Nonlinear Optics (WiSe 2017/18)
Lecture 14: December 5, 2017

9 Optical Parametric Amplifiers and Oscillators

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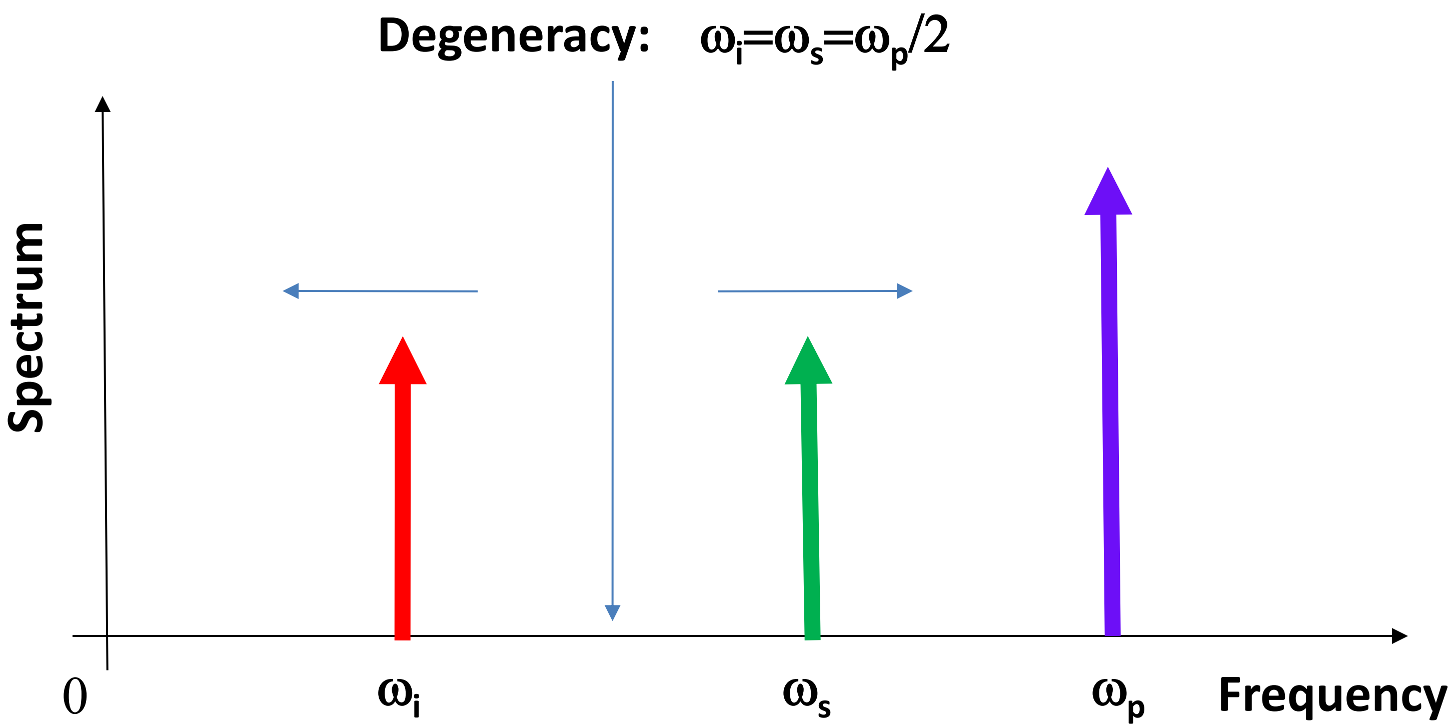
using noncollinear phase matching
9.10 Optical parametric chirped-pulse amplification (OPCPA)

[5] Largely follows the review paper by G. Cerullo *et al.*, "Ultrafast Optical Parametric Amplifiers," Rev. Sci. Instrum. 74, 1-17 (2003)

9 Optical Parametric Amplifiers and Oscillators

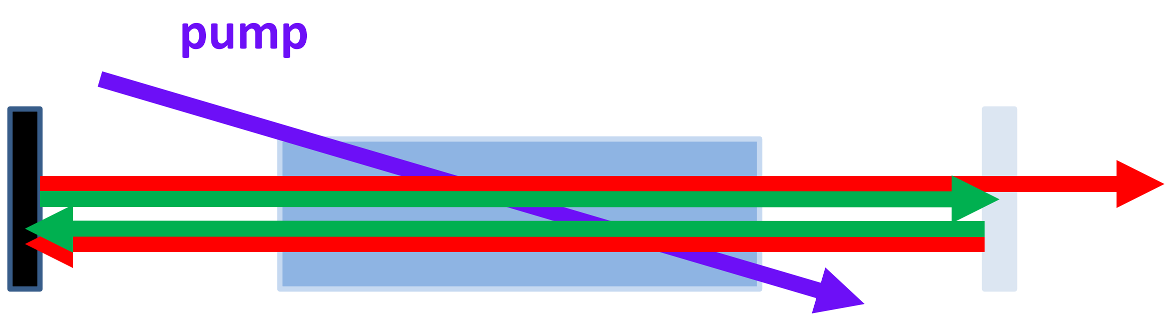
9.1 Optical Parametric Generation (OPG)



energy conservation: $$\hbar\omega_p = \hbar\omega_s + \hbar\omega_i.$$

momentum conservation: $$\hbar\vec{k}_p = \hbar\vec{k}_s + \hbar\vec{k}_i$$

Optical Parametric Oscillator (OPO)



double resonant: signal and idler resonant

single resonant: only signal resonant

Advantage: Widely tunable, both signal and idler can be used!

For OPO to operate, less gain is necessary in contrast to an OPA

Nonlinear Optical Susceptibilities

Total field: pump, signal and idler:

$$\vec{E}(\vec{r},t) = \sum_{\omega_a>0} \sum_{i=1}^{3} \frac{1}{2} \left\{ \hat{E}_i(\omega_a) e^{j(\omega_a t - \vec{k}_a \vec{r})} + c.c. \right\} \vec{e}_i.$$

Drives polarization in medium:

$$\vec{P}(\vec{r},t) = \sum_{n} \vec{P}^{(n)}(\vec{r},t)$$

Polarization can be expanded in power series of the electric field:

$$\vec{P}^{(n)}(\vec{r},t) = \sum_{\omega_b>0} \sum_{i=1}^{3} \frac{1}{2} \left\{ P_i^{(n)}(\omega_b) e^{j(\omega_b t - \vec{k}'_b \vec{r})} + c.c. \right\} \vec{e}_i.$$

Defines susceptibility tensor:

$$P_i^{(n)}(\omega_b) = \frac{\varepsilon_0}{2^{m-1}} \sum_{P} \sum_{j...k} \chi_{ij...k}^{(n)}(\omega_b : \omega_1, ..., \omega_n) E_j(\omega_1) \cdots E_k(\omega_n)$$

$$\omega_b = \sum_{i=1}^{n} \omega_i \text{ and } \mathbf{k}'_b = \sum_{i=1}^{n} \mathbf{k}_i$$

Special Cases

$$\hat{P}_i^{(2)}(\omega_3) = \varepsilon_0 \sum_{jk} \chi_{ijk}^{(2)}(\omega_3 : \omega_1, -\omega_2)\hat{E}_j(\omega_1)\hat{E}_k^*(\omega_2),$$
$$\omega_3 = \omega_1 - \omega_2 \text{ und } \mathbf{k}_3' = \mathbf{k}_1 - \mathbf{k}_2.$$

($\longrightarrow$ Difference Frequency Generation (DFG))

$$\hat{P}_i^{(2)}(\omega_2) = \varepsilon_0 \sum_{jk} \chi_{ijk}^{(2)}(\omega_2 : \omega_3, -\omega_1)\hat{E}_j(\omega_3)\hat{E}_k^*(\omega_1),$$
$$\omega_2 = \omega_3 - \omega_1 \text{ und } \mathbf{k}_2' = \mathbf{k}_3 - \mathbf{k}_1.$$

($\longrightarrow$ Parametric Generation (OPG))

$$\hat{P}_i^{(3)}(\omega_4) = \frac{6\varepsilon_0}{4} \sum_{jkl} \chi_{ijkl}^{(3)}(\omega_4 : \omega_1, \omega_2, -\omega_3)\hat{E}_j(\omega_1)\hat{E}_k(\omega_2)\hat{E}_l^*(\omega_3)$$
$$\omega_4 = \omega_1 + \omega_2 - \omega_3 \text{ und } \mathbf{k}_4' = \mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}_3.$$

($\longrightarrow$ Four Wave Mixing (FWM))

9.2 Continuous-wave OPA

Wave equation :

$$\left(\Delta - \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\right)\vec{E} = \mu_0 \frac{\partial^2}{\partial t^2}\vec{P}.$$

Include linear and second-order terms:

$$\left(\Delta - \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\right)\vec{E} = \mu_0 \frac{\partial^2}{\partial t^2}\left(\vec{P}^{(l)}(\vec{r},t) + \vec{P}^{(2)}(\vec{r},t)\right)$$

Changes group and phase velocities of waves

$k(\omega)$

Nonlinear interaction of waves

z-propagation only:

$$\vec{E}_{p,s,i}(z,t) = \mathrm{Re}\left\{E_{p,s,i}(z)\; e^{j(\omega_{p,s,i}t - k_{p,s,i}\;z)}\vec{e}_{p,s,i}\right\}$$

Wave amplitudes

$$\vec{P}^{(2)}_{p,s,i}(z,t) = \mathrm{Re}\left\{ P^{(2)}_{p,s,i}(z)\ e^{j\left(\omega_{p,s,i}t - k'_{p,s,i}z\right)}\vec{e}_{p,s,i} \right\}$$

Separate into three equations for each frequency component:

Slowly varying amplitude approximation:

$$d^2_{p,s,i}E(z)\ /dz^2 << \ 2k\ dE_{p,s,i}(z)\ /dz,$$

$$\frac{\partial E_{p,s,i}(z)}{\partial z} = -\frac{jc_0^2\omega_{p,s,i}}{2n(\omega_{p,s,i})}P^{(2)}_{p,s,i}(z)\ e^{-j\left(k'_{p,s,i} - k_{p,s,i}\right)z}$$

Introduce phase mismatch: $\Delta k = k(\omega_p) - k(\omega_s) - k(\omega_i)$

and effective nonlinearity and coupling coefficients:

$$d_{eff} = \frac{1}{2}\chi^{(2)}_{ijk}(\omega_p : \omega_s, \omega_i), \quad \kappa_{p,s,i} = \omega_{p,s,i}\ d_{eff}/(n_{p,s,i}c_0)$$

Coupled wave equations:

$$\frac{\partial E_p(z)}{\partial z} = -j\kappa_p \; E_s(z)E_i(z)\; e^{j\Delta kz} \;,$$
$$\frac{\partial E_s(z)}{\partial z} = -j\kappa_s \; E_p(z)E_i^*(z)\; e^{-j\Delta kz}, \qquad \mathbf{X} \;\; n_{p,s,i}c_0\varepsilon_0 E_{p,s,i}^*/2$$
$$\frac{\partial E_i(z)}{\partial z} = -j\kappa_i \; E_p(z)E_s^*(z)\; e^{-j\Delta kz}.$$

Intensity of waves: $$I_{p,s,i} = \frac{n_{p,s,i}}{2Z_{F_0}} \left|E_{p,s,i}\right|^2$$

Manley-Rowe relations: $$-\frac{1}{\omega_p}\frac{dI_p}{dz} = \frac{1}{\omega_s}\frac{dI_s}{dz} = \frac{1}{\omega_i}\frac{dI_i}{dz}$$

