Ultrafast Optical Physics II (SoSe 2019)

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Office hour: Tuesday, 11-12 am

Lectures: Fr 08.30 - 10.00 + 10.30 - 11.15 SemRm 4, Jungiusstr. 9
Recitations: Fr 11.15 - 12.00 SemRm 4, Jungiusstr. 9
Start: 05.04.2019
Teaching Assistants:
Dr. Andrea Trabattoni, Office O3.114, phone 040 8998 -6048,
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E-mail: andrea.cartella@cfel.de
Office hour: Thursday 9:30-11am
Giulio Maria Rossi, Office O3.125, phone 040 8998 -6383,
E-mail: giulio.maria.rossi@cfel.de
Course Secretary: Uta Freydank
O3.095, phone x-6351, E-mail: uta.freydank@cfel.de.

Class website:
https://ufox.cfel.de/teaching/summer_semester_2019/

Prerequisites: Ultrafast Optical Physics I or basic course in Electrodynamics
Required Text: Class notes will be distributed in class.
Requirements: 8 Problem Sets, Term Paper, and Term paper presentation
Collaboration on problem sets is encouraged.
Grade breakdown: Problem set (30%), Participation (30%), Term paper and presentation (40%)
Recommended Text: Ultrafast Optics, A. M. Weiner, Hoboken, NJ, Wiley 2009.

Additional References:

- Waves and Fields in Optoelectronics, H. A. Haus, Prentice Hall, NJ, 1984
- Ultrashort laser pulse phenomena: fundamentals, techniques, and applications on a femtosecond time scale, J.-C. Diels and W. Rudolph, Academic Press, 2006.
- Principles of Lasers, O. Svelto, Plenum Press, NY, 1998.
- Optical Resonance and Two-Level Atoms, L. Allen & J. H. Eberly, J. Wiley, NY, 1975.
- Fundamentals of Attosecond Science, Z. Chang, CRC Press, (2011).
- Nonlinear Optics, R. Boyd, Elsevier, Academic Press, (2008).
- Prof. Rick Trebino's course slides on ultrafast optics:
  http://frog.gatech.edu/lectures.html

Tentative Schedule

1	Kärtner 05/04/2019	**Introduction to Ultrafast Optics**
2		**Optical Pulses and Dispersion**
3	Kärtner 12/04/2019	**Linear Pulse Propagation** *Problem Set 1 Out*
4		**Nonlinear Pulse Propagation**
5	Calegari 26/04/2019	**Nonlinear Schrödinger Equation (NLSE)** *Problem Set 1 Due, Problem Set 2 Out*
6		**Pulse Compression and Dispersion Compensation Techniques**
7	Kärtner 03/05/2019	**Review of Quantum Mechanics** *Problem Set 2 Due, Problem Set 3 Out*
8		**Two-Level System and Maxwell-Bloch Equations**
9	Kärtner 10/05/2019	**Laser Rate Equations** *Problem Set 3 Due, Problem Set 4 Out*
10		**Laser CW-Operation and Q-Switching** *Distribute Term Paper Proposals*

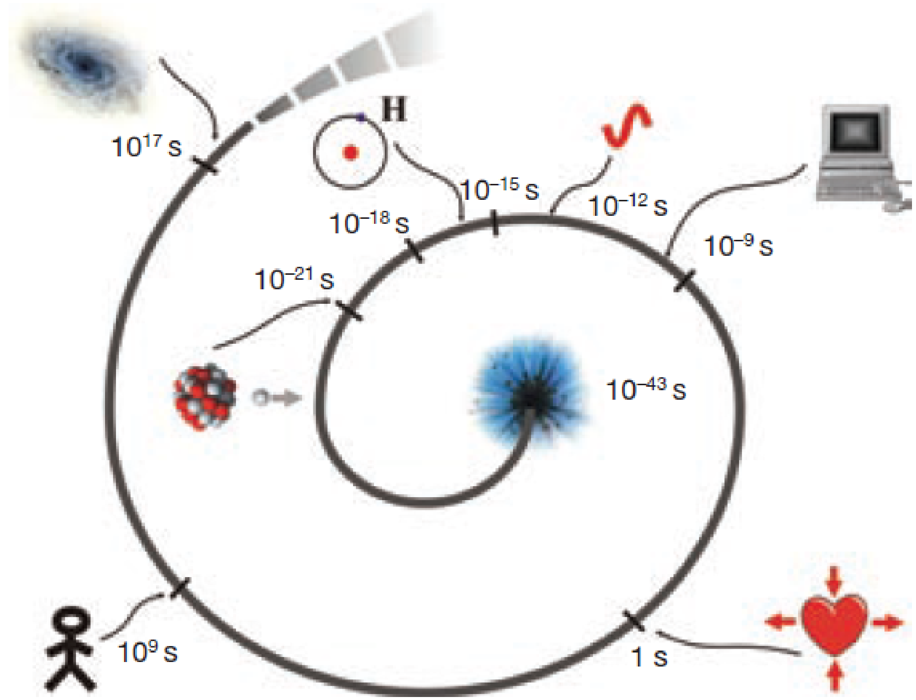
Tentative Schedule

11	Kärtner 17/05/2019	**Master Equation** *Problem Set 4 Due, Problem Set 5 Out*
12		**Active Mode-Locking**
13	Kärtner 24/05/2019	**Passive Mode-Locking with Saturable Absorbers** *Problem Set 5 Due, Problem Set 6 Out*
14		**Noise in Mode-Locked Lasers**
15	Kärtner 31/05/2019	**Femtosecond Laser Frequency Combs** *Problem Set 6 Due, Problem Set 7 Out*
16		**Pulse Amplification**
17	Calegari 07/06/2019	**Pulse Characterization I – Autocorrelation** *Problem Set 7 Due, Problem Set 8 Out*
18		**Pulse Characterization II – FROG**
19	Calegari 21/06/2019	**Second-Order Nonlinear Effects** *Problem Set 8 due*
20		**Optical Parametric Amplification**

Tentative Schedule

21	Calegari 28/06/2019	**High Harmonic Generation**
22		**Attosecond Science**
23	Calegari 05/07/2019	**Ultrafast X-Ray Sources**
24		Lab Tour: Ultrafast Optics and X-Rays Group and Attosecond Science Group
25	Kärtner Calegari 13/07/2019	**Term Paper Presentation**

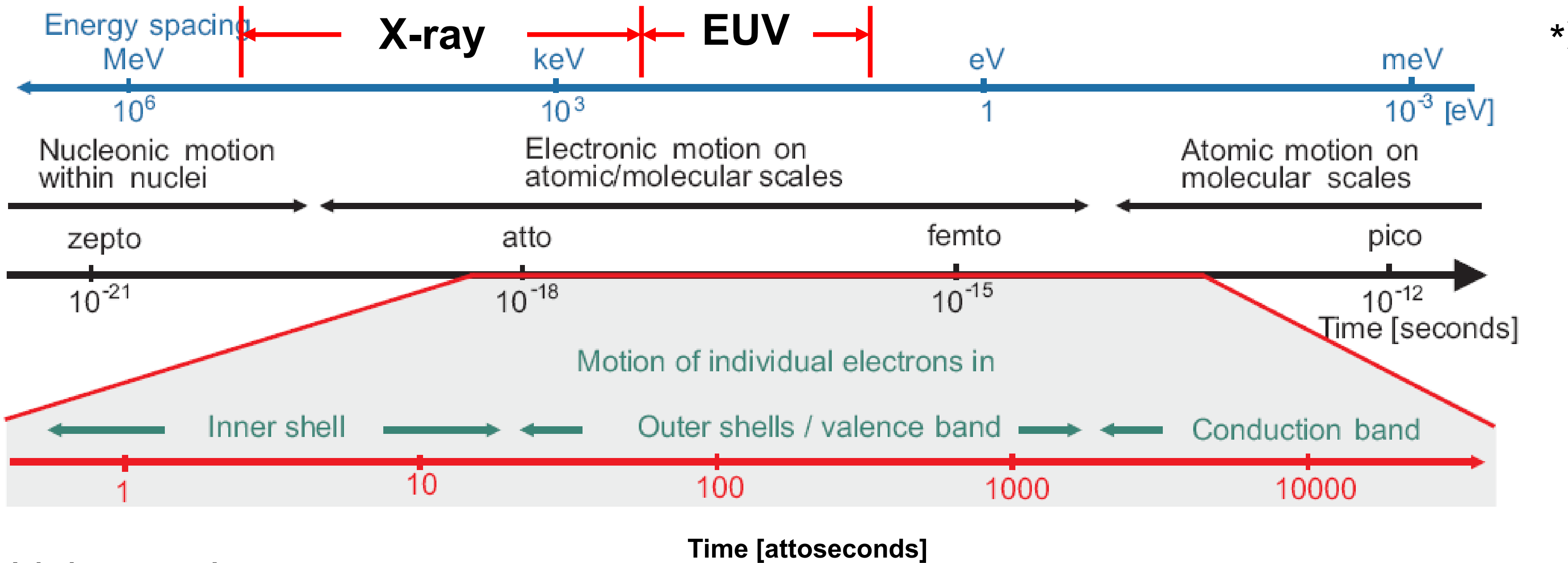
The long and short of time



Notable time scales, in seconds		Length by other measures	
5×10^{17} s	Estimated age of universe	14	billion years
2×10^{9} s	Average human lifetime	70	years
1 s	Length of a heartbeat	1	second
0.3×10^{-9} s	Current computer clock frequency	0.3	nanosecond
10^{-12} s	Length of a typical THz pulse	1	picosecond
3×10^{-15} s	Cycle length of laser	3	femtoseconds
1.5×10^{-16} s	Electron circles proton in Hydrogen atom	0.15	femtosecond
10^{-18} s	Next horizon for controllable laser pulses?	1	attosecond
10^{-21} s	Strong nuclear reactions	1	zeptosecond
10^{-43} s	Birth flash of the Big Bang		Planck time

A. E. Kaplan, "No-when: the long and short of time," Optics and Photonics News (2006, February)

Physics on femto- attosecond time scales?



Light travels:

A second: from the moon to the earth

A picosecond: a fraction of a millimeter, through a blade of a knife

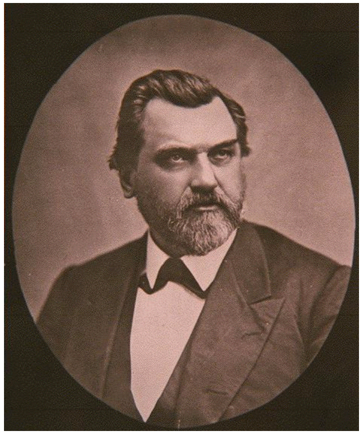
A femtosecond: the period of an optical wave, a wavelength

An attosecond: the period of X-rays, a unit cell in a solid

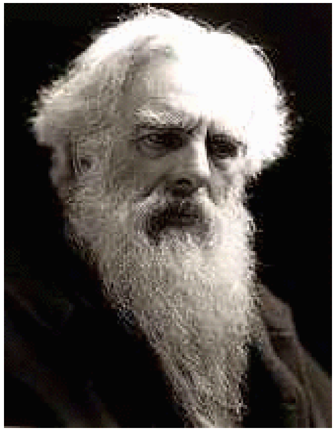
***F. Krausz and M. Ivanov, Rev. Mod. Phys. 81, 163 (2009)**

Birth of ultrafast technology

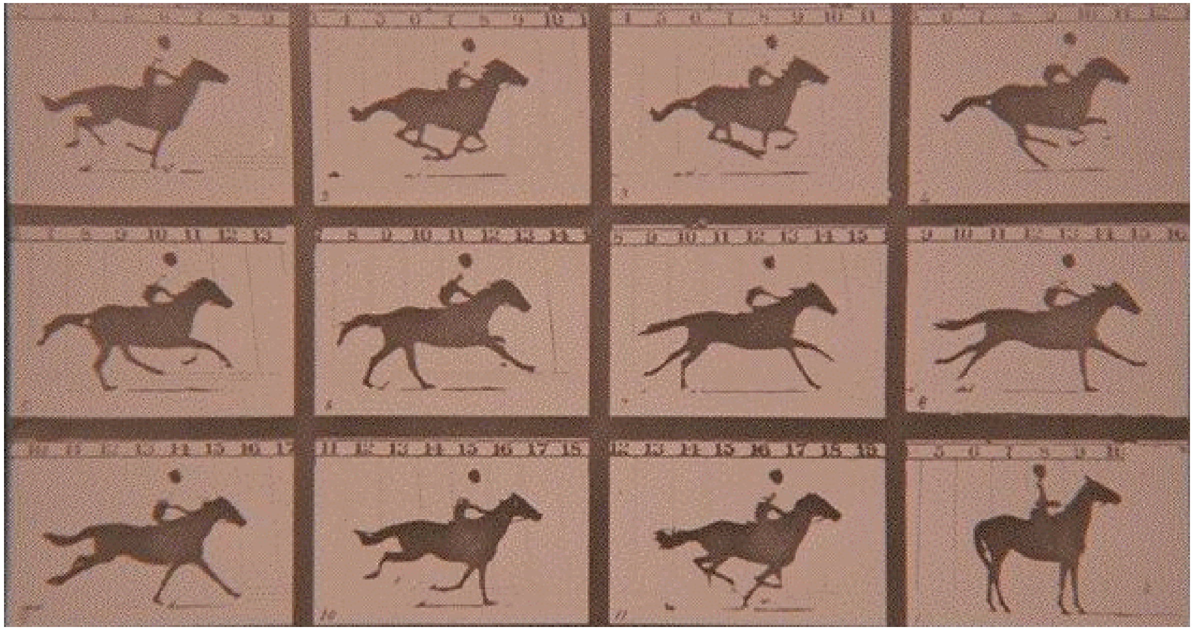
$25,000 bet: Do all four hooves of a running horse ever simultaneously leave the ground? (1872)



Leland Stanford

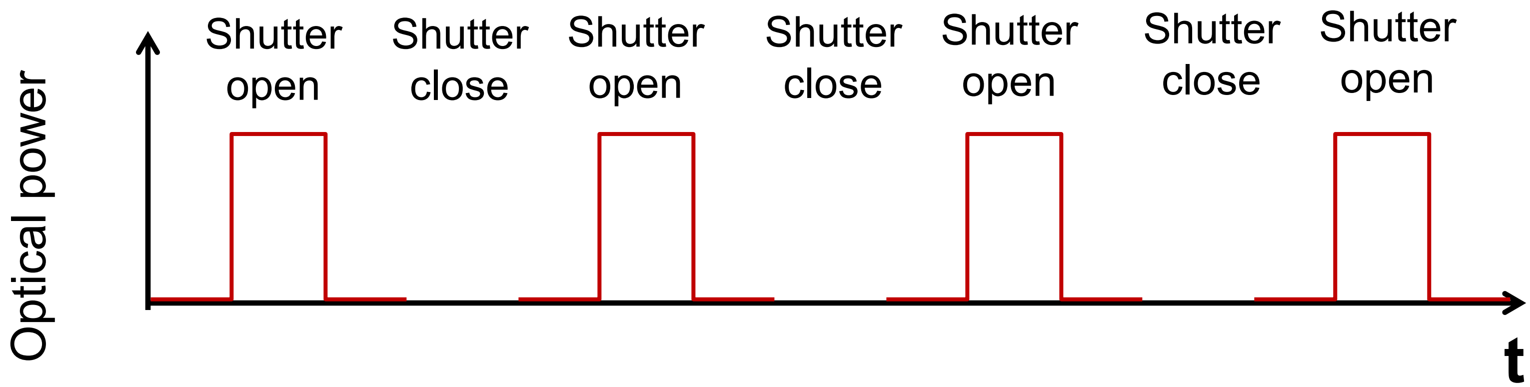


Eadweard Muybridge



What do we need to probe a fast event?

- The light signal received by the camera film is a train of optical pulse.



