Nonlinear Optics (WiSe 2017/18)

Lecture 9: November 16, 2017

Chapter 6: Acousto-optic modulators

6.1 Acousto-optic interaction

6.2 The acousto-optic amplitude modulator

6. Acousto-optic modulator
6.1 Acousto-optic interaction

In a linear acousto-optic medium, the refractive index change is proportional to the applied voltage, see Eq. (5.8). The wave equation is

$$\nabla \times \nabla \times \mathbf{E} = -\mu_0 \frac{\partial^2}{\partial t^2}\mathbf{D} = -\mu_0 \frac{\partial^2}{\partial t^2}\left(\varepsilon_0 \mathbf{E} + \mathbf{P}\right) \tag{6.1}$$

The time-varying displacement can be separated into a part described by a constant average susceptibility or refractive index n, and a time-varying contribution described by $\Delta n(r,t)$

$$\begin{aligned} \mathbf{D} &= \varepsilon_0 \mathbf{E} + \mathbf{P} = \varepsilon_0 \left(n + \Delta n(r,t)\right)^2 \mathbf{E} \\ &\approx \varepsilon_0 n^2 \mathbf{E} + 2\varepsilon_0 n \Delta n(r,t) \mathbf{E}, \end{aligned} \tag{6.2}$$

where we neglect potential higher-order terms in $\Delta n(r,t)$, i.e., $\Delta n(r,t) \ll n$. From Eqs. (6.1) and (6.2), we have

$$\nabla \times \nabla \times \mathbf{E} + \frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} = -2\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\left[\frac{\Delta n(r,t)}{n}\mathbf{E}\right] \tag{6.3}$$

with $c^2 = 1/(\varepsilon_0 \mu_0 n^2)$. We consider a plane electromagnetic wave with wave vector $\mathbf{k}_{\cancel{s}\,\mathrm{i}}$ in the x-z-plane, see Fig. 6.1.

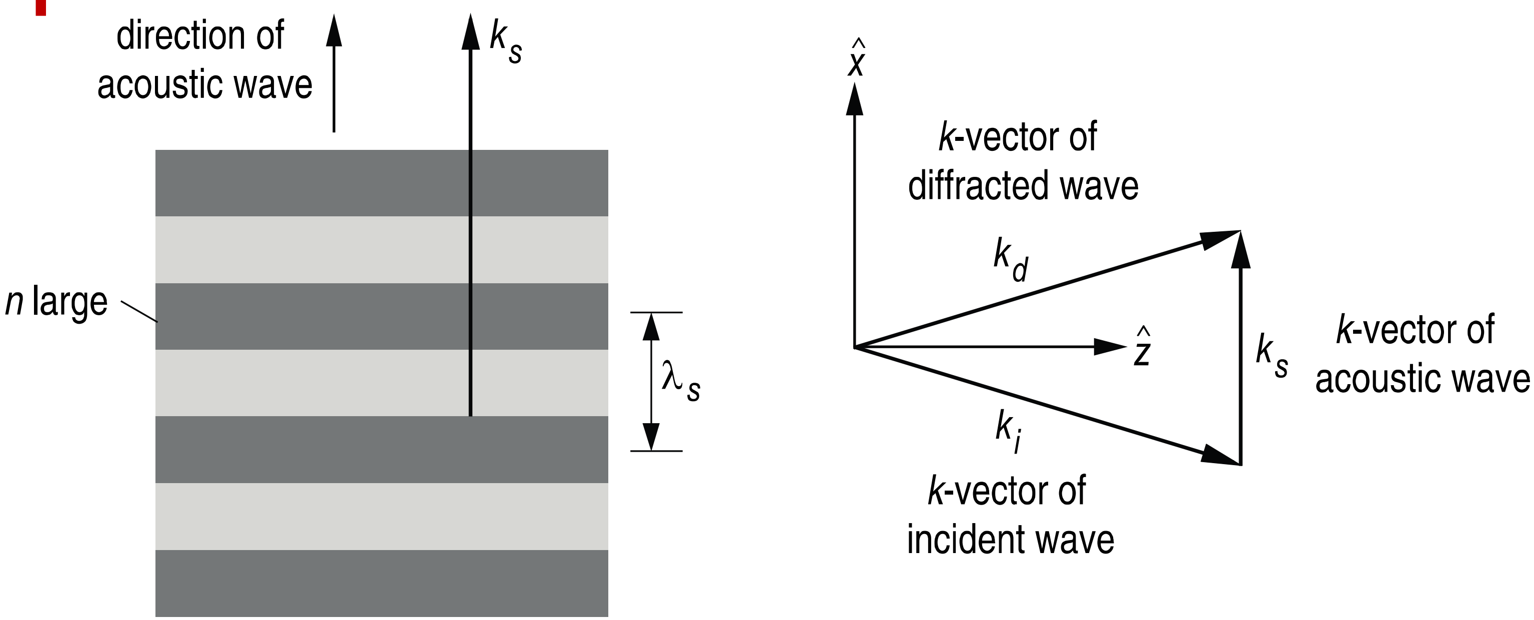

Figure 6.1: Diffraction grating in a medium generated by an acoustic wave.