9.4 Theory of Optical Parametric Amplification

Undepleted pump approximation: $E_p = const.$

$$\begin{aligned} \frac{\partial E_s(z)}{\partial z} &= -j\kappa_s\ E_p E_i^*(z)\ e^{-j\Delta k z}, \\ \frac{\partial E_i(z)}{\partial z} &= -j\kappa_i\ E_p E_s^*(z)\ e^{-j\Delta k z}. \end{aligned}$$

with:

$$E_s(z=0) = E_s(0) \qquad E_i(z=0) = 0$$

$$E_s(z)\tilde{}\,E_s(0)\ e^{gz - j\Delta k z/2} \text{ and } E_i(z)\tilde{}\,E_i(0)\ e^{gz - j\Delta k z/2}$$

$$\longrightarrow \quad \begin{vmatrix} g - j\frac{\Delta k}{2} & j\kappa_s\ E_p \\ j\kappa_i\ E_p^* & g + j\frac{\Delta k}{2} \end{vmatrix} = 0$$

$$g = \sqrt{\Gamma^2 - \left(\frac{\Delta k}{2}\right)^2}, \text{ with } \Gamma = \sqrt{\kappa_i\ \kappa_s\ \left|E_p\right|^2}.$$

gain

max. gain, when phase matched

Maximum gain

$$\boxed{\Gamma^2 = \frac{\omega_s \omega_i}{n_s n_i c_0^2} d_{eff}^2 \;\; |E_p|^2 = \frac{2Z_{F_0}\omega_s \omega_i}{n_p n_s n_i c_0^2} d_{eff}^2 \;\; \cancel{|I_p|^2}} \qquad \boxed{FOM = \frac{d_{eff}}{\sqrt{\lambda_s \lambda_i n_p n_s n_i}}}$$

General solutions:

$$E_s(z) = \{E_s(0) \cosh gz + B \sinh gz\}\, e^{-j\Delta k z/2} \qquad B = -j\frac{\Delta k}{2g} E_s(0) - j\frac{\kappa_1}{g} E_p E_i^*(0)$$

$$E_i(z) = \{E_i(0) \cosh gz + D \sinh gz\}\, e^{-j\Delta k z/2} \qquad D = -j\frac{\Delta k}{2g} E_i(0) - j\frac{\kappa_2}{g} E_p^* E_s^*(0)$$

Here:

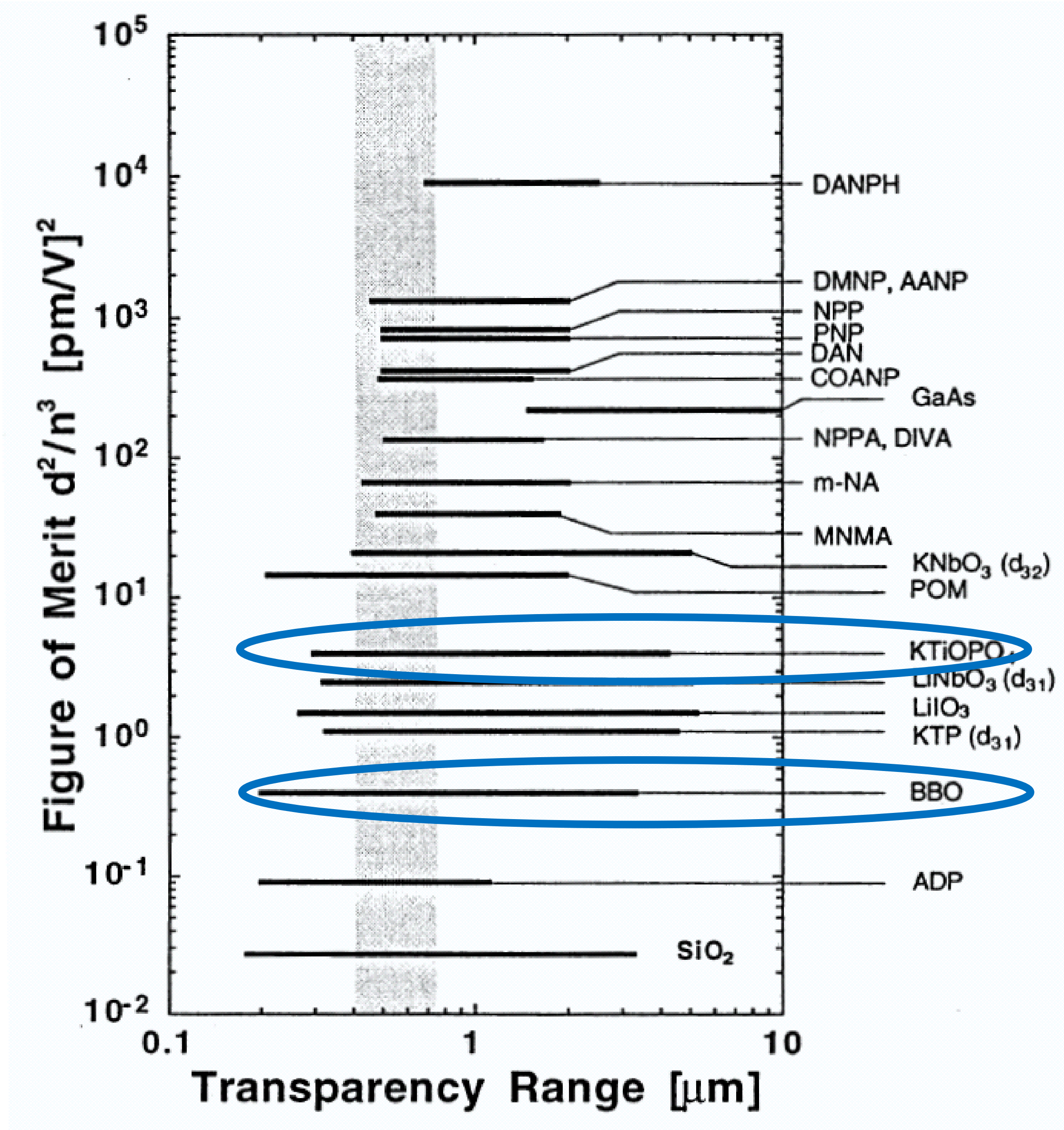
$$\begin{aligned} I_s(L) &= I_s(0) \left[1 + \frac{\Gamma^2}{g^2} \sinh^2 gL\right] \\ I_i(L) &= I_s(0)\, \frac{\omega_i}{\omega_s} \frac{\Gamma^2}{g^2} \sinh^2 gL. \end{aligned}$$

For large gain: $\Gamma L >> 1$

$$\begin{aligned} I_s(L) &= \frac{1}{4} I_s(0)\; e^{2\Gamma L}, \\ I_i(L) &= \frac{1}{4} I_s(0)\; \frac{\omega_i}{\omega_s}\; e^{2\Gamma L} \end{aligned} \quad \longrightarrow \quad G = \frac{I_s(L)}{I_s(0)} = \frac{1}{4}\; e^{2\Gamma L}$$

Figure of merit:

$$\boxed{FOM = \frac{d_{eff}}{\sqrt{\lambda_s \lambda_i n_p n_s n_i}}}$$



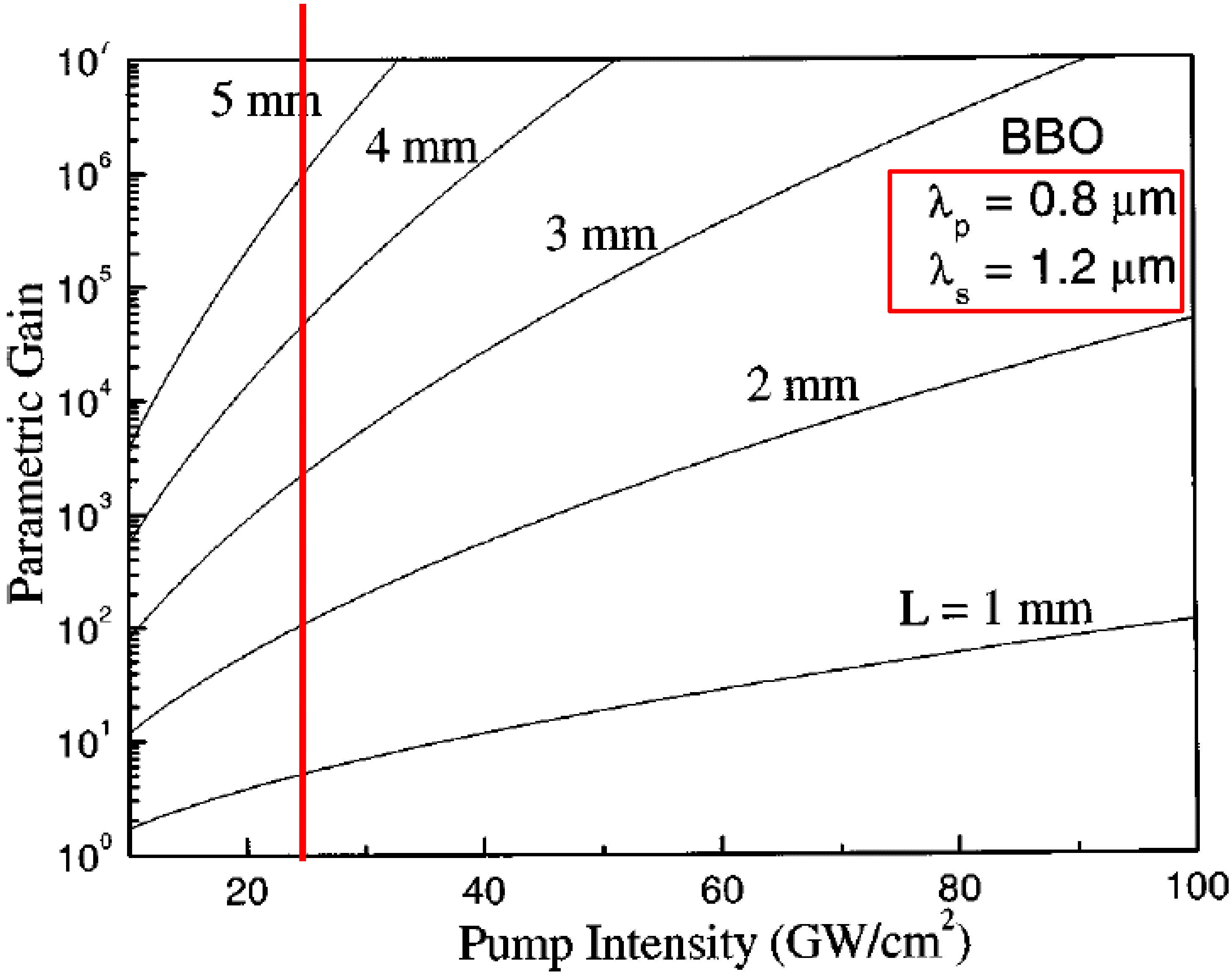


Fig. 9.3 Parametric gain for an OPA at the pump wavelength λ_p = 0.8 μm and the signal wavelength λ_s = 1.2 μm, using type-I phase matching in BBO (d_{eff} = 2 pm/V).