- We need a FASTER event to freeze the motion. Here the FASTER event is shutter opening and closing.

- If we have an optical pulse source, we can record images of a running horse in a dark room.

Early history of lasers

- 1917: *on the quantum theory of radiation* – Einstein's paper
- 1954: MASER by Charles Townes (1915—2015) *et al*.
- 1958: Charles Townes (Nobel Prize in 1964) and Schawlow (Nobel Prize in 1981) conceived basic ideas for a laser.

If you're a nobel prize winner, and 100 years old, you can comment other winners using harsh words:

Charles Townes

University of California, Berkeley, and 1964 Nobel Prize in Physics recipient

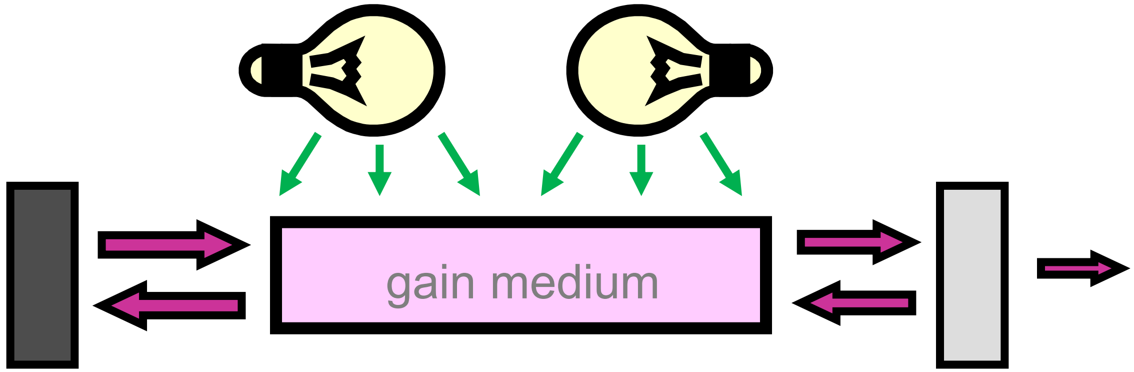
Jim Gordon was a fine person and a great scientist. He was also brave in doing research. When he worked for me as a graduate student trying to build the first maser, the chairman of the physics ~~department and the previous chairman both told him it would not work~~ and that he should stop, because the project was wasting the department's money. Both of them had Nobel Prizes, so presumably weren't stupid physicists. But Jim proceeded with his work and, about four months after they told him it wouldn't work, it did. From the maser also came the laser.

Jim didn't get the Nobel Prize with me, presumably because he was a student when the maser first worked, but I think he deserved it. He went on to do other important work. We should all celebrate him and his contributions.

Optics & Photonics News, 2014

MASER: Microwave Amplification by Stimulated Emission of Radiation
(Means of Acquiring Support for Expensive Research)

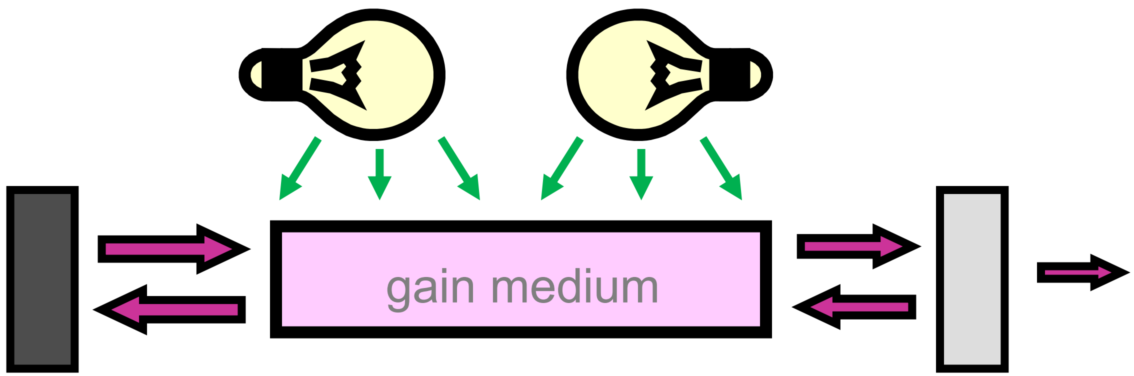
Laser basics: three key elements



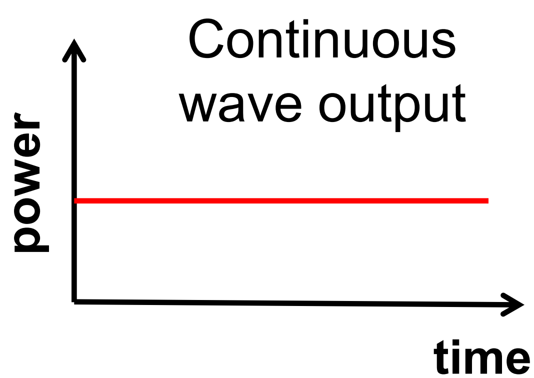
Continuous wave (CW) laser

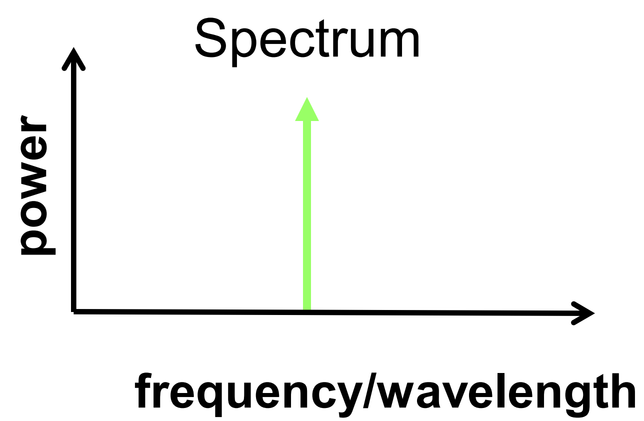
- **Gain medium**
  - Enable stimulated emission to produce identical copies of photons
  - Determine the light wavelength
- **Pump**
  - Inject power into the gain medium
  - Achieve population inversion
- **Resonator cavity**
  - make light wave oscillating to efficiently extract energy stored in the gain medium
  - Improve directionality and color purity of the light

Laser basics: three key elements

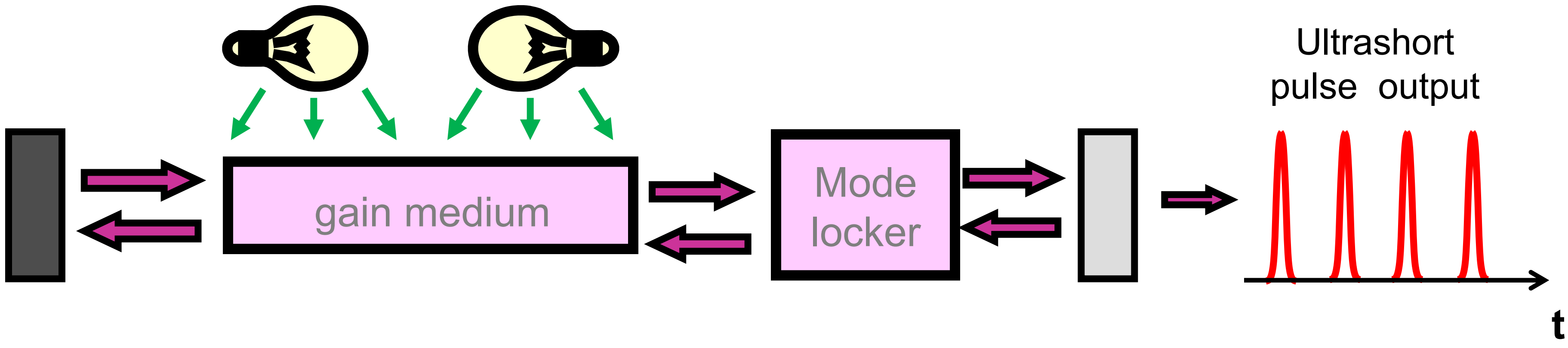


Continuous wave (CW) laser

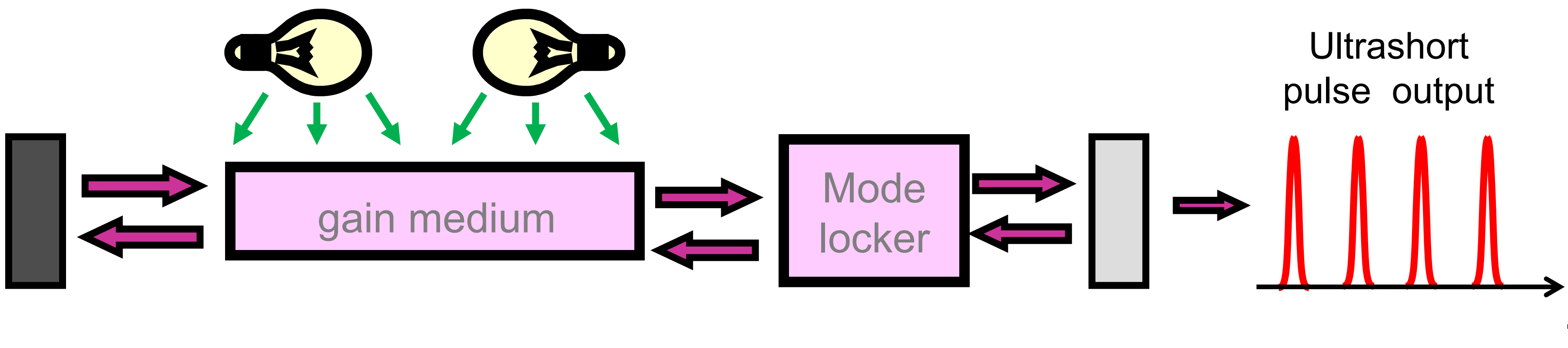


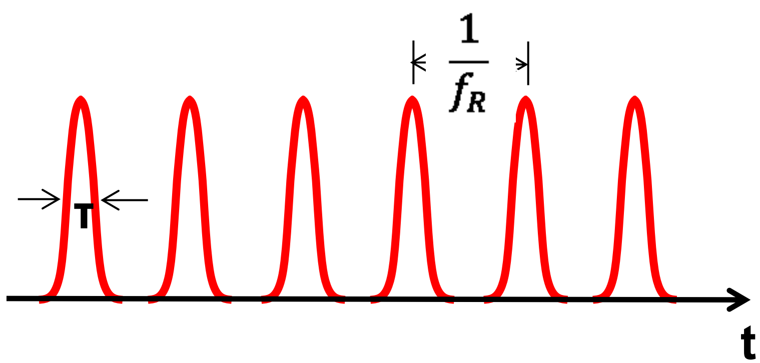


Ultrafast laser: the 4th element—mode locker



Ultrafast laser: the 4th element—mode locker

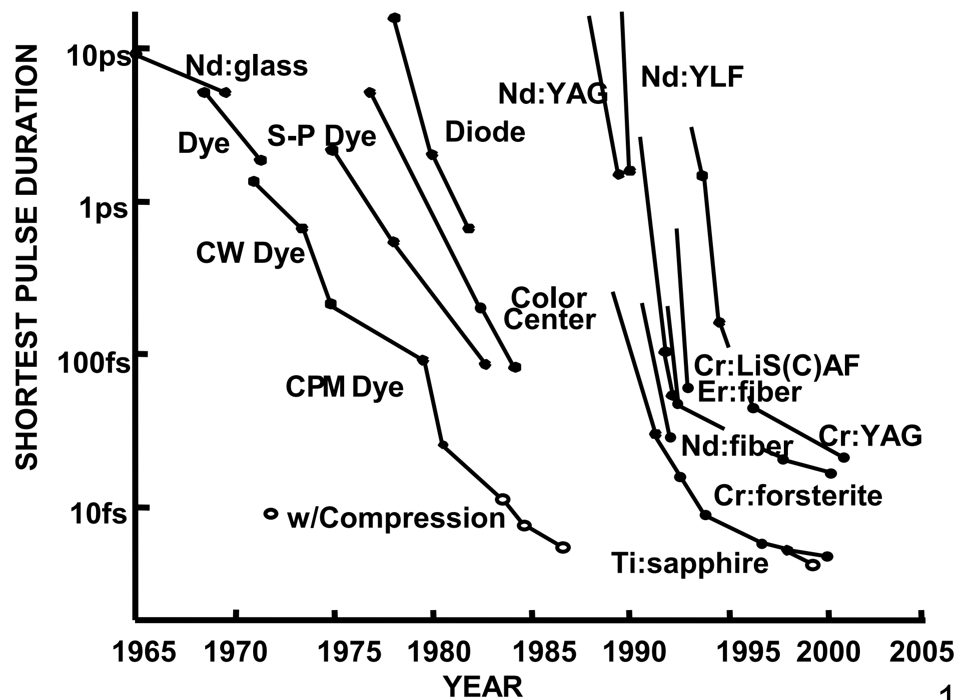




World shortest pulse: 67 attoseconds. The center wavelength is 20 nm. It is generated by high harmonic generation.

K. Zhao et al., "Tailoring a 67 attosecond pulse through advantageous phase-mismatch," Opt. Lett. 37, 3891 (2012)

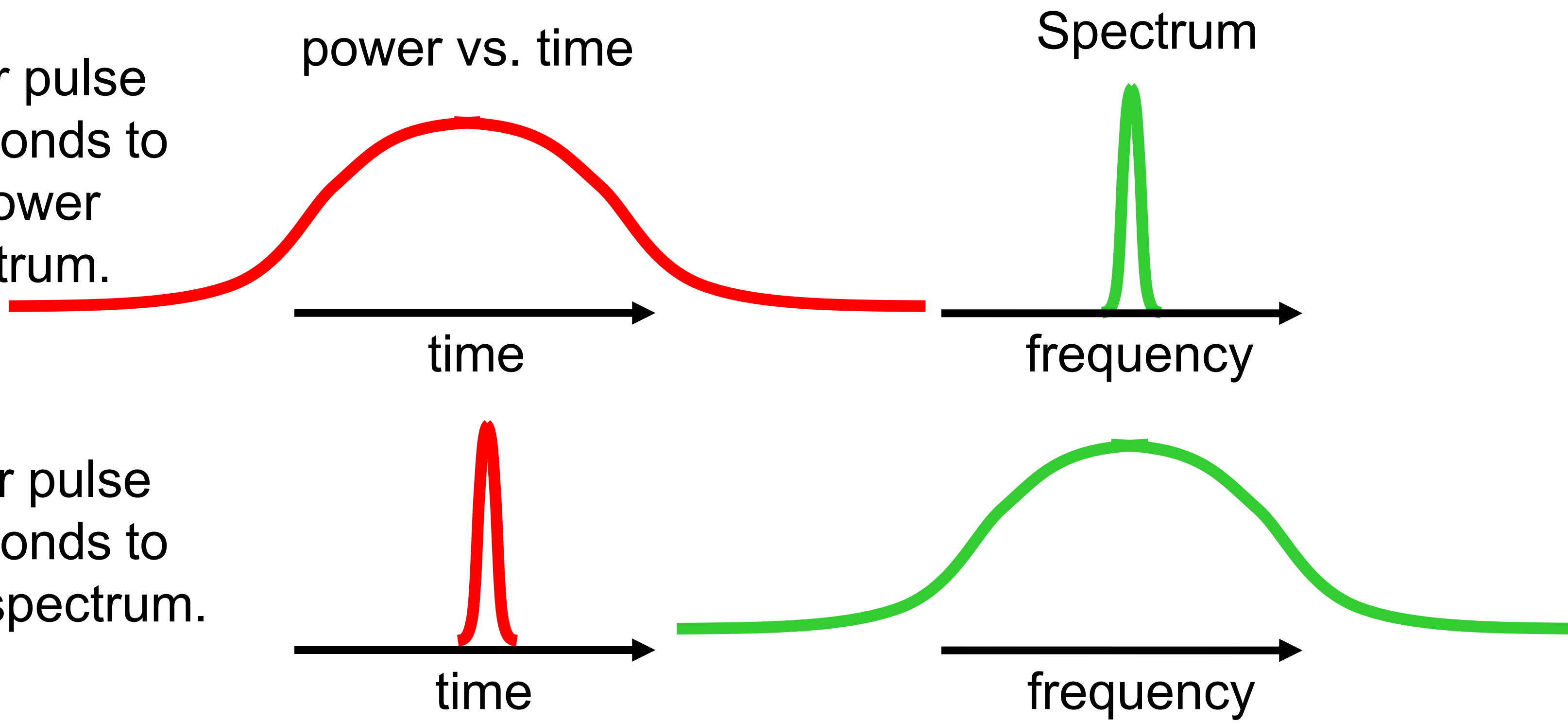
- Pulse duration T (fs – ps)



Long vs. short pulses of light

Longer pulse corresponds to narrower spectrum.