The refractive index change generated by a wave with frequency ω_s is proportional to the acoustic wave

$$\begin{aligned} \Delta n(r,t) &= \Delta\hat{n}\cos\left(\omega_s t - \mathbf{k}_s \cdot \mathbf{r}\right) \\ &= \frac{\Delta\hat{n}}{2}\left[e^{j(\omega_s t - \mathbf{k}_s \cdot \mathbf{r})} + e^{-j(\omega_s t - \mathbf{k}_s \cdot \mathbf{r})}\right]. \end{aligned} \tag{6.4}$$

$\Delta\hat{n}$ is the amplitude of the resulting refractive index wave. From Gauss law for the electric field

$$\nabla \cdot \varepsilon\mathbf{E} = \rho = \varepsilon\nabla \cdot \mathbf{E} + \mathbf{E} \cdot \nabla\varepsilon, \tag{6.5}$$

with

$$\varepsilon = \varepsilon_0\left(n + \Delta n(r,t)\right)^2 \approx \varepsilon_0\left(n^2 + 2n\Delta n(r,t)\right). \tag{6.6}$$

If the electric field is polarized along the y-direction, we obtain $\mathbf{E} \cdot \nabla\varepsilon = 0$. If there are in addition no charges ρ, then the divergence of $\mathbf{E}$ vanishes, and the

there is

$$\omega_d = \pm\omega_s + \omega_i \quad \text{and} \quad \mathbf{k}_d = \pm\mathbf{k}_s + \mathbf{k}_i.$$

Usually, the frequency of sound is much less than the optical frequency, $\omega_s \ll \omega_i$, and therefore $|\mathbf{k}_d| \simeq |\mathbf{k}_i|$. Fig. 6.1 shows the resulting $\mathbf{k}$-diagram. We solve the wave equation (6.7) approximately, assuming that the amplitudes of the incoming and diffractive waves change slowly along the z-direction

$$\begin{aligned} \mathbf{E}_i &= \mathbf{e}_y A_i(z) e^{j(\omega_i t - \mathbf{k}_i \cdot \mathbf{r})} + c.c. && (6.9) \\ \mathbf{E}_d &= \mathbf{e}_y A_d(z) e^{j(\omega_d t - \mathbf{k}_d \cdot \mathbf{r})} + c.c. && (6.10) \end{aligned}$$

If we substitute (6.7) into (6.8), and use the slowly varying envelope approximation (as in Chapter 3), we obtain

$$\begin{aligned} -\left(\mathbf{k}_d^2 - \frac{\omega_d^2}{c^2}\right) A_d(z) - 2j\mathbf{k}_d \cdot \nabla A_d(z) &\simeq -\frac{\omega_d^2}{c^2} \Delta\hat{n} A_i(z) && (6.11) \\ -\left(\mathbf{k}_i^2 - \frac{\omega_i^2}{c^2}\right) A_i(z) - 2j\mathbf{k}_i \cdot \nabla A_i(z) &\simeq -\frac{\omega_i^2}{c^2} \Delta\hat{n} A_d(z). && (6.12) \end{aligned}$$

With $\mathbf{k}_{d,i}^2 = \frac{\omega_{d,i}^2}{c^2}$ and $\mathbf{k}_d \cdot \nabla A_d(z) = k_d \cos\theta \frac{dA_d}{dz}$, we obtain

$$\begin{aligned} \frac{dA_d(z)}{dz} &\simeq -j\frac{\omega_d}{2c}\frac{\Delta\hat{n}}{\cos\theta} A_i(z) && (6.13) \\ \frac{dA_i(z)}{dz} &\simeq -j\frac{\omega_i}{2c}\frac{\Delta\hat{n}}{\cos\theta} A_d(z). && (6.14) \end{aligned}$$