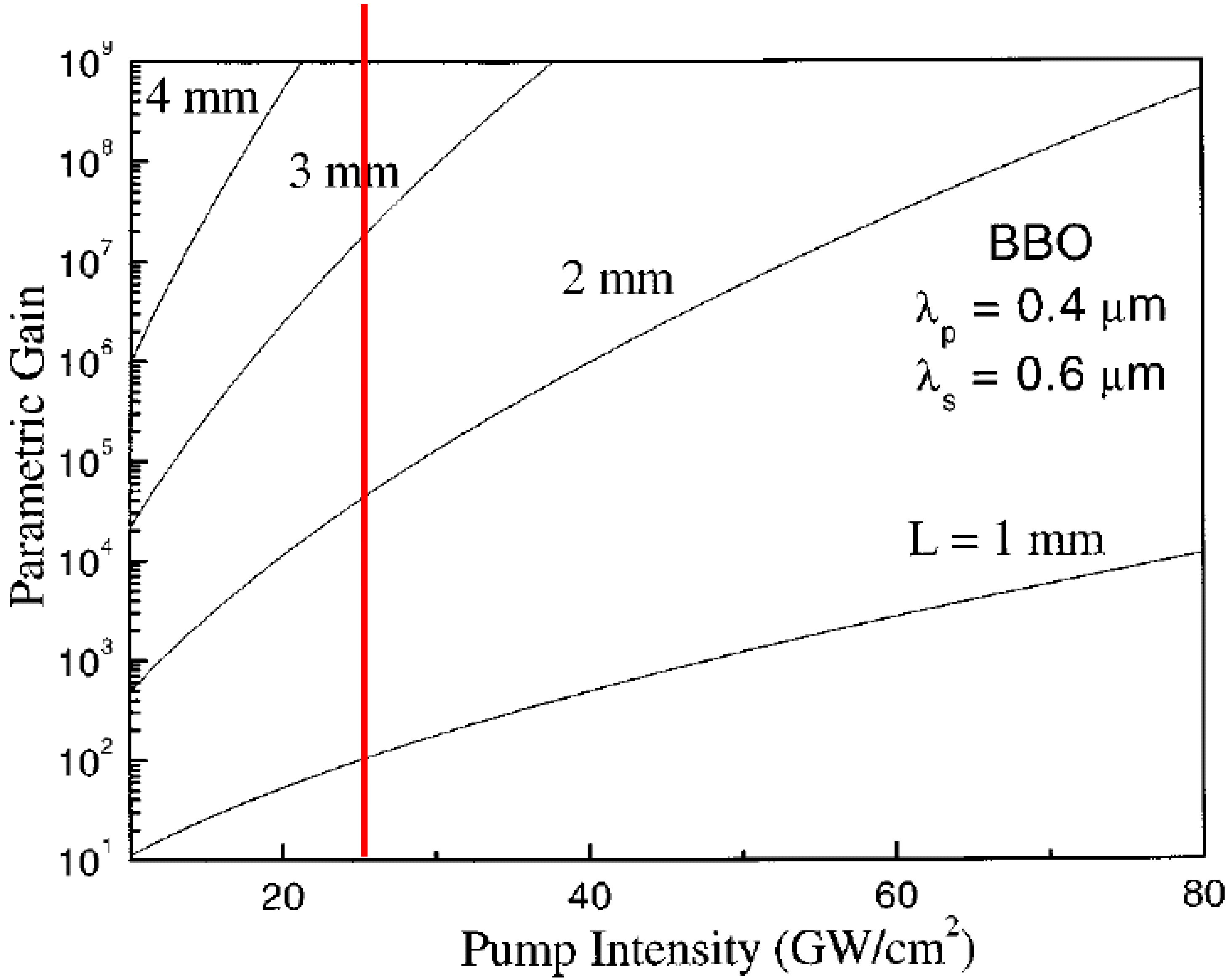


Fig. 9.4 Parametric gain for an OPA at the pump wavelength $\lambda_p = 0.4\ \mu m$ and the signal wavelength $\lambda_s = 0.6\ \mu m$, using type-I phase matching in BBO (d_{eff} = 2 pm/V).

9.4 Phase Matching

$$\Delta k = 0 \longrightarrow n_p = \frac{n_s\omega_s + n_i\omega_i}{\omega_p}$$

Uniaxial crystal: $n_e < n_o$

Type I: noncritical

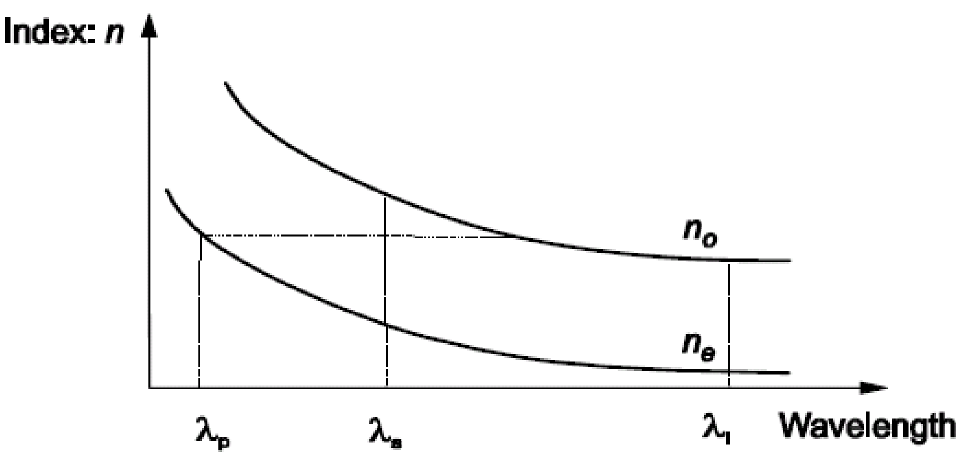


Fig. 9.5 Type-I noncritical phase matching.

Type I: critical

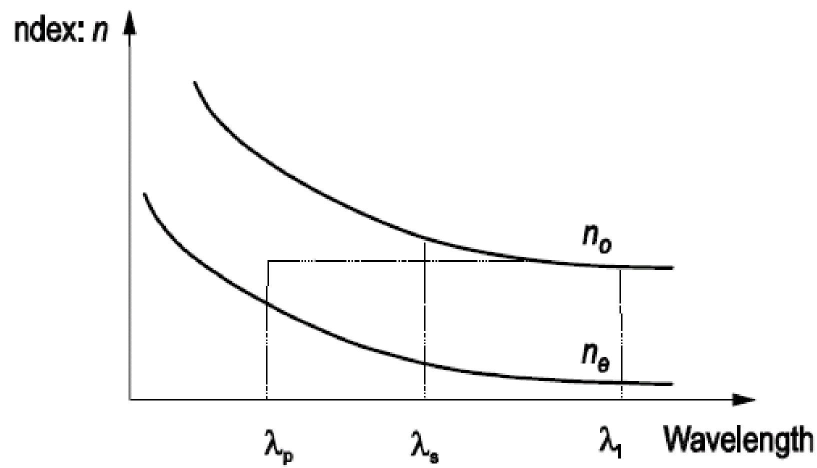
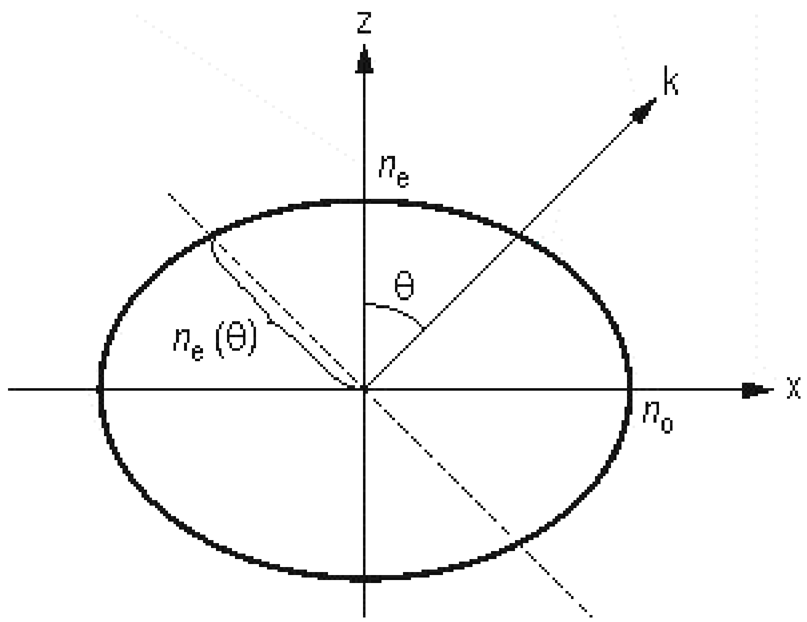


Fig. 9.6 Type-I critical phase matching by adjusting the angle θ between wave vector of the propagating beam and the optical axis.

9.4 Phase Matching



Critical Phase Matching

$$n_{ep}(\theta)\omega_p = n_{os}\omega_s + n_{oi}\omega_i$$

$$\frac{1}{n_{ep}(\theta)^2} = \frac{\sin^2\theta}{n_{ep}^2} + \frac{\cos^2\theta}{n_{op}^2}$$

$$\theta = \arcsin\left[\frac{n_{ep}}{n_{ep}(\theta)}\sqrt{\frac{n_{op}^2 - n_{ep}^2(\theta)}{n_{op}^2 - n_{ep}^2}}\right]$$

9.4 Phase Matching

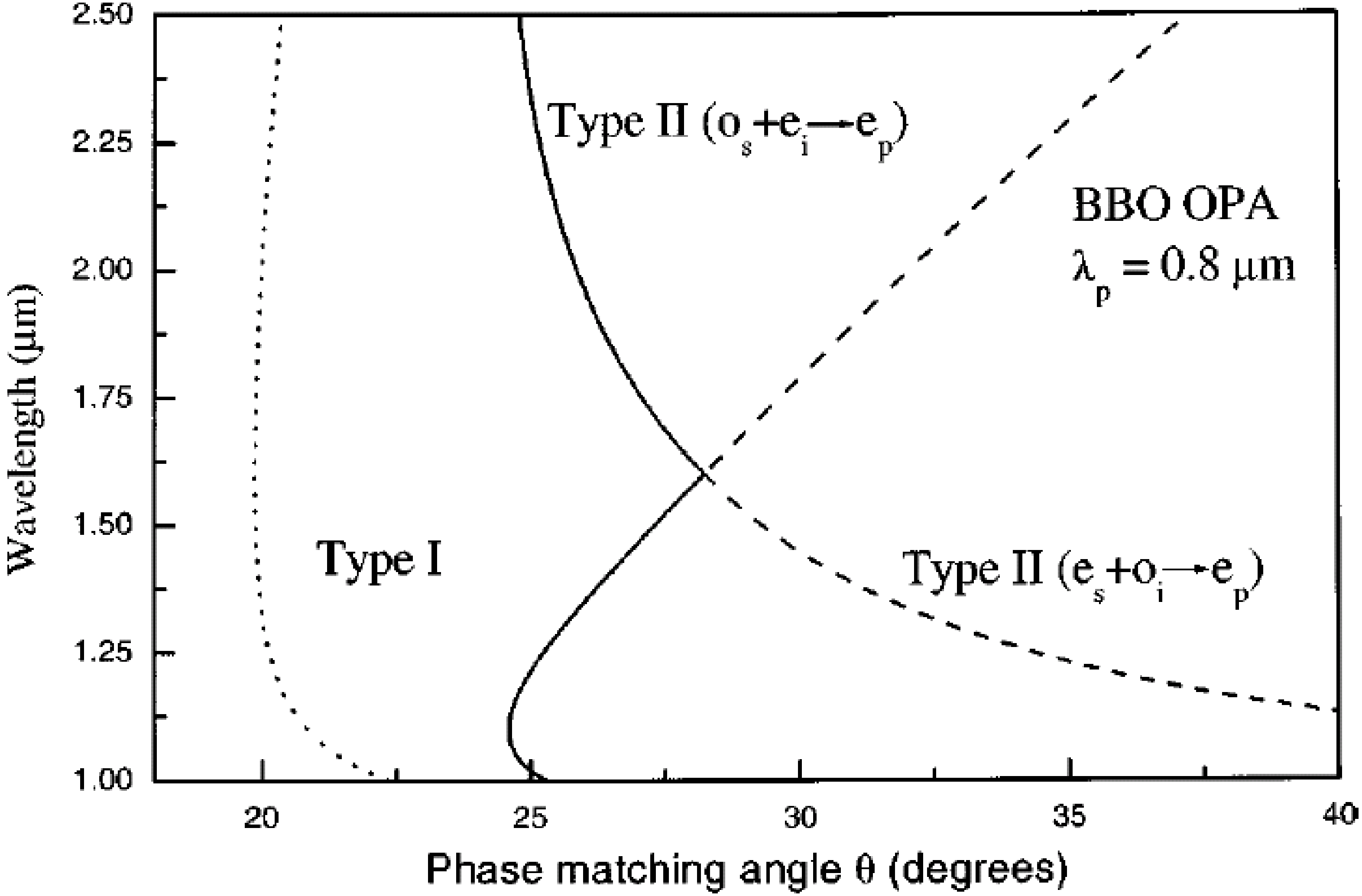


Fig. 9.7 Angle tuning curves for a BBO OPA at the pump wavelength λ_p=0.8 μm for type-I phase matching (dotted line), type-II ($o_s + e_i \rightarrow e_p$) phase matching (solid line), and type-II ($e_s + o_i \rightarrow e_p$) phase matching (dashed line).

