Nonlinear Optics (WiSe 2017/18)

Lecture 21: January 11, 2018

11 Terahertz generation and applications

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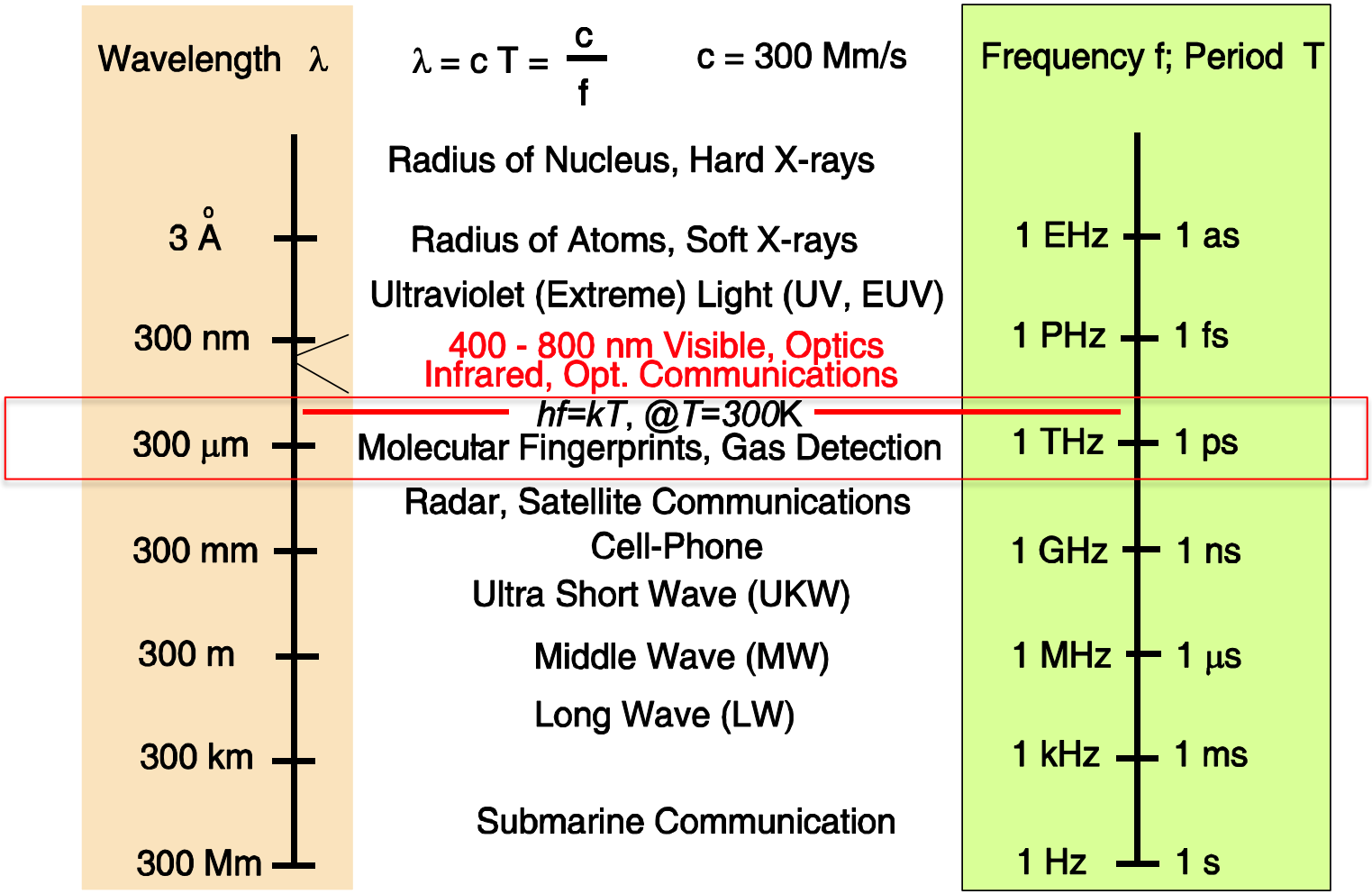
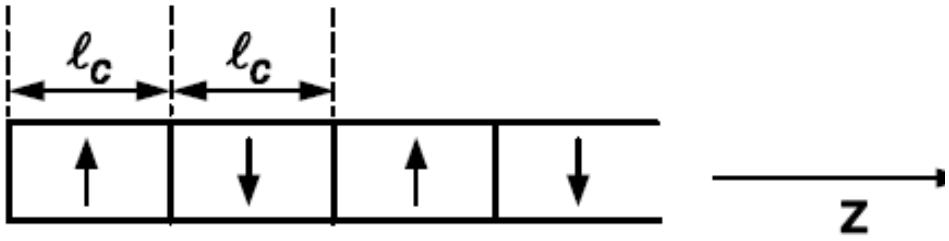
11.2.2 Optical rectification by Quasi-Phase Matching (QPM)