This set of equations describes coupled modes. However, the coupling coefficients

$$\frac{\omega_d}{2c}\frac{\Delta\hat{n}}{\cos\theta} \text{ and } \frac{\omega_i}{2c}\frac{\Delta\hat{n}}{\cos\theta}$$

are not equal, because $\omega_d \neq \omega_i$. This results from the acoustic waves that excite and drive optical waves. However, the difference is small, on the order of a millionth. Therefore, we can neglect the difference in Eqs. (6.13) and (6.14) and obtain with the coupling coefficient

$$\kappa = \frac{\omega_d}{2c}\frac{\Delta\hat{n}}{\cos\theta} \simeq \frac{\omega_i}{2c}\frac{\Delta\hat{n}}{\cos\theta} \tag{6.15}$$

and initial conditions $A_d(z) = 0$ the solution

$$\begin{aligned} A_i(z) &= A_i(0)\cos|\kappa|\,z & (6.16)\\ A_d(z) &= -jA_i(0)\sin|\kappa|\,z. & (6.17) \end{aligned}$$

The incoming wave is depleted along the propagation and transformed into a diffracted wave. The diffracted wave is slightly shifted in frequency. If the interaction length is long enough, so that $|\kappa|\,z > \pi/2$, the diffracted wave is

Applications:

Deflection of light

Frequency shifting

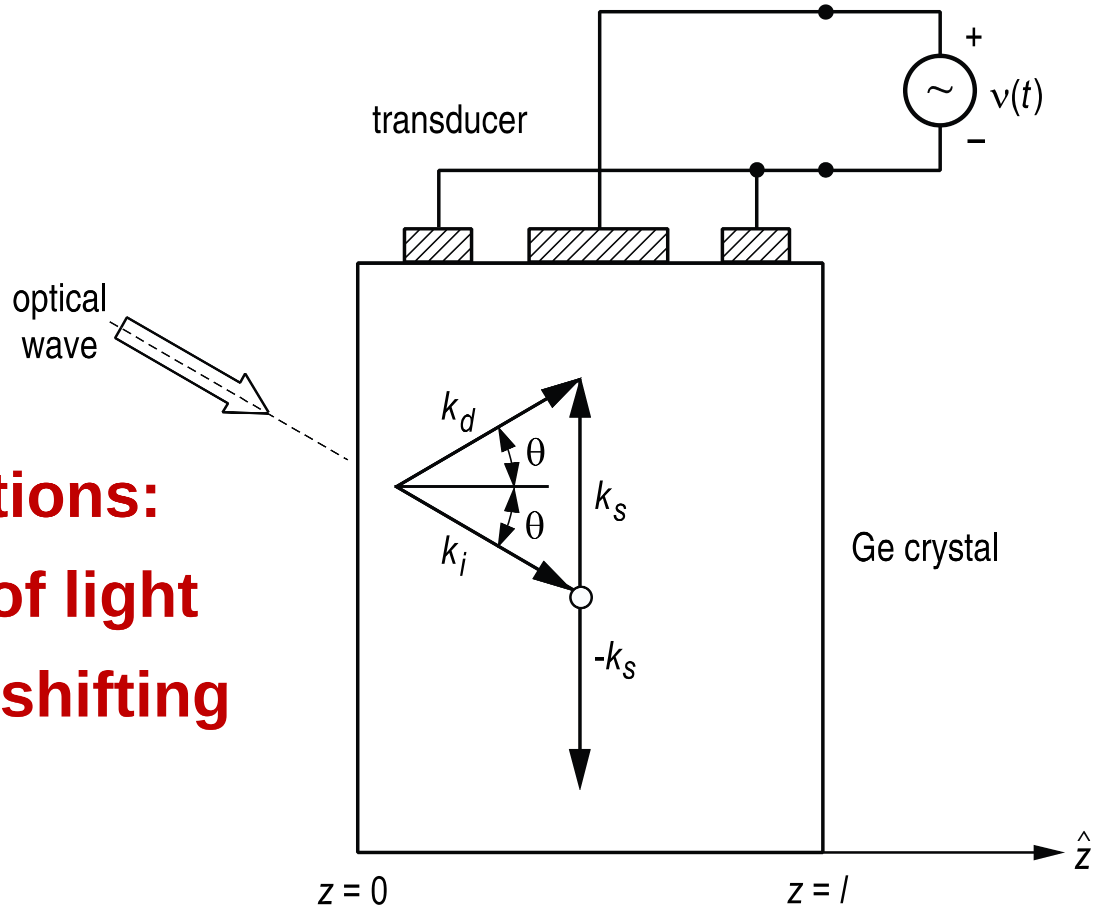

Figure 6.2: Typical **k**-diagram describing acousto-optic light diffraction at a standing acoustic wave.