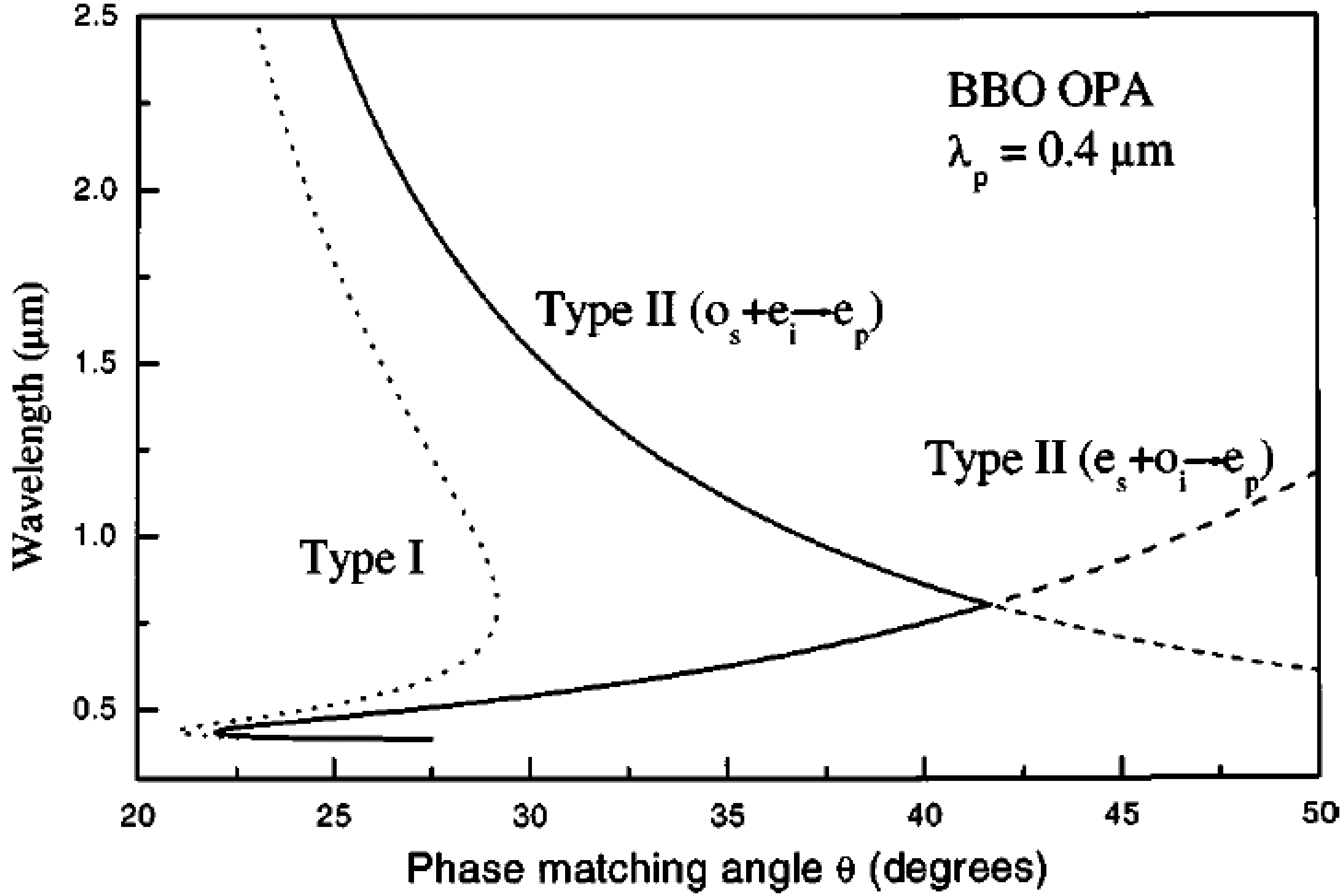


Fig. 9.8 Angle tuning curves for a BBO OPA at the pump wavelength λ_p=0.4 µm for type-I phase matching (dotted line), type-II ($o_s + e_i \rightarrow e_p$) phase matching (solid line), and type-II ($e_s + o_i \rightarrow e_p$) phase matching (dashed line).

9.5 Quasi Phase Matching

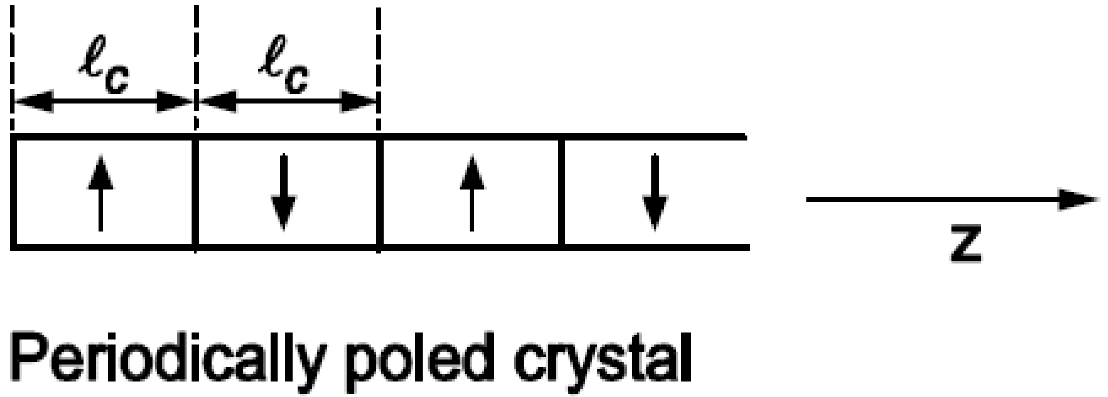


Fig.12.30: Variation of d_{eff} in a quasi phase matched material as a function of propagation distance.

$$d_{eff}(z) = \sum_{m=-\infty}^{+\infty} d_m e^{jm\kappa z}$$

$$\frac{\partial E_p(z)}{\partial z} = -j\kappa_p\, E_s(z)E_i(z)\, e^{j\Delta kz}$$

9.6 Ultrashort-Pulse Optical Parametric Amplification

$$\vec{E}_{p,s,i}(z,t) = \mathrm{Re}\left\{E_{p,s,i}(z,t)\ e^{j(\omega_{p,s,i}t - k_{p,s,i}\ z)}\vec{e}_{p,s,i}\right\}$$

Pulse envelopes

$$\begin{aligned}
\frac{\partial E_p}{\partial z} + \frac{1}{v_p}\frac{\partial E_p}{\partial t} &= -j\kappa_p\ E_s E_i\ e^{j\Delta kz}\ , \\
\frac{\partial E_s}{\partial z} + \frac{1}{v_s}\frac{\partial E_s}{\partial t} &= -j\kappa_s\ E_p E_i^*\ e^{-j\Delta kz}, \\
\frac{\partial E_i}{\partial z} + \frac{1}{v_i}\frac{\partial E_s}{\partial t} &= -j\kappa_i\ E_p E_s^*\ e^{-j\Delta kz},
\end{aligned}$$

$v_{p,s,i} = dk/d\omega|_{\omega_{p,s,i}}$ are the corresponding group velocities

$t' = t - z/v_p$

$$\begin{aligned}
\frac{\partial E_p}{\partial z} &= -j\kappa_p\ E_s E_i\ e^{j\Delta kz}\ , \\
\frac{\partial E_s}{\partial z} + \left(\frac{1}{v_s} - \frac{1}{v_p}\right)\frac{\partial E_s}{\partial t} &= -j\kappa_s\ E_p E_i^*\ e^{-j\Delta kz}, \\
\frac{\partial E_i}{\partial z} + \left(\frac{1}{v_i} - \frac{1}{v_p}\right)\frac{\partial E_s}{\partial t} &= -j\kappa_i\ E_p E_s^*\ e^{-j\Delta kz}.
\end{aligned}$$

Temporal walkoff
Group Velocity Mismatch (GVM)

Pump pulse width

$$\ell_{jp} = \frac{\tau}{\delta_{jp}}, \ \text{with } \delta_{jp} = \left(\frac{1}{v_j} - \frac{1}{v_p}\right)$$

j=s,i

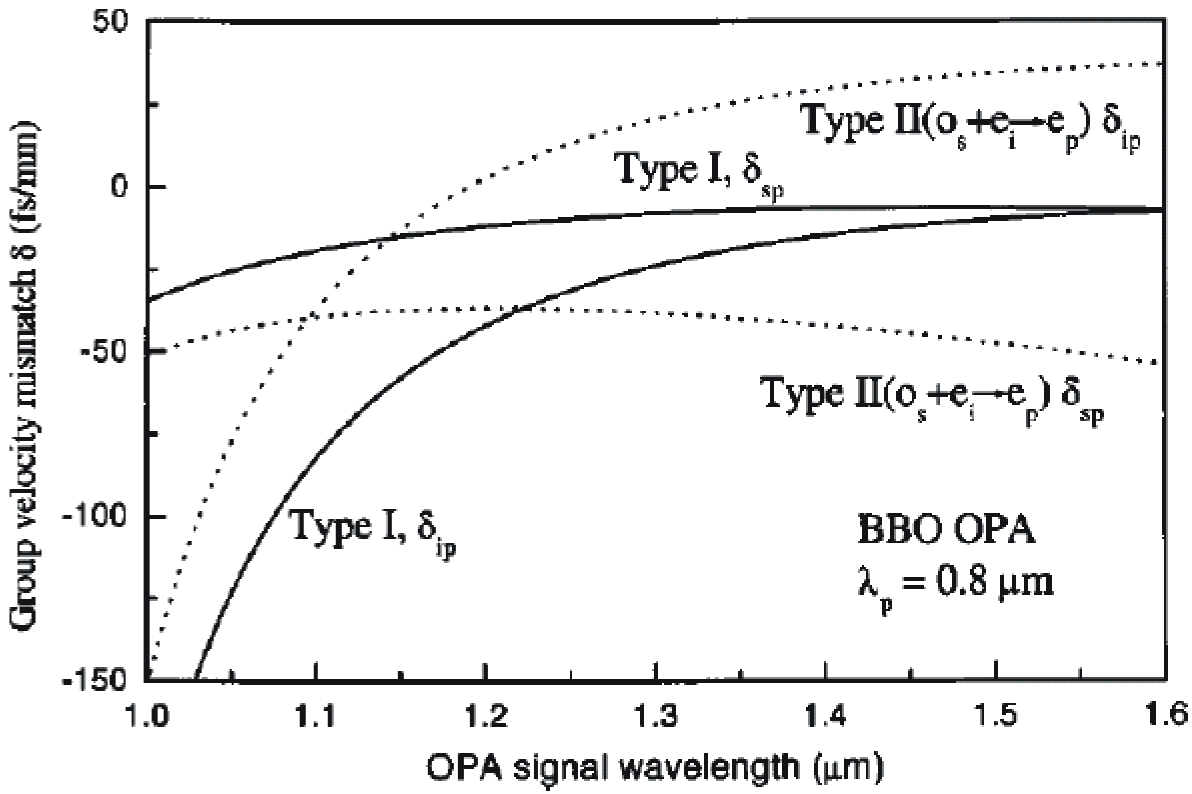


Fig. 9.9: Pump-signal (δ_{sp}) and pump-idler (δ_{ip}) group velocity mismatch curves for a BBO OPA at the pump wavelength λ_p=0.8 µm for type-I phase matching (solid line) and type-II ($o_s + e_i \rightarrow e_p$) phase matching (dashed line).