Shorter pulse corresponds to broader spectrum.



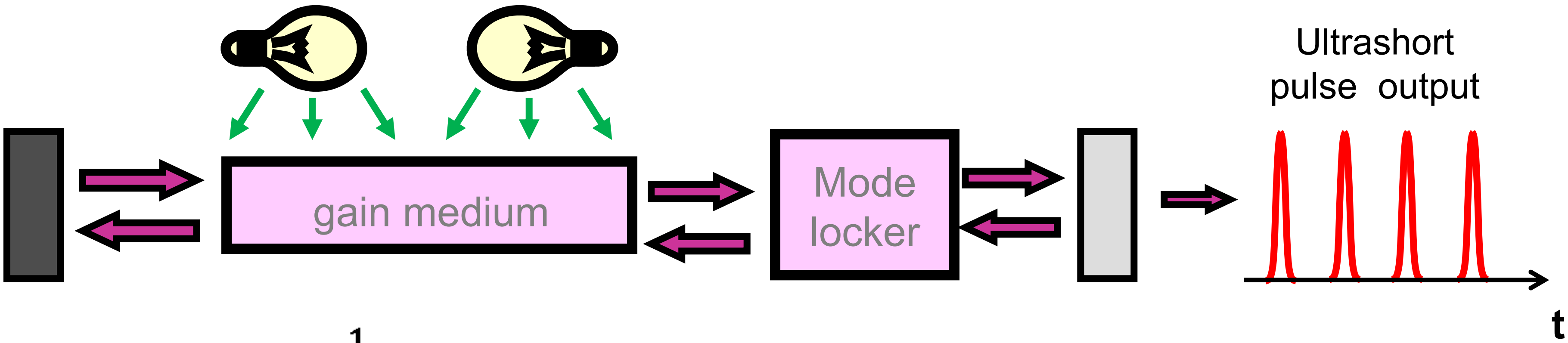
But a light bulb is also broadband.

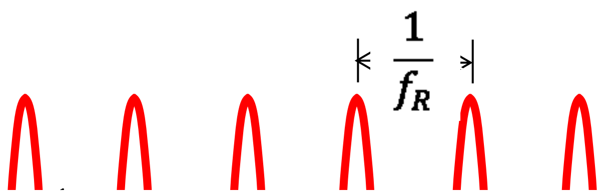
What exactly is required to make an ultrashort pulse?

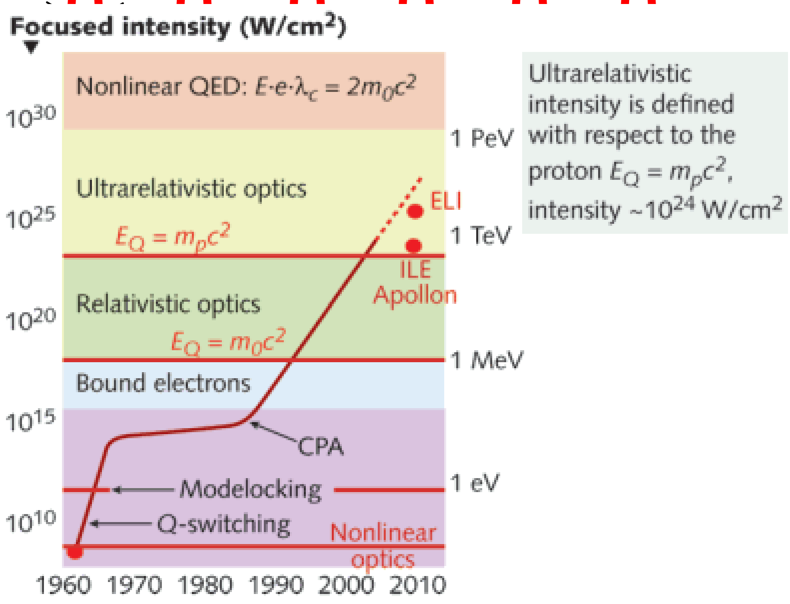
Answer: A Mode-locked Laser

Adapted from Rick Trebino's course slides

Ultrafast laser: the 4th element—mode locker

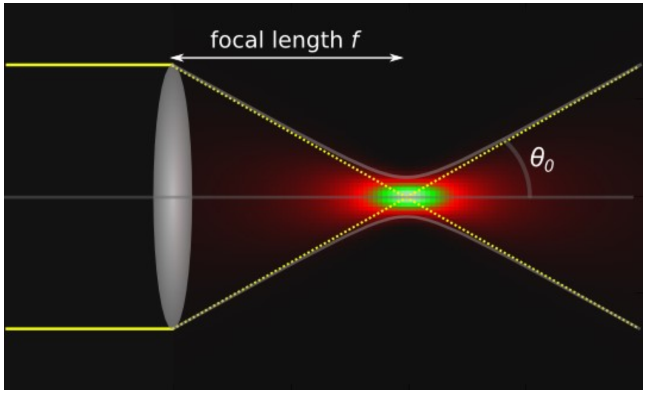




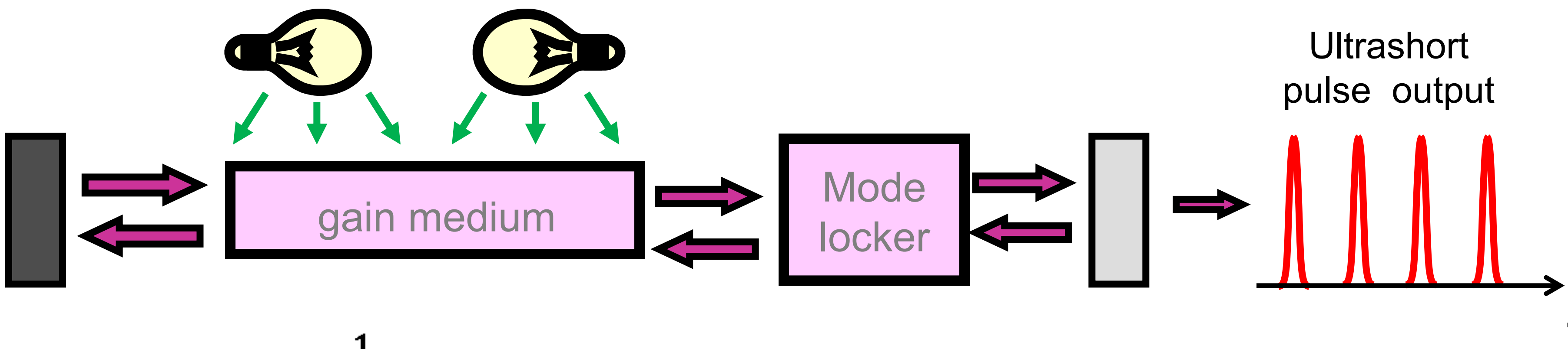


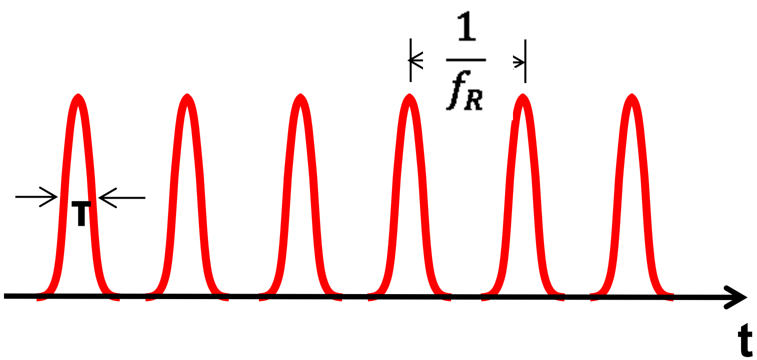
- <u>Pulse duration T (fs – ps)</u>
- <u>Pulse energy E (pJ – mJ)</u>
- <u>Peak power P_p (1 kW – 1 PW)</u>

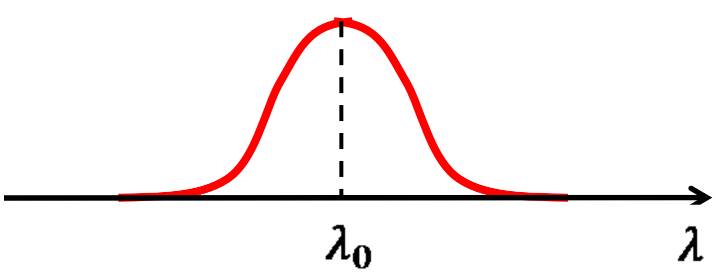
  $P_p \approx E/T$ (e.g., 1 nJ, 100 fs pulse leads to 10 kW peak power.)



Ultrafast laser: the 4th element—mode locker







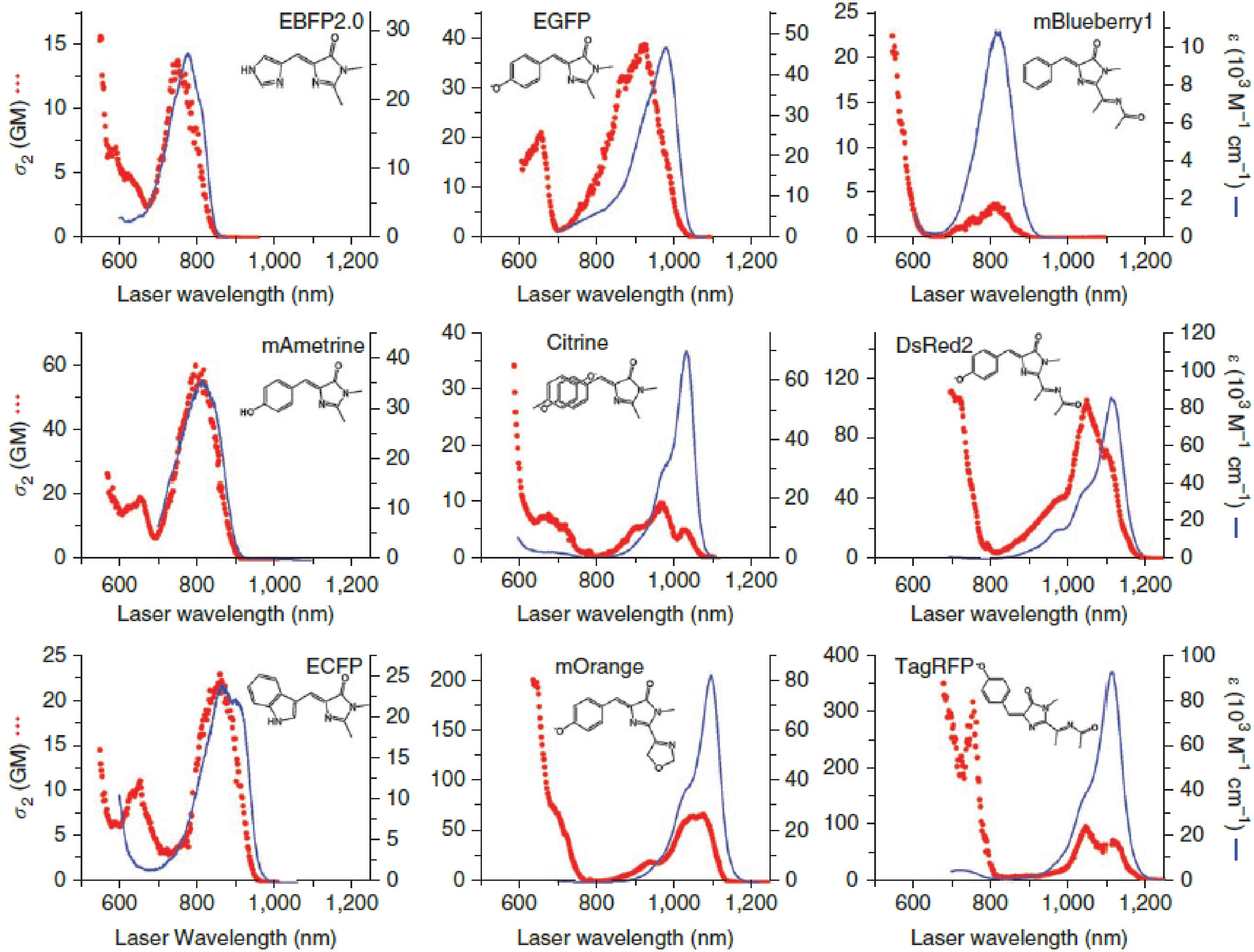
- Pulse duration T (fs – ps)
- Pulse energy E (pJ – mJ)
- Peak power P_p (1 kW – 1 PW)
  $P_p \approx E/T$ (e.g., 1 nJ, 100 fs pulse leads to 10 kW peak power.)
- Repetition rate f_R (10 MHz – 10 GHz)
- Average power P (10 mW – 100 W)
  $P = E \times f_R$ (e.g., 1 nJ, 100 MHz rep-rate laser produces 100 mW average power.
- Center wavelength λ_0 (700 nm – 2000 nm)

Examples of ultrafast solid-state laser media

Solid-state laser media have broad bandwidths and are convenient.