11 Terahertz generation and applications



0.3 – 30 THz

11.2.2 Optical rectification by Quasi-Phase Matching (QPM)



Periodically poled crystal

$$d_{eff}(z) = \sum_{m=-\infty}^{+\infty} d_m e^{jm\kappa z}$$

$$\begin{aligned}
\Delta k &= \left.\frac{\partial k_{opt}(\omega)}{\partial \omega}\right|_{\omega_0} \Omega - k_{THz}(\Omega) + m\frac{2\pi}{\Lambda} = \left(\frac{1}{v_{g,opt}} - \frac{1}{v_{p,THz}}\right)\Omega + m\frac{2\pi}{\Lambda} \\
&= \left(\frac{n_{g,opt} - n_{p,THz}}{c}\right)\Omega + m\frac{2\pi}{\Lambda} = 0 \\
\rightarrow \quad & \Lambda = m\frac{\lambda_{THz}}{n_{p,THz} - n_{g,opt}}.
\end{aligned}$$

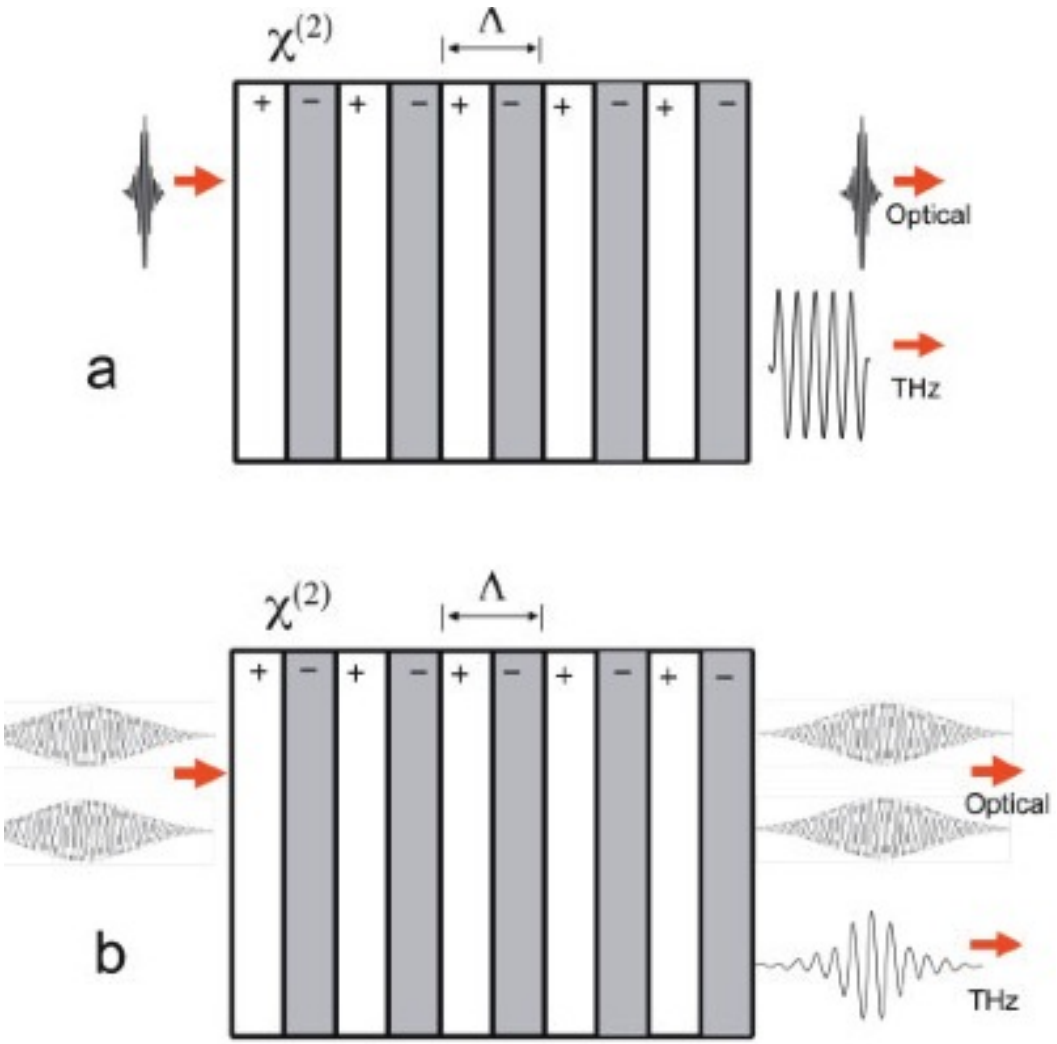


Figure 11.10: Schematic illustration of collinear THz-wave generation in a nonlin- ear crystal with periodically inverted sign of χ(2). (a) Optical rectification with femtosecond pulses, (b) difference-frequency generation with two picosecond pulses (Ω = ω3 − ω2) [10].

Plane-wave analysis of optical-to-THz conversion in QPM crystals with ultrashort pulses

$$E_{opt}(t) = Re\{E_0\ e^{-t^2/\tau^2}\ e^{j\omega_0 t}\} = \frac{1}{2}\{E_0\ e^{-t^2/\tau^2}\ e^{j\omega_0 t} + c.c.\},$$