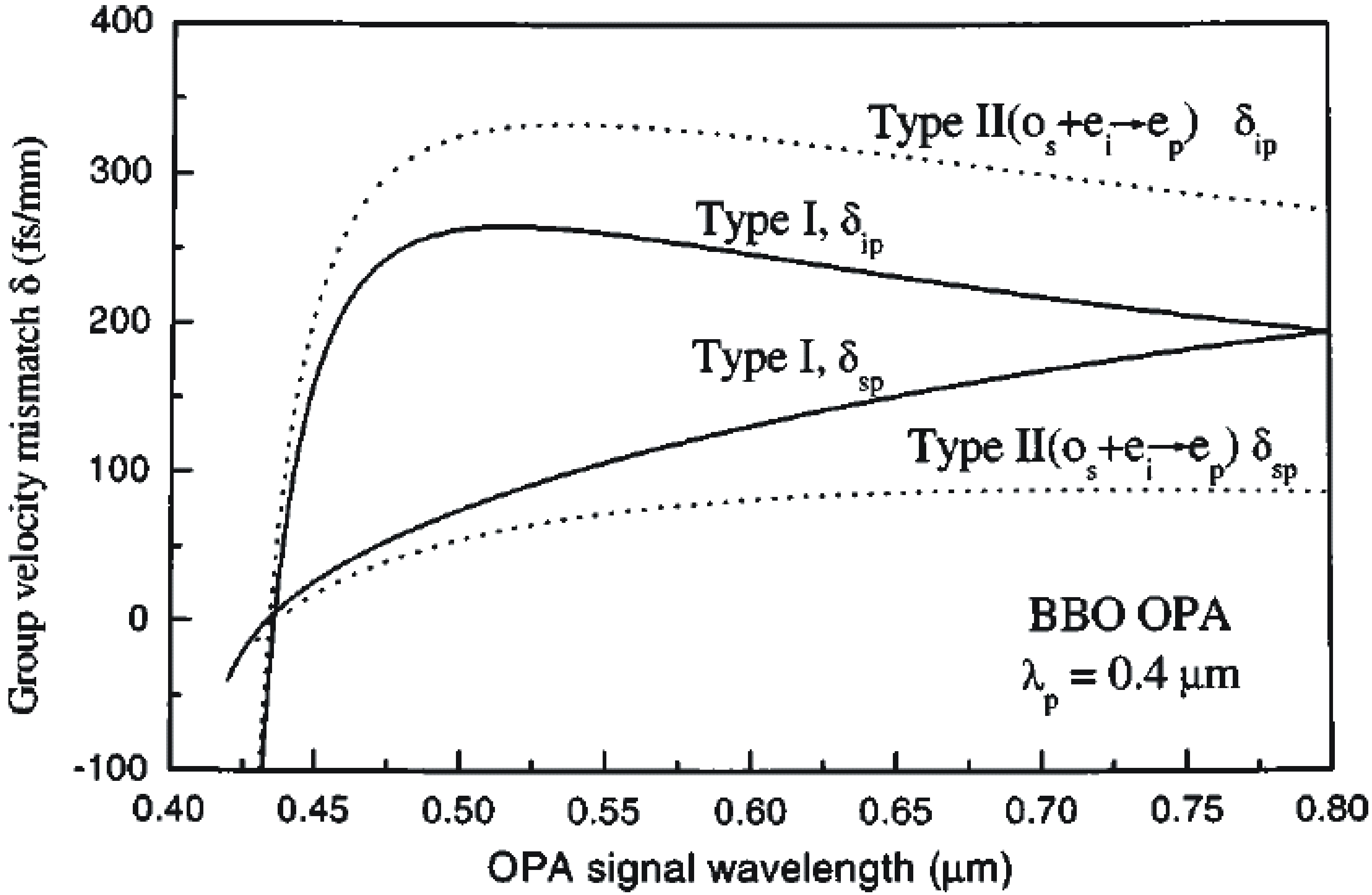


Fig. 9.10: Pump-signal (δ_{sp}) and pump-idler (δ_{ip}) group velocity mismatch curves for a BBO OPA at the pump wavelength λ_p=0.4 μm for type-I phase matching (solid line) and type-II (o_s + $e_i \rightarrow e_p$) phase matching (dashed line).

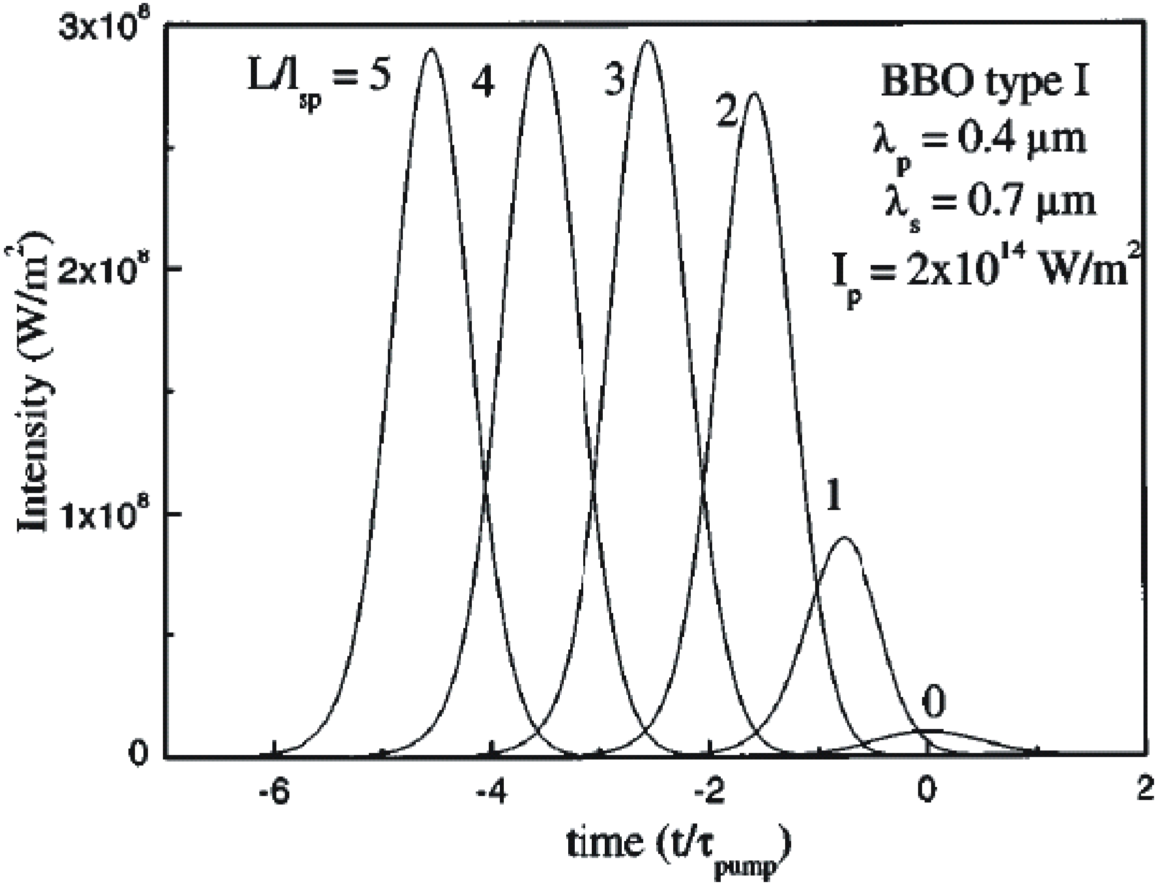


Fig. 9.11: Signal pulse evolution for a BBO type-I OPA with λ_p = 0.4 µm, λ_s = 0.7 µm, for different lengths L of the nonlinear crystal. Pump intensity is 20 GW/cm^2. Time is normalized to the pump pulse duration and the crystal length to the pump-signal pulse splitting length. [5]

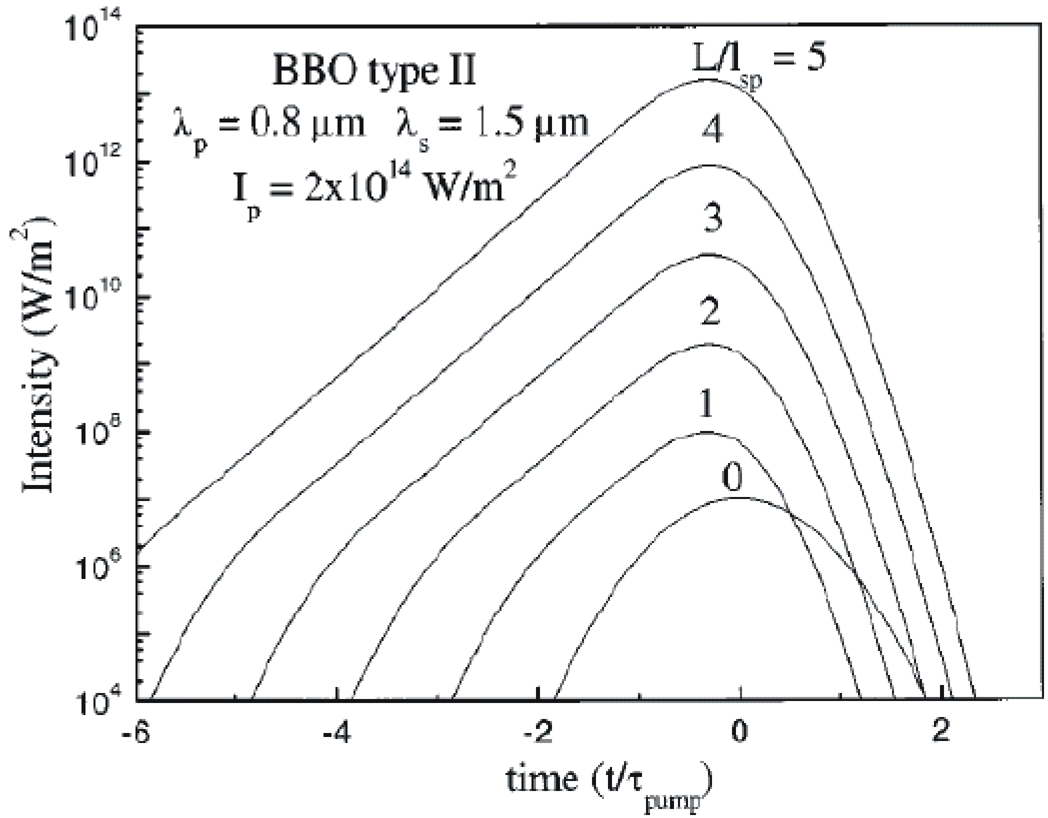


Figure 9.12: Signal pulse evolution for a BBO type-I OPA with λ_p = 0.8 µm, λ_s = 1.5 µm, for different lengths L of the nonlinear crystal. Pump intensity is 20 GW/cm^2. Time is normalized to the pump pulse duration and the crystal length to the pump-signal pulse splitting length. [5]

OPA Bandwidth

$$\omega_s \longrightarrow \omega_s + \Delta\omega \qquad \omega_i \longrightarrow \omega_i - \Delta\omega$$

$$\Delta k = -\frac{dk_s}{d\omega}\Delta\omega + \frac{dk_i}{d\omega}\Delta\omega = \left(\frac{1}{v_i} - \frac{1}{v_s}\right)\Delta\omega$$

