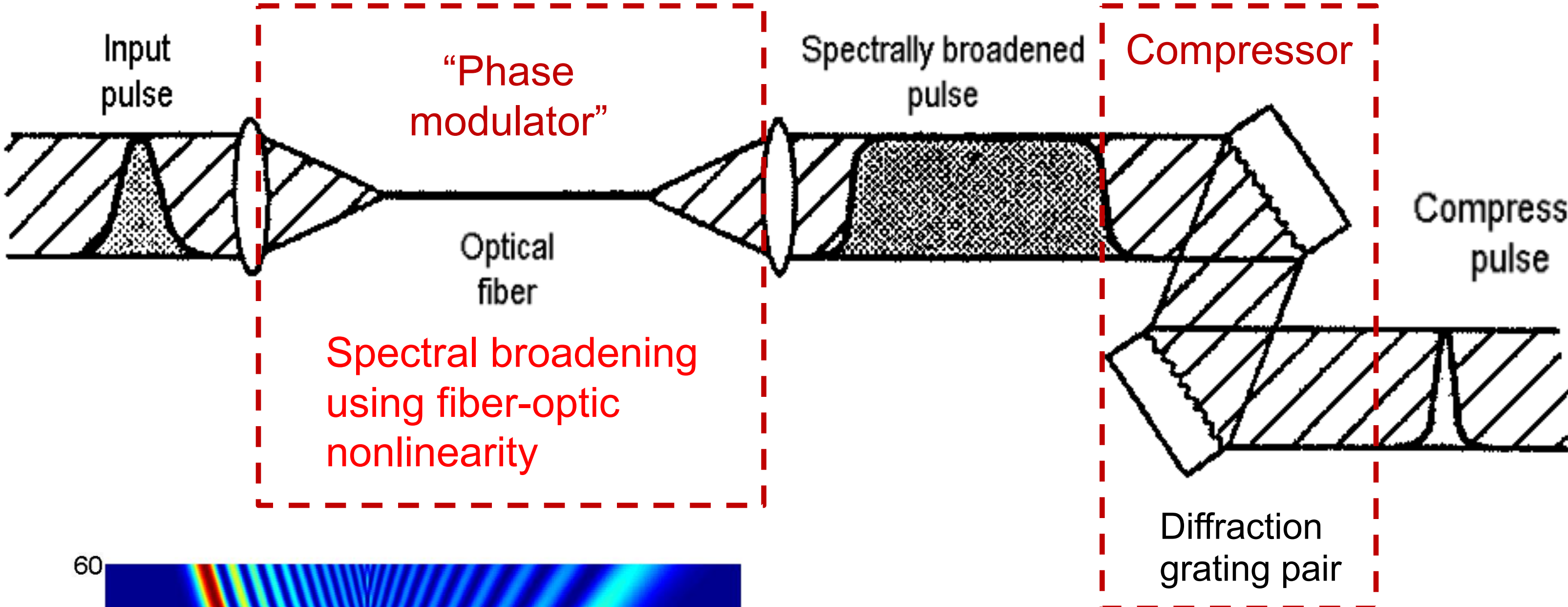
In an AOPDF, travelling acoustic wave induces variations in optical properties thus forming a dynamic volume grating.

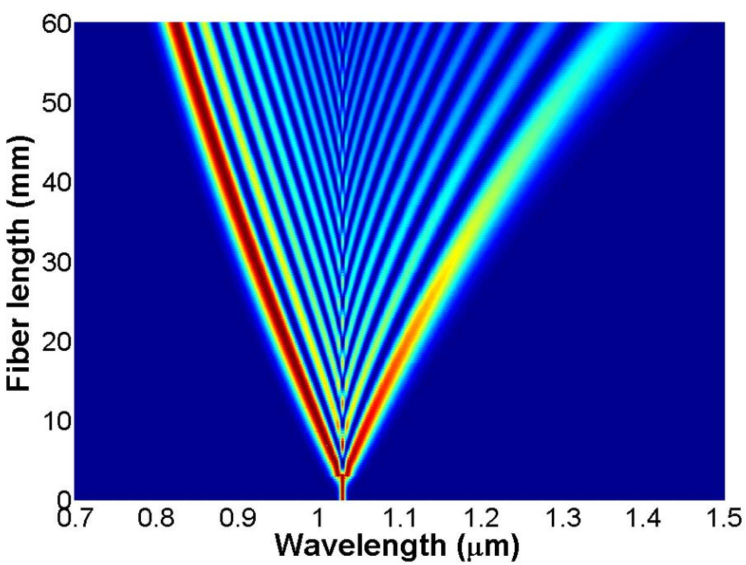
It is a programmable spectral filter, which can shape both the spectral phase and amplitude of ultrashort laser pulses.