Eqs. (6.16) and (6.17) also have a different meaning. If the frequency $\omega_s = 0$, then the optical wave interacts with a constant index grating. The diffracted wave has exactly the same frequency and the wave vectors must obey momentum conservation. Also this situation is described by the same coupled mode equations. The coupling coefficients are now identical and the total optical energy of both partial waves is conserved.

6.2 The acousto-optic amplitude modulator

The intensity is modulated, if the interaction is with a standing wave.

$$\begin{aligned}\Delta n(\mathbf{r},t) \;&=\; \Delta n \sin\omega_s t \cos(\mathbf{k}_s\mathbf{r}) = \frac{\Delta n}{4j}\left\{\exp\left[j\left(\omega_s t - \mathbf{k}_s\mathbf{r}\right)\right]\right.\\ &+\exp\left[j\left(\omega_s t + \mathbf{k}_s\mathbf{r}\right)\right] - \exp\left[-j\left(\omega_s t - \mathbf{k}_s\mathbf{r}\right)\right]\\ &\left.-\exp\left[-j\omega_s t + \mathbf{k}_s\mathbf{r}\right]\right\}.\end{aligned}$$

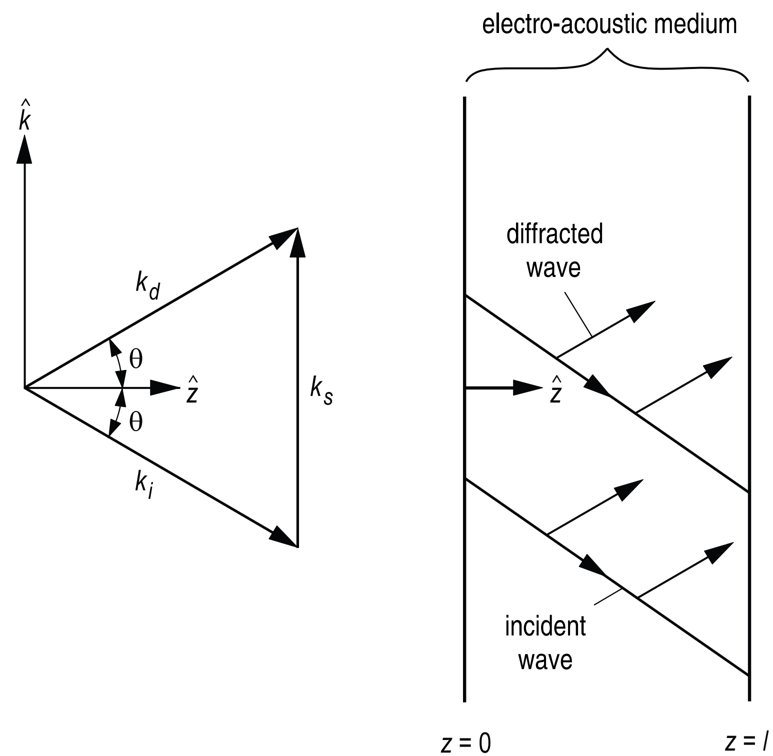

Figure 6.3: Momentum conservation in the acousto-optical amplitude modulator.

$$\omega_s << \omega_i \qquad \omega_d = \omega_i + m\omega_s$$

$$\begin{aligned} A_i(\ell) &= A_i(0)\cos\left(\frac{\omega_i}{c}\frac{\Delta n}{2\cos\theta}\ell\right) && (6.20) \\ A_d(\ell) &= -jA_i(0)\sin\left(\frac{\omega_i}{c}\frac{\Delta n}{2\cos\theta}\ell\right). && (6.21) \end{aligned}$$

Afterwards the amplitude of the wave is simply made time-dependent, i.e., $\Delta n(t) = \Delta n\,\mathrm{sin}\omega_s t$. A graphical construction of both amplitudes is attempted in Fig. 6.4.

Again with the generating function of the Bessel functions, Eqs. (5.54) and (5.55),

$$\begin{aligned} \cos(x\sin\omega_s t) &= \sum_{m\,\mathrm{even}} J_m(x)\,e^{jm\omega_s t}, && (6.22) \\ \sin(x\sin\omega_s t) &= -j\sum_{m\,\mathrm{odd}} J_m(x)\,e^{jm\omega_s t}, && (6.23) \end{aligned}$$

$$A_i(\ell) = A_i(0) \sum_{m\,\text{even}} J_m \left(\frac{\omega_i}{c} \frac{\Delta n}{2\cos\theta} \ell \right) e^{jm\omega_s t}, \tag{6.24}$$

$$A_d(\ell) = -A_i(0) \sum_{m\,\text{odd}} J_m \left(\frac{\omega_i}{c} \frac{\Delta n}{2\cos\theta} \ell \right) e^{jm\omega_s t}. \tag{6.25}$$