Laser Materials	Absorption Wavelength	Average Emission λ	Band Width	Pulse Width
Nd:YAG	808 nm	1064 nm	0.45 nm	$\sim$ 6 ps
Nd:YLF	797 nm	1047 nm	1.3 nm	$\sim$ 3 ps
Nd:LSB	808 nm	1062 nm	4 nm	$\sim$ 1.6 ps
Nd:YVO_4	808 nm	1064 nm	2 nm	$\sim$ 4.6 ps
Nd:fiber	804 nm	1053 nm	22-28 nm	$\sim$ 33 fs
Nd:glass	804 nm	1053 nm	22-28 nm	$\sim$ 60 fs
Yb:YAG	940, 968 nm	1030 nm	6 nm	$\sim$ 300 fs
Yb:glass	975 nm	1030 nm	30 nm	$\sim$ 90 fs
Ti:Al_2O_3	480-540 nm	796 nm	200 nm	$\sim$ 5 fs
Cr^{4+}:Mg_2SiO_4:	900-1100 nm	1260 nm	200 nm	$\sim$ 14 fs
Cr^{4+}:YAG	900-1100 nm	1430 nm	180 nm	$\sim$ 19 fs

Two-photon absorption properties of fluorescent proteins



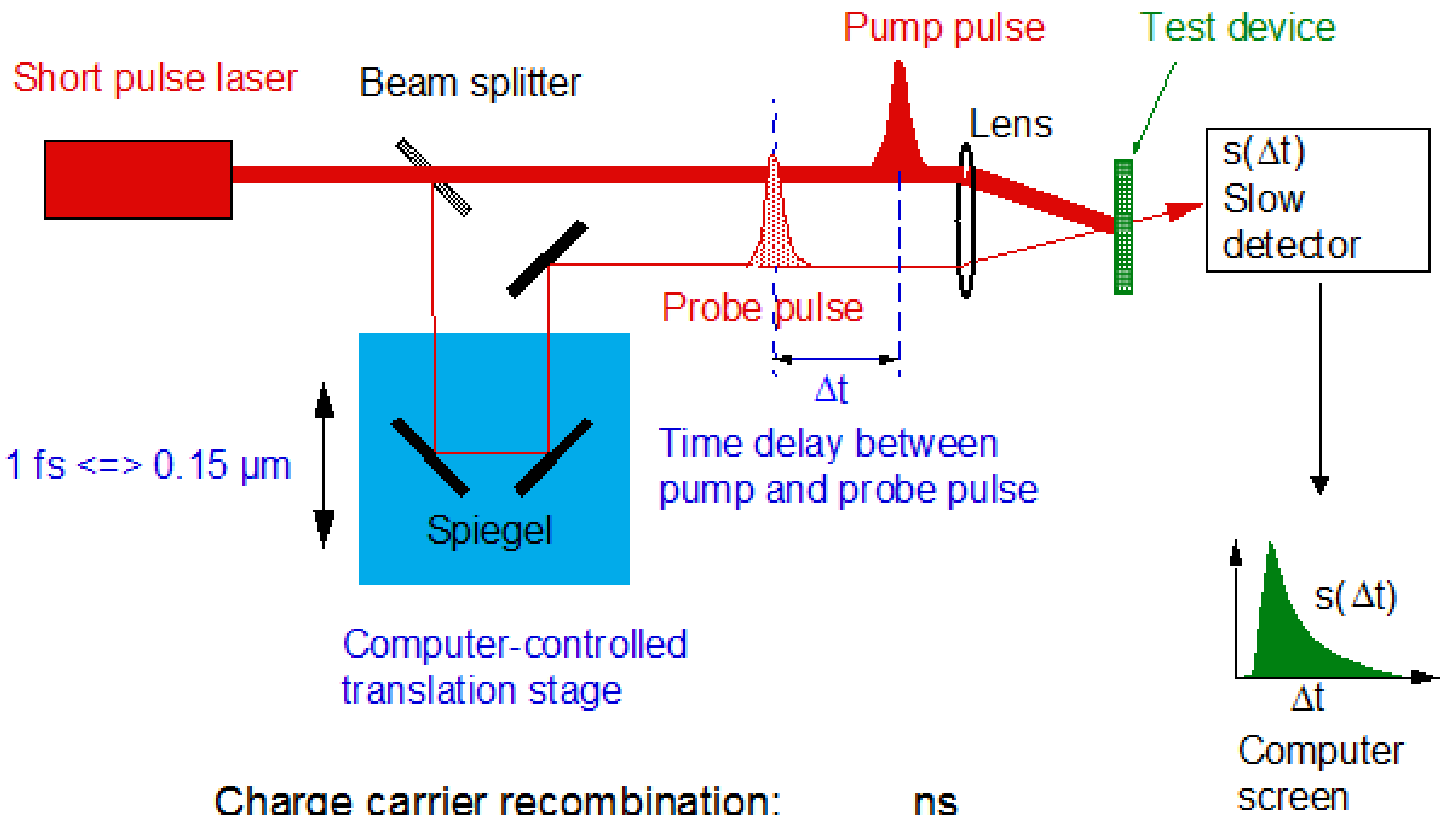
M. Drobizhev et al., "Two-photon absorption properties of fluorescent proteins" Nat. Methods 8, 393 (2011).

Main workhorse: Ti:sapphire oscillator

Typical parameters of a commercial product

- Pulse duration: ~100 fs
- Pulse energy: 1-10 nJ
- Pulse rep-rate: 50-100 MHz
- Average power: 300-1000 mW
- Center wavelength: tunable in 700-1000 nm.

Ultrafast: pump-probe spectroscopy

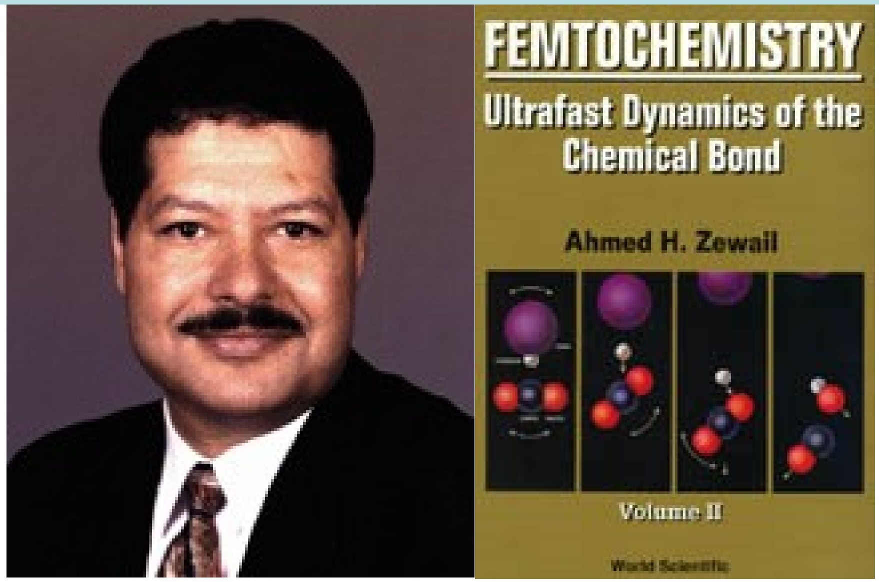


Charge carrier recombination: ns

Thermalization electrons with lattice: ps

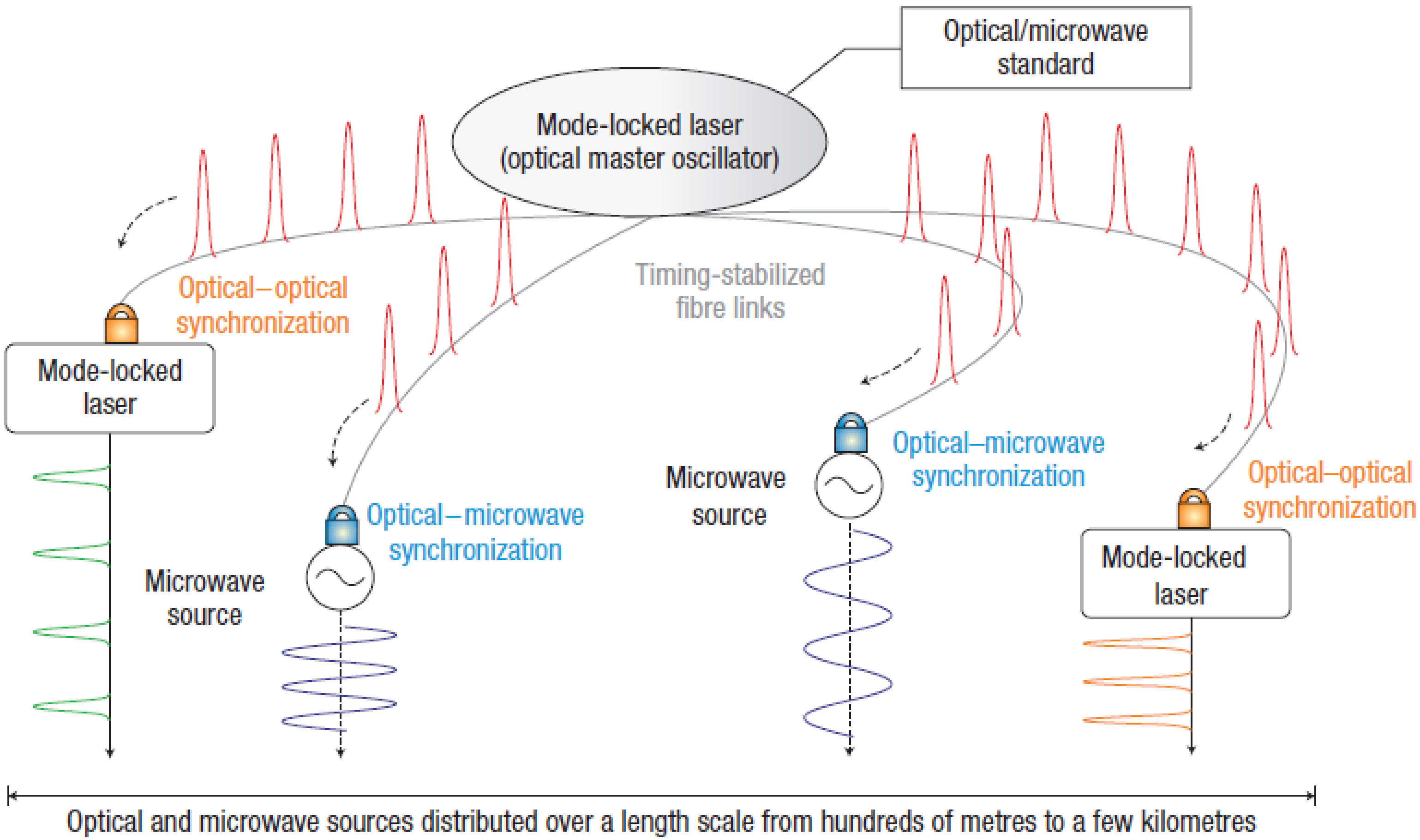
Thermalization electron gas: 10 -100 fs

Applications of ultrafast lasers: femtosecond chemistry



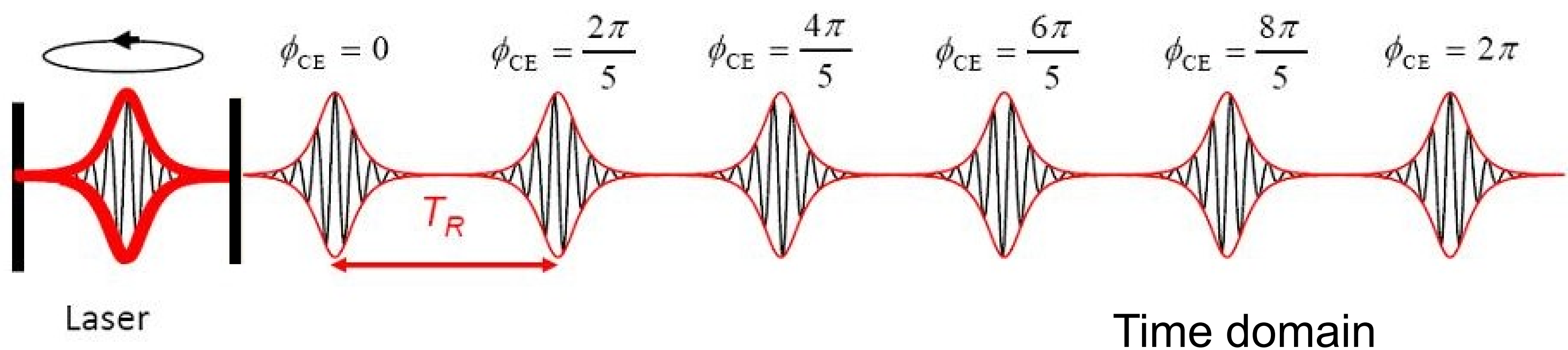
Prof. Ahmed Zewail from Cal Tech used ultrafast-laser techniques to study how atoms in a molecule move during chemical reactions (1999 Nobel Prize in Chemistry).

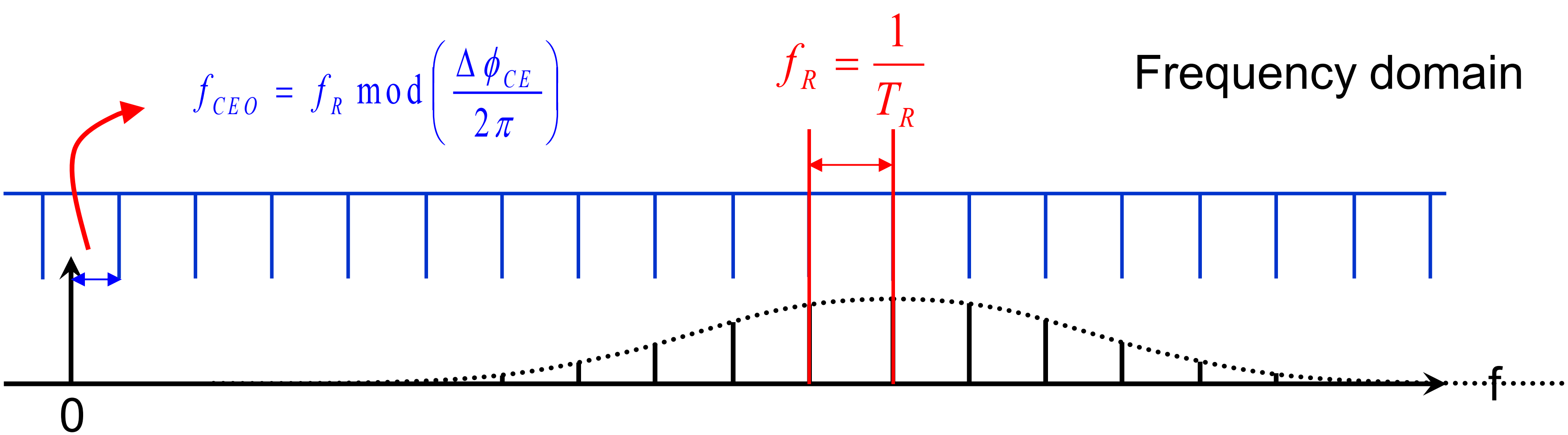
Ultra- accurate: timing distribution



Jungwon Kim et al., Nature photonics 2, 733 (2008)

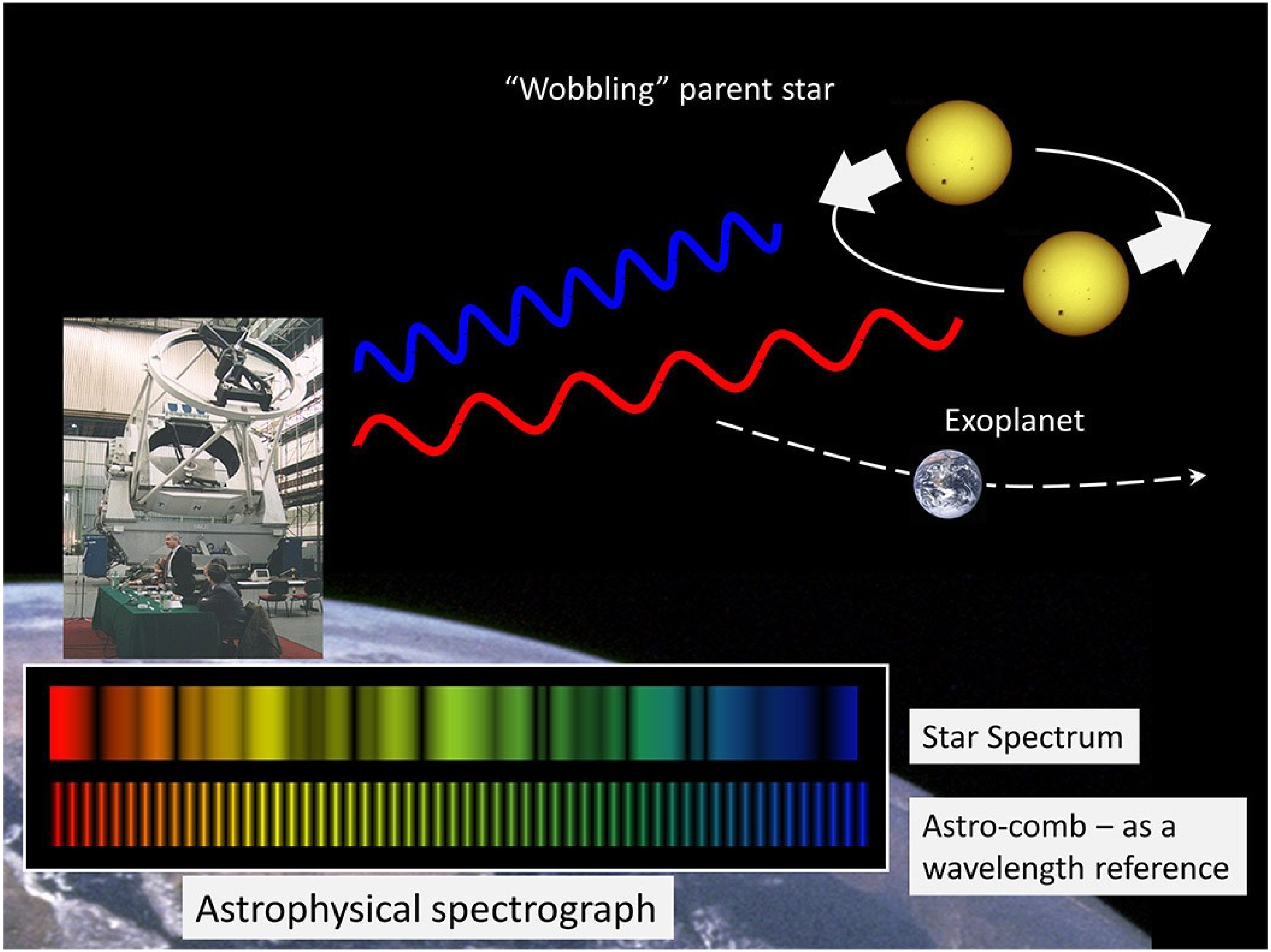
Ultra- accurate: fs laser frequency comb