$$\hat{E}_{opt}(\omega) = \frac{E_o \tau}{2\sqrt{\pi}} \exp\left(-\frac{\tau^2\omega^2}{4}\right)$$

$$\begin{aligned}\frac{d\hat{E}_{THz}(\Omega, z)}{dz} &= -j\frac{\Omega\ d_{eff}^{QPM}}{c\ n_{p,THz}}\frac{E_o^2\tau^2}{4\pi}\int_{-\infty}^{+\infty}\exp\left(-\frac{\tau^2(\omega+\Omega)^2}{4}\right)\exp\left(-\frac{\tau^2\omega^2}{4}\right)d\omega \\ &= -j\frac{\Omega\ d_{eff}^{QPM}}{c\ n_{p,THz}}\frac{E_o^2\tau}{2\sqrt{2\pi}}\exp\left(-\frac{\tau^2\Omega^2}{8}\right)\ e^{j\Delta k(\Omega)z}. \qquad (11.23)\end{aligned}$$

$$\left|\hat{E}_{THz}(\Omega, z)\right|^2 = \frac{\Omega^2\ d_{eff}^{QPM,2}}{c^2\ n_{p,THz}^2}\frac{E_o^4\tau^2}{8\pi}\exp\left(-\frac{\tau^2\Omega^2}{4}\right)\ L^2\mathrm{sinc}^2\left(\frac{\Delta k(\Omega)L}{2}\right) \qquad (11.24)$$

$$\text{with } \Delta k(\Omega) = \Delta k = \frac{n_{g,opt} - n_{p,THz}}{c} + m\frac{2\pi}{\Lambda} \qquad (11.25)$$

$$d_{eff}^{QPM} = \frac{2}{\pi} d_{eff}$$

Optical to THz conversion effficiency

$$\eta_{THz}^{PW} = \frac{\text{Fluence THz}}{\text{Fluence (pump)}}$$

$$F_{pump} = \frac{c\varepsilon_0 n_{p,opt}}{2}\int_{-\infty}^{+\infty}\left|E_{opt}\left(t,0\right)\right|^2 dt = \sqrt{\frac{\pi}{2}}\frac{c\varepsilon_0 n_{p,opt}}{2}E_o^2\tau$$