Two remarks need to be made. First, the incoming wave has only even sidebands and the diffractive wave only odd ones. I.e., the incoming wave is modulated with the frequency $2\omega_s$. The incoming wave is extinct, if the modulation depth is adjusted such that the Bessel function of zeroth order shows a zero or disappears, i.e., for

$$\frac{\omega_i}{c} \frac{\Delta n}{2\cos\theta} \ell = 2.405. \tag{6.26}$$

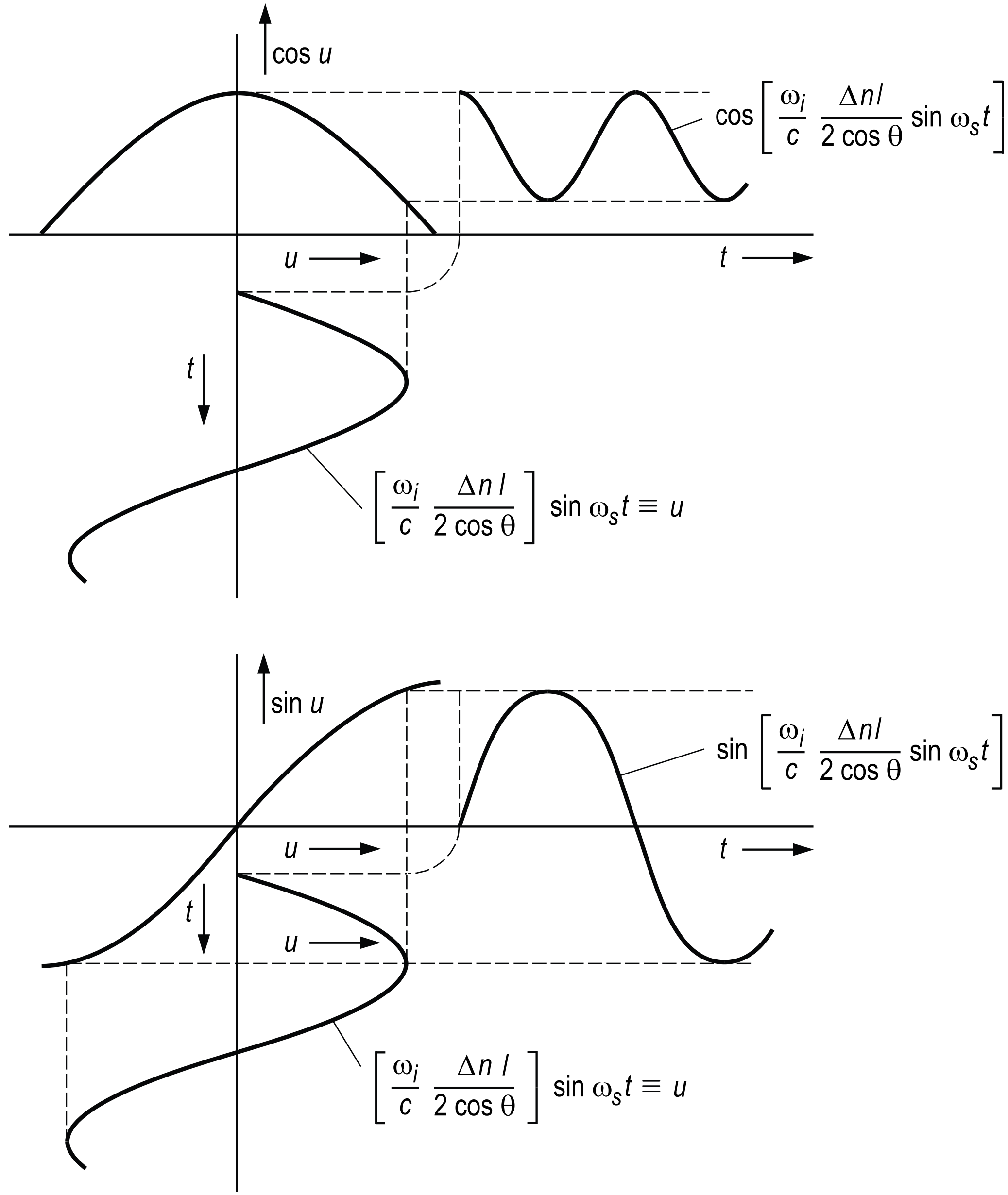

Figure 6.4: Time dependence of amplitudes for incoming and diffracted waves.