Frequency comb has two degrees of freedom: $f = nf_R + f_{CEO}$

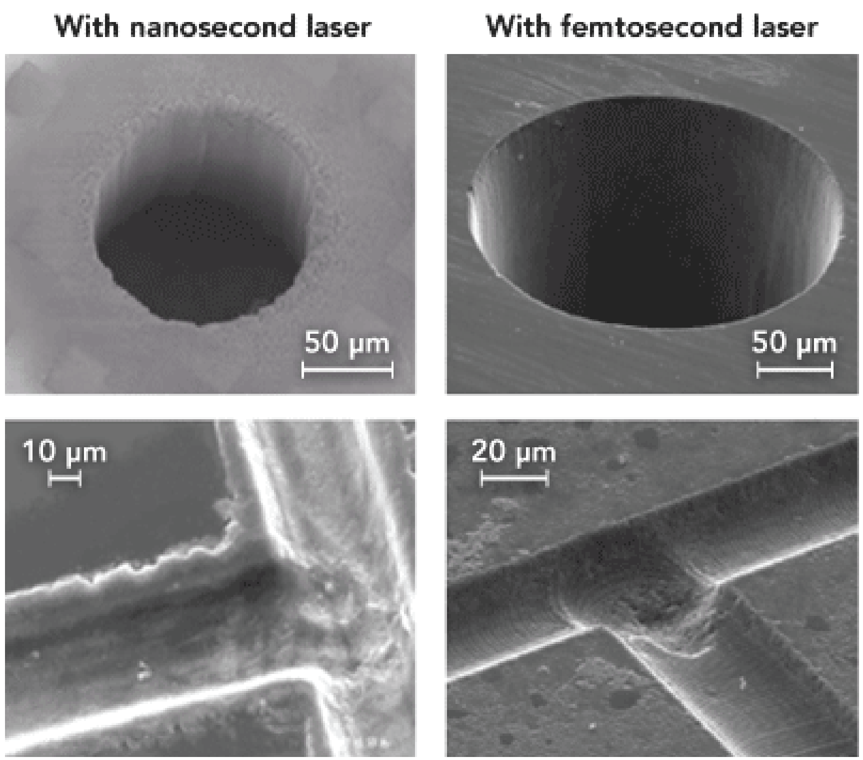
Ultra- accurate: fs laser frequency comb



C.-H. Li et al., Nature 452, 610 (2008).

Ultra- intense: femtosecond laser machining

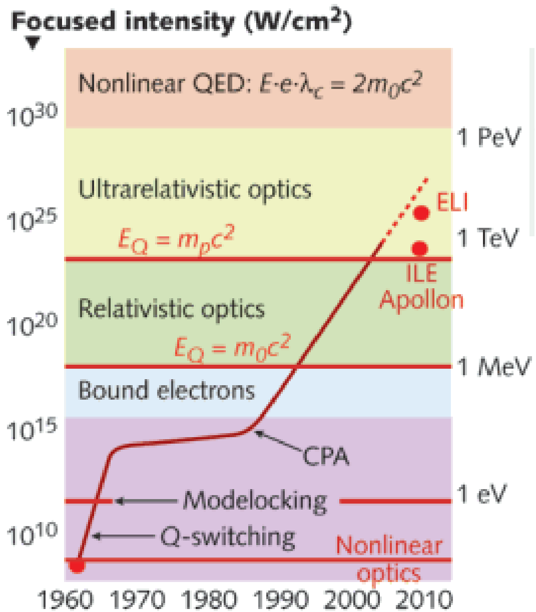
- **Sub-micron material processing:** Material milling, hole drilling, grid cutting
- **Surface structuring:** Photolithographic mask repair, surface removal or smoothing without imparting any thermal influence into the underneath sub-layers or the substrate
- **Photonics devices:** Machining of optical waveguides in bulk glasses or silica, and inscription of grating structure in fibers
- **Biomedical devices:** Use of femtosecond lasers for stent manufacture or eye surgery
- **Microfluidics:** Microfluidic channels and devices
- **Displays and solar:** Thin-film ablation, solar cell edge isolation

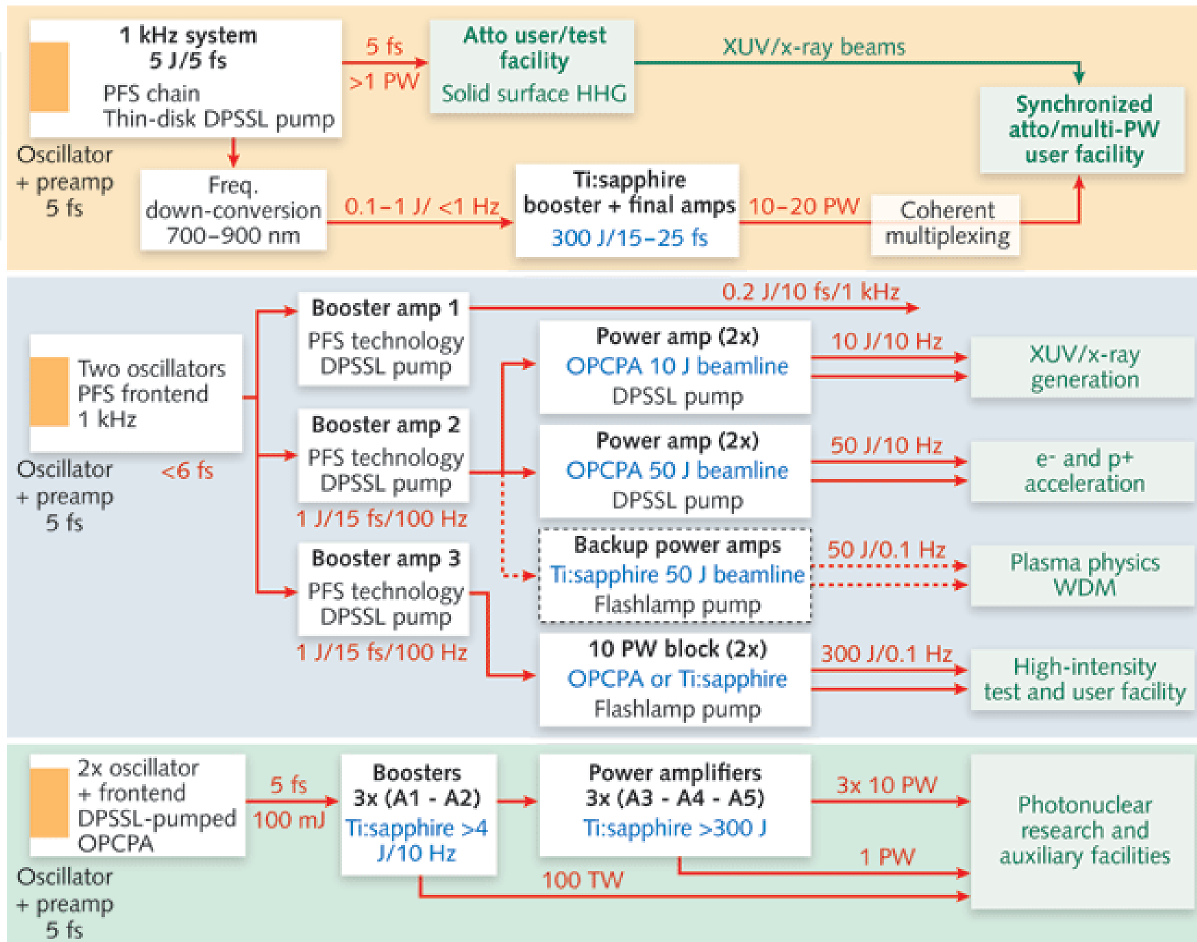


Laser processing examples on glass with a 266 nm (UV) ns-laser (left side) and with a 780 nm 100-fs laser (right side).

L. Lucas and J. Zhang, "Femtosecond laser micromachining: A back-to-basics primer," Industrial Laser Solutions (2012)

Ultra-intense: extreme light infrastructure (ELI)





ELI's goal is to explore new frontiers in physics, including trying to trigger the breakdown of the vacuum—the fabric of space-time itself.

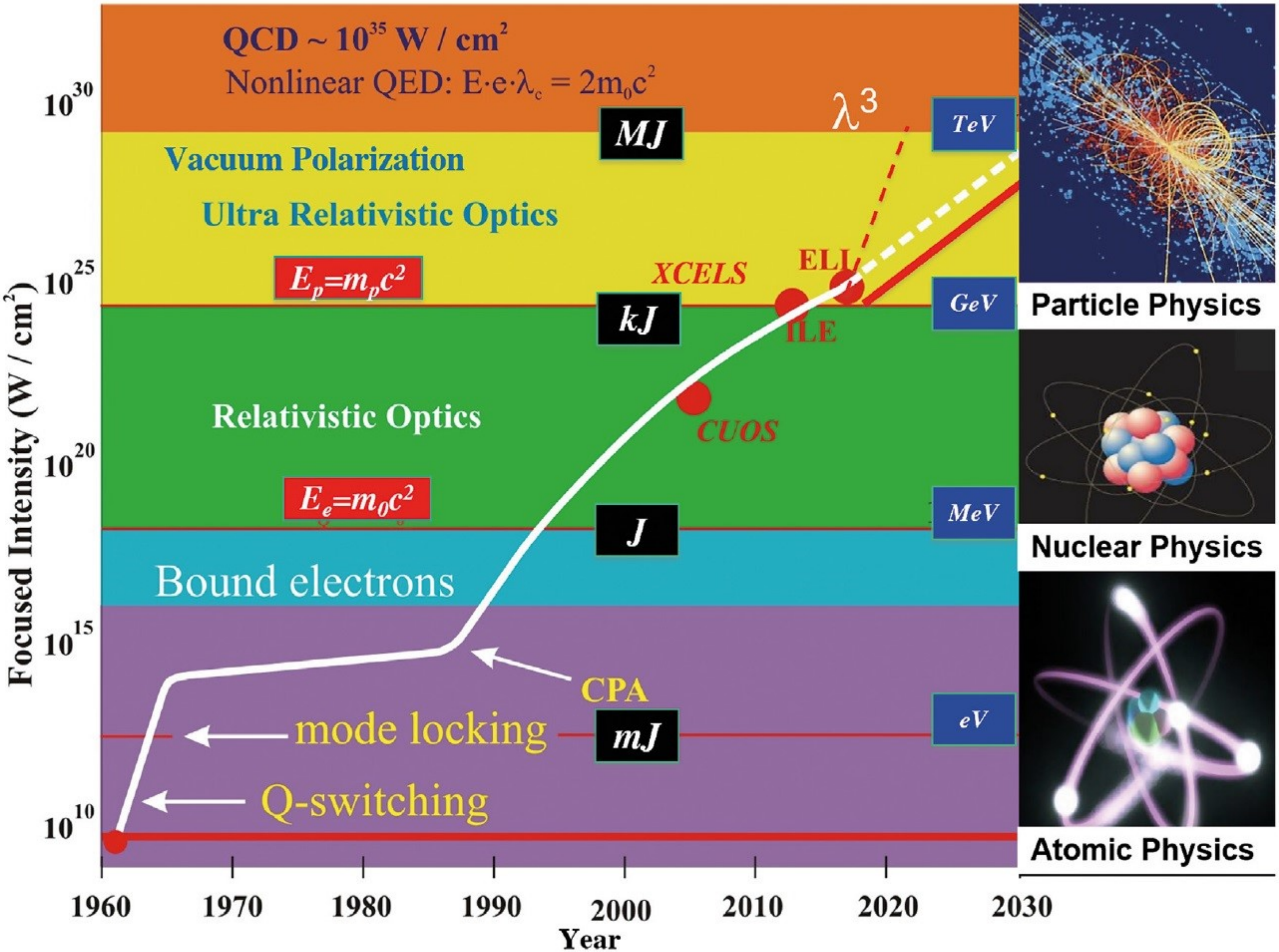
Designs of the three pillars are shown. Top: Hungarian attosecond pillar. Middle: Czech beam-line pillar. Bottom: Romanian photonuclear pillar.

J. Hecht, "Photonic frontiers: the extreme light infrastructure: the ELI aims to break down the vacuum," Laser Focus World (2011)

Nobel Prize in Physics 2018

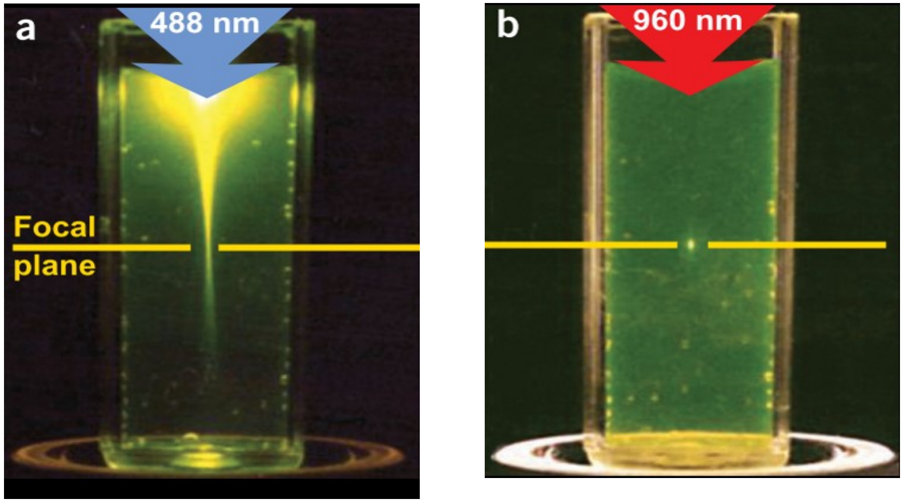
"for the optical tweezers and their application to biological systems"

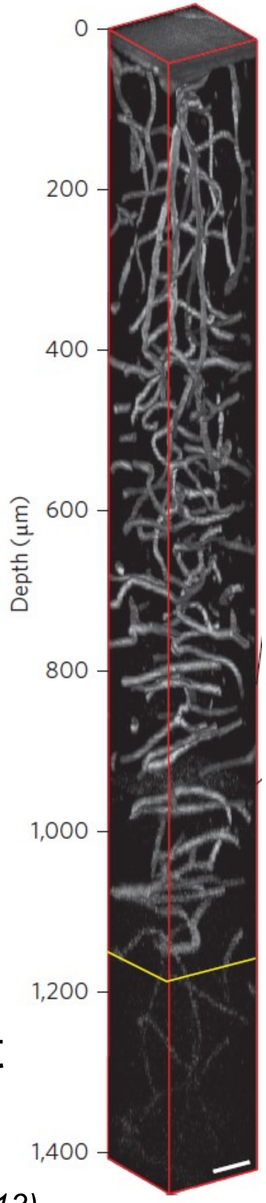
"for their method of generating high-intensity, ultra-short optical pulses“
chirped-pulse amplification (CPA)