and THz fluence

$$F_{THz} = \frac{c\varepsilon_0 n_{p,THz}}{2}\int_{-\infty}^{+\infty}\left|E_{THz}\left(t,L\right)\right|^2 dt = \frac{c\varepsilon_0 n_{p,THz}}{2}2\pi\int_{-\infty}^{+\infty}\left|\hat{E}_{THz}\left(\Omega,z\right)\right|^2 d\Omega \quad (11.28)$$

where we used Parceval's theorem

$$\int_{-\infty}^{+\infty}\left|f(t)\right|^2 dt = 2\pi\int_{-\infty}^{+\infty}\left|\hat{f}\left(\Omega\right)\right|^2 d\Omega. \quad (11.29)$$

Thus, we obtain

$$N = \frac{L}{\Lambda}$$

$$\eta_{THz}^{PW} = \frac{\Omega^2\ d_{eff}^{QPM,2}E_o^2\tau}{2\sqrt{2\pi}c^2\ n_{p,opt}n_{p,THz}}L^2\int_0^{+\infty}\exp\left(-\frac{\tau^2\Omega^2}{4}\right)\ \text{sinc}^2\left(\frac{\pi}{2}\frac{\Omega-\Omega_0}{\Delta\Omega}\right)d\Omega, \quad (11.30)$$

$$\text{with } \Omega_0 = \frac{2\pi\ c}{\Lambda\left(n_{g,opt}-n_{p,THz}\right)} \text{ and } \Delta\Omega = \frac{2\pi\ c}{L\left(n_{g,opt}-n_{p,THz}\right)} = \frac{\Lambda}{L}\Omega_0 \quad (11.31)$$

$$\eta_{THz}^{PW,sp} = \frac{\Omega^2 \; d_{eff}^{QPM,2} E_o^2 c}{2\sqrt{2\pi} c^2 \; n_{p,opt} n_{p,THz}} \; L^2 g_1(\Omega_0) \; 2 \; \Delta\Omega =$$

$$= \frac{\Omega^2 \; d_{eff}^{QPM,2} E_o^2 \tau}{c \; n_{p,opt} n_{p,THz}} \; \frac{\sqrt{2\pi}}{(n_{g,opt} - n_{p,THz})} \; L \; g(\Omega_0)$$

$$= \frac{2\Omega^2 \; d_{eff}^{QPM,2} L}{\varepsilon_0 \; c^2 \; n_{p,opt}^2 n_{p,THz} \, (n_{g,opt} - n_{p,THz})} \; g_1(\Omega_0) \; F_{pump} \quad (11.32)$$

$$\text{with } g_1(\Omega_0) = \exp\left(-\frac{\tau^2 \Omega_0^2}{4}\right) = \exp\left(-\left(\pi f_{THz} \tau\right)^2\right) \text{ and} \quad (11.33)$$

$$2 \; \Delta\Omega = \int_{-\infty}^{+\infty} \mathrm{sinc}^2\left(\frac{\pi}{2} \frac{\Omega - \Omega_0}{\Delta\Omega}\right) \, d\Omega \quad (11.34)$$