Pierre Tournois, "Acousto-optic programmable dispersive filter for adaptive compensation of group delay time dispersion in laser systems," Optics Communications ***140****, 245 (1997).*

Pulse compression: general idea

Spectral broadening followed by dispersion compensation to compress (de-chirp) the pulse





$$T_g(\omega) = \phi'(\omega_0) + \phi''(\omega_0)\Delta\omega + \frac{1}{2}\phi'''(\omega_0)\Delta\omega^2 + \frac{1}{3!}\phi''''(\omega_0)\Delta\omega^3 + \cdots$$

$$\phi''(\omega_0) = \phi''_{modulator} + \phi''_{compresssor} = 0$$

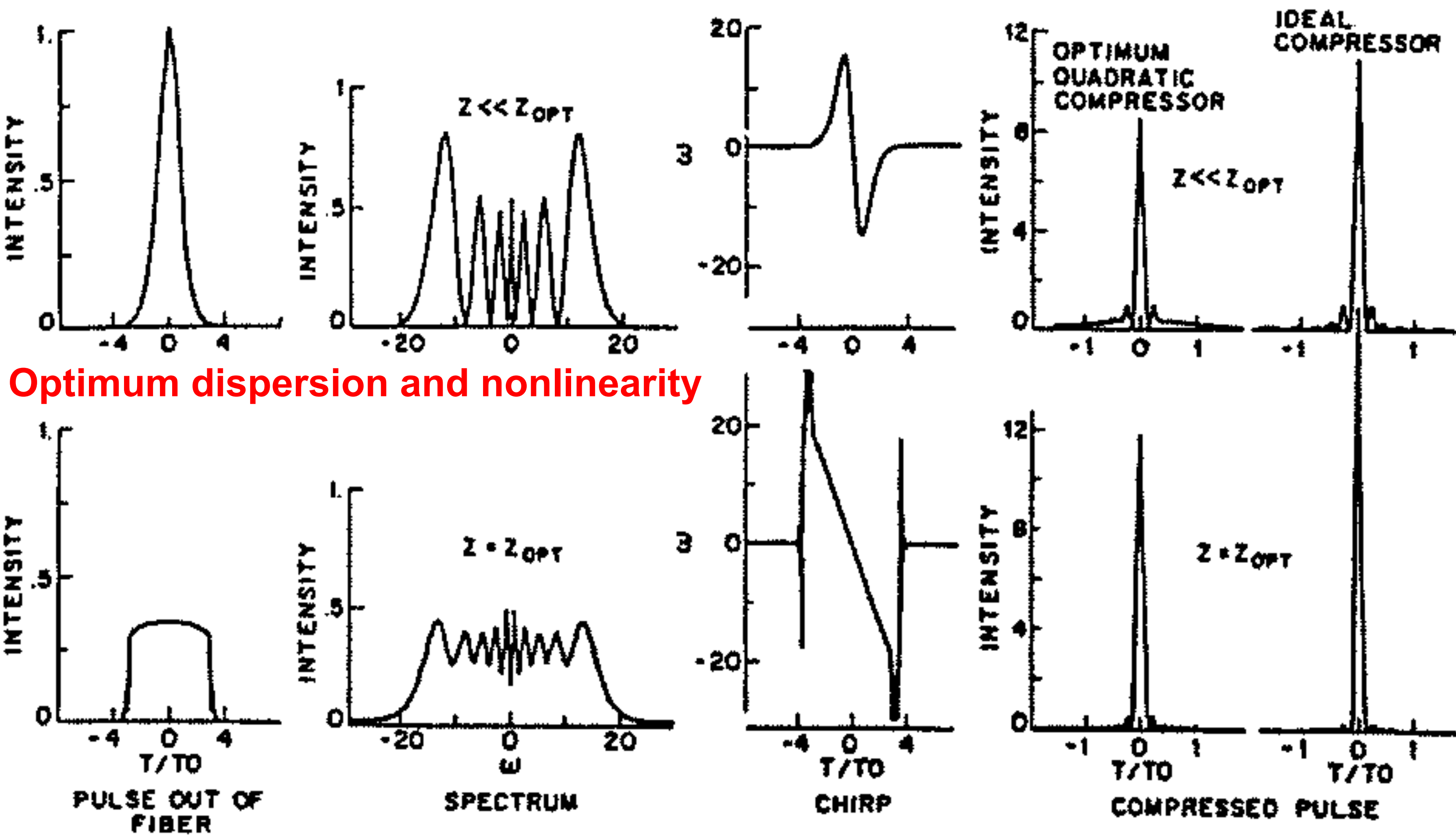
$$\phi'''(\omega_0) = \phi'''_{modulator} + \phi'''_{compresssor} = 0$$

Variable dispersion by the grating pair.