G. Mourou, J. A. Wheeler, and T. Tajima, Europhys. News **46**, 31 (2015)

Nonlinear optical microscopy

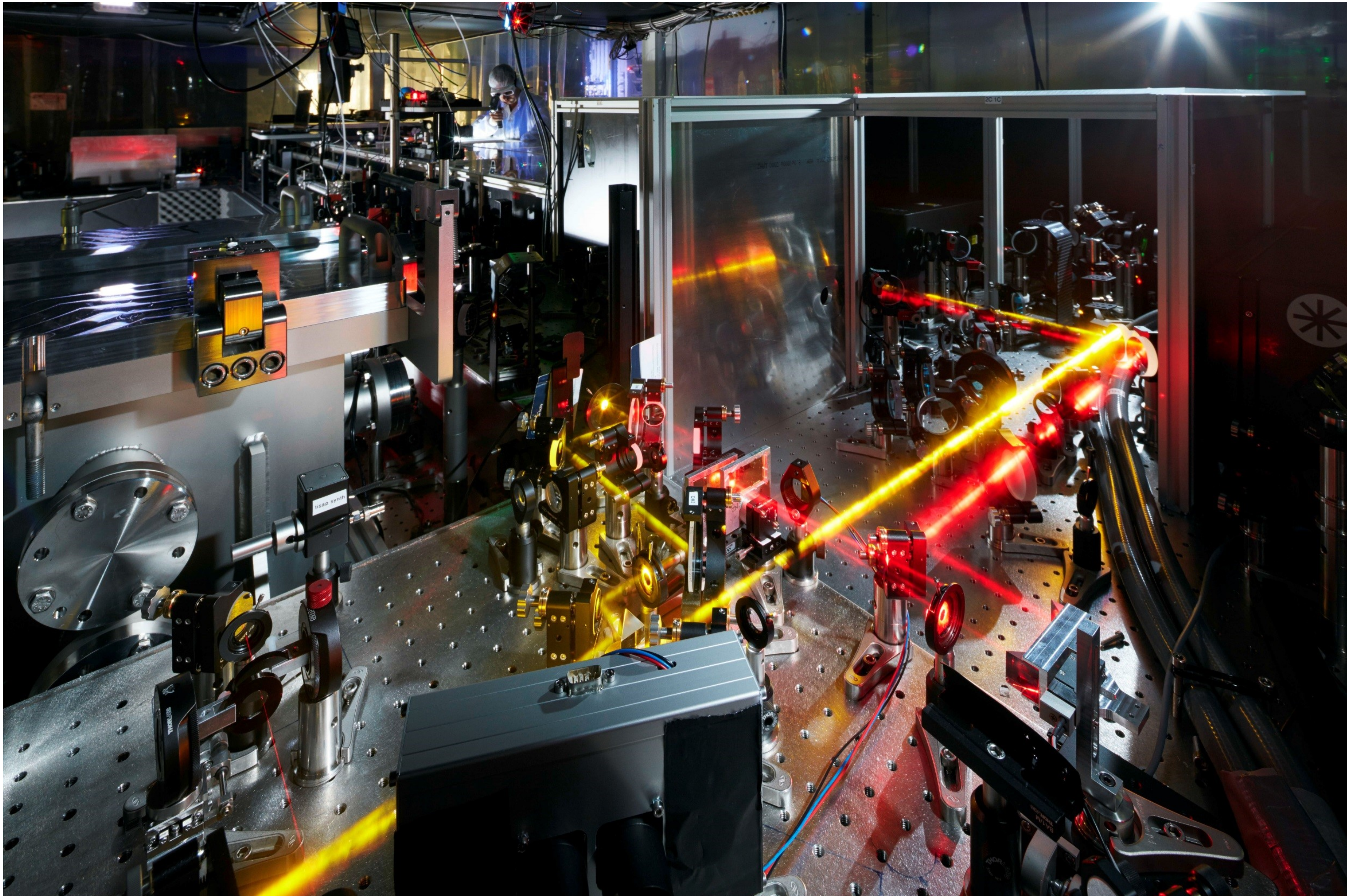




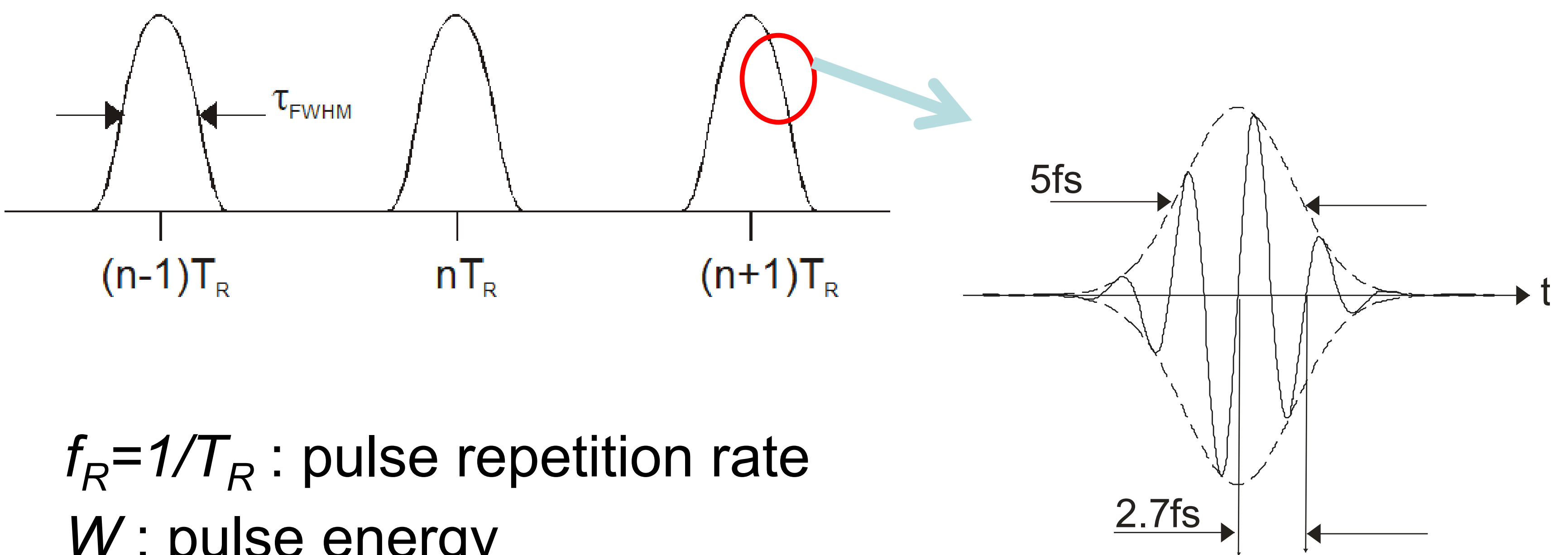
- Intrinsic sectioning ability, making 3D imaging possible.
- Longer excitation wavelength, which reduces tissue scattering and allows larger penetration depth.
- New contrast mechanisms: N-photon excitation fluorescence, Harmonic generation, Coherent Raman scattering, etc.
- Ultrashort pulses as the excitation source – many ultrafast optics technologies can be employed.

W. R. Zipfel et al., Nat. Biotechnol. 21,1369(2003). N. G. Norton et al, Nat. Photonics 7, 205 (2013).

Three channel OPA (optical parametric amplification) system: what optics lab looks like



Ultrafast lasers emit pulse train



$f_R=1/T_R$: pulse repetition rate

W : pulse energy

$P_{ave} = W/T_R$: average power

τ_{FWHM} : Full Width Half Maximum pulse width

P_p : peak power

$$P_p = \frac{W}{\tau_{\text{FWHM}}} = P_{ave}\,\frac{T_R}{\tau_{\text{FWHM}}}$$

Some physical quantities

Average power: $P_{ave} \sim 1W - 1kW$

Repetition rate: $T_R^{-1} = f_R = \mathrm{mHz} - 100\,\mathrm{GHz}$

Pulse energy: $W = 1\mathrm{pJ} - 1\mathrm{kJ}$ (Average power = Rep-rate X Pulse energy)

Pulse width (duration):

$$\tau_{\mathrm{FWHM}} = \begin{array}{ll} 5\,\mathrm{fs} - 50\,\mathrm{ps}, & \mathrm{modelocked} \\ 30\,\mathrm{ps} - 100\,\mathrm{ns}, & \mathrm{Q-switched} \end{array}$$

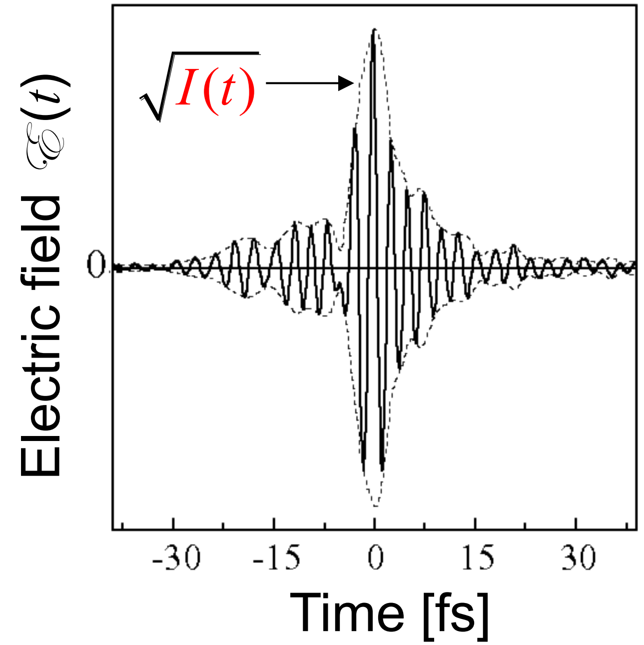
Peak power: $P_p = \dfrac{1\,\mathrm{kJ}}{1\,\mathrm{ps}} = \dfrac{1\,\mathrm{J}}{1\,\mathrm{fs}} \sim 1\,\mathrm{PW}$ (Peak power = Pulse energy / duration)

(peak) Intensity:

$$I = \frac{(Peak)\ power}{beam\ area}$$

If an optical beam with 1 PW peak power is focused to 1um^2 area, the peak intensity is 10^{23} W/cm^2.

An ultrashort laser pulse has an intensity and phase vs. time.



Neglecting the spatial dependence for now, the pulse electric field is given by:

$$E(t) \propto \frac{1}{2}\sqrt{I(t)}\exp\{j[\omega_0 t - \phi(t)]\} + c.c.$$

Intensity

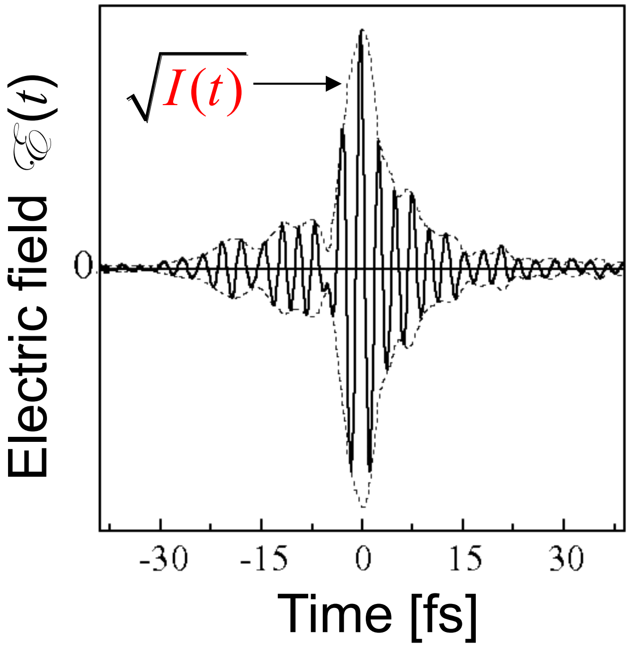
Carrier frequency

Phase

A sharply peaked function for the intensity yields an ultrashort pulse. The phase tells us the color evolution of the pulse in time.

<u>*Adapted from Prof. Rick Trebino's course slides*</u>

The real and complex pulse amplitudes



Removing the 1/2, the c.c., and the exponential factor with the carrier frequency yields the **complex amplitude, *E*(*t*),** of the pulse:

$$E(t) \propto \sqrt{I(t)}\,\exp[-j\phi(t)]$$

This removes the rapidly varying part of the pulse electric field and yields a complex quantity, which is actually easier to calculate with.

$\sqrt{I(t)}$ is often called the **real amplitude, *A*(*t*),** of the pulse.

Adapted from Prof. Rick Trebino's course slides

The Gaussian pulse

For almost all calculations, a good first approximation for any ultrashort pulse is the **Gaussian pulse** (with zero phase).

$$E(t) = E_0 \exp\left[-(t/\tau_{HW1/e})^2\right]$$
$$= E_0 \exp\left[-2\ln 2\,(t/\tau_{FWHM})^2\right]$$
$$= E_0 \exp\left[-1.38\,(t/\tau_{FWHM})^2\right]$$

- $\tau_{HW1/e}$ is the field half width at 1/e maximum, and τ_{FWHM} is the intensity full width at half maximum.
- The intensity is:

$$I(t) \propto |E_0|^2 \exp\left[-4\ln 2\,(t/\tau_{FWHM})^2\right]$$
$$\propto |E_0|^2 \exp\left[-2.76\,(t/\tau_{FWHM})^2\right]$$

Adapted from Prof. Rick Trebino's course slides

The Fourier transform

To think about ultrashort laser pulses, the Fourier Transform is essential.

$$\widetilde{E}(\omega) = \int_{-\infty}^{\infty} E(t) e^{-j\omega t} dt$$

$$E(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \widetilde{E}(\omega) e^{j\omega t} d\omega$$

We always perform Fourier transforms on the real or complex pulse electric field, and not the intensity, unless otherwise specified.

Adapted from Prof. Rick Trebino's course slides

Frequency-domain electric field

- The frequency-domain equivalents of the **intensity** and **phase** are the **spectrum** and **spectral phase**.
- Fourier-transforming the pulse electric field:

$$E(t) \propto \frac{1}{2}\sqrt{I(t)}\,\exp\{\,j[\omega_0 t - \phi(t)]\} + c.c.$$

yields:

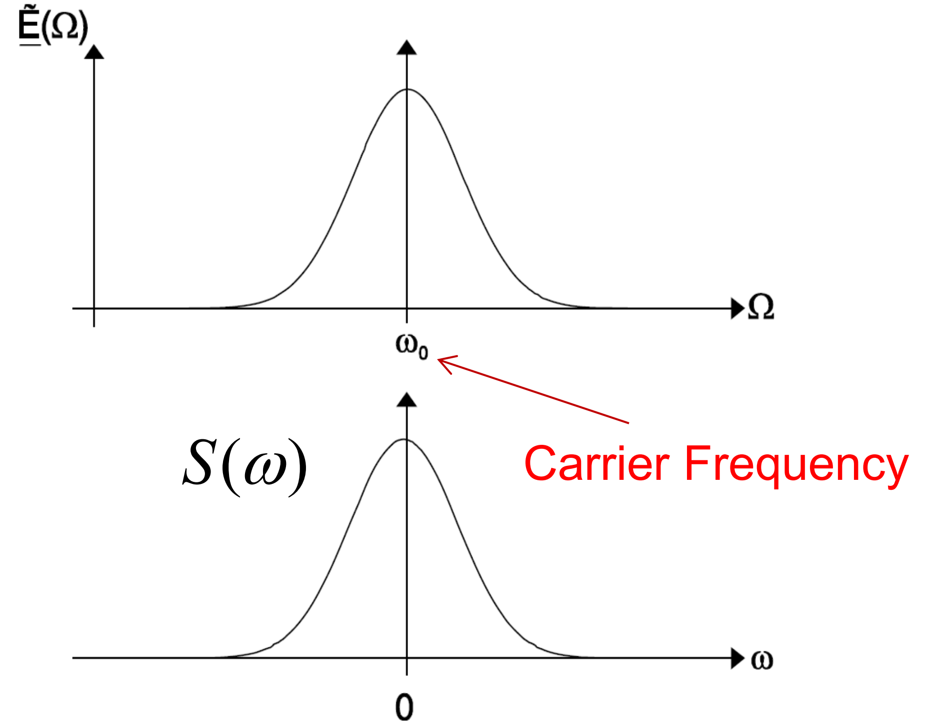
Note that ϕ and φ are different!

$$\tilde{E}(\omega) = \frac{1}{2}\sqrt{S(\omega-\omega_0)}\,\exp\{-j[\varphi(\omega-\omega_0)]\} + \frac{1}{2}\sqrt{S(-\omega-\omega_0)}\,\exp\{+j[\varphi(-\omega-\omega_0)]\}$$

The frequency-domain electric field has positive- and negative-frequency components.