Bandwidth limitation due to GVM

$$\Delta f = -\frac{2\sqrt{\ln 2}}{\pi}\sqrt{\frac{\Gamma}{L}}\frac{1}{\left|\frac{1}{v_i} - \frac{1}{v_s}\right|}$$

For signal-idler group velocity matching:

$$\Delta f = -\frac{2\sqrt[4]{\ln 2}}{\pi}\sqrt[4]{\frac{\Gamma}{L}}\frac{1}{\left|\frac{d^2k_s}{d\omega^2} + \frac{d^2k_s}{d\omega^2}\right|}.$$

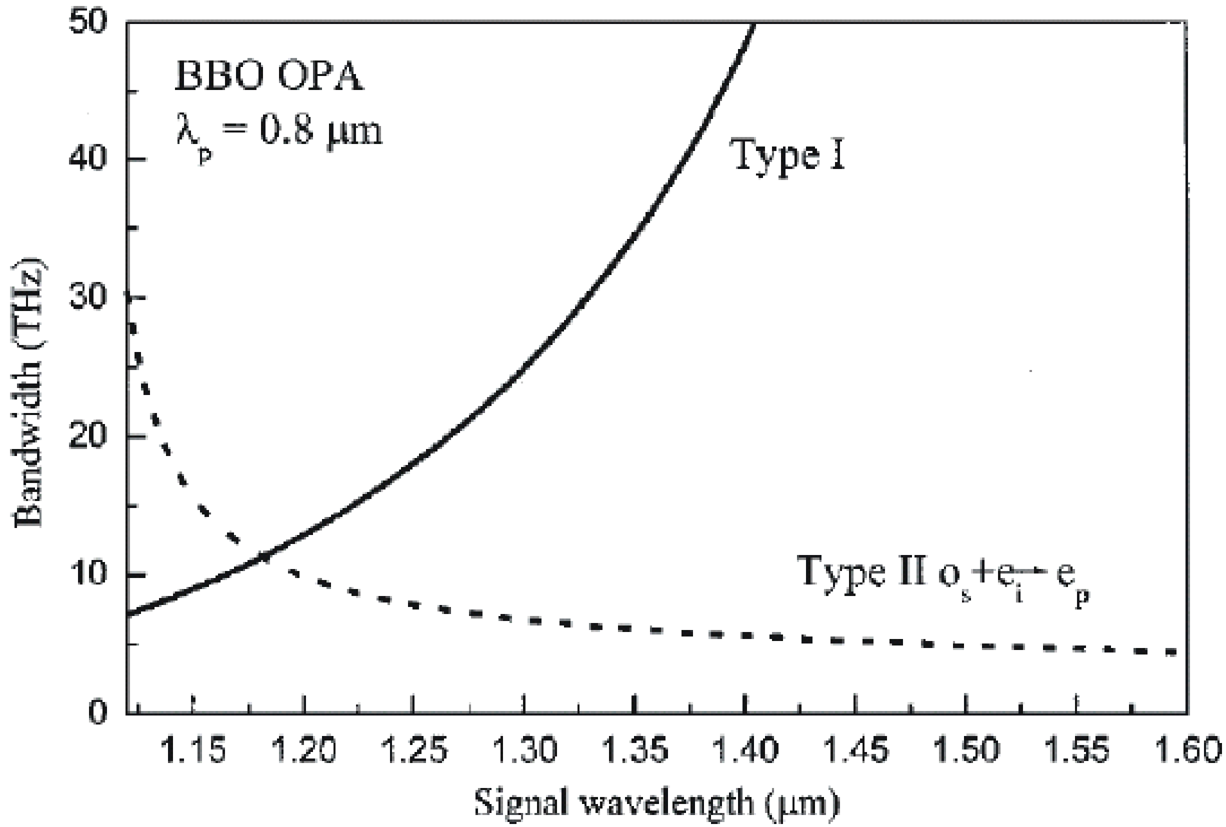


Fig. 9.13: Phase matching bandwidth for a BBO OPA at the pump wavelength λ_p=0.8 µm for type-I phase matching (solid line) and type-II ($o_s + e_i \rightarrow e_p$) phase matching (dashed line). Crystal length is 4 mm and pump intensity 50 GW/cm^2.

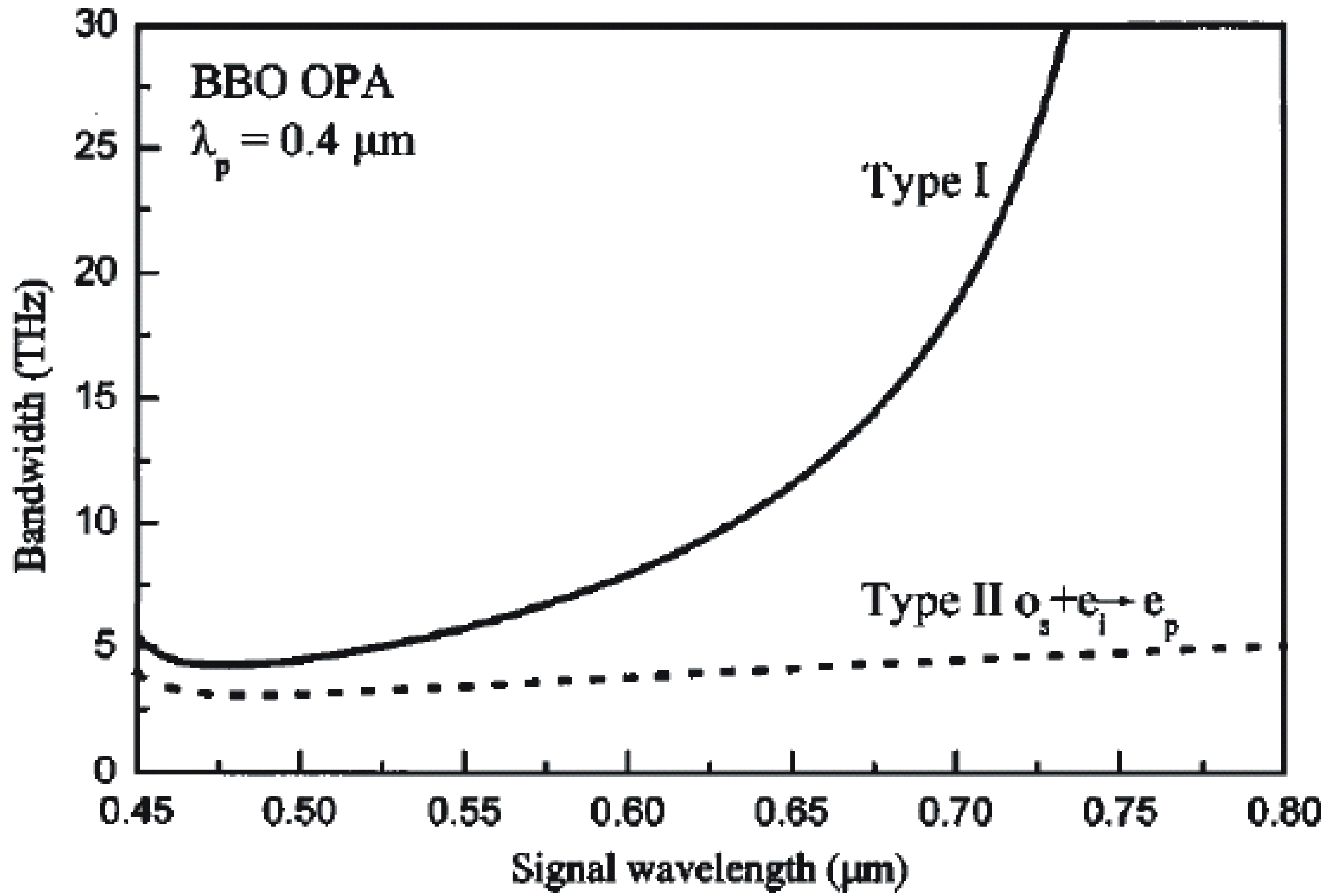


Fig. 9.14: Phase matching bandwidth for a BBO OPA at the pump wavelength λ_p=0.4 µm for type-I phase matching (solid line) and type-II ($o_s + e_i \rightarrow e_p$) phase matching (dashed line). Crystal length is 2 mm and pump intensity 100 GW/cm^2.

9.7 Optical Parametric Amplifier Designs

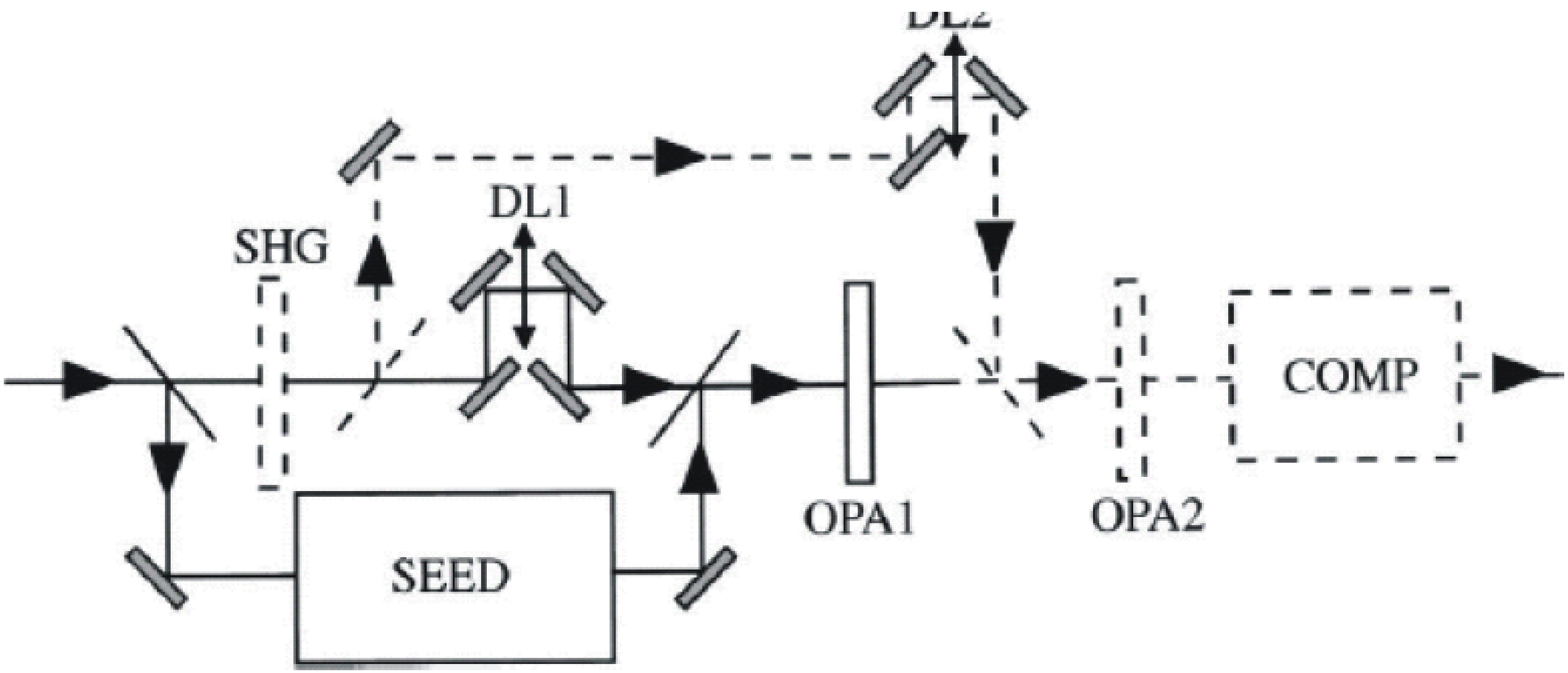


Fig. 9.15: Scheme of an ultrafast optical parametric amplifier. SEED: seed generation stage; DL1, DL2: delay lines; OPA1, OPA2 parametric amplification stages; COMP: compressor.