Dispersion matters in spectral broadening

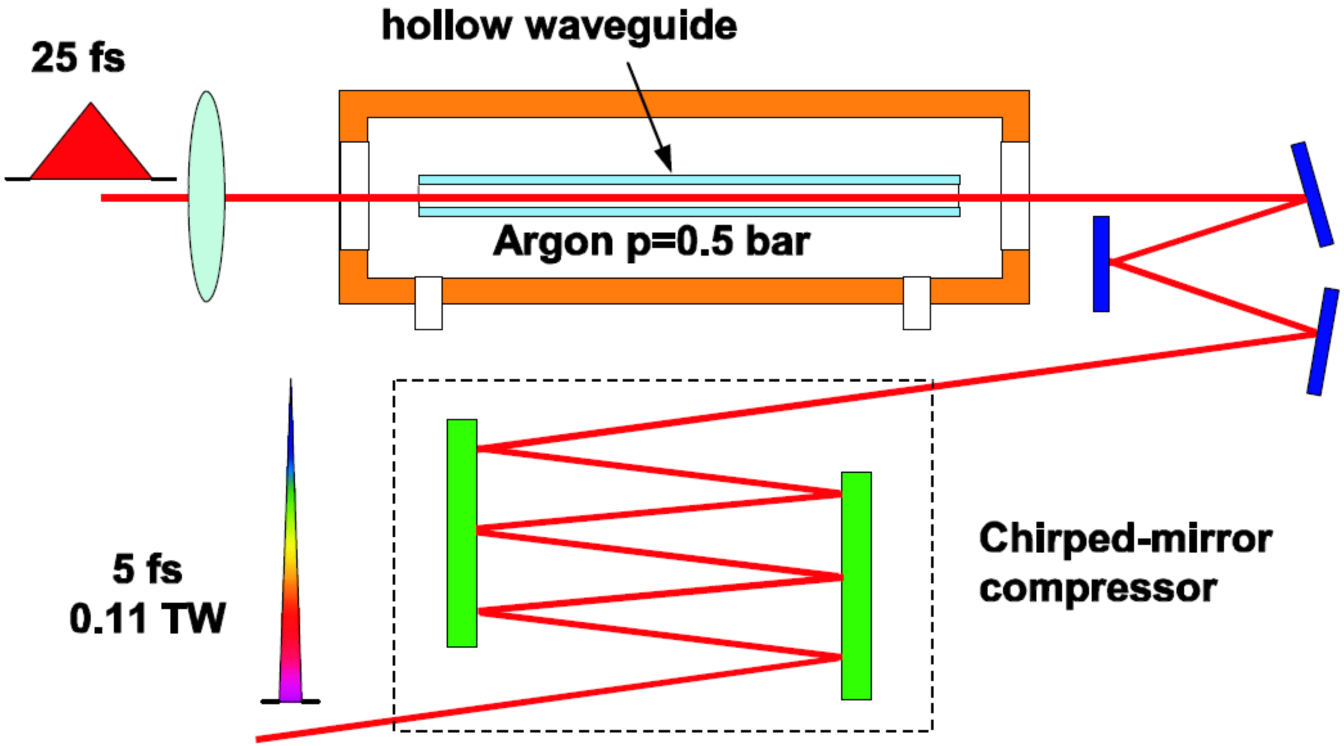
Dispersion negligible using short fiber, SPM dominates

Optimum dispersion and nonlinearity



Hollow fiber compression of mili-joule pulses

Self focusing threshold in fused silica is 4 MW. For ~100 fs pulse, the pulse energy allowed in a fused silica fiber is ~400 nJ before fiber breakdown.



The modes of the hollow fiber are leaky modes, i.e. they experience radiation loss. However, the EH_{11} mode has considerably less loss than the higher order modes and is used for pulse compression. The nonlinear index in the fiber can be controlled with the gas pressure. Typical fiber diameters are 100-500 μm and typical gas pressures are in the range of 0.1-3 bar.