Complex frequency-domain pulse field

Since the negative-frequency component contains the same information as the positive-frequency component, we usually neglect it.



We also center the pulse on its actual frequency.

So the most commonly used complex frequency-domain pulse field is:

$$\widetilde{E}(\omega) \equiv \sqrt{S(\omega)}\,\exp\{-j[\varphi(\omega)]\}$$

Thus, the frequency-domain electric field also has an intensity and phase. S is the spectrum, and φ is the spectral phase.

Adapted from Prof. Rick Trebino's course slides

Often used pulses

Pulse Shape	Fourier Transform	Pulse Width	Time-Band-width Product
$\underline{A}(t)$	$\tilde{\underline{A}}(\omega) = \int_{-\infty}^{\infty} a(t) e^{-j\omega t} dt$	Δt	$\Delta t \cdot \Delta f$
Gaussian: $\mathrm{e}^{-\frac{t^2}{2\tau^2}}$	$\sqrt{2\pi}\tau \mathrm{e}^{-\frac{1}{2}\tau^2\omega^2}$	$2\sqrt{\ln 2}\tau$	0.441
Hyperbolic Secant: $\mathrm{sech}(\frac{t}{\tau})$	$\frac{\tau}{2}\ \mathrm{sech}\left(\frac{\pi}{2}\tau\omega\right)$	$1.7627\ \tau$	0.315
Rect-function: $\begin{cases} 1, & \lvert t\rvert \leq \tau/2 \\ 0, & \lvert t\rvert > \tau/2 \end{cases}$	$\tau \frac{\sin(\tau\omega/2)}{\tau\omega/2}$	τ	0.886
Lorentzian: $\frac{1}{1+(t/\tau)^2}$	$2\pi\tau \mathrm{e}^{-\lvert\tau\omega\rvert}$	$1.287\ \tau$	0.142
Double-Exp.: $\mathrm{e}^{-\left\lvert\frac{t}{\tau}\right\rvert}$	$\frac{\tau}{1+(\omega\tau)^2}$	$\ln 2\ \tau$	0.142

Pulse width and spectral width: FWHM (full width at half maximum)

Operators used in Maxwell's Equations

The "Del" operator: $\vec{\nabla} \equiv \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right)$

The "Gradient" of a scalar function f:

$$\vec{\nabla} f \equiv \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right)$$

The "Divergence" of a vector function $\vec{G}$:

$$\vec{\nabla} \cdot \vec{G} \equiv \frac{\partial G_x}{\partial x} + \frac{\partial G_y}{\partial y} + \frac{\partial G_z}{\partial z}$$

Operators used in Maxwell's Equations

The "Laplacian" of a scalar function :

$$\nabla^2 f \equiv \vec{\nabla} \cdot \vec{\nabla} f = \vec{\nabla} \cdot \left(\frac{\partial f}{\partial x} , \frac{\partial f}{\partial y} , \frac{\partial f}{\partial z} \right)$$

$$= \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

The "Laplacian" of a vector function is the same,
but for each component :

$$\nabla^2 \vec{G} = \left(\frac{\partial^2 G_x}{\partial x^2} + \frac{\partial^2 G_x}{\partial y^2} + \frac{\partial^2 G_x}{\partial z^2} , \frac{\partial^2 G_y}{\partial x^2} + \frac{\partial^2 G_y}{\partial y^2} + \frac{\partial^2 G_y}{\partial z^2} , \frac{\partial^2 G_z}{\partial x^2} + \frac{\partial^2 G_z}{\partial y^2} + \frac{\partial^2 G_z}{\partial z^2} \right)$$

Operators used in Maxwell's Equations

The "Curl" of a vector function $\vec{G}$:

$$\vec{\nabla} \times \vec{G} \equiv \left(\frac{\partial G_z}{\partial y} - \frac{\partial G_y}{dz}, \frac{\partial G_x}{\partial z} - \frac{\partial G_z}{dx}, \frac{\partial G_y}{\partial x} - \frac{\partial G_x}{dy} \right)$$

The curl can computed from a matrix determinant :

$$\vec{\nabla} \times \vec{G} = \det \begin{bmatrix} \hat{\mathrm{x}} & \hat{\mathrm{y}} & \hat{\mathrm{z}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ G_x & G_y & G_z \end{bmatrix}$$

Maxwell's equations of isotropic and homogeneous media

Maxwell's Equations: Differential Form

Current due to free charges

Ampere's Law	$\vec{\nabla}\times\vec{H} = \dfrac{\partial\vec{D}}{\partial t} + \vec{J},$
Faraday's Law	$\vec{\nabla}\times\vec{E} = -\dfrac{\partial\vec{B}}{\partial t},$
Gauss's Law	$\vec{\nabla}\cdot\vec{D} = \rho,$
No magnetic charge	$\vec{\nabla}\cdot\vec{B} = 0.$

Free charge density

Material Equations: Bring Life into Maxwell's Equations

$$\vec{D} = \epsilon_0\vec{E} + \vec{P}, \quad \textbf{Polarization}$$

$$\vec{B} = \mu_0\vec{H} + \vec{M}. \quad \textbf{Magnetization}$$

Derivation of wave equation

Vector Identity: $\vec{\nabla}\times\left(\vec{\nabla}\times\vec{E}\right)=\vec{\nabla}\left(\vec{\nabla}\cdot\vec{E}\right)-\Delta\vec{E},$

$$\vec{\nabla}\times\left(\vec{\nabla}\times\vec{E}\right)=-\vec{\nabla}\times\frac{\partial\vec{B}}{\partial t}=-\frac{\partial}{\partial t}\left(\vec{\nabla}\times\vec{B}\right)$$

$$=-\frac{\partial}{\partial t}\left(\vec{\nabla}\times\left(\mu_0\vec{H}+\vec{M}\right)\right)=-\frac{\partial}{\partial t}\left(\mu_0\vec{\nabla}\times\vec{H}+\vec{\nabla}\times\vec{M}\right)$$

$$=-\frac{\partial}{\partial t}\left(\mu_0\left(\varepsilon_0\frac{\partial\vec{E}}{\partial t}+\frac{\partial\vec{P}}{\partial t}+\vec{J}\right)+\vec{\nabla}\times\vec{M}\right)$$

$$\Delta\vec{E}-\mu_0\frac{\partial}{\partial t}\left(\vec{j}+\epsilon_0\frac{\partial\vec{E}}{\partial t}+\frac{\partial\vec{P}}{\partial t}\right)=\frac{\partial}{\partial t}\nabla\times\vec{M}+\nabla\left(\nabla\cdot\vec{E}\right)\qquad(2.3)$$

Vacuum speed of light: $c_0=\sqrt{\frac{1}{\mu_0\epsilon_0}}.$

$$\left(\Delta-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\right)\vec{E}=\mu_0\left(\frac{\partial\vec{j}}{\partial t}+\frac{\partial^2}{\partial t^2}\vec{P}\right)+\frac{\partial}{\partial t}\vec{\nabla}\times\vec{M}+\vec{\nabla}\left(\vec{\nabla}\cdot\vec{E}\right).\qquad(2.4)$$

Derivation of wave equation

No free charges, No currents from free charges, Non magnetization

$$\left(\Delta - \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\right)\vec{E} = \mu_0\left(\cancel{\frac{\partial \vec{j}}{\partial t}} + \frac{\partial^2}{\partial t^2}\vec{P}\right) + \cancel{\frac{\partial}{\partial t}\nabla\times\vec{M}} + \nabla\left(\cancel{\nabla\cdot\vec{E}}\right). \qquad (2.4)$$

In the linear optics of isotropic media without free charges,

$$\nabla\cdot\vec{D} = 0 \longrightarrow \nabla\cdot\vec{E} = 0$$

Simplified wave equation:

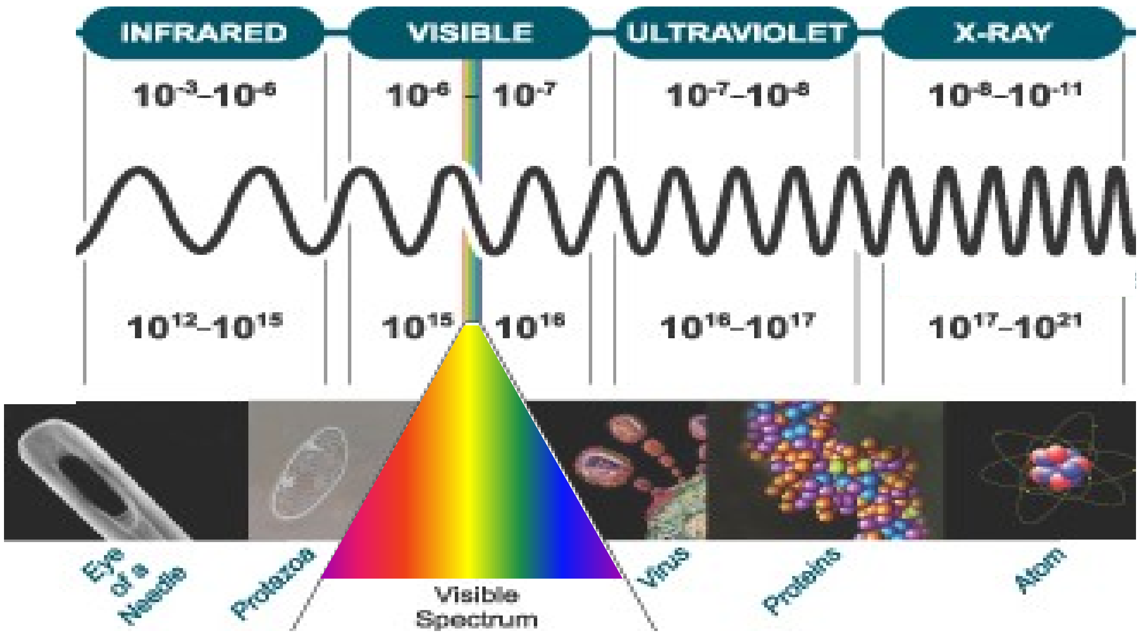
$$\left(\Delta - \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\right)\vec{E} = \mu_0\frac{\partial^2}{\partial t^2}\vec{P}$$

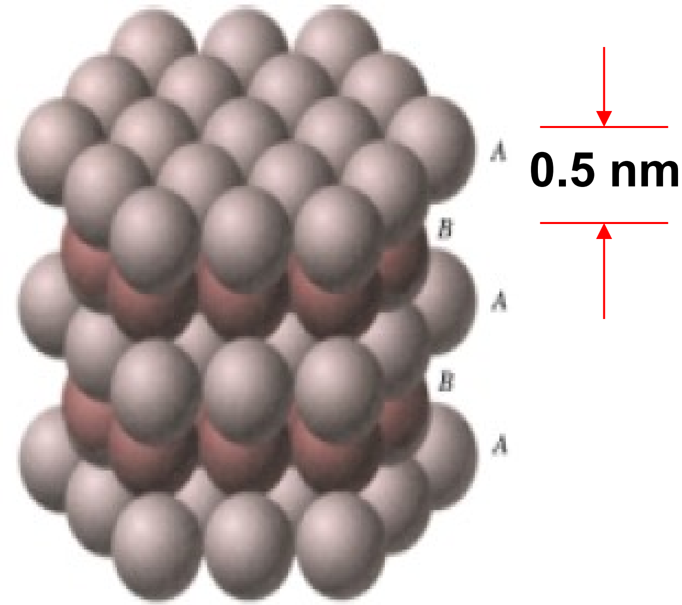
Wave in vacuum **Source term**

Laplace operator:

$$\Delta = \vec{\nabla}^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

Interaction between EM waves and materials





Wavelength of green light is about 500 nm. So the optical wave experiences an effective homogeneous medium, which is characterized by

Electric permittivity ϵ **and** **Magnetic permeability** μ

The velocity of light is different from the vacuum speed by a factor called the refractive index n

Dielectric susceptibility and Helmholtz Equation

$$\vec{P}(\vec{r},t) = \epsilon_0 \int dt' \, \chi(t-t')\, \vec{E}(\vec{r},t') \longrightarrow \tilde{\vec{P}}(\vec{r},\omega) = \epsilon_0 \tilde{\chi}(\omega) \tilde{\vec{E}}(\vec{r},\omega)$$

$$\left(\Delta - \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\right)\vec{E} = \mu_0 \frac{\partial^2}{\partial t^2}\vec{P} \longrightarrow \left(\Delta + \frac{\omega^2}{c_0^2}\right)\tilde{\vec{E}}(\omega) = -\omega^2 \mu_0 \epsilon_0 \tilde{\chi}(\omega) \tilde{\vec{E}}(\omega)$$

In a linear medium, dielectric susceptibility is independent of optical field

$$\left(\Delta + \frac{\omega^2}{c_0^2}(1+\tilde{\chi}(\omega))\right)\tilde{\vec{E}}(\omega) = 0$$

$$1+\tilde{\chi}(\omega) = n(\omega)^2$$

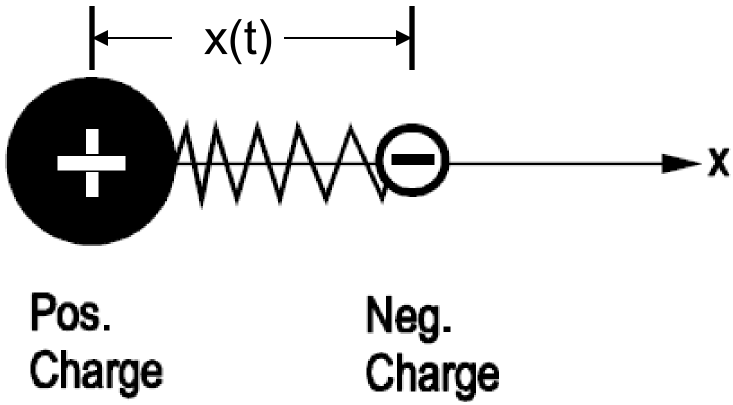
Can be complex

Refractive Index

light speed (dependent on frequency) in a medium: $c(\omega) = c_0/n(\omega)$

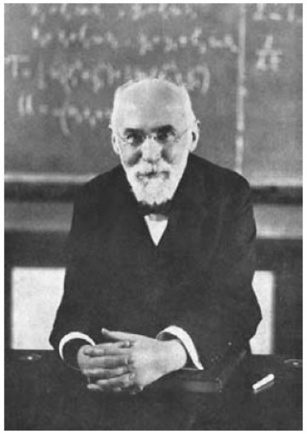
Dielectric Permittivity: Lorentz model

Lorentz Model of Matter (Atom):



Dipolmoment:

$$\vec{p}(t) = -e\underline{x}(t)$$



H. A. Lorentz (1853-1928)

$$\vec{E}(\vec{r},t) = \vec{E}(\vec{r}_A,t) = E(t)\vec{e}_x$$

$\underline{x}(t)$ is much smaller than the wavelength of electric field. Therefore we can neglect the spatial variation of the E field during the motion of the charge.