Influence of optical bandwidth

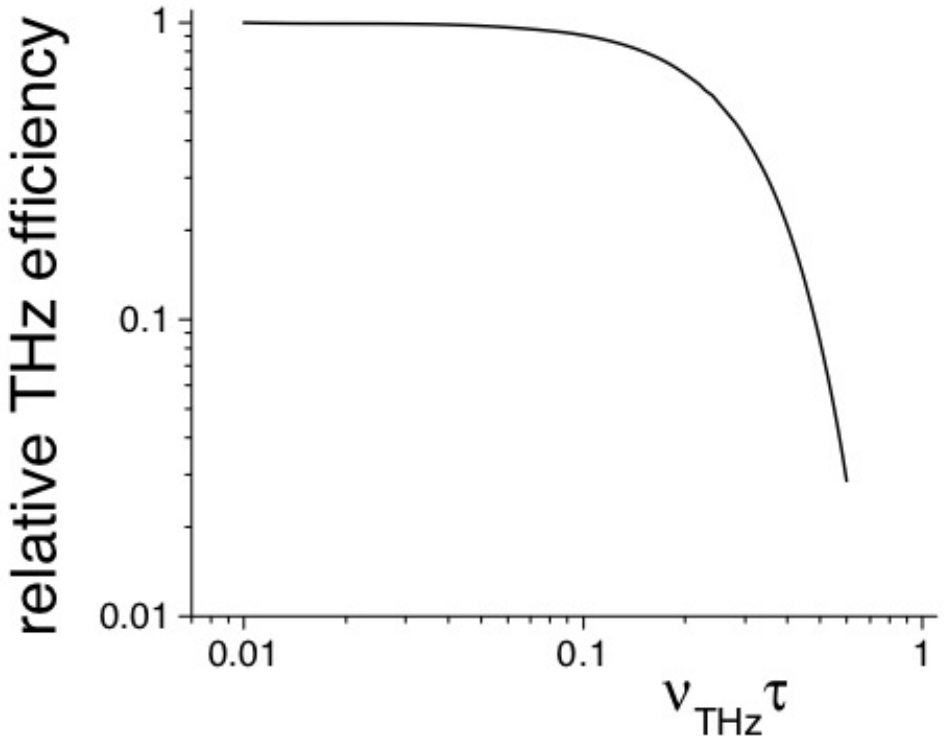


Figure 11.11: Relative THz generation reduction due to $g_1(\Omega_0)$.

For long pulses

$$\eta_{THz}^{PW,lp} = \frac{\Omega^2\ d_{eff}^{QPM,2} E_o^2 \tau}{2\sqrt{2\pi}c^2\ n_{p,opt} n_{p,THz}} L^2 \int_0^{+\infty} e^{\left(-\frac{\tau^2(\Omega-\Omega_0)^2}{4}\right)}\ \mathrm{sinc}^2\left(\frac{\pi}{2}\frac{\Omega-\Omega_0}{\Delta\Omega}\right) d\Omega, \quad (11.35)$$

$$= \frac{\Omega^2\ d_{eff}^{QPM,2} E_o^2 \tau}{2\sqrt{2\pi}c^2\ n_{p,opt} n_{p,THz}} L^2 \int_0^{+\infty} e^{\left(-\frac{\tau^2(\Omega')^2}{4}\right)}\ \mathrm{sinc}^2\left(\frac{\pi}{2}\frac{\Omega'}{\Delta\Omega}\right) d\Omega'. \quad (11.36)$$

Walk-off

$$l_w = \frac{\sqrt{\pi}c\tau}{(n_{g,opt} - n_{p,THz})}$$

$$\eta_{THz}^{PW,lp} = \frac{\Omega^2\ d_{eff}^{QPM,2} E_o^2}{2\sqrt{2\pi}c^2\ n_{p,opt} n_{p,THz}} L^2 \int_0^{+\infty} \exp\left(-\frac{y^2}{4}\right)\ \mathrm{sinc}^2\left(\frac{\sqrt{\pi}}{2}\frac{L}{l_w}y\right) dy,$$

$$\eta_{THz}^{PW,lp} = \frac{\Omega^2\ d_{eff}^{QPM,2} E_o^2}{\sqrt{2}c^2\ n_{p,opt} n_{p,THz}} \frac{l_w L}{\pi} g_2\left(\frac{2l_w}{L}\right)$$

$$g_2(x) = \int_0^{+\infty} \exp\left(-\frac{x^2\mu^2}{\pi}\right)\ \mathrm{sinc}^2(\mu)\ d\mu, \ \text{with } x = \frac{2l_w}{L}.$$