Near-IR OPA

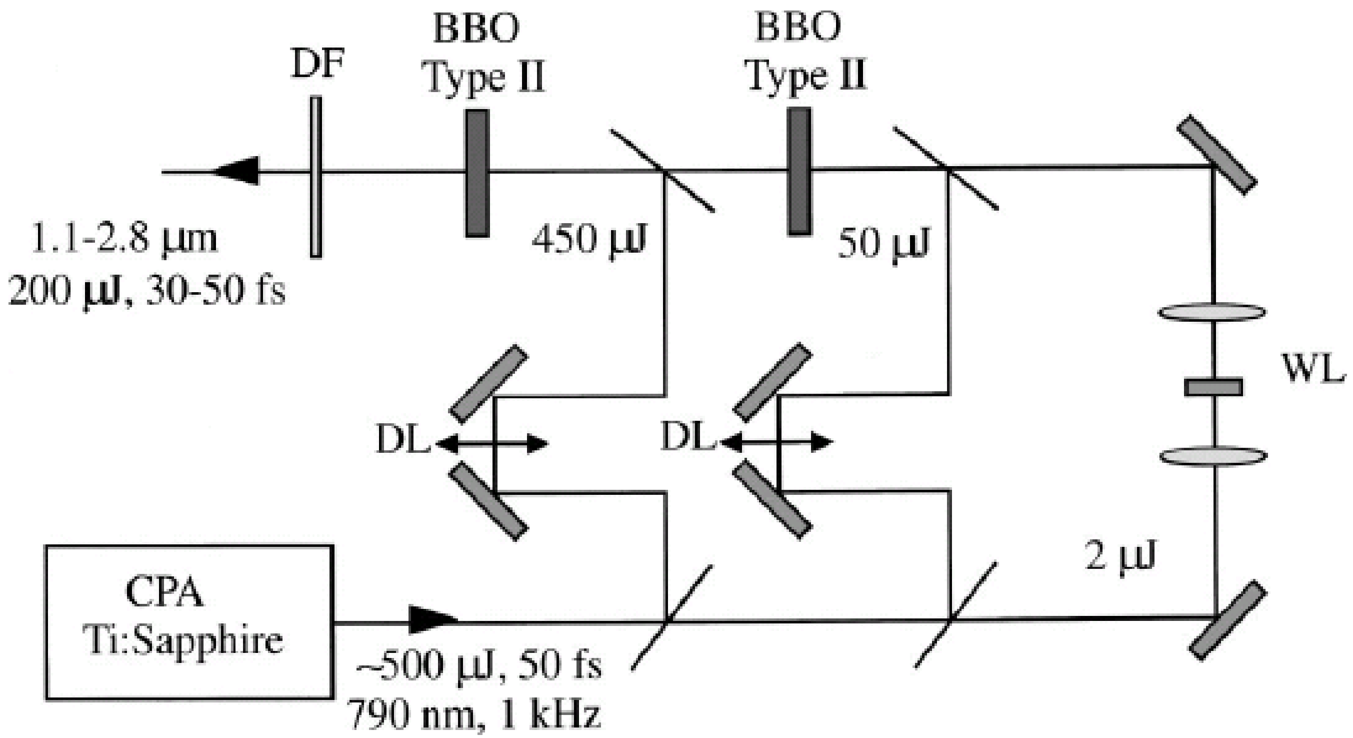


Fig. 9.16: Scheme of a near-IR OPA. DL: delay lines; WL: white light generation stage; DF: dichroic filter. [5]

9.8 Noncollinear Optical Parametric Amplifier (NOPA)

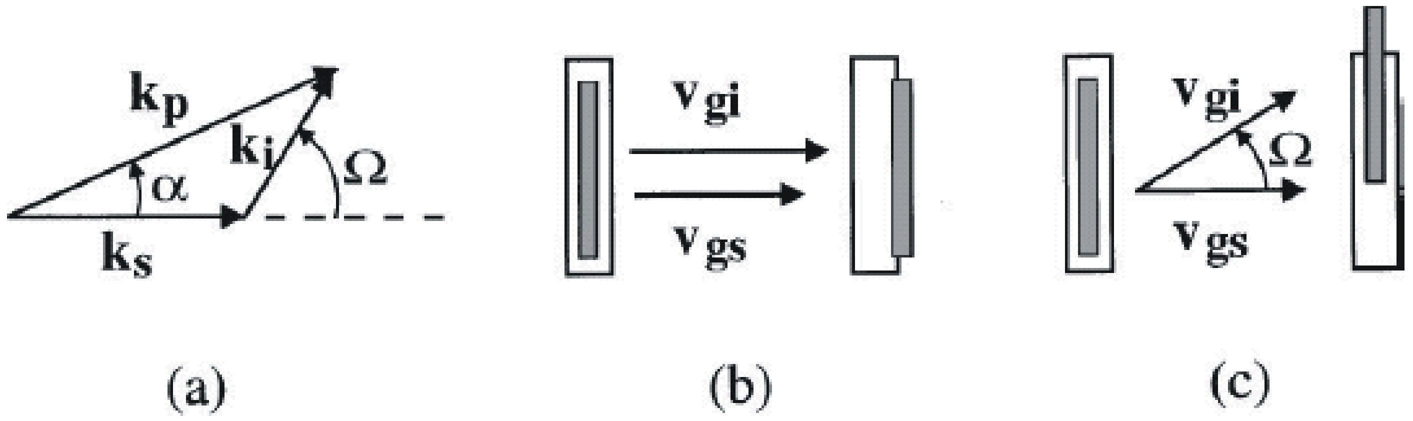


Fig. 9.17: a) Schematic of a noncollinear interaction geometry; b) representation of signal and idler pulses in the case of collinear interaction; and c) same as b) for noncollinear interaction.

Phase-matching condition: vector condition:

$$\begin{aligned}
\Delta k_{par} &= k_p \cos\alpha - k_s - k_i \cos\Omega = 0 \\
\Delta k_{perp} &= k_p \sin\alpha - k_i \sin\Omega = 0
\end{aligned}$$

Variation on phase matching condition by $\Delta\omega$

$$\Delta k_{par} = -\frac{dk_s}{d\omega_s}\Delta\omega + \frac{dk_i}{d\omega_i}\cos\Omega\ \Delta\omega - k_i \sin\Omega \frac{d\Omega}{d\omega_i}\Delta\omega = 0 \qquad \textbf{x} \quad \cos(\Omega)$$

$$\Delta k_{perp} = \frac{dk_i}{d\omega_i}\sin\Omega\ \Delta\omega + k_i \cos\Omega \frac{d\Omega}{d\omega_i}\Delta\omega = 0 \qquad \textbf{x} \quad \sin(\Omega)$$

and addition

$$\frac{dk_i}{d\omega_i} - \cos\Omega \frac{dk_s}{d\omega_s} = 0$$

Correct index

$$\boxed{v_{gs} - v_{gi}\ \cos\Omega = 0}$$

Only possible if:

$$v_{gi} > v_{gs}$$

$$\alpha = \arcsin\left[\frac{1 - \frac{v_s^2}{v_i^2}}{1 + 2v_s n_s \lambda_i / v_i n_i \lambda_s + (n_s\lambda_i / n_i\lambda_s)^2}\right]$$

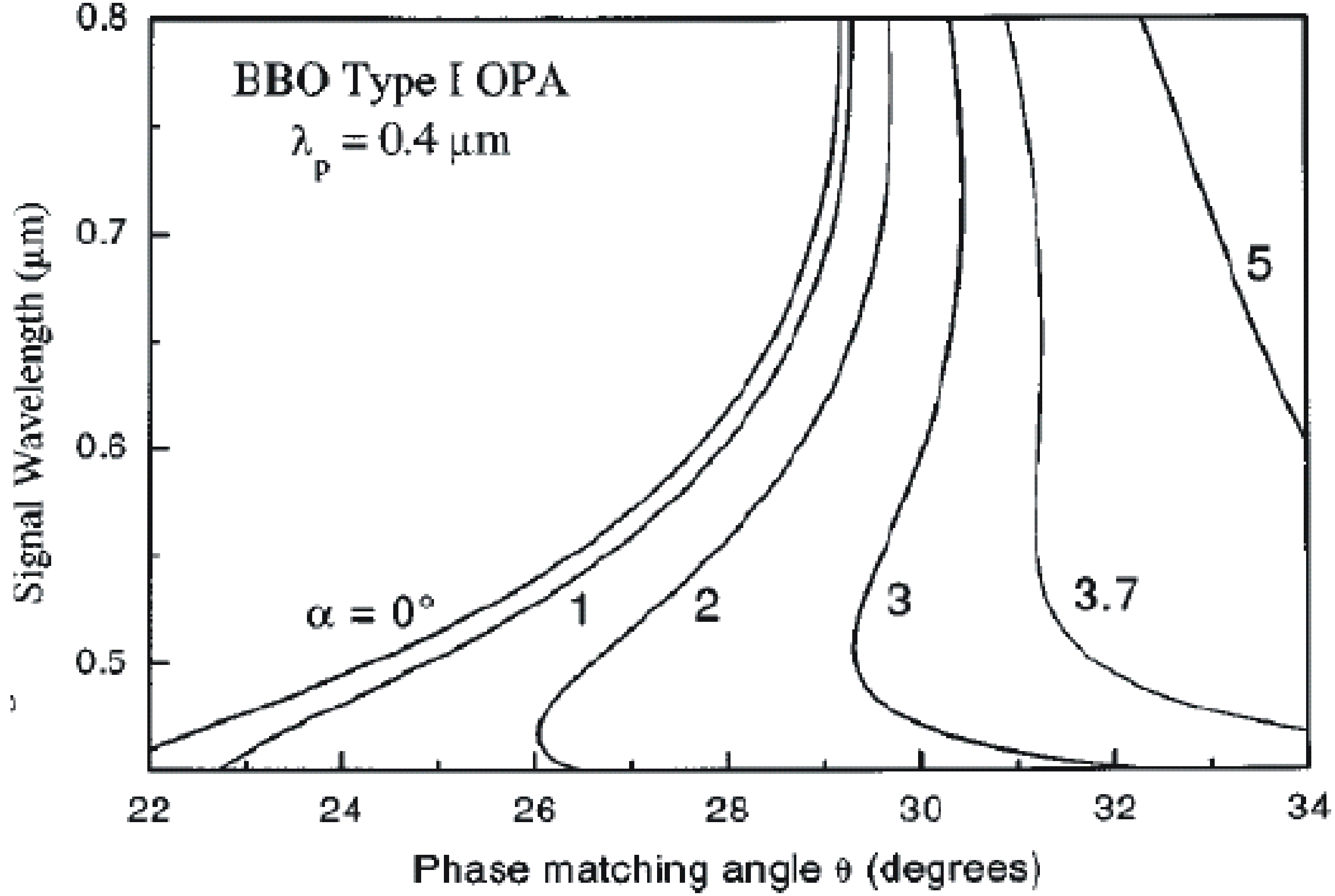
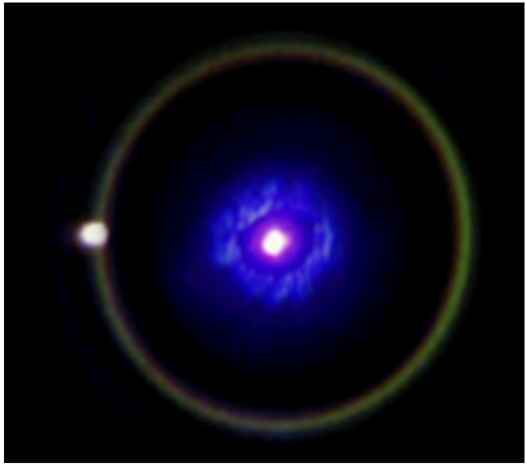


