University of Hamburg, Department of Physics

Nonlinear Optics

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Problem Set 4

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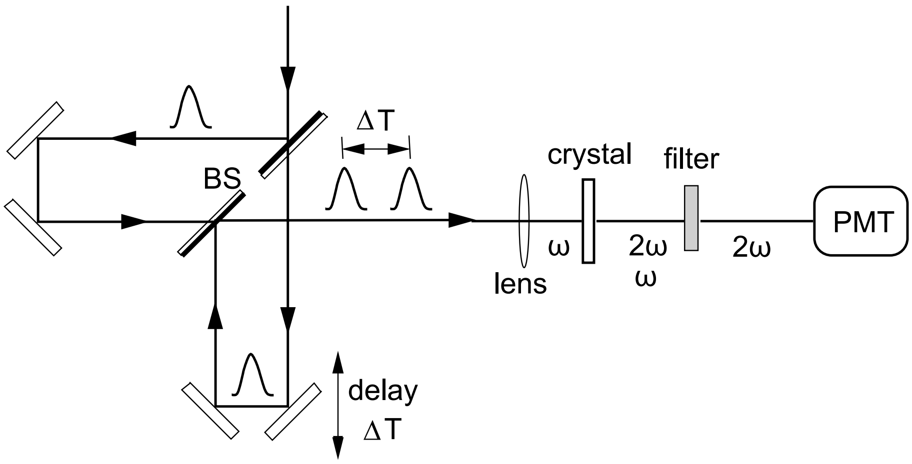
Pulse duration measurement by interferometric autocorrelation

One important application of frequency doubling is the characterization of ultrashort pulses. Transient optical signals on a picosecond and femtosecond time scale are too short to be characterized by direct electronic detection. In these cases, the short laser pulse itself is used to perform its own temporal characterization. A typical measurement scheme looks as follows:



The incident laser pulse is divided into two identical replica pulses (each with approximately half the original intensity) with a relative time delay ΔT by means of two beam splitters (BSs) rotated by 180°. Beam splitters for this purpose are typically thin (∼100-300 μm, to avoid dispersive pulse broadening) glass substrates with a metallic coating (e.g., Cr-Ni) on one side. The reflection occurs on the metallic coating. If one would use only a single beam splitter to implement an interferometer, one of the pulses would pass through the substrate only once, however, the other one would traverse the substrate three times. For very short pulses, a significant measurement error would occur. In the symmetric setup displayed above, both pulses travel identical optical paths.

The time delay ΔT is achieved using a scanning delay line consisting of two mirrors. For instance, moving the delay stage by 3 μm leads to a delay of 2×10 fs. Practically, the path-length difference can be varied with a piezo-mounted delay stage and (not shown in the figure above) permanently monitored employing an in-coupled, co-propagating HeNe laser with well-defined wavelength, that at the interferometer's drop-port creates interferences, that can simultaneously be measured using a second detector.

The resulting two replica pulses with delay ΔT are afterward focused into a nonlinear crystal. The generated second-harmonic signal is then detected (e.g., using a photomultiplier tube (PMT)) in a temporally integrated way. The pulse duration can be characterized by measuring the second-harmonic signal as function of delay ΔT. In other words, this setup measures the temporal overlap of both pulses versus ΔT. We assume that the $\chi^{(2)}$ nonlinearity has an instantaneous response, i.e., it is frequency independent in the relevant spectral range, and that we use a very thin crystal for frequency doubling. Thus, it follows

$$E_2(t, \Delta T) \approx C E^2(t, \Delta T)$$

with

$$E(t, \Delta T) = E_1(t) + E_1(t + \Delta T).$$

and

$$E_1(t) = \frac{1}{2}\tilde{E}_0(t) e^{i\omega t} + c.c.$$

Here, E_1 and E_2 denote the electric fields at the fundamental and the generated second harmonic. $\tilde{E}_0(t)$ denotes the envelope function of the ultra short laser pulse, of which we want to obtain the pulse duration.

1. Your task in the problem will be to derive an expression for the time-integrated intensity signal S_2 of the generated second-harmonic light.

   $$S_2(\Delta T) = \int_{-\infty}^{+\infty} |E_2(t, \Delta T)|^2 \, dt$$

   You will see later, that this signal encodes the pulse duration of the laser pulse we want to characterize. To start with, write down the detailed expression for $|E_2(t, \Delta T)|^2 = E_2(t, \Delta T) E_2^*(t, \Delta T)$ and then simplify it using $I_1(t) = E_1(t) E_1^*(t)$. This expression contains parts oscillating as function of ΔT, that originate from the phase delay of both replica pulses.

2. Now consider a Gaussian shaped pulse with an envelope $\tilde{E}_0(t)$ of the form

   $$\tilde{E}_0(t) = E_0 \exp\left(\frac{-t^2}{2\tau^2}\right),$$

where the pulse duration $t_p = 2\sqrt{\ln 2}\tau \approx 1.66\tau$ is the full width half maximum (FWHM) of the pulse intensity profile $I(t)$. Assuming such pulse, now solve the integral yielding $S_2(\Delta T)$ and show that **after normalization** the measured signal can be written as:

$$S_{2\,norm}(\Delta T) = 1 + 2e^{-\frac{\Delta T^2}{2\tau^2}} + 4e^{-\frac{3\Delta T^2}{8\tau^2}} \cos(\omega \Delta T) + e^{-\frac{\Delta T^2}{2\tau^2}} \cos(2\omega \Delta T)$$

Hint: You might find the technique of „completion of the square" as well as the following integral to be helpful:

$$\int_{-\infty}^{+\infty} e^{-\alpha^2 x^2} dx = \frac{\sqrt{\pi}}{2\alpha}$$

3. Now using a software of your choice plot the normalized interferometric autocorrelation signal $S_{2\,norm}(\Delta T)$ you would expect to measure for a transform-limited Gaussian pulse as given in the last subtask with a pulse duration of $t_p = 30fs$ and wavelength centered at $\lambda = 800nm$! The envelope of this signal can be easily added to the plot via setting ω to zero. Add it to the same plot.

4. Write down the expression for the full width at half maximum of the envelope of the interferometric autocorrelation trace $T_{FWHM}(t_p)$ as a function of the pulse duration t_p. With this you have finally found a direct relation between your measured signal and the pulse duration of your laser pulse!