Influence of walk-off

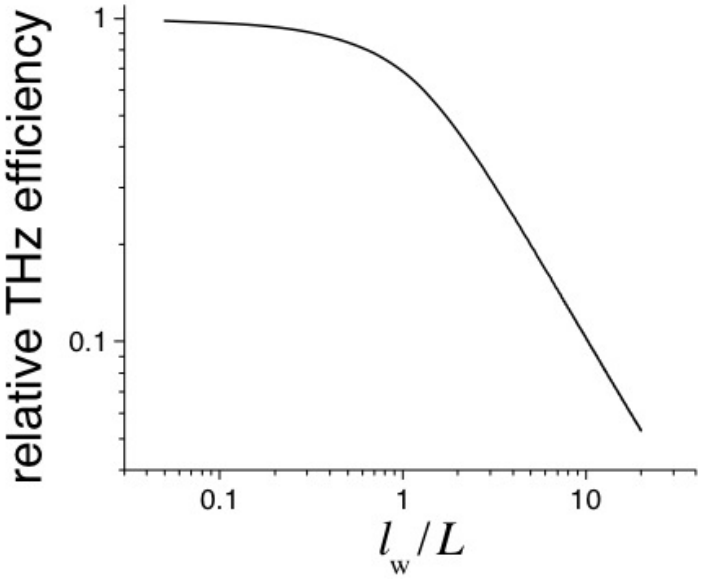


Figure 11.12: Relative THz generation reduction due to $g_2(2\frac{l_w}{L})$.

Very long pulses

$$\eta_{THz}^{PW,lp\to\infty} = \frac{\Omega^2\ d_{eff}^{QPM,2} E_o^2}{2\sqrt{2\pi}c^2\ n_{p,opt} n_{p,THz}} L^2$$

$$= \frac{\Omega^2\ d_{eff}^{QPM,2}}{\pi\varepsilon_0 c^3\ n_{p,opt}^2 n_{p,THz}} \frac{F_{pump}}{\tau} L^2,$$

Optimal length of the EO crystal

$$\eta_{THz}(L) \sim \frac{1}{\alpha_{THz}} \left[1 - e^{-\alpha_{THz} L}\right] = L_{eff}$$

$$\eta_{THz}(L) \sim g_3 L, \text{ with } g_3 = \frac{1}{\alpha_{THz} L} \left[1 - e^{-\alpha_{THz} L}\right]$$

For $L = 1/\alpha_{THz}$, $g_3 = 0.63$.

Optimal focusing

$$\eta_{THz} = \frac{U_{THz}}{U_{pump}} = g_1 \; g_3 \; \frac{2\Omega^2 \; d_{eff}^{QPM,2} L}{\varepsilon_0 \; c^2 \; n_{p,opt}^2 n_{p,THz} \left(n_{g,opt} - n_{p,THz}\right)} \; \frac{U_{pump}}{\pi w^2}$$

$$\xi = (\lambda \; L / 2\pi n_{THz} w^2), \text{ where } \lambda \text{ is the THz wavelength}$$

Ratio between crystal length and THz Rayleigh range

Enhancement factor

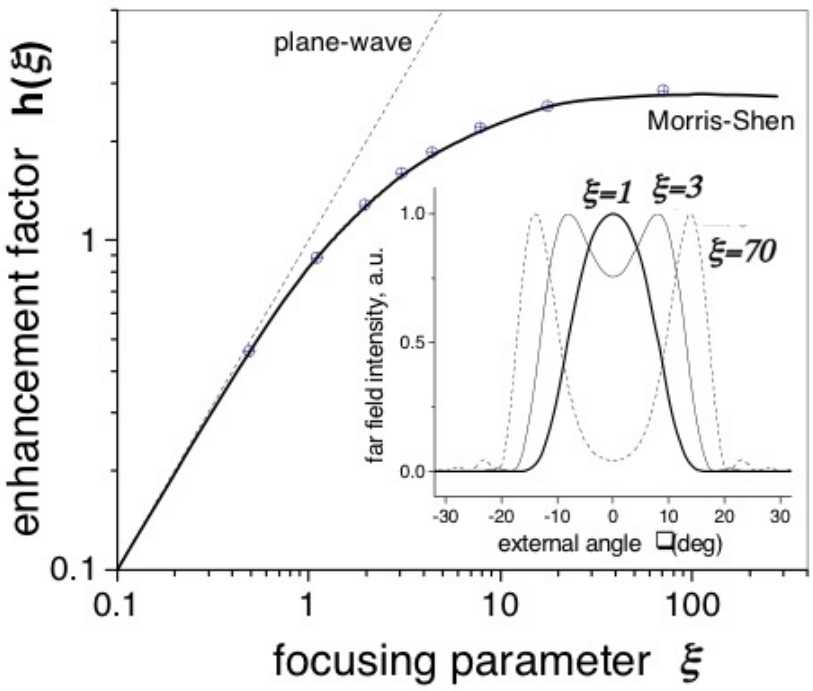


Figure 11.13: Enhancement factor h as a function of the focusing parameter ξ. Solid curve is based on Ref. [26]. Dashed curve – plane-wave approximation. Dots represent calculations based on the Green's function method. Inset: far-field THz intensity profiles at different ξ for a 1-cm-long GaAs.