Fig. 9.18: Phase-matching curves for a noncollinear type-I BBO OPA pumped at λ_p=0.4 µm, as function of the pump-signal angle α. [5]

NOPA Layout



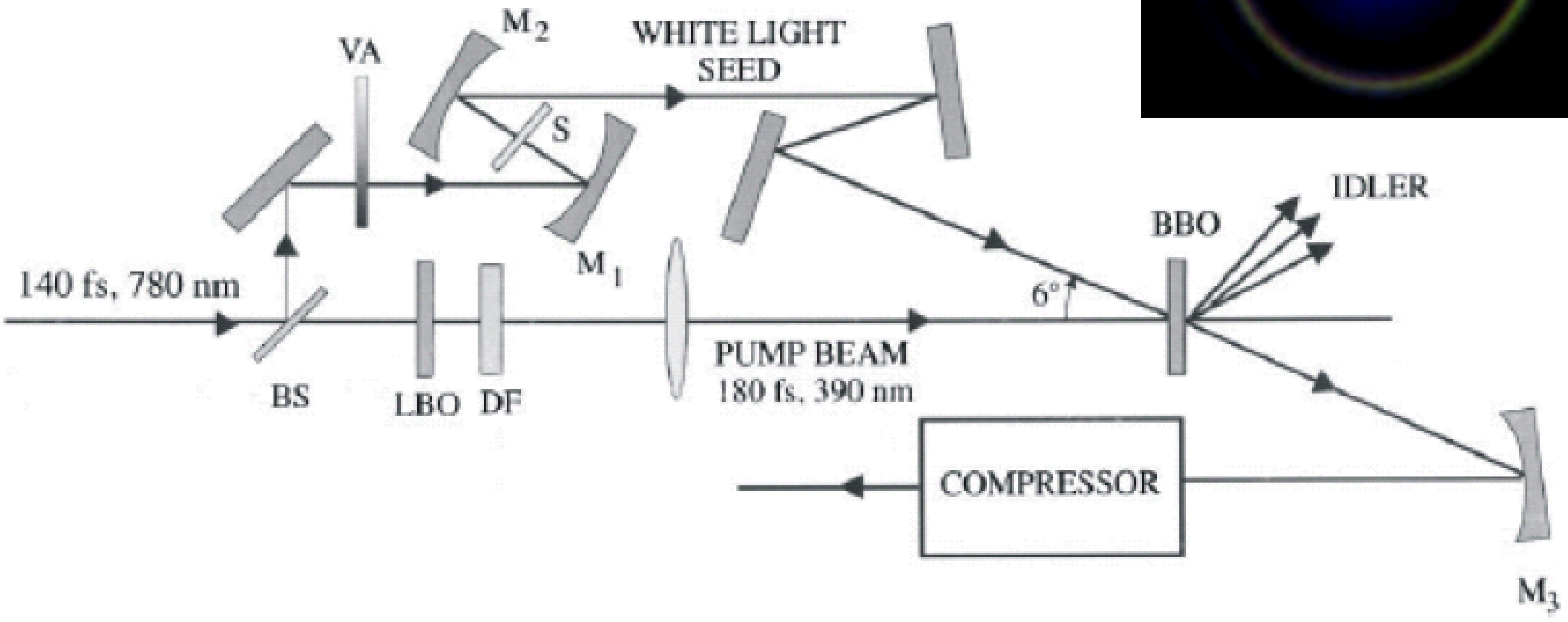
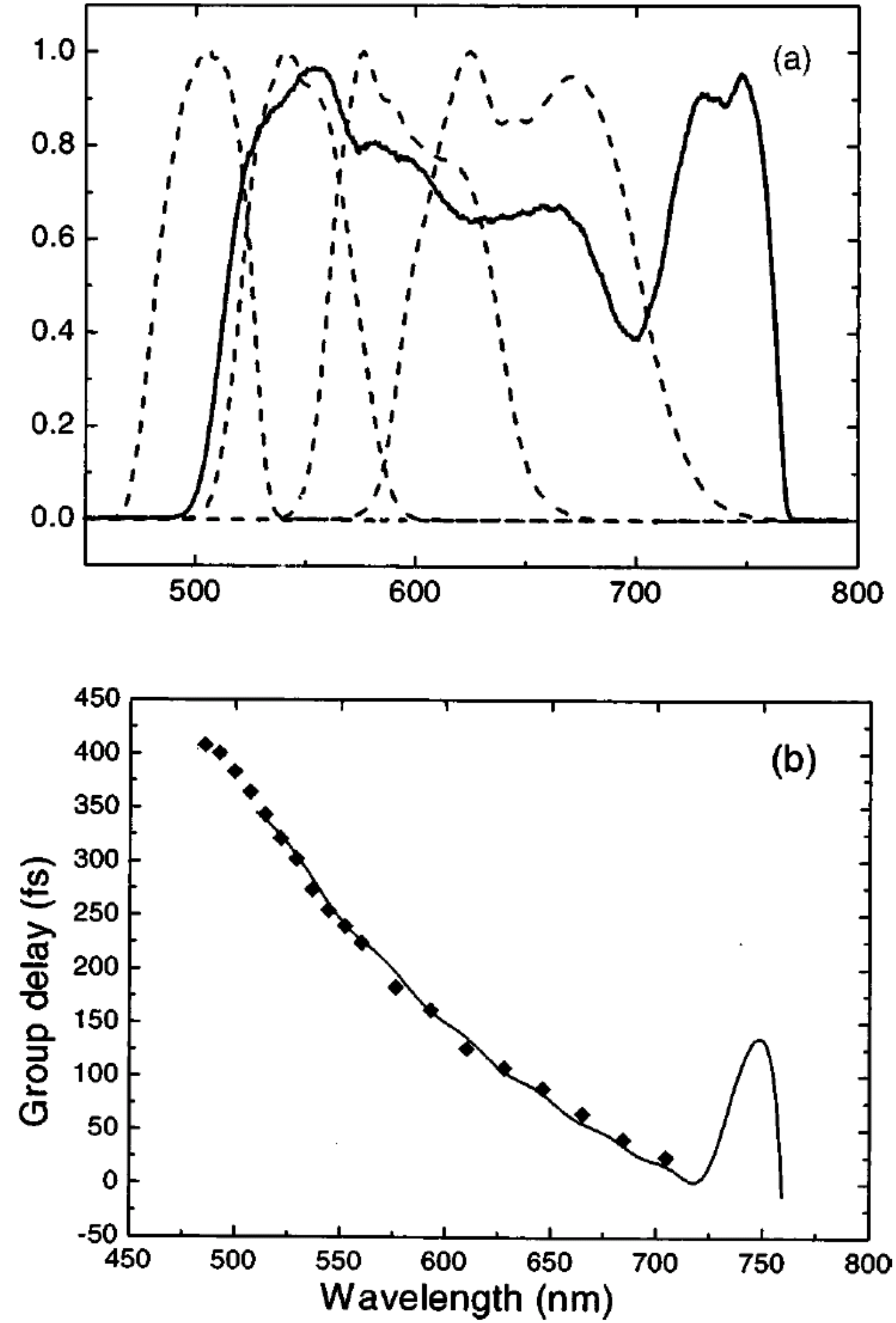


Fig. 9.19: Scheme of a noncollinear visible OPA. BS: beam splitter; VA: variable attenuator; S: 1-mm-thick sapphire plate; DF: dichroic filter; M1 ,M2 , M3 , spherical mirrors. [5]

Fig. 9.20: a) Solid line: NOPA spectrum under optimum alignment conditions; dashed line: sequence of spectra obtained by increasing the white light chirp;
b) points: measured group delay (GD) of the NOPA pulses; dashed line: GD after ten bounces on the ultrabroadband chirped mirrors.



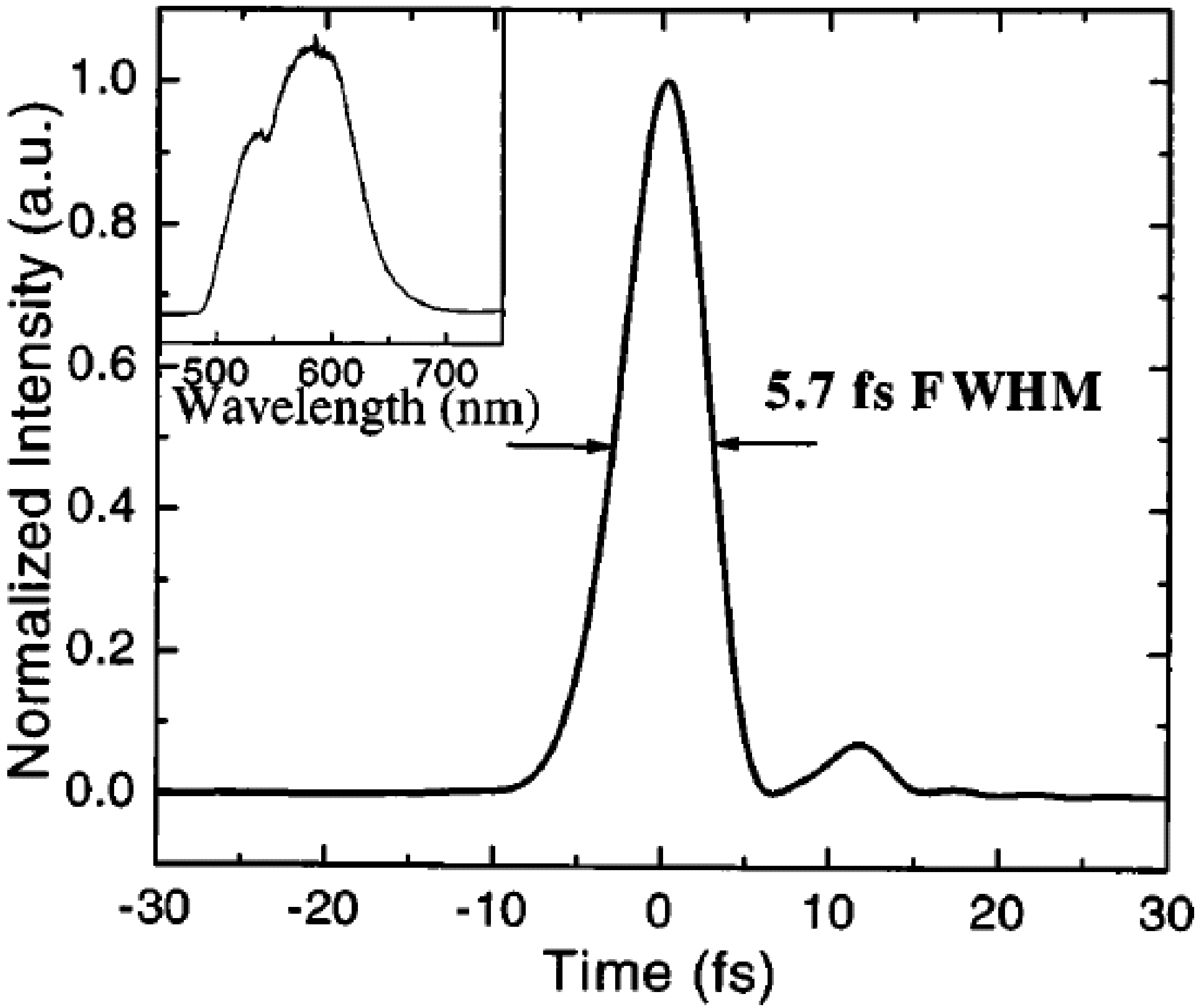


Fig. 9.21: Reconstructed temporal intensity of the compressed NOPA pulse measured by the SPIDER technique. The inset shows the corresponding pulse spectrum. [5]

9.9 Optical Parametric Chirped-Pulse Amplifier (OPCPA)

2-µm OPCPA