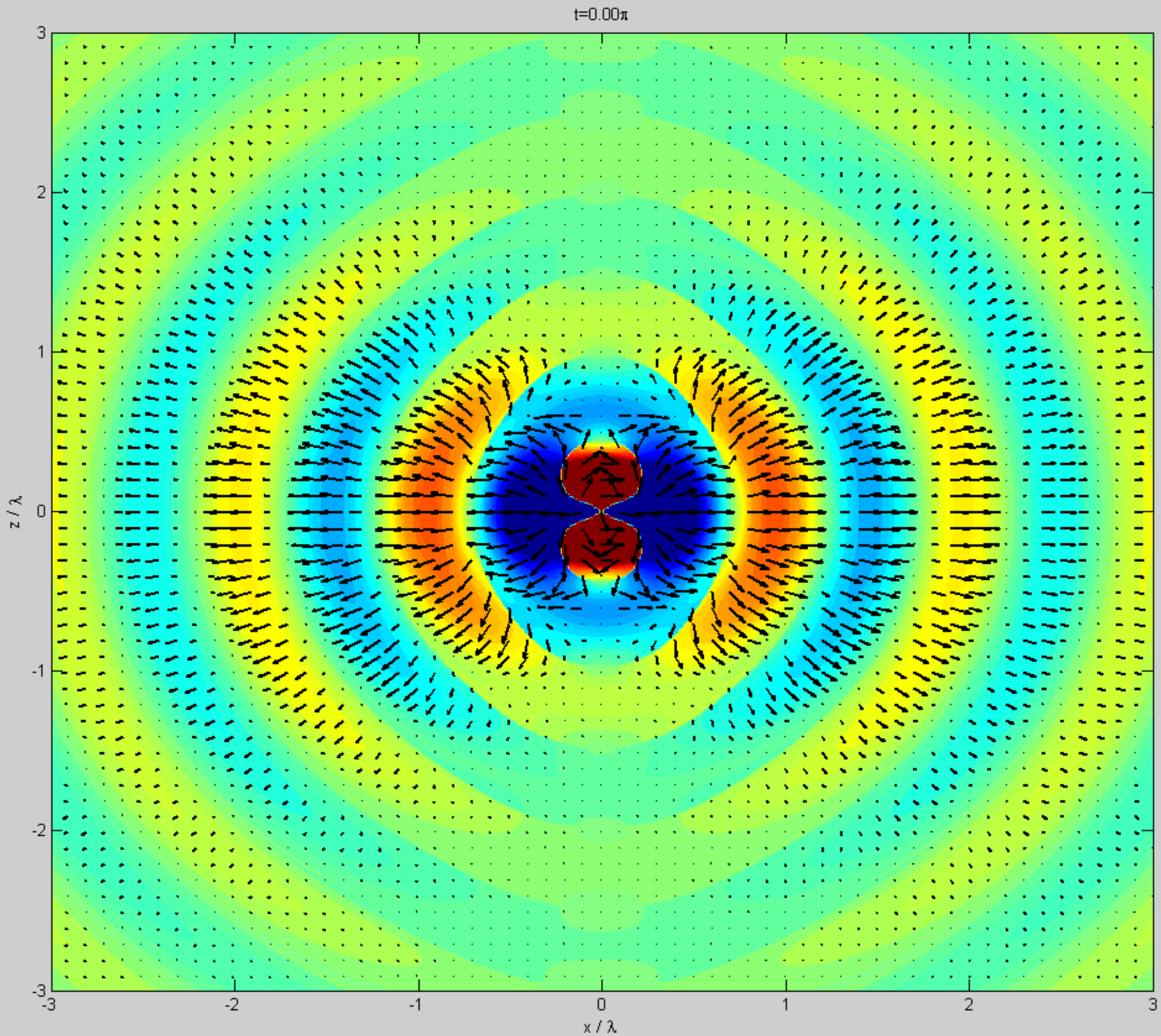
Density (# of atoms per unit volume)

$$\vec{P}(t) = \frac{\text{dipole moment}}{\text{volume}} = N \cdot \vec{p}(t)$$

Elementary Dipole

$$\underline{\tilde{\vec{P}}}(\omega) = \frac{\text{dipole moment}}{\text{volume}} = N \cdot \underline{\tilde{\vec{p}}}(\omega) = \epsilon_0 \underline{\tilde{\chi}}(\omega) \underline{\tilde{\vec{E}}}(\omega) \Rightarrow \underline{\tilde{\chi}}(\omega) = \frac{N \cdot \underline{\tilde{\vec{p}}}(\omega)}{\epsilon_0 \underline{\tilde{\vec{E}}}(\omega)}$$

Oscillating dipole moment emits new EM wave at the oscillating frequency

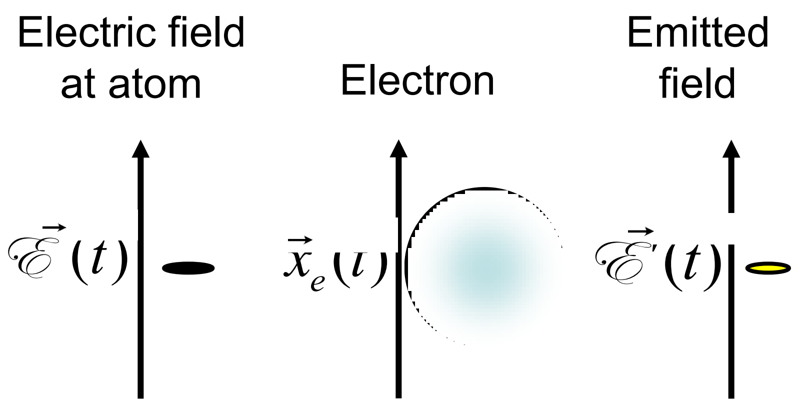


It does not emit in the oscillating direction!

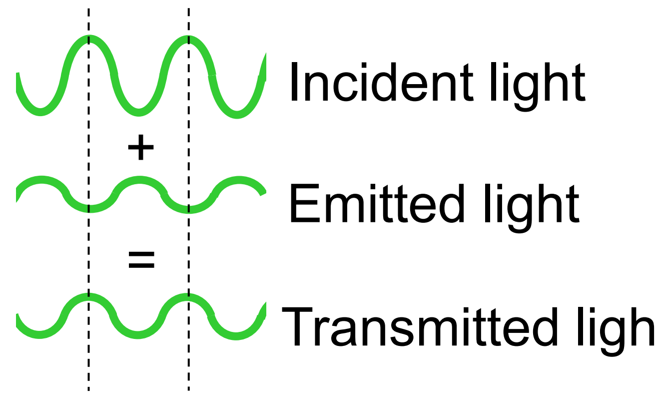
From wiki

Lorentz model of light-atom interaction

When light of frequency ω excites an atom with resonant frequency ω_0 :



On resonance ($\omega = \omega_0$)



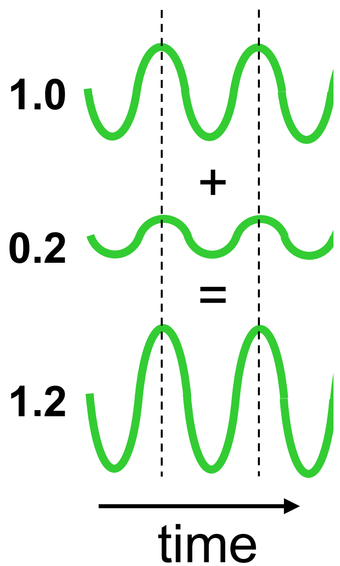
Incident Light excites electron oscillation → electron oscillation emits new light at the same frequency → incident light interferes with the new light leading to the transmitted light.

The crucial issue is the **relative phase** of the incident light and this emitted light. For example, if these two waves are ~180° out of phase, the beam will be attenuated. We call this absorption.

Interference depends on relative phase

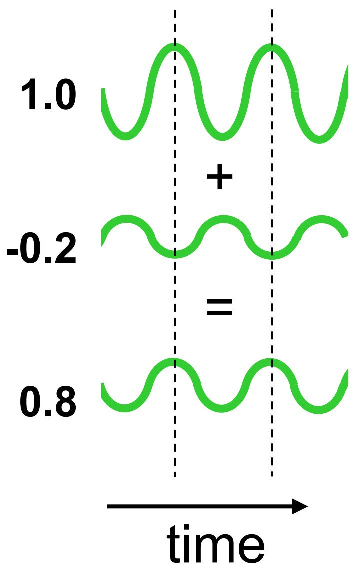
When two waves add together with the same complex exponentials, we add the complex amplitudes, $\underset{\sim}{E}_0 + \underset{\sim}{E}_0'$.

Constructive interference:



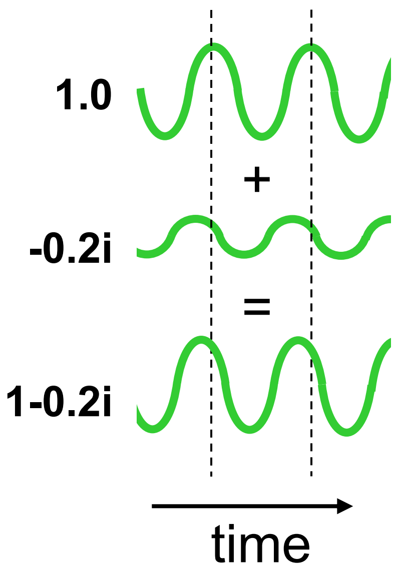
Laser

Destructive interference:



Absorption

Quadrature phase: $\pm 90^\circ$ interference:



Slower phase velocity (when accumulated over distance)

Adapted from Rick Trebino's course slides

Response to a monochromatic field

$$m\frac{d^2x}{dt^2} + 2\frac{\Omega_0}{Q}m\frac{dx}{dt} + m\Omega_0^2 x = e_0 E(t) \longleftarrow$$

force

mass

quality factor

frequency of undamped oscillator

$$\underline{E}(t) = \underline{\tilde{E}}e^{\mathrm{j}\omega t} \rightarrow \underline{x}(t) = \underline{\tilde{x}}e^{\mathrm{j}\omega t} \rightarrow \underline{p}(t) = e_0\underline{x}(t) = \underline{\tilde{p}}e^{\mathrm{j}\omega t}$$

$$\underline{\tilde{p}} = \frac{\frac{e_0^2}{m}}{(\Omega_0^2 - \omega^2) + 2\mathrm{j}\frac{\Omega_0}{Q}\omega}\underline{\tilde{E}}.$$

$$\boxed{\underline{\chi}(\omega) = \frac{N\frac{e_0^2}{m}\frac{1}{\epsilon_0}}{(\Omega_0^2 - \omega^2) + 2\mathrm{j}\omega\frac{\Omega_0}{Q}}}$$

Real and Imaginary Part of the Susceptibility

$$\underline{\tilde{\chi}}(\omega) = \tilde{\chi}_r(\omega) + \mathrm{j}\tilde{\chi}_i(\omega)$$

$$\omega_p = \left(\frac{Ne^2}{\varepsilon_0 m_0}\right)^{1/2}$$

Plasma frequency

$$\tilde{\chi}_r(\omega) = \omega_p^2 \cdot \frac{\left(\Omega_0^2 - \omega^2\right)}{\left(\Omega_0^2 - \omega^2\right)^2 + \left(2\omega\frac{\Omega_0}{Q}\right)^2}$$

$$\tilde{\chi}_i(\omega) = -\omega_p^2 \cdot \frac{2\omega\frac{\Omega_0}{Q}}{\left(\Omega_0^2 - \omega^2\right)^2 + \left(2\omega\frac{\Omega_0}{Q}\right)^2}$$

Real and Imaginary Part of the Susceptibility

$$\underline{\tilde{\chi}}(\omega) = \tilde{\chi}_r(\omega) + j\tilde{\chi}_i(\omega)$$

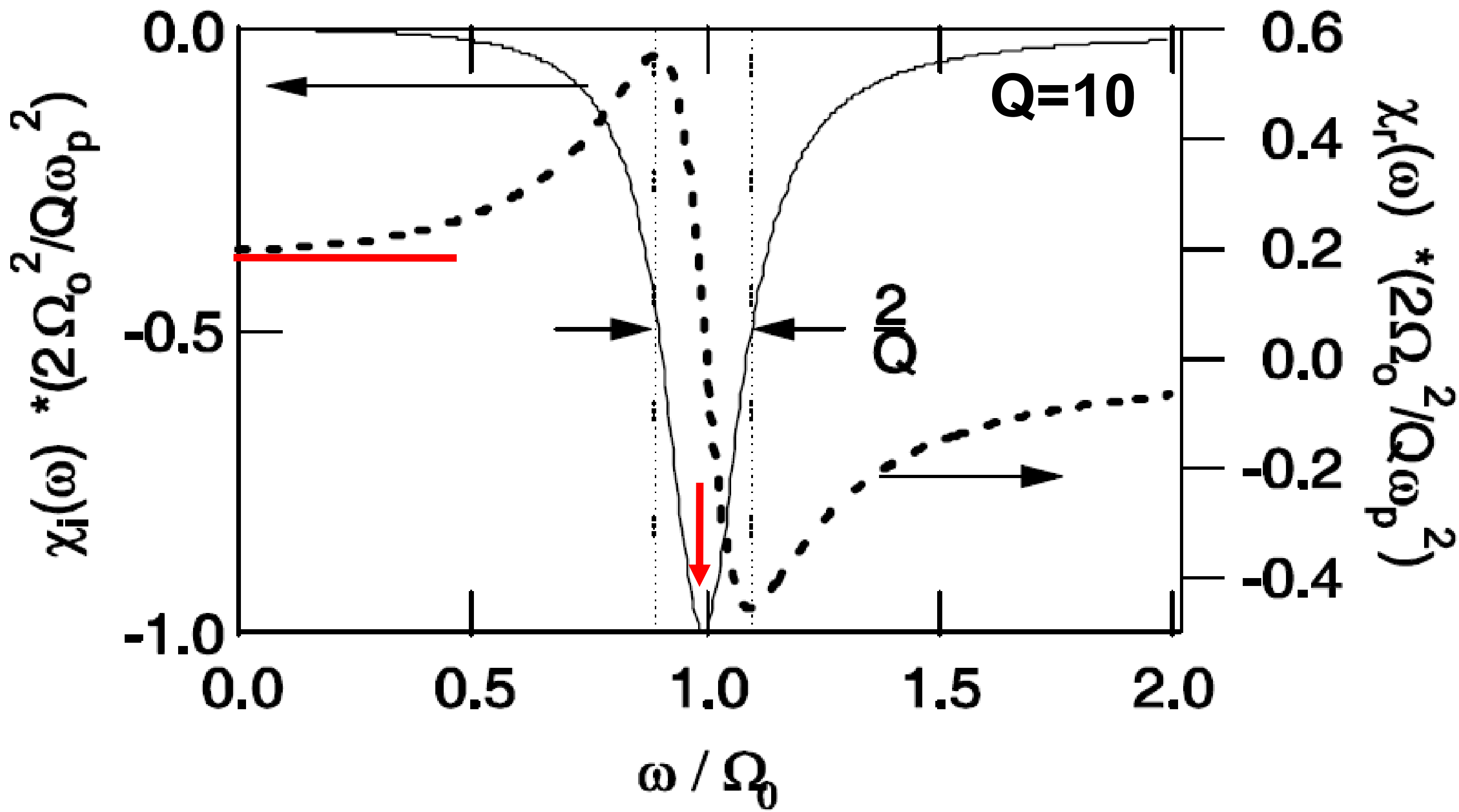


Figure 2.3: Real part (dashed line) and imaginary part (solid line) of the susceptibility of the classical oscillator model for the dielectric polarizability