Cascading and red shift

optical pulse spectrum will be red-shifted by $\Delta\omega/\omega_0 \sim \eta_{THz}$

$N = 0.5 \times$ (acceptance bandwidth) / (terahertz frequency).

$$\begin{aligned}\frac{d\Delta k}{d\omega} &= \frac{\Omega}{c}\frac{dn_{g,opt}(\omega)}{d\omega} = \frac{\Omega}{c}\frac{dn_{g,opt}(\lambda)}{d\lambda}\frac{d\lambda}{d\omega} \\ &= \frac{\Omega}{c}\frac{dn_{g,opt}(\lambda)}{d\lambda}\frac{\lambda^2}{2\pi c}\end{aligned}$$

$$\frac{d\Delta k}{d\omega} L\Delta\omega_{acc} = 2\pi$$

$$\Delta\omega_{acc} = \frac{2\pi c\omega}{L\Omega}\left(\lambda\frac{dn_{g,opt}(\lambda)}{d\lambda}\right)^{-1}.$$

Cascading and red shift

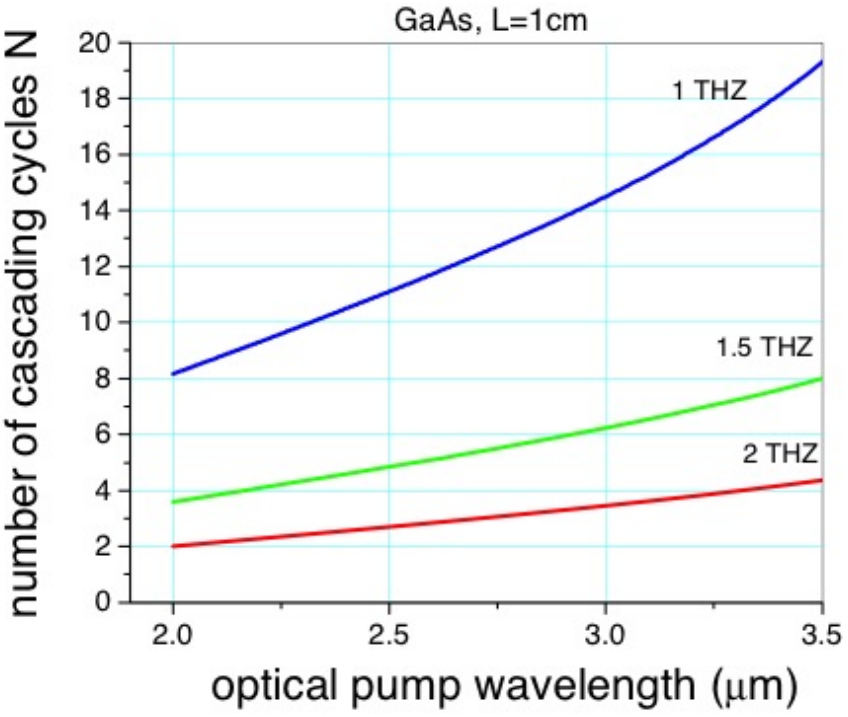


Figure 11.14: Number of THz cascading cycles as a function of THz frequency and pump wavelength for GaAs, $L = 1$ cm.

Summary

$$FOM_1 = \frac{d_{eff}^2}{n_{p,opt}\alpha_{THz}} \tag{11.45}$$

or

$$FOM_2 = \frac{d_{eff}^2}{n_{p,opt}^2(n_{p,THz} - n_{g,opt})}. \tag{11.46}$$

If the maximum propagation distance is limited by the Kerr effect, the critical FOM becomes

$$FOM_3 = \frac{\lambda_{opt}\ d_{eff}^2}{n_{p,opt}^2 n_{p,THz}\alpha_{THz} n_2}. \tag{11.47}$$